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Nuclear Catastrophe Risk Bonds in a Markov Dependent Environment

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ABSTRACT

The financing of the 2011 Fukushima disaster and the UK Hinkley nuclear power plant investment, respectively by the Japanese, and UK and Chinese governments and the private sector provide a strong motivation for this paper to explore deeper the concept of modeling and pricing Nuclear Catastrophe (N-CAT) risk bonds. Due to the magnitude of the potential liabilities and re-investments needed, the demand to develop a dependable liability coverage product that can be triggered in a case of emergency is required more than ever and it should be considered thoroughly. Thus, in the present paper, under a semi-Markov structure environment to model the relationship between claims severity and intensity, the N-CAT risk bond is further explored under various scenarios supporting further the bond sponsors, allowing them to appreciate more their significance.

24 Consequently, the new version of the N-CAT risk bond includes several absorbing and transit states
25 to make it more suitable for practitioners. Additionally, this paper employs the two most commonly
26 used interest rate models and considers four types of payoff functions. Finally, two numerical
27 examples illustrate the main findings.

28
29 **Keyword:** Nuclear Power Risk, Catastrophe Risk Bonds, Global Market, Liability, Special Purpose
30 Vehicle, Semi-Markov Environment

31 INTRODUCTION

32 Communities often experience different types of natural and human-made disasters such as
33 floods, earthquakes, hurricanes, severe storms, tornadoes, wildfires, heavy snowfalls and human-
34 caused disruptions (including also terrorist attacks and threats) which lead inevitably to numerous
35 governmental declarations and billions of US dollars in losses every year. Furthermore, the welfare
36 impact of such high-levels of disruption does not only depend on the physical characteristics of
37 the event(s) as well as its (their) direct or indirect impacts in terms of lost lives and assets, but
38 also on the aptitude of the economy to absorb, recover, reconstruct and therefore to minimize the
39 aggregate consumption losses in the short and long-run. In practice, losses and recovery costs from
40 those catastrophic (or even cataclysmic) events are typically covered by a combination of utility
41 companies, special insurance schemes and/or governments, for instance a characteristic example
42 is the coverage of the major losses from the 2011 Fukushima disaster primarily by the Japanese
43 government (Conca 2016).

44 Nuclear power plants are very popular for producing electrical energy in 31 countries (see
45 discussion in (Ayyub et al. 2016)). Among the many developed countries which have nuclear
46 plants, very recently in the UK, Hinkley Point C nuclear power plant got the greenlight from
47 the UK government to be constructed (Farrel and Macalister 2015). It will be the first nuclear
48 power station in a generation which will provide 7% of the country's electricity, and the total
49 investment cost is estimated to be more than US\$20 billion. Inevitably, this level of investment
50 brings the risk of nuclear power back into the public eye again. The enquiry which was initially

51 proposed in (Ayyub and Parker 2011) and was then emphasised in (Ayyub et al. 2016): "*how to*
52 *develop sufficient liability coverage for nuclear power risks?*" . Resources for this purpose are
53 often inadequate and require a cash reserve that could be challenging to maintain. Low penetration
54 rates for insurance leaves it up to individuals, companies and governments to shoulder the financial
55 losses arising from catastrophic events. In emerging markets with non-existent or immature legal
56 regimes, liability can lead to international tensions and potentially wars, particularly in cases of
57 cross-border exposures. Therefore, the potential financial demands on insurance and reinsurance
58 businesses make it appropriate to introduce a mechanism for individuals against nature and man-
59 made disasters.

60 Catastrophe (CAT) risk bonds (or Act-of God bonds) are securities which are born for these
61 extreme events to share the risk to another level — global financial markets as the only pool of cash
62 large enough to underwrite such losses lies in capital markets and the collection of big investors like
63 pension funds, hedge funds and sovereign wealth funds that normally invest in stocks and bonds.
64 CAT risk bonds show association between the risk deduction for insurers and are an alternative
65 source of capital for insurance companies with large risk transfer needs (Hagendorff et al. 2014).
66 On the other hand, CAT risk bonds' investors enjoy high yield coupon rates and diversification
67 effects on their investment portfolios. Furthermore, the feature of correlation of the traditional
68 stock market allows them to still gain under bad economic conditions and they reduce the barriers
69 to entry and increase the contestability of the reinsurance market (Froot 2001). These lead CAT
70 risk bonds to be the most popular insurance-linked financial securities (ILS) and their use has been
71 accelerating in the last few decades.

72 Historically, the first experimental transaction was completed in the mid-1990s after Hurricane
73 Andrew and the Northridge earthquake, which incurred insurance losses of US\$15.5 billion and
74 US\$12.5 billion, respectively, by a number of specialized catastrophe-oriented insurance and
75 reinsurance companies in the USA, including AIG, Hannover Re, St Paul Re, and USAA, (GAO
76 2002). The CAT risk bond market has boomed over the years. The issued capital has increased
77 tenfold within ten years, from less than US\$0.8 billion in 1997 to over US\$8 billion in 2007, and

78 the issuers raised more than US\$7.8 billion of new CAT risk bonds in 2015 (Artemis 2017a).
79 CAT risk bonds are inherently risky, non-indemnity-based multi-period deals, which pay a regular
80 coupon to investors at the end of each period and a final principal payment at the maturity date, if
81 no predetermined catastrophic events occur. A major catastrophe in the secured region before the
82 CAT risk bonds maturity date leads to full or partial loss of the capital.

83 The structure of CAT risk bonds, including where the capital flows from one party to another,
84 is presented in Figure 1, see also (Swiss Re Institute 2009). The issuer does not directly issue
85 the CAT risk bond, but uses a Special Purpose Vehicle (SPV) for the transaction. An SPV can
86 be interpreted as a focused insurer whose only purpose is to write one insurance contract. The
87 existence of an SPV, which is equal to a focused one-policy insurer, minimises the frictional cost of
88 capital. Furthermore, sufficient high endowment of the SPV eliminates the counterparty risk. The
89 SPV enters into a reinsurance agreement with a sponsor or counterparty (e.g., an insurer, reinsurer,
90 or government) by issuing CAT risk bonds to investors and receives premiums from the sponsor in
91 exchange for providing a pre-specified coverage. Therefore, sponsors can transfer part of the risks
92 to investors who bear the risk in return for higher expected returns. The SPV collects the capital
93 (principal and premium) and invests the proceeds into a collateral account (trust account, which is
94 typically highly related to short-term securities, e.g., Treasury bonds). The returns generated from
95 collateral accounts are swapped for floating returns based on the London Interbank Offered Rate
96 (LIBOR) in order to immunize the sponsor and the investors from interest rate risk and default risk,
97 (Cummins 2008).

98 The investors' coupon payments are made up of SPV investment returns, plus the premiums
99 from the sponsor. If no trigger event occurs during the term time of the CAT risk bonds, then
100 the collateral is liquidated at the maturity date of the CAT risk bonds and investors are repaid the
101 principal plus a compensation for bearing the catastrophe risks (solid line in Figure 1). However,
102 if a trigger event occurs before maturity, the SPV will liquidate the collateral required to make the
103 payment and reimburse the counterparty according to the terms of the catastrophe bond transaction,
104 and CAT bond investors will only receive part of the capital (dashed line in Figure 1).

105 The key parameter of a CAT risk bond transaction is the bond premium. To bear the catastrophe
106 risks, CAT risk bonds carry a 3 to 5 year maturity and compensate for a floating LIBOR coupon
107 plus a premium at a rate between 2% and 20%, see (Cummins 2008; GAO 2002). The main
108 determinants of the CAT risk bond spread/premium is the expected loss, the covered territory, the
109 sponsor, the reinsurance cycle, and the corporate bond spread (Braun 2016). (Galeotti et al. 2013)
110 modelled premiums paid by a sponsor in two parts: the expected value of loss, and a load for
111 risk margin and expenses. They compared the different premium calculation models based on the
112 basis of CAT risk bond contracts issued between April 1999 and March 2009, and recommended
113 the Wang's transformation model (Wang 2004) or the simple linear model to predict CAT risk
114 bond premiums. The key elements of pricing any CAT risk bond are the loss exceedance curve
115 and the triggers. Only when a pre-specified condition is met (e.g., a predetermined events occurs
116 and the loss exceeds a predetermined level), investors begin to lose their investment, and those
117 conditions are triggers. Triggers can be structured in many ways from a sliding scale of actual
118 losses experienced by the issuer (indemnity) to a trigger which is activated when industry wide
119 losses from an event hit a certain point (industry index trigger) to an index of weather or disaster
120 conditions, which means actual catastrophe conditions above a certain severity will trigger a loss
121 (parametric index trigger) etc., see (Swiss Re Institute 2009; Hagedorn et al. 2009; Burnecki et al.
122 2011; Johnson 2013) among others. A few CAT risk bonds use the indemnity trigger type because
123 it is subject to the highest degree of moral hazard, due to the fact that the loss is controlled by the
124 sponsor (Hagendorff et al. 2014). (Swiss Re Institute 2009) illustrated the relationship between
125 transparency and basis risk for various types of CAT risk bond triggers, also in Figure 2, and
126 investors prefer to buy the bonds with better transparency while sponsors want to minimise the
127 basis risks.

