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Keywords Aggregate Production Planning; Fuzzy optimisation; Uncertain customer demand deviation; Uncertain production output; Fuzzy sets

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A fuzzy linear programming model for aggregated production planning (APP) in the automotive industry

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Abstract

Various Aggregate Production Planning (APP) models have been proposed in the literature to determine company's production, inventory and employment levels over a finite time horizon. Majority of them are deterministic with the objective to minimise the relevant cost. Motivated by a real-world automotive supplier, this paper proposes a new fuzzy APP model which considers time required to complete operations in the production and warehouse inventory as the main indicator of the performance. The paper includes uncertainties in relevant parameters including customer demand deviations from expected values and production output, as well as uncertainty in production time, time of safety stock storing in the warehouse and time of preparation for delivery to customers. The uncertain parameters are modelled using fuzzy sets generated using historical data recorded in the supplier or based on experience of a logistics management team. Various experiments are carried out using real-world data collected in the supplier to analyse the impact that uncertainty has on APP. It is demonstrated that the developed fuzzy APP model can shorten the time required to perform the production and warehouse operations and improve the performance of the supplier.

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1. Introduction

Aggregate production planning (APP) is one of the most important part of operations management in competitive supply chains. It concerns matching supply with forecasted customer demand over a planning period, which is usually one year in practice. Generally, the aim is to determine required resources, which include production rate, warehouse levels, work force level, overtime, etc., in such a way as to meet customer demand.

In the literature, it has been assumed most often, that all the parameters which are associated with the APP process, such as customer demand, processing times, production capacities etc., are deterministic in nature (for example, [1]). Following this assumption, as a result, developed APP models have been mostly deterministic linear optimisation models with the objective to minimise the total cost consisting of production cost, inventory cost, regular pay-roll, overtime or some other cost components.

In order to handle uncertainties which characterise real world APP environments, and a randomness in customer demand, in particular, various stochastic optimisation models have been proposed [2]. Furthermore, one can find in the literature that different types of uncertainties encountered in APP problems, such as imprecise demand, production capacities with tolerance, fuzzy processing times can be specified by production managers using imprecise linguistic terms. They have led to the development of a number of fuzzy APP models and applications of fuzzy optimisation techniques[3].

All the optimisation models reported in the literature have used the total cost as a performance measure of the considered manufacturing system with the objective to minimise it. However, the cost components are different in different environments, regions or countries in which they are incurred. In order to avoid the concept of cost in the measure of APP performance, in the model proposed in this paper, different times which appear in the APP process are used to measure its efficiency. The time characteristics include the time required to manufacture demanded products, the time to store the products in the warehouse and the time to get product ready for delivery to the customer. Generally, the shorter the all these times are, the better the APP performance is. Following the idea of treating real world APP problems, it is supposed that these times are specified using imprecise linguistic terms; for example, in the specification of the production time of one product a phrase used can be “the time is *about* 4 minutes”, or “it requires *between* 0.25 and 0.3

minutes to stock one product in the warehouse" or "the preparation for delivery of one product to customer takes *between* 0.12 and 0.15 minutes", etc.

In this paper, we propose a new fuzzy model for optimal APP in the presence of uncertainty. The novelty of the model is that the objective is to minimise the fuzzy total time required for production, storing manufactured products and their preparation for delivery to the customer. We introduce uncertain factors to take into consideration uncertainty in customer demand which is forecasted and can fluctuate around these values and uncertainty in manufactured quantities. As all the time parameters listed above, customer demand deviations and the parameters which describe the output of manufacturing process are fuzzy, both the associated objective function and constraints become fuzzy, too. We adapt and apply one of the methods for transforming the fuzzy linear programming optimisation model, with the fuzzy objective function and fuzzy constraints, into a crisp optimisation model with both the crisp objective function and crisp constraints [4]. The method takes into consideration simultaneously the satisfaction degree of the fuzzy objective function value achieved and feasibility degree of constraints, and finds the balance between these two degrees. The proposed model is tested using real-world data recorded in a first tier supplier in the automotive industry. Various numerical experiments are carried out to gain a better understanding of performance of the fuzzy APP model proposed in the presence of uncertainty. The production and the safety stock plans obtained by using the fuzzy APP model are compared with real world data recorded in a 12 weeks period in the automotive supplier. The possible advantages of using the model in practice are analysed.

Novelty of the proposed APP model is as follows.

(1) A new fuzzy APP optimisation model is developed which minimises the total time required for producing, storing and preparing products for delivery. Uncertainty in customer demand deviations and quantities of manufactured products in a specified planning horizon is included in the model by introducing two fuzzy factors respectively.

(2) The model objective function and constraints are defined motivated by a real-world APP problem.

(3) A method of using collected data for generating a corresponding fuzzy set is adapted and applied in practice to obtain fuzzy production time.

(4) A methodology of transforming a fuzzy optimisation model with a fuzzy objective function and fuzzy constraints is adapted and applied to the fuzzy APP optimisation model proposed.

(5) Various experiments are carried out. Results obtained are promising and demonstrate the advantage of applying the proposed model in practice.

The paper is organised as follows. Literature review on APP models and methodologies used and methods of modelling uncertain APP parameters is presented in Section 2. Problem statement is given in Section 3. The fuzzy aggregated production and inventory planning model are described in Section 4, while Section 5 contains case study and analyses of results of different experiments carried out using the proposed model. The benefits of this research to academia and industry are discussed in Section 6 and conclusions are given in Section 7.

2. Literature review

It is well recognised in the literature that treating uncertainty in APP models in an appropriate way brings an advantage to handling real world APP problems and brings them nearer to the practice [3]. Majority of the APP models handle uncertainty using a classic probability theory approach, and consider only one type of uncertainty which is based on randomness and frequency of a random event occurrence.

Linear mixed integer programs (MIPs) were developed to solve two production planning problems with demand uncertainty [5], when the manufacturer had a flexibility to accept or reject an order. Authors considered integration of customer orders in production planning in two practical scenarios: production planning problems with limited capacity of lot size and with load dependent lead times. In the first scenario, a decision making in acceptance or rejection of customer orders depended of caused increase of expenses and in the second scenario a decision making depended of caused delays of other orders. A robust optimization approach was used and a heuristic method was proposed to find the feasible solution and then to improve it. The MIP method in production planning problems for multi-period and multi-items in make-to-order manufacturing system was used in [6]. The scenario where manufacturer split customer orders to external cooperative manufacturers in order to minimize the total production cost was considered. Three metaheuristic algorithms were applied in various experiments to obtain a Pareto optimal set.

Distribution and aggregate production planning have been considered in contemporary supply chains. They include multi-level decision making, where optimisation of one objective on each level is in conflict with optimization of the whole supply chain. Avraamidou and Pistikopoulos [7] developed a bi-level mixed integer linear programming model for a supply chain under demand uncertainty. An algorithm was developed for a multi-parametric programming problem for lower production planning level where the main parameter depended on distribution demand from the upper distribution planning level. Two different cases were analysed: deterministic constant and uncertain customer demands.

A very important issue in modern production planning is energy consumption. Today, the most manufacturers invest significant money assets to optimise and reduce energy consumption. In [8], a multi-objective linear programming problem with three objective functions including operational expense, energy expense and carbon emission, was analysed. To solve the proposed multi-objective problem, the goal attainment technique was applied. Uncertain parameters were: operational expense, energy consumption, carbon parameters, demand and maximum capacity. A robust optimization technique was used to deal with uncertainties. The results of performed experiment in steel melting manufacturing for medium-term production planning showed a very high impact of energy expense on the total expense.

Zadeh proposed a new approach to handle different types of uncertainty, by introducing the concept of fuzzy sets [9]. It has been demonstrated in the literature that fuzzy sets can be successfully applied to modelling uncertainty where available information is vague or cannot be defined precisely due to the limited knowledge. In these cases, such as APP processes, uncertainty can be described based on experts' subjective knowledge, experience and preferences, and expressed using imprecise natural language terms, such as *large*, *extra large*, *moderate*, *small enough*, etc. One can find some good examples in the literature on how fuzzy sets are applied in supply chain management problems, for example in supply chain partners' collaboration [10], in MRP (material requirement problems) [3], in serial supply chains [11], etc. Tang et al. considered both uncertainty in customer demand and production capacity and modelled them as fuzzy values in a multi-product APP model [12]. They proposed an optimisation APP model where the total cost, which included quadratic production costs and linear inventory holding costs, was minimised. The fuzzy quadratic programming model with a fuzzy objective and fuzzy constraints was transferred to a crisp Linear Programming (LP) model. Wang and Fang [13] developed a multi-products, multi-objective fuzzy linear APP model (MOFLM). The objectives considered were production capacity, manpower level, item price, customer demands and cost to subcontract. Similar APP problem was considered in [14], using parametric programming which allowed the decision maker to select a preferred aggregate plan under fuzzy demand, fuzzy capacities and financial constraints. Wang and Liang [15] developed a multi-objective linear programming APP model, where the objectives were to minimise total production cost, to minimise holding and backordering cost and to minimise rate of change in labour levels. The model was expanded in [16] by using a possibilistic linear programming (PLP) approach to modelling uncertainty in capacity, forecast demand and related operational cost. Further on, Wang and Liang [17] developed an interactive PLP model providing a choice to the decision maker to interactively change an imprecise data and parameters until a satisfactory solution was found. A fuzzy multi-objective mixed-integer non-linear programming model for a supply chain was proposed in [18]. Fuzzy customer demand was considered in three objective functions that minimised the total supply chain cost, total maximum product shortages, and the rate of changes in human resources.

The literature review of fuzzy APP models showed that these problems were often formulated as fuzzy mathematical programming models with a fuzzy objective function or fuzzy constraints or both. In order to generate crisp decisions of APP problems, a fuzzy APP model was typically transformed into a crisp optimisation model, so that a classic, well known deterministic optimisation method can be applied. Different approaches to this transformation have been proposed; for example, [19], [20] and [21]. Fuzzy optimisation models typically include: (1) a fuzzy objective function when they involve various methods of ranking fuzzy objective function values, or (2) fuzzy constraints when they involve methods of transferring fuzzy constraints into crisp constraints based on tolerance intervals or (3) both fuzzy objective function and fuzzy constraint when fuzzy objective function and fuzzy constraints are considered in the same way in such a way as to maximise the satisfaction degrees of both.

