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Fuzzy Cognitive Maps with Type 2 Fuzzy Sets

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Fuzzy Cognitive Maps with Type 2 Fuzzy Sets

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Certificate of Ethical Approval

Applicant: **Alya Al Farsi**

Project Title: A New Intelligent Method to Identify Early Stage Autism using Soft Computing Decision Modelling Approaches

This is to certify that the above named applicant has completed the Coventry University Ethical Approval process and their project has been confirmed and approved as Medium Risk

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ABSTRACT

A Fuzzy Cognitive Map (FCM) is a causal knowledge graph with feedback. Its robust characteristics make it an effective approach to reasoning and decision making in diverse application domains. However, the capabilities of a conventional FCM for modelling and reasoning real-world problems in the presence of uncertain data is limited as it relies on a Type 1 Fuzzy Set (T1FS). In this thesis, the capability of FCM for capturing more uncertainties is extended by introducing Type 2 Fuzzy Sets based on z slices (zT2FSs) which are capable of capturing higher degrees of uncertainties compared to T1FS. This extension is carried out through two stages. In Stage 1, the Interval Agreement Approach (IAA) is used to generate zT2FSs that model the weights of causal relations in the FCM. In this stage, the FCM's reasoning is carried out using an iterative reasoning algorithm with the defuzzified values of generated zT2FSs. In Stage 2, a new reasoning algorithm is introduced for the FCM which is proposed in Stage 1, where the reasoning operates with the fuzzy values of the weights without defuzzification. To demonstrate the proposed FCM in Stage 1, a new case study for early diagnosis of autism was created. Using the secondary data for an autism diagnosis, published in the literature, the accuracy of the diagnosis is increased by 5.46% when the proposed FCM is used. The results demonstrate that the proposed method outperforms conventional FCMs used for the same purpose. To demonstrate the new reasoning algorithm developed in Stage 2, the proposed FCM in Stage 1 was applied with the new reasoning algorithm in a new created case study for the evaluation of module performances across mathematical modules in a higher education institution. It was found that the results obtained have a higher correlation with domain experts' subjective knowledge than both an FCM with weights modelled using T1FS and

statistics currently used for evaluating module performances. The correlation between the domain experts and the results obtained when the proposed reasoning algorithm is applied is 0.34, while in cases when a T1FS and currently used statistics are used it is 0.08 and 0.28, respectively. In addition, sensitivity analysis is conducted to investigate the propagation of uncertainty in the proposed reasoning algorithm. The results demonstrate that the FCM that uses the new reasoning algorithm preserves the propagation of uncertainty captured from input data effectively. It was observed that changes in uncertainties of zT2FS weighted links impacted the value of the decision concept in the FCM with different degrees depending on the structure of the FCM and its links. The contributions of this research, which are obtained from the abovementioned two stages, are: (1) new extensions of FCM where the weights represented by zT2FSs outperform the conventional FCM, and (2) a new non-iterative reasoning algorithm for FCM that effectively propagates the uncertainty while reasoning and hence enhances the capability of FCM for reasoning similar to human.

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LIST OF ACRONYMS

FS	Fuzzy Set
T1FS	Type 1 Fuzzy Set
T2FS	Type 2 Fuzzy Set
IFS	Intuitionistic Fuzzy Set
GT2FS	General Type 2 Fuzzy set
IT2FS	Interval Type 2 Fuzzy set
LMF	Lower membership function
UMF	Upper membership function
FOU	Footprint of Uncertainty
zGT2FS	General Type 2 Fuzzy set based on z slices
zT2FS	Type 2 Fuzzy set based on z slices
CWW	Computing with Words
IA	Interval Approach
EIA	Enhanced Interval Approach
HMA	H Mendel Approach
FCM	Fuzzy Cognitive Map
CM	Cognitive Map
DSS	Decision Support System
MDSS	Medical Decision Support System
CFCM	Competitive Fuzzy Cognitive Map
m-FCMs	Distributed m-FCMs

RBFCM	Fuzzy rule based Fuzzy Cognitive Map
FGCM	Fuzzy Grey Cognitive Map
GST	Grey System Theory
iFCM	Intuitionistic Fuzzy Cognitive Map
IFS	Intuitionistic Fuzzy Set
DS-ET	Dempster -Shafer Evidence Theory
ECM	Evidential Cognitive Map
GCM	Granular Cognitive Maps
DCN	Cognitive Network
TFNFCM	Triangular fuzzy number FCM (TFNFCM)
TFN	Triangular fuzzy number
RCGAs	Real-coded genetic algorithm
HL	Hebian Learning
NHL	Nonlinear Hebian learning approach
GA	Genetic algorithm
PSO	Particle Swarm Optimization
DHL	Differential Hebbian Learning
BDA	Balanced Differential algorithm
E-FCM	Evolutionary FCM
zT2FCM	Fuzzy Cognitive Map with weights based on zT2FSs
ASD	Autism Spectrum Disorder
MCHAT	Modified Checklist for Autism in Toddlers
F-MCHAT	Fuzzy MCHAT

SQUH	Sultan Qaboos University Hospital
NILD	Non-Iterative Reasoning algorithm with Late Defuzzification
MPFCM	zT2FCM with NILD for evaluating the Module Performance
MASC	Department of Mathematics and Applied Sciences
MEC	Middle East College
SIS	Students Information System
TLS	Traffic Light System
MP	Module Performance
ATT	The attendance of the students in each module
CW	The total mark for the module course work
ESE	Total end semester examination result
SD	Standard Deviation of the results
PP	Pass percentage
AVE	The average of both results, CW and ESE,
$\boldsymbol{\rho}$	Pearson correlation
$\boldsymbol{R^2}$	The coefficient of the determination

PUBLICATIONS

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Chapter 1 Introduction

1.1 Background

Reasoning and decision-making processes are fundamental in major aspects of our life. Taking an optimal decision is very important as there are consequences based on this decision. The mechanism of reasoning process and its algorithm play a vital role in inferencing the more accurate decision. However, there are factors which may affect the process of reasoning and decision making, for example, the imprecise or lack of information, the hesitancy of the decision-makers (experts) based on their experiences. Indeed, in the domain where the decision is taking by a group of people, the existing uncertainties are classified into Inter- uncertainty (the uncertainty among the group of decision-makers) and intra-uncertainty (the uncertainty an individual decision-maker has)(Wagner, Miller, and Garibaldi 2013a). Therefore capturing and then handling these uncertainties; i.e. "reducing their effects"(Mendel 2001) lead to take the optimal decision/solution.

There are different types of uncertainties in different domains and each could be handled by a certain theory; for example, uncertainty which relates to randomness, and this can be handled by Probability Theory and the uncertainty which relates to fuzziness, lack of definite distinction or linguistic uncertainties, and this can be handled by Fuzzy Set Theory.

1.1.1 Fuzzy sets

Fuzzy Sets (FSs) are tools that provide a robust way to deal with uncertainties and imprecisions (Zadeh 1965). Literature reveals that there is a progression in the development of FSs since proposed in (Zadeh 1965) and different types of FSs have been defined, for example Type 1 Fuzzy Sets (T1FSs) and Type 2 Fuzzy Sets (T2FSs) . T1FS is a set which is characterised by membership function that has grades' values between 0 and 1 (Zadeh 1965), which later progress to T2FS which is characterised by membership functions that is T1FS (Zadeh 1975). Applications of FSs have a noticeable success in the domain of reasoning and decision making in presence of uncertainties as presented in (Klir and Yuan 1995). In this surge, different approaches based on FSs have been introduced for modelling in presents of uncertainties and hereafter reason to make the decision. A well-defined approach in this area is a Fuzzy Cognitive Map (FCM).

1.1.2 Fuzzy Cognitive Map

The FCM was introduced by Bart Kosko in1986 as a directed graph with feedback. The FCM consists of nodes that represent the main aspects of the modelled domain linked by weighted links that represent the causal relations between them. The reasoning using FCM relies on values of its nodes and links. These characteristics of the FCM's structure provide it with dynamicity and simplicity to understand even by a non-technical person.

FCMs have gained a noticeable attention by researchers in different domains' application. In each application, the FCM is constructed for a purpose based on one of the following functions: prediction, strategic, explanatory or a reflective purpose (Codara 1998).

1.1.2.1 Drawbacks of Fuzzy Cognitive Map

Despite the effective capabilities of FCMs, they have some drawbacks that limit their effectiveness and efficiency to handle high levels of uncertainties associated with domains of high imprecise data or a multi-meaning environment , for example as in Medicine

(Salmeron 2010) and Business (Hajek and Prochazka 2016) and thereafter hinder their abilities to reason effectively.

One of the drawback of the FCM is its disability to model real world problem of causal relations that are neither linear nor monotonic. To cope with this drawback and hence improve the performance of the FCM in reasoning, some extensions and learning algorithms have been proposed (Papageorgiou 2012). Researchers pursuing to overcome these drawbacks and hence enhance the capabilities of the Fuzzy Cognitive Map. They have been proposing several extensions and learning algorithms for the FCM. Therefore, the FCM which was introduced by Kosko, known as conventional FCM has been extended in different ways.

1.1.2.2 Extensions of Fuzzy Cognitive Map

As the weights of the causal links of the FCM are crucial for knowledge propagations while reasoning in the FCM, various studies have been conducted to improve the representation of the weight of the causal relation and hence improve its effectiveness in modelling and reasoning. Researchers attempted to use different representation for weights. For example, the study in (Miao et al. 1999) introduced Dynamic Cognitive Network (DCN) to extend the capability of FCM via enhancing the dynamicity of the causal relations among the concepts and by allowing the concepts to adjust their values based on the requirement of the system. The causal relations in DCN are represented by dynamic model based on Laplacian transformation where the causal relationships inform how long the cause will take effect and how the cause will take effect. To tackle a disability of the FCM in representing causal relations different than monotonic relation among the concepts, Carvalho and Tome, (2000) proposed Rule-based Fuzzy Cognitive Map (RBFCM), where fuzzy rules are used to determine the relations among the concepts and made the mechanism of feedback compatible with different types relations and hence reduce the complexity of modelling the system. Later, the standard reasoning mechanism of RBFCM is improved and a new mechanism is proposed in (Zdanowicz and Petrovic 2018), which makes RBFCM s more flexible for modelling the complex number.

To improve the representation of the causal relations of the Fuzzy Cognitive Map and handling their uncertainty, (Salmeron 2010) incorporated the FCM with Grey System Theory and proposed Fuzzy Grey Cognitive Map (FGCM), where imprecise relations among the concepts were represented by Grey intervals rather than fuzzy singletons as in the conventional FCM. Thus, the concepts and weights of conventional FCM are extended to grey concepts and grey weights. Comparing to the conventional FCM, the FGCM is better in dealing with unclear or absent relations between the concepts as it relies on Grey Theory, thus it expresses the existing casual relation by grey weight and the absence relation between concepts by zero.

Though the FGCM shows its effectiveness in handling imprecise data with a high level of uncertainty by using the Greyness as a measure of uncertainty, this capability is limited to be used only with little data available. The work in (Papageorgiou and Iakovidis 2013), extended the FCM to Intuitionistic Fuzzy Cognitive Map (iFCM)by introducing hesitancy to the values of its concepts and weight of the edges. The concepts and weights in the iFCM are represented by Intuitionistic Fuzzy Sets (IFSs) and conventional reasoning algorithm of the FCM is modified accordingly to be compatible with the use of IFSs. The hesitancy of outputs' concepts in the iFCM, enhances the quality of the decision-making process.

To enhance the process of aggregating the information required for constructing the FCM and for better representation of the uncertainty, the Dempster-Shafer Evidence Theory was combined with FCM and the Evidential Cognitive Map (ECM) was proposed in (Kang et al. 2012). The representation of the concepts in the ECM is more flexible than their representation in FCM, as they are represented by intervals in the ECM rather than the crisp values as in the FCM. The evidence theory was combined with the method of combining belief functions (Deng, Jiang, and Sadiq 2011) and used in the process of determining the causal relations among the concepts, which helped in representing unclear, difficult and even not existing causal relations due to ability of the ECM to measure the indeterminacy. Though the success of developing and using the ECM in some applications (Zhang et al. 2018), it required more modelling effort from the experts to combine the fuzzy aspects of the FCM and Evidential theory aspects. Moreover, it still needed to be enhanced by training the map using the appropriate learning algorithms.

The concept of the FCM was extended to the Granular Cognitive Maps (GCM) in (Pedrycz and Homenda 2014) where the links between the concepts were described using information granules. GCM succeeded in enhancing the conventional FCM by making its formation in presence of several sources more flexible.

The work in (Yesil, Dodurka, and Urbas 2014) used a fuzzy number with the triangular membership function (triangular fuzzy number) to represent the weight of interrelation between the concepts of the FCM. The representation of the proposed FCM using the triangular fuzzy number enhanced the efficiency of the conventional FCM by reducing the required number of iterations in the reasoning algorithm as the operations using the triangular membership function are easier and make the defuzzification process faster.

Indeed, it made the process of constructing the FCM more flexible when there is a need to aggregate knowledge from different experts. Though the success of the FCM using a triangular fuzzy number was recognised, the defuzzification of a triangular fuzzy number in each of its simulations process to use it in the reasoning process leads to missing some of the information captured by the triangular fuzzy number.

One study by Peter Hajak, (Hajek and Prochazka 2016) aimed to overcome the uncertainty of determining the values of concepts and weights of the FCM by extending the conventional FCM to the Interval-valued FCM, where the values of concepts and weights were represented by intervals with lower and upper values and the reasoning algorithm was modified accordingly. The proposed Interval-valued FCM gave better and results closer to the reality comparing to the conventional FCM ,but it failed to provide a dynamic inference mechanism as demonstrated in (Hajek and Froelich 2019). The Interval-valued FCM was enhanced in (Wang and Guo 2018) and the Ensemble interval-valued FCM was introduced, where the evidential reasoning was used to aggregate the ensemble maps (Yang and Xu 2013) and (Fu, Huhns, and Yang 2014). It was shown that the Ensemble interval-valued FCMs were effective in modelling complex systems with uncertainty from various fields.

To capture the uncertainty of concepts values in the FCM and hence improve its capability in the domain of decision making, the work in (Marchal et al. 2016) proposed using the non-singleton fuzzification approach to determine the values of the concept of the DCN where the concepts had dynamic causal relations. It was proved that using the non-singleton FCM provided results that matched better to the decisions made by the experts in the same field, for example, when the FCM was used as a Decision Support System for the process of production of virgin olive oil.

1.1.2.3 Learning algorithms for FCM

Learning capabilities enhance FCMs for decision making and modelling. The learning capabilities of FCMs rely on modifying the weights of the causal relations among the concepts to produce learned weights. Based on the literature (Papageorgiou 2012) , there are three categorisations of learning algorithms developed for FCMs based on the learning paradigm applied. They are Hebbian-based, population based, and hybrid, which combines the aspects of Hebbian-based- and population-based learning algorithms. These algorithms are used to train the FCMs and their success in this surge pinpointed in different applications, for example, modelling the behaviour (Koulouriotis, Diakoulakis, and Emiris 2001), control problem in the industry (Papageorgiou, E.I. Groumpos 2005), real-world problems of partner selection as in (Zhu and Zhang 2008) and prediction of Autistic disorder (Kannappan, Tamilarasi, and Papageorgiou 2011).

1.1.2.4 FCM Applications Domain

FCMs play a vital role in solving different problems in diverse paradigms. The FCMs are used as tools for prediction, classification, modelling, decision support, analysing or reasoning in different domains such as Business(Hajek and Prochazka 2016), Production Systems (Luo, Wei, and Zhang 2009a), Engineering (Jetter 2006), Medicine (Amirkhani et al. 2017) and Education (Laureano-Cruces, Ramírez-Rodríguez, and Terán-Gilmore 2004).

1.1.3 The Gap

However, in all the above mentioned extensions of the FCMs, the weight of causal relations are represent by T1FSs and hence the capability of the FCM to capture and propagate the uncertainties of knowledge is hindered. As the weights play a crucial role in knowledge propagation in the FCM and T2FSs have more advantages over T1FSs in capturing more uncertainties, this thesis contributes to the enhancement of the FCM by introducing T2FSs to its causal relations' weights and the reasoning algorithm

1.2 Motivation

As the improvement of the FCM is dependent on using the words to describe the world as emphasised in (Medsker 2012) and T2FSs are equipped with a further potential over T1FSs to model the uncertainties of words and people' perceptions, the author's target is to exploit modelling advantages of both the FCM and T2FSs in extending the conventional FCM to FCM based on T2FSs. Therefore, this extended FCM inherits the capabilities of both the standard FCM and T2FSs. That motivated the author to introduce T2FSs to represent the weights of the causal links in the FCM. Furthermore, and the iterative reasoning algorithm of the conventional FCM is enhanced to a new reasoning algorithm based on T2FSs. Thus, the conventional FCM is extended to a new proposed FCM based on T2FSs (particularly T2FSs based on z slices (zT2FSs) (Wagner et al. 2015)).

1.3 Research Questions and Objectives

The aim of this thesis is to develop extensions to the FCM that are capable of handling more uncertainties during the reasoning and infer an output that is more close to the human decision. This thesis aims to address the following questions:

- Can the FCM be extended to incorporate data driven aggregation models for handling different types of uncertainties from different sources (experts), for example: inter uncertainty of more experts on the same domain and intra uncertainty of one expert exposed over time and, hence, to facilitate a more complete information representation?

- Can the reasoning algorithms of the FCM be improved to better support a decision making process?

The main objectives of this thesis's research are as follows:

- Analyse the effectiveness of using zT2FSs in modelling of uncertainty.
- Analyse Interval Agreement Approach (IAA) and its application to generate the weights of the links in the FCM represented as zT2FSs.
- Develop an iterative reasoning algorithm for FCMs with weights represented using zT2FSs and analyse its effectiveness by creating it in novel case study
- Develop a new non-iterative reasoning algorithm for the FCMs with weights represented using zT2FSs that operates without defuzzification.
- Demonstrate that the FCM with the proposed new non-iterative reasoning algorithm outperforms the conventional FCM by creating a novel case study.
- Investigate the effectiveness and analyse the sensitivity of the new reasoning algorithm and its capability in allowing the uncertainty to propagate.

1.4 Scope

This thesis aims to extend the capability of the conventional FCM to capture more uncertainties during reasoning and therefore become more appropriate for reasoning and decision making mimicking better human (i.e. experts/ decision makers). The author proposes using zT2FSs generated using IAA to represent the weights of the FCM and then carry out reasoning with it using the standard iterative reasoning algorithm. Furthermore, the values of weights represented as zT2FSs will be used in a new reasoning process without defuzzification that will keep and operate with uncertain weights. The rationale of this is the capability of zT2FSs generated by IAA to represent the level of agreement between the experts while determining the weights' values and thus capture more uncertainties in compared to T1FSs.

In this thesis, the validation of the proposed approach to extend the conventional FCM is of two folds as follows:

- The weights of FCM are represented by zT2FSs generated by IAA. Then their defuzzified values are use in the conventional iterative reasoning algorithm of the FCM. To demonstrate this proposed FCM, a case study for Autism diagnosis is generated. The results obtained by using this FCM and the conventional iterative reasoning algorithm are compared with the results of the conventional FCM reported in the literature and used for the same purpose.
- The former proposed FCM based on zT2FSs is extended by proposing new non-iterative reasoning algorithm which uses the Type 2 fuzzy values of the weights rather than their defuzzified values. Then the validity of this new reasoning algorithm is demonstrated in the context of Module performance. A new FCM to determine Module performance is generated and results obtained are compared with other approaches used for the same purpose.

It is worth to note that the new non –iterative reasoning algorithm for the FCM is proposed for the Map with links' weights represented using zT2FSs that are generated using IAA.

1.5 Contribution

The novelties of this research are related to the FCM and its reasoning algorithm. They are presented and supported by results of two newly generated cases studies as detailed and discussed in **Chapter 5** and **Chapter 6** of this thesis**.** The contribution to the knowledge of the FCM can be outlined as follows:

- Introducing new extensions to the conventional FCM. The proposed FCMs were demonstrated in the two cases studies and the results demonstrated that the two proposed extensions of FCM outperformed the conventional FCM.
- A new non- iterative reasoning algorithm for the FCM is proposed. Its effectiveness in propagating the uncertainty during the reasoning and hence enhancing the capability of the FCM for reasoning similar to human are demonstrated.

1.6 Structure of the Thesis

Toward achieving the objectives of this research, the rest of this thesis is outlined as follows:

Chapter 2 includes a background about the FSs. It reviews the definition of the FSs, their membership functions and their main operations, such as union, intersection and complement. Furthermore, this chapter presents the types of FSs and their roles in capturing the uncertainties of the modelled domain.

Chapter 3 shows how FSs can play an essential role in Computing with Words. The chapter presents the existing fuzzy approaches for capturing the human subjective data. The differences between these approaches are discussed. The chapter emphasises the effective role of zT2FSs that are generated by IAA in capturing the uncertainties. Furthermore, a numerical example on how to use the IAA to generate z slices from interval valued data is included in the chapter.

Chapter 4 this chapter introduces the FCM and its structure. It discusses the advantages and drawbacks of the conventional FCM. The chapter presents some of the previous extensions of the conventional FCM which were introduced to overcome its limitations. Indeed, the learning algorithms which used to train the FCM are briefly presented in this chapter. Furthermore, the chapter presents some applications in which the FCM was used.

Chapter 5 presents a new extension of the FCM proposed that is called FCM based on zT2FSs (zT2FCM). This chapter shows how IAA can be used to generate zT2FSs to represent the weights of the links. A case study on Autism diagnosis is created and used to validate the approach. The chapter provides a step by step description of the proposed method.

Chapter 6 extends the proposed zT2FCM in Chapter 5 by introducing a new non- iterative reasoning algorithm where the zT2FSs values of the weights are used without defuzzification. To validate this reasoning algorithm, a novel case study on evaluating Module Performance using the FCM with zT2FSs' weights with new reasoning algorithm is generated, named Module Performance Fuzzy Cognitive Map. Indeed, this chapter includes the comparison between Module Performance FCM and other approaches used for the same purpose. For further validation, the sensitivity analysis of the proposed approach is carried out and obtained results are discussed in this chapter.

Chapter 7 summarises the contribution and results of this thesis. Indeed, it presents the future work and direction of this research.

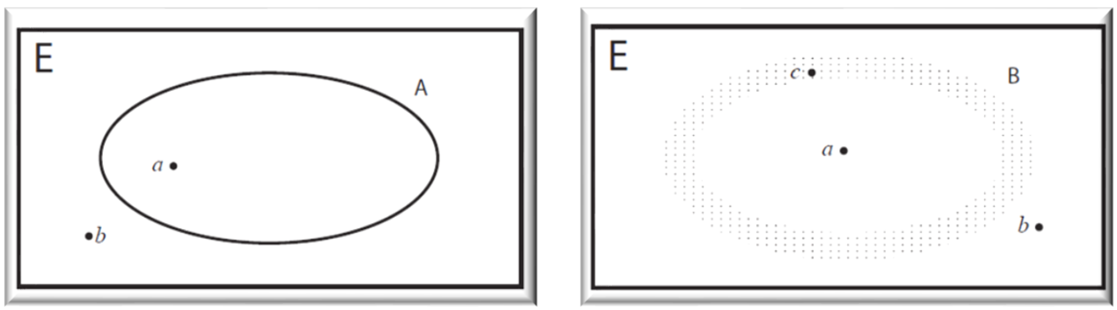
Chapter 2 Background on Fuzzy Sets

2.1 Introduction

The concept of Fuzzy sets (FS) emerged in Zadeh's seminal paper in 1965 (Zadeh 1965). Since then, FSs and Fuzzy set theory have become one of the essential areas in the domain of information processing (Pedrycz, Witold. and Gomide 1998). The Fuzzy set theory emerged after the development of Probability theory which was the essential tool for deciphering and capturing uncertainty before 1965 (Ross, T. J., & Ross 2016). It is very important to distinguish between the Fuzzy set theory and Probability theory. Probability theory is effective to manage and handle only random uncertainties, whereas Fuzzy set theory is used to handle linguistics uncertainties and sometimes the Fuzzy set theory is applicable to handle random uncertainties when the fuzzy system may use noisy measurements (Mendel 2007a). Therefore, it is worth noting here that the fuzzy set theory is not a replacement for probability theory.

The recognition of the significance of FSs in modelling uncertainty and their robust role in reducing the complexity of the model and hence increasing its credibility, pushed scientists to explore and widely use FSs. FSs have been successfully used in different applications, for example, Medicine (John and Coupland 2012), Controls (Tanaka, K. and Wang, H.O., 2001), Pattern Recognition (Papakostas et al. 2008), and Engineering (Stylios and Groumpos 2000).

FSs are the main objects of fuzzy set theory. The FS differs from a crisp set (also known as a classical set) by having membership degrees, where an element of FS is subject to a degree of belonging known as a grade of membership. Diagrams in Figure 2.1 show the difference between the precise and imprecise boundary of a classical set and a fuzzy set in universal set E (universe of discourse). For example in Figure 2.1 (a), considering an element 'a' of a crisp set A, it is clear that $a \in A$ and $b \notin A$ as'a' is inside the boundary of A and 'b' is outside the boundary. In Figure 2.1. (b) of a fuzzy set B, it is clear that $a \in B$ and $b \notin B$, but there is a doubt about belonging of 'c' to the set B, as it is partially belonging to the set B; therefore, 'c' has a partial membership to the fuzzy set B.



(a) **(b)**

Figure 2.1 Difference between the boundary of crisp set and boundary of fuzzy set

The crisp set is characterised by a function that assigns a value of either 0 or 1 to an element in the universal set, while the FS is characterised by a membership function which assigns a grade (value) of between 0 and 1 inclusively to an element in the universal set, where 0 represents no membership of the element and 1 represents the full membership of the element. Therefore, every crisp set is a fuzzy set but the reverse does not hold.

The literature reveals that FSs can be effectively used to represent and handle uncertainties and as per Mendel (Mendel 2001), handling uncertainties using fuzzy sets and fuzzy systems means the ability to model these uncertainties and reduce their effects.

The FS which was introduced in 1965 was called Type 1 Fuzzy set (T1FS) and it is capable of modelling uncertainties and imprecision (John and Coupland 2012). Later in 1975, Zadeh emphasized in his work (Zadeh 1975) that T1FSs are incapable of modelling the high level of uncertainty associated with linguistic variables. He, therefore, introduced the Type 2 Fuzzy sets (T2FSs) to cope with this deficiency.

Concepts of T1FSs and T2FSs that are related to this thesis and their different representations are presented in the following sections.

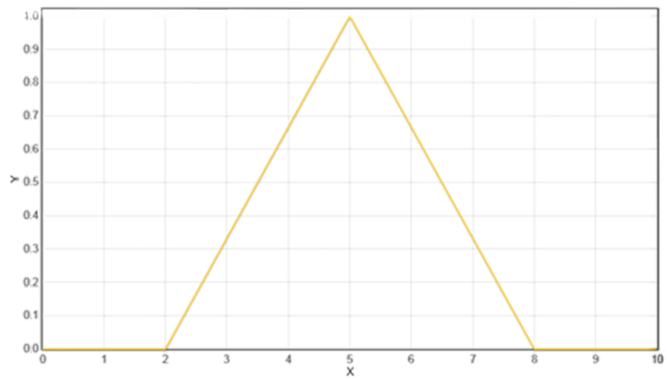
2.2 Type 1 Fuzzy Set

T1FS, which was introduced by Zadeh in 1965, is an ordinary fuzzy set. Given the universe of discourse E, a T1FS A is defined as a set of ordered pairs as follows:

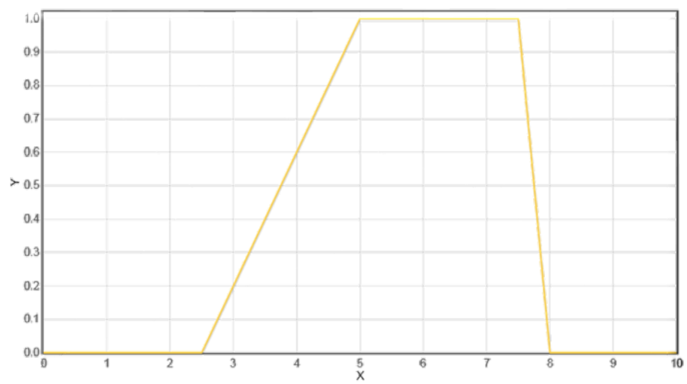
$$A = \left\{\left(x, u_A(x)\right) \mid x \in E\right\} \tag{2.1}$$

where $u_A: x \mapsto [0,1]$ is a membership function of a set A and $u_A(x)$ is the grade of the membership of an element x to the set A.

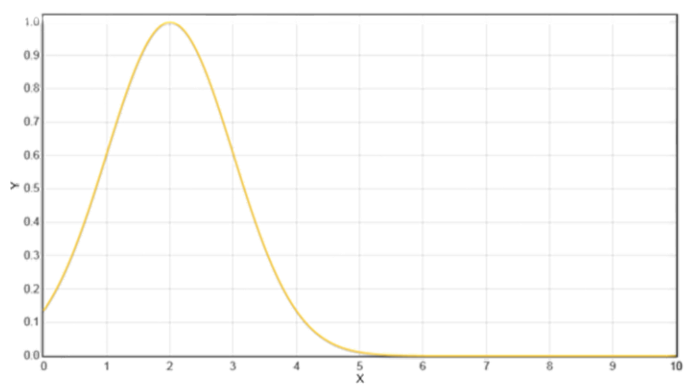
Various shapes of membership functions of T1FS have been proposed. The selection of an appropriate one relies on its adequacy of use in the required domain. The most famous shapes of T1FSs are determined by popular type-1 membership functions like triangular, trapezoidal and Gaussian as presented in Figure 2.2.



Triangular membership function



Trapezoidal membership function



Gaussian membership function

Figure 2.2 Type-1 membership functions

Each shape of these membership functions of a T1FSs can be defined by certain parameters, for example, centre and width are required parameters to define a Gaussian function (John and Coupland 2012). T1FSs and their logics have been used widely in different real-world applications for various purposes, for example, in Medicine (Navarro and Wagner 2019) to deal with the linguistic uncertainties that arise from the patients during describing the symptoms and from physicians when classifying these symptoms to determine the required therapy plan. Also, T1FSs have been used in pattern recognition (Papakostas et al. 2008) to classify the structure of the data which might be with vague boundaries.

Standard set operations such as union, intersection and complement were defined for T1FSs using their membership functions as follows:

Given two T1FS A, B with their membership functions u_A and u_B respectively in a universe of discourse X, the union $A \cup B$ is defined by the membership function $u_{A \cup B}$, where

$$u_{A \cup B} = max[u_A(x), u_B(x)] \; \forall \; x \in X \tag{2.2}$$

as depicts in Figure 2.3(a).

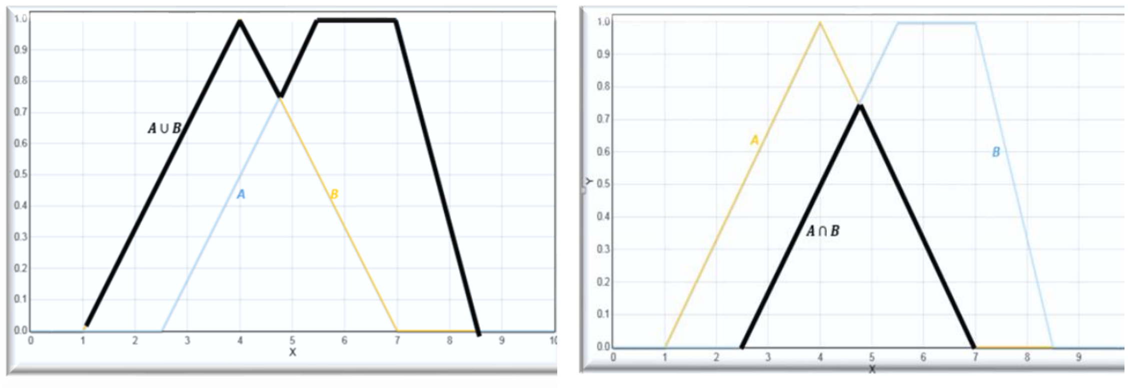
The intersection $A \cap B$ is defined by the membership function $u_{A \cap B}$, where

$$u_{A \cap B} = min[u_A(x), u_B(x)] \; \forall \; x \in X \tag{2.3}$$

as depicts in Figure 2.3(b).

The complement of A is defined by the membership function $u_{\bar{A}}$, where

$u_{\bar{A}} = 1 - u_{\bar{A}}$ (2.4)



(a) (b)

Figure 2.3 (a) The union of T1FSs (b) The intersection of T1FSs

For further information on FSs , their logics and their operations, the reader may refer to (Rossand Ross 2016) and (Klir and Yuan 1995).

Although T1FS was used successfully in various applications, there was criticism about its crisp membership function; it does not reflect any uncertainty in assigning the value to an element in the set and thus contradicts the word "fuzzy" (Mendel 2001). Indeed, this hinders the modelling ability of T1FS in the domain that involves a high level of uncertainty. This shortcoming of T1FS led to the extension of T1FSs through the introduction of a higher level of fuzzy sets where the grade of the membership is uncertain. The evolution of these higher fuzzy sets will be presented in Section 2.3.

2.3 Type 2 Fuzzy Set

In 1975, Zadeh incorporated uncertainty with the definition of the membership function of T1FS and introduced the concept of T2FS (Zadeh 1975) which is a generalization of T1FS.

Thus, if there is no uncertainty about the membership function, the T2FS is reduced to T1FS. Indeed, Zadeh proposed a generalized TnFS where, n=3, 4, 5…

Various investigations in the area of T2FSs and their logics show their ability to effectively deal with uncertainty in different real-world applications. Some of these investigations are introducing a new representation of T2FS, termed z slices and use this representation to transit from IT2FS logics and systems to T2FS logics and systems as presented in (Wagner and Hagras 2010). This transition allows to facilitate the application of T2FS in different real world applications, for example improving the combining and classification accuracy of support vector machines (Hassani et al. 2017) and measuring the agreement between the experts in decision making domain(Navarro and Wagner 2019).

