Incentivizing Proportional Fairness for Multi-Task Allocation in Crowdsensing

Lu, J., Liu, H., Jia, R., Zhang, Z., Wang, X. & Wang, J

Original citation & hyperlink:
Lu, J, Liu, H, Jia, R, Zhang, Z, Wang, X & Wang, J 2023, 'Incentivizing Proportional Fairness for Multi-Task Allocation in Crowdsensing', IEEE Transactions on Services Computing, vol. (In-Press), pp. (In-Press). https://doi.org/10.1109/tsc.2023.3325636

DOI 10.1109/tsc.2023.3325636
ESSN 1939-1374

Publisher: Institute of Electrical and Electronics Engineers

This article has been accepted for publication in IEEE Transactions on Services Computing. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSC.2023.3325636

Incentivizing Proportional Fairness for Multi-Task Allocation in Crowdsensing

Jianfeng Lu, *Member, IEEE,* Haibo Liu, Riheng Jia, *Member, IEEE,* Zhao Zhang, *Member, IEEE,* Xiong Wang, *Member, IEEE,* Jiangtao Wang, *Member, IEEE*

***Abstract*—Effective incentive mechanisms are invaluable in crowdsensing to stimulate the enthusiasm of strategic users. However, existing work focusing on multi-task allocation with the objective of purely maximizing the social utility may result in the problem of unbalanced allocation, which may damage the social fairness. This motivates us to introduce proportional fairness into the design of a novel fairness-aware incentive mechanism for the first time. Specifically, we first model the interaction of multi-task allocation in crowdsensing as a multi-requester multi-worker Stackelberg game, and then transform the fairness-aware multi-task allocation problem into a fairness-aware incentive mechanism design problem. Next, we prove that there is a unique Stackelberg equilibrium, and also show that it can be efficiently derived through cautiously proposed algorithms. Since the existing equilibrium may not be optimal, we further design a secondary allocation rule to maximize both social utility and system performance, while achieving proportional fairness at a minimum cost. Finally, extensive experiments using both synthetic and real-world datasets demonstrate the superiority of our proposed mechanism compared to the state of the arts.**

***Index Terms*—crowdsensing, incentive mechanism, proportional fairness, Stackelberg game, multi-task allocation**

I. INTRODUCTION

Crowdsensing has become a new paradigm for the collection and analysis of pervasive sensory data, far beyond the scale of what was previously possible [1, 2]. However, one of the long-standing concerns is to motivate a large population of various workers to engage in tasks and collect high-quality sensory data, as participating in crowdsensing tasks will incur costs in the process of collecting, processing, and uploading sensory data [3]. Worse yet, sensing participation also exposes workers to potential privacy leakage without satisfactorily sensing compensation [4]. Moreover, the priority goal of workers and requesters is to maximize their own utilities, so the prosperity of crowdsensing will be hindered by antisocial behaviors, such as "free-riding" and "false-reporting"[5]. Also, over-payment leads to over-sensing, while under-payment leads to under-sensing, both of which will decrease the long-term utilities of the requesters [6]. As a result, how to effectively incentivize the participation of various users [1] is the key issue for the success of crowdsensing.

Existing efforts mainly rely on monetary [7] or non-monetary [8] rewards to stimulate the enthusiasm of strategic users to undertake sensing tasks in online [9] or offline [10] scenarios. A common drawback shared by existing work is that purely maximizing the social utility may present a rather unbalanced task allocation, which would be considered extremely unfair and unacceptable for the worst-off users. Since there are many crowdsensing platforms with similar functions, a user has many choices. However, limited by time, space, sensing ability, etc., users cannot participate in all platforms. And hence, they need to strategically select the most suitable choice. Once a user suffers unfair, she may choose to leave the current platform and join another platform that can bring her higher benefit, thereby affecting the stability and long-term utility of the system. Consequently, when designing an incentive mechanism for crowdsensing, both social fairness and system efficiency need to be taken into consideration. Let us illustrate this point with a simple example.