A method which transforms a fuzzy LP model into the corresponding crisp LP model was proposed in [4]. The method handled fuzzy models with fuzzy objective linear function and fuzzy linear constraints with fuzzy parameters, where all parameters were modelled by trapezoidal membership functions. The proposed method considered two goals: improving satisfaction with a fuzzy objective function value and improving a feasibility degree of constraints. These two goals were in conflict: the higher the feasibility degree of the fuzzy constraints, i.e., the smaller the violation of the fuzzy constraints, the smaller the feasible region, and consequently, the worst the fuzzy objective function value, and, therefore, the lower satisfaction with the fuzzy objective function value. The method was searching for a balance between the feasibility degree of constraints and the satisfaction degree with the value of objective function achieved. The method proposed was iterative; in each iteration, the feasibility degree of constraints was increased and the corresponding satisfaction with the fuzzy objective function value achieved was determined. The solution with the highest combined feasibility degree and the satisfaction degree was selected. The method was demonstrated using a theoretical example.

An important question which needs to be addressed when using fuzzy sets in real-world problem is how to generate corresponding membership functions. However, there is a limited number of papers which considered this issue. Pedrycz and Gomide [22] identified experimental methods that could be used to construct a membership function based on subjective experts' estimates. Dubois and Prade [23] proposed a method for generating a membership function of an uncertain parameter based on a known probability distribution of parameter values. This method was further generalised to the case when a probability distribution of parameter values was not known, but empirical data existed [24]. It was demonstrated by applying the method in [23], that different membership functions could be generated based on different samples of data with the same probability distribution. Therefore, they proposed a method which guaranteed that the constructed membership function generated using empirical data corresponded to the unknown probability distribution with a given confidence level.

The review of the published APP models showed that they did not consider and analysed the material flow time in the APP problems. All of the APP models have been developed to minimize operational cost in manufacturing, dealing with impacts on production, inventory or delivery costs. The most influential factors have been assumed or theoretically defined and incorporated in the developed APP models. Furthermore, many of the models have been validated theoretically only, without their testing in real world environments. However, in some industrial sectors, the material flow time is a very important factor and cannot be neglected, because it has a big impact on the total measure of manufacturer performance. A typical example is an automotive industry. Further on, most of the developed fuzzy APP models include fuzzy parameters with triangular or trapezoidal membership functions, due to their easy interpretability and simplicity of calculations. We consider a real world APP problem in the automotive industry and develop a fuzzy LP model which considers a material flow as the measure of performance. Further on, we use real-world historical data to generate a membership function of uncertain unit production time, which is not triangular or trapezoidal, but piece-wise linear. Therefore, we have to adapt a method proposed in [4] in such a way as to handle the piece-wise fuzzy parameters of the APP model.

3. Problem statement

A problem is to generate the optimum aggregate production and inventory plan for a supplier for a given planning time horizon. The supplier operates in a "make-to-order" manner and has to prepare a production and inventory plan in such a way as to satisfy customer demand and optimise an associated performance measure in the considered time horizon.

However, due to habitual changes in the market, it is supposed that customer demand fluctuates around forecasted values in an uncertain way. For example, customer demand over the planning horizon can be around 10% higher or lower than the forecasted demand. This means that the aggregate production and inventory planning has to be carried out in the presence of uncertainty in customer demand. It is further supposed that the production capacity is limited. In addition, it is believed that the number of manufactured products in a time period depends on external and internal factors including the available labour force level, efficiency of the labour, percentage of manufactured products which are not of the required standard, possible machine breakdown, a custom percentage of faulty input parts which cannot be used in production, etc. All the factors listed above are uncertain and cannot be specified precisely. They can be estimated based on the subjective supplier's management team experience.

The manufactured products are stored in the warehouse. Customer demand is satisfied by using the stock available in the warehouse. In order to satisfy fluctuated customer demand, the supplier keeps "a safety stock" in the warehouse called "days-of-inventory". The safety stock is determined in such a way as to cover forecasted demand of the given number of days.

The planning time horizon is discretised into a series of subsequent discrete time periods. The APP determines 3 quantities to be generated for each time period in the planning time horizon: (1) optimal production quantity to be manufactured, (2) the safety stock quantity that should be kept in the warehouse and (3) the quantity that should be delivered to the customer.

If the same production line is used for manufacturing of different products for more than one customer, an efficient use of the production line is of paramount importance for the production process. This requires the development of a good measure of performance of the production and inventory plan as a whole. In this paper, the focus of the production process modelling is placed on different products demanded by different customers that require the same production line. Therefore, our view point is that the total performance of the production and inventory planning can be measured by the time required to satisfy customer demand. It is calculated as the sum of production time needed to manufacture required number of products, storing the manufactured products in the warehouse and preparing the planned amount of products for delivery in each time period within the considered time horizon.

A closer investigation shows that there are different sources of uncertainty which affect production and inventory planning, including:

- customer demand fluctuations around forecasted values; customer demand has to be fully satisfied in each time period either using the products manufactured in that time period or available safety stock,
- quantity of manufactured products,
- unit production time affected by the factors listed above,
- time required to store a product in the warehouse including time for picking a full container at a packaging place of production line by a forklift and time for the forklift driving to the warehouse and storing it in a pallet rack, and
- time required to prepare one container for delivery to the customers based on time of printing picking list of containers in the warehouse requested for delivery, collection of the containers and moving them to a shipment area with forklift, printing and attaching shipping labels to the containers, scanning and loading of the containers into trucks.

All these uncertainties have to be taken into account when generating the optimal production and inventory plan.

4. Fuzzy aggregated production and inventory planning

4.1. Notation

The following notation is used:

i – index of a time period in a planning horizon, *i = 1,…,n,*

D_i – customer demand in period *i*, *i = 1,…,n,*

$\tilde{n}_p$ – fuzzy number of products manufactured per unit time, with a linear piece wise membership function,

$\tilde{t}_p$ – fuzzy production time per unit of product (in minutes), with a linear piece wise membership function $\tilde{t}_p = (t_{p\,1}, t_{p\,2}, t_{p\,3}, \dots)$,

$\tilde{t}_s$ – fuzzy warehouse storing time per unit of product (in minutes), with trapezoidal membership function $\tilde{t}_s = (t_{s\,1}, t_{s\,2}, t_{s\,3}, t_{s\,4})$,

$\tilde{t}_t$ – fuzzy preparation time for shipping to customer per unit of product (in minutes), with trapezoidal membership function $\tilde{t}_t = (t_{t\,1}, t_{t\,2}, t_{t\,3}, t_{t\,4})$,

$\widetilde{w}_i^d$ - fuzzy factor for uncertain customer demand deviation from forecasted value in period *i*, *i = 1,…,n,* with triangular membership function $\widetilde{w}_i^d = (w_{i\,l}^d, w_{i\,m}^d, w_{i\,u}^d)$,

$\widetilde{w}_i^p$ - fuzzy factor for uncertain production quantity output in period *i*, *i = 1,…,n,* $\widetilde{w}_i^p = (w_{i\,l}^p, w_{i\,m}^p, w_{i\,u}^p)$,

T^l – minimum "days of inventory" in the warehouse,

T^u – maximum "days of inventory" in the warehouse,

C – machine capacity.

Decision variables:

P_i – quantity manufactured in period *i*,

Ss_i – safety stock in period *i*,

Q_i – quantity delivered to customer in period *i*.

4.2. Fuzzy APP LP model

The problem is formulated as a fuzzy LP model. Two fuzzy factors, $\widetilde{w}_i^d$ and $\widetilde{w}_i^p$, are introduced to model uncertainty in a change of customer demand and uncertainty in manufactured quantity in each time period *i*, *i* = 1,…,*n*, respectively. Therefore, customer demand and production quantities manufactured in each time period *i* are calculated as the products $\widetilde{w}_i^d D_i$ and $\widetilde{w}_i^p P_i$, respectively.

The objective is to minimize the total material lead time $\tilde{Z}$ including the production time $\tilde{t}_p\ P_i$, warehouse time $\tilde{t}_s\ Ss_i$ required for storing safety stock of manufactured products and time for preparation of delivery to customers $\tilde{t}_t\ Q_i$, as follows:

(1) $\min \tilde{Z} = \sum_{i=1}^{n} \tilde{t}_p P_i + \tilde{t}_s Ss_i + \tilde{t}_t Q_i.$

The following constraints are considered:

Uncertain customer demand $\widetilde{w}_i^d D_i$ in each time period *i* is satisfied using the uncertain production $\widetilde{w}_i^p P_i$ or safety stock Ss_i:

(2) $Ss_i + \widetilde{w}_i^p P_i \geq \widetilde{w}_i^d D_i,\ i = 1, \dots, n$

The safety stock Ss_{i+1} in each time period $i + 1$ is equal to the stock in the previous period Ss_i increased by uncertain production in the previous period, $\widetilde{w}_i^p P_i$, and reduced by uncertain customer demand, i.e., quantity delivered to the customer in the previous period, $\widetilde{w}_i^d D_i$:

(3) $Ss_{i+1} = Ss_i + \widetilde{w}_i^p P_i - \widetilde{w}_i^d D_i$, *i* = 1,…,*n*

Installed machine capacity C produces uncertain $\widetilde{w}_i^p P_i$ units per period *i*:

(4) $\widetilde{w}_i^p P_i \geq 0$, *i* = 1,…,*n*

(5) $C \geq \widetilde{w}_i^p P_i$, *i* = 1,…,*n*

The safety stock Ss_i in period *i* is defined by a supplier's target to cover between T^l and T^u days of uncertain customer demand $\widetilde{w}_i^d D_i$ in that period:

(6) $Ss_i \geq T^l \widetilde{w}_i^d D_i$, *i* = 1,…,*n*

(7) $T^u \widetilde{w}_i^d D_i \geq Ss_i$, *i* = 1,…,*n*

The delivery Q_i in each period *i* must be equal to uncertain customer demand $\widetilde{w}_i^d D_i$ in order to operate with the maximum service level - 100%.

(8) $Q_i = \widetilde{w}_i^d D_i$, *i* = 1,…,*n*

Decision variables P_i, Ss_i and Q_i in each time period *i* are non-negative:

(9) P_i, Ss_i, $Q_i \geq 0$, *i* = 1,…,*n*.

4.3. Modelling uncertainty using fuzzy sets based on historical data

The objective function includes 3 time related parameters, $\tilde{t}_p$, $\tilde{t}_t$ and $\tilde{t}_s$, that are very difficult to specify precisely in practice. Therefore, we modelled them using fuzzy sets. Data about number of manufactured pieces in a given time period are typically recorded by the supplier and are used to determine the fuzzy time for manufacturing one product. However, typically, there are no recorded data on time of warehouse inventory preparation $\tilde{t}_s$ and time of preparation of delivery to customers $\tilde{t}_t$. The precise data is not practical to evaluate, because of many unmeasurable causatives, such as different number of products in packaging unit and different speed of forklifts. These time evaluations can be specified by the Logistic expert and it is convenient to specify them using imprecise linguistic terms. These imprecise linguistic terms are modelled using fuzzy sets with trapezoidal membership functions.

