

# A sustainable inventory model with controllable carbon emissions for green-warehouse farms

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## **A sustainable inventory model with controllable carbon emissions in green-warehouse farms**

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# **A sustainable inventory model with controllable carbon emissions in green-warehouse farms**

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## **A sustainable inventory model with controllable carbon emissions in green-warehouse farms**

### **Abstract**

Global warming becomes a sensitive issue and many countries try to control CO<sub>2</sub> emissions by investing in many projects and promoting green industry practices. This study proposes a sustainable price-reliant demand inventory model under the effect of controllable carbon emission to reduce CO<sub>2</sub> emissions from a farm warehousing activity. This study extends previous research that framed a sustainable inventory system with greenhouse facilities and controlling carbon emissions by green investment. This study involves an energy-efficient green technology investment in a two-warehouse inventory system to curb carbon emissions during the transportation of products from the owned and rented warehouses and also to the end customers. Two warehouses are frequently used in business to avoid stock out situations and make the business more profitable. Case 1 represents the model with allowable shortages and Case 2 stands for without shortages. Pricing strategies and a hybrid payment scheme are applied throughout this study to make the business more profitable as well as to entice new customers. The retailer has to prepay some portion of the purchasing cost, in which the payment scheme comes to an end with multiple installments. A non-instantaneous deterioration is considered in this study to show the freshness lifetime of the products with a constant backorder. A nonlinear model is framed, and the solution procedure is suggested. A real case study is introduced and provides a connection with the anticipated model for the readers. The examples are executed which is followed by a sensitivity analysis to reveal the model validity and conclusions.

**Keywords:** Carbon emission reduction; two green-warehouse farms; deterioration; price-reliant demand; multiple prepayments

## **A sustainable inventory model with controllable carbon emissions in green-warehouse farms**

### **1. Introduction**

Many countries control CO<sub>2</sub> emissions by investing in many projects and promoting green industry practice. This practice, such as sustainable production and green supply chain, help industrial sectors sustaining their financial performance while simultaneously promoting environmental conduct. To support these practices, various studies have been carried out to provide practical guidance for the industry to reduce its environmental impacts, such as greenhouse gas emissions, water pollution, and solid waste. Many are focusing on the sustainable supply chain of the agricultural industry. An agriculture supply chain has many distinctive challenges on environmental issues as a result of long distribution networks and natural product degradation or deterioration. For example, Alvarez & Orozco (2013) examined a long distribution network of agricultural products, while Coley et al. (2009) concerned about carbon emission accounting between local vegetables and long distribution systems. They identified emissions from packing activity, cold storage, transport to the regional hub, and finally the transport to the customer's location.

Greenhouse farming is one of the agricultural technologies developed in many countries. It produces vegetables, fruits, and flowers in a controlled environment. However, this technology has some emission consequences from the products themselves, and from controlling the best conditions for plant growth and preserving the products. Based on this situation, Taleizadeh et al. (2020) incorporated emissions from product handling and warehousing, distribution, and obsolete materials to optimize the inventory decision of a greenhouse. Meanwhile, Mishra et al. (2020b) took a greenhouse flower retailer as a case to optimize the inventory decision that aims on reducing the total carbon emissions. A greenhouse maintains the freshness level of the product to a certain level before starting to experience degradation which is called a non-instantaneous deterioration. Recently, Datta (2017) and Mishra et al. (2020a) combined the benefit of a green technology investment and a careful inventory decision in a production environment. Green or clean technology is a result of science and technology innovation that emit less emission, consume less energy and resources, and increase the usability of a product through remanufacturing and recycling. Hence, sustainable inventory management of perishable agricultural products with optimum adoption of green technology is a big challenge.

In many real logistics systems, retailers may have limited warehouse capacity. Hence, many retailers rent another warehouse as it is more efficient compared to moving or building a new larger warehouse. In this two-warehouse system, many works have been published assuming a limitless capacity of the rented warehouse (Shah & Cárdenas-Barrón, 2015; Tiwari et al., 2016; Chakraborty et al., 2018; Jonas, 2019; etc.). In addition to its benefits, this system can lead to increased transportation and other material handling activities. Hence, it potentially increases the emissions from transportation among the warehouses. It has motivated our study to provide a sustainable managerial decision method for a greenhouse retailer of a non-instantaneous deterioration agricultural product under a situation of the two-warehouse system. This study considers an energy-efficient green technology investment in a two-warehouse inventory system to curb carbon emissions during the transportation of products. Hence, we introduce the term of a two green-warehouse system. This study is significant as the

retailer is requested for an advance payment from the supplier and that sales depend on the selling price. Moreover, only a certain percentage of demand shortages are backlogged. From our literature review, no work examined this kind of sustainable inventory decision. Taleizadeh et al. (2020) studied a sustainable economic order quantity (EOQ) of a greenhouse farm but did not consider the effect of item deterioration, two-warehouse system, and green technology investment. Tiwari et al. (2019) incorporated carbon emission reduction as the objective of an EOQ model assuming an unlimited single warehouse and non-instantaneous deteriorating items. Mishra et al. (2020b) developed an EOQ model for non-instantaneous deteriorating items with a carbon emissions effect but did not allow a shortage, green technology investment, and has no warehouse limitation.

Considering the possible increase in carbon emissions from a two-warehouse system, this study presents a sustainable EOQ model that is valuable in creating a guide to managerial decision making by adjusting replenishment decisions and green technology investment to build a two green-warehouse inventory system for non-instantaneous deteriorating items. This study problems are as follows.

- How long is the optimum inventory replenishment cycle that will maximize the total profit under the effect of controllable carbon emissions?
- How much is the optimum green technology investment that will maximize the total profit? How do the investments affect total profit and emissions?
- How do the deterioration-free time, shortages policy, advance payment, and pricing strategies affect the profit of the green warehouse retailer?

The remaining part of this study are as follows. The literature that develops the logic of this study in section 2. Section 3 and 4 explain the complete model and theoretical development. Then, section 5 gives some special cases of the developed model. Section 6 and 7 illustrate and discuss the model characteristic through numerical examples and sensitivity analyses, and finally, Section 8 summarizes the study.

## **2. Literature review**

This section presents the past research that becomes the foundation of our study. We discussed the development of the traditional inventory system, also some challenges in sustainable inventory systems.

### **2.1. Traditional inventory system**

Inventory management has focused on the economic aspect since its development in the early 20th century. Inventory management is a challenge in all types of industries because it is closely related to various kinds of costs. Inventory costs occur due to various activities of ordering, shipping, storing, quality control, and waste disposal, and are influenced by many factors such as demand pattern, payment method, and product deterioration. Customer demand is the basic driver of business life. Usually, the demand of a customer depends on the selling price (Lou et al., 2015; Datta, 2017; Taleizadeh et al., 2020; etc.). Datta (2017) and Taleizadeh et al. (2020) considered a linear demand function with a coefficient that will reduce the demand when the price increases. Hence, a business must set an optimal price or plans a discount program to maintain customer demand.

Prepayment (or advance payment) has become a common business practice for many years. Therefore, prior studies have incorporated the effect of this practice on the EOQ model. Zhang et al. (2014) studied an EOQ model where they allowed advance payment and proved that if the entire payment could be paid in advance, then, the customer replenishment cycle has no relationship with the payment length. Taleizadeh (2014a) developed a deteriorating EOQ model in a purchasing inventory system under multiple prepayments. He discussed EOQ models with or without a shortage under prepayments. He found that when the customer chooses a lower period for prepayments, then, the total profit of the system rises. Teng et al. (2016) presented the deteriorating products with an expiration date, then, proved that their model was more realistic than the other. Wu et al. (2017) assumed advance-cash-credit payment strategies and incorporated some important facts to determine the fraction of cycle time so that total profit is maximized.

Deterioration has a certain effect on the inventory management of decayable products such as vegetables, fruits, flowers, gasoline, and chemical products (Nahmias, 1982; Ferguson & Koenigsberg, 2007). By nature, deterioration reduces products' quality and quantity. Researchers are continuously investigating the effect of this deterioration rate. Recently, Mishra et al. (2019) investigated the effect of price and stock-dependent demand on a deteriorating inventory model with trade credit policy and resulted in the process of finding cycle time, the maximum price for an item, and maximum profit. Other research focused on a non-instantaneous deterioration in which the degradation occurred after a certain fresh time (Ouyang et al., 2006; Tiwari et al., 2016; Mashud et al., 2018; Hasan et al., 2020). Certain preservation technology may extend the freshness period (Dye, 2013; Mashud et al., 2019; Mashud et al., 2020a). However, this technology may have a certain effect on the environment Mishra et al. (2020b).