128 CAT risk bonds can be structured to provide per-occurrence cover, so exposure to a single
129 major loss event (currently US\$ 12,932.41 million which accounts for 55.6%) or to provide annual
130 aggregate cover, exposure to multiple event triggers over each annual risk-period (Woo 2004;
131 Artemis 2017b). Some CAT risk bonds transactions work on a multiple loss approach and so are

132 only triggered (or portions of the deals are) by second and subsequent events. This means that
133 sponsors can issue a deal that will only be triggered by a second landfalling hurricane to hit a
134 certain geographical location, for example.

135 Despite the rising popularity, the number of previous studies devoted to CAT risk bond modeling
136 and pricing is relatively limited. Some notable models have been based on: quasi Monte Carlo
137 (Vaugirard 2003; Albrecher et al. 2004) and indifference pricing techniques (Young 2004), entropy
138 based models (Ling and Jun 2009), a simple robust model (Jarrow 2010), a representative agent
139 pricing approach (Cox and Pedersen 2000; Shao et al. 2015), premium calculation models (Galeotti
140 et al. 2013), a mixed approximation method (Ma and Ma 2013), a Bayesian pricing model (Ahrens
141 et al. 2014), a cluster analysis approach (Constantin et al. 2014), a multifactor pricing model
142 (Gomez and Carcamo 2014), modeling using multifractal processes (Hainaut and Boucher 2014),
143 fuzzy based approaches (Nowak and Romaniuk 2013b; Nowak and Romaniuk 2017), and with
144 Cox-Ingersoll-Ross interest rate models (Nowak and Romaniuk 2016).

145 Some notable applications have included: modeling of tropical cyclones (Daneshvaran and
146 Morden 2004), systemic risks in agriculture for the case of Georgia cotton (Vedenov et al. 2006),
147 transportation assets and feasibility analysis for bridges (Sircar et al. 2009), calibration using
148 Chinese earthquake loss data (Wu and Zhou 2010), models for earthquakes (Penalva Zuast 2002;
149 Zimbidis et al. 2007; Tao et al. 2009; Härdle and Cabrera 2010; Ahrens et al. 2014; Shao et al.
150 2015), modeling of tornado occurrence in the USA (Hainaut and Boucher 2014), exposure to
151 currency exchange risk (Lai et al. 2014), seismic risk management of insurance portfolio (Goda
152 2015), hedging of flood losses (Tetu et al. 2015), and temperature-based agricultural applications
153 (Karagiannis et al. 2016) among others.

154 Recently, Shao et al. (Shao et al. 2017) modeled the dependence of the claim inter-arrival time
155 on the claim size for the aggregate claims as a semi-Markov process. As it has been discussed
156 in (Shao et al. 2017), there are quite a few applications where the Markov-dependent structure
157 has been applied. For instance, (Janssen and Manca 2007; Janssen and Limnios 1999) provided
158 plenty of applications in queueing theory, insurance mathematics, reliability and maintenance and

159 fluid mechanics. (Reinhard 1984; Asmussen and Rolski 1992; Lu and Li 2005) focus on modeling
160 and computing Semi-Markov processes in ruin theory. Moreover, (Ayyub et al. 2016) proposed
161 nuclear catastrophe risk bonds (also known as N-CAT) for the very first time, addressing the nuclear
162 liability conventions and the current liability limitations, for more details see (Ayyub and Parker
163 2011). This N-CAT risk bond utilised the indemnity trigger with lowest basis risks to the sponsor,
164 however, it has lowest transparency for investors. In order to prevent the intent of manipulating the
165 N-CAT risk bonds prices by deliberately triggering a nuclear catastrophe, the N-CAT risk bonds
166 writer should specify in the contract that man-made accidents directly caused by the reimbursement
167 beneficiaries (normally government) are excluded. Although very unlikely, this extra term in the
168 N-CAT risk bonds contract provides safeguard against such behaviour.

169 In the present paper, a complete analysis of N-CAT risk bonds is presented by implementing
170 three main extensions compared with the previous papers (Ayyub et al. 2016; Shao et al. 2017).
171 First, the authors embed a flexible interest rate model framework. Thus, a sensitivity analysis
172 based on the classical Vasicek and Cox-Ingersoll-Ross models is provided (Nowak and Romaniuk
173 2013a). Then, the authors construct model in a Markov-dependent environment (Shao et al. 2017)
174 and the authors generalise the transition matrix with w transit states and r absorbing states (Ayyub
175 et al. 2016). Finally, by employing four payoff functions including an issuer default model, two
176 illustrative numerical examples are provided.

177 The contents of this paper are organized as follows. The Modelling N-CAT Risk Bond section
178 presents the pricing model of CAT risk bonds including: assumptions, probability structure,
179 valuation method, interest rate processes, aggregate claims processes, and the payoff functions.
180 Explicit closed form solutions are shown in Theorems 2.1 to 2.4. The section Numerical Examples:
181 Analysis and Discussion illustrates the numerical examples of the N-CAT risk bonds pricing
182 formulae and compares the effect size for varying interest rates, time to maturity and threshold
183 levels, accordingly.

184 **MODELLING N-CAT RISK BOND**

185 Following closely (Cox and Pedersen 2000; Shao et al. 2015; Ayyub et al. 2016; Shao et al.

2017), in the present paper, the N-CAT risk bonds is priced under the following assumptions: (i) an arbitrage-free investment market exists with an equivalent martingale measure, (ii) the financial market behaves independently of the occurrence of catastrophes, and (iii) the interest rate changes can be replicated using existing financial instruments.

Probabilistic structure and valuation theory

Let $0 < T < \infty$ be the maturity date of the continuous time trading interval $[0, T]$. The market uncertainty is defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \in [0, T]}, \mathbb{P})$, where \mathcal{F}_t is an increasing family of σ -algebras, which is given by $\mathcal{F}_t = \mathcal{F}_t^{(1)} \times \mathcal{F}_t^{(2)} \subset \mathcal{F}$, for $t \in [0, T]$, where $\mathcal{F}_t^{(1)}$ represents the investment information (e.g., past security prices and interest rates) available to the market at time t and $\mathcal{F}_t^{(2)}$ represents the catastrophic risk information (e.g., insured property losses). The financial risk variables and the catastrophic risk variables can be modelled on $(\Omega^{(1)}, \mathcal{F}^{(1)}, (\mathcal{F}_t^{(1)})_{t \in [0, T]}, \mathbb{P}^{(1)})$ and $(\Omega^{(2)}, \mathcal{F}^{(2)}, (\mathcal{F}_t^{(2)})_{t \in [0, T]}, \mathbb{P}^{(2)})$, respectively. Moreover, define two filtrations $\mathcal{A}^{(1)}$ ($\mathcal{A}_t^{(1)} = \mathcal{F}_t^{(1)} \times \{\emptyset, \Omega^{(2)}\}$ for $t \in [0, T]$) and $\mathcal{A}^{(2)}$ ($\mathcal{A}_t^{(2)} = \{\emptyset, \Omega^{(1)}\} \times \mathcal{F}_t^{(2)}$ for $t \in [0, T]$). It is proved by Lemma 5.1 (Cox and Pedersen 2000) that the σ -algebras $\mathcal{A}_t^{(1)}$ and $\mathcal{A}_t^{(2)}$ are independent under the probability measure \mathbb{P} . Thus, an $\mathcal{A}_T^{(\kappa)}$ measurable random variable X on $(\Omega = \Omega^{(1)} \times \Omega^{(2)}, \mathcal{F}, (\mathcal{F}_t)_{t \in [0, T]}, \mathbb{P})$ (or an $\mathcal{A}^{(\kappa)}$ adapted stochastic process Y) is said to depend only on the financial risk variables ($\kappa = 1$) or catastrophic risk variables ($\kappa = 2$).