4.4. From the fuzzy APP optimisation model to a crisp APP optimisation model

We applied a method developed by Jimenez et al [4] to transform the fuzzy APP model into a crisp APP model. We adapted it in such a way as to handle fuzzy parameters in the objective function with piece-wise and trapezoidal membership functions. Relevant fuzzy sets definitions and Jimenez et al method are given in Appendices A and B, respectively.

The transformation includes 3 steps as follows.

Step 1. The decision maker specifies the feasibility degree β of constraint satisfaction he/she is ready to accept. Let us assume that the lowest feasibility degree that the decision maker is ready to consider is *Neither acceptable nor unacceptable solution* - β = 0.5. of course, it can be changed to any other feasibility degree β from interval [0, 1].

The crisp optimisation model is solved iteratively for each feasibility degree β = 0.5., 0.6, …, 0.9, 0.95, 0.99 and 1 where each solution is β-feasible, i.e., the minimum of feasibility achieved for all constraints is β. The β-feasible solution P_i, Ss_i and Q_i, *i* = 1,…, *n* are found as follows.

First, fuzzy parameters $\tilde{t}_p$, $\tilde{t}_s$ and $\tilde{t}_t$ in the objective function are mapped into their crisp expected values, as defined in Appendix A. They are calculated as the middle points of the

Expected intervals. For example, Expected interval $EI(\tilde{t}_p)$ of the fuzzy unit processing time $\tilde{t}_p$ is

$EI(\tilde{t}_p) = [E_1^{t_p}, E_2^{t_p}] = [\frac{1}{2}(t_{p1} + t_{p2}), \frac{1}{2}(t_{p3} + t_{p4})]$.

Then, the expected value $EV(\tilde{t}_p)$ of the fuzzy unit processing time is calculated as:

$EV(\tilde{t}_p) = \frac{1}{2}(E_1^{t_p} + E_2^{t_p})$.

Expected intervals $EI(\tilde{t}_t)$ and $EI(\tilde{t}_{ps})$ of $\tilde{t}_s$ and $\tilde{t}_t$, and their Expected values, $EV(\tilde{t}_t)$ and $EV(\tilde{t}_s)$, respectively, are determined in the same way. In this way the fuzzy objective function is transformed into the crisp objective.

Each fuzzy constraint (2) to (8) in the proposed model is transformed into the crisp constraint using formulae given in Appendix A as follows:

(10) $Ss_i + \left[(1-\alpha)E_2^{w_i^p} + \alpha E_1^{w_i^p}\right]P_i \geq \left[\alpha E_2^{w_i^d} + (1-\alpha)E_1^{w_i^d}\right]D_i$, $i = 1,\ldots,n$

where $E_1^{w_i^p} = \frac{1}{2}(w_{i\,1}^p + w_{i\,2}^p)$, $E_2^{w_i^p} = \frac{1}{2}(w_{i\,3}^p + w_{i\,4}^p)$ and $E_1^{w_i^d} = \frac{1}{2}(w_{i\,1}^d + w_{i\,2}^d)$, $E_2^{w_i^d} = \frac{1}{2}(w_{i\,3}^d + w_{i\,4}^d)$.

(11) $\left[(1-\alpha)E_2^{w_i^p} + \alpha E_1^{w_i^p}\right]P_i = Ss_{i+1} - Ss_i + \left[(1-\alpha)E_2^{w_i^d} + \alpha E_1^{w_i^d}\right]Q_i$, $i = 1,\ldots,n$

where $E_1^{w_i^p} = \frac{1}{2}(w_{i\,1}^p + w_{i\,2}^p)$, $E_2^{w_i^p} = \frac{1}{2}(w_{i\,3}^p + w_{i\,4}^p)$ and $E_1^{w_i^d} = \frac{1}{2}(w_{i\,1}^d + w_{i\,2}^d)$, $E_2^{w_i^d} = \frac{1}{2}(w_{i\,3}^d + w_{i\,4}^d)$.

(12) $\left[(1-\alpha)E_2^{w_i^p} + \alpha E_1^{w_i^p}\right]P_i \geq 0$, $i = 1,\ldots,n$

where $E_1^{w_i^p} = \frac{1}{2}(w_{i\,1}^p + w_{i\,2}^p), E_2^{w_i^p} = \frac{1}{2}(w_{i\,3}^p + w_{i\,4}^p)$

(13) $C \geq \left[(1-\alpha)E_2^{w_i^p} + \alpha E_1^{w_i^p}\right]P_i$, $i = 1,\ldots,n$

where $E_1^{w_i^p} = \frac{1}{2}(w_{i\,1}^p + w_{i\,2}^p), E_2^{w_i^p} = \frac{1}{2}(w_{i\,3}^p + w_{i\,4}^p)$

(14) $Ss_i \geq T^{\,l} \left[\alpha E_2^{w_i^d} + (1-\alpha)E_1^{w_i^d}\right] D_i$, $i = 1,\ldots,n$

where $E_1^{w_i^d} = \frac{1}{2}\left(w_{i\,1}^d + w_{i\,2}^d\right)$, $E_2^{w_i^d} = \frac{1}{2}\left(w_{i\,3}^d + w_{i\,4}^d\right)$

(15) $T^{\,u} \left[\alpha E_2^{w_i^d} + (1-\alpha)E_1^{w_i^d}\right] D_i \geq Ss_i$, $i = 1,\ldots,n$

where $E_1^{w_i^d} = \frac{1}{2}\left(w_{i\,1}^d + w_{i\,2}^d\right)$, $E_2^{w_i^d} = \frac{1}{2}\left(w_{i\,3}^d + w_{i\,4}^d\right)$

(16) $Q_i = \left[\alpha E_2^{w_i^d} + (1-\alpha)E_1^{w_i^d}\right] D_i$, $i = 1,\ldots,n$

where $E_1^{w_i^d} = \frac{1}{2}\left(w_{i\,1}^d + w_{i\,2}^d\right)$, $E_2^{w_i^d} = \frac{1}{2}\left(w_{i\,3}^d + w_{i\,4}^d\right)$

The solution of the above crisp optimisation problem are decision variables P_i, Ss_i and Q_i, $i = 1,\ldots,n$ obtained for each feasibility degree β = 0.5., 0.6, …, 0.9, 0.95, 0.99, 1. The corresponding fuzzy values of the objective function are calculated as

$\tilde{Z}(\beta) = \sum_{i=1}^{n}(\tilde{t}_p P_i + \tilde{t}_t Q_i + \tilde{t}_s Ss_i)$, β = 0.5., 0.6, …, 0.9, 0.95, 0.99, 1.

The formula for multiplication of a scalar and a fuzzy set is given in Appendix A.

<u>Step 2.</u> The decision maker specifies tolerance thresholds to obtained fuzzy objective function values achieved for different β-satisfaction of constraints. The shortest time $\underline{Z}$ will be achieved for the lowest constraints' satisfaction β = 0.5 and the longest time $\overline{Z}$ for the highest constraints' satisfaction β = 1. We assume that the tolerance function $\tilde{G}$ is linear between these two tolerance thresholds, the shortest time $\underline{Z}$ and the longest time $\overline{Z}$. The membership function is:

$$\mu_{\tilde{G}}(z) = \begin{cases} 1, & z < \underline{Z} \\ \dfrac{\overline{Z} - z}{\overline{Z} - \underline{Z}}, & \underline{Z} \leq z \leq \overline{Z} \\ 0, & z > \overline{Z} \end{cases}$$

We propose the following formula to calculate tolerance $K_{\tilde{G}}\,(\tilde{Z}(\beta))$ to obtained objective function value $\tilde{Z}(\beta)$ when the feasibility of constrains is β, as illustrated in Figure 1. The formula provides a good estimation of the tolerance and is easier to implement in practice compared to the formula given in Appendix B.

$$K_{\tilde{G}}\left(\tilde{Z}(\beta)\right) = \frac{\overline{Z} - EV(\tilde{Z}(\beta))}{\overline{Z} - \underline{Z}}, \beta = 0.5., 0.6, \dots, 0.9, 0.95, 0.99, 1$$

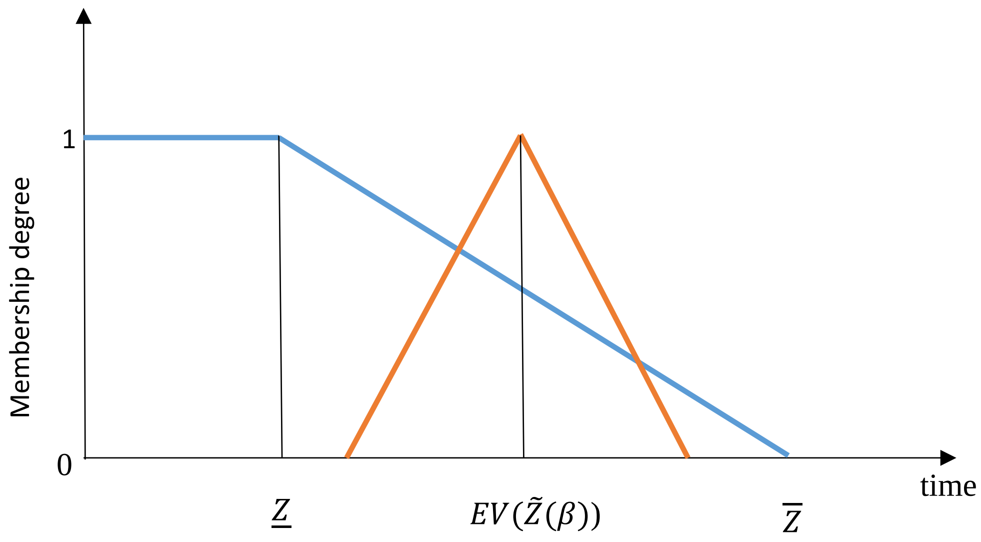

Figure 1. Tolerance function G to obtained objective function value $\tilde{Z}(\beta)$

<u>Step 3.</u> Balance between the feasibility degree of constraints β and the satisfaction degree of solution, $K_{\tilde{G}}\left(\tilde{Z}(\beta)\right)$, is calculated as:

$$\beta \cdot K_{\tilde{G}}\left(\tilde{Z}(\beta)\right)$$

The solution P_i, Ss_i, Q_i, $i = 1,\dots,n$ which achieves the highest balance $\max_{\beta=0.5,0.6,\dots,0.9,0.95,0.99,1} \beta \cdot K_{\tilde{G}}\left(\tilde{Z}(\beta)\right)$, is recommended.

The flow chart of the proposed APP model is given in Figure 2.