Example 1. *Suppose in a micro jobs website that rewards for completing simple tasks, such as Amazon Mechanical Turk, Freelancer, Upwork, etc., there are two micro jobs τ_1 and τ_2 with service fees of \$20 and \$10, and four workers w_1, w_2, w_3, and w_4 with unit costs of 1, 2, 4, and 5, respectively [2]. The platform has six possible solutions that can assign two tasks to four workers (To prevent free-riding and idle tasks, the platform will preclude assigning a single worker or all four workers to a single task.). As depicted in Table I, let u_i and $\overline{u_i}$ denote the utility gain for worker w_i and the maximum utility he can get across all solutions, respectively [3]. On the one hand, in pursuit of maximizing the social utility (i.e., $\sum_{i=1}^{4} u_i$),*

This work was supported in part by the National Natural Science Foundation of China (No. 62372343, 62272417, 62072411), in part by the Zhejiang Provincial Natural Science Foundation of China (No. LR21F020001), in part by the Key Research and Development Program of Hubei Province (No. 2023BEB024), and in part by the fund from Hubei Province Key Laboratory of Intelligent Information Processing and Real-time Industrial System (Wuhan University of Science and Technology). (Corresponding author: Riheng Jia)

Jianfeng Lu is with the School of Computer Science and Technology, Wuhan University of Science and Technology, Wuhan 430065, China (e-mail: lujianfeng@wust.edu.cn).

Haibo Liu, Riheng Jia, and Zhao Zhang are with the School of Computer Science and Technology, Zhejiang Normal University, Jinhua 321004, China (e-mail: lhbzjnu@163.com; rihengjia@zjnu.edu.cn; zhaozhang@zjnu.cn).

Xiong Wang is with the School of Computer Science and Technology, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: xiongwang@hust.edu.cn).

Jiangtao Wang is with the Center for Intelligent Healthcare, Coventry University, CV1 5FB Coventry, U.K. (e-mail: jiangtao.wang@coventry.ac.uk).

[1] For convenience, we refer to both workers and requesters as users, which is a general term for them. When it comes to distinguishing individuals, we will refer to them as workers or requesters, respectively.

[2] worker w_i's sensing cost is equal to the product of her unit cost c_i and sensing time t_i, where t_i can be calculated according to Eq. (16), and the reward that w_i should be payed can be calculated according to Eq. (7), and j represents the task number. For the convenience of readers' understanding, j is omitted here.

[3] u_i is equal to the reward she received minus the sensing cost she offered, which can be directly calculated according to Eq. (20).

Table I
COMPARISON BETWEEN THE ABSOLUTE UTILITY AND RELATIVE LOSS IN TASK ALLOCATION SOLUTIONS.

No	Solution	u_1	u_2	u_3	u_4	$\sum_{i=1}^{4} u_i$	$\frac{\overline{u_1}-u_1}{u_1}$	$\frac{\overline{u_2}-u_2}{u_2}$	$\frac{\overline{u_3}-u_3}{u_3}$	$\frac{\overline{u_4}-u_4}{u_4}$	$\sum_{i=1}^{4} \frac{\overline{u_i}-u_i}{u_i}$
1	$\{\tau_1 \to \{w_1, w_2\}, \tau_2 \to \{w_3, w_4\}\}$	8.89	2.22	3.09	1.97	16.17	0.56	3.59	1.00	1.00	**6.15**
2	$\{\tau_1 \to \{w_3, w_4\}, \tau_2 \to \{w_1, w_2\}\}$	4.44	1.11	**6.17**	**3.95**	15.67	2.13	8.19	**0.00**	**0.00**	10.32
3	$\{\tau_1 \to \{w_1, w_3\}, \tau_2 \to \{w_2, w_4\}\}$	12.80	5.10	0.80	0.82	19.52	0.09	1.00	6.71	3.82	11.62
4	$\{\tau_1 \to \{w_1, w_4\}, \tau_2 \to \{w_2, w_3\}\}$	**13.89**	4.44	1.11	0.56	**20.00**	**0.00**	1.30	4.56	6.05	11.91
5	$\{\tau_1 \to \{w_2, w_4\}, \tau_2 \to \{w_1, w_3\}\}$	6.40	**10.20**	0.40	1.63	18.63	1.17	**0.00**	14.43	1.42	17.02
6	$\{\tau_1 \to \{w_2, w_3\}, \tau_2 \to \{w_1, w_4\}\}$	6.94	8.89	2.22	0.28	18.33	1.00	0.15	1.78	13.11	16.04

the fourth solution is the optimal choice. On the other hand, seeking to minimize total relative loss (i.e., $\sum_{i=1}^{4} \frac{\overline{u_i}-u_i}{u_i}$), the first solution is much fairer than the fourth solution. Because the benefits of workers w_1 and w_2 are based on the damage of workers w_2 and w_3. This example reveals that the mere pursuit of social utility maximization may disproportionately hurt the utilities of some workers.

