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Author Accepted Manuscript PDF deposited in Coventry University’s Repository

Original citation:
https://doi.org/10.1177/09544070211069666

DOI 10.1177/09544070211069666
ISSN 0954-4070
ESSN 2041-2991

Publisher: Sage

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Improving Correlation Accuracy of Crashworthiness Applications by Combining the CORA and MADM Methods

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Abstract: With the increasing use of Computer Aided Engineering, it has become vital to be able to evaluate the accuracy of numerical models. This research poses the problem of selection of the most accurate and relevant correlation solution to a set of corridor variations. Specific methods such as CORA, widely accepted in industry, are developed to objectively evaluate the correlation between monotonic functions, while the Minimum Area Discrepancy Method, or MADM, is the only method to address the correlation of non-injective mathematical variations, usually related to force / acceleration vs. displacement problems. Often, it is not possible to differentiate objectively various solutions proposed by CORA, which this paper proposes to answer. This research is original, as it proposes a new innovative correlation optimization framework, which can select the best CORA solution by including MADM as a subsequent process. The paper and the methods are rigorous, having used an industry standard driver airbag computer model, built virtual test corridors and compared the relationship between different CORA and MADM ratings from 100 Latin Hypercube samples. For the same CORA value of ’1’ (perfect correlation), MADM was capable to objectively differentiate between them. The paper has recommended the MADM settings n=1; m=2 or n=3; m=2 for a congruent relationship with CORA. As MADM is performed subsequently, this new framework can be implemented in already existing industrial processes and provide automotive manufacturers and Original Equipment Manufacturers (OEM) with a new tool to generate more accurate computer models.

Keywords: MADM, CORA, Correlation, FvD, Minimum Area Discrepancy Method

Nomenclature:

<table>
<thead>
<tr>
<th>MADM</th>
<th>Correlation rating value (Minimum Area Discrepancy Method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{model}$</td>
<td>Area between the CAE model and the average signal</td>
</tr>
<tr>
<td>$A_{upper}$</td>
<td>Area between average signal and the upper test tolerance (+1 or +2 standard deviations)</td>
</tr>
<tr>
<td>$A_{lower}$</td>
<td>Area between average signal and the lower test tolerance (-1 or -2 standard deviations)</td>
</tr>
<tr>
<td>$R$</td>
<td>Ratio between $A_{model}$ and the standard average between $A_{upper}$ and $A_{lower}$</td>
</tr>
<tr>
<td>OEM</td>
<td>Original Equipment Manufacturer</td>
</tr>
</tbody>
</table>

1. Introduction

The development of computing capability has led to an ever-increasing use of numerical modelling in science and engineering. It is essential to validate any numerical model in order to ensure credibility of the results. Usually, the response of a model is compared to that of the represented system for a set of (physical) experimental test configurations. Physical testing relies on instrumentation most often in the form of accelerometers or force transducers, which measure accelerations and forces respectively. These outputs can in turn be post-processed and compared against a set of engineering criteria, usually based on a mean response, which is bound by one or two standard deviations. Such type of correlation targets are common in biomechanics [1][2][3][4][5]. Readings from transducers as a function of time can be correlated using the “CORrelation and Analysis” software (CORA) [6][7] and the “Enhanced Error Assessment of Response Time Histories”

http://mc.manuscriptcentral.com/jauto
(EEARTH) [8] ratings. The CORA and EEARTH methods were developed to evaluate the correlation of a Computer Aided Engineering (CAE) time history signal to a reference experimental signal. These methods are extensively used in industry and consider amplitude, phase shift, size and corridor constraints. These methods are however, designed to evaluate time history signals and are not applicable to Force vs. Displacement (FvD) signals, which are frequently used in experimental validation datasets within the field of crash safety and biomechanics. CORA only works for monotonic functions, and as such fails to be precise on the rebound phase of an impact, as the ordinate of the FvD function then has more than one value. Barbat [9] listed a set of seven relevant criteria to evaluate the quality of a time history rating method, being objective, generic, robust, symmetric, simple and provide physical meanings and ratings under uncertainty. All these are met by CORA, making it a trusted correlation widely used industry for industrial applications.

A new method named the Minimum Area Discrepancy Method for Force Displacement Response Correlation (MADM) [10] calculates a correlation coefficient for FvD signals in order to rate how close they are to a given test response. The MADM correlation criteria utilises an area method and aims to maximise area overlap; a complete (maximum) overlap indicates perfect correlation. Figure 1 (a) contains an example of numerical analysis results which can be overlaid on the test data represented by the “average” (b), “lower” (c) and “upper” (d) test-corridors. The problem can be split in three shaded zones, which will represent the difference between the CAE and the test about the mean whilst considering the spread of datasets.