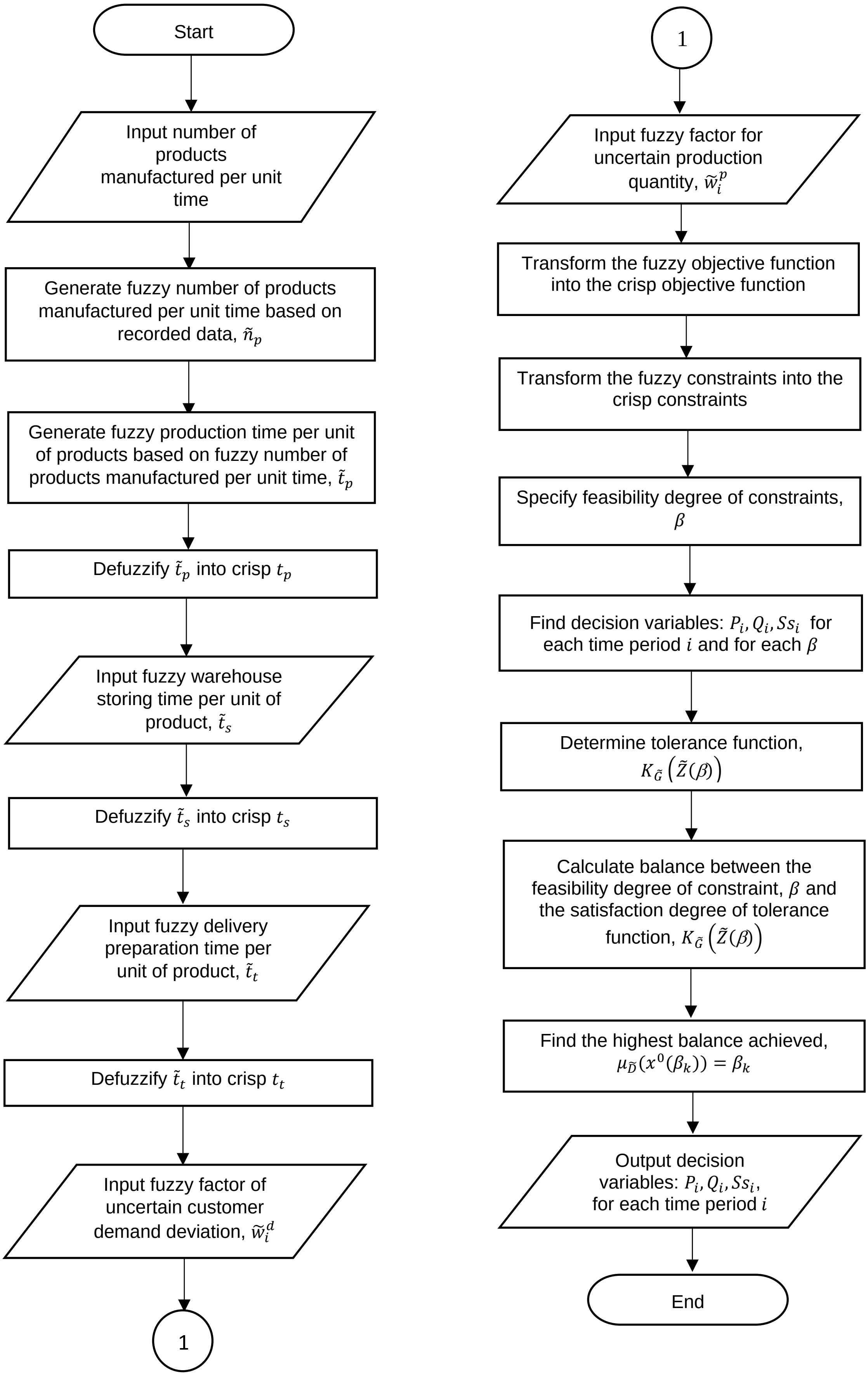

Figure 2. The flow chart of the proposed APP model

5. Case study

We considered a first tier supplier in the automotive industry located in Serbia, which has become an increasingly important industrial sector in the recent years. The factory supplies window regulators to a number of European car manufacturers. We analysed one production line which manufactures multi products for two different customers. All products belong to the same product family. They are packed in two types of plastic containers specified by the customers.

The developed fuzzy APP model is applied to determine the minimal time required for production and logistics processes. This time is crucial for efficient management of the main activities in the factory. If the factory can manufacture and deliver the same quantity of products in a shorter period of time than its competitors, it becomes more competitive in the market.

The planning horizon is selected to be a period of 12 weeks. Customer demand forecast for 12 weeks is a typical mid-term forecast used in the automotive industry for production planning. A longer period of customer demand has huge uncertainty and is not reliable for sustainable production planning.

We carried out and analysed 6 experiments including:

(1) a benchmark case,

(2) different uncertainties in production output,

(3) different uncertainties in customer demand deviation,

(4) different strategies in safety stock keeping, and

(5) comparison of benchmark results with real data recorded in the factory.

5.1. Benchmark case

Data collection is carried out in the factory in the period of 12 weeks. The measuring of unit production time, $\tilde{t}_p$, is performed including all products manufactured for the two customers on the considered production line. The tool used is the counter installed on the production line to record the number of products manufactured in 1 minute. The data are then used to create a fuzzy set $\tilde{n}_p$ with a piece-wise linear membership function, as shown in

Figure 3. The method applied to generating the membership function is explained in Appendix C.

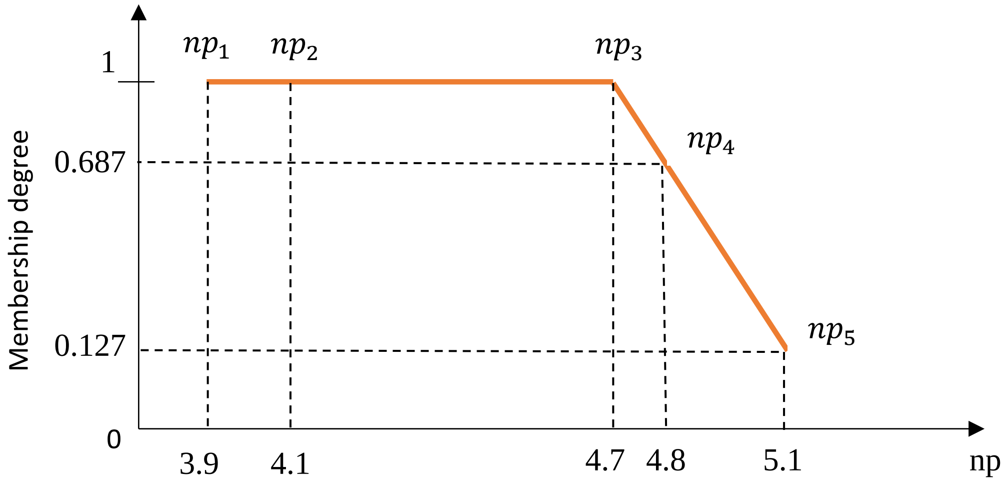

Figure 3. Fuzzy set of manufactured products per one minut $\tilde{n}_p$

We used the Expected interval method and adapted it in order to defuzzify the piece-wise linear fuzzy set $\tilde{n}_p$ as follows:

$EI(\tilde{n}_p) = [E_1^{n_p}, E_2^{n_p}]$, where the Expected value of the left side $E_1^{n_p}$ is

$$E_1^{n_p} = \int_0^1 \left[n_{p\,1} + (n_{p\,2} - n_{p\,1})\,\mathrm{r}\right] \mathrm{dr} = \frac{n_{p\,1} + n_{p\,1}}{2} = \frac{3.9 + 4.1}{2} = 4$$

The Expected value of the right side $E_2^{n_p}$ contains three partial integrals for each part of the peace wise linear membership function, as follows:

$$E_2^{n_p} = \int_{0.687}^{1} [n_{p\,4} + (n_{p\,3} - n_{p\,4})r]dr + \int_{0.126}^{0.687} \left[n_{p\,5} + \left(n_{p\,4} - n_{p\,5}\right) r\right] dr + \int_0^{0.126} n_{p\,5}\, dr$$

$$= \int_{0.687}^{1} [4.8 + (4.7 - 4.8)r]dr + \int_{0.126}^{0.687} [5.1 + (4.8 - 5.1)\, r]\, dr + \int_0^{0.126} 5.1\, dr = 4.91$$

The obtained Expected interval of number of produced product units during one minute, $EI(\tilde{n}_p)$, is:

$EI(\tilde{n}_p) = [4, 4.91]$

Expected value $EV(\tilde{n}_p)$ of the obtained Expected interval is:

$EV(\tilde{n}_p) = \frac{1}{2}\,(4 + 4.91) = 4.46.$

It means that the number of manufactured products during 1 minute on the production line is 4.46. Therefore, the defuzzified unit production time is $t_p = 1/EV(\tilde{n}_p) = 0.224$ minutes.

However, there are no recorded data on time of safety stock storing, $\tilde{t}_s$ and time of delivery preparation, $\tilde{t}_t$. The measuring is done by warehouse staff under supervision of the Logistic Manager. Safety stock storing time, $\tilde{t}_s$, is measured for 100 sampled stored containers. It is modelled by a trapezoidal fuzzy set:

$\tilde{t}_s = (0.020, 0.023, 0.028, 0.04).$

The defuzzified value is obtained using the Expected value of fuzzy set:

$EV(\tilde{t}_s) = \frac{1}{2}\,(E_1^{t_s} + E_2^{t_s}) = \frac{1}{4}(0.020 + 0.023 + 0.028 + 0.04) =$

0.03 minutes per product.

The measuring of shipment preparation time, $\tilde{t}_t$,was performed in the shipping area in the factory warehouse using a sample of 100 shipments including all the logistics operations needed for the shipment preparation for both containers' types. The shipment preparation time is specified as trapezoidal fuzzy set

$\tilde{t}_t = (0.075, 0.077, 0.082, 0.086).$

The defuzzified value is obtained using the Expected value of fuzzy set:

$EV(\tilde{t}_t) = \frac{1}{2}\,(E_1^{t_t} + E_2^{t_t}) = \frac{1}{2}(0.075 + \ 0.077 + \ 0.082 + \ 0.086) =$

0.08 minutes per product.

Fuzzy sets $\tilde{t}_s$ and $\tilde{t}_t$ are given in Figure 4.