A review of the literature in this area reveals the gradual transition from using T1FSs to T2FSs in the last two decades. The reason for this transition is that T1FSs are incapable of handling a high level of uncertainty because they are characterised by crisp membership functions. Conversely, T2FSs are characterised by fuzzy membership functions and thus they are capable of modelling a high level of uncertainty where determining the precise membership of the fuzzy set is difficult or even impossible.

Given the universe of discourse E, the T2FS Ã is defined in the three dimensional space as a set of ordered pairs as follows:

$$\tilde{A} = \left\{ \left(\left(x, u_{\tilde{A}}(x) \right), \mu_{\tilde{A}}\left(x, u_{\tilde{A}}(x) \right) \right) \mid \forall x \in E \ and \ \forall \ u_{\tilde{A}}(x) \in J_x, J_x \subseteq [0,1] \right\} \qquad (2.5)$$

where $0 \leq \mu_{\tilde{A}} \leq 1$ is a type-2 membership function of Ã.

At each x, $u_{\tilde{A}}$ is called a primary membership function and it is the domain of $\mu_{\tilde{A}}$ which is called a secondary membership function. It is noticeable from this definition that T2FS has three dimensions for the representation, compared to T1FS which has two dimensions.

Based on the literature, this T2FS is known as the General Type 2 Fuzzy set (GT2FS) to distinguish it from other representations of T2FS as will be presented later in this chapter.

It is clear from the former definition of T2FS that a degree of the belonging of an element (membership) to T2FS is represented by T1FS as there is uncertainty about the degree of membership, while the membership degree in T1FS is certain and is represented by a crisp value (singleton). Therefore, the T2FS could be viewed as T1FS where the membership of each element is a T1FS in the universe of discourse (John and Coupland 2012).

Although T2FSs provide a robust framework to handle uncertainties compared to T1FSs, its third dimension requires more effort for computations and that makes the representation and use of their systems difficult. Therefore, most studies undertaken 20 years ago only focussed on the theoretical concepts of T2FSs. After that, there was a noticeable transition in the study of both theoretical and applied types of research of T2FSs, especially after defining the concept of type reduction by Karnik and Mendel (Karnik, Mendel, and Liang 1999). Type reduction of FS is a process in which the higher order of FS is reduced to ordinary FS (T1FS) which then defuzzify to a single value which represent the uncertainty that captured in the set.

One of the challenges of T2FS development is to provide a framework that allows the comparison between T1FS and T2FS (John and Coupland 2012) in order to help users decide which sets to use. This comparison framework is important for users as T1FS is easier in terms of computation efforts compared to T2FSs. In the surge to cope with this challenge, (Lynch, Hagras, and Callaghan 2007), (Lynch, Hagras, and Callaghan 2005) and (Melgarejo R. and Peña-Reyes 2004), used a hardware implementation such as Field Programmable Gate Array to offer a type reduced set, but that required more computations too.

The third dimension is vital to T2FS as it provides an additional degree of freedom to represent uncertainties (Mendel and John 2002), but it complicates the process of determining the parameters of T2FSs as they are characterised by three-dimensional type 2 fuzzy membership functions. To reduce this difficulty, two main representations of T2FS were introduced, namely the vertical-slice representation and wavy-slice representation. The vertical-slice representation is used more for computational purposes, whereas the wavy-slice is more used for theoretical purposes (Mendel 2007a). The vertical slice representation relies on dividing the T2FS into vertical slices in such a way that their union represents the original T2FS itself. The wavy slice representation is known as Mendel-John Representation Theorem (Mendel and John 2002) and it relies on dividing the T2FS into simpler embedded T2FS; therefore, wavy slices are avoided for computational purposes. To minimize the computational efforts required to deal with T2FSs and hence employ them in fuzzy systems, several vertical representations were proposed, for example, Interval Type 2 Fuzzy set (IT2FS) (Mendel 2001), Geometric representation (Coupland and John 2007), Alpha-plane (Mendel and Liu 2008) and z Slices (Wagner and Hagras 2010).

Later, Mendel introduced the wavy slice to represent the third dimension of T2FS which is more suitable to be used in the theoretical representation (Mendel 2007a).

Note that the secondary membership function at each x in T2FS (2.5) is a vertical slice which is a two dimensional plane between $u_{\tilde{A}}$ and $\mu_{\tilde{A}}$ (Mendel and John 2002). The union of all the vertical slices represents the T2FS. This representation, introduced by (Mendel 2001), helped to simplify and reduce the complexity of T2FS computations.

As per (Mendel 2007a) and (Mendel, Liu, and Zhai 2009), IT2FS and Alpha-plane could be interpreted by both vertical slice and wavy slice representations.

The IT2FS and z slices are widespread representations of T2FS as the computations of their third dimension are less complex comparing to other representations of T2FS (Wagner and Hagras 2008), and, therefore, and for the purpose of this thesis, these two types of T2FSs will be presented only.

2.4 Interval Type 2 Fuzzy Set

IT2FS, introduced by Mendel (Mendel 2001), provides the membership grades within an interval; therefore, IT2FS has a wider scope for capturing uncertainty compared to T1FS. The IT2FS is generated by blurring a T1FS (up and down/ right and left) which produces two type-1 membership functions: lower membership function (LMF) and upper membership function (UMF). The area bounded by LMF and UMF captured all the uncertainty of IT2FS and is known as the footprint of uncertainty (FOU) and the IT2FS is defined by its FOU.

Blurring directions result in different types of secondary membership functions. Indeed, the blurring notion gives the membership functions of the IT2FS a view that they are type-1 membership functions whereas their membership grades are type 1 fuzzy sets. The third dimension of IT2FS set is fixed as 1, hence IT2FS is a GT2FS where the third dimension is fixed as 1. Therefore, when $\mu_{\tilde{A}}(x, u_{\tilde{A}}(x)) = 1$ in (2.5), the T2FS Ã becomes an IT2FS.

Given the universe of discourse E, the IT2FS $\tilde{A}$ is defined in the three dimensional space as a set of ordered pairs as follows:

$$\left\{\tilde{A} = \left((x, u_{\tilde{A}}(x)), 1\right) \mid \forall x \in E \; and \; \forall \, u_{\tilde{A}}(x) \in J_x, J_x \subseteq [0,1]\right\} \quad (2.6)$$

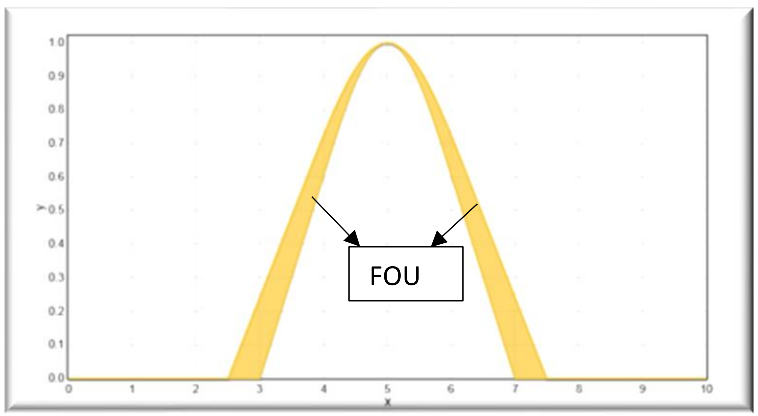


Figure 2.4 IT2FS and its FOU

The IT2FS is known as first- order uncertainty fuzzy set and GT2FS is known as second-order uncertainty fuzzy set. Figure 2.4 presents an IT2FS in two dimensions.

IT2FSs are used in different applications, and their success in modelling systems with high uncertainty is noticeable. Some of these applications are presented in (Mendel 2007a), (Liu and Mendel 2008) and (Coupland, Mendel, and Wu 2010).

In spite of the fact that IT2FSs offer a wide scope for capturing uncertainty and accurately representing them, several studies have suggested the necessity of moving to an alternative representation of T2FS where the third dimension is used to capture more uncertainties. The fuzzy set then becomes more capable of an accurate representation of uncertainties required for knowledge representation, approximate reasoning and decision making. Hence efforts were made to enhance the representation of GT2FSs and limit their complexity. Coupland and John (Coupland and John 2007), represented the GT2FS using the geometric representation like polygons and polylines which made the execution of computational operations faster. Later, the T2FSs were represented using alpha planes as introduced by(Mendel and Liu 2008), which overcame the complicated computation that is associated with set operations and type reduction for the T2FS.

In order to further reduce the immense computations required for GT2FS and effectively use the third dimension in capturing more uncertainty, z Slices representation of GT2FSs (zT2FS) and their fuzzy logics were introduced by (Wagner and Hagras 2008) .The z slices representation allows the smooth transition from IT2FS to GT2FS and helps to utilise the third dimension for capturing higher uncertainty, but it keeps the level of complexity at the level of interval T2FS. The following sections include more details about zT2FSs and their applications.

2.5 The z Slices Type 2 Fuzzy Set

A zT2FS is generated by slicing the third dimension z of GT2FS into a series of vertical slices Z_i each at level z_i where $0 < z_i < 1$. These slices called z slices and each slice Z_i is equivalent to an IT2FS but its third dimension is equal to z_i not 1. Hence, (2.5) is modified to define z slices Z_i as follows:

$$Z_i = \left\{ \left(\left(x, u_i(x)\right), z_i \right) \mid \forall x \in E \; and \; \forall \, u_i(x) \in J_x, J_x \subseteq [0,1] \right\} \tag{2.7}$$

In the discrete universe of discourse, there is a finite number of z slices and hence,

$$zT2FS = \sum_{0\le i\le I} Z_i \qquad (2.8)$$

and in the continuous universe of discourse, there is an infinite number of z slices and hence

$$zT2FS = \int_{0\le i\le I} Z_i \quad I \to \infty \qquad (2.9)$$

Note that $\sum$ and $\int$ represent the union of the slices.

For example, as shown in part (a) of Figure 2.5, a point x' has membership function u_0 of values within an interval $[l_0, r_0]$ in the second dimension y. Then the interval $[l_0, r_0]$ is sliced in third dimension to 4 vertical slices Z_i each at level z_i where $0 < z_i < 1$ as presented in Figure 2.5(b). Hence $0 < \mu_0(x')< 1$ in the third dimension is a set of

4 z slices.

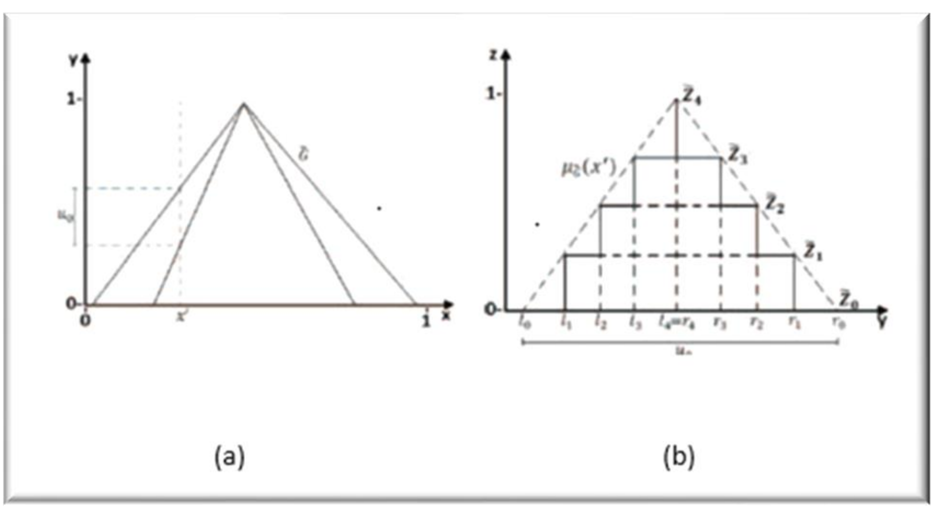


Figure 2.5 (a) Front view of zT2FS (b) Third dimension at x of zT2FS

Figure 2.6 presents a 3 dimension view for a zT2FS of three slices.

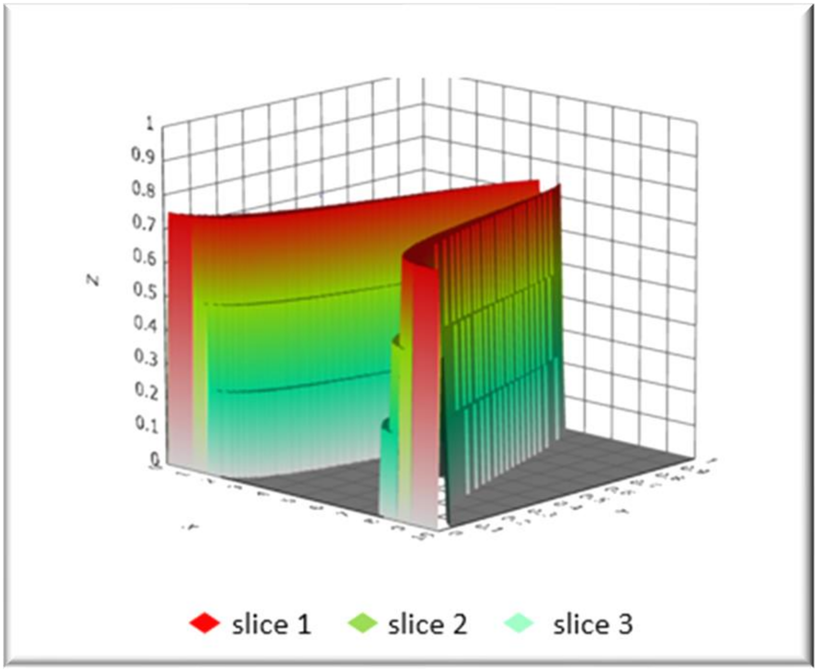


Figure 2.6 3D view for zT2FS of 3 slices

The z slices model GT2FS to a degree of accuracy which only depends on the number of z slices i.e. as the number of z slices is increased, the accuracy of inference is increased and this effectively facilitates the use of GT2FS in the application of fuzzy logic systems as proved in (Wagner and Hagras 2008). Indeed, the concept of Multilevel agreement is developed for z slices (Wagner and Hagras 2011) and thereafter z slices play a robust role to model Multilevel Agreement in different real applications. The reason for this success is that z slices offer an additional degree of freedom to model the uncertainties associated with the opinions of decision makers in the third dimension. The work of (Wagner and Hagras 2010) provides a complete representative framework of zT2FSs and their fuzzy logic system.

the GT2FS, in particular zT2FSs were used in different application, for example,(Bilgin et al. 2016) (Hassani et al. 2017) (Adams and Bank 2020) where T1FSs was not possible to use in these applications which have high level of uncertainty that well captured by zT2FSs.

2.5.1 Defuzzification

In applications of fuzzy systems, the output needs to be crisp (Mendel 2001) and this is accomplished by defuzzification. Defuzzification is a method in which the fuzzy set is represented by a crisp number. There are different types of defuzzifications, but the most used are, centroid (centre of gravity), weighted average, and mean of maxima. for a purpose of this thesis, the author discusses the centroid method, particularly for zT2FS for further details on other methods, the reader may refer to (Roychowdhury and Pedrycz 2001).

Defuzzification of T2FS relies on reducing it to T1FS which makes the computation of its centroid easier. The process of defuzzifying the T1FS and IT2FS is easier than in T2FS, as the latter involves several embedded T1FSs. Thus computing the centroid of T2FS could be intensive if there are enormous numbers embedded T1FS (Mendel 2001). To reduce the complexity of computing the centroid of T2FSs, some approaches, such as the random sampling (Karnik and Mendel 2001) and vertical slice-centroid-type reduction (Lucas, Centeno, and Delgado 2007) have been proposed particularly for IT2FSs.

The zT2FSs are defuzzified using a centroid which represents the linear combination of centroids of all z slices, as proved in Theorem 1 in (Wagner and Hagras 2010). Thus, the centroid of z T2FS which has N slices Z_i , where each Z_i is at level z_i, $1 \leq i \leq N$ is calculated as follows:

$$C_{\mathrm{zT2FS}} = \sum_{i=1}^{N} z_i / C_{Z_i} \tag{2.10}$$

where C_{Z_i} is the centroid of Z_i slice.

2.6 Conclusion

Using the three-dimensional view the following fuzzy sets have been defined:

If the membership grade of each element x in the universe of discourse is a singleton (a crisp value) in the second dimension y and there is no uncertainty about its value, i.e. the third dimension value is fixed to be $z = 1$, then the results set is T1FS.

If the membership grade of each element x in the universe of discourse is a crisp interval in the second dimension y and there is no uncertainty about its values, i.e. the third dimension values corresponding to the interval values are fixed to be $z = 1$, then the resultant set is IT2FS.

If the membership grade of each element x in the universe of discourse is a T1FS of support belonging to $[0, 1]$, i.e. the values of z belong to $[0, 1]$, then the resultant set is GT2FS.

Figure 2.7 shows by using the three dimensions view, the diffrence between the value of input p in a universe of discourse using T1FS, IT2FS and GT2FS.

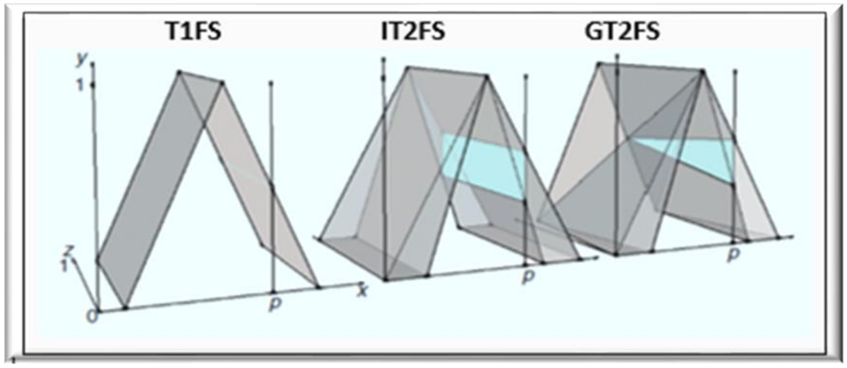


Figure 2.7 Value of input p using three types of Fuzzy Sets

The type of the data and the level of imprecision and uncertainty within them inform the decision of whether to use T1FS or T2FS to model a specific system. As the level of uncertainties and imprecision increase, the T2FSs have more leverage compared to T1FS to offer a suitable paradigm to represent the system (Robert and Coupland 2008).

Several representations for GT2FSs have been proposed to reduce the complexity of using it in the applications, for example, IT2FS and z T2FS are more used representations of GT2FS.

The literature review (McCulloch, Ellerby and Wagner 2019) reveals that IT2FS and the zT2FS are widely used via specific approaches, for example, in Interval Approach and Interval Agreement Approach to model the agreement between the decision makers about the word/ concept meaning and/or an aspect of the designed system, as presented in

Chapter 3.

Chapter 3 Fuzzy sets based approaches to capturing human subjective data

3.1 Introduction

Fuzzy logic systems are based on rules governed by words with uncertain meaning. Based on this precept, Zadeh (Zadeh 1975) introduced the concept of ‘Computing with Words’ (CWW), as a process where words were used for computing rather than numbers. He emphasized that fuzzy sets and fuzzy logics were powerful methodologies for CWW. The concept of CWW and Mendel’s notion that “words mean different things to different people” (Mendel 1999) demonstrate the potential of a fuzzy set that is capable of capturing the semantic uncertainties of a word and hence strengthening the process of decision making where words or perspectives are used rather than numbers. There are notable works that have used fuzzy sets to model these words and /or concepts for reasoning as a part of the CWW applications. Some of these applications used T1FSs (Herrera et al. 2009), (Herrera, Herrera-Viedma, and Martínez 2000) and (Lawry 2001) whilst some CWW applications used T2FSs (Mendel and Wu 2010) (Miller et al. 2012). The literature reveals that proficiency of T2FS is higher than of T1FS for accurately capturing the uncertainties of meaning and opinions associated with the word. The reason is that T2FSs are characterized by an additional degree of freedom which allows them to handle more uncertainty and hence provide more accuracy in the representation. In this line, the work of (Mendel 2007b) presented two approaches to collect data about words or concepts from a group. They are: the person-membership function approach and interval endpoints approach. Both approaches represented the semantic uncertainties using IT2FSs in combination with statistics in what has been called fuzzistics.

There are various techniques which have been developed for accurately capturing people's opinions about words and concepts using surveys tools to create a fuzzy-based model. There are two types of uncertainties that can be captured using the survey tool. They are intra-uncertainty (uncertainty a person has during iterative surveys) and inter-uncertainty (uncertainty a group of people have during iterative surveys). The works of (Liu and Mendel 2008), (Coupland, Mendel, and Wu 2010), (Miller et al. 2012) and (Wagner et al. 2015), proposed approaches to create a fuzzy model from interval-valued survey data where each interval described a specific word or a participant's beliefs. These approaches are based on aggregating the intervals that capture uncertainties of the participants' responses to generate fuzzy sets that represent the agreement/ consensus of their responses. The widely used techniques in the surge of creating fuzzy agreement models from surveys are Interval Approach (IA) and its extension (Liu and Mendel 2008). Enhanced Interval Approach (EIA) (Coupland, Mendel and Wu 2010) and Interval Agreement Approach (IAA) (Wagner et al. 2015). Each of these approaches takes different method to generating FSs and making different assumption about the nature of the data. For example, IA, EIA and their extensions create IT2FS from interval-valued survey data, where IAA uses interval values data obtained from a survey to construct zT2FS.

More about IA, EIA and IAA will be included in the following sections.

3.2 Interval Approach and its Extensions

The IA proposed in (Liu and Mendel 2008) is an approach towards modelling uncertainties in survey data. It maps each response's intervals to T1FS and then represents the union of all T1FSs as an IT2FS. The IA comprises two parts: data part and fuzzy part. In the data part, there are two main steps where the collected intervals of responses are reprocessed and the statistics of the remaining interval data (after the reprocessing) are computed. The step of reprocessing of the data includes the following four stages: (1) removing the outliers, (2) removing bad data (data that is not relative to the analysis), (3) removing the intervals which do not overlap with other intervals (known as non- reasonable intervals) and (4) removing intervals that do not fit the tolerance threshold functions. The step of statistics in data part includes assigning a probability distribution to each of the remaining intervals after the step of reprocessing. The fuzzy part is executed through nine stages (Liu and Mendel 2008) as follows: (1) the intervals are mapped to appropriate T1FSs (usually with triangle, left or right shoulder membership function) using the mean and the standard deviation of the probability distribution of the interval created in the data part. (2) Then measures of uncertainty of T1FSs are established and they are the mean and standard deviation of the corresponding probability distributions. (3) Calculate the measures of uncertainty (mean and standard deviation) of created T1FSs. (4) Then the mean and standard deviation of T1FS is equated to each interval. (5) Establishing the nature of FOU, where each interval is classify either to triangular, left or right shoulder FOU. (6) Compute the embedded T1FS. (7) Remove the T1FSS that outside the desired range. 8) Aggregate T1FSs to create to IT2FS. (9) Compute FOU which capture the uncertainty of IT2FS (Mendel 2007a) by determining the LMF and UMF of FOU based on representation theorem for an IT2FS as presented in Chapter 2. Therefore, the sets generated by IA model the inter-uncertainty and intra -uncertainty of surveys' respondents in FOU.

Later, the IA was enhanced by the EIA method (Coupland, Mendel, and Wu 2010), where the statistical test in data part is stricter compared to IA, and by improving the process of computing the apex of the LMF in fuzzy part. Both methods, IA and EIA, capture both intra and inter uncertainties in the produced mathematical model in FOU of IT2FS, but in EIA, the resulting IT2FSs are less wide than those which result using IA and the height of their membership function is higher compared to the IT2FS generated by IA. Thus EIA overcomes the limitation of IA which produces too imprecise fuzzy sets to be of use in inference.

It is worth noting that the data part of both IA and EIA includes data reprocessing, where outliers are removed using a Box and Whisker Test (Walpole et al. 1993). However, this may lead to loss of some important information as it is not always the case that outliers are bad data or do not include useful information. The reason for reprocessing of survey data in IA and EIA is because both methods attempt to capture the meaning of words by the survey and thus there are some responses which are not useful to consider. Detailed descriptions of the IA and EIA are presented in (Liu and Mendel 2008) and (Coupland, Mendel, and Wu 2010) respectively.

The work of (Hao and Mendel 2016) proposed the HM Approach (HMA) which shares with EIA the stages of pre-processing of the data but it enhanced EIA by assigning a high membership value to the intervals with high agreement among the respondents of the survey to be more useful and provide more accurate representation of the data.

3.3 Interval Agreement Approach

The IAA is proposed in (Wagner et al. 2015) as a tool to create an agreement model based on T1FSs or zT2FSs from interval-valued survey data. This approach produces a data-driven model that captures both intra and inter uncertainties associated with the modelled domain. The literature (Miller et al. 2012, Wagner et al. 2015) demonstrates that IAA outperforms IA and EIA because IAA creates the fuzzy sets solely from the data without reprocessing or removing the outliers as in IA and EIA. Therefore, the generating agreement model includes all the information obtained from the data. Further, the IAA distinguishes between intra-uncertainty and inter-uncertainty as each uncertainty is represented on a different axis, compared to the IA and EIA methods where all the uncertainties are captured in FOU. This distinction fortifies the quality of the model generated from survey data.

Former characterisations make IAA an ideal and useful approach in the domain of reasoning and decision making. The work in (Wagner et al. 2015) extended IAA's capability from modelling the uncertainty of crisp intervals as in (Miller et al. 2012) to model intervals with uncertain endpoints. The intervals are obtained as survey responses from a single source as well as multiple sources without the need for data pre-processing and/or outliers' removal. The IAA method reduces any need of assumption during the creation of the model as it generates the desired fuzzy sets from the available data without a requirement to choose a specific fuzzy set type with, for example, triangular or Gaussian membership function. Consequently, the resultant fuzzy sets will capture more uncertainties and hence more potential information (Klir and Folger 1988).

The IAA includes two phases, in which the result of each phase captures a type of model uncertainty including intra and inter-uncertainty. These phases are:

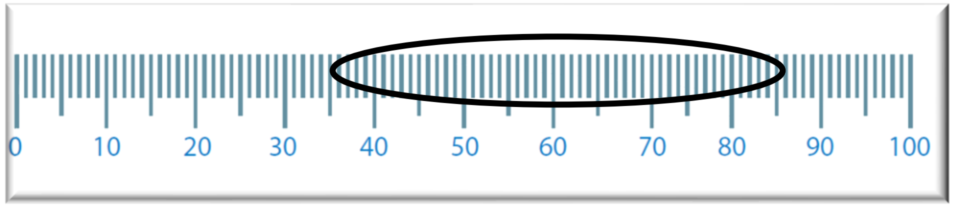
Phase 1. Creating T1FSs in which each of them includes intra- uncertainty of each of the survey respondents.

Phase 2. Aggregate former T1FSs to generate the z slices that capture inter- uncertainty of all the survey respondents.

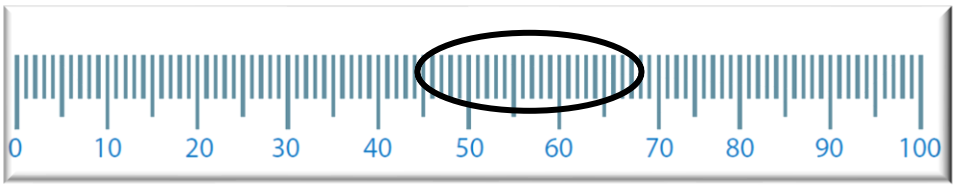
Thus the generated zT2FS agreement model contains both intra and inter respondent uncertainty.

Each type of uncertainty are modelled on two separate axes and thus, the IAA allows for an additional degree of freedom for each uncertainty as there are three dimensions for representing and handling the uncertainty.

The survey used to collect the required data to generate the zT2FS agreement model using IAA is designed so that each question's response is collected by drawing an ellipse on a Likert scale (Likert 1932) ranging from 0 to 100 or the respondent may provide the answer as an interval (e.g. $[a, b]$, where $a \geq 0, b \leq 1$) . The respondents can thus express their uncertainties about the answers to each question as an interval rather than a crisp value. The width of the interval reflects the extent to which the respondents are certain about their responses. For example, wide intervals show that respondents are less certain (more uncertain) about their responses as presented in Figure 3.1 (a), where narrower intervals reflect that respondents are more certain (less uncertain) about their responses as presented in Figure 3.1 (b).



(a) More uncertain response



(b) Les uncertain response

Figure 3.1 Uncertainty represented by intervals

Based on the literature, (Wagner, Miller and Garibaldi 2013a) it can be assumed that multiple iterations of the survey on the same topic offer more accurate information about the surveyed topic as the surveyed experts may have better information or their level of understanding of the concepts increases over time. Thus for a purpose of IAA and to capture more intra-uncertainty of experts, there are multiple iterations of the designed survey.

For N respondents of a survey (experts) who are surveyed S times, the IAA's phases are executed as follows. In the first phase, it is assumed that each expert answers using S intervals that are aggregated to generate a T1FS to capture an expert intra-uncertainty. The generated T1FS is modelled in a 2 dimensional space (x and y axis) with the membership degree $y_i = \frac{i}{S}$, where $i \in \{1,2,\dots,S\}$ as follows: union of all the intervals has a membership degree $y_1 = \frac{1}{S}$, union of all 2-tuple intersections of intervals has an associated membership degree $y_2 = \frac{2}{S}$, union of all 3-tuple intersections of intervals has an associated membership degree $y_3 = \frac{3}{S}$, and so on for membership degrees $y_4,\ldots, y_S$. Note that $y_S = 1$. It is worth noting here that the primary membership degree y_i reflects the number of overlapped intervals of an expert's responses. Therefore, the T1FS that models intra-uncertainty of the agreement in S intervals responses from a particular respondent (for example, the respondent A, who provides A_i intervals and $i \in \{1,2,\ldots,S\}$) is characterised by a primary membership function which calculated as follows:

$$u(A) = y_1/\bigcup_{i_1=1}^{S} A_{i_1} + y_2/\bigcup_{i_1=1}^{S-1}\bigcup_{i_2=1+i_1}^{S}\left(A_{i_1}\cap A_{i_2}\right) + y_3 /\bigcup_{i_1=1}^{S-2}\bigcup_{i_2=1+i_1}^{S-1}\bigcup_{i_3=1+i_1}^{S}\left(A_{i_1}\cap A_{i_2}\cap A_{i_3}\right) + \cdots + y_n/\bigcup_{i_1}^{1}\cdots\bigcup_{i_S}^{S}\left(A_{i_1}\cap\cdots\cap A_{i_S}\right) \tag{3.1}$$

where ∪and ∩ denote the union and intersection of the intervals respectively and + denotes the union operation that is used to find the standard union between the created T1FSs.

At the end of Phase 1, there are N T1FSs generated. In Phase 2, N T1FSs generated in Phase 1 are aggregated to produce N z slices using a similar process as in Phase 1, where each slice Z_j has a membership degree of z_j on z axis, where $j \in \{1,2,\ldots,N\}$ and $z_j = \frac{j}{N}$ is the secondary membership degree that reflects the overlap of the above-mentioned N T1FSs , representing the level of agreement among the N experts as per the agreement principle (Wagner and Hagras 2011). Therefore the secondary membership function of generated zT2FS, B_j , from aggregating the N T1FSs, where $j \in \{1,2, \dots, N\}$ is calculated as follows:

$$\mu(B_j) = z_1 / \bigcup_{j_1=1}^{N} B_{j_1} + z_2 / \bigcup_{j_1=1}^{N-1} \bigcup_{j_2=1+j_1}^{N} (B_{j_1} \cap B_{j_2}) + z_3 / \bigcup_{j_1=1}^{N-2} \bigcup_{j_2=1+j_1}^{N-1} \bigcup_{j_3=2+j_1}^{N} (B_{j_1} \cap B_{j_2} \cap B_{j_3}) + \cdots + z_n / \bigcup_{j_1}^{1} \cdots \bigcup_{j_N}^{N} (B_{j_1} \cap \cdots \cap B_{j_N}) \tag{3.2}$$

Note that the number of slices is equivalent to the number of experts, capturing experts' inter-uncertainty in form of zT2FSs. It is clear from the above mentioned phases that the zT2FS agreement model, namely Z, which is generated using IAA, captures both intra and inter uncertainties of experts and combines all the slices. Therefore, as N experts surveyed to generate Z model using IAA, there are N slices (Z_j), where $j \in \{1,2, \dots, N\}$. Hence,

$$Z = \bigcup_{j=1}^{N} Z_j \tag{3.3}$$

3.3.1 Illustrative example of using IAA to generate zT2FS

For the purpose of this thesis, this section presents a numerical example on how IAA is used to produce zT2FSs agreement model from crisp interval valued survey data.

Assume that 3 participants A, B and C are surveyed twice for answering a survey designed as explained earlier in Section 3.3 and the extracted intervals from the survey are as shown in Table 3.1.

Table 3.1 Response intervals of the participats

Participants	**1st survey**	**2nd survey**
A	[0.50, 0.85]	[0.41,0.78]
B	[0.22, 0.88]	[0.15,0.89]
C	[0.61,0.71]	[0.29,0.90]

Hence, by using the construction process detailed in the earlier section, $N = 3, S = 2$, $y_1 = \frac{1}{2} = 0.5$ and $y_2 = \frac{2}{2} = 1$, the IAA's phases are executed as follows:

In Phase 1, three T1FSs are created where each captures the intra-uncertainty of each of N participants. Each of these T1FSs is characterized by its membership u. Therefore, T1FSs for the participants A, B and C are defined by the membership functions $u(A), u(B)$and $u(C)$ respectively; their membership degree values are $y_1 = 0.5$ and $y_2 = 1$.