The presence of catastrophic risks that are uncorrelated with the underlying financial risks leads us to consider an incomplete market, and there is no universal theory addressing all aspects of pricing (Young 2004). The benchmark to price uncertain cash flow under an incomplete framework is the representative agent. For valuation purposes, similar to (Merton 1976), the authors assume that under the risk-neutral pricing measure \mathbb{Q} , the overall economy depends only on financial risk variables. This is a fairly natural approximation because the global economic conditions and other securities traded on capital markets are only marginally influenced by localized catastrophes, for more information and justification see (Cox and Pedersen 2000; Merton 1976; Doherty 1997; Lee and Yu 2002; Ma and Ma 2013; Gürtler et al. 2016). According to Lemma 5.2 (Cox and Pedersen 2000), under an assumption that the aggregate consumption is $\mathcal{A}^{(1)}$ adapted (assumption (ii)), for

213 any random variable X that is $\mathcal{A}_T^{(2)}$ measurable,

$$214 \quad E^{\mathbb{Q}}[X] = E^{\mathbb{P}}[X]. \quad (1)$$

215 Thus, a $\mathcal{A}^{(2)}$ -adapted aggregate loss process $\{L(t) : t \in [0, T]\}$ retains its original distributional
 216 characteristics after changing from the historical estimated actual probability measure \mathbb{P} to the
 217 risk-neutral probability measure \mathbb{Q} . The σ -algebras $\mathcal{A}_T^{(1)}$ and $\mathcal{A}_T^{(2)}$ are independent under the risk-
 218 neutral probability measure \mathbb{Q} . In an arbitrage-free market (assumption (i)) at any time t , the price
 219 of an attainable contingent claim with payoff $\{P(T) : T > t\}$ can be expressed by the fundamental
 220 theorem of asset pricing in the following form:

$$221 \quad V(t) = \mathbb{E}^{\mathbb{Q}} \left(e^{-\int_t^T r(s) ds} P(T) | \mathcal{F}_t \right), \quad (2)$$

222 see (Delbaen and Schachermayer 1994). Similar to (Shao et al. 2017), the authors assume partic-
 223 ular types of payoff functions. Thus, the authors denote the CAT risk bonds price process by
 224 $\{V^{(\varrho)}(t) : t \in [0, T]\}$, which is characterized by the aggregate loss process $\{L(t) : t \in [0, T]\}$, and
 225 the payoff functions $P_{CAT}^{(\varrho)}$, where $\varrho = 1, 2, 3, 4$. For each $t \in [0, T]$, the process $\{N(t) : t \in [0, T]\}$
 226 describes the number of claims that occur until time t . In addition, define the spot interest rate
 227 process by $\{r(t) : t \in [0, T]\}$ and let $\{W(t) : t \in [0, T]\}$ be a standard Brownian motion.

228 Interest rate process

229 There are different types of interest rates, such as government and interbank rates. Zero-
 230 coupon rates can be either from government rates which are usually deduced by bonds issued
 231 by governments or from interbank rates which are exchanged deposits between banks. The most
 232 important interbank rate usually considered as a reference for contracts is the LIBOR rate, fixed
 233 daily in London. For the purpose of bond prices, all kinds of interest rate models are feasible.
 234 The first stochastic interest rate model was proposed by (Merton 1973), followed by the pioneering
 235 approach of (Vasicek 1977) and some other classical models, such as (Dothan 1978; Cox et al.
 236 1985; Ho and Lee 1986; Hull and White 1990; Black et al. 1990). This section provides analysis for

237 two spot interest rate dynamics: Vasicek and Cox-Ingersoll-Ross (CIR) models, with the explicit
 238 solution for the value of zero-coupon bonds, which are widely used in the financial literature.
 239 Other forms of spot interest rates can also be used in the N-CAT risk bond model, but require
 240 computational calculations for the zero-coupon bond value.

241 *Vasicek model*

242 The instantaneous short rate has the following stochastic process under the risk-neutral measure
 243 \mathbb{Q} :

244
$$dr(t) = a(b - r(t))dt + \sigma dW(t), \quad (3)$$

245 where $\{W(t) : t \in [0, T]\}$ is a standard Wiener process under \mathbb{Q} . The terms a and b are, respectively,
 246 mean reversion speed and mean reversion level of the short rate. The price of a zero-coupon bond
 247 at time t with maturity time T is:

248
$$B_V(t, T) = A(t, T)e^{-B(t, T)r(t)}, \quad (4)$$

249 where

250
$$B(t, T) = \frac{1 - e^{-a(T-t)}}{a}, \quad (5)$$

251
$$A(t, T) = \exp\left(\frac{(B(t, T) - T + t)(a^2b - \sigma^2/2)}{a^2} - \frac{\sigma^2 B(t, T)^2}{4a}\right). \quad (6)$$

252 *Cox–Ingersoll–Ross (CIR) model*

253 The short-rate dynamics $\{r(t) : t \in [0, T]\}$ under the risk-neutral measure \mathbb{Q} can be expressed
 254 as follows:

255
$$dr(t) = k[\theta - r(t)]dt + \sigma\sqrt{r(t)}dW(t), \quad (7)$$

256 with the condition

257
$$2k\theta > \sigma^2, \quad (8)$$

258 where $\{W(t) : t \in [0, T]\}$ is a standard Brownian motion, and $r(0)$, k , θ and σ are positive constants.
 259 CIR model is an extension of Vasicek model where the standard deviation factor changes over time.
 260 It also fixes the Vasicek model shortcoming on theoretically possibility of a negative interest rate.
 261 (Nowak and Romaniuk 2013a) compared the CAT bond prices under the assumption of the spot
 262 interest rate described by the Vasicek, Hull-White, and CIR models. Readers can refer to (Brigo
 263 and Mercurio 2007) for more information on interest rate dynamics. Assume a constant $\lambda_r(t)$ which
 264 represents the market price of risk, and price a pure-discount T-bond at time t by the following
 265 equalities:

$$266 \quad B_{CIR}(t, T) = A(t, T)e^{-B(t, T)r(t)}, \quad (9)$$

267 where

$$268 \quad A(t, T) = \left[\frac{2\gamma e^{(k+\lambda_r+\gamma)(T-t)/2}}{2\gamma + (k + \lambda_r + \gamma) (e^{(T-t)h} - 1)} \right]^{\frac{2k\theta}{\sigma^2}}, \quad (10)$$

$$269 \quad B(t, T) = \left[\frac{2 \left(e^{(T-t)\gamma} - 1 \right)}{2\gamma + (k + \lambda_r + \gamma) (e^{(T-t)\gamma} - 1)} \right], \quad (11)$$

$$270 \quad \gamma = \sqrt{(k + \lambda_r)^2 + 2\sigma^2}. \quad (12)$$

271 **Aggregate claims process**

272 In the classical actuarial literature, (Bowers Jr. et al. 1986) stated that risk models are charac-
 273 terised by the following two stochastic processes: the claim number process (or frequency), which
 274 counts the claims; the claim amounts process or severity, which determines the size of losses when a
 275 claim occurs. Previous literature on CAT risk bonds assumed that these two processes are mutually
 276 independent. However, because the independence assumption is restrictive in many applications,
 277 the relationship between the claim sizes and the inter-arrival times between the events process
 278 is considered when modeling the aggregate losses of CAT risk bonds, and the first experimental
 279 analysis was conducted by (Shao et al. 2017; Ayyub et al. 2016). This paper completes those
 280 models, and introduces additional flexibility for practitioners to implement in a real world CAT risk

281 bond deal.

282 The aggregate loss process $\{L(t) : t \in [0, T]\}$ is defined as a function of two independent
283 variables, claim number process $\{N(t) : t \in [0, T]\}$ and claim sizes $\{X_n : n \in \mathbb{N}^+\}$:

$$284 \quad L(t) = \sum_{n=1}^{N(t)} X_n, \quad (13)$$

285 with the convention that $L(t) = 0$ when $N(t) = 0$, and $X_0 = N(0) = 0$ almost surely (a.s.). The
286 value of the total loss process $L(t)$ is typically calculated by the bond issuer to determine whether
287 or not it met the predetermined level of the trigger event specified in the bond contract.

288 Similar to (Ayyub et al. 2016; Shao et al. 2017), the authors also consider a semi-Markovian
289 dependence structure in continuous time, where the process $\{J_n, n \geq 0\}$ represents the successive
290 type of claims or environment states taking their values in $J = \{1, \dots, w, w + 1, \dots, w + r\}$.
291 However, this is an extension of (Ayyub et al. 2016) to a more general case with $(w + r)$ states. For
292 notational convenience, denote $W = \{1, 2, \dots, w\}$, and $O = \{w + 1, w + 2, \dots, w + r\}$, therefore,
293 $J = W + O$. Here states W are called the work of the system, referring to the incident and
294 accident risks events; and states O (absorbing states) are defined as the failure of the system,
295 where the N-CAT risk bonds system terminates when a major accident risk event occurs, leading
296 the bonds to exercise immediately. the authors call states O absorbing states because once the
297 system reaches those states, the system is unable to escape and will stay there forever. Bond issuers
298 can structure multiple absorbing states in their contract to establish a CAT risk bonds which will
299 exercise immediately in different predetermined situations. The transition matrix $\mathbf{P} = (p_{ij}, i, j \in J)$
300 can be written as

$$\mathbf{P} = \begin{pmatrix} \mathbf{W} & \mathbf{R} \\ \mathbf{0} & \mathbf{I}_r \end{pmatrix} = \begin{pmatrix} p_{11} & \cdots & p_{1w} & p_{1(w+1)} & \cdots & \cdots & p_{1(w+r)} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ p_{w1} & \cdots & p_{ww} & p_{w(w+1)} & \cdots & \cdots & p_{w(w+r)} \\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & \vdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, \quad (14)$$

where $\sum_{j=1}^{w+r} p_{ij} = 1, i \in J$, and \mathbf{I}_r is an r -by- r identity matrix. To interpret this claim based N-CAT risk bond structure more precisely: the bond should not be exercised before expiration if and only if all events occurred stay in the incident or accident level (the event state i stays in the period of work of the system W), and the probability having the next event in state j ($j \in J$) is p_{ij} . If a major accident occurs (a state O event), the N-CAT risk bond contract will terminate immediately, i.e. the system will stay in the state O . Also, $J_{N(T)}$ is the state where the last claim stays at the exercise date.