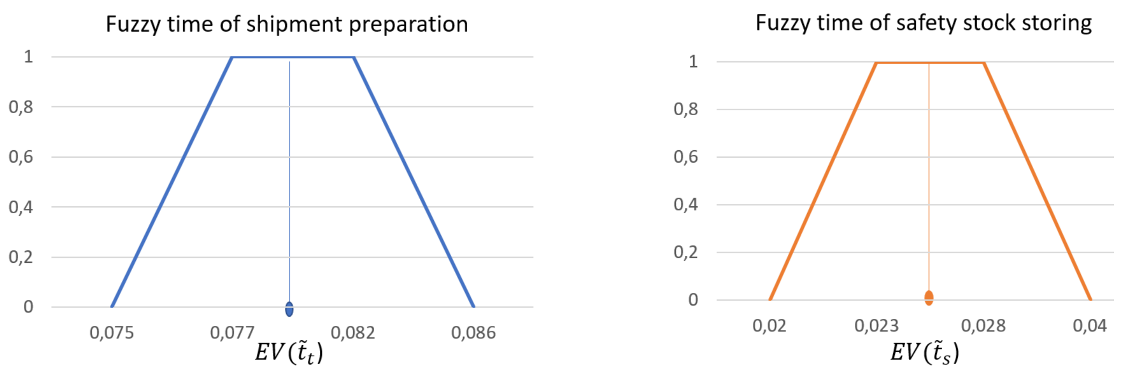

Figure 4. Fuzzy sets of safety stock storing time and shipment preparation time

The uncertainties in production and customer demands are analysed with the factory's logistics management team. Based on their experience, deviation of 10% is used to represent uncertainty in the production output. Therefore, fuzzy factor $\widetilde{w}_i^p$ of production output in each week *i*, *i* = 1,...,12, is triangular fuzzy set is:

$$\widetilde{w}_i^p = (0.9, 1, 1.1)$$

Customer demand is considered for all the products of both customers. Data collection is performed by the logistics management team. They use an advanced ERP software for integration of customer demands, production planning and material planning. All data are transparent and easy to extract for further analysis with other tools. Customer demand data are collected for 2 periods:

(1) 12 weeks realised before the planning period and

(2) 12 weeks of the planning period.

Using data on customer demand ordered in 12 weeks before the planning period, a standard deviation is calculated as:

$$\sigma = \sqrt{\frac{1}{12} \sum_{i=-1}^{-12} (D_i - \overline{D})^2} = 1869,$$

where $\overline{D}$ is the average demand recorded in 12 weeks before the planning period.

Then, deviation f_i of customer demand D_i from the forecasted demand, in each week *i*, *i*=1,...,12, is calculated as:

$$f_i = \frac{D_i}{\sigma}$$

The 12 weeks deviations obtained are:

$$0.19, 0.20, 0.23, 0.14, 0.22, 0.33, 0.15, 0.19, 0.18, 0.18, 0.18, 0.18.$$

In consultation with the logistics expert, it is decided to consider 10% of determined deviations of customer demand. Therefore, fuzzy factor of customer demand deviation, $\widetilde{w}_i^d$, is set to be triangular fuzzy set $(1 - f_i, 1, 1 + f_i)$. Fuzzy factors w_i^d calculated for each week *i*, i = 1,...,12 are given in Table 1.

Table 1. Fuzzy factors w_i^d of customer demands for each week *i*, i = 1,...,12 in the planning horizon

$\tilde{w}_1^d = (0.81,1,1.19)$	$\tilde{w}_7^d = (0.85,1,1.15)$
$\tilde{w}_2^d = (0.80,1,1.20)$	$\tilde{w}_8^d = (0.81,1,1.19)$
$\tilde{w}_3^d = (0.77,1,1.23)$	$\tilde{w}_9^d = (0.82,1,1.18)$
$\tilde{w}_4^d = (0.86,1,1.14)$	$\tilde{w}_{10}^d = (0.82,1,1.18)$
$\tilde{w}_5^d = (0.78,1,1.22)$	$\tilde{w}_{11}^d = (0.82,1,1.18)$
$\tilde{w}_6^d = (0.67,1,1.33)$	$\tilde{w}_{12}^d = (0.82,1,1.18)$

The minimum "days of inventory" in the warehouse is T^l = 3 days and the maximum "days of inventory" in the warehouse is T^u = 5 days. Machine capacity is *C* =19000.

The objective function value is calculated for every feasibility degree β, β = 0.5., 0.6, ..., 0.9, 0.95, 0.99, 1. Table 2 shows the results obtained including feasibility degree β, the sum of decision variables P_i, Ss_i and Q_i, *i*=1,...,12, within the planning horizon of 12 weeks, the fuzzy objective function values with trapezoidal membership function (z_1, z_2, z_3, z_4), degree of tolerance $\mu_{\tilde{G}}(z)$ to achieved objective function value, balance $K_{\tilde{G}}(Z(\beta))$ and the optimal crisp objective function value z. The maximum balance value is $K_{\tilde{G}}(Z(\beta)) = 0.4225$. The balance between feasibility degree of constraints β and satisfaction degree of the objective value $\mu_{\tilde{G}}(z)$ is achieved with $\beta = 0.8$.

Table 2. Results of the benchmark case

Feasibility β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(z)$	Balance $K_{\tilde{G}}(Z(\beta))$	Objective function value z
	$\sum P_i$	$\sum Ss_i$	$\sum Q_i$	z_1	z_2	z_3	z_4			
0.5	188776	123372	194883	54839	58009	62338	68889	0.733	0.3665	61382
0.6	194107	126022	198581	56235	59489	63927	70646	0.666	0.3995	62948
0.7	199546	128672	202279	57654	60992	65541	72429	0.598	0.4183	64538
0.8	**205098**	**131323**	**205977**	**59094**	**62519**	**67180**	**74241**	**0.528**	**0.4225**	**66153**
0.9	210766	134141	209674	60562	64074	68851	76089	0.458	0.4118	67799
0.95	213644	135620	211523	61305	64863	69698	77027	0.422	0.4006	68634
0.99	215968	137007	213003	61909	65503	70386	77791	0.393	0.3886	69313
1	216552	137365	213372	62061	65664	70559	77983	0.385	0.3852	69484

The optimal objective function value for β = 0.8 is 66153 minutes for 12 weeks. On weekly level it is 5513 minutes, and daily it is 1103 minutes (considering 5 working days/week, i.e., 18.4 hours/day). An overview of the average required time for all considered operations on daily level is presented in Table 3.

Table 3. Optimal average required time on daily level

Production time (hours)	Safety stock storing time (hours)	Shipment preparation time (hours)	Total time (hours)
12.8	1	4.6	18.4

It is very important for practice to analyse the results of the optimal solution of the fuzzy APP model for each week in the planning horizon. The customer satisfaction is 100%, which means that the customer demand is delivered completely. The longest time for all 3 activities is required in week 4 when the largest quantity Q_4 is delivered to the customers. However, it is achieved although quantity kept in the safety storage, Ss_4, and quantity manufactured, P_4 are not the largest quantities in 12 weeks periods. The largest quantity P_i is manufactured in week 3, and the largest quantity of safety storage Ss_i is kept in week 1. The objective function values given in Table 4 are expressed as average time required daily for all three considered operations; they are expressed in hours.

Table 4. Benchmark case: optimal solution P_i, Ss_i and Q_i, i=1,…,12, when $\beta = 0.8$

Week i	P_i	Ss_i	Q_i	Objective function value z
1	10070	17031	17031	13.7
2	16585	9768	16279	17.7
3	19588	9576	14419	19.4
4	19089	14156	23594	21.9
5	12564	9079	15132	14.3
6	17272	6135	10224	16.2
7	19443	12664	21107	21.3
8	18106	10417	17361	19.1
9	18245	10618	17697	19.4
10	18264	10618	17697	19.4
11	18262	10637	17729	19.4
12	17610	10622	17704	18.9
Σ	205098	131323	205977	18.4

5.2. Different uncertainties in production output

The management team in the factory proposed 10% potential deviation of the production output. It is very important to understand the impact of uncertainty of production output on production planning. We carried out two experiments:

1. Uncertainty in production output w_i^p, *i* = 1,…,12, is 50% smaller than in the benchmark case, with the triangular fuzzy set (0.95, 1, 1.05).
2. Uncertainty in production output w_i^p, *i* = 1,…,12, is 50% higher than in the benchmark case, with the triangular fuzzy set (0.85, 1, 1.15).

Results obtained for each feasibility degree in the first experiment are given in Table 5.

Table 5. Results obtained when there is 50% less uncertainty in production output

Feasibility β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(z)$	Balance $K_{\tilde{G}}(Z(\beta))$	Objective function value z
	$\sum P_i$	$\sum Ss_i$	$\sum Q_i$	z_1	z_2	z_3	z_4			
0.5	188776	123372	194883	54839	58009	62338	68889	0.716	0.3580	61382
0.6	193131	126022	198581	56040	59281	63706	70402	0.654	0.3927	62729
0.7	197531	128672	202279	57251	60563	65083	71926	0.592	0.4147	64086
0.8	**201975**	**131323**	**205977**	**58470**	**61854**	**66470**	**73461**	**0.530**	**0.4240**	**65452**
0.9	206464	134141	209674	59701	63159	67873	75014	0.467	0.4202	66834
0.95	208726	135620	211523	60322	63816	68580	75797	0.435	0.4133	67531
0.99	210544	137007	213003	60824	64349	69153	76435	0.409	0.4052	68096
1	211000	137365	213372	60950	64483	69297	76595	0.403	0.4028	68237

Feasibility degree β =0.8 has the highest balance index, $K_{\tilde{G}}(Z(\beta)) = 0.4240$. Optimal decision variables' values are $\sum_{i=1}^{12} P_i$=201975, $\sum_{i=1}^{12} Ss_i$=131323 and $\sum_{i=1}^{12} Q_i$=205977 products. The obtained optimal objective function value *z* is 65452 minutes. The average required total time for all considered operations in the factory is 5454 minutes on weekly level, and on daily basis it is 1090 minutes (18.2 hours). Table 6 presents the average required optimal time on daily level.

Table 6. Optimal average required time on daily level when there is 50% less uncertainty in production output

Production time (hours)	Safety stock storing time (hours)	Shipment preparation time (hours)	Total time (hours)
12.6 h	1 h	4.6 h	18.2 h

In the following experiment 2, the optimal solution is obtained for feasibility degree, $\beta = 0.7$ when the highest balance index, $K_{\tilde{G}}(Z(\beta)) = 0.4220$, is achieved. The cumulative optimal decision variables are $\sum_{i=1}^{12} P_i$=201604, $\sum_{i=1}^{12} Ss_i$=128672 and $\sum_{i=1}^{12} Q_i$=202279 products, as presented in Table 7. In this experiment, when uncertainty in production output is higher, the sum of production quantities manufactured in 12 weeks is higher for each feasibility degree β compared to the results obtained when the production output has lower uncertainty.