The T1FSs of the participants A, B and C (denoted by $u(A), u(B)$ and (C)) are defined as follows:

$$u(A) = 0.5/([0.50, 0.85] \cup [0.41, 0.78]) + 1/([0.50, 0.85] \cap [0.41, 0.78])$$

$$u(A) = (0.5/[0.41, 0.85] + 1/[0.50, 0.78]),$$

$$u(B) = 0.5/([0.22, 0.88] \cup [0.15, 0.89]) + 1/([0.22, 0.88] \cap [0.15, 0.89])$$

$u(B) = (0.5/[0.15, 0.89] + 1/[0.22, 0.88])$ and

$$u(C) = 0.5/([0.61, 0.71] \cup [0.29, 0.90]) + 1/([0.61, 0.71] \cap [0.29, 0.90])$$

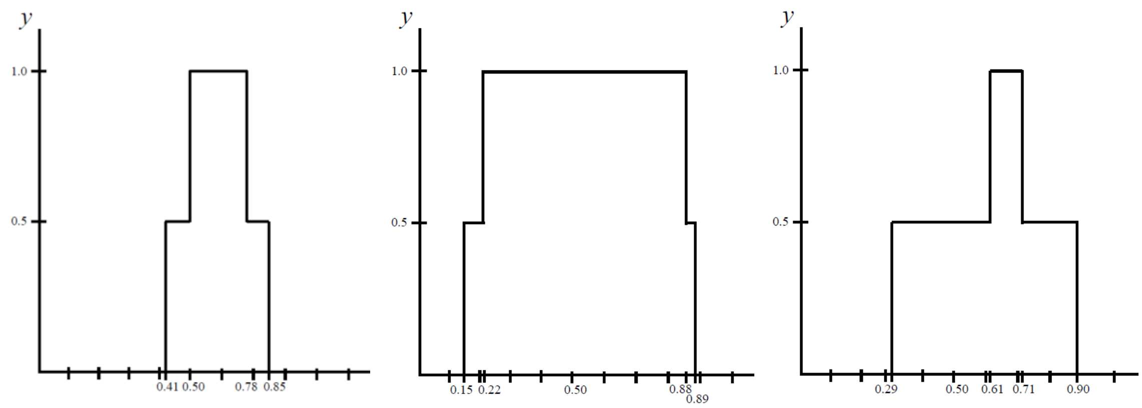
$$u(C) = (0.5/[0.29, 0.90] + 1/[0.61, 0.71]).$$

The details of intra-uncertainty captured by T1FS are summarised in Table 3.2.

Table 3.2 Intra uncertainty of each participant

Participants	**1st survey**	**2nd survey**	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$
A	[0.50, 0.85]	[0.41,0.78]	[0.41, 0.85]	[0.50,0.78]
B	[0.22, 0.88]	[0.15,0.89]	[0.15, 0.89]	[0.22,0.88]
C	[0.61,0.71]	[0.29,0.90]	[0.29,0.90]	[0.61,0.71]

Figure 3.2 depicts the generated T1FSs that capture the intra-uncertainty of the participant A, B and C.



A B C

Figure 3.2 T1FSs of the participants' responses

In Phase 2 of IAA method, T1FSs which captured intra- uncertainty of experts A, B and C that are produced in Phase 1 are aggregated to produce z slices of zT2FSs. As in this example, there are three participants; then, as explained earlier in Section 3.3 there are three slices Z_1, Z_2 and Z_3 at level $z_1 = \frac{1}{3}, z_2 = \frac{2}{3}$ and $z_3 = 1$, respectively. Three slices Z_1, Z_2 and Z_3 are created as follows:

$Z_1 = \frac{1}{3} / \left(y_1/([0.41\ ,0.85] \cup\ [0.15, 0.89]\ \cup [0.29,0.90]) + y2/([0.50,0.78] \cup [0.22,0.88] \cup [0.61,0.71])\right)$

$$\boldsymbol{Z_1 = \frac{1}{3} / \left(\frac{1}{2}/([0.15, 0.90]) + 1/([0.22, 0.88])\right)}$$

$Z_2 = \frac{2}{3} / \Big(y_1/\big(([0.41\ ,0.85] \cap\ [0.15,0.89])\ \cup ([0.41, 0.85] \cap [0.29, 0.90]) \cup ([0.15, 0.89] \cap [0.29, 0.90])\big)\Big) + y_2\ /\big(([0.50, 0.78] \cap\ [0.22, 0.88])\ \cup ([0.50\ ,0.78] \cap [0.61, 0.71]) \cup ([0.22, 0.88] \cap [0.61, 0.71])\big)$

$$\mathbf{Z_2} = \frac{\mathbf{2}}{\mathbf{3}}/\left(\frac{\mathbf{1}}{\mathbf{2}}/([\mathbf{0.29}, \mathbf{0.89}]) + \mathbf{1}/([\mathbf{0.50}, \mathbf{0.78}])\right)$$

$$Z_3 = 1/\big(y_1/([0.41, 0.85] \cap [0.15, 0.89] \cap [0.29, 0.90]) + \mathbf{y_2}/([0.50, 0.78] \cap [0.22, 0.88] \cap [0.61, 0.71])\big)$$

$$\boldsymbol{Z_3} = \mathbf{1}/\left(\frac{\mathbf{1}}{\mathbf{2}}/([\mathbf{0.41}, \mathbf{0.85}]) + \mathbf{1}/([\mathbf{0.61}, \mathbf{0.71}])\right)$$

The result of the previous calculations of z slices are summarised in Table 3.3.

Table 3.3 Inter uncertainty of all participants

Slice	$\boldsymbol{z_i}$	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$
Z_1	$1/3$	$[0.15, 0.90]$	$[0.22, 0.88]$
Z_2	$2/3$	$[0.29, 0.89]$	$[0.50, 0.78]$
Z_3	1	$[0.41, 0.85]$	$[0.61, 0.71]$

The level in the third dimension represents the level of agreement among the participants. The generated fuzzy set $\boldsymbol{Z}$ contains the three slices, and is defined as follows:

$$\boldsymbol{Z} = \boldsymbol{Z_1} \cup \boldsymbol{Z_2} \cup \boldsymbol{Z_3}$$

Figure 3.3(a) and Figure 3.3 (b) depict the 2D and 3D visualization of the aggregated slices.

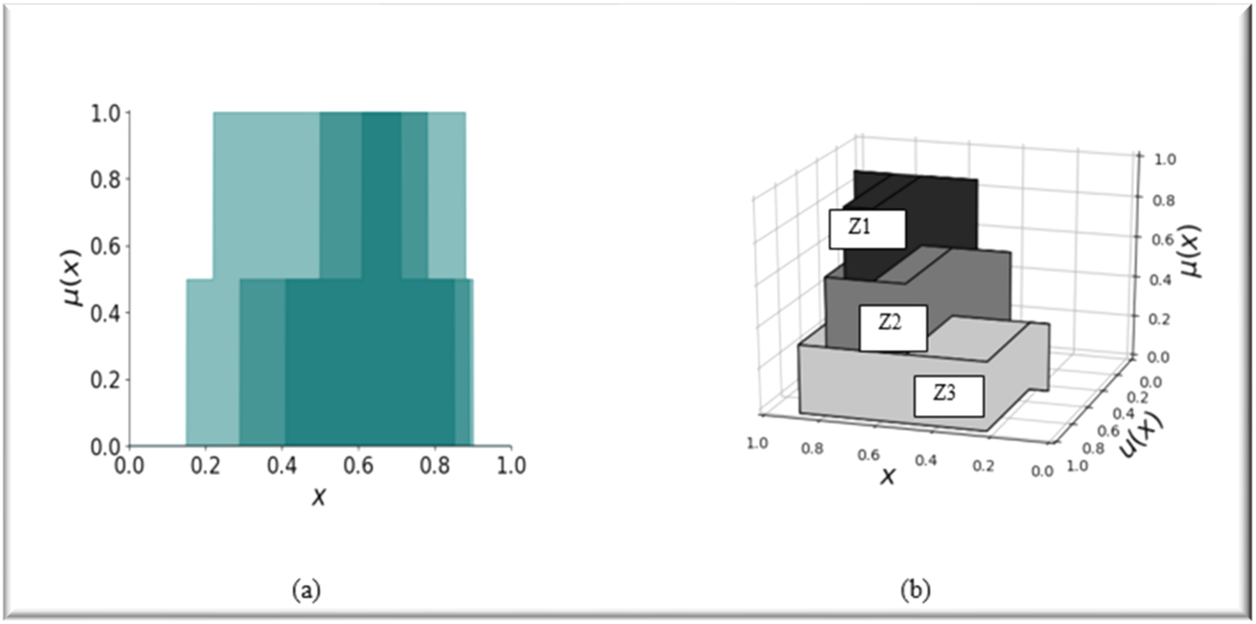


Figure 3.3 Aggregation of Z slices

The centroid of the set Z (C_Z), is obtained using (2.10) as follows:

$$C_Z = \frac{\left(\frac{1}{3}\right) C_{Z_1} + \left(\frac{\mathbf{2}}{\mathbf{3}}\right) C_{Z_2} + (\mathbf{1}) C_{Z_3}}{\left(\frac{1}{3} + \frac{2}{3} + 1\right)} = 0.6231$$

where:

$$C_{Z_1} = \left(\frac{(0.5)\left(\frac{0.15 + 0.9}{2}\right) + (1)\left(\frac{0.22 + 0.88}{2}\right)}{(0.5 + 1)} \right) = 0.5417,$$

$$C_{Z_2} = \left(\frac{(0.5)\left(\frac{0.29 + 0.89}{2}\right) + (1)\left(\frac{0.50 + 0.78}{2}\right)}{(0.5 + 1)} \right) = 0.6233$$

and

$$C_{Z_3} = \left(\frac{(0.5)\left(\frac{0.41 + 0.85}{2}\right) + (1)\left(\frac{0.61 + 0.71}{2}\right)}{(0.5 + 1)} \right) = 0.65$$

Note that C_{Z_1}, C_{Z_2} and C_{Z_3} are centroids of the slices $\boldsymbol{Z_1}, \boldsymbol{Z_2}$ and $\boldsymbol{Z_3}$ respectively.

3.4 Application of IAA

IAA has been used in different applications as a potential approach to accurately model uncertainties that exist in the information. one of the practical application of IAA presented in (Wagner, Miller, and Garibaldi 2013b), where the IAA used to create word and concept model with similarity measures to evaluate services of restaurants. Other applications of IAA are presented in (Navarro et al. 2016) (Navarro et al. 2017),where the IAA used to model the perceptions of linguistics from different groups (e.g. doctors and patients) in medical domain to show similarity and difference between understanding the word by the groups.

As presented in Section 3.3, IAA has a potential to accurately model the uncertainties of interval valued surveys in the generated zT2FS model. Therefore, it is clear that this generated fuzzy model captures more uncertainties around the accurate answers of survey's questions (McCulloch, Ellerby, and Wagner 2019). This push the author to utilise IAA for modelling the weights of link of Fuzzy Cognitive Map and its reasoning algorithm as present in Chapter 5 and Chapter 6 of this thesis. Thus the proposed FCM inherent the advantages of IAA.

3.5 Conclusion

The above-mentioned sections discussed approaches to using interval-valued data to create models capable of capturing more uncertainty of the collected data and then using it for reasoning and making decisions to arrive at correct decisions.

IA is used as a method to create a model capturing uncertainties about the meaning of words. However, generated IT2FSs are wide with a low height of membership values, and thus its capability to offer better and useful representation is hindered. To overcome the limitation of IA, the EIA was developed. The EIA enhanced IA by refining the reprocessing of the data and IT2FSs thus generated are less wide and have higher membership value compared to IA. However, the EIA may skip assigning maximum grades of membership to more overlapping intervals (where there is more agreement among survey respondents). To resolve this shortcoming, HMA. IA is proposed whose extensions reprocess the data that may lead to loss of useful information as it is not always the case that outliers include poor information.

The IAA generates zT2FSs based agreement model which captures the uncertainties of the complete data without any reprocessing. Thus, the representation of the generated model is closer to the original data. The IAA shows clearly the agreement, partial agreement or disagreement among the survey participants. Indeed, the IAA captures the standard deviation of the raw data. Based on these discussions, it can be concluded that IAA is ideal for generating an agreement/ consensus model capturing uncertainties of opinions and perceptions of survey participants. It can be used for reasoning, especially in survey contexts where there is a high level of uncertainty.

Chapter 4 Fuzzy Cognitive Map

4.1 Introduction

The concept of a Fuzzy Cognitive Map (FCM) emerged in 1986 by Kosko (Kosko 1986) as an extension of Cognitive Map (CM) introduced by Axelrod (Axelrod 1976). Kosko introduced fuzziness in the causal relations among the concepts of CM and then extended it to FCM. The motivation for that extension is the fact that the causal relations of a modelled system involve uncertainty, especially if they are based on classifications or causes which cannot always be represented by singletons (crisp values) unlike fuzzy sets which are more capable of such representation. Indeed, the propagation of uncertainty in the reasoning process of a CM could not be described accurately (Wang and Guo 2018). The FCM extends the CM in two aspects; first, by representing the interrelation by fuzzy values and second, by introducing the dynamicity of the system (Gray, Zanre, and Gray 2014). Thus, the FCM evolves with time.

The FCM is a soft computing paradigm that combines the properties of a CM and Fuzzy Logics that are required for knowledge representation and reasoning. Over the past three decades, the FCM has attracted a considerable research attention and it has been used in various fields (Papageorgiou et al. 2006). The reasons for that are dynamicity of the FCM, its learning capabilities and the perceivable meaning of its structure that can be understand by non-technical people

As emphasised by Codara, the FCM plays the role of one of the following four functions based on the purpose that the FCM is designed for; (1) explanatory: focus on modelling the system based on the experts' understanding and their directions for the appropriate decision, (2) reflective, focus on checking the adequacy of the modelled situation and do the early changes if required (3) prediction: focus on prediction decisions or actions for the future and (4) strategic: focus on providing more accurate explanation for a complex situation,.

The FCM has demonstrated its capability as an effective approach for decision making (Dabbagh and Yousefi 2019), (Iakovidis and Papageorgiou 2011), prediction (Goeman, Vandierendonck, and De Bosschere 2001), (Papageorgiou and Froelich 2012), classification (Papageorgiou and Froelich 2012), (Oikonomou and Papageorgiou 2013) and modelling (Papageorgiou and Salmeron 2014a), (Stylios and Groumpos 2004a). FCMs have been used in different applications, for instance, Agriculture (Markinos et al. 2007), (Jayashree et al. 2015), Engineering (Stylios and Groumpos 2000), Medicine (Amirkhani et al. 2017), Management and Business (Xirogiannis and Glykas 2004), Politics (Neocleous and Schizas 2012) and Education (Tan and Wu 2011), (Mendonca et al. 2015).

The flexibility of the FCM is represented in its ability to allow for removing and/or adding concepts as per further mapping required to enhance the resolution of the system modelled by the FCM (Groumpos 2010a).

Different FCMs developed to model a specific domain can be merged to generate a single FCM which can incorporate the advantages of the individual FCMs. Thus the FCM allows the synthesis of the knowledge and perceptions from different experts and/or stakeholders of the modelled domain and hence the reasoning and inference capabilities of the generated FCM is enhanced (Stach, Kurgan, and Pedrycz 2010).

Although Koskos' (1986) FCM (known as the conventional FCM) was used in various applications, for example: engineering (Stylios and Groumpos 2004b), medicine (Iakovidis and Papageorgiou 2011), education (Lq et al. 2008), its drawbacks were seen in some applications requiring a high level of uncertainty and /or where the relations of the modelled domain are nonlinear and/or non-monotonic. This motivated researchers to enhance the conventional FCM by introducing new extensions and learning algorithms to address these drawbacks.

This chapter aims to define the structure of FCM and its components. It discusses the drawbacks of the conventional FCM and presents some of FCM's extensions proposed to overcome these drawbacks. Additionally, this chapter presents some of the learning algorithms used to train the FCM. Furthermore, this chapter presents FCM's applications in some domains.

4.2 Fuzzy Cognitive Map structure

The structure of a FCM consists of nodes and weighted and directed links between them. The nodes represent the main aspects of the modelled system and are known as concepts of the FCM. The links represent the direction of causal interrelations among the nodes and the weights of the links are represented by fuzzy values that reflect the degree of the causality and/ or influence among the linked concepts. Therefore, the mathematical structure of the FCM comprises m concepts C_i, $i = 1,2, \dots, m$ which are linked by the weighted and directed edges e_{ij}, where each edge e_{ij} of a causal relation from a cause concept j to effects concept i, $i = 1, \dots, m$ and $j = 1, \dots, m-1$, has a causal weight $W_{i,j}$, as presented in Figure 4.1

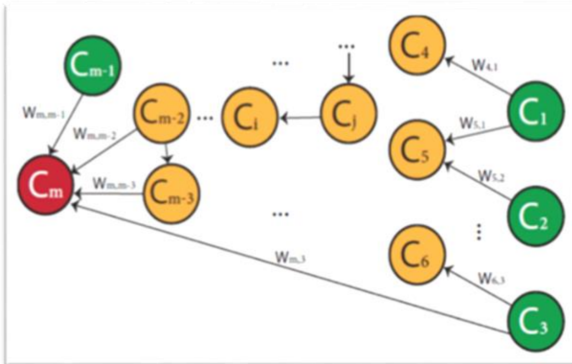


Figure 4.1 The structure of FCM

If the concept C_j does not affect the concept C_i then $W_{i,j} = 0$. As concept C_i does not affect itself, $W_{i,i} = 0$ for all $i = 1,2, \dots, m$. The values of all concepts fall in the interval [0,1] but the value of weights fall in the interval $[-1,1]$ as the weights represent the degree and the direction of influence between the concepts.

Therefore the relation between a concept C_j and the concept C_i in the FCM is described by one of the following:

1. Positive relation, if $W_{i,j} > 0$.
2. Negative relation, if $W_{i,j} < 0$.
3. No relation, if $W_{i,j} = 0$.

The weights $W_{i,j}$ can be arranged in a connection matrix W (known as weight matrix) as follows:

$$W = \begin{bmatrix} W_{1,1} & W_{1,2} & \dots & W_{1,(m-1)} \\ W_{2,1} & W_{2,2} & \dots & W_{2,(m-1)} \\ & & \dots & \\ W_{m,1} & W_{m,2} & \dots & W_{m,(m-1)} \end{bmatrix} \tag{4.1}$$

Cause and effect concepts are classified based on the impact among them as following:

- Input concepts: These are concepts that impact other concepts although they are not influenced by other concepts. The green nodes in Figure 4.1 represent input concepts of the FCM.
- Intermediate concepts: These are concepts that impact other concepts and are influenced by other concepts. The orange nodes in Figure 4.1 represent intermediate concepts of the FCM.
- Decision concepts: These are concepts which are only influenced by other concepts, but do not impact other concepts and represent the output of the modelled system. The red node in Figure 4.1 is the decision concept of the FCM. The FCM can has more than one decision concept and that depends on the modelled system.

During the process of designing the FCM for a specific problem, the determination of the required concepts and the causal relations among them is tackled by experts who sometimes combine their experiences about the aspects of the modelled system with the existing system's related historical data (Papageorgiou 2011a). It is worth noticing that different topologies of an FCM can be obtained from different groups of experts who are involved in creating the FCM for the same problem (Penn et al. 2013). The reason for this is that different experts might have different opinions based on their experiences and knowledge and thus identify their own criteria for a specific problem. This leads to variations in how the topology of the FCM is defined and, therefore, current research is looking at ways to consolidate multi- criteria group decision making processes in the generation of FCMs (Penn et al. 2013).

After the identification of the FCM concepts that are required to model the system, the experts describe each causal relation between the concepts by linguistic values. These are then aggregated to produce the overall weight which is defuzzified to a numeric value by the defuzzification method of Centre of Gravity.

After the construction of the FCM, the concepts of FCM interact with each other to produce the value of the output concept (the decision concept) using the following iterative reasoning algorithm:

$$C_i^{(k+1)} = f\left(C_i^{(k)} + \sum_{\substack{j=1 \\ i \neq j}}^{m-1} C_j^{(k)} * W_{i,j} \right) \tag{4.2}$$

where $C_i^{(k+1)}$ is the value of the concept C_i at time $k + 1$ and $C_i^{(k)}$ is the value of the concept C_i at time k of the iterative reasoning process; $W_{i,j}$ is the weight of the link from concept C_j to concept C_i; and f is the sigmoid threshold function which is required to squash the value of the concept between 0 and 1 and it is calculated as follows:

$$f(x) = \frac{1}{1 + e^{-mx}} \tag{4.3}$$

where $1 \leq m \leq 5$ is typically determined empirically by the FCM's designers and $f(x)$ has a value between 0 and1 (Miao and Liu 2000).

In the situation where the concept values could fall within the interval$[-1,1]$, the threshold function f could be calculated as follows:

$$f(x) = tanh\ x \tag{4.4}$$

The process of reasoning in (4.2) involves iteratively updating the concept values until $C_i^{(k+1)} - C_i^{(k)} < e$, where e is the residual value. This process causes the FCM to converge to a steady state and its output value is used to inference the decision concept.

It is worth mentioning that (4.2) is the most common function used for reasoning in the FCM and it extends the reasoning algorithm (4.5), which is proposed by Kosko, (1986) by including the value of the concept C_i at time k in evaluating its value at time $k + 1$ (Stylios and Groumpos 2004a).

$$C_i^{(k+1)} = f\left(\sum_{\substack{j=1 \\ i \neq j}}^{m-1} C_j^{(k)} * W_{i,j}\right) \tag{4.5}$$

The mechanism of establishing the FCM allows for evolving its structure through the modification of concepts and the links between them and thus the process of enhancing the model of the system based on the FCM becomes flexible (Miao et al. 2001) (Groumpos 2010b). Indeed, that model gains the dynamic property that is inherent in the FCM.

The FCMs has been used as Decision Support Systems (DSS) for different applications. A review of the literature reveals that the FCM architectures that have been as DSS are as follows:

1. FCMs based on learning algorithms.

These FCM types are trained by learning algorithms and thus their connection matrix is updated to enhance the DSS to generate a good decision. These architectures of FCMs have been used in medical applications; for example, non-linear hebbian learning algorithm used to train FCM used for the autism prediction (Kannappan, Tamilarasi, and Papageorgiou 2011) and the unsupervised active Hebbian learning algorithm used in classifying tumor grade of urinary bladder (Papageorgiou et al. 2006).

2. Competitive FCMs.

The Competitive FCM (CFCM) classifies the concepts of FCM into two types: factor concepts and decision concepts. The factor concepts stand for the main aspects of the DSS and their values are updated dynamically during the process of interaction of the CFCM (Stylios et al. 2008). Thus factor concepts of the CFCM are a combination of input and intermediate FCM concept types as mentioned earlier in this section. In the CFCM, decision concepts represent the output concepts. The structure of the CFCM reveals that it is capable of a differential decision where the decision-making process is complex and involves other considerations of interrelations among the concepts. It worth noting here that reasoning process of CFCM is the same as in the standard FCM. Some of the applications where CFCM is used are diagnosis of the knee injuries as presented in (Anninou, Groumpos, and Polychronopoulos 2013) and for dyslexia and language impairment as presented in (Stylios et al. 2007).

3. Distributed m-FCMs

A distributed m-FCM is used to model a complex system which consists of other complex sub-systems. These subsystems have some common aspects between them. Each subsystem represents a concept of m-FCM and the inter-relation between these subsystems stands for the links of the m-FCM. The m-FCM integrates the subsystems of the complex system and their relationships to infer the final decision for the main complex system. The distributed m-FCM is effective for differential diagnosis and it showed success in diagnosis in the domain of speech and language pathology as demonstrated in (Stylios et al. 2007).

4.3 Advantages and Drawbacks of Fuzzy Cognitive Maps

The FCM has been widely recognised and adopted due to its simplicity and flexibility in construction that allows it to enhance the model representation and to generate the output value by adding and /or removing concepts when required. Indeed, the low computation time of the FCM and its ability to combine different views from different experts about the modelled system during the construction process credits its rapid recognition. Furthermore, its dynamicity and capability of learning empower it to be an efficient and effective tool that can be used in diverse domains. Although the FCM has these advantages, including its strong mathematical structure, it is not capable of modelling real-world problems of causal relations that are neither linear nor monotonic (Papageorgiou and Salmeron 2013). In addition to this drawback, the conventional FCM is represented by singletons, thus its ability to represent and control knowledge with high randomness and uncertainties between the concepts is hindered. Moreover, conventional FCMs cannot handle more than one interrelationship between the concepts and it cannot model a grey domain environment (an environment with multi-meaning). To overcome these drawbacks and limitations and improve the performance of FCM, various extensions and learning algorithms have been proposed that target either its causal weights, design or reasoning process as presented in Section 4.4 and Section 4.5.

4.4 Fuzzy Cognitive Map Extensions

The extensions of FCM can be classified into three categories based on the types of drawbacks it aims to overcome (Papageorgiou and Salmeron 2014a). For example, to overcome the drawbacks of modelling the uncertainty and handling more relations between the concepts, FCM has been extended to Evidential FCM, Intuitionistic FCM, Fuzzy Grey Cognitive Map and Interval-valued FCM. To solve dynamicity issues, the FCM has been extended to Dynamic Cognitive Map and Dynamic Random FCM. To overcome drawbacks related to rule-based knowledge representation, Fuzzy rules-based FCM and Fuzzy Rules Incorporated with FCM have been proposed.

For a purpose of this thesis, following sections present some FCM extensions that attempt to enhance the representation of the weights of the causal links in the FCM for a better representation of the knowledge and increase its capability to capture more uncertainties.

4.4.1 Fuzzy rule based Fuzzy Cognitive Map

Fuzzy rule based FCM (RBFCM) was introduced by Carvalho (Carvalho and Tome 2000) to tackle the disability of conventional FCMs to model real-world problems involving causal relations that are neither linear nor monotonic. The RBFCM enhances the conventional FCM by introducing fuzzy causal relations and new operations, named fuzzy carry accumulation to cope with the accumulative nature of causality as the traditional fuzzy operations do not implement the causality as in the FCM. Later on, the effect of time was introduced to RBFCM (Carvalho and Tomé 2001) to improve the representation of FCM dynamicity. For that purpose a concept of B-Time is defined in the RBFCM to enhance the resolution of the system's simulations after incorporating the time with the iterative reasoning process of FCM.

The reasoning mechanism of RBFCM was enhanced by (Zdanowicz and Petrovic 2018) for a better impact accumulation. Although the RBFCM enhances the qualitative representation of the knowledge about causal relations by using fuzzy sets, its effectiveness in representing the linguistic nature of some phenomena is limited, as RBFCM relies on T1FS which has limited potential to deal with linguistic uncertainties.

4.4.2 Fuzzy Grey Cognitive Map

Fuzzy Grey Cognitive Map (FGCM) combines the advantages of both FCM and Grey System Theory (GST). It was proposed by Salmeron (Salmeron 2010) where its usefulness to forecast using small, incomplete and uncertain data sets was demonstrated. The grey number's accurate value is unknown but it falls within a known interval of values, thus in general it is known as a grey interval.

Concepts in the FGCM are represented by grey variables and the grey interrelations among them are represented by directed causal links. The weights of causal links are represented by grey intervals, which enhance the capability of the FGCM to represent the uncertain knowledge. The FGCM can represent relations among the concepts even if there is no causal relation and/ or unknown intensity, where the conventional FCM ability is hindered to measure the intensity of the existing causal relation. The FGCM shows success in the decision-making domain, for example it used as DSS for planning radiotherapy process

(Salmeron and Papageorgiou 2012) and (Papageorgiou and Salmeron 2012). Although the FGCM is adapted to handling uncertainty, it is incapable of modelling dynamic and nonlinear relations. Its capacity to model the domain of big data is also limited.

4.4.3 Intuitionistic Fuzzy Cognitive Map

Intuitionistic Fuzzy Cognitive Map (iFCM) is a fusion of Intuitionistic Fuzzy Sets (IFSs) and FCM. It was proposed by Iakovidis and Papageorgiou (2011) as a tool for modelling in domains that involve uncertainty, imprecision or missing information. IFS's membership function determine the membership and non-membership degrees of an element in the universe of discourse and in addition to that it define hesitancy degree that express the indeterminacy degree of the membership of the element (Atanassov 1999). The iFCM extends the FCM by introducing IFSs to represent the weights of causal relations among the concepts. Thus, the weight of a causal link in the iFCM is represented by the hesitancy weight and influence weight which are aggregated and then defuzzified by the method of Centre of Gravity. The influence weight defines the fuzzy relation between the concepts, where the hesitancy weight define the hesitancy of the experts to define this relation. Therefore, (4.2) is reformulated to combine the former weights as mentioned in (Iakovidis and Papageorgiou 2011). Introducing hesitancy to define the weights of the links support experts when there is difficulty to express their hesitancy about the relationships between the concepts. The hesitancy weight of that relation is assumed to be 0.5 rather than 0. This iFCM, known as iFCM-I, was extended later to iFCM-II by introducing IFSs to represent the concept values when the modellers are hesitant to determine the values of the concept. To compensate for the problem of missing values, the concepts with unknown values have to be given a value of 0.5 instead of zero, so the difference between the exact value of the concept (which its value unknown) and 0.5 is belong to [0, 0.5]. Indeed, in iFCM-II, the iterative reasoning algorithm is modified to be compatible with the IFSs that introduced for both concepts and weights as presented in (Papageorgiou and Iakovidis 2013) . (Papageorgiou and Iakovidis 2013) had experimentally demonstrated that iFCM-II outperforms iFCM-I especially in medical domain as DSS for determining the criticality of pneumonia infection as its results match physicians' decisions more accurately than FCM and iFCM-I. Although iFCM is less affected by missing input data compared to the conventional FCM, the reasoning process of iFCM for each iteration involves complex computation due to the complex operations of IFSs. Hence, complexity is the cost as the number of iterations to address the problem is between 17 and 24 (Papageorgiou and Iakovidis 2013).

4.4.4 Evidential Cognitive Map

To cope with the limitation of the FCM in handling uncertainties from different sources, Dempster-Shafer proposed the evidence theory, DS-ET (Gordon and Shortliffe 1984). It has been incorporated with the FCM into the Evidential Cognitive Map (ECM) (Kang et al. 2012). Due the ability of DS-ET to fuse and represent imprecise and uncertain knowledge, the ECM extends the capability of FCM to handle and aggregate modelling information from different experts. The concepts in the ECM are represented by intervals and the weights of interrelated links among them are estimated based on the causal relation that could be described by one of the following four cases in the DS-ET (Kang et al. 2012):

- Positive causal relation
- Negative causal relation
- No causal relation

- No idea about the causal relation

Although the ECM outperforms the FCM in handling imprecision from different sources, incorporating belief aspects of DS-ET to FSs while dealing with high conflict between the knowledge from different sources is difficult and decreases its effectiveness.

4.4.5 Granular Cognitive Map

As a part of the surge to cope with the deficiency of the FCM in handling knowledge from different sources as in the ECM, (Pedrycz and Homenda 2014) introduced granular computing (Bargiela et al. 2003a) (Bargiela et al. 2003b) and (Bargiela and Pedrycz 2005) to the FCM and extended it to the Granular Cognitive Maps (GCM). The links between the concepts of GCM are described using intervals and T1FS as information granules and that made the construction of the CM flexible by incorporating several sources and developing more alignment with the reality of the modelled system. In GCM the reasoning algorithm (4.2) is reformulated, where the weights $W_{i,j}$ replaced by their granular counterparts and the operations of addition and multiplication include information granules and defined accordingly. Note that the output of GCM is information granules

4.4.6 Interval- valued Fuzzy Cognitive Map

(Hajek and Prochazka 2016) aimed to overcome the uncertainty of determining the values of concepts and weights of the FCM by extending the conventional FCM to Interval-valued FCM, where the values of concepts and weights are represented by intervals with lower and upper values and the reasoning algorithm is modified accordingly. The proposed Interval-valued FCM gave a closer result to the reality compared to the conventional FCM and iFCM as interval- valued FCM allows for additional freedom to capture more uncertainties while determining the values of concepts and weights and thus it will offer more accurate result to the reality. though this advantage of Interval Valued FCM, it failed to provide a dynamic inference mechanism (Hajek and Prochazka 2016).

4.4.7 Dynamic Cognitive Network

To alleviate the deficiency of conventional FCMs in representing the dynamicity of causal influence among the concepts effectively, Miao and Liu introduced the dynamic causal relations and extended the FCM to Cognitive Network (DCN) (Miao et al. 1999)(Miao et al. 2001).

The DCN allows concepts to adjust their values based on the requirement of the modelled system, where these values can be FSs or intervals. Indeed, the influence of cause concept on an effect concept is represented by the dynamic system which makes the process of modelling a causal system more flexible. The former characterisations of DCN made it an optimal tool to be used in analysing stock markets (Miao et al. 2001). However, the DCM's ability to model some systems is limited because it is difficult to incorporate their dynamicity properties, which rely on Laplacian framework with the iterative reasoning algorithm of FCM.

4.4.8 Triangular Fuzzy Number Fuzzy Cognitive Map

The FCM has been extended to Triangular fuzzy number FCM (TFNFCM) in (Yesil, Dodurka, and Urbas 2014), where the weight of causal relations among the concepts are represented by a triangular fuzzy number (TFN) and its reasoning algorithm is modified accordingly. Hence, the ability of the TFNFCM to represent the uncertainties about the influences between the concepts of modelled system is higher compared to the FCM which is characterised by fuzzy singletons (crisp) or interval weights. The TFNFCM improves the capability of conventional FCM to capture more uncertainties during the construction but its iterative reasoning algorithm which execute via four simulations requires early defuzzification of the TFN through iterations. This may lead to loss of information and restricts the full propagation of uncertainty during the reasoning process.

4.5 Learning algorithms

Learning algorithms have been developed as methodologies to improve the FCM by modifying its connection matrix which represents the causal relations among the concepts. The trained FCM becomes more flexible and enables modelling of nonlinear relations. Hence training makes it optimal for decision making, prediction and modelling (Papageorgiou 2012)(Papageorgiou and Salmeron 2014b).

The learning algorithms that are used to evolve the FCM consider weights from three directions: historical data, experts' intervention, and the production of weight matrices by combining expert knowledge and historical data (Stach et al. 2005).