Social fairness has been a perennial and venerable topic of social welfare, and it involves comparing one's utility gained with that of the others [11, 12]. Recently, a variety of social fairness criteria have been proposed in the literature [13], and different approaches have been developed for guaranteeing the social fairness in resource allocation [14], machine learning [15], computer vision [16], etc. Note that, social fairness is not uniquely defined in the literature, as it depends heavily on the specific problem setting and also on individuals' perceptions of fair solutions [12, 17]. In this paper we consider fairness concepts in a general multiple crowdsensing task allocation problem. Although maximizing social utility is essential, a fairer criterion should be the relative utility gain rather the absolute utility gain, since simply pursuing social utility maximization may disproportionately hurt the utilities of some users, as shown in Example 1. Despite the importance of social fairness, research effort on multi-task allocation in crowdsensing rarely combines social fairness with system efficiency to date. The main shortcoming in most of existing studies is that the solution is chosen by a central decision maker, without considering users' active roles. Actually, rational and selfish workers will strategically select tasks and determine the level of contributions to maximize their own utilities. Also, strategic requesters will engage in vicious price competition in order to compete for limited worker resources. The aforementioned misbehaviors will inevitably damage the benefits of some users, and the worse-off users may be disappointed with the crowdsensing platform, directly reducing its competitiveness against other platforms. Therefore, how to incentivize the participation of various users, without giving an impression that only a few users are benefiting, remains a significant challenge in practice.

With the above challenges and gaps in mind, this paper aims to maximize the social utility of crowdsensing while maintaining fairness at a minimum cost. To accomplish this mission, we introduce proportional fairness (PF) into the development of a novel fairness-aware incentive mechanism for crowdsensing. One reason to choose PF is that simply enforcing equality across various users potentially disproportionately hurts those users with better performance, so a more appropriate criterion should be the relative utility gain rather than the absolute one [11]. Another reason for such a choice is that PF is a common fairness metric, which can effectively deal with the conflict between system efficiency and social fairness [12, 18]. To the best of our knowledge, studies on fair allocation of limited and strategic user pool in the presence of a knapsack-like "capacity" constraint has still been under-explored so far. We believe that studying this topic is timely since fairness plays a vital role in the success of crowdsensing, without which the participation from the worse-off users will inevitably be discouraged, thereby affecting sustainable healthy collaboration in such an ecosystem. Although few work investigates fairness-aware incentive mechanisms for crowdsensing [19–21], these are usually based on empirical models without accurate mathematical models to formulate and quantify the fairness. In contrast, we initiate the study of strategy-proof and PF-aware incentive mechanism for multi-task allocation in crowdsensing for the first time, and show that our mechanism maintains PF at a minimum cost.

Our main contributions are summarized as follows:

- In terms of idea, to maximize the social utility of crowdsensing while maintaining fairness at a minimum cost, we introduce PF into the design of a novel fairness-aware incentive mechanism for the first time. This idea can not only reduce excessive intervention on the crowdsensing platform, but also is consistent with the risk neutral pricing theory [22], which is particularly suitable for studying the strategic choices of rational and selfish users in crowdsensing.
- In terms of approach, we model the interaction of multi-task allocation in crowdsensing as a multi-leader multi-follower Stackelberg game. More importantly, we prove that there only exists a unique Stackelberg equilibrium that can be efficiently characterized through our proposed Nash equilibrium calculation algorithms. Furthermore, we design a secondary allocation rule to maximize the social utility while maintaining PF at a minimum cost. Based on the rigorous integration of the above work, we can design the optimal fairness-aware incentive mechanism for crowdsensing.
- In terms of experimental verification, we conduct extensive performance evaluations on a synthetic and four real-world datasets to further demonstrate the superiority of our proposed fairness-aware incentive mechanism. Compared with three baselines, our proposed mechanism can maximize not only social utility but also system

performance under the premise of achieving PF at a minimum cost.

In the rest of this article, we first review related work in Section II, and then introduce the preliminaries and formulates the fairness-aware multi-task allocation problem in Section III. Followed by the development of a fairness-aware incentive mechanism in Section IV, we describe the design details of our mechanism in Section V. We conduct experimental evaluations in Section VI, and finally draw the conclusion in Section VII.

II. Related Work

Although much effort has been devoted to investigating incentive mechanisms for crowdsensing task allocation in the literature [23–25], they usually focus on how to improve the social utility of crowdsensing task allocation. A common drawback shared by existing work is that an optimal solution with the objective of optimizing the social utility may result in the problem of unbalanced allocation, which may damage the social fairness. Accordingly, it is critical to address the issue of fairness-aware incentive mechanisms in crowdsensing in a principled manner, as users who suffer unfairness would negatively affect their active participation and sensory data quality, and even leave the system, thereby affecting the stability and long-term utility of the system. In the literature, a few papers investigate incentive mechanism for crowdsensing with fairness considerations, the social fairness research in crowdsensing can be divided into *custom fairness* and *axiomatic fairness* according to the differences in scene modes and the relevance of application purposes.