Many retailers implemented a two-warehouse policy in managing their inventories. The retailer has an owned warehouse (OW) with a certain capacity. If the number of items exceeds this capacity, a rented warehouse (RW) is acquired to keep the excess items safe. Usually, the buyer is served from the RW first, then, from the OW. Using these concepts, Shah & Cárdenas-Barrón (2015) established an inventory model where the retailer's credit policy and ordering decisions were analyzed when the supplier offered the retailer a discount on cash or a fixed credit period. Jaggi et al. (2015) settled a deteriorating inventory model with imperfect items under a two-warehouse policy. They introduced a new inventory system with a two-warehouse policy for deteriorating imperfect products. Tiwari et al. (2016) extend the model assuming a trade credit policy and inflation for non-instantaneous deteriorating products under shortages and determined optimal replenishment policies by taking several numerical examples that maximize the optimal profit.

In addition, Majumder et al. (2016) projected an integrated inventory model with a two-level credit period. Xu et al. (2017) deliberated a two-warehouse inventory system over a finite time horizon by comparing different dispatching systems for declining products and proved the uniqueness and existence of optimal solutions. Chakraborty et al. (2018) extended Jaggi et al.'s (2015) model that allowed three-parameter Weibull distribution deterioration and ramp-type demand under the delay-in-payments in an inflationary environment. Jonas (2019) studied a two-warehouse inventory system with backorder under trade credit policy and diminishing payment conditions while Mashud et al. (2020b) added advance payment with the same

attributes in a two-warehouse system. Research on the two-warehouse inventory model is continuously attracting people. Recently, Chandra (2020), Khan et al. (2020), and Atabaki et al. (2020) explored two-warehouse inventory systems for deteriorating items with different demand types but no one concern about the environmental effect of the system and no one introduced green technology adoption.

## **2.2. Sustainable inventory system**

Green supply chain management is studied to provide practical guidance on how the industry is adapting to environmental preservation. Srivastava (2007) studied relevant green supply chain literature and supplies several fruitful future directions. The industry can harmonize its operations with various existing regulations. A green supply chain will maintain industrial efficiency, reliability, quality, and performance while reducing the ecological impact. Ghosh & Shah (2012) developed a green supply chain inventory model from the perspective of a retailer and a manufacturer and calculated the total profit of the system. Swami & Shah (2013) studied the coordination between a manufacturer and a retailer for a vertical supply chain model using green products. The study demonstrated that the proportion of the optimal greening cost calculated by the supply chain members is the same as the proportion of their greening cost ratios and green sensitivity ratios. Zailani et al. (2015) adopted the green innovation policy and checked the impacts on firm performance. Zhu & He (2016) investigated the green product design issues and found that the existence of double marginalization consequence has an anti-intuitive effect on the greenness of products.

Moreover, Wahab et al. (2011) proposed a sustainable inventory with emission reduction under environmental circumstances and focused to control the emission rate in inventory transportation. Chen et al. (2013) considered a model to reduce the emission cost and to modify order quantities and implemented their model for controlling several environmental regulations. Chen & Hao (2015) studied the effect of optimal pricing under the emission tax mechanism for a firm by recovering that more reduction percentage leads to more profit. Datta (2017) combined the benefit of a green technology investment and a careful inventory decision in a production environment. Bartolini et al. (2019) derived a model for green warehousing. Wee & Daryanto (2020) studied low-carbon supply chain inventory models with a carbon emission tax system and allowed backorder. Tiwari et al. (2019) developed a green inventory lot-sizing model for non-instantaneous deteriorating items from a retailer's perspective. The retailer has an unlimited single warehouse and offers a delay-in-payment. Recently, Mishra et al. (2020b) proposed a sustainable inventory model with a controllable emission and allowed the model to control deterioration. The result showed the benefit of green technology and offered a greenhouse model from the retailer perspective that used only one warehouse; hence, two warehouse inventory systems are indeed important to control the emission by using green technology.

Prior studies examined carbon emissions reduction through inventory management decisions. However, a sustainable inventory system needs to consider a two-warehouse model as it is a common business practice. Moreover, the adoption of green technology gives an environmental advantage. This study investigates a new managerial situation and extends the previous studies by assimilating the effect of carbon emission, partial backloging, and advance payment with price-dependent demand. The objective is to examine the consequence of green



technology investment and cycle time for a two green-warehouse retailer. Table 1 presents the prior studies on this topic.

**(INSERT Table 1 here)**

Table 1. Overview of the related study

Author(s)	Two-warehouse inventory system	Payment scheme	Price-dependent demand	Non-instantaneous deterioration	Partially backlogged	Green investment	Carbon Emission
Chen et al. (2013)	x	x	x	x	x	x	✓
Taleizadeh et al. (2014a)	x	Advance	x	x	x	x	x
Jaggi et al. (2015)	✓	x	x	✓	x	x	x
Chen & Hao (2015)	x	x	x	x	x	x	✓
Lou et al. (2015)	x	x	✓	x	x	✓	x
Tiwari et al. (2016)	✓	x	x	✓	✓	x	x
Teng et al. (2016)	x	Advance	x	x	✓	x	x
Datta (2017)	x	x	✓	x	x	✓	✓
Xu et al. (2017)	✓	x	x	✓	x	x	x
Chakraborty et al. (2018)	✓	x	x	x	✓	x	x
Wu et al. (2018)	x	Advance-cash-credit	x	x	x	x	x
Taleizadeh et al. (2020)	x	x	✓	x	✓	x	✓
Yu (2019)	✓	x	x	✓	x	x	x
Tiwari et al. (2019)	x	x	✓	✓	x	x	✓
Mishra et al. (2020b)	x	Advance	x	✓	x	✓	✓
Wee & Daryanto (2020)	x	x	x	x	✓	x	✓
This study	✓	Advance	✓	✓	✓	✓	✓

✓ = Yes; x = No

### 3. Model development

This study works under several assumptions:

- a. The inventory system has a constant lead-time with an infinite replenishment rate.
- b. An infinite planning horizon for the whole system is considered.
- c. Shortage is partially backlogged with a rate of  $\eta$  (see Chakraborty et al., 2018; Taleizadeh et al., 2020).
- d. Considering the linear effect of the price ( $p$ ) on customer demand (see Taleizadeh et al., 2020), the demand level is considered as:
- e.  $D = \begin{cases} (a - bp) & \text{when } I(t) \geq 0 \\ \eta(a - bp) & \text{when } I(t) < 0 \end{cases}$  for a constant market size  $a$  and shape parameter  $b$ .
- f. Considering a non-instantaneous deterioration phenomenon (see Jaggi et al., 2015; Tiwari et al., 2019; etc.), the non-deterioration period lies in the interval  $[0, t_1]$  and the deterioration occurs during the interval  $[t_1, t_2]$  at a constant rate  $\alpha$  at the OW and  $\beta$  at the RW.
- g. The supplier requests prepayment in  $n$  installments. Then, the retailer takes a loan from a financial institution with a certain interest rate. A similar procedure is used throughout this study, where the cyclic capital cost of prepayments is calculated similarly to Taleizadeh et al. (2013) and Lashgari et al. (2018).

- h. The retailer has a proposal in the direction of a greener logistic system by investing capital in advanced technology ( $G$ ) such as energy-efficient systems, renewable energy sources, and so on. However, there is an upper limit based on the retailer's budget for a green technology project. The fraction of the emission reduction is  $F = \xi(1 - e^{-\chi G})$  for  $\xi$  is the portion of carbon emissions when green technology is invested and  $\chi$  redirects the ability of greener technology in decreasing emissions.  $F = \xi(1 - e^{-\chi G}) \Rightarrow G = -(1/\chi) [\ln(1 - F/\xi)]$ , similar to the relation anticipated by Lou et al. (2015). Hence,  $F$  becomes zero when  $G = 0$  and inclines to  $\xi$  when  $G \rightarrow \infty$ . Lou et al. (2015) took  $\xi = 1$ . With an investment  $G$ , the retailer can decrease emissions while the investment cost function  $F(G)$  is incessantly differentiable with the condition  $F'(G) > 0$ ,  $F''(G) < 0$ . This function is used in literature to formulating various investment policies and which is considered by Lou et al. (2015).