The literature reveals that the learning algorithms that are used to train the FCM could be categorised based on the type of the knowledge used as follows:

1. Population-based learning algorithms which rely on existing historical data in the application domain to find the optimal weights matrix of FCM (Papageorgiou 2012). These algorithms trained the FCM with less/no intervention from experts. Some of these algorithms are Genetic algorithm (GA) (Mateou, Moiseos, and Andreou 2005), Particle Swarm Optimization (PSO) (Parsopoulos et al. 2003),

Genetic Strategies (Koulouriotis, Diakoulakis, and Emiris 2001) and Memetic algorithm (Petalas et al. 2005)

2. Hebbian- type learning algorithms modify the weight matrix iteratively based on the experiences of experts in such a way as to converge to the desired target. Some of these algorithms are Differential Hebbian Learning (DHL) (Dickerson and Kosko n.d.), which was improved to Balanced Differential algorithm (BDA) (Dickerson and Kosko n.d.), Real-coded genetic algorithm (RCGAs) and nonlinear Hebian learning approach (NHL) (Papageorgiou, Stylios, and Groumpos 2003).
3. Hybrid algorithms combine aspects of Hebbian algorithms and population algorithms. Hybrid algorithms modify the weight matrix based on both the experts' knowledge and the existing historical data. For example, NHL-RCGAs (Zhu and Zhang 2008) is a hybrid algorithm that combines NHL and RCGAs. Indeed, NHL-Differential Evolution (Papageorgiou, E.I. Groumpos 2005) is a hybrid learning algorithm used to train the FCM's connection matrix by combining differential evolution (DE) with NHL.

Each category of former learning algorithms has its advantages and disadvantages, which therefore determines its suitability for training a specific FCM. For instance, Population-based learning algorithms provide a high quality learned model in the context of dynamicity, but they require complicated calculations which consume the time and may provide non interpretable solution. Hebbian- type learning algorithms are fast as they rely on experts' knowledge but sometimes these algorithms update the weights while reasoning considering only the influence between each pair of the concepts and neglect the impact of other concepts. Hybrid algorithms inherit the advantages of Population and Hebbian algorithms and (Papageorgiou 2012) emphasizes that hybrid approaches are more practical and reasonable for designing FCMs due to their capabilities for offering accurate representation of FCMs and enhancing their reasoning capabilities. Learning algorithms have achieved promising results in training the FCMs in different applications (Papageorgiou 2011b) , (Froelich and Juszczuk 2009) and (Luo, Wei, and Zhang 2009b). For example, GA have been used to train FCM that used for pattern recognition as in (Papakostas et al. 2008) through adjusting the weights matrix to improve the required classification process. The Active Hebbian learning used to train the FCM that used to simulate the evaluation of credit risk of the companies in (Zhai, Chang, and Zhang 2009) and the results demonstrated the enhancement of the FCM capabilities for the classifications. Indeed, NHL- Differential Evolution algorithm has been used to train the FCM in different applications and the results showed the effectiveness of NHL-Differential Evolution in enhancing the FCM's capabilities for reasoning and modelling as emphasised in (Papageorgiou, E.I. Groumpos 2005).

In addition to the above-mentioned extensions of FCMs and learning algorithms to enhance conventional FCM capabilities, current research is focussed on exploring ways to consolidate multi-criteria group decision-making processes in the generation of FCMs to overcome the issue of imprecise data of different criteria. For this purpose, interval intuitionistic fuzzy sets are incorporated with FCM (Hajek and Froelich 2019).

4.6 Applications Areas

FCMs emerged in different domains to solve different applications' problems as revealed by the literature (Papageorgiou 2011a) This noticeable surge toward using FCMs in these applications is due to the advantages of FCMs which are represented in their learning capabilities, easiness to understand by non-technical people and their effectiveness in reasoning as humans in diverse domains.

As an FCM is designed for a practical purpose, it has been playing a role in prediction, classification, modelling, decision support, analyzing and reasoning.

Following are selected widely used applications' domains where the conventional FCM, trained FCM or an extension of FCM has been used for any of the formerly mentioned purposes.

4.6.1 Medicine

In the medical domain, there is a high chance for medical errors to have happened, due to lack of required information, the existence of huge data/ medical records to be analysed or doctors' uncertainties due to their lack of experience or hesitancy in taking the decision. Thus there is a necessity to design an efficient and effective Medical Decision Support System (MDSS) to support the doctors in decision making.

The FCM and its extensions emerged in the Medical domain as an effective tool for MDSS to effectively reason in the presence of imprecise or incomplete data. The literature (Amirkhani et al. 2017) revealed that the structure of FCMs that have been used in medical areas have one of the three architectures of FCM, namely CFCM, m-FCM and the trained FCM as mentioned in Section 4.2. Some of FCM's extensions are used in the medical domain's application. For example, iFCM has been used to predict Pneumonia risk (Iakovidis and Papageorgiou 2011) and its effectiveness to predict in the existence of missing output was proved. However, relatively complex mathematical calculations of IFs that represent its concept and weight values hinder its use, particularly when there are many experts involved in the designing process. A trained FGCM is also used in the medical field to model the radio therapy process (Papageorgiou and Salmeron 2012) using its effectiveness to model the experts' uncertainty. A FGCM could handle small data of the domain, which is not always the case in the medical domain, also the FGCM requires experts' feedback to determine the number of grey concepts and weights. Therefore, the capability of FGCM in the medical domain is limited. Also, FCMs which are trained by learning algorithms have been used for different purposes in the medical domain as mentioned in Section 4. Although the effective performance of these trained FCMs in the medical domain were recorded, they are limited in some applications as they used to update the weights' matrix based on initial weights of the causal relations that were obtained using historical data without considering the experts' opinions (Amirkhani et al. 2017) and hence more information about the modelled system were lost.

4.6.2 Business

The FCM gained momentum in the business field due to its capability as a tool for prediction, analysis and reasoning, which are core requirements for the planning process of any business. The study in (Wei, Lu, and Yanchun 2008) used the FCM to model and evaluate trust dynamics in the virtual enterprises and though there was an acceptable performance in evaluating the trust, there was a necessity to consider the fuzzy setting when dealing with the weights of the causal relation. However, this was neglected when creating the FCM. This was considered as a limitation of FCM in this application.

The FCM was trained by a Genetic Algorithm to analyse the what-if process of forward - backwards problem in the supply chain domain (Kim et al. 2008). Though this proposed FCM showed high accuracy, there was a source of inaccuracy that came from the inadequate weight learning time. An interval-valued FCM was proposed in (Hajek and

Prochazka 2016) to cope with the uncertainties while reasoning in the business domain. In this study, the weights of causal relations of the FCM were represented by intervals to capture more uncertainties in determining these causal relations. The interval-valued FCMs were used in two case studies, namely supplier selection and performance modelling, where the outputs were close to the real decisions. Despite this success, Interval-valued FCM fails to address the issue of using interval values of weights and concepts with dynamic inference mechanisms as there was a lack of well-defined operations required to deal with interval values of the concepts and weights.

4.6.3 Education

The study in (Lq et al. 2008) uses a FCM to learn style recognition and prove the efficiency of the FCM in this application. In (Cai et al. 2010), the FCM was extended to an Evolutionary FCM (E-FCM) by introducing probability of the causal relations between the FCM'S concepts, and then used to create a game for science learning. The E-FCM outperformed the conventional FCM by allowing different update time schedule for each concept, so it evolved in a dynamic way. Furthermore, real time variable state was simulated. Also, a conventional FCM trained by Hebian learning algorithm (HL) was created to design a game based learning system in (Luo, Wei, and Zhang 2009a). The rationale of training this FCM by HL for this application was disability of the FCM to get new knowledge from the available data.

4.6.4 Engineering

FCMs are widely used in engineering, particularly for modelling and control. For example, the work in (Stylios and Groumpos 2004b) used a FCM to control a supervisory system. The FCM was trained by nonlinear Hebian algorithm for modelling an industrial process control problem (Papageorgiou, Stylios, and Groumpos 2006). Indeed, the hierarchal architecture of FCM was used to model online design and self fine-tuning of Fuzzy Logic Controllers to generate controllers which were capable to produce the optimal controller (De Tre, Hallez, and Bronselaer 2014).

In addition to applications domains mentioned above, an FCM and its extensions were used for different purposes in different domains, for example telecommunication (Li et al. 2009), information technology (Bueno and Salmeron 2008) and solar energy (Jetter and Schweinfort 2011). Table 4.1 presents some of Application domains of FCMs and the purpose of using them in the specific domains.

Table 4.1 Application domains of FCM

Application Domain	Purpose (problem solving)
Medicine	Prediction, decision making, knowledge representation, classification and reasoning (Amirkhani et al. 2017)
Business	Prediction, modelling and analysis (Wei, Lu, and Yanchun 2008) and (Kim et al. 2008)
Education	Modelling (Cai et al. 2010)
Engineering	Control and modelling (Stylios and Groumpos 2004) and
Environment and Agriculture	Prediction and classification (Markinos et al. 2007)
Information Technology	Analysis and Modelling (Aguilar, J. and Contreras, J. 2010)
Solar Energy	Modelling (Jetter, A. and Schweinfort, W 2011)

4.6.5 Discussions

Though the success of the conventional FCM and its extensions in diverse applications, it can be noticed that they have drawbacks. Some of these drawbacks are represented in the disability of the FCM to model efficiently the subjective opinions of the experts about the causal relations among the concepts, as these opinions may change over time or vary depending on the level of expertise that experts have. Indeed, there is a need to measure the uncertainties associated with weights and concepts. Further, the designers of the FCM have to select a slope value in the threshold function (4.3) and such selection depends on the designers' preferences and ultimately affect the number of iterations during the reasoning using (4.2) (Papageorgiou 2011a). Moreover, the discrete values that represent the FCM's concepts' values and links' weights (defuzzified values of T1FSs which were used initially to represent concepts and weights), hinder the capability of the existing FCMs to handle the uncertainties of the modelled system efficiently.

Former drawbacks highlight the necessity to incorporate new approaches to the conventional FCM and extend it to overcome these drawbacks, particularly with respect to discrete values of the concepts and weights that are incapable to represent the uncertainties and the iteration process during the reasoning. Motivated by the advantages of IAA in generating zT2FSs which capture well the inter and intra- uncertainties while modelling the data, the author proposed in this thesis using the zT2FSs to represent the weight in the FCM and then use their values without defuzzification in the reasoning process, as presented in Chapter 5 and Chapter 6 of this thesis.

4.7 Conclusion

The FCM is a soft computing tool with feedback that has a promising application in the domain of modelling and reasoning. Although the extensions of FCMs that are summarised in Table 4.2 achieved a noticeable success in enhancing the FCM's reasoning capabilities in the presence of imprecise or missed information, their structures rely on T1FSs which hinder their ability to capture a high level of uncertainty and to aggregate information from different sources associated with real-world application domains.

It is worth noting that the strength of the knowledge propagation in the FCM and hence the accuracy of the output relies on the weights of the causal links. Thus, the weights of the causal links of the FCM is crucial. Motivated by former findings and to extend the conventional FCM, this thesis proposes a Type 2 Fuzzy Cognitive Map (T2FCM). First, this involves incorporating zT2FSs to represent the causal relation which enhances the acquisition of information about this relation and enhances its qualitative representation. Second, this requires a development of a new non-iterative reasoning algorithm that is compatible with the use of zT2FSs. The ability of the proposed FCM to mimic human reasoning will be demonstrated through sensitivity analysis.

Table 4.2 FCM Extensions

FCM's extension	How and why is the extension carried out
Fuzzy Rule Based Fuzzy Cognitive Map (FRBFCM) (Carvalho and Tome 2000)	To model causal relations that are neither linear nor monotonic by fuzzy if-then ruled and new operations
Fuzzy Grey Cognitive Map (FGCM) (Salmeron 2010)	To model weights by grey intervals to be effective in forecasting using small, incomplete and uncertain data
Intuitionistic Fuzzy Cognitive Map (i-FCM) (Papageorgiou and Iakovidis 2013)	To extends the FCM by IFSs to represent the weights of causal relations among the concepts modelling in domains that involve uncertainty and hesitancy of modelers
Evidential Cognitive Map (ECM) (Kang et al. 2012).	To handle the limitation of the FCM in handling uncertainties from different sources by introducing DS-ET
Granular Cognitive Map (GCM) (Pedrycz and Homenda 2014)	To cope with the deficiency of the FCM in handling knowledge from different sources by introducing granules (intervals and T1FSs) to represent the link's weights)
Interval Valued FCM (Hajek and Prochazka 2016)	To use intervals to represent the values of concepts and weights to overcome the uncertainty of determining the values of concepts and weights of the FCM
Dynamic Cognitive Network (DCN) (Miao et al. 1999)	To introduce dynamic causal relations to represent the dynamicity of causal influence effectively
Triangular Fuzzy Number Fuzzy Cognitive Map (TFNFCM) (Yesil, Dodurka, and Urbas 2014)	To use TFN to represent weights of causal relations and hence capture more uncertainties

Chapter 5 Fuzzy Cognitive Map Enhanced by Type 2 Fuzzy Set based on z slices

5.1 Introduction

In reviewing the literature, it can be noticed that the structure of existing FCMs relies on T1FSs to represent the weight of causal relations among the concepts. Therefore, the ability of FCM to handle high levels of uncertainties is hindered during the construction, reasoning and then production of accurate results. To overcome these shortcomings and to address the research question of this thesis, the author introduced a Fuzzy Cognitive Map based on zT2FSs (zT2FCM) as an extension of the conventional FCM, where zT2FSs are used to represent the weights of the causal relations. These zT2FSs are generated using IAA and hence they capture all the information and uncertainties of experts' opinions about the weights without any data reprocessing. This extension of the conventional FCM is a part of the novelty of this thesis.

This chapter aims to present the proposed zT2FCM and details the process of its construction. To demonstrate and analyse the effectiveness of the proposed zT2FCM, a case study of Autism diagnosis, i.e. to identify early a risk of developing Autism is used. A detailed description of the conduct of this case study is presented in this chapter. Additionally, a method for collecting data for the case study is presented. Also, an overview about some aspects related to the domain of the case study are presented to explicate the construction process of the proposed z T2FCM for the case study.

5.2 Construction of Fuzzy Cognitive Maps based on zT2FSs

Based on the fundamentals of the FCM's structure mentioned in Section 4.2, to design zT2FCM to model a problem or a system, the first step is to identify the main aspects of the system that required to be modelled. Then, these aspects are represented by the concepts of the zT2FCM. After that, the causal relations among the concepts are identified. As mentioned in Chapter 4, the identification of the concepts and the interrelations among them relies on the experts' knowledge and/ or existing historical data about the modelled system. After identifying the concepts and the causal relations between them, the weights of the links in zT2FCM are determined. For this purpose, an interval-valued questionnaire is designed in such a way that the aim of each question is to identify the weight of a single relation of zT2FCM. Hence, the number of questions in the questionnaire is equal to the number of the existing links in the zT2FCM.

For the zT2FCM that comprises m concepts, the weights $W_{i,j}$, where $i = 1,2,3,..m$ and $j = 1,2,3,...m - 1$ are identified as follows:

N experts responded to the questionnaire that is designed for identifying the weights of the L causal links in the zT2FCM. The questions were designed to support the collection of the required data from the experts. For example, each question may require a single response or multiple responses based on given criteria of the question (e.g. the question has subsections). The experts offer their responses to each question and / or its subsections by drawing an ellipse on a Likert scale (Likert 1932) ranging from 0 to 100 to determine the weight which reflects the strength of the causality of the link between the two concepts mentioned in that question. Hence, for a single link, there are either N or tN (if the question has t subsections) ellipses, in which each extracted response interval represents the uncertainty of the corresponding expert in determining the weight of the link. The process of collecting the responses to the questions may be repeated after a specific period of the time, to capture the intra-uncertainty of experts in determining the weights of the links. The inclusion of experts' intra-uncertainty was considered as they might possibly had better insights about a topic in their area of specialization (Wagner, Miller, and Garibaldi 2013a).

After this step, the response intervals were converted to T1FSs and then aggregated across each question using IAA as detailed in Section 3.3 to capture the intra-uncertainty and inter- uncertainty of the experts about assigning the weights of the links between two concepts. The output of IAA is a zT2FS. The produced zT2FS represents a fuzzy agreement model of the weight of the relation between the two concepts which corresponds to that question. The generated fuzzy agreement model captures both the intra and inter-uncertainties of the experts with respect to the weight of the relation. zT2FSs were used to represent the weights in zT2FCM because of their ability to capture more uncertainties compared to the T1FSs as they offer additional degree of freedom for modelling by using the third dimension. Note that the third dimension of the generated zT2FSs which represent the weights in the zT2FCM, reflects the level of agreement among the experts in determining the weights of the causal relations between the concepts.

After the zT2FS is generated, it is defuzzified by centroid to a crisp value using (2.10). The obtained crisp value represents the weight of the link, i.e. weight of the causal relation between the linked two concepts.

By following these steps, the weights of the zT2FCM's links are obtained using the response intervals from N experts for each question in the questionnaire.

After the identifications of the concepts and determining the weights of the links between them the construction of zT2FCM is completed. The zT2FCM is ready to receive the concepts' values and to carry on the reasoning using the iterative formula (4.2). In this way, the output value is inferred.

It is worth noting that in the zT2FCM the conventional FCM is enhanced via representing its weights by zT2FSs, but its reasoning algorithm remains the same as in the conventional FCM where the crisp values (centroids) of zT2FSs are used for weights' values.

To demonstrate the use and effectiveness of the zT2FCM, a case study on Autism diagnosis was created. The zT2FCM is proposed as a decision tool to identify early the risk of developing autism among children. The following sections detail and discuss this case study and the questionnaire created for the purpose of collecting data required to represent weights in the generated zT2FCM.

Furthermore, a comparison between the results obtained using the zT2FCM and the conventional FCM, used for the same purpose, is provided in order to demonstrate the accuracy of the proposed zT2FCM over the FCM.

5.3 Case Study: Autism Diagnosis using Fuzzy Cognitive Map based on zT2FSs

As mentioned earlier in this chapter, to demonstrate the proposed zT2FCM, the author created a case study where zT2FCM can be used for early diagnosis of a child with an Autism Spectrum Disorder (ASD). The main aim of this case study is to demonstrate that representing the weights of FCM's links by zT2FSs enhances its capability for handling and aggregating uncertain information from different sources and hence improves the ability of FCM in mimicking human decisions effectively. Required aspects to understand this case study are presented in Section 5.3.1 and process of construction zT2FCM for Autism Diagnosis is presented in Section 5.3.2 and Section 5.3.3.

5.3.1 Background

5.3.1.1 Autism

ASD is a complex neurodevelopmental disorder that appears in the early years of a child's life. Children with ASD tend to have abnormalities represented by impairments in the social interaction and communication, severely restricted interests and highly repetitive behaviour (Birx et al. 2011). Their understanding capacity of non-verbal activities is lower than that of normal children and this restricts their communication and interaction with others in the society. Early diagnosis of Autism can help Autistics to reach their developmental potential, engage with others, and integrate into society. The process of diagnosing ASD is challenging due to the existence of different qualitative and quantitative data sources that need to be elicited and analysed in order to diagnose the severity of the condition. Moreover, the different opinions of stakeholders such as therapists, parents, and doctors may vary and need to be taken into account. The aforementioned reasons suggest the necessity to create a decision model based on combining key indicators contributing to a diagnosis, which can be used to identify early signs, type and severity of the ASD. It was shown in the literature that a conventional FCM can be trained by learning algorithms and used for classifying the risk of developing ASD (Kannappan, Tamilarasi, and Papageorgiou 2011) and (Papageorgiou and Kannappan 2012). The author decided to create zT2FCM for the same problem to demonstrate its capability in compare to conventional FCM to capture uncertainties about ASD's diagnosis and classifications and hence offers an output that is more accurate and closer to doctors' decisions.

5.3.1.2 The Modified Checklist for Autism

The Modified Checklist for Autism in Toddlers (MCHAT) is a screening tool recognized by the American Academy of Paediatrics as a tool for diagnosing if a child between 16 months and 30 months of age is at risk of developing Autism (Birx et al. 2011). MCHAT is a questionnaire containing 20 questions that require crisp inputs namely yes/no regarding the behaviour, unique skills and difficulties of the child. Based on the responses of parents on the MCHAT, the physicians follow subsequent evaluation flow charts to reach a decision on diagnosis.

This decision can be imprecise and intuitive in nature, based on the perception and experiences of the given physicians. These procedures can also be time consuming with a high degree of information loss in the assessment procedure due to its dependence on crisp inputs. To overcome the shortcomings of the existing MCHAT, the responses to each question are modified to three options a, b and c in (Kannappan, Tamilarasi, and Papageorgiou 2011), each refer to frequency of experienced symptoms as presented in Appendix 1 instead of dichotomous options (e.g. a. Certainly not, b. At times and c. Always). For the purpose of this case study, the MCHAT used in (Kannappan, Tamilarasi, and Papageorgiou 2011) (known as Modified MCHAT) is extended and modified again in order to allow the experts to express the fuzzy nature of their decisions based on parents' responses; this fuzziness should factor into decisions for diagnosis of autism. The extended and modified MCHAT questionnaire for a purpose of this case study is called Fuzzy

MCHAT (F-MCHAT). It is created and used to identify the concepts of the proposed zT2FCM and to determine the weights in zT2FCM.

The F-MCHAT questionnaire is given in the Appendix 2. It comprises 20 questions that are designed to collect the experts' decisions with respect to diagnosing the development of autism in the child based on the responses of his/her parents to the questions of the modified MCHAT.

It is worth noting here that the respondents to F-MCHAT (presented in Appendix 2) which created for the case study of this thesis are the experts (doctors) assuming that parents respond to corresponding modified MCHAT (presented in Appendix 1)questions and each of the three options for each question, where the respondents to standard MCHAT and modified MCHAT in (Kannappan, Tamilarasi, and Papageorgiou 2011) are the parents.

For example, in F-MCHAT each question requires each of the experts to represent his/her decision about the possibility of autism development considering the response of the parent to the corresponding question in modified MCHAT. The parents response is either option a, b or c. The experts express their decision by drawing an ellipsis on Likert scale, ranging from 0 to indicate no possibility for autism, to 1 to indicate the certainty of the child developing autism. For example, to identify the weight of the relation between the concept "enjoy being swung and decision concept "Autism Diagnosis", each experts need to express his/her decision by drawing ellipsis on three Likert scales if the parents' response to the corresponding modified MCHAT question is a. certainly not, b. At times and c. Always as shown in Figure 5.1.

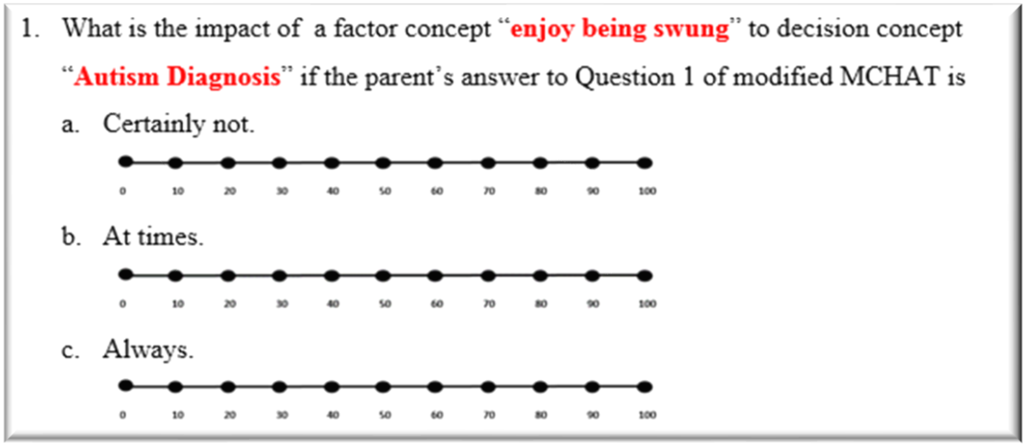


Figure 5.1 Sample question

The motivation for this extension and modification of modified MCHAT to F-MCHAT is to capture all the uncertainties about the diagnostic decision, taking into account all the possible responses from the parents about the frequency and/or degree of symptoms experienced by their child. Indeed, collecting the responses in term of the intervals gives the experts a chance to express their uncertainty about the diagnostic decisions.

It is worth noting here that for each option of a question in F-MCHAT used to determine the weight of the causal relation between zT2FCM's concepts, the number of responses by experts is equal to the number of experts who respond to the questionnaire F-MCHAT. Therefore, for option 'a' of each question, there are *N* responses that would be aggregated to produce intra- uncertainty model based on T1FS on option 'a'; similar is the case for option 'b' and option 'c'. Then the three produced T1FSs are aggregated to capture inter-uncertainty of the experts for the options using IAA. The generated zT2FS represents the fuzzy agreement model of the weight of the link between the two concepts which mention in the question.

5.3.2 The concepts of zT2FCM for Autism Diagnosis

To identify the concepts of zT2FCM for this case study, the F-MCHAT was used as a standard tool to derive the required concepts. F-MCHAT comprises questions for doctors on 20 major aspects that can inform decisions on the classification of Autism. Therefore, the proposed zT2FCM contains 20 inputs concepts and one concept as an output concept representing the decision concept. These concepts are listed in Table 5.1, where each $C_i, i = 1,2, \dots, 21$ is a symptom of Autism diagnosis and corresponds to a concept of the proposed zT2FCM. For example, C_1 is a concept correspond to the symptom “Enjoy being swung” and C_{21} is the decision concept that correspond to the Autism Diagnosis.

As each question in F-MCHAT contributes to determining the diagnosis of Autism by the doctors based on the parents' response on these 20 aspects in the corresponding modified MCHAT, there is a link from each input concept to the decision concept.

Therefore, the constructed zT2FCM which is designed for Autism diagnosis contains 21 concepts and 20 links as shown in Figure 5.2. Based on these concepts and links, this designed zT2FCM is a competitive FCM (refer to Section 4.2) that focuses on emphasising the influence between each cause concept and the decision concept only with no cyclic relations among the concepts (Stylios et al. 2007).

Table 5.1 Concepts of zT2FCM used for Autism diagnosis

C1	Enjoy being swung
C2	Take an interest in other children
C3	Climbing on things
C4	Pretend to be other things
C5	Pointing with index finger
C6	Indication of interest
C7	Bringing objects to parents
C8	Walking
C9	Oversensitive to noise
C10	Smile in response to parents face
C11	Imitate
C12	Respond to the name
C13	Looking at a toy when pointing
C14	Eye contact
C15	Look at things you are looking at
C16	Unusual finger movement near his/her face
C17	Attract your attention
C18	Deafness
C19	Understanding what others say
C20	Look to your face to check reaction
C21	Autism Diagnosis

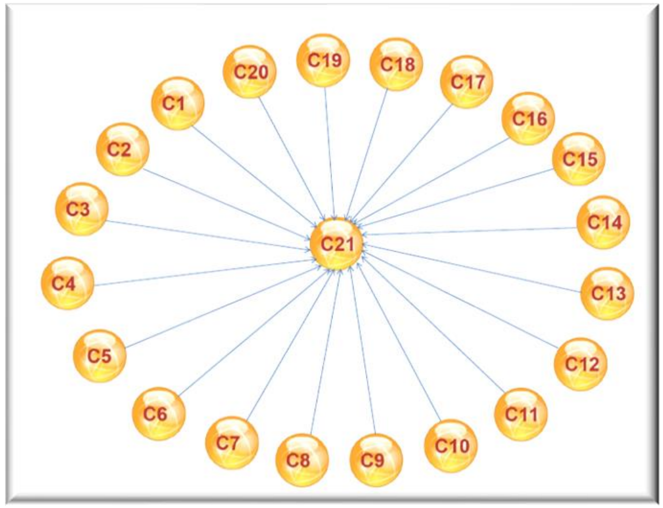


Figure 5.2 Structure of zT2FCM for Autism Diagnosis

5.3.3 The weights of zT2FCM for Autism diagnosis

Based on the process of constructing zT2FCM mentioned in Section 5.2, in order to determine the weights of the 20 links in constructed zT2FCM and for the purpose of this case study, three doctors from Sultan Qaboos University Hospital (SQUH) in Oman responded to the F-MCHAT questionnaire about the risk of developing Autism.

Following the concept of responding to the F-MCHAT mentioned in Section 5.3.1.2, to determine the weight of the link from the concept C_j, $\text{j} = 1, \ldots, 20$ to the decision concept C_{21}, each of the three doctors provided a response to each question by drawing ellipses on Likert scales to determine the impact of the value of the concept C_j given in options 'a', 'b' and 'c' on the decision concept C_{21}. Note that each option represents the parents' response on the corresponding question in modified MCHAT. In other words, the doctors determine the impact of the concept C_j on the concept C_{21} in a specific question Q_j in F-MCHAT if the parents response to Q_j in modified MCHAT is 'a', 'b' or 'c'.

Figure 5.3 is an example of a doctor's response to a question 1 in F-MCHAT which presented in Figure 5.2.

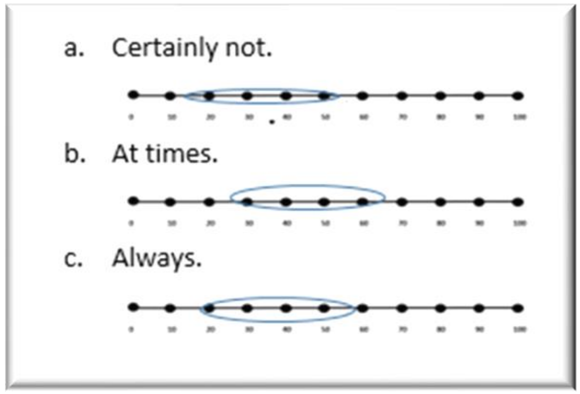


Figure 5.3 Example of a doctor response

The intervals were then extracted from the ellipses given by all the doctors. For each question, there were a total of nine intervals representing the responses of the three doctors to that question, where each doctor gives his/her opinions for all three options: 'a', 'b' and 'c'. These intervals represent the doctors' opinions about the relation between the causal concept and the decision concept. The response intervals of the three doctors for the 20 questions are presented in Appendix 3.

Collecting the opinions as intervals gives the doctors a greater chance to express their uncertainties about assigning the weights of the causal relations due to imperfect information. Thus, the width of the interval reflects the uncertainty that a doctor has in answering a question. For example, a narrow interval represents less uncertainty (more certainty), where a wider interval represents more uncertainty (less certainty) in answering a question.

After that, for each question Q_j, the nine intervals, three replies for each of the three options from the three doctors, were aggregated using IAA. A zT2FS is generated, which represents the doctors' agreement model about the weight of the link from the concept C_j and the decision concept C_{21}determined by question Q_j. Note here and as mentioned earlier, the number of the questions in F-MCHAT is equal to the number of causal concepts. The zT2FS weight is generated using IAA as follows:

For each Q_j, the responses of the three doctors D1, D2 and D3 are collected across each of the three options: 'a', 'b' and 'c'. Hence, for the option 'a', there are three intervals, noted as $D1_{a_j}, D2_{a_j}, D3_{a_j}$, for the option 'b' there are three intervals noted as $D1_{b_j}, D2_{b_j}, D3_{b_j}$ and for the option 'c', there are three intervals, noted as $D1_{c_j}, D2_{c_j}, D3_{c_j}$ (i.e. $D1_{a_1}$, represents the response interval to the option 'a' of Q_1 by D1 and so on).

For each option 'a', 'b' and 'c', the responses of the three doctors were collected and aggregated using the first phase of IAA detailed in Section 3 where *S*=3 to generate a T1FS, namely M, with membership degreesy_1, y_2 andy_3, where $y_1 = \frac{1}{3}$ for the union of all three intervals cross each option, $y_2 = \frac{2}{3}$ for union of intersection of all 2-tuple intervals cross each option and $y_3 = 1$ is for the intersections of the three intervals for each option. This T1FS, M captured the option's intra uncertainty as shown in Figure 5.4.

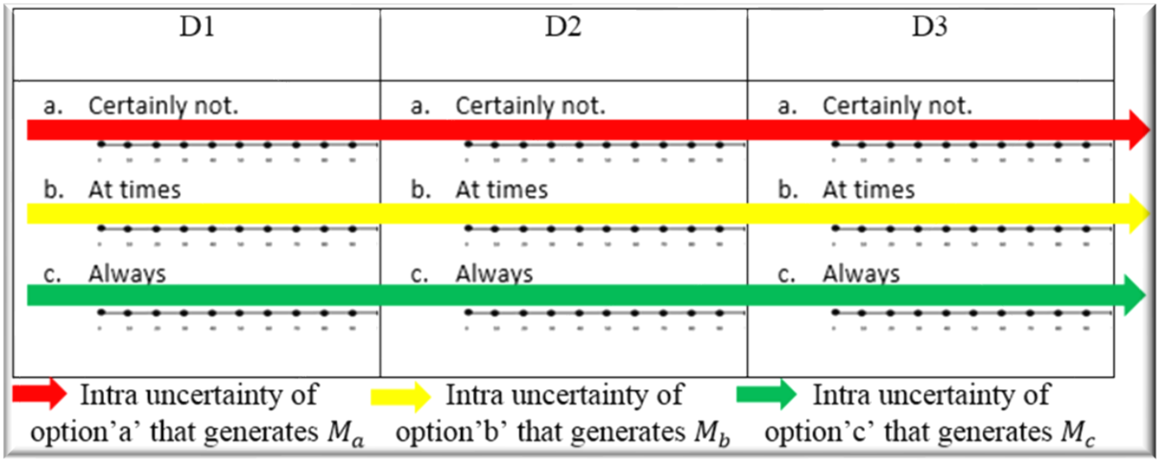


Figure 5.4 Capturing intra-uncertainty of each option

Therefore, for each question, three T1FSs are generated, namely M_a, M_b and M_c, which captures the uncertainty of the responses of the three doctors to the question's options 'a', 'b' and 'c', respectively. They are created as follows:

$$M_a = \frac{1}{3}/(D1_a \cup D2_a \cup D3_a) + \frac{2}{3}/\big((D1_a \cap D2_a) \cup (D1_a \cap D3_a) \cup (D2_a \cap D3_a)\big) + 1/(D1_a \cap D2_a \cap D3_a) \tag{5.1}$$

$$M_b = \frac{1}{3}/(D1_b \cup D2_b \cup D3_b) + \frac{2}{3}/\big((D1_b \cap D2_b) \cup (D1_b \cap D3_b) \cup (D2_b \cap D3_b)\big) + 1/(D1_b \cap D2_b \cap D3_b) \tag{5.2}$$

$$M_c = \frac{1}{3} / (D1_c \cup D2_c \cup D3_c) + \frac{2}{3} / \big((D1_c \cap D2_c) \cup (D1_c \cap D3_c) \cup (D2_c \cap D3_c)\big) + 1/(D1_c \cap D2_c \cap D3_c) \tag{5.3}$$

After that, M_a, M_b and M_c, generated for each question are aggregated using the second phase of IAA defined in Section 3.3, where $N = 3$. Their aggregation produces zT2FS that captures the inter uncertainty of the doctors' responses to all the question's options. Indeed the produced zT2FS represents the weight W of the link between the two concepts, the causal concept and the decision concept, determined by the corresponding question as follows:

$$W = \frac{1}{3} / (M_a \cup M_b \cup M_c) + \frac{2}{3} / \big((M_a \cap M_b) \cup (M_a \cap M_c) \cup (M_b \cap M_c)\big) + 1/(M_a \cap M_b \cap M_c) \tag{5.4}$$

The generated W in (5.4) is a zT2FS of the three slices, namely Z_1, Z_2 and Z_3, where

$$Z_1 = \frac{1}{3} / (M_a \cup M_b \cup M_c) \tag{5.5}$$

$$Z_2 = \frac{2}{3} / \big((M_a \cap M_b) \cup (M_a \cap M_c) \cup (M_b \cap M_c)\big) \tag{5.6}$$

$$Z_3 = 1/(M_a \cap M_b \cap M_c) \tag{5.7}$$

Therefore,

$$W = Z_1 \cup Z_2 \cup Z_3 \tag{5.8}$$

Here, W represents the fuzzy agreement model considering the responses of the three doctors about the weight of the link between the causal concept and decision concept. It is worth noting here that the third dimension reflects the level of agreement between the doctors on the relation between the causal concept and the decision concept.