(a) Typical correlation problem. CAR model in red to compare to tests average, upper and lower

(b) $A_{model}$. Area between the CAE model and the average signal
A parameter ‘R’ is calculated to represent the level of discrepancy between the numerical model and the experimental test data using the average, lower and upper test corridors. The ratio ‘R’ is calculated as per Equation 1:

$$R = \frac{A_{\text{model}}}{\frac{A_{\text{upper}} + A_{\text{lower}}}{2}}$$

Equation 1: Calculation of R: area ratio overlap between the CAE model and the test corridors

The ratio ‘R’ is then normalised and referred to as the MADM correlation number, as per Equation 2, which is subsequently adjusted to suit the desired degree of correlation, by changing the values of ‘n’ and ‘m’ from Equation 2.

$$MADM = \frac{1}{1 + nR^m}$$

Equation 2: MADM Generic Form

The values of ‘n’ and ‘m’ represent the normalisation “rate” of MADM (Figure 3 in the methodology section). In a previous publication [10], the values n=1; m=2 where used, however no other parameter combinations were studied, which this paper will investigate. The MADM method is the only computational method able to address the correlation of FvD problems [10], when only the FvD curve and its corridors are available. Such problem are present in human cadaveric tests literature [11][12][13][14][15], where no time dependant data is available.

From an engineering perspective, CORA and MADM approaches the correlation of CAE to physical test data in significantly different ways: CORA is time dependant while MADM studies area differences between FvD curves, hence an energy perspective. It is therefore of interest to investigate how these two methods respond to a complex engineering problem (here a CAE airbag model), and whether any synergies between these methods can be exploited to improve current industrial correlation methods.
2.0 Method

Using four steps, this paper will critically evaluate the performance of CORA and MADM applied to airbag design, including exploration of whether any synergies can be exploited between these two correlation techniques.

The CAE model chosen is a driver airbag obtained from the standard MADYMO examples [16], which will be validated against corridors representing a range of physical tests. This computer replicated an airbag physical test, investigating its deployment response when subjected to a 30kg mass dropping at 6.0m/s (Figure 2).

**Figure 2: MADYMO Driver Airbag Model**

**Step 1: Corridor generation:**

Both correlation methods need physical data which require to be converted into test corridors. In industry, corridors are obtained by performing 3 to 5 tests, which are then fed into the CORA routines for corridor extraction and the calculation of the mean. The most common option is the use of constant corridor widths. Typically, a share of the global absolute maximum is used as width, whilst the mean is calculated form the test data. 0.05 is a common “share” of the global absolute maximum (inner corridor) and 0.50 for the outer corridor [6]. As physical test data were not available for the MADYMO CAE driver airbag, Design of Experiments (DOE) coupled with numerical simulation was used to generate a representative inner corridor of +/- 1 sigma and an outer corridor of +/- 2 sigma. The DOE was created by varying the inflator temperature, the vent discharge coefficient (CDEX) as well as the airbag diameter (Table 1).
Airbag feature (variable) | Variable range
--- | ---
Inflator Temperature: | +/-10%
CDEX (Vent discharge coefficient): | +/-10%
Airbag Diameter: | +/-5%

Table 1: Airbag design variables

The variables range from Table 1 is used to generate a DOE of 100 Latin Hypercube samples (LHC) which in turn will result in the creation of artificial test corridors. The median as well as +/- 1 and 2 standard deviations will be calculated and used as inner and outer corridors for CORA and the +/-1 sigma for lower and upper MADM corridors.

Step 2: Computation of the CORA ratings
For each of the 100 LHC samples, the CORA rating will be computed against the corridors computed in Step 1. As the CORA rating depends on the choice of the weight used, it is proposed to use two weight settings: one more aligned with OEMs practices [17] and another one based on evenly distributed weights. The CORA settings are listed in Table 2.

<table>
<thead>
<tr>
<th>Impactor Signal</th>
<th>Typical Industry setting for airbag correlation (OEM Setting) [17]</th>
<th>Corridor Setting (evenly distributed weights)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighting</td>
<td>Corridor Method</td>
<td>Cross correlation Method</td>
</tr>
<tr>
<td>Acceleration</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>Velocity</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Stroke</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: CORA settings for the study

Step 3: Computation of the MADM ratings
For each of these 100 LHC samples, the MADM rating will be computed. As the MADM rating, depends on the choice of the ‘n’ and ‘m’ coefficients different coefficients will be considered. The evolution of MADM ratings (between 0 and 1), as function of ‘n’ and ‘m’, is presented in Table 3. These weights were carefully defined in order to investigate the effect of the rate (how fast the MADM rating was changing) and the spread (how wide the MADM rating changes for the range of R) on the airbag correlation rating. The values of ‘n’ and ‘m’ are listed in Table 3, and the MADM evolution is displayed in Figure 3.
Table 3: MADM 'n' and 'm' values for the study (*configuration used in [10])

<table>
<thead>
<tr>
<th>Coefficient configuration</th>
<th>MADM ‘n’ Values</th>
<th>MADM ‘m’ values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1*</td>
<td>2*</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>D</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>0.5</td>
<td>2.0</td>
</tr>
<tr>
<td>F</td>
<td>0.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>

It can be observed in Figure 3 that for cases A and B, the range of MADM ratings vary from 1 to 0.5, while for cases C, D, E, and F it varies from 1 to 0.7. Amongst C, D, E and F scenarios, case C has overall the slowest rate of evolution, which means that if the R value changes, then the MADM value also increases slowly. For the same value of ‘n’ the greater the value ‘m’, the greater the rate of evolution, for the same range. In conclusion, ‘n’ affects the range and ‘m’ the rate.