The following notations are also used in model development.

<i>Notations</i>	<i>Descriptions</i>
$R$	backorder quantity
$Q$	order quantity
$CCC$	the cyclic capital cost
$TCR$	reduced transportation cost
$SR$	sales revenue
$X$	total annual cost
$c_1$	the minimum transportation cost needs to run the transport
$d$	distance traveled from OW to RW and from RW to customer

$t_v$	variable transportation cost, which is equal to fuel price
$c_2$	fuel consumption when the truck is empty
$c_3$	supplementary fuel consumption of the truck per ton of payload
$m$	product weight
$\aleph$	trip number
$e_1$	carbon emission cost produced by the vehicle
$e_2$	extra carbon emission cost for transporting one-unit of item
$O_r$	ordering cost
$a$	a constant market size
$b$	shape parameter
$c_p$	purchasing cost
$C_{ho}$	holding cost for OW
$C_{hr}$	holding cost for RW
$c_s$	per unit backorder cost
$L$	total lead-time for delivery of the product
$I_p$	interest pay
$\sigma$	the fraction of purchasing cost
$S_r$	capacity of OW
	<i>Decision variables</i>
$G$	green technology investment
$t_1$	the deterioration-free time
$T$	cycle time

In this study, two realistic cases are studied where Case 1 shows a two green-warehouse farm with shortages of products while Case 2 illustrates the view with no shortages of products.

### 3.1. Case 1 (Shortages is allowed)

A two green-warehouse inventory model is proposed where a retailer is requested by the supplier to deposit a  $\sigma$  portion of the purchasing cost before the transport of the products while the process should be anticipated in  $n$  equal multiple installments in the range of the lead time  $L$  at equal intervals. At the receiving of the products, at time  $t = 0$ , the retailer is asked to submit the remaining portion of  $(1-\sigma)$  of the purchasing cost. The retailer then kept some of the highly demanded products in the OW and others in the RW, transported by a truck. This transportation needs some cost which has a fix and variable transportation cost including carbon emission cost. CO<sub>2</sub> emission is a destructive thing for the environment, so green technology is used to reduce the amount. The technology is employed in a precise way so that the retailer gets benefited. While receiving the lot,  $R$  units of items are used to satisfy the partial backlogged demand and the on-hand inventory level afterward drops to  $S$ . The purchasable products of  $S_r$  units are reserved in the OW while the residual portion of the purchasable units  $(S-S_r)$  is kept in the RW.

The holding cost in RW is greater than the OW because RW is owned by the other party and may have better service. Throughout the time interval  $[0, t_1]$ , the inventory level in the RW decreases according to the demand function  $D = (a-bp)$  and a constant deterioration  $\beta$ . The

products in the RW are depleted first and become zero at  $t = t_1$  (see Figure 1). In the case of the OW, the stock is decreasing during  $[0, t_1]$  following a constant deterioration rate  $\alpha$ . Then, in the time interval  $[t_1, t_2]$ , the stock level in OW reduces owing to the joint consequence of customer demand  $D$  and deterioration  $\alpha$  and falls to zero at time  $t = t_2$ . The shortages are taken place during  $[t_2, T]$  at a constant rate  $\eta$ . The total scenario of the inventory is presented in Figure 1. In Figure 1, the lead time is presented by  $L$ , where the installment interval is  $L/n$  as the number of installments is  $n$ . The yellow shaded area is for the remaining portion of purchase cost which is made at the time of receiving the products while the blue shaded area denotes the portion of purchase cost which is made before receiving the products.

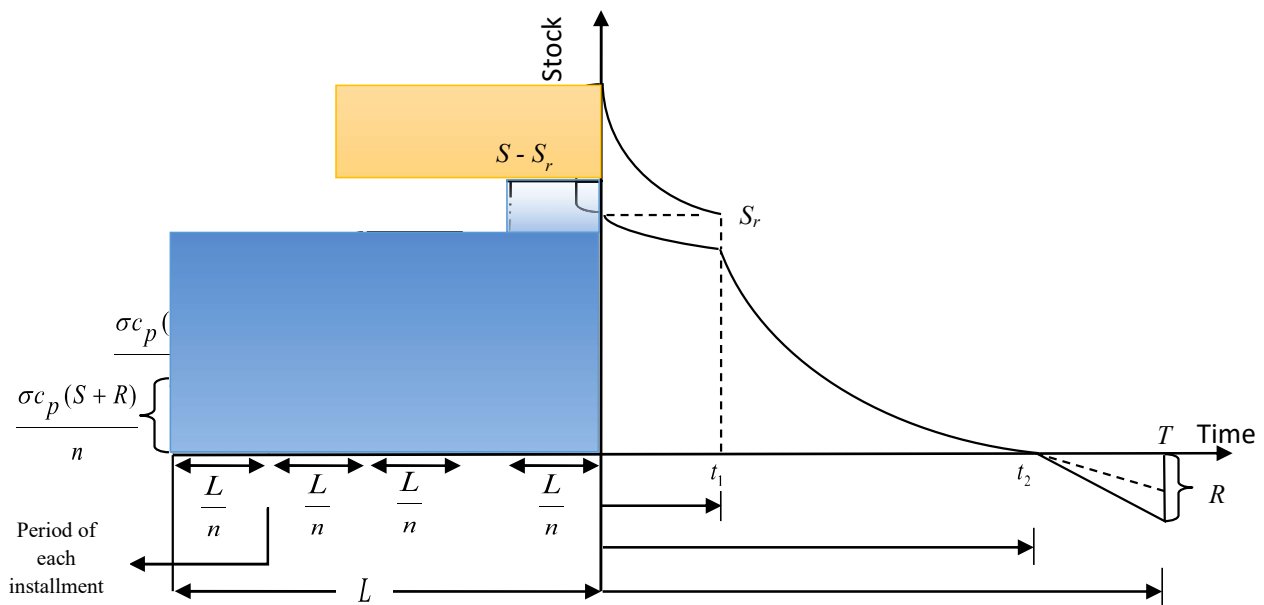


Figure 1. Advance payment and two-warehouse inventory system

At any moment  $t$ , the inventory level  $q_r(t)$  in the RW is defined by the resulting differential equations (DEs):

$$\frac{dq_r}{dt} + \beta q_r = -D, \quad 0 \leq t \leq t_1 \quad (1)$$

Using boundary conditions

$$q_r(0) = S - S_r \text{ and } q_r(t_1) = 0 \quad (2)$$

the solution of Equation (1) is

$$q_r = \frac{D}{\beta} [e^{\beta(t-t_1)} - 1], \quad 0 \leq t \leq t_1 \quad (3)$$

Now, plugging the value  $q_r(0) = S - S_r$  in the above Equation (3), we have

$$S = S_r + \frac{D}{\beta} (e^{\beta t_1} - 1) \quad (4)$$

This study refers to Chang (2004), Taleizadeh (2014a), and Taleizadeh (2014b) for assuming a small consequence of the deterioration rate, hence the same procedure is used, which is

$$e^{\beta t_1} \approx 1 + \beta t_1 + \frac{\beta^2 t_1^2}{2}$$

Thus, equation (4) is transferred as

$$S = S_r + Dt_1 \left( 1 + \frac{\beta t_1}{2} \right) \quad (5)$$

Again, the level of inventory denoted by  $q_o(t)$  in the OW at any time  $t$  is labeled thru the subsequent DEs:

$$\frac{dq_o}{dt} + \alpha q_o = 0, \quad 0 \leq t \leq t_1 \quad (6)$$

$$\frac{dq_o}{dt} + \alpha q_o = -D, \quad t_1 \leq t \leq t_2 \quad (7)$$

$$\frac{dq_o}{dt} = -\eta D, \quad t_2 < t \leq T \quad (8)$$

$$\text{With } q_o(0) = S_r; q_o(t_2) = 0; q_o(T) = -R \quad (9)$$

Once more, at the point  $t = t_1$  and  $t = t_2$  the inventory level  $q_o(t)$  is continuous.