Define $\{T_n, n \in \mathbb{N}^+\}$ to be the epoch time of the n th claim. Suppose that $0 < T_1 < T_2 < \dots < T_n < T_{n+1} < \dots, T_0 = U_0 = 0$ a.s., and let $U_n = T_n - T_{n-1}$ ($n \in \mathbb{N}^+$) denote the sojourn time in state J_{n-1} . Assume that the trivariate process $\{(J_n, U_n, X_n); n \geq 0\}$ is a semi-Markovian dependency process defined by the matrix $\mathbf{Q} = (Q_{ij}, i, j \in J)$:

$$Q_{ij}(t, x) = \mathbb{P}(J_n = j, U_n \leq t, X_n \leq x | (J_k, U_k, X_k), k = 1, 2, \dots, n-1, J_{n-1} = i). \quad (15)$$

Assuming that the random variable $J_n, n \geq 0$ and the two-dimensional random variable $(U_n, X_n), n \geq 1$ are conditionally independent, then

$$\begin{aligned}
G_{ij}(t, x) &= \mathbb{P}(U_n \leq t, X_n \leq x | J_0, \dots, J_{n-1} = i, J_n = j) \\
&= \begin{cases} Q_{ij}(t, x)/p_{ij}, & \text{for } p_{ij} > 0, \\ \mathbb{1}\{t \geq 0\} \mathbb{1}\{x \geq 0\}, & \text{for } p_{ij} = 0, \end{cases} \tag{16}
\end{aligned}$$

316 where $\mathbb{1}\{\cdot\}$ denotes an indicator function. Denote now

$$317 \quad G_{ij}(t, \infty) = \mathbb{P}(U_n \leq t | J_0, \dots, J_{n-1} = i, J_n = j), \tag{17}$$

$$318 \quad G_{ij}(\infty, x) = \mathbb{P}(X_n \leq x | J_0, \dots, J_{n-1} = i, J_n = j), \tag{18}$$

$$319 \quad H_i(t, x) = \mathbb{P}(U_n \leq t, X_n \leq x | J_0, \dots, J_{n-1} = i) = \sum_{j=0}^m p_{ij} G_{ij}(t, x), \tag{19}$$

$$320 \quad H_i(t, \infty) = \mathbb{P}(U_n \leq t | J_0, \dots, J_{n-1} = i), \tag{20}$$

$$321 \quad H_i(\infty, x) = \mathbb{P}(X_n \leq x | J_0, \dots, J_{n-1} = i). \tag{21}$$

322 Assuming that the sequences $\{U_n, n \geq 1\}$, $\{X_n, n \geq 1\}$ are conditionally independent and given the
323 sequence $\{J_n, n \geq 0\}$, then

$$324 \quad G_{ij}(t, x) = G_{ij}(t, \infty)G_{ij}(\infty, x), \forall t, x \in \mathbb{R}, \forall i, j \in J. \tag{22}$$

325 Thus, the semi-Markov kernel \mathbf{Q} can be expressed as the following product

$$326 \quad Q_{ij}(t, x) = p_{ij}G_{ij}(t, \infty)G_{ij}(\infty, x), \forall t, x \in \mathbb{R}, \forall i, j \in J. \tag{23}$$

327 Let L_n be the successive total claims amount after the arrival of the n th claim. Then, the joint
328 probability of the process $\{(J_n, T_n, L_n); n \geq 0\}$ can be denoted as

$$329 \quad \mathbb{P}[J_n = j, T_n \leq t, L_n \leq x | J_0 = i] = Q_{ij}^{*n}(t, x), \tag{24}$$

$$330 \quad \mathbb{P}[J_n = j, T_n \leq t, L_{n-1} \leq x | J_0 = i] = \tilde{Q}_{ij}^{*n}(t, x), \tag{25}$$

331 where $i, j \in J$. It is crucial to introduce the process $\tilde{Q}_{ij}^{*n}(t, x)$ because when a major accident
 332 occurs (a state O event), the N-CAT risk bonds need to be exercised immediately regardless of
 333 the size of this particular event. These two n -fold convolution matrices $\mathbf{Q}^{*n} = (Q_{ij}^{*n}, i, j \in J)$ and
 334 $\tilde{\mathbf{Q}}^{*n} = (\tilde{Q}_{ij}^{*n}, i \in J, j \in 0)$ can be valued recursively by the following two parts:

$$335 \quad Q_{ij}^{*0}(t, x) = \begin{cases} [1 - G_{ij}(0, \infty)] [1 - G_{ij}(\infty, 0)], & \text{if } i = j, \\ 0, & \text{elsewhere,} \end{cases} \quad (26)$$

$$336 \quad Q_{ij}^{*1}(t, x) = Q_{ij}(t, x), \quad \dots \quad (27)$$

$$Q_{ij}^{*n}(t, x) = \mathbb{P}[J_n = j, \dots, J_1 = W, L_n \leq x, T_n \leq t | J_0 = i] = \sum_{k=1}^w \int_0^t \int_0^x Q_{kj}^{*(n-1)}(t - t', x - x') dQ_{ik}(t', x'). \quad (28)$$

337 and

$$338 \quad \tilde{Q}_{ij}^{*0}(t, x) = 0, \quad (29)$$

$$339 \quad \tilde{Q}_{ij}^{*1}(t, x) = Q_{ij}(t, \infty), \quad \dots \quad (30)$$

$$\tilde{Q}_{ij}^{*n}(t, x) = \mathbb{P}[J_n = 0, J_{n-1} = W, \dots, J_1 = W, L_{n-1} \leq x, T_n \leq t | J_0 = i] = \sum_{k=1}^w \int_0^t Q_{ik}^{*(n-1)}(t - t', x) d(Q_{kj}(t', \infty)). \quad (31)$$

340 Moreover, suppose that a sequence of probabilities $(\Pi_1, \dots, \Pi_{w+r})$ exists (assume that $\Pi_{w+1} =$
 341 $\dots = \Pi_{w+r} = 0$, a.s.), representing the starting probability distribution for the embedded Markov
 342 Chain $\{J_n; n \geq 0\}$, $\Pi_1 + \Pi_2 + \dots + \Pi_w = 1$ and $\Pi_1, \Pi_2, \dots, \Pi_w \in [0, 1]$.

343 The following probabilities are essential for pricing N-CAT risk bonds. At time t , for the

344 predetermined threshold level D ($D \geq 0$),

$$345 \quad F_1(t, D) = \mathbb{P}(L(t) \leq D, J_{N(T)} = W) = \sum_{i=1}^w \sum_{j=1}^w \Pi_i \sum_{n=0}^{\infty} \int_0^t (1 - H_j(t - t', \infty)) dQ_{ij}^{*n}(t', D), \quad (32)$$

$$346 \quad F_2(t, D) = \mathbb{P}(L(t) \leq D, J_{N(T)} = O) = \sum_{i=w+1}^{w+r} \sum_{i=1}^w \Pi_i \sum_{n=0}^{\infty} Q_{ij}^{*n}(t, D), \quad (33)$$

$$347 \quad F_3(t, D) = \mathbb{P}(J_{N(T)} = O) = \sum_{i=w+1}^{w+r} \sum_{i=1}^w \Pi_i \sum_{n=0}^{\infty} \tilde{Q}_{ij}^{*n}(t, D), \quad (34)$$

$$348 \quad F_4(t, D) = \mathbb{P}(L(t) > D, J_{N(T)} = W) = 1 - F_1(t, D) - F_3(t, D), \quad (35)$$

$$349 \quad F_5(t, D) = \mathbb{P}(L(t) \leq D) = F_1(t, D) + F_2(t, D), \quad (36)$$

350 which are the probability of a total loss less than the threshold level and that the last event is not
 351 a major accident, the probability of a total loss less than the threshold level and that the last event
 352 is a major accident, the probability of a major accident occurring, the probability of a total loss
 353 greater than the threshold level and that the last event is not a major accident, and the probability of
 354 a total loss less than the threshold level, respectively. In the N-CAT risk bonds pricing model, the
 355 process changes its state at every claim instance based on the transition matrix \mathbf{P} , with the claim
 356 size distribution dependent on the future state. While, the arrival time before the next catastrophic
 357 claim U_n depends on the severity of the current event X_n , for all $n = 0, 1, 2, \dots$

358 **Payoff functions**

359 This section illustrates the most common payoff functions for CAT risk bonds, (Shao et al.
 360 2017) and two-trigger type payoff structure for T time maturity one-period CAT risk bonds. This
 361 paper only discusses one-period bonds in this paper because multi-period coupon bonds can be
 362 treated as a portfolio of zero-coupon bonds with different maturities. Define a hypothetical zero
 363 coupon N-CAT risk bonds with face value Z at the maturity date, as follows:

$$364 \quad P_{CAT}^{(1)} = \begin{cases} Z, & \text{for } L(T) \leq D, \\ \eta Z, & \text{for } L(T) > D, \end{cases} \quad (37)$$

365 where $L(T)$ is the total insured loss value at the expiry date T , D denotes the threshold value agreed
 366 in the bond contract, and η ($\in [0, 1)$) is the fraction of the principle Z , which the bondholders must
 367 pay when a trigger event occurs.