By repeating these steps for all the questions, the weights of the zT2FCM's links as zT2FSs are determined as given in Appendix 4.

Thereafter, each of the zT2FSs is defuzzified using (2.10) to obtain the crisp value of the corresponding weight between the causal and decision concepts.

5.3.3.1 Illustrative example of determining the weight between two concepts in the zT2FCM for Autism Diagnosis

This section presents an illustrative example of determining the weight between two concepts in the proposed zT2FCM for Autism Diagnosis using the doctors' responses to F-MCHAT. For example, to determine the weight $W_{21,14}$ of the directed link from the concept C_{14} (Eye contact) to the decision concept C_{21} (Autism Diagnosis), the three doctors responded to Q_{14} which presented in Figure 5.5.

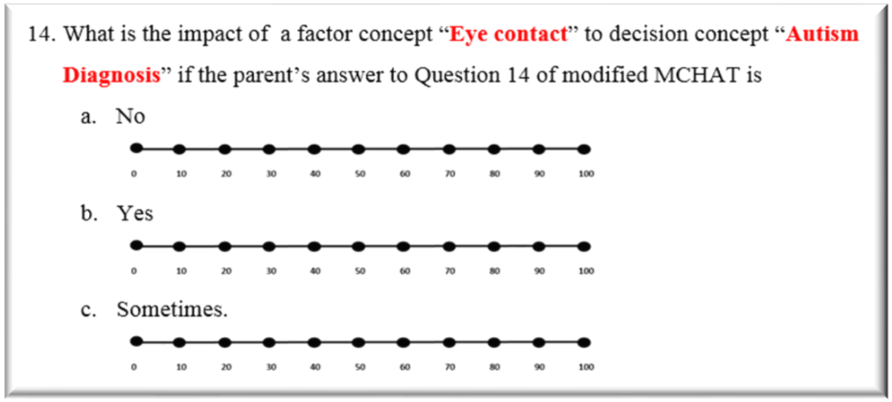


Figure 5.5 Q_{14} in F-MCHAT

Then the interval responses of the three doctors D1, D2 and D3 to Q_{14} , are extracted as presented in Table5.2.

Table 5.2 the doctors responses to Q_{14}

Q_{14}	D1	D2	D3
a	[0.32, 0.55]	[0.27, 0.45]	[0.35, 0.57]
b	[0.55, 0.75]	[0.42, 0.68]	[0.32, 0.55]
c	[0.17, 0.37]	[0.41, 0.65]	[0.37, 0.55]

After the data collection, the three T1FSs, namely M_a , M_b andM_c , which capture the intra-uncertainty for options 'a', 'b' and 'c', respectively are calculated using (5.1), (5.2) and (5.3) respectively as follows:

$$M_a = \frac{1}{3}/ ([0.32, 0.55] \cup [0.27, 0.45] \cup [0.35, 0.57]) + \frac{2}{3}/\big(([0.32, 0.55] \cap [0.27, 0.45]) \cup ([0.32, 0.55] \cap [0.35, 0.57]) \cup ([0.27, 0.45] \cap [0.35, 0.57])\big) + 1/([0.32, 0.55] \cap [0.27, 0.45] \cap [0.35, 0.57])$$

$$M_a = \frac{1}{3}/ [0.27,0.57] + \frac{2}{3}/[0.32, 0.55] + 1/[0.35, 0.45]$$

$$M_b = \frac{1}{3}/ ([0.55, 0.75] \cup [0.42, 0.68] \cup [0.32, 0.55]) + \frac{2}{3}/\big(([0.55,0.75] \cap [0.42,0.68]) \cup ([0.55,0.75] \cap [0.32, 0.55]) \cup ([0.42, 0.68] \cap [0.32, 0.55])\big) + 1/([0.55,0.75] \cap [0.42,0.68] \cap [0.32,0.55])$$

$$M_b = \frac{1}{3}/ [0.32, 0.75] + \frac{2}{3}/[0.42, 0.68] + 1/[0.55, 0.55]$$

$$M_c = \frac{1}{3}/ ([0.17, 0.37] \cup [0.41, 0.65] \cup [0.37, 0.55]) + \frac{2}{3}/\big(([0.17,0.37] \cap [0.41,0.65]) \cup ([0.17,0.37] \cap [0.37,0.55]) \cup ([0.41,0.65] \cap [0.37, 0.55])\big) + 1/([0.17, 0.37] \cap [0.41, 0.65] \cap [0.37, 0.55])$$

$$M_c = \frac{1}{3}/ [0.17, 0.65] + \frac{2}{3}/[0.41, 0.55]$$

The three T1FSs, M_a , M_b and M_c were then aggregated using (5.4) to capture the inter-uncertainty of the responses to the three options. Hence, the weight $W_{21,14}$ was generated as the zT2FS containing three slices Z_1, Z_2 and Z_3 at $z_1 = \frac{1}{3}$, $z_2 = \frac{2}{3}$ and $z_3 = 1$, respectively as follows:

$$Z_1 = \frac{1}{3} / \left(\frac{1}{3} / ([0.27, 0.57] \cup [0.32, 0.75] \cup [0.17, 0.65]) + \frac{2}{3} / ([0.32, 0.55] \cup [0.42, 0.68] \cup [0.41, 0.55]) + 1/([0.35, 0.45] \cup [0.55, 0.55]) \right)$$

$$\boldsymbol{Z_1} = \frac{\mathbf{1}}{\mathbf{3}} / \left(\frac{\mathbf{1}}{\mathbf{3}} / ([0.17, 0.75]) + \frac{\mathbf{2}}{\mathbf{3}} / ([0.32, 0.68]) + \mathbf{1}/([0.35, 0.45] \cup [0.55, 0.55]) \right)$$

$$Z_2 = \frac{2}{3} / \Big(y_1 / \big(([0.27,0.57] \cap [0.32,0.75]) \cup ([0.27,0.57] \cap [0.17,0.65]) \cup ([0.32,0.75] \cap [0.17,0.65]) \big) + y_2 / \big(([0.32,0.55] \cap [0.42,0.68]) \cup ([0.32,0.55] \cap [0.41,0.55]) \cup ([0.42,0.68] \cap [0.41,0.55]) \big) + y_3 / ([0.35,0.45] \cap [0.55,0.55])$$

$$\mathbf{Z_2} = \frac{\mathbf{2}}{\mathbf{3}} / \left(\frac{\mathbf{1}}{\mathbf{3}} / ([0.27, 0.65]) + \frac{\mathbf{2}}{\mathbf{3}} / ([0.41, 0.55]) \right)$$

$$Z_3 = 1/\big(y_1 / ([0.27,0.57] \cap [0.32,0.75] \cap [0.17,0.65]) + \mathbf{y_2} / ([0.32,0.55] \cap [0.42,0.68] \cap [0.41,0.55]) + y_3 / ([0.35, 0.45] \cap [0.55,0.55]) \big)$$

$$\boldsymbol{Z_3} = \mathbf{1} / \left(\frac{\mathbf{1}}{\mathbf{3}} / ([0.32, 0.57]) + \frac{\mathbf{2}}{\mathbf{3}} / ([0.42, 0.55]) \right)$$

The result of the previous calculations are presented in Table 5.3 where $\emptyset$ indicates the empty intersections of intervals. It reflects the agreement/disagreement among the experts in deciding the weight $W_{21,14}$

Table 5.3 $W_{21,14}$

Slice of $W_{21,14}$	z_i	$y_1 = 1/3$	$y_2 = 2/3$	$y_3 = 1$
Z_1	$z_1 = 1/3$	$[0.17, 0.75]$	$[0.32, 0.68]$	$[0.35, 0.45] \cup [0.55, 0.55]$
Z_2	$z_2 = 2/3$	$[0.27, 0.65]$	$[0.41, 0.55]$	$\emptyset$
Z_3	$z_3 = 1$	$[0.32, 0.57]$	$[0.42, 0.55]$	$\emptyset$

As mentioned earlier, $W_{21,14}$ presented in Table 5.3, is a zT2FS, where $W_{21,14} = Z_1 \cup Z_2 \cup Z_3$ is a fuzzy agreement model that captures the uncertainty of the doctors in assigning the weight of the link from C_{14} (the causal concept, Eye Contact) to C_{21} (the decision concept, Autism Diagnosis) based on the responses of the doctors to Q_{14}.

After that the generated zT2FS is defuzzified to calculate the crisp value of the weight $W_{21,14}$.

Using (2.10) as follows:

$$C_{W_{21,14}} = \frac{\left(\frac{1}{3}\right)(0.4808) + \left(\frac{\mathbf{2}}{\mathbf{3}}\right)(0.2366) + (\mathbf{1})(0.2358)}{\left(\frac{1}{3} + \frac{2}{3} + 1\right)} = 0.2769$$

where $(0.4808), (0.2366)$ and (0.2358) are centroids of the slices $\boldsymbol{Z_1}, \boldsymbol{Z_2}$ and $\boldsymbol{Z_3}$**,** respectively.

Hence $W_{21,14} = 0.2769$ **.** By following the abovementioned procedure for all the links from $\boldsymbol{C_j}$ to $\boldsymbol{C_{21}}$, where $=$ $1,2, \ldots ,20$, all weights $W_{21,j}$ are calculated as listed in Table 5.4.

Table 5.4 $W_{21,j}$

j	1	2	3	4	5	6	7	8	9	10
$W_{21,j}$	0.0875	0.0847	0.1271	0.1	0.0917	0.1854	0.1306	0.209	0.1028	0.1083
j	11	12	13	14	15	16	17	18	19	20
$W_{21,j}$	0.1319	0.1063	0.1	0.2769	0.2215	0.0681	0.1271	0.1458	0.1478	0.0938

Now the zT2FCM for Autism Diagnosis was constructed as in Figure 5.6 to prepare it for receiving the concept values and then conduct reasoning to determine the value of the decision concept.

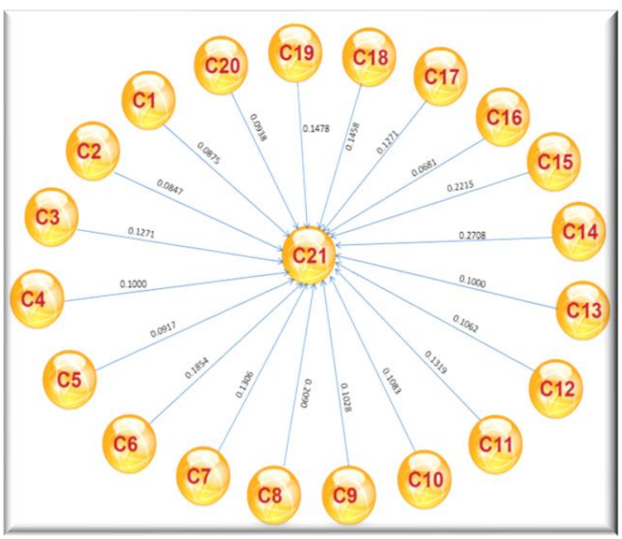


Figure 5.6 zT2FCM for Autism Diagnosis

Based on Figure 5.6, it can be noticed that the link of the weight $W_{21,14}$ is the strongest link and the link of the weight $W_{21,2}$ is the weakest link and therefore the concept C_{14}, Eye contact, has more impact on the Autism diagnosis while the concept C_2, Take interest in other children, has less impact on the Autism diagnosis.

5.3.4 Results of Simulations

The zT2FCM that was constructed for Autism Diagnosis is used to predict the risk of developing autistic disorder as follows:

In each iteration, the influence of the causal concepts to the decision concept within zT2FCM is determined by the conventional iterative reasoning algorithm (4.2) to obtain the value of decision concept C_{21} as follows:

$$C_{21}^{(k+1)} = f\left(C_{21}^{(k)} + \sum_{j=1}^{20} C_j^{(k)} * W_{21,j}\right) \tag{5.9}$$

where f here is the threshold function (4.3) and $m = 1$, as it is, based on (Miao and Liu 2000), the best option for classification reasoning which is used for a purpose of this case study.

The process of reasoning in (5.9) iteratively updates the concept values until zT2FCM converges to a steady state. Its output value is used to infer the decision about developing the risk of autism. The value of decision concept C_{21} classifies the risk of developing the Autism (Autism Diagnosis) as in Table 5.5

Table 5.5 Classification of risk of developing the Autism based on C_{21} value

C_{21} **value**	**Classification**
$0 \leq C_{21} \leq 0.33$	No Autism
$0.33 < C_{21} \leq 0.50$	Probable Autism
$0.50 < C_{21} \leq 1$	Definite Autism

This classification follows the same classification for decision concept C_{21} in (Kannappan, Tamilarasi, and Papageorgiou 2011) for a comparison purposes as it will explained in this section and Section 5.5.3.

The constructed zT2FCM that presented in Figure 5.6 was used to predict the risk of developing Autism for the same 40 datasets in (Kannappan, Tamilarasi, and Papageorgiou 2011). It used the same initial values of these datasets for the concepts C_1 to C_{20} to calculate the decision concept C_{21} which has an initial value of 0.

It worth noting that for each dataset, there are three decisions about the classification for risk of developing Autism using three different approaches. They are:

- Decision by the doctors regarding the case depending on standard MCHAT's responses by the parents (conventional approach).
- Decision based on output value of decision concept using the conventional FCM in (Kannappan, Tamilarasi, and Papageorgiou 2011)
- Decision based on output value of decision concept using proposed zT2FCM.

For example, the initial values for C_1 to C_{20} of one of the cases in (Kannappan, Tamilarasi, and Papageorgiou 2011) were used as initial values of the factor concepts in the proposed zT2FCM, where the initial value of C21 is 0 as listed in Table 5.6. These values are then iterated using (5.9) as shown in Table 5.7. The concepts reached equilibrium after seven iterations. The decision concept C21 of the proposed zT2FCM resulted in a final value of 0.9356 which falls in the Definite Autism category, based on the thresholds for classification given in Table 5.5. Actually, this case was classified as Definite Autism by the doctors and it classified as Probable Autism using conventional FCM. Hence, it can be concluded that the result of the reasoning for this case using the proposed zT2FCM matched the diagnosis of the doctors for the same case.

Table 5.6 An Example for initial values of concepts

Concept	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Initial value	0.3	0.55	0.6	0.2	0.69	0.73	0.86	0.1	0.57	0.4
Concept	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
Initial value	0.5	0.62	0.6	0.71	0.9	0.15	0.25	0.45	0.49	0.62

Table 5.7 An example for iterations of initial values

Iteration	1	2	3	4	5	6	7
C_1	0.3000	0.5744	0.6398	0.6547	0.6581	0.6588	0.6590
C_2	0.5500	0.6341	0.6534	0.6578	0.6588	0.6590	0.6590
C_3	0.6000	0.6457	0.656	0.6584	0.6589	0.6590	0.6590
C_4	0.2000	0.5498	0.6341	0.6534	0.6578	0.6588	0.6590
C_5	0.6900	0.666	0.6606	0.6594	0.6591	0.6591	0.6591
C_6	0.7300	0.6748	0.6626	0.6598	0.6592	0.6591	0.6591
C_7	0.8600	0.7027	0.6688	0.6612	0.6595	0.6592	0.6591
C_8	0.1000	0.525	0.6283	0.6521	0.6575	0.6587	0.6590
C_9	0.5700	0.6388	0.6545	0.6580	0.6588	0.659	0.6590
C_{10}	0.4000	0.5987	0.6454	0.6560	0.6584	0.6589	0.6590
C_{11}	0.5000	0.6225	0.6508	0.6572	0.6586	0.6590	0.6590
C_{12}	0.6200	0.6502	0.6571	0.6586	0.6589	0.6590	0.6590
C_{13}	0.6000	0.6457	0.656	0.6584	0.6589	0.6590	0.6590
C_{14}	0.7100	0.6704	0.6616	0.6596	0.6592	0.6591	0.6591
C_{15}	0.9000	0.7109	0.6706	0.6616	0.6596	0.6592	0.6591
C_{16}	0.1500	0.5374	0.6312	0.6528	0.6576	0.6587	0.6590
C_{17}	0.2500	0.5622	0.6370	0.6541	0.6579	0.6588	0.6590
C_{18}	0.4500	0.6106	0.6481	0.6566	0.6585	0.6589	0.6590
C_{19}	0.4900	0.6201	0.6502	0.6571	0.6586	0.6589	0.6590
C_{20}	0.6200	0.6502	0.6571	0.6586	0.6589	0.6590	0.6590
C_{21}	0.6591	0.6590	0.659	0.6590	0.6590	0.6590	0.9356

The above-mentioned procedure of simulation was performed with the same dataset of 40 diagnosed cases used for Autism Diagnosis in the study (Kannappan, Tamilarasi, and Papageorgiou 2011). The collected simulation using zT2FCM resulted in 22 cases of Definite Autism, 11 cases of Probable Autism and three cases of No Autism.

5.3.5 Comparison of zT2FCM and a conventional FCM for Autism diagnosis

To demonstrate the effectiveness of the proposed zT2FCM that relies on zT2FSs' weights compared to a trained FCM that relies on T1FSs' weights (Kannappan, Tamilarasi, and Papageorgiou 2011) when both of the FCMs are used for the same purpose, the accuracy of their classification ability as the doctors (experts) is calculated.

In the dataset used for the 40 diagnosed cases by the doctors in (Kannappan, Tamilarasi, and Papageorgiou 2011) for developing Autism's risk , there are 23 reported cases diagnosed as definite Autism, 13 cases diagnosed as probable Autism and 4 cases diagnosed as not Autism.

The results of using FCM with the same dataset for the same purposes as presented in the (Kannappan, Tamilarasi, and Papageorgiou 2011) resulted in 20 cases of Definite Autism, 10 cases of Probable Autism and 3 cases of No Autism. As mentioned earlier in Section 5.3.4, using the proposed zT2FCM with the same dataset resulted in 22 cases of Definite Autism, 11 cases of Probable Autism and 3 cases of Not Autism.

Table 5.8 presents summery of the number of the cases that reported as definite Autism, probable Autism and not Autism using diagnosis of the doctors, conventional FCM and zT2FCM

Table 5.8 Results of Autism diagnosis using the three approaches

	Doctors (ground of truth)	**FCM**	**zT2FCM**
Autism	23	20	22
Probable Autism	13	10	11
No Autism	4	3	3

Therefore, the accuracy of the classification (AC) using conventional FCM and proposed zT2FCM for developing Autism's risk is calculated respectively as follows:

$$AC\ of\ FCM = \left(\frac{\frac{20}{23} + \frac{10}{13} + \frac{3}{4}}{3}\right) \times 100 = 79.63\% \qquad (5.10)$$

$$AC\ of\ z\,T2FCM = \left(\frac{\frac{22}{23} + \frac{11}{13} + \frac{3}{4}}{3}\right) \times 100 = 85.09\% \qquad (5.11)$$

The summary of AC results using the conventional FCM in (Kannappan, Tamilarasi, and Papageorgiou 2011) and the proposed zT2FCM comparing to doctors' decisions is presented in Table 5.9.

Table 5.9 Summary of AC results

	FCM	**zT2FCM**
Autism	$20/23$	$22/23$
Probable Autism	$10/13$	$11/13$
No Autism	$3/4$	$3/4$

Based on (5.10) and (5.11), it is clear that the classification accuracy of the zT2FCM is higher in this case study than the classification accuracy of the FCM. Indeed, it can be noticed that the FCM in (Kannappan, Tamilarasi, and Papageorgiou 2011)failed in diagnosing seven out of the 40 cases, whereas zT2FCM failed in diagnosing only four cases out of the 40 cases; hence the accuracy of diagnosis (AD) of the FCM and the z T2FCM in comparison to the actual diagnosis by the doctors (decision makers) is calculated, respectively, as follows:

$$AD\ of\ FCM = \left(\frac{33}{40}\right) * 100 = 82.5\% \tag{5.12}$$

$$AD\ of\ zT2FCM = \left(\frac{36}{40}\right) * 100 = 90\% \tag{5.13}$$

Therefore, the AD achieved by the zT2FCM is higher than the AD achieved by the FCM.

The abovementioned results demonstrate that the zT2FCM that relies on zT2FS outperforms the conventional FCM in producing results with high accuracy and more close to the decision makers.

The concise results of this chapter have been published and presented at (Al Farsi et al. 2017)

5.4 Conclusion

This chapter proposed incorporating IAA to determine the weights of a FCM and represent them using zT2FSs. In this way the conventional FCM is extended to the zT2FCM. The proposed approach improved the accuracy of the FCM weights and has therefore enhanced the capability of the FCM to offer more accurate output.

To demonstrate the proposed zT2FCM and analyse its effectiveness, a case study on Autism Diagnosis was conducted. This involved improving the traditional existing approach of MCHAT to form an interval valued questionnaire for the purpose of collecting required data to generate the weights of zT2FCM. In this chapter, the MCHAT has improved to F-MCHAT. The F-MCHAT has increased the expressivity and reduced the complexity of the MCHAT. Furthermore, it improved the accuracy of the modelling weights of the zT2FCM.

The results reported in this chapter using zT2FCM were compared with results using standard MCHAT by the doctors and conventional FCM for the same purpose of Autism

Diagnosis. The results of the case study showed that zT2FCM is more accurate than FCM for Autism diagnosis. Based on the results obtained in this chapter, it can be concluded that zT2FCM outperform conventional FCM in capturing more uncertainties and reasoning as decision makers.

Chapter 6 A Non-Iterative Reasoning Algorithm for Fuzzy Cognitive Maps using Type 2 Fuzzy Sets and Late Defuzzification (NILD)

6.1 Introduction

In Chapter 5, the extension of the FCM to the zT2FCM by representing the weights of its causal links through zT2FSs was explicated. The results demonstrated that the zT2FCM can capture more uncertainties from different sources and, hence, it is capable of performing more appropriate reasoning in the presence of uncertainty compared to the conventional FCM. This motivated the author to explore possibilities for a further improvement of the zT2FCM in order to increase its capability for reasoning in the presence of uncertainties or imprecise information. To this end, the following questions are raised: What is the need to defuzzify the zT2FS values of the weights to crisp values? Can weights represented using zT2FSs be used in reasoning and then defuzzify the final output at the end of the reasoning process, i.e., carry out late defuzzification? Does late defuzzification enhance FCM capabilities for handling higher orders of uncertainties about the modelled system and hence improve the reasoning models for supporting the decision making process? Towards answering these questions and achieving the objectives of this thesis, a Non-Iterative Reasoning Algorithm (NILD) for zT2FCM is developed. The new NILD reasoning algorithm that is has been developed relies on late defuzzification of the fuzzy values of weights.

To demonstrate the capabilities of the proposed new reasoning, a case study on Module Performance has been conducted and the proposed zT2FCM with NILD is verified.

This chapter presents the structure (Topology) of the zT2FCM considered and its new reasoning algorithm NILD. The chapter also details the process of creating the case study that was conducted for the purpose of demonstrating the proposed reasoning algorithm NILD. In this case study, a zT2FCM with proposed NILD is constructed for evaluating the performance of a module and it called MPFCM. Indeed, for the purposes of the further validation of NILD reasoning algorithm, this chapter includes results of comparison of the MPFCM, the conventional FCM, existing statistical method and the experts' decisions that have been used for the same purpose. In addition, the sensitivity analysis has been used to demonstrate the reasoning capability of NILD, and the propagation of the uncertainties in MPFCM.

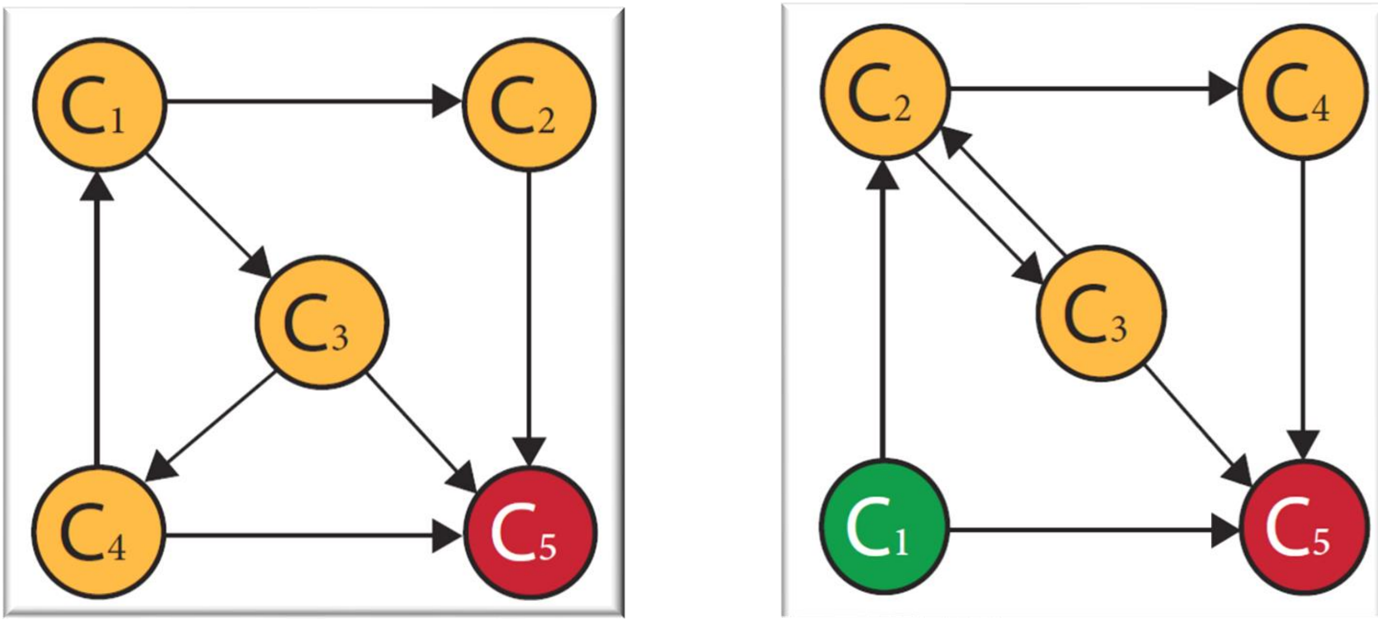
6.2 Topology of zT2FCM with the proposed NILD

The structure of zT2FCM with proposed NILD is the same as the generic structure of FCM that was mentioned in Section 4.2. Therefore, there are m concepts $C_i,\ i = 1,2, \dots m$ in zT2FCM which represent the main aspects of the modelled system. These concepts are linked by directed links that have zT2FSs causal weights $W_{i,j}$, where $W_{i,j}$ is the causal weight of the link directed from a concept C_j $j = 1,2, \dots, m-1$ to a concept C_i , $i = 1, 2, \dots m$. The concepts of zT2FCM and the directed links between them are identified by the experts of the modelled domain. Initially the values of C_i are crisp values between 0 and 1, but the values of $W_{i,j}$ in zT2FCM are zT2FSs as proposed in Chapter 5. An interval valued survey is designed, where the response intervals are extracted and aggregated using IAA as detailed in Chapter 2 to determine the weight of the causal relations between the linked concepts. The weights of zT2FCM capture all the uncertainties the experts have about the causal relations between the concepts.

As stated earlier, the zT2FCM with NILD is an extension of the conventional FCM, in particular zT2FCM by reasoning using the actual zT2FSs that represent weights. For this purpose, the conventional reasoning algorithm (4.2) is modified to a new reasoning algorithm named NILD to accommodate the fuzzy sets representation of the weights. NILD is a non- iterative algorithm that relies on if- then relations existing in the FCM which are neglect while using the conventional iterative reasoning. Therefore, the topology of zT2FCM with NILD is crucial to start reasoning using NILD. For this purpose and based on the classification of FCM's concepts and the relations between them that is defined in Section 4.2, zT2FCM may have one of the following topologies:

- Topology1. There are all three types of the concepts: input concepts, intermediate concepts and decision output as depicted in Figure 4.1
- Topology 2. There are intermediate concepts and output concept, but no input concepts as depicted in Figure 6.1(a).
- Topology 3. All concepts are inputs and there is one decision concept as the proposed zT2FCM depicted in Figure 5.2 and used for identifying the Autism Risk developing.
- Topology 4. There are cyclic relations as in Figure 6.1(b).

After determining the concepts and the zT2FSs weights of the causal links in zT2FCM, initial values of the concepts are received and the NILD reasoning is performed. The details of the proposed NILD considering the listed topologies are presented in Section 6.3.



(a) **(b)**

Figure 6.1 Examples of z T2FCM's Topology

6.3 Reasoning Algorithm NILD

NILD is developed by the author to incorporate the zT2FSs that represent the values of the weights in the reasoning process of the zT2FCM. The NILD comprises three phases, as follows:

Phase 1

In this phase, the values of all causally linked concepts with the exception of decision concept C_m, are evaluated. In other words, all the values of the concepts C_j (antecedent), $j = 1, \dots, m-1$and the concept C_i (consequent), $i = 1, \dots, m-1$ of the if- then causal relation within the zT2FCM are evaluated, excluding the output concept C_m. In this phase, the topology of zT2FFCM determines which concept in the existing causal relations the NILD is starting from as follows:

If zT2FFCM has Topology 1, start the reasoning from an input concept which has the highest initial value comparing to other existing input concepts and causes an effect to an intermediate concept. Therefore, the input concept has to be in the antecedent of if- then relation. After that, evaluate the values of the concepts of the causal relations followed from the former relation until there is no relation other than relations with the decision concept. Then, do repeat the same steps for other input concepts if they exist. Using this procedure the values of all the zT2FFCM's concepts are calculated, except a value of the decision concept.

If zT2FFCM has Topology 2, start the reasoning from the causal relation in which an intermediate concept with the highest value among the rest of the other intermediate concepts represents the antecedent part of the if- then relation of the causal relation. Then follow the consequences of this causal relation until there is no relation other than relations with the decision concept. After that, do repeat the same steps for other intermediate concepts that are not involved in the previous evaluation. Using this procedure the values of all the FCM's concepts are calculated, except a value of the decision concept.

The question which may raise here if in Topology 1 handling the rest of the concepts after determining the input concept to start NILD is either input or intermediate concepts, why do I need Topology 2 then? The answer is just to determine the starting point of reasoning using NILD.

If zT2FFCM has Topology 3, start the reasoning from the input concept with the highest value. Actually as there is no intermediate concept which may affected by the input concept, the reasoning in this topology may start from any input concept to the decision concept and then continue with the rest concepts.

If zT2FFCM has Topology 4, first the resultant of weights W' within each cyclic relation between any two concepts, $C_i \ and \ C_j$, has to be found. The resultant W' is calculated as intersection of the two zT2FSs which represent the weights of the two links between $C_i \ and \ C_j$, namely $W_{i,j}$ and $W_{j,i}$. This intersection is defined in Table 6.3 given later in this section. The rationale of evaluating the resultant relation as intersection is that both relations are between the same concepts, but in the opposite direction, which means that both concepts have a causal relation between themselves. In this way the cyclic relations between $C_i \ and \ C_j$ are reduced to a single relation with the weight W'. It will be directed from the concept with a higher value among them. For example:

- If $C_i > C_j$, then the causal relation will be directed from $C_i \ to \ C_j$ with the weight $W'_{j,i}$
- If $C_i < C_j$, then the causal relation will be directed from $C_j \ to \ C_i$ with the weight $W'_{i,j}$

By applying the former steps, the Topology 4 of the zT2FCM becomes either Topology 1 or Topology 2. Hence, the reasoning is continued according to the resulted topology as mentioned earlier.

It is worth noting, that the topology of the zT2FCM determines the path of the reasoning algorithm and its starting step.

After determining the initial relation to start the reasoning with, the reasoning is carried out for each if – then causal relation directed from causal concepts $C_j, j = 1 \dots m-1$ to the affected concept $C_i, = 1 \dots m$, by calculating pre and post values of concepts C_i as follows:

$$\boldsymbol{C}_{\boldsymbol{i}}^{(\boldsymbol{post})} = \boldsymbol{C}_{\boldsymbol{i}}^{(\boldsymbol{pre})} + \sum_{\substack{\boldsymbol{j=1} \\ \boldsymbol{i \neq j}}}^{(\boldsymbol{m-1})} \left(\boldsymbol{C}_{\boldsymbol{j}} \star \boldsymbol{W}_{\boldsymbol{ij}}\right) \tag{6.1}$$

where $C_i^{(pre)}$ is the pre value of concept C_i before it is affected by concept C_j and $C_i^{(post)}$ is the post value of concept C_i after it is affected by concept C_j. Note that $\sum$ indicates the aggregation (union) considering the impact of all causal concepts on an affected concept. For the purpose of NILD algorithm, the union operator is defined in a novel way as follows:

Let two zT2FSs A and B be given, where each of them is generated using IAA. The number of slices of A and B is depending on the number of the participants and the number of survey's iterations. For the purpose of this definition, let us assume that A and B are generated using IAA based on responses in the form of intervals from three participants surveyed twice. Hence, A and B can be represented as presented in Table 6.1.