Now, with Equation (9), it is possible to rewrite Equations (6), (7) and (8) as

$$q_o(t) = S_r e^{-\alpha t} \approx S_r \left( 1 - \alpha t + \frac{\alpha^2 t^2}{2} \right), \quad 0 \leq t \leq t_1$$

$$(10) \quad q_o(t) = \frac{D}{\alpha} [e^{\alpha(t_2-t)} - 1], \quad t_1 < t \leq t_2$$

$$\approx q_o(t) = D(t_2 - t) \left[ 1 + \frac{\alpha(t_2 - t)}{2} \right] \quad (11)$$

$$q_o(t) = \eta D(T - t) - R, \quad t_2 < t \leq T \quad (12)$$

Now, using the relation of continuity at the points  $t = t_1$  and  $t = t_2$ , we have

$$S_r e^{-\alpha t_1} = \frac{D}{\alpha} [e^{\alpha(t_2-t_1)} - 1] \quad (13)$$

$$\text{and } R = \eta D(T - t_2) \quad (14)$$

Hence, per cycle order quantity is

$$Q = S + R$$

$$= S_r + Dt_1 \left( 1 + \frac{\beta t_1}{2} \right) + \eta D(T - t_2) \quad (15)$$

Now, the inventory associated costs for Case 1 of this model is listed as follows:

a. Ordering cost:  $O_r$

b. Purchasing cost:

$$c_p(S + R) = c_p \left[ S_r + Dt_1 \left( 1 + \frac{\beta t_1}{2} \right) + \eta D(T - t_2) \right] \quad (16)$$

c. Holding cost:

$$\begin{aligned}
& c_{hr} \int_0^{t_1} q_r(t) dt + c_{ho} \int_0^{t_1} q_o(t) dt + c_{ho} \int_{t_1}^{t_2} q_o(t) dt \\
& \approx \frac{1}{2} c_{hr} D t_1^2 + c_{ho} S_r t_1 \left[ 1 - \frac{\alpha t_1}{2} \right] + \frac{c_{ho} D}{2} (t_2 - t_1)^2
\end{aligned} \tag{17}$$

d. Shortage cost:

$$-c_s \int_{t_1}^T q_o(t) dt = \frac{1}{2} c_s \eta D (T - t_2)^2 \tag{18}$$

e. Cyclic capital cost:

The cyclic capital cost is the same as the procedure for the purchaser formerly receiving the order in Taleizadeh et al. (2013), Teng et al. (2016), Wu et al. (2017), and Lashgari et al. (2018) (see Assumption (f)), as presented in Figure 1.

$$\begin{aligned}
CCC &= \left( \frac{I_p \sigma c_p}{n} Q \times n \times \frac{L}{n} \right) + \left( \frac{I_p \sigma c_p}{n} Q \times (n-1) \times \frac{L}{n} \right) \\
&+ \left( \frac{I_p \sigma c_p}{n} Q \times (n - (n-2)) \times \frac{L}{n} \right) + \left( \frac{I_p \sigma c_p}{n} Q \times (n - (n-1)) \times \frac{L}{n} \right) \\
&= \left( \frac{I_p \sigma c_p}{n} Q \times \frac{L}{n} \right) [n + (n-1) + \dots + 2 + 1] \\
&= \frac{n+1}{2n} I_p L \sigma c_p Q
\end{aligned} \tag{19}$$

f. The transportation cost comprises of a fixed and variable transport cost together with carbon emission cost. Here, the emission cost hinges on the delivery quantity ( $Q$ ). The variable emission cost is calculated based on the weight of the shipment and it relates to the size of the shipment. In this proposed model, the shipment size  $Q$  is transported or shipped from OW to RW and from RW to the customer and the transporting/shipping cost is paid by the retailer. Here  $2d$  is added as the distance is counted for the reverse process also (back and forth).

The emission cost has a dependency on the delivery quantity ( $Q$ ) of the product or the vehicle payload (this is defined as truck capacity). Here,  $t_v$  is the variable transportation cost, which is multiplied by total vehicle fuel consumption  $c_2$ . Here, the additional vehicle fuel consumption  $c_3$  is needed for one distance when the truck is loaded with the products, so the distance  $d$  is multiplied. Similarly, the additional carbon emission cost is applicable when the truck is full of product, not when the free truck is returning.

Hence, the transportation cost is

$$TNC = \frac{n}{T} [c_1 + (2dt_v c_2 + dt_v c_3 mQ) + (2de_1 + de_2 Q)] \tag{20}$$

g. The green technology investment cost is the total cycle length ( $T$ ) multiply by the total investment in green technology ( $G$ ), which is

$$GTI = GT \tag{21}$$

h. With the implementation of green technology (Please see Assumption (g)), the new/reduced transportation cost is

$$TCR = \frac{\$}{T} \left[ c_1 + (2dt_v c_2 + dt_v c_3 mQ) + (2de_1 + de_2 Q)(1 - \xi(1 - e^{-\xi G})) \right] \quad (22)$$

Consequently, the total cost  $X = \frac{1}{T}$  [Ordering cost + Purchasing cost + Holding cost + Shortage cost + Capital cost + reduced Transportation cost+ Green technology investment cost], i.e.,

$$X = \frac{1}{T} \left[ O_r + c_p Q + \frac{1}{2} c_{hr} D t_1^2 + c_{ho} S_r t_1 \left[ 1 - \frac{\alpha t_1}{2} \right] + \frac{c_{ho} D}{2} (t_2 - t_1)^2 + \frac{n+1}{2n} I_p L \sigma c_p Q + \frac{1}{2} c_s \eta D (T - t_2)^2 + \$ \left[ c_1 + (2dt_v c_2 + dt_v c_3 mQ) + (2de_1 + de_2 Q)(1 - \xi(1 - e^{-\xi G})) \right] + GT \right] \quad (23)$$

Now, the sales revenue of Case 1 is calculated as

$$SR = p [Dt_2 + \eta D(T - t_2)] \quad (24)$$

So, the total profit is

$$\Delta_{TP} = \frac{1}{T} \left[ \begin{array}{l} p [Dt_2 + \eta D(T - t_2)] \\ - \left[ O_r + c_p Q + \frac{1}{2} c_{hr} D t_1^2 + c_{ho} S_r t_1 \left[ 1 - \frac{\alpha t_1}{2} \right] + \frac{c_{ho} D}{2} (t_2 - t_1)^2 + \frac{1}{2} c_s \eta D (T - t_2)^2 \right. \\ \left. + \frac{n+1}{2n} I_p L \sigma c_p Q + \$ \left[ c_1 + (2dt_v c_2 + dt_v c_3 mQ) + (2de_1 + de_2 Q)(1 - \xi(1 - e^{-\xi G})) \right] + GT \right] \end{array} \right] \quad (25)$$

Meanwhile, the system without carbon emission tax is

$$\delta_{TP} = \frac{1}{T} \left[ \begin{array}{l} p [Dt_2 + \eta D(T - t_2)] \\ - \left[ O_r + c_p Q + \frac{1}{2} c_{hr} D t_1^2 + c_{ho} S_r t_1 \left[ 1 - \frac{\alpha t_1}{2} \right] + \frac{c_{ho} D}{2} (t_2 - t_1)^2 + \frac{1}{2} c_s \eta D (T - t_2)^2 \right. \\ \left. + \frac{n+1}{2n} I_p L \sigma c_p Q + \$ \left[ c_1 + (2dt_v c_2 + dt_v c_3 mQ) \right] + GT \right] \end{array} \right] \quad (26)$$

### 3.2 Case 2 (Shortages is not allowed)

An inventory model is formulated when shortages are not allowed In this case. When shortages occur, some customers bothered to wait, so any sort of shortage may bring some loss in goodwill. It has a profound impact on a sustainable business, therefore retailers may not allow this situation. In this case, if this study puts  $t_2 \approx T$ , i.e.,  $R = 0$  in Case 1, the resultant profit function will generate an inventory model without shortages as depicted in Figure 2. In Figure 2, the left side shows the equal prepayments in lead-time  $L$ , while after time,  $t = 0$ , the retailer started to sell the products. First, the rented warehouse products finished at  $t = t_1$  and then started to deplete the owned warehouse products. However, all the demands are satisfied within the cycle  $T$ ; thus, no shortages arise in this case.