368 The next payoff function with a multi-threshold value is given by

$$369 \quad P_{CAT}^{(2)} = \sum_{k=1}^l \eta_k Z \quad \forall D_{k-1} < L(T) \leq D_k, \quad (38)$$

370 where $\eta_1 = 1 > \eta_2 > \dots > \eta_l \geq 0$ and $D_0 = 0 < D_1 < \dots < D_l = D$. In general, an investor's rate
 371 of return is inversely proportional to the total catastrophe claims.

372 Another payoff function with a coupon payment at the maturity date, if the trigger has not
 373 occurred, is of the form

$$374 \quad P_{CAT}^{(3)} = \begin{cases} Z + C, & \text{for } L(T) \leq D, \\ Z, & \text{for } L(T) > D, \end{cases} \quad (39)$$

375 where $C > 0$ is the coupon payment level.

376 The two-trigger type payoff function is defined by the following structure:

- 377 1. If at expiry time T , $L(T) \geq D$ ($D \geq 0$) and $J_{N(T)} = W$, that is, the total loss is greater than
 378 a predefined level and no major accident occurred prior to T , the bond holder will lose part
 379 of the capital and receive $\eta_2 Z$ ($\eta_2 > 0$);
- 380 2. If a major accident (state O event) ($J_k \in \{w + 1, w + 2, \dots, w + r\}$) occurs before the expiry
 381 date T , the N-CAT risk bonds expires immediately and the bond holder will receive a partial
 382 amount of their principle $\eta_3 Z$ (normally $0 < \eta_3 < \eta_2$);
- 383 3. Otherwise the bond holder will receive the face value Z .

384 Formally, the payoff function described above is given mathematically by

$$385 P_{CAT}^{(4)} = \begin{cases} Z, & \text{for } L(T) \leq D \text{ and } J_{N(T)} = W, \\ \eta_2 Z, & \text{for } L(T) > D \text{ and } J_{N(T)} = W, \\ \eta_3 Z, & J_{N(T)} = O. \end{cases} \quad (40)$$

386 According to (Shao et al. 2017), the bondholders' payoffs are also determined by the bond
 387 issuers' leverage ratio which is the indicator of the financial risk. In this paper, assume that F_{De}
 388 is the probability of a certain financial institute defaulting in a given period, while bondholders
 389 receive 0 if their bond seller is unable to repay their obligation, which is the worst case scenario.

390 Pricing N-CAT risk bonds

391 This section derives the price of N-CAT risk bonds using the standard tool of a risk-neutral
 392 valuation measure with the payoff functions mentioned above. N-CAT risk bond prices at time t
 393 paying principal Z at time to maturity T are given in the following Theorems 2.1 to 2.4, see also
 394 (Shao et al. 2017; Ayyub et al. 2016; Cox and Pedersen 2000).

395 **Theorem 2.1.** *Let $V^{(1)}(t)$ be the prices of the T -maturity zero-coupon N-CAT risk bond with face*
 396 *value Z under the risk-neutral measure \mathbb{Q} at time t with payoff function $P_{CAT}^{(1)}$, as defined in Eq.*
 397 *(37). Then,*

$$398 V^{(1)}(t) = B(t, T)Z(\eta + (1 - \eta)(F_1(T - t, D) + F_2(T - t, D)))(1 - F_{De}), \quad (41)$$

399 where $F_1(T - t, D)$ and $F_2(T - t, D)$ represent the probabilities given in Eqs. (32) and (33),
 400 respectively, pure discounted bond price $B(t, T)$ is the zero-coupon bond value, and F_{De} is the
 401 probability of a bond issuer defaulting.

402 *Proof.* (Cox and Pedersen 2000) stated that the payoff function is independent of the financial risks

403 variable (interest rate) under the risk-neutral measure \mathbb{Q} . Then, according to Eq. (2),

$$404 \quad V^{(1)}(t) = \mathbb{E}^{\mathbb{Q}} \left(e^{-\int_t^T r_s ds} P_{CAT}^{(1)}(T) | \mathcal{F}_t \right) = \mathbb{E}^{\mathbb{Q}} \left(e^{-\int_t^T r_s ds} | \mathcal{F}_t \right) \mathbb{E}^{\mathbb{Q}} \left(P_{CAT}^{(1)}(T) | \mathcal{F}_t \right). \quad (42)$$

405 Using the closed form solution of the zero-coupon bond price, $\mathbb{E}^{\mathbb{Q}} \left(e^{-\int_t^T r_s ds} \right) = B(t, T)$ as discussed
 406 in Interest Rate Process section, where $B(t, T) = B_V(t, T)$ or $B_{CIR}(t, T)$ in this paper, and this can
 407 be easily substituted depending on the choice of the interest rate model. Together with Eq. (1), the
 408 above equation can be rewritten as

$$409 \quad B(t, T) \mathbb{E}^{\mathbb{P}} \left(P_{CAT}^{(1)}(T) | \mathcal{F}_t \right). \quad (43)$$

By simply applying the payoff function Eq. (37) and rearranging the formula, the N-CAT risk bond price can be formulated as

$$\begin{aligned} V^{(1)}(t) &= B(t, T) \mathbb{E}^{\mathbb{P}} [(Z \mathbb{1}\{L(T) \leq D\} + \eta Z \mathbb{1}\{L(T) > D\})(1 - F_{De}) | \mathcal{F}_t] \\ &= B(t, T) (Z \mathbb{P}(L(T) \leq D) + \eta Z \mathbb{P}(L(T) \geq D)) (1 - F_{De}) \\ &= B(t, T) Z (F_5(T, D) + \eta (1 - F_5(T, D))) (1 - F_{De}), \end{aligned} \quad (44)$$

410 The result follows by some rearrangement. □

411 The proofs of Theorems 2.2 and 2.3 follow the same procedure of Theorem 2.1.

412 **Theorem 2.2.** Let $V^{(2)}(t)$ be the prices of the T -maturity zero-coupon N-CAT risk bond with face
 413 value Z under the risk-neutral measure \mathbb{Q} at time t with payoff function $P_{CAT}^{(2)}$, as defined in Eq.
 414 (38). Then,

$$415 \quad V^{(2)}(t) = B(t, T) Z \sum_{k=1}^l \eta_k (F_5(T - t, D_k) - F_5(T - t, D_{k-1})) (1 - F_{De}), \quad (45)$$

416 where $F_5(T - t, D)$ represents the probabilities given in Eq. (36), pure discounted bond price $B(t, T)$
 417 is the zero-coupon bond value, and F_{De} is the probability of a bond issuer defaulting.

418 **Theorem 2.3.** Let $V^{(3)}(t)$ be the prices of the T -maturity coupon N -CAT risk bond with face value
 419 Z under the risk-neutral measure \mathbb{Q} at time t with payoff function $P_{CAT}^{(3)}$, as defined in Eq. (39).
 420 Then,

$$421 \quad V^{(3)}(t) = B(t, T)(Z + CF_5(T - t, D))(1 - F_{De}), \quad (46)$$

422 where $F_5(T - t, D)$ represents the probabilities given in Eq. (36), pure discounted bond price $B(t, T)$
 423 is the zero-coupon bond value, and F_{De} is the probability of a bond issuer defaulting.

424 **Theorem 2.4.** Let $V^{(4)}(t)$ be the value of the T -maturity zero-coupon N -CAT risk bond with face
 425 value Z under the risk-neutral measure \mathbb{Q} at time t with payoff function $P_{CAT}^{(4)}$, as defined in Eq.
 426 (40). Then,

$$427 \quad V^{(4)}(t) = B(t, T)Z(\eta_2 + (1 - \eta_2)F_1(T - t, D) + (\eta_3 - \eta_2)F_3(T - t, D))(1 - F_{De}), \quad (47)$$

428 where $F_1(T - t, D)$ and $F_3(T - t, D)$ represent the probabilities given in Eqs. (32) and (34),
 429 respectively, pure discounted bond price $B(t, T)$ is the zero-coupon bond value, and F_{De} is the
 430 probability of a bond issuer defaulting.