In order for the proposed definition of aggregation (union) of A and B, $A + B$, to generate a zT2FS that is compatible with the new reasoning algorithm, it is defined as presented in Table 6.2.

Table 6.1 zT2FSs: A and B

Slice of set A	Level of the slice	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$
Z_1	$z_1 = \frac{1}{3}$	$[a_1,b_1]$	$[c_1,d_1]$
Z_2	$z_2 = \frac{2}{3}$	$[e_1,f_1]$	$[g_1,h_1]$
Z_3	$z_3 = 1$	$[i_1,j_1]$	$[k_1,l_1]$

Slice of set B	Level of the slice	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$
Z_1	$z_1 = \frac{1}{3}$	$[a_2,b_2]$	$[c_2,d_2]$
Z_2	$z_2 = \frac{2}{3}$	$[e_2,f_2]$	$[g_1,h_2]$
Z_3	$z_3 = 1$	$[i_2,j_2]$	$[k_2,l_2]$

Table 6.2 The union of two zT2FSs

Slice	Level of the slice	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$
Z_1	$z_1 = \frac{1}{3}$	$[a_1,b_1] \cup [a_2,b_2]$	$[c_1,d_1] \cup [c_2,d_2]$
Z_2	$z_2 = \frac{2}{3}$	$[e_1,f_1] \cup [e_2,f_2]$	$[g_1,h_1] \cup [g_2,h_2]$
Z_3	$z_3 = 1$	$[i_1,j_1] \cup [i_2,j_2]$	$[k_1,l_1] \cup [k_2,l_2]$

Further on, a new operator that represents the compatibility of a causal node value A with the causal weight B, denoted by $\star$ is defined, where A and B are zT2FSs given in Table 6.1, as presented in Table 6.3.

Table 6.3 The compatibility * of two zT2FSs

Slice	Level of the slice	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = \frac{1}{3}$	$[a_1, b_1] \cap [a_2, b_2]$	$[c_1, d_1] \cap [c_2, d_2]$
Z_2	$z_2 = \frac{2}{3}$	$[e_1, f_1] \cap [e_2, f_2]$	$[g_1, h_1] \cap [g_2, h_2]$
Z_3	$z_3 = 1$	$[i_1, j_1] \cap [i_2, j_2]$	$[k_1, l_1] \cap [k_2, l_2]$

To find $C_i^{(post)}$ in (6.1), the following steps are performed:

Step 1:

For $j = 1 \ldots m-1$, where C_j affects C_i Find $C_j \star W_{ij}$

If C_j is singleton Then

If $C_j \in W_{ij}$ (value C_j belongs to any x-interval of W_{ij} slices) Then

$S = (C_j \star W_{ij})$

where the resultant S is a zT2FS that includes those intervals of W_{ij} that include C_j

Else $S = 0$

End If

Else If C_j is zT2FS

create resultant zT2FS S where each x-interval of S is the intersection of the corresponding x-interval of C_j and x- interval of W_{ij} (see Table 6.3)

End If

End for

Step 2: Find ${C_i}^{(pre)} + S$. (Note that + indicates the union i.e. the aggregation of impacts).

This step depends on the resultant zT2FS S calculated in Step 1 as considered in the following two cases:

Case 1: S has a slice where $y = 1$ and $z = 1$

If ${C_i}^{(pre)}$ belongs to x-interval of S where $y = 1$ and $z = 1$ Then

$S = {C_i}^{(pre)} + S$

${C_i}^{(post)} = S$

Else x-interval of S where $y = 1$ and $z = 1$ is extended to include ${C_i}^{(pre)}$ as follows:

Let us assume that x-interval of S where $y = 1$ and $z = 1$ is an interval (a, b):

If ${C_i}^{(pre)} < a$ Then

x-interval of S becomes $S^* = ({C_i}^{(pre)}, b)$

If ${C_i}^{(pre)} > b$ Then

x-interval of S becomes $S^* = (a, {C_i}^{(pre)})$

End If

$S^* = {C_i}^{(pre)} + S$

${C_i}^{(post)} = S^*$

Note that in Case 1, ${C_i}^{(post)}$ becomes zT2FS.

Case 2: S does not have a slice where $y = 1$ and $z = 1$

Add to S a slice$((C_i{}^{(pre)}, C_i{}^{(pre)}),1,1)$

$M = C_i{}^{(pre)} + S$, where M is a zT2FS, and $C_i{}^{(pre)} + S$ is calculated as union of two zT2FSs as defined above.

$C_i{}^{(post)} = M$

The former steps, Step 1 and Step 2, show that after C_j impacts C_i, the value of $C_i{}^{(post)}$ is either a singleton (crisp), or zT2FS. In Phase 1 the values of all concepts except the decision concept are calculated.

Phase 2

The result of the reasoning algorithm in Phase 1 is used to determine the value of the decision concept C_m as follows (note that initial value of $C_m = 0$).

The value of decision concept C_m is determined based on the weights W_{mi} of the link from the concepts $C_i^{(post)}, i = 1,2, \dots, m-1$, (determined in Phase 1), to decision concept C_m. Therefore, it is calculated as:

$$\boldsymbol{C_m^{(post)}} = \underbrace{\boldsymbol{C_m^{(pre)}}}_{=\boldsymbol{0}} + \sum_{\boldsymbol{i=1}}^{(\boldsymbol{m-1})} (\boldsymbol{C_i} \star \boldsymbol{W_{mi}}) \quad (6.2)$$

where $\sum$ of z 2TFSs is defined above. Therefore, $C_m^{(post)}$ is a z T2Fs.

Phase 3

By applying **Phase 1** and **Phase 2**, the uncertainty of both concepts and weights are captured by postponing any defuzzification until the end of the reasoning. The zT2FS that represents the post value of C_m is defuzzified at the end of the reasoning by using the centroid defuzzification method for z slices as in (2.10).

In conventional reasoning of FCMs, the values of weights and concepts are crisp due to early defuzzification. Thus, most of information captured in zT2FSs may be lost. The late defuzzification of the proposed reasoning algorithm NILD supports preserving and propagating information and input uncertainties until the end of the reasoning process to affect the value of the decision outcome. To demonstrate the effectiveness of proposed NILD, it used with zT2FCM that created for evaluating Module Performance as presented in Section 6.4.

6.4 Case Study: Evaluating Module Performance

To demonstrate the effectiveness and accuracy of NILD, the author created a case study for evaluating the performance of Mathematical modules offered by the Department of Mathematics and Applied Sciences (MASC) at Middle East College (MEC) in Oman. In this case study NILD and other approaches that can be used to evaluate the performance of the modules, including statistical approach used by MEC and conventional FCM and their effectiveness are compared to the experts' decision about the performances of the same module.

Section 6.4.1 includes preliminaries of aspects that are required in order to understand the case study. Section 6.4.2 illustrates the process of developing zT2FCM with the proposed NILD for the evaluation of module performance, and the validation of it is presented in Section 6.5.

6.4.1 Background

6.4.1.1 Module Performance

In an academic institution, determining the Module Performance (MP) is a very important indicator that influences students' progression on a course. Therefore, each institution has subjective quantitative and qualitative mechanisms to evaluate the MP and usually they use percentage of MP to indicate the level of performance i.e., 0% means the performance of students in the module is poor, where 100% means excellent performance. In most institutions, the decision about the MP relies on simple statistics of the modules, such as marks average and standard deviation. However, this may not capture the importance and causal influence of different factors affecting the module performance as well as account for the subjective decision makers' (lecturers) points of view related to it. FCMs have the potential to capture the interplay of these factors. Therefore, NILD ability to extend and operate with this capability which facilitates more effective capture, aggregation and reasoning of lectures subjective opinions for determining MP is analysed.

6.4.1.2 Students Information System (SIS) and Traffic Light System (TLS) at MEC

In MEC, results that students achieve in each of their taught modules are recorded in the Student Information System (SIS). At the end of each semester, students' results for a given module are recorded and the following statistical summaries are calculated: CW - the total mark for the module course work, ESE – total end semester examination result, AVE - the average of both results, CW and ESE, SD - standard deviation of the results, PP - pass percentage, and ATT - the attendance of the students in each module, i.e. percentage of attended module lectures by the students. At MEC, the MP is evaluated using Traffic Light System (TLS) that is based on PP and SD calculated for each module in SIS. TLS classifies the MP into three colour codes: green, amber and red using the ranges for PP and SD as given in Table 6.4.

Table 6.4 TLS colour code system and intervals

Colour	Pass percentage PP	Standard deviation SD	Module performance MP
Green	$\geq 90\%$	$8 - 12$	$66.6 \leq \text{MP} \leq 100$
Amber	$80\% - 89\%$	$5 - 8$ or $12 - 16$	$33.33 < \text{MP} < 66.6$
Red	$< 80\%$	< 5 or > 16	$0 \leq \text{MP} \leq 33.33$

As per the practice at MEC, one of the colours in the TLS is assigned to a module, taking into account both PP and SD achieved. In other words, the results need to fall within both the PP and SD ranges of a colour. In the case the result does not fall in both ranges of PP and SD of a colour, the SD is taken into account to assign the performance colour code to the module. For example, if a module has PP of 72% and SD of 27.49, then both conditions are satisfied and the performance classification of the module is red. However, if a module has a PP of 81.25% and SD of 16.55, then the performance colour code of the module is still classed as red rather than amber, because of the value of the SD.

For the purpose of this research, to make the results obtained from TLS compatible for comparison between results obtained by using zT2FCM with NILD evaluating MP, a conventional FCM and Experts' evaluation , the scale of the resulting MP score obtained by using TLS, ranging from 0 to 100, is split into three equal length sized intervals. The colour of each module is mapped to a corresponding interval as shown in Table 6.4.

6.4.2 Development of Fuzzy Cognitive Map using type 2 fuzzy sets for Module Performance – MPFCM

As mentioned earlier in this chapter, as a step to validate the proposed NILD reasoning algorithm, a case study to evaluate MP at MEC is generated, where zT2FCM with NILD is proposed by the author to evaluate the performance of mathematical modules offered by MASC. and it called MPFCM as it used for evaluating MP. For this purpose, three lecturers from MASC at MEC are polled to define the required concepts of the MPFCM and identify the relations among them as will be detail in Section 6.4.2.1 and Section 6.4.2.2.

6.4.2.1 Concepts of MPFCM

To build MPFCM, three lecturers were polled to define the concepts of MPFCM that were required for determining the MP and the causal relations among them. The lecturers agreed that the required concepts are Attendance of the students (ATT), Coursework Results of the Module (CW) and the results of the End Semester Examination (ESE). Indeed, the lecturers agreed the following causal relations among these three defined concepts and the decision concept MP:

1. The ATT has a direct effect on the module results in CW and ESE, but it does not has direct effect on the MP.
2. There is no effect between the results of CW and ESE, but each of them has a direct effect on the MP.

Therefore, based on the classification of the FCM's concepts which are mentioned in Section 4.2, the concept ATT is an input concept, CW and ESE are intermediate concepts and MP is the decision concept. Figure 6.2 presents the structure of MPFCM that comprises of these concepts and the causal relations between them.

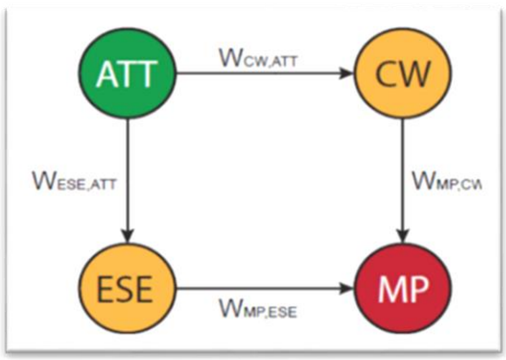


Figure 6.2 MPFCM

6.4.2.2 Weights of MPFCM

The next step after identification of the concepts and the causal relations between them is the determination of the weight of the causal relations, namely $W_{ESE,ATT}$, $W_{CW,ATT}$, $W_{MP,CW}$ and $W_{ESE,MP}$. For that purpose, an interval-valued survey is designed to collect the lecturers' opinions about each causal relation in MPFCM. As there are four causal relations (see Figure 6.2), the survey contains four questions , given in Appendix 5, where each of the question aims to determine to which level the concept C_j affects the concept C_i in a specific causal relation. Each of the three lecturers has to respond to each question by drawing an ellipse on a Likert scale, and, hence, the intervals which reflect the causal relations between the two concepts in the question are identified. Figure 6.3 shows an example of one of the survey's question that is used to determine the weight of the causal relation between CW and MP, and Table 6.5 shows the extracted intervals from the survey's responses by the three lecturers to this question.

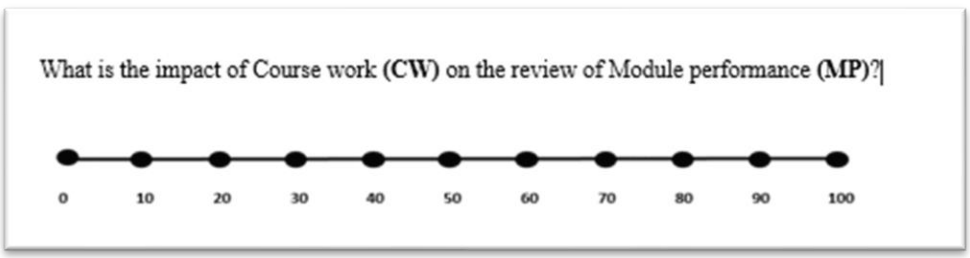


Figure 6.3 An Example of the question to determine weight of causal relation from CW to MP

Table 6.5 Intervals Responses to determine the weight of causal relation from CW to MP

	First Iteration of the Survey	Second Iteration of the Survey
Lecturer 1	[0.38, 0.72]	[0.45, 0.65]
Lecturer 2	[0.60, 0.88]	[0.60, 0.80]
Lecturer 3	[0.30, 0.75]	[0.40, 0.65]

The rationale of collecting the responses as intervals is to allow capturing more uncertainty about assigning the weight of the causal relation. To capture the intra uncertainties of the lecturers in providing the responses' intervals, the survey is circulated among the same lecturers after four weeks and the intervals extracted again from the lecturers for the second iteration as shown in the example presented in Table 6.5. The collected responses' intervals of the three lecturers to each of the four questions in both iterations are presented in Appendix 6.

Referring to Section 3.3, for determining the weight as zT2FS of each causal relation in MPFCM, the author applies IAA with $N = 3,\ S = 2$ as three lecturers are surveyed twice and, therefore, $y_1 = \frac{1}{2}$ and $y_2 = 1$. It is worth noticing that the generated zT2FS of each causal relation represents the fuzzy agreement model of its weight, which captures both the inter-uncertainty and intra-uncertainty of the lecturers about assigning the weight of the causal relation. Indeed, the third dimension of the resulted zT2FS represents the level of the agreement between the lecturers about assigning the weight of the causal relation.

For example, Table 6.6 presents the weight $W_{MP,CW}$ of the causal relation from CW to MP as zT2FS generated by IAA using intervals represent the lecturers' responses to determine this causal relation (presented in Table 6.5). The reader may refer to the synthetic example in Section 3.3.1 for further illustration on generating z slices using IAA. The representation of the MPFCM's weights as zT2FSs are presented in Appendix 7.

Table 6.6 Weight $W_{MP,CW}$

Slice of $\boldsymbol{W_{MP,CW}}$	$\boldsymbol{z_i}$	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$
$\boldsymbol{Z_1}$	$\frac{1}{3}$	$[0.30, 0.88]$	$[0.40, 0.80]$
$\boldsymbol{Z_2}$	$\frac{2}{3}$	$[0.38, 0.75]$	$[0.45, 0.65]$
$\boldsymbol{Z_3}$	1	$[0.60, 0.72]$	$[0.60, 0.65]$

Once, the weights of MPFCM are determined, the concepts values of each module can be input and then the reasoning using NILD can be carried out to infer the decision about the performance of the module.

6.4.2.3 Application of NILD to MPFCM

Reasoning using NILD in MPFCM to infer MPs is validated using thirty mathematical modules that are selected by the author. Their statistic recorded in SIS are used as input values of the MPFCM's concepts. For example, the input values of the concepts required for determining MP of Module 22 using MPFCM which obtained from SIS are presented in Table 6.7. The inputs values for all modules are presented in Appendix 8.

Table 6.7 inputs values of Module 22

Module 22	
ESE	0.72
CW	0.796
ATT	0.81

For each module, the values of the ESE, CW and ATT are input to MPFCM and the reasoning is performed using NILD and the outcome of the concept MP is achieved to represent the evaluation of the performance of the module. Thereafter, the resulted zT2FSs of MP of all the thirty modules are defuzzified and their crisp centroids are calculated. Note that the value MP is zT2FS that captures all the uncertainties about the MP; the reasoning process incorporates the initial values of the concepts and weights, without any reprocessing. This is the main aim of the new NILD algorithm that avoids early defuzzification as it leads to the loss of information that should be kept in reasoning. Section 6.4.2.4, includes an example to illustrate how an MP decision outcome is achieved using NILD for one module, namely Module 22. The results for evaluating the performance of each of the thirty modules are presented in Appendix 9. Additionally, their centroids are presented in Table 6.9 as S_1

6.4.2.4 Example

Data required for the reasoning algorithm to determine the MP for Module 22, was collected from the SIS (refer to Table 6.7) as follows: $CW = 0.796, ESE = 0.72$ and $ATT = 0.81$. Note that these values represent the pre values of the MPFCM's concepts CW, ESE and ATT, respectively. Recall that the weights $W_{CW,ATT}$,$W_{ESE,ATT}$, $W_{MP,CW}$

and $W_{MP,ESE}$ were generated as illustrated in Section 6.4. Using MPFCM shown in Figure 6.2 and following NILD algorithm i.e., the phases detailed in Section 6.3, post values of $ATT^{(post)}$, $CW^{(post)}$, $ESE^{(post)}$, and MP are obtained as follows:

$ATT^{(post)} = ATT^{(pre)} = 0.81$, as ATT is input concept that is not affected by other concept (refer to Section 6.3) and thus its post value equals to its pre value.

$CW^{(post)} = CW^{(pre)} + (ATT^{(post)} \star W_{CW,ATT})$,

$CW^{(pre)} = 0.796$ and as $ATT^{(post)} \in W_{CW,ATT}$, then based on Phase 1, Step 1 of NILD algorithm, $(ATT^{(post)} \star W_{CW,ATT})$ and $CW^{(post)}$ become z T2FS as shown in Table 6.8.

Table 6.8 Calculation of $(ATT^{(post)} \star W_{CW,ATT})$ and $CW^{(post)}$

$\boldsymbol{ATT^{(post)} \star W_{CW,ATT}}$				$\boldsymbol{CW^{(post)}}$			
Slice	Level of the slice	$y_1 = 0.5$	$y_2 = 1$	Slice	Level of the slice	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = \frac{1}{3}$	[0.47,0.82]	ϕ	Z_1	$z_1 = \frac{1}{3}$	[0.47,0.82]	ϕ
Z_2	$z_2 = \frac{2}{3}$	ϕ	ϕ	Z_2	$z_2 = \frac{2}{3}$	ϕ	ϕ
Z_3	$z_3 = 1$	ϕ	ϕ	Z_3	$z_3 = 1$	ϕ	[0.796,0.796]

Therefore, using Case 2 of Step 2 of the NILD algorithm, $CW^{(post)}$ becomes a zT2FS as presented in Table 6.8.

$ESE^{(post)}$ is calculated as follows:

$$ESE^{(post)} = ESE^{(pre)} + (ATT^{(post)} \star W_{ESE,ATT})$$

As $ATT^{(post)} \notin W_{ESE,ATT}$ based on Step 1 in Phase 1 of the NILD algorithm,

$ATT^{(post)} \star W_{ESE,ATT} = 0$ and, therefore,

$$ESE^{(post)} = 0.72 + 0 = 0.72.$$

Following Phase 2 of NILD algorithm, the MP of Module 22 is calculated as follows:

$$MP^{(post)} = MP^{(pre)} + \left((CW^{(post)} \star W_{MP,CW}) + (ESE^{(post)} \star W_{MP,ESE})\right).$$

where $MP^{(pre)} = 0$. By following Step 1 in Phase 1 of the reasoning algorithm, the z slices of $CW^{(post)} \star W_{(MP,CW)}$ and $ESE^{(post)} \star W_{(MP,ESE)}$ are obtained as follows:

$\boldsymbol{CW^{(post)} \star W_{MP,CW}}$			
Slice	Level of the slice	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = \frac{1}{3}$	[0.47,0.82]	ϕ
Z_2	$z_2 = \frac{2}{3}$	ϕ	ϕ
Z_3	$z_3 = 1$	ϕ	ϕ

$\boldsymbol{ESE^{(post)} \star W_{MP,ESE}}$			
Slice	Level of the slice	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = \frac{1}{3}$	[0.38,0.90]	[0.55, 0.80]
Z_2	$z_2 = \frac{2}{3}$	[0.50,0.85]	[0.65, 0.75]
Z_3	$z_3 = 1$	[0.58, 0.77]	[0.68, 0.72]

Following Phase 2 and Phase 3 of the reasoning algorithm (union operation defined in Table 6.2), MP is calculated as zT2FS and defuzzified, respectively as follows:

	Slice	**Level of the slice**	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$	**Centroid of the slice**	**Overall centroid**
$\boldsymbol{MP}$	Z_1	$z_1 = \frac{1}{3}$	[0.38,0.90]	[0.55,0.80]	$\frac{1}{3}/0.6633$	0.686944
	Z_2	$z_2 = \frac{2}{3}$	[0.50,0.85]	[0.65,0.75]	$\frac{2}{3}/0.6917$	
	Z_3	$z_3 = 1$	[0.58,0.77]	[0.68,0.72]	$1/0.6917$	

Hence, $MP^{(post)} = 0.686944444$.

Therefore, the performance of the module 22 is MP = 68.69%

6.5 Validation of NILD

To validate the proposed NILD for zT2FCM, the author compared its effectiveness to match experts' decision comparing to the conventional FCM where weights are represented using T1FSs and statistical methods that are used in SIS, described above. The following methods are applied to the problem of evaluating module performances and compared with evaluations generated by MPFCM and NILD algorithm:

T1FCM: Construct an FCM that has the same topology and same purpose as MPFCM, but it relies on T1FSs weights (T1FCM). These weights are obtained from a single second iteration of the survey given in Appendix 5. The standard iterative reasoning algorithm (4.2) is used. It is worth noting that the centroids of these T1FSs are used as the weights of T1FCM. The rationale of using the second iteration is that, the author believes that the lecturers may have better understanding and information about the survey's questions in the second iteration compared to the first iteration. The values of the input concepts of the constructed T1FCM are obtained by SIS.

Lecturers' evaluation: Collect the opinions of selected lecturers about selected module performances given the values of the module statistics obtained by SIS. The subjective opinions of the lecturers are collected through an interval survey designed for this purpose presents in Appendix 10. The lecturers provided the intervals' answers based on their experiences on evaluating the performance of selected mathematical modules. After the responses' intervals are collected, they are aggregated using IAA. In this way the fuzzy agreement model of each module is created as a T2FS, which captures lecturers' opinions on the MP. These generated T2FSs are defuzzified and their centroids are recorded.

TLS used in MEC: Obtain evaluation of MPs of the selected modules delivered at MEC's Mathematical department from SIS.

The results obtained from each of the above-mentioned methods are collected and comparison is conducted using Pearson correlation ρ, where ρ between the results of the Lecturers' evaluation and the results of MPFCM with NILD, results of T1FCM and results of TLS at MEC are calculated as present in Section 6.5.4.

The motivation of using the model of lecturers' opinion on modules' performance as a benchmark for this comparison is of two folds; first, there is no ground of truth for this evaluation, and, second, the aim is to determine which of the listed methods generates decisions that are more correlated to humans' decisions.

6.5.1 T1FCM

As pointed earlier in Section 6.5, the results of the MPFCM will be compared with the reasoning's results obtained using the T1FCM created for the same purpose of the modules' performances. T1FCM has the same structure as the MPFCM, but the weights in T1FCM are represented by T1FSs obtained from a single second iteration of the survey that used to determine the weights of MPFCM (the survey presented in Appendix 5). The rationale for selecting the responses of a second iteration is the author assume that by the time when the second survey is conducted, the lecturers may have better understanding about the survey's questions and hence their responses are with less uncertainty.

The intervals' responses of the three lecturers for the second iteration of the survey are collected as presented in Appendix 6. Taking into account each of the four questions Q_s, where $s = 1,2,3,4$, the response's intervals are aggregated using the first phase of IAA (presented in Section 3.3) to generate T1FS of the corresponding question's weight as follows:

Given that the response interval of lecturer l to answer question Q_s is $I^{l}_{Q_s}$, where $l = 1,2,3$ as there are three lecturers, the T1FS generated for each Q_s ($T1FS_{Q_s}$) is calculated as follows;

$$T1FS_{Q_s} = \frac{1}{3}/\left(I^1_{Q_s} \cup I^2_{Q_s} \cup I^3_{Q_s}\right) + \frac{2}{3}/\left(\left(I^1_{Q_s} \cap I^2_{Q_s}\right) \cup \left(I^1_{Q_s} \cap I^3_{Q_s}\right) \cup \left(I^2_{Q_s} \cap I^3_{Q_s}\right)\right) + 1/\left(I^1_{Q_s} \cap I^2_{Q_s} \cap I^3_{Q_s}\right), s = 1, \dots, 4 \tag{6.3}$$

The complete calculations of generating the four $T1FS_{Q_s}$ that represent the four weights of causal links in T1FCM are presented in Appendix 11.

Subsequently, the centroid of each generated T1FSs is calculated to be used in the iterative reasoning process (See Appendix 11).

In this way, the T1FCM is prepared for reasoning; it receives the inputs from the SIS of the same thirty modules used in PMFCM using the iterative reasoning algorithm (4.2). The results of this reasoning process are captured in Table 6.9 as S_2

6.5.2 Agreement model of the lectures' opinions

As stated in Section 6.5, to compare the effectiveness of MPFCM over other methods for evaluating module performance, the author aims to compare their results of reasoning with the decision of humans performed for the same purpose.

For this reason, a fuzzy agreement model of the lecturers on the performance of each of the thirty modules, based on the values provided by SIS and subjective lecturers' responses to the survey questions is generated using IAA. These 30 agreement models represent the benchmark of the MPs for this case study as there is no ground truth for assessing the performance of the modules.

An interval-valued survey is designed to collect the opinions of the three lecturers, who polled earlier to design the MPFCM, about the MP given the same inputs values provided for MPFCM. For example, each lecturer is asked to express his/her view about a specific MP given the module values of ESE, CW, and ATT. The survey designed for this purpose is provided in Appendix 10. Figure 6.4 shows a survey questions that is used to generate the agreement model about the MP of Module 22.

How do you evaluate the module performance, if the result summary of a module at the end of the semester is as follows

1. End Semester Examination result is 72%
2. Course Work result is 79.6% and
3. Attendance percentage is 81%.

Figure 6.4 A Question to determine the MP of Module 22 based on the teachers experiences

Thereafter, the intervals of the responses from the three lecturers are obtained for each module. They are aggregated using IAA to generate fuzzy agreement models that reflect and capture the inter uncertainties of the lecturers about their decision on MPs of the selected modules.

Those agreement models are used as benchmark in this case study; their centroids S_3 are calculated as listed in Table 6.9.

6.5.3 Statistical view on MP using TLS

As a further step in demonstrating the potential benefits of using NILD for reasoning more similar to lecturers' reasoning, the MP of the same thirty modules based on TLS are collected as described earlier in Section 6.4.1.2. It is worth reminding that TLS relies on the statistical attributes PP and SD only to appraise MP.

After collecting data for the modules using the TLS and SIS system and determining the modules' colours and the corresponding intervals, the centroid of each interval is calculated as the midpoint of the corresponding interval. The results of MP using TLS in SIS are presented in Table 6.9 as S_4.

6.5.4 Results and Comparison

As mentioned earlier the lecturers' opinions are used as the benchmark to perform the comparison with the results of the methods presented in Section 6.4 and Section 6.5 to evaluate the MP. Applying the methods listed above, namely MPFCM, T1FCM, Lecturers' agreement and TLS results i.e., output values that represent MP of each of the 30 modules are obtained and presented in Table 6.9

Table 6.9 Centroids of the methods' results

Module	MPFCM with NILD algorithm (S_1)	T1FCM with iterative reasoning (S_2)	lecturers' opinions (S_3)	Traffic Light System (S_4)
1	0.4363889	0.8636053	0.6773333	0.1666667
2	0.4513889	0.8592155	0.679375	0.1666667
3	0.6697222	0.8584359	0.67075	0.1666667
4	0.2727778	0.8584454	0.5774167	0.1666667
5	0.635	0.8585328	0.6420833	0.1666667
6	0.1105556	0.857563	0.6165	0.1666667
7	0.4430556	0.8581518	0.7150833	0.1666667
8	0.4513889	0.8595942	0.7938333	0.5
9	0.2905556	0.8583725	0.557125	0.1666667
10	0.2808333	0.8588445	0.70205	0.1666667
11	0.2875	0.858565	0.739525	0.5
12	0.2393176	0.8576333	0.5529167	0.1666667
13	0.1622222	0.8599275	0.5348667	0.1666667
14	0.4363889	0.8587458	0.6812667	0.1666667
15	0.4363889	0.8586817	0.6975833	0.5
16	0.2952778	0.8570444	0.8038167	0.1666667
17	0.2783333	0.8582224	0.7750833	0.1666667
18	0.6088889	0.8583158	0.6920333	0.1666667
19	0.285	0.8583158	0.459225	0.1666667
20	0.4141667	0.8579079	0.5869167	0.1666667
21	0.2783333	0.8581532	0.64475	0.5
22	0.6869444	0.8574953	0.7572833	0.1666667
23	0.2783333	0.8584507	0.613175	0.1666667
24	0.2783333	0.8577901	0.588175	0.1666667
25	0.5068556	0.85739	0.6063833	0.1666667
26	0.635	0.8577283	0.6372417	0.1666667
27	0.2808333	0.8581545	0.6507833	0.5
28	0.1827778	0.857078	0.6360917	0.1666667
29	0.4513889	0.8585852	0.6360917	0.5
30	0.2905556	0.8577709	0.5411667	0.1666667

For this purposes, the Pearson correlation coefficient ρ is used to evaluate the strength of the relation between the results produced from the lecturers' opinions (S_3) and the results of the other three methods, namely MPFCM with NILD algorithm (S_1), T1FCM with iterative reasoning (4.2) (S_2) and TLS (S_4).

Generally, Pearson correlation test is conducted to determine the strength of the relation between two variables X and Y as present in Table 6.10

Table 6.10 Correlation values

$\boldsymbol{\rho_{X,Y}}$	**Strength of the relation**
$0 < \rho < 0.3$	Low
$0.3 \leq \rho < 0.5$	Medium
$0.5 \leq \rho \leq 1$	High

where

$$\rho_{X,Y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{6.4}$$

$\rho_{X,Y}$ is correlation coefficient between the variables X and Y,

x_i and y_i are values of X variable and Y variable respectively in the sample and

$\bar{x}$ and $\bar{y}$ are means of the values of the X variable and Y variable respectively.

Therefore, by using (6.4), the values of ρ_{S_1,S_3}, ρ_{S_2,S_3} and ρ_{S_4,S_3} are calculated and the results are presented in Table 6.11

Table 6.11 Correlations results

$\boldsymbol{\rho_{(S_1,S_3)}}$	$\boldsymbol{\rho_{(S_2,S_3)}}$	$\boldsymbol{\rho_{(S_4,S_3)}}$
0.34	0.08	0.28

These results indicate that MPFCM has a moderate correlation of 0.34 with the lecturers' opinions. It is the highest correlations comparing to the low correlation of 0.28 and 0.08 between the lecturers' opinions and each of TLS and T1FCM, respectively.

Above mentioned results of correlation indicate that the proposed NILD algorithm applied to MPFCM has more potential to imitate the lecturers decisions comparing to TLS and T1FCM.

To explore the ability of NILD in preserving and propagating the uncertainties derived from the changes in uncertainty of participants' responses, a sensitivity analysis is conducted as presented in Section 6.6.

6.6 Sensitivity analysis

In order to check if the proposed NILD allows for the uncertainty propagation, a sensitivity analysis is conducted as is detailed in this Section 6.6. The conducted sensitivity analysis relies on investigating how the value of the decision concept in zT2FCM with NILD is affected if there is a change in uncertainties of each of the weight in zT2FCM. The coefficient of the determination R^2 is used to address to which level is the proposed zT2FCM with NILD sensitive to the change in the uncertainties.

To further investigate and understand how proposed NILD algorithm operates in the presence of changes FCM's weights and how it allows for the uncertainty to propagate, a sensitivity analysis is performed. For this purpose, the MPFCM is used to conduct the sensitivity analysis. The uncertainty of each weight in the MPFCM is changed by δ, where δ= 0.01, 0.025, 0.05, 0.07, 0.1. The change of the weights is created by extending the response intervals for each question from each lecturer by the above mentioned values of δ and then aggregated using IAA to produce the corresponding weights in the form of zT2FSs as described in Section 6.4. This emulated increasing the uncertainty of each weight as the widths of the intervals are increasing (Miller et al. 2012).

The process of performing the sensitivity analysis was carried out in the following steps:

Step 1: The width of response intervals of each question from the lecturers are changed by δ keeping the centre of the interval the same, i.e., boundaries of the interval are extended by δ. Consequently, the uncertainty of the intervals are increased. It is worth noting here that in case the boundary value becomes less than 0, it is fixed as 0, and in case it is more than 1, it is fixed to be 1. The rationale of this is that weights' values generated from intervals represent the percentage of the MP which belong to interval [0,100] that is normalised to a value in interval [0, 1]. The response intervals of the lecturers after changing their width by δ are presented in Appendix 12.

Step2: The produced intervals in Step 1 are aggregated as to produce zT2FS for each weight of MPFCM as detailed in Section 6.4.2.2. The generated weights after each change of the responses' intervals by δ are presented in Appendix 13. In this step, the values of the weights are prepared to be used in MPFCM for reasoning on the MP.