#### 4. Theoretical development

In the theoretical development section, we will prove the concavity to show the availability of an optimum solution (concavity narrates to the rate of change of a function's derivative) for Case 1 and then for Case 2. Concavity of profit function and convexity of cost function has the same meaning. So, in this case, the convexity of cost function has been proved theoretically for our convenience.

##### 4.1. Case 1 (Shortage is allowed)

The cost function for Case 1,  $X$  is given by Equation (23). Now, to get the cycle length  $T$  from the above equation one needs to continuously two times differentiate Equation (23) with regard to decision variable  $T$  as follows

$$\frac{dX}{dT} = \frac{-1}{T^2} \left[ \begin{aligned} & \left[ O_r + c_p Q + \frac{1}{2} c_{hr} D t_1^2 + c_{ho} S_r t_1 \left[ 1 - \frac{\alpha t_1}{2} \right] + \frac{c_{ho} D}{2} (t_2 - t_1)^2 + \frac{1}{2} c_s \eta D (T - t_2)^2 + \right. \\ & \left. \frac{n+1}{2n} I_p L \sigma c_p Q + \aleph \left[ c_1 + (2dt_v c_2 + dt_v c_3 m Q) + (2de_1 + de_2 Q) (1 - \xi (1 - e^{-\alpha G})) \right] + GT \right] \\ & + \frac{1}{T} [2\eta(n+1)D + c_s \eta D (T - t_2) + G] \end{aligned} \right] \quad (30)$$

$$\frac{d^2 X}{dT^2} = \frac{2}{T^2} \left[ \begin{aligned} & \left[ O_r + c_p Q + \frac{1}{2} c_{hr} D t_1^2 + c_{ho} S_r t_1 \left[ 1 - \frac{\alpha t_1}{2} \right] + \frac{c_{ho} D}{2} (t_2 - t_1)^2 + \frac{1}{2} c_s \eta D (T - t_2)^2 + \frac{n+1}{2n} I_p L \sigma c_p Q \right. \\ & \left. + \aleph \left[ c_1 + (2dt_v c_2 + dt_v c_3 m Q) + \left( 2de_1 + de_2 \left( S_r + Dt_1 \left( 1 + \frac{\beta t_1}{2} \right) (1 - \xi (1 - e^{-\alpha G})) \right) \right) \right] \right] - \\ & \left[ \frac{1}{2} [2\eta(n+1)D + c_s \eta D (T - t_2)] \right. \\ & \left. + \frac{1}{T} [c_s \eta (a - bp) + 2G] \right] \end{aligned} \right] \quad (31)$$

As the second derivative gives the positive values for  $T > 0$  (checked by Mathematica 9.0), so, the total cost function is convex.

Further, the optimum period length ( $T^*$ ) is derived by

$$T^* = T^*(t_1) = \sqrt{\frac{\gamma(t_1)}{\phi_4}} \quad (32)$$

Proof: See **Appendix A**

Also, the optimum  $t_1^*$  is derived by

$$t_1^* = \frac{-\phi_3}{2\phi_2}, \quad (33)$$

Proof: See **Appendix B**

Finally, by substituting the values, we acquire the optimum value of the cost and profit function.

## 4.2. Case 2 (Shortages is not allowed)

The concavity test for Case 2 is very much akin to Case 1, so to avoid the redundancy for the readers we have omitted the theoretical development for Case 2 whereas, a numerical example is presented later to show its concavity numerically.

## 5. Cases information

Some cases could be identified concerning the proposed model:

- a. If anyone put  $t_2 \approx T$ , i.e.,  $R = 0$ , Case 1 is transferred to an inventory model without shortages (Case 2). When  $\eta = 1$  (unit backlogging rate), the shortages are fully backlogged in the anticipated model.
- b. For transforming the two-warehouse system into a single warehouse system one needs to put  $S - S_r = 0$ .
- c. If  $S - S_r = 0$ , i.e., the stock at rented warehouse becomes zero (single warehouse case) and if there is no investment in green technology, i.e.,  $G = 0$ , together with  $t_1 = 0$  then the projected model converted to a single warehouse problem like the model of Wu et al. (2017).
- d. If one considers constant demand  $D$ ,  $S - S_r = 0$  and  $G = 0$  with the partial and full trade-credit policy then the suggested model has a form of Lashgari et al. (2016).
- e. The recommended model is similar to the model of Teng et al. (2016) with time-dependent deterioration, constant demand and  $S - S_r = 0$ ,  $M = 0$ ,  $I_e = 0$ ,  $I_p = 0$ , and  $G = 0$ .
- f. When  $S - S_r = 0$ ,  $\alpha = 0$ ,  $\beta = 0$ ,  $C_b \rightarrow \infty$ ,  $L = 0$ , and constant demand with the deterioration period  $t_1 = 0$ , more importantly, if there is no investment in green technology, then the model of Musa et al. (2012) is be degenerated from this model.
- g. When  $S - S_r = 0$ ,  $M = 0$ ,  $I_e = 0$ ,  $I_p = 0$ , and  $G = 0$  and demand is constant under the non-instantaneous period  $t_1 = 0$ , the study generates Taleizadeh et al. (2013) model.
- h. When the green technology investment becomes zero then the model of Taleizadeh (2014a) together with Zia et al. (2015) and Wu et al. (2017) are easily be degenerated by putting the values of  $S - S_r = 0$ ,  $t_1 = 0$ , and constant demand.

## 6. Numerical illustrations

The numerical illustrations are considered to elucidate the original model while the truncated model is used only for theoretical derivation. In subsection 6.1, a solution algorithm is presented for Case 1 while to give more logic in favor of the projected model a physical case study has been carried out with a numerical illustration. This study takes the following parametric values in appropriate units.

### 6.1. Algorithm (Case 1)

Step 1. Plugging all the required key parameters in Lingo 15 software.

Step 2. Set  $G = 0$ .

Step 3. Calculate  $T^*$  and  $t_1^*$  from Equations (32) and (33). Find  $\Delta_{TP}(t_1, T)$  from Equation (25).

Step 4. Put  $G = G + 1$  and recurrently use Step 3 until attaining the maximum value of  $\Delta_{TP}(t_1, T)$ .

Step 5. Compute  $\Delta_{TP}(t_1, T)$  and  $TCR$ .

Step 6. Stop.

## 6.2. A real case with a special example for the proposed model (Case 1)

A case study is being presented here, where a two green-warehouse farm is set up for testing the growth of different deteriorating vegetables (Figure 3) under the Department of Agriculture Engineering at Hajee Mohammad Danesh Science and Technology University, Dinajpur, Bangladesh. We have adopted their experimental values in our model as our study is similar to their experiment although we have studied in two different branches of research. The procedure to study a single greenhouse farm is comparable to Mishra et al. (2020b) and Taleizadeh et al. (2020). We have talked to the authority (greenhouse retailer) for the data and they agreed to provide us some realistic data which are very relevant to our proposed model. When the products are ready to transport, then the greenhouse retailer uses a truck to transport it from the warehouse to the warehouse or retailer's house, and for transportation-related data, he provides some data from Volvo Truck Company of Bangladesh which are mostly used during their research.



Figure 3. A greenhouse farm for vegetable garden

**Example 1: (Case 1)** This example illustrates the case when shortages are allowed, and green technology investment is absent. The two green-warehouse system has the following data: per order ordering costs  $O_r = \$500$ ,  $a = 70$ ,  $b = 0.5$ , numerous items purchase cost is  $c_p = \$35$ , OW holding cost per unit item  $c_{ho} = \$1$ , RW holding cost per unit item  $c_{hr} = \$2$ , backorder cost  $c_s = \$24/\text{unit}$ , deterioration rate in OW  $\alpha = 0.01$ , deterioration rate in RW  $\beta = 0.05$ , the value of the backlogged parameter is  $\eta = 5$ , lead-time  $L = 0.5$ , number of installments  $n = 5$ , interest paid due to loan  $l_p = \$0.07$ , the fractional portion which needs to be prepaid before receiving the product  $\sigma = 0.7$ , and the capacity of OW  $S_r = 150$  units.