Proof. Similar to the proof of Theorem 2.1, we have:

$$\begin{aligned} V^{(4)}(t) &= B(t, T)\mathbb{E}^{\mathbb{P}}\left(P_{CAT}^{(4)}(T)|\mathcal{F}_t\right) \\ &= B(t, T)\mathbb{E}^{\mathbb{P}}\left[(Z\mathbb{1}\{L(T) \leq D, J_{N(T)} = W\} + \eta_2 Z\mathbb{1}\{L(T) > D, J_{N(T)} = W\}\right. \\ &\quad \left.+ \eta_3 Z\mathbb{1}\{J_{N(T)} = O\})(1 - F_{De})|\mathcal{F}_t\right] \\ &= B(t, T)Z(\mathbb{P}(L(T) \leq D, J_{N(T)} = W) + \eta_2 \mathbb{P}(L(T) > D, J_{N(T)} = W) \\ &\quad + \eta_3 \mathbb{P}(J_{N(T)} = O))(1 - F_{De}) \\ &= B(t, T)Z(F_1(T, D) + \eta_2 F_4(T, D) + \eta_3 F_3(T, D))(1 - F_{De}). \end{aligned} \quad (48)$$

431 The result follows by some rearrangement. □

432 NUMERICAL EXAMPLES: ANALYSIS AND DISCUSSION

433 Due to limitations in obtaining real data for the determination and calibration of some of the
434 many parameters involved in the pricing process of the N-CAT risk bond, thus for illustration
435 purposes of the theoretical findings, the following two numerical examples are discussed. In
436 the insurance industry, bond issuers can judge their situation and choose a suitable model (more
437 accurately: number of states and the payoff structure) applicable for them. It is important to note
438 that the choice of the model and distribution are crucial in N-CAT risk bond pricing because
439 they affect the bond price significantly. However, the method of selecting a better model and the
440 numerical algorithm are beyond the scope of this paper. Interested readers can refer to (Shao et al.
441 2017) for more details.

442 **A generalized example**

443 This section considers a general example of N-CAT risk bonds, applying the pricing formula in
444 the previous section as an illustration.

445 As in (Ayyub et al. 2016), in this particular example, the authors adopt the same number of
446 states and distributions. Thus, there are 4 states in the period of work of the system ($w = 4$) and
447 1 absorbing state ($r = 1$). The inter-arrival time distribution $G_{ij}(t, \infty)$ is defined to be a Poisson
448 process with parameter λ_i , determined by the state where the system starts. Here, arbitrarily
449 choose $\lambda_i = 10, 30, 5, 20$ for $i = 1, 2, 3, 4$, respectively. The claim size distribution $G_{ij}(\infty, x)$ is
450 assumed to follow a lognormal distribution with mean μ_j and variance σ_j , determined by the state
451 where the system ends. Similarly, assume that $\mu_j = 2, 1, 2.5, 3, 1.5$ and $\sigma_j = 1, 0.8, 1.5, 1.2, 1.5$ for
452 $j = 1, 2, 3, 4, 5$, respectively. Moreover the transition matrix $\mathbf{P} = (p_{ij})$ is given by

$$453 \quad \mathbf{P} = \begin{pmatrix} 0.397 & 0.3 & 0.2 & 0.1 & 0.003 \\ 0.4 & 0.096 & 0.3 & 0.2 & 0.004 \\ 0.4 & 0.4 & 0.199 & 0.1 & 0.002 \\ 0.2 & 0.2 & 0.5 & 0.098 & 0.001 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (49)$$

454 and the stationary distribution $(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = (0.348, 0.261, 0.264, 0.127)$. The parameters of

455 the interest rate model are calibrated from the same data set as in (Shao et al. 2017) (i.e., 3-month
 456 maturity US monthly treasury bill data for the period of 1994–2013). Assume that in both models,
 457 the initial short-term interest rate r_0 is 0.3% and the market price of risk λ_r is a constant -0.01 .
 458 The parameters of the Vasicek model are $\{a, b, \sigma\} = \{0.0790, 3.48\%, 1.28\%\}$, and the parameters
 459 of the CIR model are $\{k, \theta, \sigma\} = \{0.0388, 3.78\%, 5.32\%\}$.

460 To analyze the N-CAT risk bond price sensitivity in terms of the maturity and threshold level,
 461 this paper calculates the bond values with the face value of US\$1,000 for $T = [0.5, 2]$ years to
 462 maturity and threshold level $D = [100, 1600]$ in millions of US\$. The payoff function's parameters
 463 are $\eta = 0.5$, $\eta_1 = 1$, $\eta_2 = 0.5$, $\eta_3 = 0.25$, $C = 0.1$ and $F_{De} = 0.1\%$. For the case when $P_{CAT}^{(2)}$,
 464 $D_1 = 100$ US\$ in millions, D_2 varies within the range of $[100, 1600]$, and $D_3 = \infty$.

465 A benefit of the CIR model over the Vasicek model to analyze the spot interest rate is that CIR
 466 prevents the interest rate falling below zero, which applies to the majority of the real world interest
 467 rate situations. However, according to Figures 3 and 4, the actual differences in the zero-coupon
 468 bond prices, implementing the formula in the Interest Rate Process section, between those two
 469 models are relatively small. N-CAT risk bond issuers can employ other interest rate models in their
 470 contract, but the change is highly likely to be non-significant compared to the bond value proposed
 471 in this paper, see also (Nowak and Romaniuk 2013a). Therefore, from this point onwards, only
 472 results based on the CIR interest rate model will be shown to save space.

473 Figure 5 illustrates the value of the N-CAT risk bonds with face value US\$1000 for the payoff
 474 functions $P_{CAT}^{(1)}$, $P_{CAT}^{(2)}$, $P_{CAT}^{(3)}$ and $P_{CAT}^{(4)}$ with threshold level D and time to maturity T under the
 475 stochastic interest rate assumptions. Comparing across the sub-figures in Figure 5, the bond values
 476 depend heavily on the choice of the payoff function. The value of a zero-coupon bond is normally
 477 less than its face value, as is indicated in Figures 5a, 5b, 5d; while for a coupon bond, the bond
 478 value is greater than the face value, see Figure 5c. For all payoff functions, the prices of N-CAT
 479 risk bonds ($V(t)$) decrease with the increase of the time to maturity (T); while the bond prices ($V(t)$)
 480 increase with the increase of the threshold level (D).

Another example featuring the International Nuclear and Radiological Event Scale (INES)

The International Atomic Energy Agency (IAEA) introduced a worldwide tool – the INES Scale – for communicating the safety significance or damage severity of nuclear and radiological events, see (International Atomic Energy Agency (IAEA) 2013) and Figure 6 for more details. The pyramid on the left-hand side of Figure 6 classifies nuclear-related events on the scale of level 0 to level 7, and the severity of an event falling within one level is about ten times greater than in the previous level. While the right-hand side of Figure 6 generally describes the events in terms of a range of impacts, including people and the environment, radiological barriers and control and defence-in-depth. NCAT risk bond issuers are encouraged to use the INES Scale as a general guidance in bond contract design. In this example, assume the number of states corresponds to the INES Scale level (5 states in the period of work of the system $w = 5$ for risk levels 1 to 5, and 2 absorbing states $r = 2$ for risk levels 6 and 7).

The inter-arrival time distribution $G_{ij}(t, \infty)$ is defined to be a Poisson process with parameter λ_i , determined by the state where the N-CAT risks bonds system starts. Here, arbitrarily choose $\lambda_i = 5, 20, 10, 30, 40$ for $i = 1, 2, 3, 4, 5$, respectively. Again, the authors omitted the analysis of the effect on CAT risk bonds between different $G_{ij}(t, \infty)$ distributions. The claim size distribution $G_{ij}(\infty, x)$ is assumed to follow a lognormal distribution with mean μ_j and variance σ_j , determined by the state where the system ends. Similarly, assume that $\mu_j = 0, 1, 2, 3, 4, 5, 6$ and $\sigma_j = 0.25, 0.5, 1, 1.5, 2, 10, 20$ for $j = 1, 2, 3, 4, 5, 6, 7$, respectively. In general, claims with a higher risk level (or which are more severe) tend to receive more losses, and have more chance to

501 experience an extreme event. Moreover the transition matrix $\mathbf{P} = (p_{ij})$ is given by

$$502 \quad \mathbf{P} = \begin{pmatrix} 0.4989 & 0.25 & 0.15 & 0.06 & 0.04 & 1 \times 10^{-3} & 1 \times 10^{-4} \\ 0.25 & 0.3978 & 0.2 & 0.1 & 0.05 & 2 \times 10^{-3} & 2 \times 10^{-4} \\ 0.3 & 0.2 & 0.2967 & 0.1 & 0.1 & 3 \times 10^{-3} & 3 \times 10^{-4} \\ 0.35 & 0.25 & 0.15 & 0.1956 & 0.05 & 4 \times 10^{-3} & 4 \times 10^{-4} \\ 0.35 & 0.3 & 0.15 & 0.1 & 0.0945 & 5 \times 10^{-3} & 5 \times 10^{-4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (50)$$

503 and the stationary distribution $(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5) = (0.3, 0.25, 0.2, 0.15, 0.1)$. Therefore, in this
 504 example, there is only a small chance to have a level 6 or level 7 severity accident compared to
 505 the chances of having a level 1 to 5 severity event. Additionally, the parameters of the interest rate
 506 model and the payoff functions are the same as in the previous example.

507 The value of the N-CAT risk bonds with face value US\$1000 for the payoff functions $P_{CAT}^{(1)}$,
 508 $P_{CAT}^{(2)}$, $P_{CAT}^{(3)}$ and $P_{CAT}^{(4)}$ with threshold level $D = 1000$ and time to maturity $T = 1$ under the stochastic
 509 interest rate assumptions are 961.51, 780.77, 1088.10 and 953.05, respectively. Comparing with
 510 the previous example (980.51, 636.87, 1091.90 and 969.76, respectively), the CAT risk bond prices
 511 in the new example are lower than in the previous example as it is more risky (includes more risks
 512 with higher possible losses). Similarly, with additional coupon payment, the bond price $V^{(3)}$ is
 513 more valuable than the face value; the simple zero-coupon price $V^{(1)}$ depreciates in value with extra
 514 layers of discount on face value ($P_{CAT}^{(2)}$) and additional default risks ($P_{CAT}^{(4)}$) in the payoff functions,
 515 as a result of $V^{(2)}$ and $V^{(4)}$, respectively.