Table 7. Results obtained when there is 50% more uncertainty in production output

Feasibility β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(z)$	Balance $K_{\tilde{G}}(Z(\beta)$	Objective function value z
	$\sum P_i$	$\sum Ss_i$	$\sum Q_i$	z_1	z_2	z_3	z_4			
0.5	188776	123372	194883	54839	58009	62338	68889	0.749	0.3744	61382
0.6	195092	126022	198581	56432	59698	64151	70892	0.677	0.4060	63169
0.7	**201604**	**128672**	**202279**	**58065**	**61429**	**66009**	**72944**	**0.603**	**0.4220**	**65000**
0.8	208320	131323	205977	59739	63204	67912	75047	0.527	0.4217	66876
0.9	215250	134141	209674	61458	65028	69870	77210	0.449	0.4043	68806
0.95	218799	135620	211523	62336	65959	70869	78315	0.409	0.3890	69791
0.99	221679	137007	213003	63051	66718	71684	79218	0.377	0.3733	70595
1	222405	137365	213372	63231	66909	71889	79446	0.369	0.3689	70797

The obtained optimal objective function value *z* is 65000 minutes. On weekly level, the required total time for all considered operations in the factory is 5417 minutes, and on daily basis it is 1083 minutes (18.1 hours). Table 8 presents the average required time on daily level obtained in the optimal solution.

Table 8. Optimal average required time on daily level when there is 50% more uncertainty in production output

Production time (hours)	Safety stock storing time (hours)	Shipment preparation time (hours)	Total time (hours)
12.6 h	1 h	4.5 h	18.1 h

It might be interesting to notice that the optimal solution in the first experiment, when uncertainty in production output is smaller, is obtained for feasibility degree β = 0.8, while it is obtained for feasibility degree β = 0.7, in the second experiment, when uncertainty in production output is higher. Comparison of the objective function values for different feasibility degrees β practically will not give comparable data. Therefore, the objective function values obtained for the same feasibility degree $\beta = 0.8$ are compared and presented in Table 9.

Table 9. Comparison of objective function values for same feasibility degree, $\beta = 0.8$

Feasibility degree β	Uncertanty in production output	Objective function value for 12 weeks	Objective function value on daily level
0.8	50% smaller	65452	1090
0.8	50% higher	66876	1115

Results showed that there is a difference in required time for operations' activities if we consider the same feasibility degree of the constraints. For example, the difference is 25 minutes on daily level. It means if uncertainty in production output is 50% higher than in the benchmark case, the required total time for all considered operations is 25 minutes longer every day. This means it is 9.2 hours longer per month which corresponds to 1.5 shifts more per month. For the factory environment, this is considered as considerable longer time.

5.3. Different uncertainties in customer demand deviation

In the automotive industry, customers can change their demand due to updates of their own production plans. In order to better understand the impact that uncertainty in deviation in customer demands has on production planning, two experiments are carried out as follows:

1. Uncertainty in customer demand deviation w_i^d, *i* = 1,...,12 is 50% smaller than in the benchmark case, with the triangular fuzzy set $(1 - 0.05f_i, 1, 1 + 0.05f_i)$.
2. Uncertainty in customer demand deviation w_i^d, *i* = 1,...,12 is 50% higher than in the benchmark case, with the triangular fuzzy set $(1 - 0.15f_i, 1, 1 + 0.15f_i)$.

Results obtained for each feasibility degree β in the first experiment are given in Table 10. The optimal solution is obtained for feasibility degree β=0.9 with the highest balance index $K_{\tilde{G}}\big(Z(\beta)\big) = 0.4351$. Optimal decision variables values are $\sum_{i=1}^{12} P_i$=203704, $\sum_{i=1}^{12} Ss_i$=128672 and $\sum_{i=1}^{12} Q_i$=202279 products. The obtained optimal objective function value *z* is 65471 minutes. It means that the required total time for all considered operations in the factory is 5456 minutes on weekly level, and on daily level, it is 1091 minutes (18.2 hours). One can notice that the time is shorter compared to the Benchmark case. Table 11 presents the optimal average required time on daily level.

Table 10. Results obtained when there is 50% less uncertainty in customer demand deviation

Feasibility β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(z)$	Balance $K_{\tilde{G}}(Z(\beta))$	Objective function value z
	$\sum P_i$	$\sum Ss_i$	$\sum Q_i$	z_1	z_2	z_3	z_4			
0.5	188776	123372	194883	54839	58009	62338	68889	0.688	0.3441	61382
0.6	192395	124697	196732	55728	58951	63350	70006	0.638	0.3830	62379
0.7	196088	126022	198581	56632	59910	64378	71141	0.588	0.4113	63392
0.8	199856	127347	200430	57550	60885	65423	72295	0.536	0.4288	64423
0.9	**203704**	**128672**	**202279**	**58485**	**61876**	**66486**	**73469**	**0.483**	**0.4351**	**65471**
0.95	205658	129335	203203	58958	62378	67024	74063	0.457	0.4340	66002
0.99	207235	129865	203943	59340	62783	67458	74543	0.435	0.4311	66430
1	207632	129998	204128	59436	62885	67567	74663	0.430	0.4300	66537

Table 11. Optimal average required time on daily level when there is 50% less uncertainty in customer demand deviation

Production time (hours)	Safety stock storing time (hours)	Shipment preparation time (hours)	Total time (hours)
12.7 h	1 h	4.5 h	18.2 h

In the following experiment, when uncertainty in customer demand deviation is 50% higher, the optimal solution is obtained for a lower feasibility degree $\beta = 0.7$. The balance index achieved is $K_{\tilde{G}}(Z(\beta)) = 0.4259$. Optimal decision variables are $\sum_{i=1}^{12} P_i$=208692, $\sum_{i=1}^{12} Ss_i$=136555 and $\sum_{i=1}^{12} Q_i$=212099 products. One can notice that the quantity manufactured $\sum_{i=1}^{12} P_i$, the safety stock kept $\sum_{i=1}^{12} Ss_i$ and the quantity delivered to the customers $\sum_{i=1}^{12} Q_i$ are higher when customer deviation is more uncertain. Also, the optimal obtained objective function value *z* is higher - 67595 minutes in 12 weeks. On weekly level it is 5633 minutes, and on daily basis 1127 minutes (18.8 hours). Table 12 presents the results obtained while Table 13 shows the optimal average required time on daily level.

Table 12. Results obtained when there is 50% more uncertainty in customer demand deviation

Feasibility β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(z)$	Balance $K_{\tilde{G}}(Z(\beta))$	Objective function value z
	$\sum P_i$	$\sum Ss_i$	$\sum Q_i$	z_1	z_2	z_3	z			
0.5	194543	127965	201169	56556	60027	64293	70854	0,765	0,3826	63307
0.6	201546	131912	206634	58445	62034	66443	73227	0,688	0,4126	65425
0.7	**208692**	**136555**	**212099**	**60377**	**64087**	**68646**	**75664**	**0,608**	**0,4259**	**67595**
0.8	215986	141933	217564	62353	66189	70902	78167	0,527	0,4216	69818
0.9	223431	150321	223029	64420	68392	73277	80828	0,441	0,3972	72159
0.95	227212	156518	225761	65505	69552	74534	82253	0,396	0,3762	73399
0.99	230266	161935	227947	66388	70497	75559	83419	0,359	0,3555	74409

1	231033	163452	228494	66613	70738	75821	83718	0,350	0,3496	74667

Table 13. Optimal average required time on daily level when there is 50% more uncertainty in customer demand deviation

Production time (hours)	Safety stock storing time (hours)	Shipment preparation time (hours)	Total time (hours)
13 h	1.1 h	4.7 h	18.8 h

In these two experiments with different uncertainties in customer demand deviations, the optimal solutions are obtained for different feasibility degrees of constraints β = 0.9 and β = 0.7, respectively. Comparison of the objective function values obtained for different feasibility degrees, will not give comparable data. Therefore, the objective function values obtained for the same feasibility degree in both experiments, $\beta = 0.9$, are compared, as presented in Table 14.

Table 14. Comparison of objective function values for the same feasibility degree, $\beta = 0.9$

Feasibility degree β	Uncertanty in customer demand deviation	Objective function value for 12 weeks	Objective function value on daily level
0.9	50% smaller	65471	1091
0.9	50% higher	72159	1203

Comparing the results obtained, one can notice a considerable difference on daily level for the average required total time. The difference is 112 minutes. It means if uncertainty in customer demands is 50% higher than in the benchmark case, the required total time for all considered operations is 112 minutes (1.87 h) longer every day.

5.4. Different strategies in safety stock keeping

In these experiments, we analysed the impact that different strategies in safety stock keeping have on production planning. Days of Inventory (DOI) is a parameter typically used in the automotive industry to specify how many products to keep in the warehouse and it is expressed in days of sales. Of course, DOIs used has a big impact on the financial efficiency of companies.

Therefore, we carried out two experiments considering different levels of safety stock:

1. Safety stock is between 2 and 4 days, i.e., T^l = 2 and T^u = 4 days,
2. Safety stock is between 1 and 3 days, i.e., T^l = 1 and T^u = 3 days.

It is worth reminding that in the benchmark case the safety stock level is between 3 and 5 days, i.e., T^l = 3 and T^u = 5 days.

In the first experiment, the optimal solution is obtained for feasibility degree $\beta = 0.8$, with the highest balance index $K_{\tilde{G}}(Z(\beta)) =$ 0.4214. Furthermore, $\sum_{i=1}^{12} P_i$=208610, $\sum_{i=1}^{12} Ss_i$=93283 and $\sum_{i=1}^{12} Q_i$=205977 products. The obtained optimal objective function value is 65886 minutes for 12 weeks. On weekly level, the average required total time for all considered operations in the factory is 5491 minutes, and on daily basis, it is 1098 minutes (18.3 hours). Tables 15 and 16 present the results obtained and the average required time on daily level obtained using the optimal solution.

Table 15. Results obtained when the safety stock is between 2 and 4 days

Feasibility β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(z)$	Balance $K_{\tilde{G}}(Z(\beta)$	Objective function value z
	$\sum P_i$	$\sum Ss_i$	$\sum Q_i$	z_1	z_2	z_3	z			
0.5	191997	85748	194883	54731	57829	62017	68189	0.738	0.3688	61061
0.6	197423	87844	198581	56135	59316	63612	69947	0.669	0.4014	62633
0.7	202959	90240	202279	57568	60834	65241	71745	0.599	0.4192	64238
0.8	**208610**	**93283**	**205977**	**59036**	**62391**	**66913**	**73598**	**0.527**	**0.4214**	**65886**
0.9	214378	96796	209674	60537	63984	68626	75498	0.453	0.4077	67574
0.95	217307	98806	211523	61302	64795	69500	76470	0.415	0.3946	68435
0.99	219673	100532	213003	61920	65452	70207	77258	0.385	0.3810	69132
1	220268	100963	213372	62076	65617	70384	77455	0.377	0.3772	69307

Table 16. Optimal average required time on daily level when the safety stock is between 2 and 4 days

Production time (hours)	Safety stock storing time (hours)	Shipment preparation time (hours)	Total time (hours)
13 h	0.7 h	4.6 h	18.3 h

In the following experiment, when the safety stock is kept to cover between 1 and 3 days of customer demand, the optimal solution is obtained for the same feasibility degree $\beta = 0.8$, with the slightly higher balance index, $K_{\tilde{G}}(Z(\beta)) =$ 0.4234. Furthermore, $\sum_{i=1}^{12} P_i$= 212121, $\sum_{i=1}^{12} Ss_i$=65353 and $\sum_{i=1}^{12} Q_i$=205977 products. Table 17 presents the results obtained. We can conclude that when a lower safety stock is kept, in the first experiment 93238, while in the second experiment 65353, the production has to increase, from 208610 to 212121, respectively.