Now, for transportation and carbon emission costs calculation, the additional data are  $c_1 = \$0.1/\text{shipping}$ ,  $c_2 = 0.75\text{liter}/100 \text{ km}$  (Volvo truck report for the long haul),  $c_2 = 2.4\text{liter}/100 \text{ km}/\text{ton}$  payload (Volvo truck report for the long haul),  $d = 100 \text{ km}$ ,  $t_v = \$0.1/\text{liter}$ ,  $m = 5 \text{ kg}/\text{unit}$ ,  $e_1 = \$2.35/\text{km}$ ,  $e_2 = \$1.3/\text{unit}/\text{km}$ .

By using the proposed algorithm, the optimal solutions obtained from Lingo 15 software with the above-given data set are  $t_1^* = 7.752$  days,  $T^* = 50.498$  days, and  $\Delta_{TP} = \$1258.278$ .

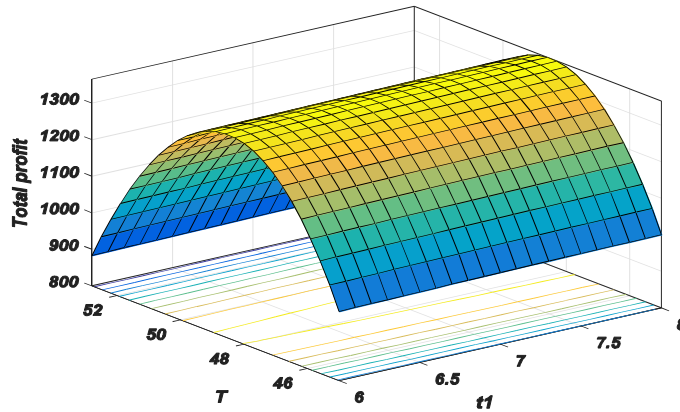


Figure 4. Graphical illustration of concavity of total profit function without green technology investment

The above Figure 4 displays the concavity of the profit function respecting the decision variables and the maximum value exhibit is \$1258.278 with a total optimal cycle length of 50.498 days and the deterioration free time is 7.752 days.

**Example 2: (Case 1)** This example illustrates the impact of green technology investment in the proposed model. Taking the green technology investment in the model with  $\xi = 0.5$  and  $\chi = 0.7$  with all the other data the same as Example 1, then one gets the following Figure 5 where the total profit is \$1276.133 for  $G = 4$ .

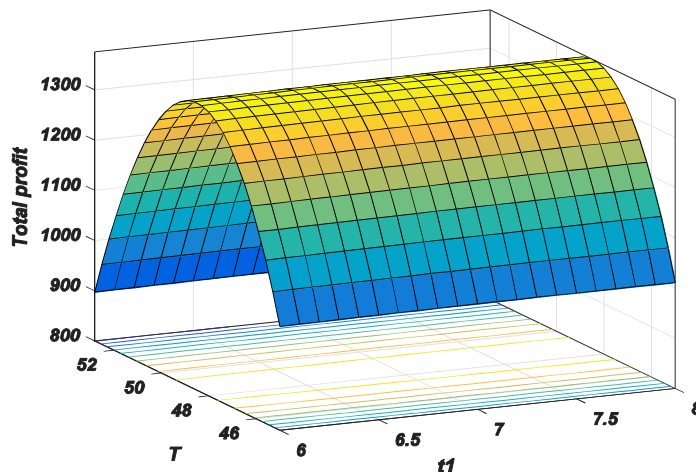


Figure 5. Graphical illustration of concavity of total profit function with green technology investment

Table 2. Optimum solution with green technology investment

$G$	$t_1$	$T$	$\Delta_{TP}(t_1, T)$
-----	-------	-----	-----------------------

0	7.752	50.498	1258.278
1	7.752	50.493	1268.992
2	7.752	50.491	1273.809
3	7.752	50.490	1275.698
4	7.752	50.489	1276.133
5	7.752	50.489	1275.845
6	7.752	50.489	1275.199

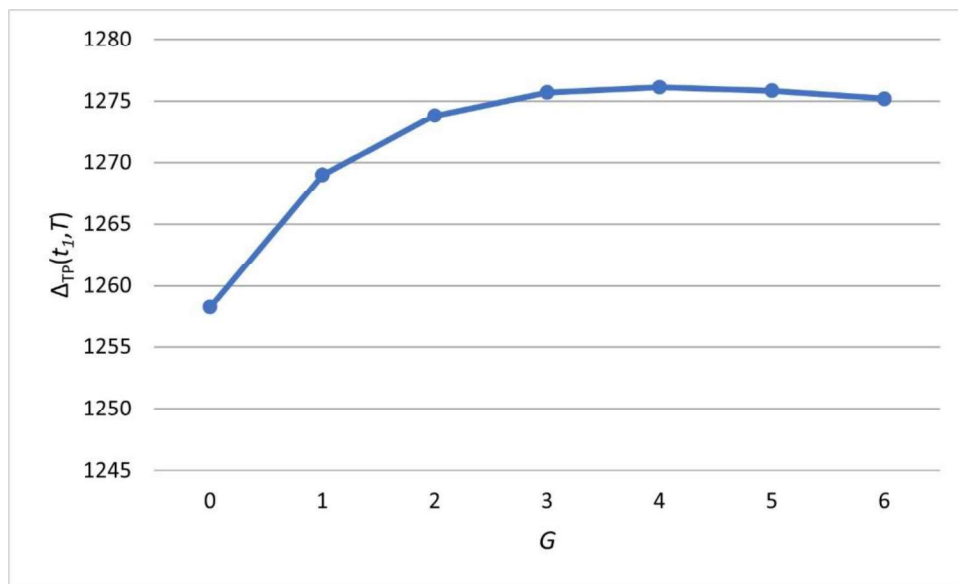


Figure 6. Different values of total profit with the change in green technology investment

The total profit for different values of green technology investments  $G$  is given in Table 2. From Figure 6, it is clear that the intensifications of green technology investment ( $G$ ) result in an increase in total profit for a certain level. After the optimum point of green technology investment, the profit declines although the value of  $G$  is increased.

**Example 3. (Case 2)** (When  $t_2 \approx T$ , i.e.,  $R = 0$ ) In this case, there is no shortage allowed in the proposed model. Then, taking the same values as stated in Example 1 with  $t_2 \approx T$ , i.e.,  $R = 0$ , one gets the following values.

Table 3 presents the result with and without investment in green technology for different cases of shortages. The total profit is higher for the case with shortages, that is \$1276.133. Further, they have the same optimum green technology investment level. The optimum total profit is achieved when  $t_1^* = 7.752$  days,  $T^* = 50.4899$  days, and  $G = 4$ .

Table 3. Total profit with or without shortages and investment in green technology

$G$		$T$	With Shortages	Without shortages

	$t_1$		$\Delta_{TP}(t_1, T)$	$\Delta_{TP}(t_1, T)$
Without investment in Green Technology				
0	7.752	50.4980	1258.278	1196.450
With investment in Green Technology				
1	7.752	50.4937	1268.992	1207.522
2	7.752	50.4915	1273.809	1212.517
3	7.752	50.4904	1275.698	1214.494
4	7.752	50.4899	1276.133	1214.972
5	7.752	50.4896	1275.845	1214.706
6	7.752	50.4895	1275.199	1214.070

#### Example 4. (For Both Cases)

In this example, we discuss the effect of green technology investment on carbon emissions and transportation costs. Table 4 reveals the impacts of green technology on environmental emissions. The higher the investment results in lower emission cost which indicates a lower emissions amount.