Step 3: Reasoning on MPFCM using NILD is performed to determine the value of the decision concept MP for each module considering one changed weight from the weights produced in Step 2 and keeping the remaining weights the same. For example, the value of $W_{MP,CW}$ is used when its uncertainty is changed by $\delta = 0.01$ and the values of the remaining weights, $W_{CW,ATT}$, $W_{ESE,ATT}$ and $W_{MP,ESE}$ are kept the same, as obtained in Section 6.4.2.2.

Step 4: Step 3 is repeated for the rest of the weights for each change of the uncertainty in the weights using parameter δ.

6.6.1 Example for process of performing the sensitivity analysis

This example presents how to perform the sensitivity analysis using above-mentioned steps when the uncertainty of $W_{MP,CW}$ is changed by $\delta = 0.01$.

Step 1, the width of response intervals from the lecturers to determine the weight of causal relation from CW to MP which presented in Table 6.5 becomes as follows:

	First Iteration	**Second Iteration**
Lecturer 1	[0.37, 0.73]	[0.44, 0.66]
Lecturer 2	[0.59, 0.89]	[0.59, 0.81]
Lecturer 3	[0.29, 0.76]	[0.39, 0.66]

Step 2, the intervals which obtained in Step1 are aggregated to produce zT2FS that represent the value of $W_{MP,CW}$ when its uncertainty is changed by $\delta = 0.01$as follows:

Slice	Level	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = 1/3$	$[0.29, 0.89]$	$[0.39, 0.81]$
Z_2	$z_2 = 2/3$	$[0.37, 0.76]$	$[0.44, 0.66]$
Z_3	$z_3 = 1$	$[0.59, 0.73]$	$[0.59, 0.66]$

Step 3, the value of $W_{MP,CW}$ which obtained in Step 2 are used in reasoning on MPFCM using NILD, where the values of the remaining weights, $W_{CW,ATT}$, $W_{ESE,ATT}$ and $W_{MP,ESE}$ are kept the same, as obtained in Section 6.4.2.2. Then the results of this reasoning process are collected for the 30 Modules as presented in Appendix 14.

Step 4, repeat Step 3 for the rest of the weights for each change of the uncertainty in the weights using parameter δ.

The results of sensitivity analysis using former Steps with all weights are presented in Appendix 14. For this purpose, a software is developed and implemented using MATLAB as presented in Appendix 15.

The observations and discussions on these results are provided in next Section 6.6.2.

6.6.2 Results and Discussions

The following observations and conclusions are made:

- The change of uncertainty in $W_{ESE,ATT}$ by $\delta = 0.01$ and 0.025 does not affect the decision concept value. Changing the uncertainty by $\delta = 0.05, 0.7, 0.1$ changes the decision concept's value only for a small number of modules, for example Module 6, Module 12, Module 16, Module 20, Module 22, Module 25, Module 26, Module 28 and Module 30 (refer to Appendix 14). The Pearson correlation coefficient between the original values of the decision concept and its values after changing the uncertainty of $W_{ESE,ATT}$ is 1 when $\delta = 0.01$ and 0.025 , which represents a strong association. However, it starts declining slightly when $\delta = 0.05$, 0.7 and 0.1, from 0.97 to 0.9 as these changes in uncertainty in $W_{ESE,ATT}$ affected some modules, as mentioned earlier.

- The change of uncertainty in $W_{CW,ATT}$ by $\delta = 0.01$ affects the decision concept value for some modules, but with higher increases from $\delta = 0.025$ to $\delta = 0.1$, the decision concept's value is changed for almost 90% of the modules (refer to Appendix 14). The Pearson correlation coefficient between the original values of the decision concept and its values after changing the uncertainty of $W_{CW,ATT}$ is 0.98, when δ = 0.01 and it start declining when δ is increased, but it still reflects a high association. Although both weights $W_{ESE,ATT}$ and $W_{CW,ATT}$ link input concept ATT and intermediate concepts CW and ESE, it can be observed that the impact of changing uncertainty in $W_{CW,ATT}$ is a fraction higher than the impact when $W_{ESE,ATT}$ is changed. The reason is that the level of agreement (modelled by overlapping intervals) in $W_{ESE,ATT}$ is a fraction higher than the agreement in $W_{CW,ATT}$, as it is presented in Appendix 7. Hence, the influence of increasing uncertainty in $W_{ESE,ATT}$ is a fraction smaller than the influence of increasing uncertainty in $W_{CW,ATT}$.

- The change of uncertainty in $W_{MP,CW}$ by $\delta = 0.01$ and 0.025 slightly affects the decision concept value for some modules, but by increasing δ further, there is a noticeable change in the decision concept's values for most modules. The Pearson correlation coefficient between the original values of the decision concept and its values after changing the uncertainty in $W_{MP,CW}$ is 0.93, when δ = 0.01. It declines by increasing δ to reach 0.32 when δ = 0.1, hence the correlation between the values declines from a high (0.93) to a medium value (0.32).
- The change of uncertainty in $W_{MP,ESE}$ affects the decision concept's value for most modules. By increasing δ, the Pearson correlation coefficient between the original values of the decision concept and its values is declined from 0.91 to 0.69, thus it could be concluded that this association is still high. However, the impact of changing the uncertainty in $W_{MP,CW}$ is higher than the impact when uncertainty in $W_{MP,ESE}$ is changed. This could be justified as follows. There is more uncertainty in the agreement (width of the intervals is higher) in $W_{MP,CW}$ than in $W_{MP,ESE}$. Hence, increasing the uncertainty in $W_{MP,CW}$ led to more overlap with other intervals generated from intersection and union of the concept (CW^{POST}) and weight $W_{MP,CW}$ during NILD using (6.1) and consequently there was a more effect on the value of decision concept.
- The impact of changing uncertainty in weights of direct links to the decision concept, such as $W_{MP,CW}$ and $W_{MP,ESE}$, is higher than the impact of changing uncertainty in weights of links between input and intermediate concepts, such as weights $W_{ESE,ATT}$ and $W_{CW,ATT}$. However, the effect of changing uncertainty in $W_{CW,ATT}$ (where there was less agreement in its zT2FS and, therefore, more

uncertainty) is combined with the effect of changing uncertainty in $W_{MP,CW}$, and the decision concept value is therefore, affected more.

The coefficient of determination R^2 is also calculated to determine to what level the outputs are affected by changes in input uncertainties based on linear regression. The coefficient of determination R^2 is calculated between the original values of MP, when there are no changes in weights, and its values when changes by δ are applied. Table 6.12 presents the values of R^2 between the original values of MP and its values when there is a change of δ in a weight. The value of R^2 is used to determine the percentage of modules with values of MP not affected by changing the uncertainty of a specific weigh, and consequently, the percentage of the modules which are affected $(1 - R^2)$. For example, R^2 between the original values of MP and its values when there is a change in weight $W_{CW,ATT}$ by δ=0.05, is 0.845 as presented in Table 6.12. This indicated that with a change in uncertainty of weight $W_{CW,ATT}$ by 5%, MP of 84.5% of the 30 modules (around 25 modules) remained the same, while it changed for 15.6% modules (5 modules) when uncertainty in $W_{CW,ATT}$ is changed by δ=0.05.

Table 6.12 Values of R^2

$\boldsymbol{\delta}$	$\boldsymbol{W_{ESE,ATT}}$	$\boldsymbol{W_{CW,ATT}}$	$\boldsymbol{W_{MP,CW}}$	$\boldsymbol{W_{MP,ESE}}$
0.010	1	0.958	0.869	0.824
0.025	1	0.908	0.659	0.740
0.050	0.952	0.845	0.369	0.540
0.070	0.789	0.747	0.227	0.530
0.100	0.814	0.747	0.105	0.476

From Table 6.12, we can observe the following:

1) In most of the cases, as δ is increasing, R^2 is decreasing ($1 - R^2$ is increasing). Therefore, as the uncertainty in the weights increase, the change in MP's values increase too.

2) When uncertainty in $W_{(ESE,ATT)}$ and $W_{(CW,ATT)}$ is changed by δ, from 0.01 to 0.1 (this represents a change in uncertainty from 1% to 10%), there is a slight change in the output value of MP, but the regression was still high; the values of R^2 decrease from 1 to 0.814 and from 0.958 to 0.747, when these uncertainties change to $W_{ESE,ATT}$ and $W_{CW,ATT}$, are applied, respectively. Therefore, we can conclude that MP is less sensitive to changes in uncertainty in the weights $W_{ESE,ATT}$ and $W_{CW,ATT}$.

3) By increasing the uncertainty in $W_{MP,CW}$ and $W_{MP,ESE}$, there is a considerable change in the output values of MP. The values of R^2 drop from 0.869 to 0.105 and from 0.824 to 0.476, when uncertainty changes in $W_{MP,CW}$ and $W_{MP,ESE}$ are increased from 1% to 10%, respectively. Hence, we can conclude that the value of MP is more sensitive to the changes in uncertainty in the weights $W_{MP,CW}$ and $W_{MP,ESE}$ compared to changes in uncertainty in the weights $W_{ESE,ATT}$ and $W_{CW,ATT}$.

The previous observations showed that the proposed NILD algorithm propagated well the uncertainty in weights and the decision concept values obtained were sensitive to changes in the uncertainty of FCM's weights. Greater sensitivity to uncertainty changes in weights of a direct link to the decision concept was observed. It was also observed that a considerable change in the value of the decision concept could occur when the cause concept was intermediate. In this case, the intermediate concept was previously affected by other concepts while uncertainty was propagated between other affected concepts and finally to the decision concept.

6.7 Conclusion

This chapter proposed a new approach to reasoning in FCMs where the weights of links are zT2FSs generated by IAA. NILD can be applied to zT2FCMs of different topology. New operations in NILD are defined in such a way as to make the reasoning compatible with zT2FS. NILD preserves the captured uncertainties throughout the causal reasoning process by delaying the defuzzification to the end of the process. This makes the new reasoning algorithm more robust in comparison to the conventional iterative reasoning methods which rely on early defuzzification, where information/uncertainties maybe lost.

To evaluate the effectiveness of the proposed reasoning algorithm, a case study of evaluating module performance was conducted using real data about module performance obtained from MASC in MEC. This data is used to construct a new FCM, MPFCM, with the decision concept that represents module performance. The new reasoning algorithm NILD is used to determine performance of 30 selected modules. The experiments demonstrated the ability of the proposed NILD to produce results that were more correlated to experts' decisions compared to the results obtained by using TLS currently used at MEC and a new FCM with T1FSs weights (T1FCM) created for the same purpose. This underpins the ability of NILD to better mimic human reasoning in the presence of intra and inter uncertainties in the opinions of domain experts. Sensitivity analysis is conducted to further validate NILD and analyses its ability to help propagate input uncertainties within the FCMs structure which impact the decision concept. The results of the sensitivity analysis showed that both changes in uncertainties in zT2FS represented weighted links direct from intermediate concepts to the decision concept and from input to intermediate concepts impacted the value of the decision concept to different degrees. Indeed, the results demonstrated that NILD algorithm enabled a propagation of uncertainty in weights which affected outcome decisions.

The results of the conducted experiments analysed in this chapter demonstrated that NILD is capable to capture and propagate uncertainties while reasoning and thus determines decisions that mimic human reasoning in the presents of uncertainty. Consequently, zT2FCM with NILD can outperform the conventional FCM in reasoning.

Chapter 7 Conclusions

7.1 Conclusion

This thesis presents extensions to the conventional FCM by introducing zT2FSs to model the weights of the links among the concepts in order to improve its capabilities to capture more uncertainties and reasoning to infer an output closer to the human decision. The literature revealed that the weights of the causal relations in the FCM play a robust role in making the FCM more effective for reasoning and knowledge representation. So far the weights of the conventional FCM and its extensions rely either on crisp values or T1FSs and this hinder the ability of FCM to capture high level of uncertainties during the construction and / or reasoning. As demonstrated in chapter 3, IAA is an effective method that generates a fuzzy agreement model based on zT2FSs. It is capable for capturing more uncertainties about human subjective opinions on a particular matter. Motivated by this, this thesis is focused on incorporating the use of zT2FSs generated by IAA to the FCM.

As mentioned earlier in Chapter 1, the aim of this thesis was of two folds as follows:

1. Extending the conventional FCM to zT2FCM that is capable for capturing more uncertainties,
2. Enhance the reasoning of zT2FCM by introducing new reasoning algorithm NILD, where the reasoning is carried out with weights values as zT2FSs without defuzzification.

This aim was achieved by meeting the following objectives of the thesis:

- Analyse the effectiveness of using zT2FSs in modelling of uncertainty.

In Chapters 2, the types of FSs were reviewed and the effectiveness of T2FSs in modelling higher uncertainties comparing to T1FSs was highlighted. Also it was emphasised that zT2FSs was an effective representation of T2FSs which could be used in different applications' domains when FSs are required for modelling. Later in Chapter 3, the effectiveness of IAA comparing to other approaches, namely IA and EIA in capturing uncertainties of human opinions using surveys was demonstrated. Additionally, the capability of IAA to generate zT2FSs that represented fuzzy agreement model which included intra and inter-uncertainties of experts while modelling was emphasised in Chapter 3.

- Analyse Interval Agreement Approach (IAA) and its application to generate the weights of the links in the FCM represented as zT2FSs.

Chapter 4 highlighted that using T1FSs to represent FCM's weights was a drawback of conventional FCM which hindered its capabilities for capturing different types of uncertainties from different sources (e.g. experts). This restricts the performance of FCM to provide an accurate decision output. In Chapter 5, the author used IAA to generate zT2FSs from the responses of an interval valued survey to represent the weights of the links between the concepts and the zT2FCM was proposed. The generated zT2FSs captured inter and intra -uncertainties of the experts about the weights of the causal relations among the concepts.

- Develop an iterative reasoning algorithm for FCMs with weights represented using zT2FSs (zT2FCM) and analyse its effectiveness by using it in a novel case study.

To demonstrate the effectiveness of zT2FCM comparing to the conventional FCM, a novel case study for Autism diagnosis was created in Chapter 5 and a zT2FCM for Autism diagnosis is proposed. F-MCHAT was created and then used to collect intervals which reflected doctors' opinions about the weights of the causal relations among the concepts of proposed zT2FCM. Then IAA was used to generate zT2FSs to represent the fuzzy agreement models of the weights of causal links. Afterwards these zT2FSs were defuzzified to crisp values and used in the iterative reasoning algorithm (4.2) to diagnose 40 cases. The results of this proposed zT2FCM for Autism diagnosis and the results of an FCM created for the same purpose (Kannappan, Tamilarasi, and Papageorgiou 2011) using the same 40 cases were compared to the doctors' decisions on the Autism diagnosis for the same cases. The doctor's decisions were used as a benchmark to test the accuracy of the diagnosis achieved by the conventional FCM and the zT2FCM proposed. The results of zT2FCM provided accuracy of 85.09% compared with the doctors' decisions which was more than 79.63% of the accuracy that FCM had compared with the doctors' decisions. Hence, the zT2FCM outperformed the FCM in inferring as the human.

- Develop a new non-iterative reasoning algorithm for the FCMs with weights represented using zT2FSs that operates without defuzzification.

Motivated by the result presented in Chapter 5 that showed that the zT2FCM outperformed a conventional FCM in providing decisions closer to the decision makers, a new non-iterative reasoning algorithm, NILD, was proposed for zT2FCMs in Chapter 6; the values of weights represented as zT2FSs were using in the reasoning process without defuzzification. For this purposes new operations were introduced and defined in Chapter 6, which were needed for using the zT2FSs while reasoning.

- Demonstrate that the zT2FCM with the proposed new non –iterative reasoning algorithm outperforms the conventional FCM in a novel created case study.

To achieve this objective, a novel case study to evaluate MP of mathematical modules offered my MASC at MEC was created as presented in Chapter 6. In this case study, a zT2FCM with NILD, named MPFCM, and a conventional FCM with weights represented using T1FS – T1FCM were created to evaluate MP of 30 modules offered by MASC. Also, the MP of these modules obtained by the system TLS used in MASC, based on the statistical approach, were collected. Indeed, a fuzzy agreement model of lecturers on MP was created for each of these 30 modules. The results of these agreement models were used as a benchmark to compare MP of the modules obtained by applying MPFCM, T1FCM and TLS. To demonstrate that zT2FCM outperformed the conventional FCM and other statistical approaches used for the same problem the correlations between the results of the lecturers' agreement models and results of MPFCM, T1FCM and TLS were calculated. The obtained results showed that MPFCM was more correlated to lecturers (decision makers) decisions compared to T1FCM and TLS, as presented in Chapter 6. This underpinned the ability of the zT2FCM with the reasoning algorithm NILD to better mimic human reasoning in the presence of intra and inter uncertainties in the opinions of domain subjects than the FCM with a standard iterative reasoning method. It is worth to emphasis again here that the new non –iterative reasoning algorithm for the FCM is proposed for FCMs with links' weights represented using zT2FSs that are generated using IAA.

- Investigate the effectiveness and analyse the sensitivity of the new reasoning algorithm and its capability in allowing the uncertainty to propagate

To investigate the capability of the proposed NILD for propagating the uncertainty, the sensitivity analysis had been conducted in Chapter 6. For this purpose, the uncertainty of the lecturers responses to determine the original weights of the causal relations in MPFCM was changed systematically. After each change the intervals were aggregated using IAA and the changed weights were produced. The results obtained were analysed and it was observed that both changes in uncertainties of zT2FS based weighted links direct from intermediate concepts to the decision concept and from input to intermediate concepts impacted the value of the decision concept albeit to different degrees. The results demonstrated that NILD algorithm enabled a propagation of uncertainty which affected outcome decisions. Sensitivity analysis was conducted using a software developed for reasoning in MPFCM using NILD. This software was implemented using MATLAB as presented in Appendix 15.

The novelties of this research and the results obtained from two newly generated cases studies toward achieving above-mentioned objectives, contribute to extend the FCM that relies on zT2FSs. This extension enhances FCM's capabilities for capturing more uncertainties and facilitate more complete information representation. Indeed, the capability of the proposed new reasoning algorithm to propagate the uncertainties improved the FCM to better support a decision making process.

7.2 Future Work

The future work on the proposed zT2FCM with NILD algorithm could be carried out in three directions as follows:

1. Representation of the concepts' values using zT2FSs.

The zT2FCM with NILD algorithm could be further enhanced by representing its concepts' values using zT2FSs. Then the NILD algorithm has to be enhanced to accommodate the new representations of uncertain values of FCM's concepts and to reason when both concept values and weights are modelled using zT2FSs.

2. Development of learning for zT2FCM.

Despite the results obtained in Chapter 5 that zT2FCM outperformed the FCM in providing results more close to decision makers, there is a demand in many applications domains to create an FCM based on historical data. As part of this surge, one of the future direction for the work presented in this thesis is to train the zT2FCM which is created based on historical data by one of the available learning algorithms (Papageorgiou 2012).

3. Application of zT2FCM with NILD algorithm in new domains.

In this thesis, the effectiveness of using the proposed zT2FCM has been evaluated in two case studies in two different domains, namely Medicine and Education. Motivated by the outcomes and results of these two case studies, the zT2FCM with NILD algorithm could be applied in other applications domains where its usefulness has not been validated so far. The future work on the application of zT2FCM with NILD algorithm has to underpin the application where the effectiveness of zT2FCM in capturing more uncertainties from different sources can be utilised and effectiveness of its reasoning can be proved by providing better decision.

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APPENDIX 1. Modified MCHAT

(To be answer by child's parents)

1. Does your child enjoy being swung, bounced on your knee, etc.?
   a. Certainly not.
   b. At times.
   c. Always.

2. Does your child take an interest in other children?
   a. Yes, quite a lot
   b. Yes, sometimes
   c. no or very little

3. Does your child like climbing on things, such as upstairs?
   a. Yes, often
   b. No
   c. Very rarely.

4. Does your child ever pretend, for example, to talk on the phone or take care of dolls, or pretend other things?
   a. No
   b. Yes
   c. Slightly true

5. Does your child ever use his/her index finger to point, to ask for something?
   a. Never
   b. Certainly
   c. Rarely

6. Does your child ever use his/her index finger to point, to indicate interest in something?
    a. Never.
    b. Certainly
    c. Rarely

7. Does your child ever bring objects over to you (parent) to show you something?
    a. No doesn’t do it
    b. Have for himself
    c. Does it

8. Does your child walk?
    a. With other help
    b. No very little
    c. By himself

9. Does your child ever seem oversensitive to noise? (e.g., plugging ears)
    a. I don’t think so
    b. Rarely
    c. usually does

10. Does your child smile in response to your face or your smile?
    a. No
    b. Yes
    c. Sometimes

11. Does your child imitate you? (e.g., you make a face-will your child imitate it?
   a. Quite a lot
   b. Does not
   c. Once a while

12. Does your child respond to his/her name when you call?
   a. Yes ,always
   b. Yes, sometimes
   c. No, very little

13. If you point at a toy across the room, does your child look at it?
   a. Rarely does this
   b. Yes , this is typical
   c. Once in a while

14. Does your child look you in the eye for more than a second or two?
   a. No
   b. Yes
   c. Sometimes.

15. Does your child look at things you are looking at?
   a. Definitely not
   b. Yes, sometimes
   c. Often

16. Does your child make unusual finger movements near his/her face?
    a. Yes, often and for rather long periods
    b. Very rarely
    c. No

17. Does your child try to attract your attention to his/her own activity?
    a. Yes
    b. No
    c. Slightly True

18. Have you ever wondered if your child is deaf?
    a. Yes
    b. Definitely not.
    c. Not sure

19. Does your child understand what people say?
    a. Yes, understands
    b. Rarely understands
    c. Very little or no understanding

20. Does your child look at your face to check your reaction when faced with something unfamiliar?
    a. Yes, definitely.
    b. Yes, slightly true.
    c. Doesn’t look

APPENDIX 2. F-MCHAT

(To be answer by doctors to determine the weights of zT2FCM for Autism Diagnosis)

1. What is the impact of a factor concept "**enjoy being swung**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 1 of modified MCHAT is
   a. Certainly not.
   b. At times.
   c. Always.
2. What is the impact of a factor concept "**Take an interest in other children**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 2 of modified MCHAT is

   a. Yes, quite a lot
   b. Yes, sometimes
   c. no or very little

3. What is the impact of a factor concept "**Climbing on things**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 3 of modified MCHAT is

   a. Yes, often
   b. No
   c. Very rarely.

4. What is the impact of a factor concept "**Pretend to be other things**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 4 of modified MCHAT is

   a. No
   b. Yes
   c. Slightly true

5. What is the impact of a factor concept "**Pointing with index finger**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 5 of modified MCHAT is

   a. Never
   b. Certainly
   c. Rarely

6. What is the impact of a factor concept "**Indication of interest**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 6 of modified MCHAT is

   a. Never.
   b. Certainly
   c. Rarely

7. What is the impact of a factor concept "**Bringing objects to parents**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 7 of modified MCHAT is

   a. No doesn't do it
   b. Have for himself
   c. Does it

8. What is the impact of a factor concept "**Walking**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 8 of modified MCHAT is

   a. With other help
   b. No very little
   c. By himself

9. What is the impact of a factor concept "**Oversensitive to noise**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 9 of modified MCHAT is

    a. I don't think so
    b. Rarely
    c. usually does

10. What is the impact of a factor concept "**Smile in response to parents face**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 10 of MCHAT is

    a. No
    b. Yes
    c. Sometimes

11. What is the impact of a factor concept "**Imitate**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 11 of modified MCHAT is

    a. Quite a lot
    b. Does not
    c. Once a while

12. What is the impact of a factor concept "**Respond to the name**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 12 of modified MCHAT is

    a. Yes ,always
    b. Yes, sometimes
    c. No, very little

13. What is the impact of a factor concept "**Looking at a toy when pointing**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 13 of modified MCHAT is
    a. Rarely does this
    b. Yes , this is typical
    c. Once in a while

14. What is the impact of a factor concept "**Eye contact**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 14 of modified MCHAT is
    a. No
    b. Yes
    c. Sometimes.

15. What is the impact of a factor concept "**Look at things you are looking at**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 15of modified MCHAT is
    a. Definitely not
    b. Yes, sometimes
    c. Often

16. What is the impact of a factor concept "**Unusual finger movement near his/her face**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 16 of modified MCHAT is
    a. Yes, often and for rather long periods
    b. Very rarely
    c. No

17. What is the impact of a factor concept "**Attract your attention**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 17 of modified MCHAT is

a. Yes
b. No
c. Slightly True

18. What is the impact of a factor concept "**Deafness**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 18 of modified MCHAT is
    a. Yes
    b. Definitely not.
    c. Not sure

19. What is the impact of a factor concept "**Understanding what others say**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 19 of modified MCHAT is
    a. Yes, understands
    b. Rarely understands
    c. Very little or no understanding

20. What is the impact of a factor concept "**Look to your face to check reaction**" to decision concept "**Autism Diagnosis**" if the parent's answer to Question 20 of modified MCHAT is
    a. Yes, definitely.
    b. Yes, slightly true.
    c. Doesn't look

APPENDIX 3. Response intervals of SQUH's doctors to F-MCHAT

Q_1	D1	D2	D3
a	[0.20, 0.45]	[0.15, 0.35]	[0.05, 0.25]
b	[0.05, 0.25]	[0.35, 0.60]	[0.40, 0.70]
c	[0, 0.15]	[0.50, 0.70]	[0.65, 0.85]

Q_2	D1	D2	D3
a	[0, 0.15]	[0.05, 0.30]	[0.05, 0.25]
b	[0.45, 0.65]	[0.20, 0.50]	[0.45-0.65]
c	[0.75, 0.95]	[0.50, 0.65]	[0.65-0.85]

Q_3	D1	D2	D3
a	[0.35, 0.55]	[0.15, 0.40]	[0.15, 0.25]
b	[0.10, 0.35]	[0.25, 0.50]	[0.35, 0.55]
c	[0, 0.15]	[0.70, 0.95]	[0.75, 0.95]

Q_4	D1	D2	D3
a	[0.65, 0.85]	[0.05, 0.35]	[0.15, 0.35]
b	[0.25, 0.45]	[0.35, 0.55]	[0.50, 0.75]
c	[0, 0.15]	[0.60, 0.80]	[0.85, 1]

Q_5	D1	D2	D3
a	[0.65, 0.85]	[0.15, 0.35]	[0.05, 0.30]
b	[0.40, 0.55]	[0.50, 0.70]	[0.55, 0.75]
c	[0, 0.25]	[0.10, 0.30]	[0.05, 0.25]

Q_6	D1	D2	D3
a	[0.65, 0.85]	[0.50, 0.80]	[0.55, 0.75]
b	[0.45, 0.65]	[0.50, 0.75]	[0.75, 0.95]
c	[0.05, 0.25]	[0.20, 0.45]	[0.20, 0.45]

Q_7	D1	D2	D3
a	[0.65, 0.85]	[0.55, 0.75]	[0.35, 0.55]
b	[0.40, 0.65]	[0.30, 0.60]	[0.45-0.65]
c	[0.05, 0.30]	[0.15, 0.35]	[0.20-0.45]

Q_8	D1	D2	D3
a	[0.75, 0.95]	[0.70, 0.95]	[0.35, 0.55]
b	[0.05, 0.25]	[0.40, 0.65]	[0.35, 0.55]
c	[0.25, 0.50]	[0.45, 0.65]	[0.35, 0.60]

Q_9	D1	D2	D3
a	[0.10, 0.35]	[0.25, 0.45]	[0.15, 0.35]
b	[0, 0.15]	[0.10, 0.35]	[0.05, 0.25]
c	[0.55, 0.75]	[0.40, 0.65]	[0.60, 0.85]

Q_{10}	D1	D2	D3
a	[0.65, 0.85]	[0.75, 0.90]	[0.60, 0.85]
b	[0.05, 0.25]	[0.20, 0.45]	[0.05, 0.30]
c	[0.25, 0.45]	[0.40, 0.65]	[0.45, 0.70]

Q_{11}	D1	D2	D3
a	[0.75, 0.95]	[0.65, 0.85]	[0.75, 0.95]
b	[0.35, 0.55]	[0.15, 0.30]	[0.05, 0.25]
c	[0.20, 0.45]	[0.25, 0.50]	[0.45, 0.65]

Q_{12}	D1	D2	D3
a	[0-0.25]	[0.15, 0.35]	[0, 0.15]
b	[0.35, 0.55]	[0.25, 0.50]	[0.35, 0.55]
c	[0.75, 0.95]	[0.60, 0.85]	[0.75, 0.95]

Q_{13}	D1	D2	D3
a	[0.65, 0.85]	[0.55, 0.75]	[0.75, 0.95]
b	[0.10, 0.30]	[0.25, 0.45]	[0.05, 0.25]
c	[0.35, 0.55]	[0.35, 0.55]	[0.35, 0.55]

Q_{14}	D1	D2	D3
a	[0.32, 0.55]	[0.27, 0.45]	[0.35, 0.57]
b	[0.55, 0.75]	[0.42, 0.68]	[0.32, 0.55]
c	[0.17, 0.37]	[0.41, 0.65]	[0.37, 0.55]

Q_{15}	D1	D2	D3

a	[0.60, 0.85]	[0.65, 0.85]	[0.55, 0.75]
b	[0.35, 0.55]	[0.40, 0.70]	[0.35, 0.55]
c	[0.35, 0.60]	[0.10, 0.35]	[0.35, 0.55]

Q_{16}	D1	D2	D3
a	[0.55, 0.75]	[0.55, 0.80]	[0.75, 0.90]
b	[0, 0.15]	[0.15, 0.35]	[0.15, 0.35]
c	[0.05, 0.25]	[0.20, 0.45]	[0, 0.10]

Q_{17}	D1	D2	D3
a	[0.05, 0.30]	[0.15, 0.35]	[0.05, 0.25]
b	[0.75, 0.95]	[0.55, 0.80]	[0.75, 0.95]
c	[0.25, 0.45]	[0.20, 0.40]	[0.25, 0.50]

Q_{18}	D1	D2	D3
a	[0.70, 0.95]	[0.65, 0.85]	[0.75, 0.95]
b	[0.05, 0.30]	[0.15, 0.35]	[0.35, 0.60]
c	[0.25, 0.50]	[0.05, 0.25]	[0.05, 0.25]

Q_{19}	D1	D2	D3
a	[0.15, 0.35]	[0.25, 0.35]	[0.05, 0.25]
b	[0.55, 0.75]	[0.50, 0.75]	[0.35, 0.55]
c	[0.75, 0.95]	[0.65, 0.85]	[0.75, 0.95]

Q_{20}	D1	D2	D3
a	[0.05, 0.25]	[0.10, 0.25]	[0.15, 0.35]
b	[0.25, 0.45]	[0.25, 0.50]	[0.25, 0.45]
c	[0.65, 0.90]	[0.50, 0.75]	[0.65, 0.85]

APPENDIX 4. zT2FSs weights of zT2FCM used for Autism Diagnosis

$W_{21,1}$	y_1	y_2	y_3
z_1	[0, 0.85]	[0.15, 0.70]	[0, 0.25]
z_2	[0.05, 0.70]	∅	∅
z_3	[0.05, 0.45]	∅	∅

$W_{21,2}$	y_1	y_2	y_3
z_1	[0, 0.95]	[0.05, 0.85]	[0.05, 0.5]
z_2	[0.20, 0.65]	∅	∅
z_3	∅	∅	∅

$W_{21,3}$	y_1	y_2	y_3
z_1	[0, 0.95]	[0.15, 0.95]	∅
z_2	[0.10, 0.55]	[0.25, 0.40]	∅
z_3	[0.15, 0.55]	∅	∅

$W_{21,4}$	y_1	y_2	y_3
z_1	[0,1]	[0.15, 0.55]	∅
z_2	[0.05, 0.85]	∅	∅
z_3	[0.25, 0.75]	∅	∅

$W_{21,5}$	y_1	y_2	y_3
z_1	[0, 0.85]	[0.05, 0.70]	[0.10, 0.25]
z_2	[0.05, 0.75]	[0.15, 0.25]	∅
z_3	∅	∅	∅

$W_{21,6}$	y_1	y_2	y_3
z_1	[0.05, 0.95]	[0.2, 0.8]	[0.2, 0.75]
z_2	[0.5, 0.85]	[0.55, 0.65]	∅
z_3	∅	∅	∅

$W_{21,7}$	y_1	y_2	y_3

z_1	[0.05, 0.85]	[0.15, 0.75]	[0.20, 0.6]
z_2	[0.30, 0.65]	∅	∅
z_3	[0.35, 0.45]	∅	∅

$W_{21,8}$	y_1	y_2	y_3
z_1	[0.05, 0.95]	[0.35, 0.95]	[0.45, 0.50]
z_2	[0.25, 0.65]	[0.40, 0.55]	∅
z_3	[0.35, 0.65]	∅	∅

$W_{21,9}$	y_1	y_2	y_3
z_1	[0, 0.85]	[0.05, 0.75]	[0.1, 0.65]
z_2	[0.10, 0.45]	[0.15, 0.25]	∅
z_3	∅	∅	∅

$W_{21,10}$	y_1	y_2	y_3
z_1	0.05, 0.90	[0.05, 0.85]	[0.20, 0.85]
z_2	0.25, 0.70	∅	∅
z_3	∅	∅	∅

$W_{21,11}$	y_1	y_2	y_3
z_1	[0.05, 0.95]	[0.15, 0.95]	[0.75, 0.85]
z_2	[0.20, 0.55]	∅	∅
z_3	∅	∅	∅

$W_{21,12}$	y_1	y_2	y_3
z_1	[0, 0.95]	[0, 0.95]	[0.35, 0.85]
z_2	[0.25, 0.35]	∅	∅
z_3	∅	∅	∅

$W_{21,13}$	y_1	y_2	y_3
z_1	[0.05, 0.95]	[0.10, 0.85]	[0.35, 0.55]
z_2	[0.35, 0.45]	∅	∅
z_3	∅	∅	∅

$W_{21,14}$	y_1	y_2	y_3
z_1	[0.17, 0.75]	[0.32, 0.68]	[0.35, 0.45]∪[0.55, 0.55]

z_2	[0.27, 0.65]	[0.41, 0.55]	∅
z_3	[0.32, 0.57]	[0.42, 0.55]	∅

$W_{21,15}$	y_1	y_2	y_3
z_1	[0.10, 0.85]	[0.35, 0.85]	[0.40, 0.75]
z_2	[0.35, 0.70]	[0.35, 0.55]	∅
z_3	[0.55, 0.6]	∅	∅

$W_{21,16}$	y_1	y_2	y_3
z_1	[0, 0.90]	[0.05, 0.80]	∅
z_2	[0, 0.35]	[0.15, 0.25]	∅
z_3	∅	∅	∅

$W_{21,17}$	y_1	y_2	y_3
z_1	[0.05, 0.95]	[0.05, 0.95]	[0.15, 0.85]
z_2	[0.20, 0.35]	[0.25, 0.30]	∅
z_3	∅	∅	∅

$W_{21,18}$	y_1	y_2	y_3
z_1	[0.05, 0.95]	[0.05, 0.95]	[0.75, 0.85]
z_2	[0.05, 0.50]	[0.15, 0.25]	∅
z_3	∅	∅	∅

$W_{21,19}$	y_1	y_2	y_3
z_1	[0.05, 0.95]	[0.15, 0.95]	[0.75, 0.85]
z_2	[0.65, 0.67]	∅	∅
z_3	∅	∅	∅

$W_{21,20}$	y_1	y_2	y_3
z_1	[0.05, 0.90]	[0.10, 0.85]	[0.15, 0.75]
z_2	[0.25, 0.35]	∅	∅
z_3	∅	∅	∅

APPENDIX 5. Questions to determine the weight of the interrelation among the concepts

1. What is the impact of Attendance on the End Semester result?
2. What is the impact of Attendance on Course work result?
3. What is the impact of Course work on the review of Module performance?
4. What is the impact of End semester on the review of Module performance?