Step 4: CORA rating and MADM comparison:
CORA rating and MADM are overlaid on the same graph and their relationships analysed.
3.0 Results

3.1 Corridor Generation.

Following the 100 LHC computations, the corridors for the acceleration vs time and displacement vs time (both to be used by CORA) were created, as illustrated in Figure 4, and then combined to generate the corridors for the force vs displacement signal (to be used by MADM), Figure 5.

3.2 CORA OEM and Corridor Settings for Airbag Validation.

The OEM CORA and MADM ratings are overlaid in the same graph. Their relationship is also compared against the correlation line $y=x$ (Figure 6). A regression line (red) is drawn to evaluate the relationship between these two methods.
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Figure 6: Relationship between CORA and MADM for an OEM CORA setting

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Figure 7: Relationship between CORA and MADM for and standard Corridor setting

4.0 Discussion

Looking at Figure 6 and Figure 7, it can be observed that when MADM rating range is small (cases C, D, E and
F), then the MADM response does not grow as fast as the one of CORA, leading to the best fit curve to cross over the y=x correlation boundary. It is further reinforced for cases C and D where the rate of MADM growth is slowest compared to E and F, leading to an average correlation slope between the two methods of around 0.2 and 0.4, compared to 0.7 and 0.9-1.0. In such instances the MADM and CORA rating values alternate, which is not desirable, as consistency of interpretation between the two methods is preferred. Consequently, it is best to use the wider range option setting for MADM, i.e. A and B to provide a consistent correlation relationship between the two methods. It can be noticed that, in all the cases, the linear interpolation is positive, which means that CORA and MADM correlation predictions are congruent with each other. For the ‘n’ and ‘m’ values chosen in A and B, the MADM responses are usually lower than the CORA values, which means that MADM is more discriminating than CORA. Following this analysis, the ‘n’ and ‘m’ values in A and B are more adequate for future studies.

It can be noted in Figure 6 and Figure 7 that many samples from the DOE have obtained a CORA value of ‘1’, i.e. perfect correlation, while their MADM rating was different. It is then possible to select amongst these solutions the best MADM option. This can be illustrated using the arbitrary choice of MADM setting n=1; m=2, scenario A in Figure 8.

**MADM Setting n=1; m=2 vs. typical CORA Setting**

![MADM Setting n=1; m=2 vs. typical CORA Setting](image)
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Figure 8: Effect of MADM response for curves with CORA OEM setting rating of 1

Figure 8 clearly highlights that the CAE prediction’s diverge from the mean the lower the MADM value, albeit the CORA value being ‘1’. This is evident for the area in the return of the signal where the shape departs from the mean at (0.2; 0). Some changes are also visible in the (0.2; 200) area where a poorer MADM value deviates from the mean. Note that the findings are the same for scenario B (n=2; m=3).

Consequently, if CORA is an excellent and proven tool to correlate time domain signals, it is possible, at the same time, to convert these time domain signals into a displacement (timeless) space, run a MADM analysis to support the best choice of the CORA solution. It could also be envisaged in the future to perform a multi-objective optimisation to maximise both ratings.

The proposed correlation process can be summarised in Figure 9.
This approach is suitable for all crashworthiness applications, as it is an add-on an existing proven method based on CORA. This new framework can easily be implemented in an industrial setting and can improve the accuracy of computer models compared to real life.

5.0 Conclusions

In this paper, a new approach to improve the correlation of a crashworthiness application is proposed by combining the time dependent correlation features of CORA with the Minimum Area Discrepancy Method (MADM) approach (energy based). The research has shown that, in the case of a crashworthiness example of a driver airbag correlation, CORA could provide more than one optimum solution. By computing these optimum ratings using MADM, it is possible to rank these CORA correlation propositions objectively and provide best solution from the samples provided. The study has shown that in all cases MADM and CORA ratings are congruent and that MADM values were consistently lower than CORA for settings A (n=1; m=2) and B (n=3; m=2), which are recommended. For settings A and B, the MADM rating values are always lower than CORA, making MADM a more discriminative correlation method. The paper concludes that the current industrial correlation methods could be improved by adding a final loop of MADM rating. It is believed that this new framework can easily be implemented in an industrial setting and can improve the accuracy of human body model developments as well as designing safer vehicles.

6.0 MADM software

The MADM software is available by contacting the authors.
This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declarations of interest: none.
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