Table 17. Results obtained when the safety stock is between 1 and 3 days

Feasibility β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(z)$	Balance $K_{\tilde{G}}(Z(\beta)$	Objective function value z
	$\sum P_i$	$\sum Ss_i$	$\sum Q_i$	z_1	z_2	z_3	z_4			
0.5	195219	54479	194883	54750	57795	61874	67744	0.746	0.3731	60916
0.6	200739	57622	198581	56194	59326	63520	69568	0.676	0.4054	62538
0.7	206372	60883	202279	57663	60885	65194	71424	0.604	0.4226	64189
0.8	**212121**	**65353**	**205977**	**59180**	**62496**	**66929**	**73358**	**0.529**	**0.4234**	**65899**
0.9	217990	72675	209674	60777	64197	68771	75437	0.450	0.4052	67715
0.95	220971	76619	211523	61591	65065	69711	76499	0.410	0.3893	68642
0.99	223378	79775	213003	62246	65763	70468	77354	0.377	0.3736	69388
1	223983	80564	213372	62411	65939	70658	77568	0.369	0.3692	69575

The obtained optimal objective function value *z* is very similar to the previous experiment - 65899 minutes for 12 weeks. On weekly level the average required total time for all considered operations in the factory is nearly the same - 5492 minutes, and on daily basis it is the same - 1098 minutes (18.3 hours). Table 18 presents the optimal average required time on daily level obtained. We concluded that keeping the smaller stock level causes higher production and, consequently, higher production time, while the storing time for safety stock is reduced. However, the total time in both experiments remains the same.

Table 18. Optimal average required time on daily level when the safety stock is between 1 and 3 days

Production time (hours)	Safety stock storing time (hours)	Shipment preparation time (hours)	Total time (hours)
13.2 h	0.5 h	4.6 h	18.3 h

The overview of the optimal average time required for every week in the planning horizon, when the safety stock is between 1 and 3 days and 2 and 4 days, is presented in Table 19. The largest difference is recorded in week 6 when the average daily required time for all considered operations is 1.3 hours longer if safety stock level is higher. Still, comparing the total average required time for the whole planning horizon of 12 weeks, one can conclude that there is no difference between two strategies of safety stock level keeping.

Table 19. Optimal average time required for every week in the planning horizon, $\beta = 0.9$

Week i	Objective function value z for the safety stock between 1 and 3 days, (hours)	Objective function value z for the safety stock between 2 and 4 days, (hours)	Difference in objective function values, (hours)
1	13.3	13.5	-0.2
2	17.0	17.2	-0.2
3	18.8	19.1	-0.3
4	21.7	21.9	-0.2
5	15.2	14.8	0.4
6	13.5	14.8	-1.3
7	20.8	21.1	-0.3
8	19.4	18.8	0.6
9	19.8	19.0	0.8
10	19.9	19.3	0.6
11	20.1	20.1	0
12	20.2	20.2	0
∑	18.3	18.3	0

5.5. Comparison of experiments results with real data from the factory

The real-world data, including, production quantity P_i, safety stock level Ss_i and quantity delivered to the customers Q_i, in each week *i*, *i*=1,…,12 of the considered planning horizon, are recorded in the factory (Table 20). They are compared with results of the fuzzy APP model in the following way: P_i and initial safety stock SS_1 are obtained using the fuzzy APP model and Q_i is considered to be real-world delivery $Q*_i$. The safety stock level Ss_i , *i*=2,…,12 for the rest of the planning period is then determined using the formula:

$Ss_{i+1} = Ss_i + P_i - Q *_i, i = 2, \dots, 12$

Table 20. Collected data in the factory

Week, i	Safety stock level, Ssi	Produced quantity, Pi	Delivered quantity, Qi
1	10000	17450	16516
2	10934	17800	15744
3	12990	17750	16877
4	13863	18350	16122
5	16091	17740	22030
6	11801	17850	11398
7	18253	15950	15130
8	19073	16840	16888
9	19025	17530	21980

10	14575	17440	15492
11	16523	17530	16258
12	17795	18360	16374

The total time of all three activities considered including production time, safety stock storing time and shipment preparation time are recorded in the practice and compared with the time obtained in all the experiments, as shown in Figure 5. The time recorded in the factory is 67822 minutes and it is presented as continuous line. The optimal value of objective function in each experiment is presented in the bar. As it can be seen, the objective function values obtained in each experiment is shorter than in the practice, apart from the experiment when uncertainty in customer demand deviation is higher than in the benchmark case. Interestingly, the shortest times are obtained when there is higher and lower uncertainty in production output, 2659 and 2361 minutes in 12 weeks, that corresponds to 44 minutes and 39 minutes shorter time on daily basis, respectively. These improvements in times are considered to be of high importance to the factory.

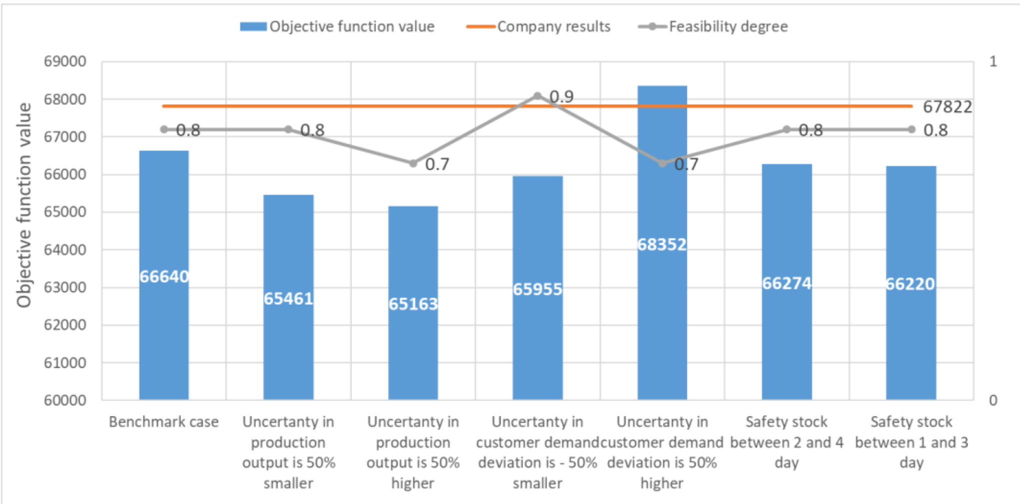

Figure 5. Comparison of real time recorded in the factory and results of the experiments

5.6. Advantages and disadvantages of the proposed fuzzy APP model

Advantages and disadvantages of the proposed model are listed in the table below (Table 21) considering different aspects including applicability of the model in practice, the use of fuzzy sets to modelling uncertain parameters, fuzzy optimisation method and modelling of fluctuations in customer demand.

Table 21. Advantages and disadvantages of the proposed model

Aspect	Advantages	Disadvantages
Practical application	Provides a solution for practical APP problems in enterprices	Requires collection of data in enterprices
Use of fuzzy sets	Generates fuzzy sets using real world data Applicable also when fuzzy sets are subjectively determined	Based on a complex procedure of using real world data to generate fuzzy sets Generated fuzzy sets might not be trapezoidal, but with piece-wise linear membership funcations
Fuzzy optimisation	Provides balance between objective function and constraints	/
Fluctuation in customer demand	Based on statistical analysis of historical data in the previous period of the testing period	/

6. Benefits to academia and industry

Benefits of the APP model proposed are both academic and industrial. First, a novel APP model is proposed that considers a new measure of performance based on the total time of production, store and delivery preparation of demanded products. Two uncertain parameters, demand and production quantity, which are very important in APP are modelled using fuzzy factors and embedded in the fuzzy APP optimisation model. Second, three methods are adapted and combined into one robust APP framework including: 1) the fuzzy optimisation method which transform a fuzzy APP LP model into a crisp APP LP model in such a way as to balance achieved feasibility degree of constraints and satisfaction with the obtained total time, 2) a method for generating a fuzzy set of unit processing time based on real-world data and 3) a novel defuzzification method for fuzzy sets with piece-wise membership functions. Third, the fuzzy APP model is successfully validated using real world data collected in a supplier's factory in the automotive industry. This validation proves that fuzzy sets and the fuzzy optimisation APP model can be successfully applied in practice. Various experiments carried out give a new insight into the APP problems of the automotive industry which is characterised by a high level of uncertainty. In all the experiments with changing uncertainties in production output, uncertainties in customer demand deviation, strategies in safety stock keeping, the results outperformed the real-world results, apart from the experiment the uncertainty in customer demand deviation is 50% higher. The application of the fuzzy APP model shortened the total material flow time and in this way improved the supplier performance.

7. Conclusion

A new fuzzy APP model is developed which minimises the total time required to manufacture products, store them in a warehouse and prepare for delivery to customers. The uncertainties included in the model are deviations in customer demand and production outputs. Furthermore, the unit production time, unit stocking time in the warehouse and unit preparation for delivery are uncertain as well.

The model is applied to a real-world supplier in the automotive industry. Various experiments are carried out in order to obtain an insight into the impact of uncertainty on production planning. Using real-world data it is demonstrated the uncertainty in production output and deviation in customer demand can have various impacts. First, uncertainty in the APP parameters can influence how well the APP constraints are satisfied; the more uncertainty in the APP parameters, such as production output, the smaller the feasibility of the constraints. Furthermore, uncertainty in the APP parameters can reduce the APP performance; for example, higher uncertainty in production output can increase the time required for all activities. Second, uncertainty in the APP parameters impacts the decision made in production planning. For example, higher uncertainty in customer deviation increases the production and safety stock kept in the warehouse. It is demonstrated that the strategy of keeping higher safety stock can require the same total time; however, in this case less time can be spent in production but more in the safety stock storing. Finally, using the real-world collected data it is demonstrated that the developed fuzzy APP model can improve the factory’s performance measured by the time to carry out the required operations. Results obtained using the proposed APP model are better compared to the practical results; the total material flow time is shorter using the proposed APP model. Practical application of the APP model in the factory would contribute to optimised production and inventory plan with higher customer satisfaction with the service level. Finally, the cash flow in the factory can be much improved.

The future work will be carried out in the following directions: (1) to forecast customer demand based on historical data and link the forecasts with the fuzzy APP model, and (2) to analyse the performance of the fuzzy APP model compared to some standard planning strategies, such as keeping different levels of safety stock, changing production capacities used, etc.