Table 4. Green technology and reduction on emission cost

$G$	Original transportation cost ( $TC$ )	Reduced transportation cost ( $TCR$ )	Original carbon emission cost	Reduced carbon emission cost	With Shortages	Without shortages
					$\Delta_{TP}(t_1, T)$	$\Delta_{TP}(t_1, T)$
Without investment in Green Technology						
0	2425.500	2425.500	2350.000	2350.000	1258.278	1196.450
With investment in Green Technology						
1	2425.500	1833.988	2350.000	1758.488	1268.992	1207.522
2	2425.500	1540.251	2350.000	1464.751	1273.809	1212.517
3	2425.500	1394.386	2350.000	1318.886	1275.698	1214.494
4	2425.500	1321.952	2350.000	1246.452	1276.133	1214.972
5	2425.500	1285.982	2350.000	1210.482	1275.845	1214.706
6	2425.500	1268.120	2350.000	1192.620	1275.199	1214.070

Table 4 shows that when the green investment ( $G$ ) increases, the emission decreases. However, the profit for both with and without shortages is maximum when the value of green investment ( $G$ ) equals four. The most striking thing about the investment is that a continuous investment reduces the carbon emission significantly while it does not always mean a greater profit. A significant reduction in transportation cost is also noticed with the intensifications in green investment ( $G$ ). To secure a marginal profit, the retailer must maintain an optimum balance among these attributes, viz., green investment, carbon emission, transportation cost, etc.

## 7. Sensitivity investigation and managerial implication

This study presents the consequence of a given factor respecting the decision variables for Case 1 while the same procedure is applicable for Case 2. A sensitivity investigation is performed by fluctuating the values of the factors from +20% to -20% and the outcomes are presented in Table 5. The value of  $G$  is set as zero when there is no investment in technology and, in contrast, the value of  $G$  is four when the maximum output is given by the technology.

Table 5. Sensitivity investigation for the case mentioned above

Parameter	% changes	$t_1^*$	$T^*$	When $G = 0$	When $G = 4$	% changes in $\Delta_{TP}^*$
				$\Delta_{TP}^*$	$\Delta_{TP}^*$	
$t_v$	-20	7.752	50.498	1258.575	1276.430	1.420
	-10	7.752	50.498	1258.426	1276.281	1.420
	10	7.752	50.498	1258.129	1275.984	1.420
	20	7.752	50.498	1257.981	1275.836	1.420
$d$	-20	7.752	50.494	1267.883	1281.367	1.060
	-10	7.752	50.496	1263.080	1278.750	1.240
	10	7.752	50.500	1253.476	1273.516	1.600
	20	7.752	50.502	1248.674	1270.898	1.780
$c_p$	-20	8.692	50.856	1369.412	1387.113	1.290
	-10	8.215	50.677	1312.716	1330.493	1.350
	10	7.303	50.318	1206.093	1224.027	1.490
	20	6.867	50.137	1156.159	1174.172	1.560
$\sigma$	-20	7.999	50.594	1287.257	1305.070	1.380
	-10	7.875	50.546	1272.686	1290.520	1.400
	10	7.630	50.450	1244.033	1261.909	1.440
	20	7.509	50.401	1229.950	1247.847	1.460
$c_2$	-20	7.752	50.498	1258.813	1276.668	1.420
	-10	7.752	50.498	1258.694	1276.549	1.420
	10	7.752	50.498	1258.456	1276.311	1.420
	20	7.752	50.498	1258.337	1276.192	1.420
$n$	-20	8.166	50.655	1316.514	1329.943	1.020
	-10	7.968	50.581	1288.394	1304.031	1.210
	10	7.519	50.407	1226.305	1246.389	1.640
	20	7.270	50.308	1192.630	1214.955	1.870
$e_1$	-20	7.752	50.495	1267.585	1281.070	1.060
	-10	7.752	50.496	1263.031	1278.602	1.230
	10	7.752	50.500	1253.624	1273.717	1.600



	20	7.752	50.501	1248.971	1271.196	1.780
$L$	-20	7.999	50.594	1287.257	1305.070	1.380
	-10	7.875	50.546	1272.686	1290.520	1.400
	10	7.630	50.450	1244.033	1261.909	1.440
	20	7.509	50.401	1229.950	1247.847	1.460
$\chi$	-20	7.752	50.489	1258.278	1275.071	1.330
	-10	7.752	50.498	1258.278	1275.551	1.370
	10	7.752	50.498	1258.278	1276.053	1.410
	20	7.752	50.498	1258.278	1276.199	1.420

Table 5 shows that green technology investment (when  $G = 4$ ) always beneficial for the retailer indicated by a positive value of the % changes in  $\Delta_{TP}$ .

- As variable transportation cost  $t_v$  increases, the total profit with and without using green technology is inversely changed. In realistically, if the transportation cost is increased notably, then the total profit is reduced. The most striking thing is that there is no effect on the optimum non-deteriorating ( $t_1^*$ ) and replenishment ( $T^*$ ) periods. The retailer has to hold the products in a short period to reduce the holding cost, but the decrease should not exceed the increase in the transportation cost. Further, the % changes in  $\Delta_{TP}$  remain constant.
- An increase in distance results in a decrease in profit. The long-distance runs by the vehicle will consume more fuel, consequently more costs related to transportation which will then simultaneously rises the total cost and decrease the profit. When the distance is progressing, the corresponding cost is growing rapidly and consequently more promptly diminishing the profit. However, the noticeable thing is that the % changes in  $\Delta_{TP}$  increase significantly which indicates the increasing benefit of green technology investment.
- It is observed that when the purchasing cost is shrinking, the profit of the system is swelling. However, the important factor is the ratio of the purchasing cost which a retailer needs to prepay before delivery of the product. If the retailer has to compensate an almost full portion of the purchasing cost in terms of advance then he cannot utilize his money in other sectors (e.g. bank or any business) to earn more revenue.
- From Table 5, it is detected that the characteristics of lead-time  $L$  and installments  $n$  are almost similar. When the lead-time is bigger, the total profit of the system is lower. As the installments are equally spaced, so, when the lead-time increases then the retailer gets more time to prepay the purchased amount which results in a reduction in profit as in every installment the retailer needs to pay the original amount with some interest. It has a similar observation noticed in Mashud et al. (2020b).
- The intensification in the amount of carbon emission cost will affect the total profit of the system. More interestingly, green technology is working effectively when the carbon emission cost increases as the % changes in  $\Delta_{TP}$  increase significantly. Another thing is that when the vehicle's fuel consumption increases then the total cost increases, and accordingly, the total profit decreases.

- f. When the efficiency of greener technology  $\chi$  increases, simultaneously the total profit also increases. The higher the efficiency, then the higher is the profit.

## 8. Conclusions

This study investigates the sustainable inventory model for retailers with a two-warehouse system that sells non-instantaneous deteriorating items. This carbon emission related model gives an insight to the retailer how to use the transport system more efficiently from the owned warehouse to the rented warehouse and then to the customer with minimum emission of carbon for securing the maximum profit. It also proposes a new idea in the investment strategy by introducing a new investment in green technology which simultaneously reducing the emission of CO<sub>2</sub> and swelling the profit of the greenhouse system. Besides these environmental issues, the advance payment scheme is implemented to give insights for the greenhouse retailer about the supplier requirement. Initially, the retailer needs to deposit the purchase cost as a prepayment to order the product, and later, the rest of the amount must be provided at the time of receiving the product. This model extends the idea of Mishra et al. (2020b) by incorporating a two-warehouse system in place of a single warehouse with green technology investment and shortages (Case 1) and without shortages (Case 2).

The result has proved the benefit of green technology investment indicated by a reduction in emission cost and a positive value of the %changes in total profit. An optimum level of green technology investment must be specified to achieve a maximum profit. The benefit from the implementation of green technology in transportation is affected by the changes in delivery distance and emission cost of the vehicle. Further, the numerical results show a 1.42% increase in profit for the case with shortages and a 1.55% intensifications in profit for the case without shortages. Incorporating carbon emission reduction in the system, and carbon emission cost into the model, helps greenhouse retailer to increase the total profit and protecting the environment. The emission has been reduced with green technology implementation while it also reduces transportation cost. The fraction of emission reductions depends on the efficiency of the invested green technology.

Nevertheless, the proposed model has some limitations. So far, it is considered that carbon is emitted due to transportation, but in the real system, the emission is also possible to be emitted during the holding of the products in both warehouses. Hence, future research may consider this source of emission. We only discuss two significant issues of triple bottom line (3BL) sustainability, namely the economic and environmental aspects of the inventory system. Hence, it is suggested to include the third issue of social implications to society and other sustainability parameters such as energy consumption in the future study. Some other interesting extension of this model is also possible in numerous ways by reducing its ordering cost as well as introducing preservation technology which controls the deterioration rate. This model is for price-sensitive demand; hence one can further investigate by incorporating stochastic or time-varying demand.