516 CONCLUSIONS

517 This paper set out to explore the concept of modeling N-CAT risk bonds under various scenarios,
 518 and to help bond sponsors to set a fair price in their contract. The motivation behind this work was to
 519 protect those liability limited regions against the huge economic losses caused by the nuclear power
 520 plant faults. Moreover, there is increasing attention in this area because of the 2011 Fukushima

521 disaster and the UK Hinkley nuclear power plant. In our approach a complete N-CAT risk bond
522 model is proposed as an easily applicable solution for practitioners, filling the gap between the
523 theoretical study and the real world application.

524 The aggregate claims process is one of the most popular indicators as the CAT risk bond trigger.
525 This paper employs a semi-Markov structure to model the dependence of the claim intensity on
526 the severity. In addition, this is the very first paper which includes absorbing states in the Markov
527 process and presents a generalised model with w transit states to indicate the work of the system
528 and r absorbing states to indicate the stop of the system in the CAT risk bonds literature. In any
529 real world application, bond issuers can use any interest rate model they prefer to obtain a pure
530 zero-coupon bond value. However, this might require numerical approximation, because there is
531 not always a closed form solution for a given interest rate model. This paper employed the two most
532 commonly used interest rate models as illustrations. Moreover, four types of payoff structure are
533 proposed in this paper. It is proved that given the same time to maturity and threshold level, different
534 payoff structures can suggest significantly different prices. Additionally, the driving factors of the
535 N-CAT risk bond value are the length of the CAT risk bond contract and the level of the trigger
536 threshold value, i.e., the longer the time to maturity and the smaller the threshold level, the lower
537 the value of the bond. This work is also applicable to other catastrophe risks events.

538 Although, in the present paper, a model is proposed with the flexibility of different interest
539 rates, aggregate claims, payoff structures and the underlying distributions, the relationship between
540 the nuclear power risks and the financial market risks is not considered in our framework. In the
541 literature (Gürtler et al. 2016; Ragin and Halek 2016) examined the impact of natural catastrophes
542 and financial crises on the CAT risk bond premiums. It would be interesting though to consider the
543 case of terrorism as a future extension of the current model (Allison 2005; Kunreuther et al. 2005).
544 This is still a very challenging area to address in future research.

545
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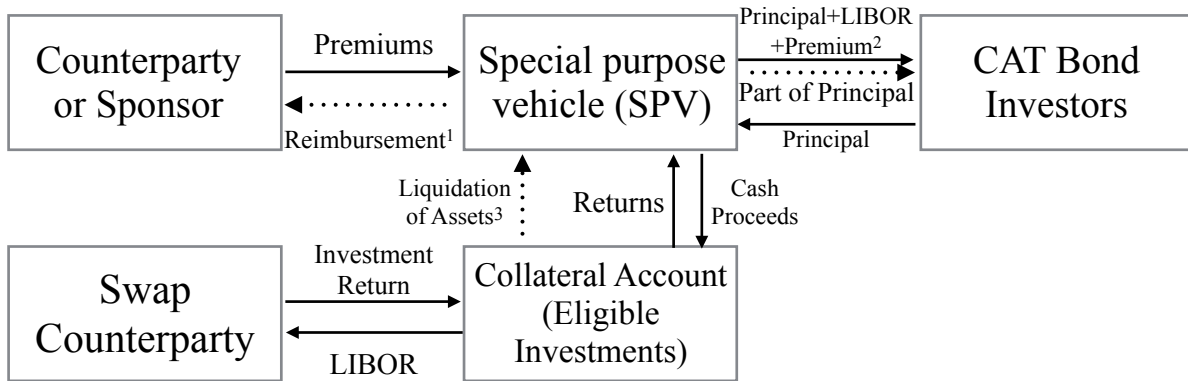
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¹ Event Contingent

² At maturity

³ Event contingent or at maturity

Fig. 1. The process for CAT risk bonds is described.

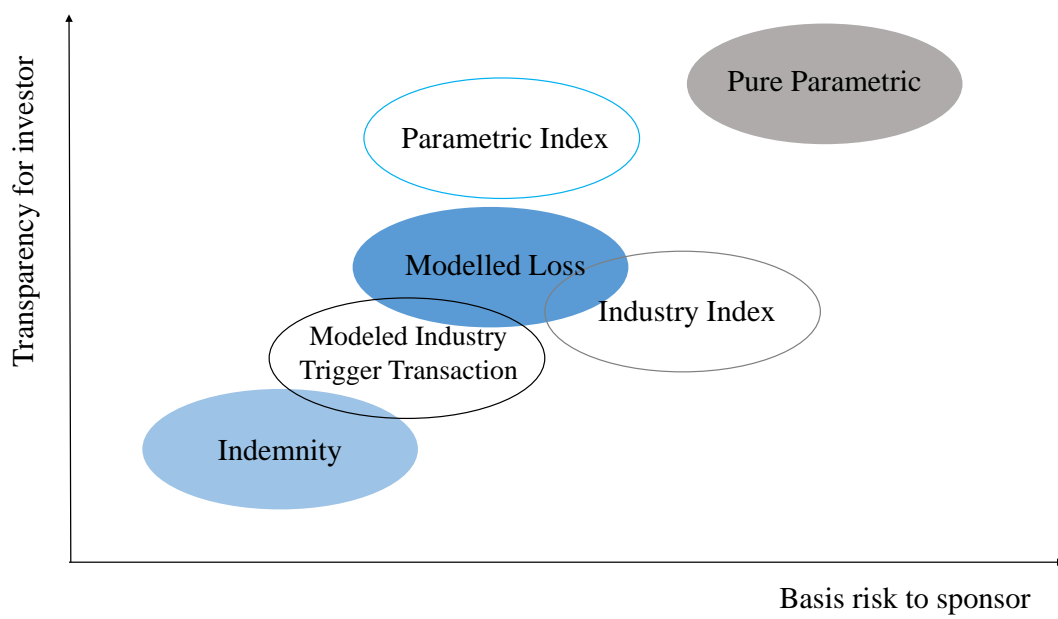


Fig. 2. The transparency and basis risk for various types of triggers.

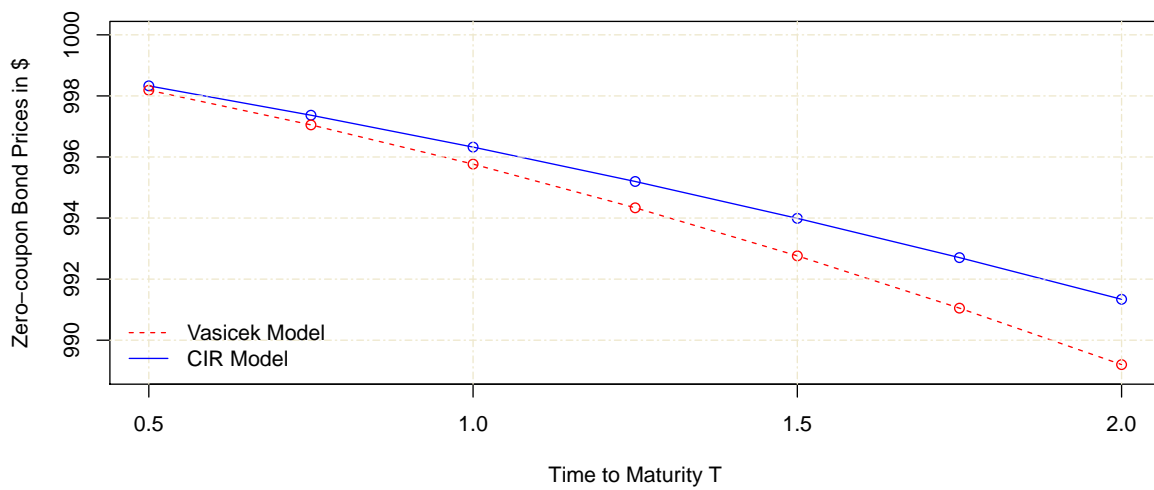


Fig. 3. Zero-coupon bond prices with the face value US\$1000, time to maturity between 0.5 years to 2 years, when interest rate follows a Vasicek or CIR model.