APPENDIX 6. Intervals Responses of the three lecturers to each of the four questions in both iterations of the survey for determine the weights of causal relation in MPFCM

Question 1	First Iteration of the Survey	Second Iteration of the Survey
Lecturer 1	[0.15, 0.35]	[0.15, 0.45]
Lecturer 2	[0.37, 0.72]	[0.40, 0.70]
Lecturer 3	[0.38, 0.75]	[0.40, 0.75]

Question 2	First Iteration of the Survey	Second Iteration of the Survey
Lecturer 1	[0.12, 0.38]	[0.05, 0.40]
Lecturer 2	[0.28, 0.42]	[0.20, 0.40]
Lecturer 3	[0.50, 0.82]	[0.47, 0.80]

Question 3	First Iteration of the Survey	Second Iteration of the Survey
Lecturer 1	[0.38, 0.72]	[0.45, 0.65]
Lecturer 2	[0.60, 0.88]	[0.60, 0.80]
Lecturer 3	[0.30, 0.75]	[0.40, 0.65]

Question 4	First Iteration of the Survey	Second Iteration of the Survey
Lecturer 1	[0.38, 0.77]	[0.55, 0.75]
Lecturer 2	[0.68, 0.85]	[0.50, 0.72]
Lecturer 3	[0.65, 0.90]	[0.58, 0.80]

APPENDIX 7. Weights of MPFCM as z T2FSs

Weight	Slice	Level	$y_1 = 0.5$	$y_2 = 1$
$W_{ESE,ATT}$	Z_1	$z_1 = 1/3$	$[0.15, 0.75]$	$[0.15, 0.35] \cup [0.40, 0.75]$
	Z_2	$z_2 = 2/3$	$[0.37, 0.72]$	$[0.4, 0.7]$
	Z_3	$z_3 = 1$	$[0.38, 0.45]$	ϕ
$W_{CW,ATT}$	Z_1	$z_1 = 1/3$	$[0.05, 0.42] \cup [0.47, 0.82]$	$[0.12, 0.40] \cup [0.50, 0.80]$
	Z_2	$z_2 = 2/3$	$[0.2, 0.4]$	$[0.28. 0.38]$
	Z_3	$z_3 = 1$	ϕ	ϕ
$W_{MP,CW}$	Z_1	$z_1 = 1/3$	$[0.30, 0.88]$	$[0.40, 0.80]$
	Z_2	$z_2 = 2/3$	$[0.38, 0.75]$	$[0.45, 0.65]$
	Z_3	$z_3 = 1$	$[0.60, 0.72]$	$[0.60, 0.65]$
$W_{MP,ESE}$	Z_1	$z_1 = 1/3$	$[0.38, 0.9]$	$[0.55, 0.8]$
	Z_2	$z_2 = 2/3$	$[0.5, 0.85]$	$[0.65, 0.75]$
	Z_3	$z_3 = 1$	$[0.58, 0.77]$	$[0.68, 0.72]$

APPENDIX 8. Values of inputs values of the modules

Module 1	
ESE	0.6704
CW	0.7388
ATT	0.8817

Module 7	
ESE	0.6688
CW	0.7690
ATT	0.8579

Module 13	
ESE	0.339
CW	0.7462
ATT	0.9920

Module 19	
ESE	0.4112
CW	0.5212
ATT	0.8700

Module 25	
ESE	0.6256
CW	0.6264
ATT	0.8024

Module 2	
ESE	0.7246
CW	0.8232
ATT	0.9374

Module 8	
ESE	0.7470
CW	0.8650
ATT	0.9663

Module 14	
ESE	0.6774
CW	0.7196
ATT	0.9020

Module 20	
ESE	0.6618
CW	0.5006
ATT	0.8400

Module 26	
ESE	0.6993
CW	0.6391
ATT	0.8269

Module 3	
ESE	0.7016
CW	0.7092
ATT	0.8789

Module 9	
ESE	0.5484
CW	0.5952
ATT	0.8742

Module 15	
ESE	0.7450
CW	0.6808
ATT	0.8972

Module 21	
ESE	0.5602
CW	0.7142
ATT	0.8580

Module 27	
ESE	0.6407
CW	0.6791
ATT	0.8580

Module 4	
ESE	0.4994
CW	0.6838
ATT	0.8796

Module 10	
ESE	0.7650
CW	0.6777
ATT	0.9094

Module 16	
ESE	0.7700
CW	0.8262
ATT	0.7776

Module 22	
ESE	0.72
CW	0.7960
ATT	0.8100

Module 28	
ESE	0.5656
CW	0.7378
ATT	0.7800

Module 5	
ESE	0.6826
CW	0.6426
ATT	0.8861

Module 11	
ESE	0.5886
CW	0.7878
ATT	0.8885

Module 17	
ESE	0.8368
CW	0.7200
ATT	0.8631

Module 23	
ESE	0.5313
CW	0.6740
ATT	0.8800

Module 29	
ESE	0.7436
CW	0.8280
ATT	0.8900

Module 6	
ESE	0.5124
CW	0.7134
ATT	0.8149

Module 12	
ESE	0.45167
CW	0.6112
ATT	0.8200

Module18	
ESE	0.798
CW	0.6256
ATT	0.8700

Module 24	
ESE	0.5256
CW	0.6682
ATT	0.8314

Module 30	
ESE	0.5198
CW	0.5690
ATT	0.8287

APPENDIX 9. zT2FSs representations of MP of the modules using MPFCM

Module 1				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.4363889
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 2				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.55,0.80]	0.451388889
Z_2	$z_2 = 2/3$	[0.50,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 3				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.669722222
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	[0.68,0.72]	

Module 4				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.2727778
Z_2	$z_2 = 2/3$	[0.38,0.75]	$\emptyset$	
Z_3	$z_3 = 1$	[0.60,0.72]	$\emptyset$	

Module 5				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.635
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.45,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$[0.6, 0.65] \cup [0.68, 0.72]$	

Module 6				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.38,0.90]	$\emptyset$	0.110555556
Z_2	$z_2 = 2/3$	[0.50,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	$\emptyset$	$\emptyset$	

Module 7				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.443055555555556
Z_2	$z_2 = 2/3$	[0.50,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 8				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.55,0.80]	0.451388888888889
Z_2	$z_2 = 2/3$	[0.50,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 9				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.290555555555556
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.45,0.65]	
Z_3	$z_3 = 1$	$\emptyset$	$\emptyset$	

Module 10				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.280833333333333
Z_2	$z_2 = 2/3$	[0.38,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	[0.58,0.87]	$\emptyset$	

Module 11				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	
Z_2	$z_2 = 2/3$	[0.50,0.85]	$\emptyset$	0.2875
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 12				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.38,0.90]	$\emptyset$	
Z_2	$z_2 = 2/3$	$\emptyset$	$\emptyset$	0.239317556
Z_3	$z_3 = 1$	$\emptyset$	[0.61128, 0.611286	

Module 13				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.88]	[0.40,0.80]	0.162222222222222
Z_2	$z_2 = 2/3$	[0.38,0.75]	$\emptyset$	
Z_3	$z_3 = 1$	$\emptyset$	$\emptyset$	

Module 14				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.88]	[0.55,0.80]	0.436388888888889
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 15				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.9]	[0..4,0.8]	0.436388888888889
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 16				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.38,0.9]	[0..5,0.8]	0.295277777777778
Z_2	$z_2 = 2/3$	[0.50,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 17				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.278333333333333
Z_2	$z_2 = 2/3$	[0.38,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	[0.60,0.72]	$\emptyset$	

Module 18				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.608888888888889
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.45,0.65]	
Z_3	$z_3 = 1$	[0.60,0.72]	[0.60,0.65]	

Module 19				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.285
Z_2	$z_2 = 2/3$	[0.38,0.75]	[0.45,0.65]	
Z_3	$z_3 = 1$	$\emptyset$	$\emptyset$	

Module 20				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.414166666666667
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.45,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 21				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.278333333333333
Z_2	$z_2 = 2/3$	[0.38,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	[0.60,0.77]	$\emptyset$	

Module 22				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.38,0.90]	[0.55,0.80]	0.686944444444445
Z_2	$z_2 = 2/3$	[0.50,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	[0.68,0.72]	

Module 23				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.278333333333333
Z_2	$z_2 = 2/3$	[0.38,0.85]	∅	
Z_3	$z_3 = 1$	[0.60,0.72]	∅	

Module 24				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.278333333333333
Z_2	$z_2 = 2/3$	[0.38,0.85]	∅	
Z_3	$z_3 = 1$	[0.60,0.72]	∅	

Module 25				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.50,0.80]	0.506855555555556
Z_2	$z_2 = 2/3$	[0.50,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	[0.58,0.77]	[0.6264, 0.6264]	

Module 26				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.635
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.45,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	[0.60, 0.65] ∪ [0.68, 0.72]	

Module 27				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.30,0.90]	[0.40,0.80]	0.280833333333333
Z_2	$z_2 = 2/3$	[0.38,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 28				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.38,0.90]	[0.50,0.80]	0.182777777777778
Z_2	$z_2 = 2/3$	[0.50,0.85]	$\emptyset$	
Z_3	$z_3 = 1$	$\emptyset$	$\emptyset$	

Module 29				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.38,0.90]	[0.55,0.80]	0.451388888888889
Z_2	$z_2 = 2/3$	[0.50,0.85]	[0.65,0.75]	
Z_3	$z_3 = 1$	[0.58,0.77]	$\emptyset$	

Module 30				
Slice	Level	$y_1 = 0.5$	$y_2 = 1$	Centroid
Z_1	$z_1 = 1/3$	[0.38,0.90]	[0.55,0.80]	0.290555555555556
Z_2	$z_2 = 2/3$	[0.38,0.85]	[0.45,0.65]	
Z_3	$z_3 = 1$	$\emptyset$	$\emptyset$	

APPENDIX 10. A Question to determine the module performance based on the teachers experiences

How do you evaluate the module performance, if the result summary of a module at the end of the semester is as follows:

1. End Semester Examination result is ---%
2. Course Work result is---% and
3. Attendance percentage is ---%

APPENDIX 11. Weights of T1FCM for evaluating MP

$W_{ESE,ATT}$	$y_1 = \frac{1}{3}$	$y_2 = \frac{2}{3}$	$y_3 = 1$	Centroid
	[0.15, 0.75]	[0.40, 0.70]	[0.40, 0.45]	0.470833333

$W_{CW,ATT}$	$y_1 = \frac{1}{3}$	$y_2 = \frac{2}{3}$	$y_3 = 1$	Centroid
	[0.05,0.40]∪[0.47,0.80]	[0.20, 0.40]	∅	0.171666667

$W_{MP,CW}$	$y_1 = \frac{1}{3}$	$y_2 = \frac{2}{3}$	$y_3 = 1$	Centroid
	[0.40, 0.80]	[0.45, 0.65]	[0.60, 0.65]	0.595833333

$W_{MP,ESE}$	$y_1 = \frac{1}{3}$	$y_2 = \frac{2}{3}$	$y_3 = 1$	Centroid
	[0.50, 0.80]	[0.55, 0.75]	[0.58, 0.72]	0.65

APPENDIX 12. The response intervals of the lecturers after changing their width by δ

Change in the response intervals of each question from the lecturers by $\delta = 0.01$

	iteration 1	iteration 2
Q1	[0.14, 0.36]	[0.14, 0.46]
	[0.36, 0.73]	[0.39, 0.71]
	[0.37, 0.76]	[0.39, 0.76]

	iteration 1	iteration 2
Q2	[0.11, 0.39]	[0.04,0.41]
	[0.27, 0.43]	[0.19, 0.41]
	[0.49,0.83]	[0.46, 0.81]

	iteration 1	iteration 2
Q3	[0.37, 0.73]	[0.44, 0.66]
	[0.59, 0.89]	[0.59, 0.81]
	[0.29, 0.76]	[0.39, 0.66]

	iteration 1	iteration 2
Q4	[0.37,0.78]	[0.54, 0.76]
	[0.67,0.86]	[0.49, 0.73]
	[0.64, 0.91]	[0.57,0.81]

Change in the response intervals of each question from the lecturers by $\delta = 0.025$

	iteration 1	iteration 2
Q1	[0.125, 0.375]	[0.125, 0.475]
	[0.345, 0.745]	[0.375,0.725]
	[0.355, 0.775]	[0.375,0.775]

	iteration 1		iteration 2	
Q2	[0.095,	0.405]	[0.025	0.425
	[0.255,	0.445]	[0.175	0.425
	[0.475,	0.845]	[0.445	0.825

	iteration 1	iteration 2
Q3	[0.355, 0.745]	[0.425,0.675]
	[0.575, 0.905]	[0.575,0.825]
	[0.275, 0.775]	[0.375,0.675]

	iteration 1	iteration 2
Q4	[0.355, 0.795]	[0.525,0.775]
	[0.655,0.875]	[0.475, 0.745]
	[0.625, 0.925]	[0.555,0.825]

Change in the response intervals of each question from the lecturers by $\delta = 0.05$

	iteration 1	iteration 2
Q1	[0.10, 0.40]	[0.10, 0.50]
	[0.32, 0.77]	[0.35, 0.75]
	[0.33, 0.80]	[0.35, 0.8]

	iteration 1	iteration 2
Q2	[0.07, 0.43]	[0, 0.45]
	[0.23, 0.47]	[0.15, 0.45]
	[0.45, 0.87]	[0.42, 0.85]

	iteration 1	iteration 2
Q3	[0.33, 0.77]	[0.40, 0.70]
	[0.55, 0.93]	[0.55, 0.85]
	[0.25, 0.80]	[0.35, 0.70]

	iteration 1	iteration 2
Q4	[0.33, 0.82]	[0.5,0.80]
	[0.63, 0.90]	[0.45,0.77]
	[0.60, 0.95]	[0.53,0.85]

Change in the response intervals of each question from the lecturers by $\delta = 0.07$

	iteration 1		iteration 2	
Q1	[0.08	0.42]	[0.08	0.52]
	[0.3	0.79]	[0.33	0.77]
	[0.31	0.82]	[0.33	0.82]

	iteration 1	iteration 2
Q2	[0.05,0.45]	[0,0.47]
	[0.21,0.49]	[0.13,0.47]
	[0.43,0.89]	[0.40, 0.87]

	iteration 1	iteration 2
Q3	[0.31,0.79]	[0.38,0.72]
	[0.53,0.95]	[0.53,0.87]
	[0.23,0.82]	[0.33,0.72]

	iteration 1	iteration 2
Q4	[0.31,0.84]	[0.48, 0.82]
	[0.61,0.92]	[0.43, 0.79]
	[0.58,0.97]	[0.51, 0.87]

Change in the response intervals of each question from the lecturers by $\delta = 0.1$

	iteration 1	iteration 2
Q1	[0.05,0.45]	[0.05,0.55]
	[0.27,0.82]	[0.30,0.80]
	[0.28,0.85]	[0.30,0.85]

	iteration 1	iteration 2
Q2	[0.02,0.48]	[0,0.5]
	[0.18,0.52]	[0.10,0.5]
	[0.40,0.92]	[0.37,0.9]

	iteration 1	iteration 2
Q3	[0.280.82]	[0.35,0.75]
	[0.5,0.98]	[0.5,0.9]
	[0.2,0.85]	[0.3,0.75]

	iteration 1	iteration 2
Q4	[0.28,0.87]	[0.45,0.85]
	[0.58,0.95]	[0.40,0.82]
	[0.55,1]	[0.48,0.90]

APPENDIX 13. The generated weights after each change of the responses' intervals by δ

Weight $W_{ESE,ATT}$

$W_{ESE,ATT}$	y1=0.5	y2=1
z1=1/3	[0.14, 0.76]	[0.14, 0.36]∪[0.39, 0.76]
z2=2/3	[0.36, 0.73]	[0.39, 0.71]
z3=1	[0.37, 0.46]	∅

$W_{ESE,ATT}$	y1=0.5	y2=1
z1=1/3	[0.125,0.775]	[0.125, 0.775]
z2=2/3	[0.345, 0.745]	[0.375, 0.725]
z3=1	[0.355, 0.475]	∅

$W_{ESE,ATT}$	y1=0.5	y2=1
z1=1/3	[0.1, 0.8]	[0.1,0.8]
z2=2/3	[0.32, 0.77]	[0.35, 0.75]
z3=1	[0.33, 0.5]	[0.35, 0.4]

$W_{ESE,ATT}$	y1=0.5	y2=1
z1=1/3	[0.08,0.82]	[0.08,0.82]
z2=2/3	[0.30,0.79]	[0.33,0.77]
z3=1	[0.31,0.52]	[0.33,0.42]

$W_{ESE,ATT}$	y1=0.5	y2=1
z1=1/3	[0.05,0.85]	[0.05,0.85]
z2=2/3	[0.27,0.82]	[0.3,0.8]
z3=1	[0.28,0.55]	[0.3,0.45]

Weight $W_{CW,ATT}$

$W_{CW,ATT}$	**y1=0.5**	**y2=1**
z1=1/3	[0.04, 0.43]∪[0.46, 0.83]	[0.11, 0.41]∪[0.49, 0.81]
z2=2/3	[0.19, 0.41]	[0.27, 0.39]
z3=1	∅	∅

$W_{CW,ATT}$	**y1=0.5**	**y2=1**
z1=1/3	[0.025, 0.845]	[0.095, 0.425] ∪ [0.475, 0.825]
z2=2/3	[0.175, 0.425]	[0.255, 0.405]
z3=1	∅	∅

$W_{CW,ATT}$	**y1=0.5**	**y2=1**
z1=1/3	[0,0.87]	[0.07,0.87]
z2=2/3	[0.15,0.47]	(0.23, 0.43) ∪ (0.45, 0.45)
z3=1	[0.42,0.45]	∅

$W_{CW,ATT}$	**y1=0.5**	**y2=1**
z1=1/3	[0,0.89]	[0.05, 0.87]
z2=2/3	[0.13,0.49]	[0.21, 0.47]
z3=1	[0.40,0.47]	[0.43, 0.45]

$W_{CW,ATT}$	**y1=0.5**	**y2=1**
z1=1/3	[0,0.92]	[0.02,0.9]
z2=2/3	[0.1,0.52]	[0.18,0.5]
z3=1	[0.37,0.5]	[0.4,0.48]

Weight $W_{MP,CW}$

$W_{MP,CW}$	y1=0.5	y2=1
z1=1/3	[0.29,0.89]	[0.39, 0.81]
z2=2/3	[0.37,0.76]	[0.44, 0.66]
z3=1	[0.59,0.73]	[0.59, 0.66]

$W_{MP,CW}$	y1=0.5	y2=1
z1=1/3	[0.275,0.905]	[0.375,0.825]
z2=2/3	[0.355,0.775]	[0.425,0.675]
z3=1	[0.575,0.745]	[0.575,0.675]

$W_{MP,CW}$	y1=0.5	y2=1
z1=1/3	[0.25,0.93]	[0.35,0.85]
z2=2/3	[0.33,0.80]	[0.4,0.70]
z3=1	[0.55,0.77]	[0.55,0.7]

$W_{MP,CW}$	y1=0.5	y2=1
z1=1/3	[0.23,0.95]	[0.33,0.87]
z2=2/3	[0.31,0.82]	[0.38,0.72]
z3=1	[0.53,0.79]	[0.53,0.72]

$W_{MP,CW}$	y1=0.5	y2=1
z1=1/3	[0.20, 0.98]	[0.30, 0.90]
z2=2/3	[0.28, 0.85]	[0.35, 0.75]
z3=1	[0.5, 0.82]	[0.50, 0.75]

Weight $W_{MP,ESE}$

$W_{MP,ESE}$	y1=0.5	y2=1
z1=1/3	[0.37,0.91]	[0.54,0.81]
z2=2/3	[0.49,0.86]	[0.64,0.76]
z3=1	[0.57,0.78]	[0.67,0.73]

$W_{MP,ESE}$	y1=0.5	y2=1
z1=1/3	[0.355,0.925]	[0.525,0.825]
z2=2/3	[0.475,0.875]	[0.625,0.775]
z3=1	[0.555,0.795]	[0.655,0.745]

$W_{MP,ESE}$	y1=0.5	y2=1
z1=1/3	[0.33,0.95]	[0.5,0.85]
z2=2/3	[0.45,0.9]	[0.6,0.8]
z3=1	[0.53,0.82]	[0.63,0.77]

$W_{MP,ESE}$	y1=0.5	y2=1
z1=1/3	[0.31,0.97]	[0.48,0.87]
z2=2/3	[0.43,0.92]	[0.58,0.82]
z3=1	[0.51,0.84]	[0.61,0.79]

$W_{MP,ESE}$	y1=0.5	y2=1
z1=1/3	[0.28,1]	[0.45,0.9]
z2=2/3	[0.40,0.95]	[0.55,0.85]
z3=1	[0.48,0.87]	[0.58,0.82]

APPENDIX 14. Results of Sensitivity Analysis

Values of MP when $W_{ESE,ATT}$ is change by δ and rest weights are remaining same:

Module	Output of MP with original values of weights	Output of MP when the uncertainty of $W_{ESE,ATT}$ changed by $\delta=0.01$	Output of MP when the uncertainty of $W_{ESE,ATT}$ changed by $\delta=0.025$	Output of MP when the uncertainty of $W_{ESE,ATT}$ changed by $\delta=0.05$	Output of MP when the uncertainty of $W_{ESE,ATT}$ changed by $\delta=0.7$	Output of MP when the uncertainty of $W_{ESE,ATT}$ changed by $\delta=0.1$
1	0.43638888	0.436389	0.436389	0.436389	0.436389	0.436389
2	0.45138888	0.451389	0.451389	0.451389	0.451389	0.451389
3	0.66972222	0.451389	0.669722	0.669722	0.669722	0.669722
4	0.27277777	0.272778	0.272778	0.272778	0.272778	0.272778
5	0.635	0.635	0.635	0.635	0.635	0.635
6	0.110556	0.110556	0.110556	0.110556	0.108333	0.1825
7	0.44305555	0.443056	0.443056	0.443056	0.443056	0.443056
8	0.45138888	0.451389	0.451389	0.451389	0.451389	0.451389
9	0.29055555	0.290556	0.290556	0.290556	0.290556	0.290556
10	0.28083333	0.280833	0.280833	0.280833	0.280833	0.280833
11	0.2875	0.2875	0.2875	0.2875	0.2875	0.2875
12	0.23931755	0.239318	0.239318	0.239318	0.466818	0.1825
13	0.16222222	0.162222	0.162222	0.162222	0.162222	0.162222
14	0.43638888	0.436389	0.436389	0.436389	0.436389	0.436389
15	0.43638888	0.436389	0.436389	0.436389	0.436389	0.436389
16	0.29527777	0.295278	0.295278	0.105556	0.177222	0.335278
17	0.27833333	0.278333	0.278333	0.278333	0.278333	0.278333
18	0.60888888	0.608889	0.608889	0.608889	0.608889	0.608889
19	0.285	0.285	0.285	0.285	0.285	0.285
20	0.41416666	0.414167	0.414167	0.414167	0.414167	0.283611
21	0.27833333	0.278333	0.278333	0.278333	0.278333	0.278333
22	0.68694444	0.686944	0.686944	0.686944	0.460833	0.4225
23	0.27833333	0.278333	0.278333	0.278333	0.278333	0.278333
24	0.27833333	0.278333	0.278333	0.278333	0.278333	0.272222
25	0.50685555	0.506856	0.506856	0.506856	0.317133	0.3913
26	0.635	0.635	0.635	0.635	0.635	0.615161
27	0.28083333	0.280833	0.280833	0.280833	0.280833	0.280833
28	0.18277777	0.182778	0.182778	0.105556	0.177222	0.335278
29	0.45138888	0.451389	0.451389	0.451389	0.451389	0.451389
30	0.29055555	0.290556	0.290556	0.290556	0.290556	0.284444

Values of MP when $\boldsymbol{W_{CW,ATT}}$ is change by δ and rest weights are remaining same:

Module	Output of MP with original values of weights	Output of MP when the uncertainty of $W_{CW,ATT}$ changed by δ=0.01	Output of MP when the uncertainty of $W_{CW,ATT}$ changed by $\delta = 0.025$	Output of MP when the uncertainty of $W_{CW,ATT}$ changed by $\delta = 0.05$	Output of MP when the uncertainty of $W_{CW,ATT}$ changed by $\delta = 0.7$	Output of MP when the uncertainty of $W_{CW,ATT}$ changed by $\delta = 0.1$
1	0.43638888	0.436389	0.436389	0.436389	0.451389	0.451389
2	0.45138888	0.451389	0.451389	0.451389	0.451389	0.451389
3	0.66972222	0.669722	0.669722	0.669722	0.684722	0.684722
4	0.27277777	0.272778	0.272778	0.272778	0.033333	0.033333
5	0.635	0.635	0.635	0.635	0.675156	0.675156
6	0.11055555	0.110556	0.179167	0.175	0.175	0.175
7	0.44305555	0.443056	0.443056	0.443056	0.443056	0.443056
8	0.45138888	0.451389	0.451389	0.451389	0.451389	0.451389
9	0.29055555	0.290556	0.290556	0.290556	0.108333	0.108333
10	0.28083333	0.280833	0.280833	0.280833	0.280833	0.280833
11	0.2875	0.2875	0.2875	0.2875	0.295833	0.295833
12	0.23931755	0.239318	0.307933	0.303767	0.303767	0.303767
13	0.16222222	0.162222	0.162222	0.162222	0.162222	0.162222
14	0.43638888	0.436389	0.436389	0.436389	0.436389	0.436389
15	0.43638888	0.436389	0.436389	0.436389	0.436389	0.436389
16	0.29527777	0.294722	0.291667	0.2875	0.491267	0.491267
17	0.27833333	0.278333	0.278333	0.175	0.175	0.175
18	0.60888888	0.608889	0.608889	0.383533	0.383533	0.383533
19	0.285	0.285	0.285	0.1	0.1	0.1
20	0.41416666	0.414167	0.451389	0.443056	0.443056	0.443056
21	0.27833333	0.278333	0.278333	0.175	0.175	0.175
22	0.68694444	0.683611	0.680556	0.676389	0.676389	0.676389
23	0.27833333	0.278333	0.278333	0.278333	0.108333	0.108333
24	0.27833333	0.278333	0.108333	0.295	0.175	0.175
25	0.50685555	0.503522	0.500467	0.4963	0.4963	0.4963
26	0.635	0.676794	0.674572	0.666239	0.666239	0.666239
27	0.28083333	0.280833	0.280833	0.2875	0.2875	0.2875
28	0.18277777	0.182222	0.179167	0.175	0.175	0.175
29	0.45138888	0.451389	0.451389	0.451389	0.451389	0.451389
30	0.29055555	0.110556	0.108333	0.175	0.175	0.175

Values of MP when $\boldsymbol{W_{MP,CW}}$ is change by δ and rest weights are remaining same:

Module	Output of MP with original values of weights	Output of MP when the uncertainty of $W_{MP,CW}$ changed by δ=0.01	Output of MP when the uncertainty of $W_{MP,CW}$ changed by $\delta = 0.025$	Output of MP when the uncertainty of $W_{MP,CW}$ changed by $\delta = 0.05$	Output of MP when the uncertainty of $W_{MP,CW}$ changed by $\delta = 0.7$	Output of MP when the uncertainty of $W_{MP,CW}$changed by $\delta = 0.1$
1	0.43638888	0.435556	0.434028	0.430556	0.429444	0.602778
2	0.45138888	0.451111	0.4425	0.4425	0.4425	0.427778
3	0.66972222	0.668889	0.667361	0.663889	0.607778	0.602778
4	0.27277777	0.2725	0.272222	0.602778	0.602778	0.602778
5	0.635	0.633056	0.642361	0.605556	0.607778	0.602778
6	0.11055555	0.110556	0.110556	0.110556	0.348356	0.348356
7	0.44305555	0.442778	0.454444	0.430556	0.429444	0.427778
8	0.45138888	0.451111	0.450833	0.450833	0.4425	0.4425
9	0.29055555	0.608056	0.606944	0.605556	0.604444	0.602778
10	0.28083333	0.28	0.278472	0.605556	0.604444	0.602778
11	0.2875	0.287222	0.286667	0.2775	0.273889	0.272222
12	0.23931755	0.239322	0.239322	0.239322	0.239322	0.239322
13	0.16222222	0.162222	0.162222	0.272222	0.272222	0.602778
14	0.43638888	0.435556	0.434028	0.430556	0.607778	0.602778
15	0.43638888	0.435556	0.434028	0.611111	0.607778	0.602778
16	0.29527777	0.295278	0.295278	0.295278	0.299722	0.295278
17	0.27833333	0.2725	0.276389	0.275	0.604444	0.602778
18	0.60888888	0.608056	0.606944	0.605556	0.604444	0.602778
19	0.285	0.284722	0.284444	0.284444	0.284444	0.602778
20	0.41416666	0.412222	0.409444	0.405278	0.401944	0.602778
21	0.27833333	0.2775	0.276389	0.275	0.604444	0.602778
22	0.68694444	0.686944	0.686944	0.686944	0.686944	0.686944
23	0.27833333	0.2775	0.606944	0.605556	0.604444	0.602778
24	0.27833333	0.2775	0.606944	0.605556	0.604444	0.602778
25	0.50685555	0.506856	0.506856	0.506856	0.506856	0.506856
26	0.635	0.633056	0.642361	0.614444	0.607778	0.602778
27	0.28083333	0.28	0.278472	0.605556	0.604444	0.602778
28	0.18277777	0.183333	0.182778	0.182778	0.182778	0.428711
29	0.45138888	0.451111	0.450833	0.4425	0.4425	0.430278
30	0.29055555	0.289722	0.288611	0.605556	0.604444	0.602778

Values of MP when $\boldsymbol{W_{MP,ESE}}$ is change by δ and rest weights are remaining same:

Module	Output of MP with original values of weights	Output of MP when the uncertainty of $W_{MP,ESE}$ changed by δ=0.01	Output of MP when the uncertainty of $W_{MP,ESE}$ changed by $\delta = 0.025$	Output of MP when the uncertainty of $W_{MP,ESE}$ changed by $\delta = 0.05$	Output of MP when the uncertainty of $W_{MP,ESE}$ changed by $\delta = 0.7$	Output of MP when the uncertainty of $W_{MP,ESE}$ changed by $\delta = 0.1$
1	0.43638888	0.671111	0.673194	0.676667	0.679444	0.683056
2	0.45138888	0.685	0.686944	0.686111	0.686667	0.686944
3	0.66972222	0.671111	0.673194	0.676667	0.679444	0.683056
4	0.27277777	0.279167	0.280417	0.2825	0.288056	0.294167
5	0.635	0.6375	0.644583	0.655	0.663333	0.671944
6	0.11055555	0.110556	0.110556	0.185556	0.298056	0.298056
7	0.44305555	0.443889	0.678472	0.680556	0.682222	0.684167
8	0.45138888	0.451667	0.452083	0.686111	0.686667	0.686944
9	0.29055555	0.291944	0.294028	0.41	0.412778	0.416389
10	0.28083333	0.282222	0.439444	0.676667	0.679444	0.683056
11	0.2875	0.288333	0.289583	0.291667	0.448889	0.684167
12	0.23931755	0.239989	0.239989	0.314989	0.110556	0.185556
13	0.16222222	0.162222	0.162222	0.164167	0.164722	0.165
14	0.43638888	0.671111	0.673194	0.676667	0.679444	0.683056
15	0.43638888	0.437778	0.673194	0.676667	0.679444	0.683056
16	0.29527777	0.295833	0.296667	0.6825	0.686944	0.686944
17	0.27833333	0.279167	0.280417	0.285278	0.290556	0.449722
18	0.60888888	0.610278	0.610972	0.635	0.64	0.671944
19	0.285	0.285278	0.292639	0.286389	0.286944	0.298333
20	0.41416666	0.416667	0.65375	0.66	0.665	0.671944
21	0.27833333	0.279722	0.284306	0.287778	0.290556	0.449722
22	0.68694444	0.686944	0.686944	0.686944	0.686944	0.686944
23	0.27833333	0.279167	0.281806	0.287778	0.290556	0.294167
24	0.27833333	0.279167	0.281806	0.285278	0.290556	0.294167
25	0.50685555	0.506856	0.662411	0.506856	0.686944	0.686944
26	0.635	0.6375	0.64125	0.655	0.663333	0.671944
27	0.28083333	0.437778	0.439861	0.676667	0.679444	0.683056
28	0.18277777	0.180833	0.296667	0.298056	0.298056	0.453611
29	0.45138888	0.451667	0.685417	0.686111	0.686667	0.686944
30	0.29055555	0.291389	0.292639	0.2975	0.412778	0.416389

Appendix 15. MATLAB Program used for Sensetivity Analysis