## Appendix A

Putting the first-order derivative in Equation (30) equal to zero, i.e.,  $\frac{dX}{dT} = 0$ , produces the optimum period length ( $T$ ).

The cost function  $X$  is rewritten as  $X = \frac{1}{T}(\phi_1 + \phi_2 t_1^2 + \phi_3 t_1) + \phi_5 + \phi_4 T$  (A.1)

Where,

$$\phi_1 = O_r + c_p S_r + c_p \eta D t_2 + \frac{1}{2} c_{ho} D t_2^2 + \frac{1}{2} c_s \eta D t_2^2 + \frac{n+1}{2n} I_p L \sigma c_p Q + \aleph \left[ c_1 + (2dt_v c_2 + dt_v c_3 m Q + 2de_1 + de_2 Q)(1 - \xi(1 - e^{-\xi})) \right] \quad (A.2)$$

Since all the parameters are a positive number and the customer demand  $D = a - bp$  is decreasing but always a positive one,  $\phi_1 > 0$  always.

$$\phi_5 = c_s \eta D + \frac{n+1}{2n} I_p L \sigma c_p \eta D + \aleph [\eta D dt_v c_3 m + de_2 \eta D - c_s \eta D t_2] \quad (A.3)$$

$\phi_5 > 0$  is for the reason described above, the term  $c_s \eta D t_2$  is itself defining a negative number as shortages are starting from the points  $t_2$ , hence,  $-c_s \eta D t_2$  is a positive term.

$$\phi_4 = \frac{1}{2} c_s \eta D + 2G \quad (A.4)$$

$\phi_4 > 0$  is for all the similar reasons described for  $\phi_1$ .

$$\phi_2 = \frac{1}{2} c_p D \beta + \frac{1}{2} c_{hr} D + \frac{1}{2} c_{ho} (D - \alpha) + \frac{n\beta}{2} [D + de_2 D] \quad (A.5)$$

$\phi_2 > 0$  is for all similar reasons described for  $\phi_1$  whereas, the term  $(D - \alpha) > 0$  also as  $\alpha$  is the constant deterioration rate happens in OW and to exist in an inventory model, in reality, demand is always greater than deterioration.

$$\phi_3 = c_p D \beta + c_{ho} Q + \frac{n+1}{n} I_p L \sigma c_p D \beta + \aleph [dt_v c_3 m D + de_2 D] \quad (A.6)$$

$\phi_3 > 0$  is for all the similar reasons described for  $\phi_1$ .

Not only analytically, but it is also be checked by using Mathematica that all the terms described above are strictly positive.

Equation (23) can also be written as  $X(T, t_1) = \frac{1}{T}(\gamma(t_1)) + \phi_4 T + \phi_5$  (A.7)

where,  $\gamma(t_1) = (\phi_1 + \phi_2 t_1^2 + \phi_3 t_1)$

The target is to launch the circumstance in which Equation (A.1) gives a distinctive interior minimizer. Now, the derivative of  $X(T, t_1)$  regarding  $T$  gives,

$$\frac{\partial X(T, t_1)}{\partial T} = -\frac{\gamma(t_1)}{T^2} + \phi_4 T \quad (A.8)$$

This equals to zero only when  $T$  fulfill

$$T^* = T^*(t_1) = \sqrt{\frac{\gamma(t_1)}{\phi_4}} \quad (A.9)$$

## Appendix B

The discriminant of  $\gamma(t_1)$  is,

$$\begin{aligned} \phi_3^2 - 4\phi_2\phi_1 = & \left( c_p D\beta + c_{ho} Q + \frac{n+1}{n} I_p L\sigma c_p D\beta + \aleph [dt_v c_3 mD + de_2 D] \right)^2 - \\ & \left( 2c_p D\beta + \frac{1}{2} c_{ho} D + 2c_{ho} (D - \alpha) + 2n\beta [D + de_2 D] \right) \\ & \left( O_r + c_p S_r + c_p \eta Dt_2 + \frac{1}{2} c_{ho} Dt_2^2 + \frac{1}{2} c_3 \eta Dt_2^2 + \frac{n+1}{2n} I_p L\sigma c_p Q + \aleph [c_1 + (2dt_v c_2 + dt_v c_3 mQ + 2de_1 + de_2 Q)] \right) \end{aligned} \quad (B.1)$$

[Note:  $\phi_1, \phi_2, \phi_3$  all are individually positive quantity (described above). If we multiply  $\phi_2$  with  $\phi_1$ , this leads to an enormous quantity that itself contains all terms of  $\phi_3$  with addition to more terms. Therefore, the quantity  $\phi_3^2 - 4\phi_2\phi_1$  is a negative one. Instead of analytical continuation, we checked it by using Mathematica also, but for simple readability, we have added the analytic arguments only.]

As the discriminant of  $\gamma(t_1)$  is negative,  $\gamma(t_1)$  has no roots. Thus,  $\gamma(t_1)$  is either positive or negative. As, in the compact interval  $[0, 1]$  the value of  $\gamma(0) = \phi_1 > 0$ , and  $\gamma(t_1)$  is strictly positive. Thus equation (A.4) provides, for each  $t_1$ , a distinctive  $T^* = T^*(t_1)$  that minimizes the cost function assumed by equation (A.1).

Replacing the expression for  $T^*(t_1)$  in equation (A.4) into equation (A.1) gives:

$$\widehat{X}(t_1) = X(t_1, T^*(t_1)) = \frac{\gamma(t_1)}{\sqrt{\gamma(t_1)}} \sqrt{\phi_4} + \phi_4 \sqrt{\frac{\gamma(t_1)}{\phi_4}} + \phi_5 = 2\sqrt{\gamma(t_1)\phi_4} + \phi_5 \quad (B.2)$$

This denotes the minimal possible cost for all value of  $t_1$ ,  $\widehat{X}(t_1)$  is continuous and consumes one or more local minima on the compact interval  $[0, 1]$ , among those the smallest one is the global minimum of the cost function. To catch these minima, continuously first and second-order derivatives of  $\widehat{X}(t_1)$  with respect to  $t_1$  is needed and which is respectively:

$$\begin{aligned} \frac{d\widehat{X}}{dt_1} &= \sqrt{\phi_4} \frac{\gamma'(t_1)}{\sqrt{\gamma(t_1)}}, \\ \frac{d^2\widehat{X}}{dt_1^2} &= \sqrt{\phi_4} \frac{[2\gamma''(t_1)\gamma(t_1) - (\gamma'(t_1))^2]}{2(\gamma(t_1))^{\frac{3}{2}}} \end{aligned} \quad (B.3)$$

For all  $t_1$ , we have,

$$\frac{d^2\widehat{X}}{dt_1^2} = \sqrt{\phi_4} \frac{[2\gamma''(t_1)\gamma(t_1) - (\gamma'(t_1))^2]}{2(\gamma(t_1))^{\frac{3}{2}}} = \sqrt{\phi_4} \frac{[4\phi_2(\phi_1 + \phi_2 t_1^2 + \phi_3 t_1) - (2\phi_2 t_1 + \phi_3)^2]}{2(\gamma(t_1))^{\frac{3}{2}}} = \frac{\sqrt{\phi_4} [4\phi_2\phi_1 - \phi_3^2]}{2(\gamma(t_1))^{\frac{3}{2}}} \quad (B.4)$$

Because  $\phi_1, \phi_2, \phi_3, \phi_4$  and  $\gamma(t_1)$  are all greater than zero, thus,  $\widehat{X}(t_1)$  is convex and putting its first differentiation

$\frac{d\hat{X}}{dt_1} = \sqrt{\phi_4} \frac{\gamma'(t_1)}{\sqrt{\gamma(t_1)}}$  if this study puts this expression equal to zero then it provides a global minimum. As  $\sqrt{\phi_4}$  and  $\gamma(t_1)$  are always positive,  $\gamma'(t_1)$  have to be equal to zero, that means,  $\gamma'(t_1) = 2\phi_2 t_1 + \phi_3 = 0$ . So, this study is proposed as  $t_1^* = \frac{-\phi_3}{2\phi_2}$ ,

(B.5)

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