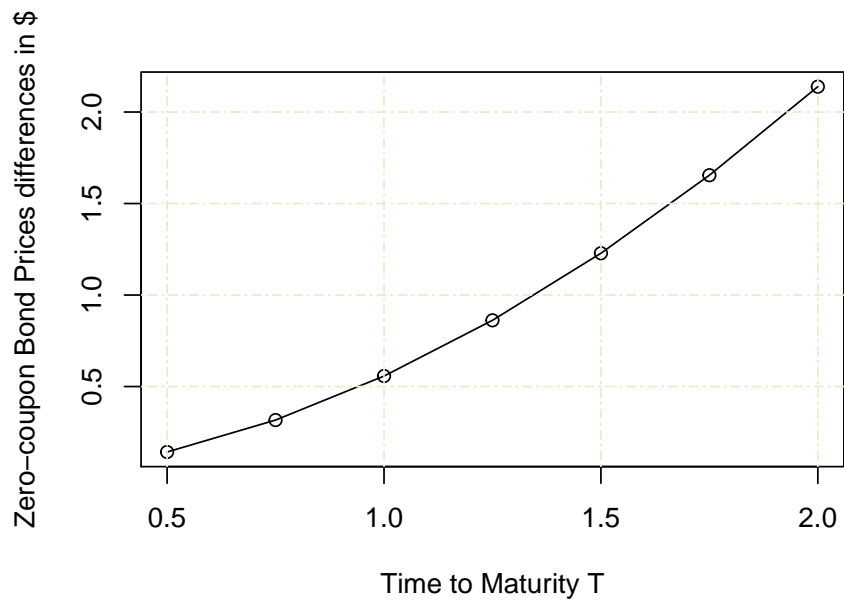
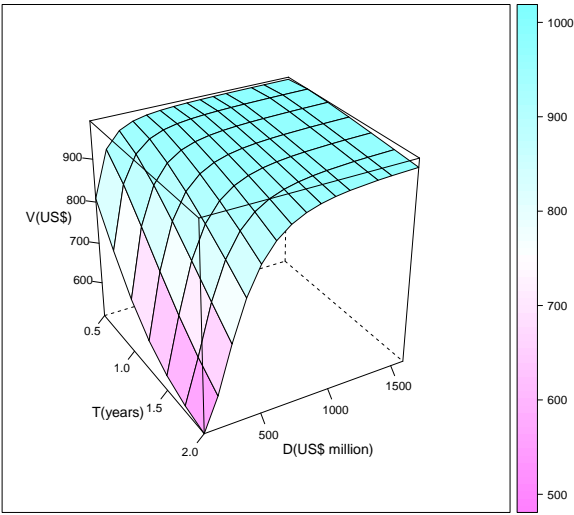
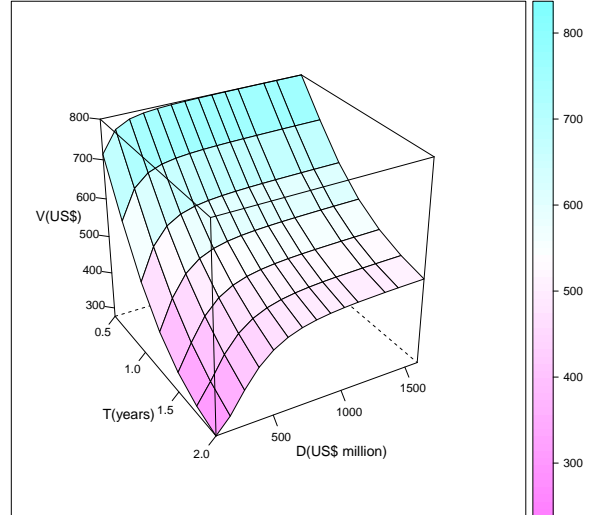


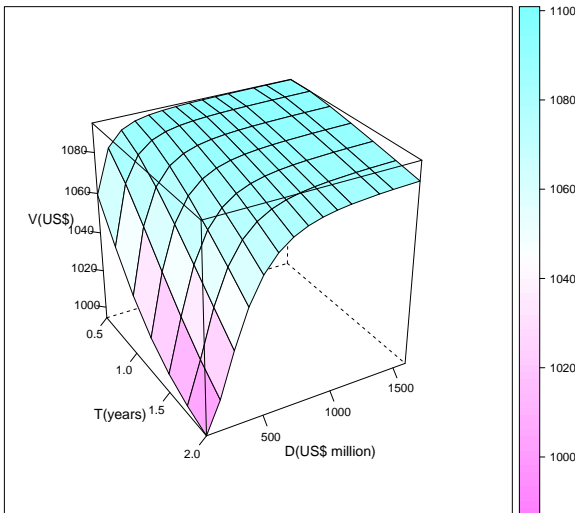
Fig. 4. The relative change between the two prices presented in Fig.3.



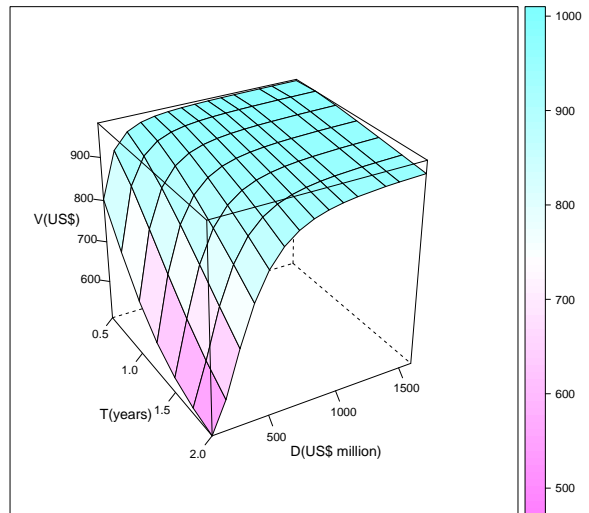
(a) The price of N-CAT risk bonds $V^{(1)}(t)$ featuring the payoff function $P_{CAT}^{(1)}$.



(b) The price of N-CAT risk bonds $V^{(2)}(t)$ featuring the payoff function $P_{CAT}^{(2)}$.



(c) The price of N-CAT risk bonds $V^{(3)}(t)$ featuring the payoff function $P_{CAT}^{(3)}$.



(d) The price of N-CAT risk bonds $V^{(4)}(t)$ featuring the payoff function $P_{CAT}^{(4)}$.

Fig. 5. Value of N-CAT risk bonds (z-coordinate axes) with face value US\$1000 under the lognormal, the non-homogenous Poisson process and CIR interest rate assumptions. Here, time to the maturity (T) decreases by the left axes and threshold level (D) increases by the right axes.

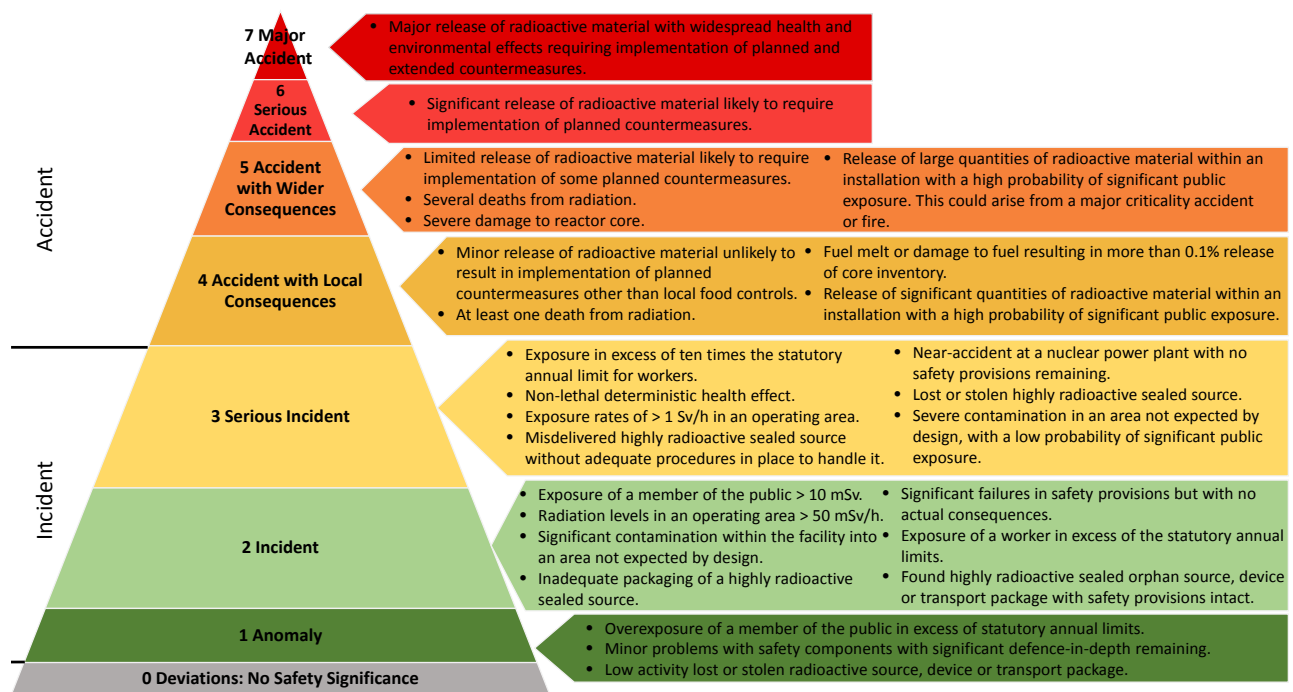


Fig. 6. The international nuclear and radiological event scale (INES) and general description.