Appendix A. Relevant definitions of fuzzy sets

Definition of trapezoidal fuzzy set and multiplication with scalar [22]

Fuzzy set $\tilde{a}$ is trapezoidal, $\tilde{a} = (a_1, a_2, a_3, a_4)$, if its membership function $\mu_{\tilde{a}}$ is

$$\mu_{\tilde{a}}(x) = \begin{cases} f_a\,(x) & a_1 \leq\ x < a_2 \\ 1, & a_2 \leq\ x \leq a_3 \\ g_a\,(x), & a_3 <\ x \leq a_4 \end{cases}$$

Where $f_a(x)$ and $g_a(x)$ are increasing and decreasing functions, respectively. If $a_2 = a_3$,then $\tilde{a}$ is triangular fuzzy set.

Multiplication of trapezoidal fuzzy set $\tilde{a} = (a_1, a_2, a_3, a_4)$ and scalar r is defined as $\tilde{a}\ \cdot r = (a_1 \cdot r, a_2 \cdot r, a_3\ \cdot r, a_4 \cdot r)$.

Definition of Expected interval $EI(\tilde{a})$ of fuzzy set $\tilde{a}$ [25], [26]

$$EI(\tilde{a}) = [E_1^a, E_2^a] = \left[\int_0^1 f_a^{-1}(r)dr, \int_0^1 g_a^{-1}(r)dr\right]$$

Definition of Expected value $EV(\tilde{a})$ of fuzzy set $\tilde{a}$

$$EV(\tilde{a}) = \frac{1}{2}\ (E_1^a + E_2^a)$$

If fuzzy set $\tilde{a}$ is trapezoidal, then Expected interval and Expected value are:

$$EI(\tilde{a}) = \left[\frac{1}{2}(a_1 + a_2), \frac{1}{2}(a_3 + a_4)\right]$$

$$EV(\tilde{a}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$$

Comparison of two fuzzy sets $\tilde{a}$ and $\tilde{b}$ [27]

For any two fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the membership function of degree in which $\tilde{a}$ is bigger than $\tilde{b}$, $\tilde{a} \geq_\beta \tilde{b}$, is:

$$\mu_{\geq}(\tilde{a},\tilde{b}) = \begin{cases} 0 & if\ E_2^a - E_1^b < 0 \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & if\ 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & if\ E_1^a - E_2^b > 0 \end{cases}$$

Fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are indifferent if $\mu_{\geq}(\tilde{a},\tilde{b}) = 0.5$. If $\tilde{a}$ is bigger or equal to $\tilde{b}$ at least in degree β, it can be presented as $\tilde{a} \geq_{\beta} \tilde{b}$.

<u>β - feasibility of constraints</u>

Given decision vector x ∈ R^n is feasible in degree β if

$$\min_{i=1,\ldots,m} \{\mu_A(\tilde{a}_i x, \tilde{b}_i)\} = \beta$$

where $\tilde{a}_i = (\tilde{a}_{i1},\ldots,\tilde{a}_{in})$. It can be presented as $\tilde{a}_i x \geq_{\beta} \tilde{b}_i, i = 1,\ldots,m$. Referring to the above membership function $\mu_{\geq}(\tilde{a},\tilde{b})$ it can be formulated as:

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \geq \beta, i = 1,\ldots,m$$

Or

$$\left[(1-\beta)E_2^{a_i x} + \beta E_1^{a_i x}\right] x \geq (1-\beta)E_1^{b_i} + \beta E_2^{b_i}, i = 1,\ldots,m.$$

Appendix B. Transformation of fuzzy LP model into a crisp LP model

The fuzzy LP problem is

minimise $z = \tilde{c}\, x$,

where c̃ is vector of fuzzy parameters of the objective function

subject to $x \in \aleph_{\beta}(\tilde{A},\tilde{b}) = \{x \in R^n | \tilde{a}_i x \geq_{\beta} \tilde{b}_i, \quad i = 1,\ldots,m, x \geq 0\}$

The method is based on determining a balance between feasibility degree of constraints and degree of satisfaction to the objective function value [4].

Feasibility degree β_0 that the decision maker is ready to accept is given semantically, with associated feasibility degree, from Unacceptable solution β_0=0, Practically unacceptable solution β_0 =0.2, up to Practically acceptable solution β_0 =0.9 and Completely acceptable solution β_0 =1. Feasibility degrees $\beta_k \in [\beta_0, 1]$ are considered iteratively as follows.

<u>Step 1.</u> Crisp LP problem is solved for each β_k feasible solution:

minimise $\mathrm{EV}(\tilde{c})\mathrm{x}$

subject to $x \in \aleph_{\beta_k}(\tilde{A},\tilde{b}) = \{x \in R^n | \tilde{a}_i x \geq_{\beta_k} \tilde{b}_i, \quad i = 1,\ldots,m, x \geq 0\}$

Optimal solution of this crisp LP problem is crisp vector $x^0(\beta_k)$.

The decision maker defines his/her tolerance to achieved fuzzy value of the objective function $\tilde{G}$ by specifying the lowest and the highest boundaries $\underline{G}$ and $\overline{G}$, respectively. The membership function of the fuzzy tolerance $\tilde{G}$ is linear function:

$$\mu_{\tilde{G}}(z) = \begin{cases} 1 & if\ z < \underline{G} \\ \partial \in [0,1] & if\ \underline{G} \leq z \leq \overline{G} \\ 0 & if\ z > \overline{G} \end{cases}$$

Step 2. Degree of tolerance $K_{\tilde{G}}\ \tilde{G}$ to achieved fuzzy value $\tilde{z}^0(\beta_k)$ is calculated based on the centre of gravity defuzzification method:

$$K_{\tilde{G}}(\tilde{z}^0(\beta_k)) = \frac{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\beta_k)}(z) \cdot \mu_{\tilde{G}}(z) dz}{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\beta_k)}(z) dz}$$

Step 3. Decision $\widetilde{D}$ to be made is calculated as the balance between the tolerance to the fuzzy objective function value $K_{\tilde{G}}(\tilde{z}^0(\beta_k)$ and β_k − acceptable optimal solution for each β_k:

$$\mu_{\widetilde{D}}(x^0(\beta_k)) = \beta_k \cdot K_{\tilde{G}}(\tilde{z}^0(\beta_k)$$

Finally, to obtain the crisp solution of the fuzzy LP problem, *x**, it is proposed to take the solution with the highest membership degree of fuzzy set $\widetilde{D}$:

$$\mu_{\widetilde{D}}(x^*) = \max_{\beta_k}\{\beta_k * K_{\tilde{G}}(\tilde{z}^0(\beta_k))\}$$

Appendix C. Generation of fuzzy unit processing time using real-world data

In order to generate fuzzy set $\widetilde{np}$ which represents an uncertain number of products manufactured on the considered production line, real-world data on a number of products manufactured during one minute is recorded. The measurement is repeated 1000 times. Each time, a number of manufactured products is plotted as represented in Figure 6.

Based on the collected data, 4 intervals of data are identified, including intervals $[3.9, 4.1], [4.1, 4.7], [4.7, 4.8], [4.8, 5.1]$. The probabilities of identified intervals of data are calculated based on the frequency $p_i = n_i/N$, *i* = 1,…,4, where n_i is the number of obtained data in one interval, and *N* is the total number of collected data; in this case *N*=1000, and $p = (p_1, p_2, p_3, p_4) = (0.348, 0.318, 0.208, 0.126)$. However, the probability distribution obtained is based on one sample of collected data only. Different fuzzy sets cane generated from different samples of data which have the same probability distribution. Therefore, we applied a method proposed in [24], which generates the unique fuzzy set based on the collected data as follows.

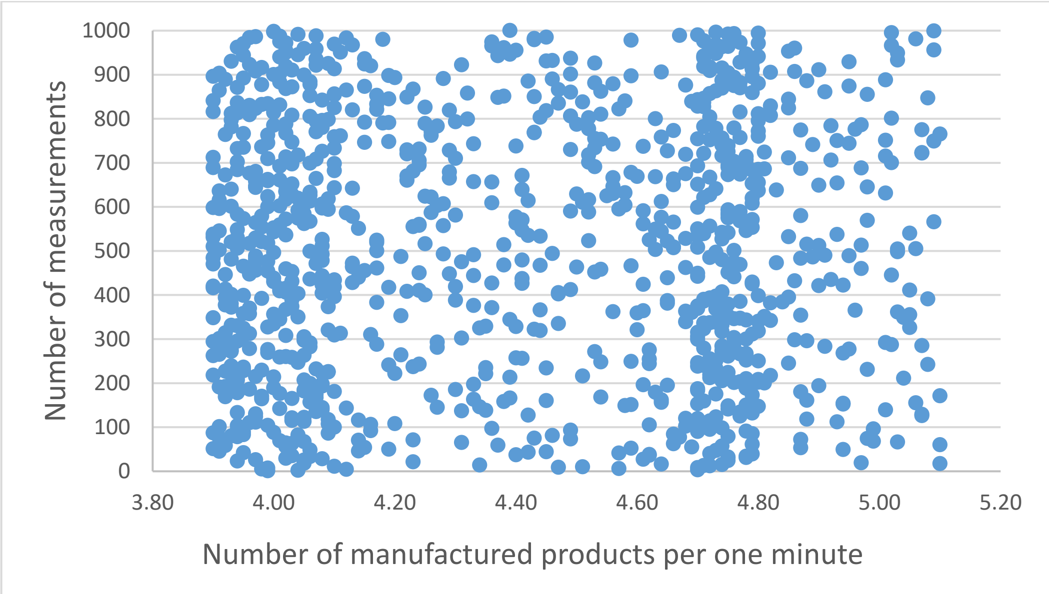

Figure 6. Numbers of manufactured products per one minute recorded on the production line

We consider all 4 intervals of data simultaneously in order to determine parameters of the corresponding unknown probability distribution of data. Parameters p_i, *i*=1,…,4 belong to ranges $[p_i^-, p_i^+]$ with the same confidence level $1-\alpha$, where we set $\alpha = 0.1$. This means that the probability that the true value of parameter p_i belongs to range $[p_i^-, p_i^+]$ is $1-\alpha$. Ranges $[p_i^-, p_i^+]$ are given in Table 22.

Table 22. Probability distribution ranges $[p_i^-, p_i^+]$ for intervals *i* = 1,…,4 of recorded data

Number of interval i	p_i^-	p_i^+
1	0.275	0.348
2	0.313	0.389
3	0.208	0.275
4	0.079	0.126

Finally, the membership function which dominates all the probability distributions presented in Table 22 is determined. It is also an uncertain process where uncertainty is expressed with confidence level $1-\alpha$ that the membership function will dominate the unknown probability distribution.

The membership function obtained is a peace wise linear function with the membership degrees given in Figure 3.

Acknowledgment

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