Custom fairness means that researchers design a fairness principle and measure the sufficiency of it on this basis. For instance, Korn *et al.* [32] declared that their work is the first to consider multi-dimensional fairness for data provider selection in the crowdsensing system. However, their proposed fairness factor is only quantified from three aspects, including data quality, lost frequency, and submission time of each provider. Furthermore, they considered a scenario consisting of single requester and multiple workers, but only considered the fairness of workers. In contrast, our game model consists multiple requesters and multiple workers, and our mechanism maintains PF for both requesters and workers at a minimum cost. Similarly, Liu *et al.* [33] considered the fairness of tasks and users, where each task should be allocated according to the allocation frequency, and each user's limited ability should be fully utilized to handle tasks. However, they did not provide a formal definition of fairness, which is actually two independent constraints aimed at minimizing allocation costs and fully utilizing users' abilities. Earlier, Zhu *et al.* [21] considered the case where malicious competition would affect the fairness of the bidding process, combined a reverse auction and a Vickrey auctions to design an incentive mechanism for crowdsensing, and proved that the designed mechanism can satisfy five economic properties such as computational efficiency, individual rationality, budget-balance, truthfulness, and honesty. They therefore believed that the abovementioned properties can improve the fairness of the bidding process and the quality of sensory data. On reward-fair, Goel *et al.* [34] defined a notion of fairness of rewards, that is, the expected reward of each worker is only directly proportional to the accuracy of her reported answers. Wang et al [35] demanded that the higher rewards for participating users with higher local model quality in model training, and no rewards for non-participating users. On cost-fair, Sun *et al.* [36] concentrated on the cost-fair task allocation which aims to balance the sensing costs undertaken by all users as much as possible when assigning tasks to users, and satisfy the data reliability required by the requester. Wang *et al.* [37] suggested that service providers acquire accurate sensing data at a minimum cost and workers receive the high return by contributing the least efforts. On scheduling fairness, Li *et al.* [38] ensured that all users have opportunities to participate in performing tasks and earn rewards. And Song *et al*. [39] took the fairness of users and tasks into consideration, while the fairness of the user refers to the full use of each user's ability to process the task, and the fairness of the task means that each task will have a certain frequency of assignment according to its characteristics. The commonality of the abovementioned studies is that the concept of fairness is based on empirical models, and there is a lack of accurate mathematical models to formulate and quantify it.

Compared with custom fairness, axiomatic fairness is more rigorous and objective because it formalizes the concept of social fairness based on mathematical theory. For instance, Li *et al.* [19] applied the decoy effect and fairness preference theory from behavioral economics to the design of an incentive mechanism for crowdsensing. However, the determination of the key parameters of their model leads to high computational complexity, and the boundary of the decoy task is difficult to be determined. Wang *et al*. [18] used Lyapunov function to handle discontinuous coverage to optimize PF in worker distribution, thereby ensuring that low-value tasks also remain fully competitive be in a long period. Nevertheless, the solution of PF is simply converted into the maximal logarithm function in [40], and the fairness requirement of requesters is also ignored. Different from the above work, we are the first to study strategy-proof fairness-aware incentive mechanism for crowdsensing. In particular, we indirectly and precisely achieve PF-aware multi-task allocation in crowdsensing through the optimal design of the incentive mechanism.

III. Problem Formulation

A. System Model

As illustrated in Fig. 1, a Multi-Task Allocation (MTA) system consists of a platform, a set $\mathcal{R} = \{r_1, \cdots, r_m\}$ of requesters, and a set $\mathcal{W} = \{w_1, \cdots, w_n\}$ of workers in the system. Each requester $r_j \in \mathcal{R}$ can post a sensing task $\tau_j \in \mathcal{T}$ to the crowdsensing platform, where $\mathcal{T} = \{\tau_1, \cdots, \tau_m\}$ denotes the set of all publicized sensing tasks. Requester r_j needs to pay sufficient reward to compensate the incurred costs of workers when participating in the sensing activity. When tasks are described and publicized by the crowdsensing platform, each worker $w_i \in \mathcal{W}$ can determine her sensing plan strategy $\pi_i = (s_i, t_i)$, where s_i is the selected task and t_i means the sensing time. In general, a task is completed by

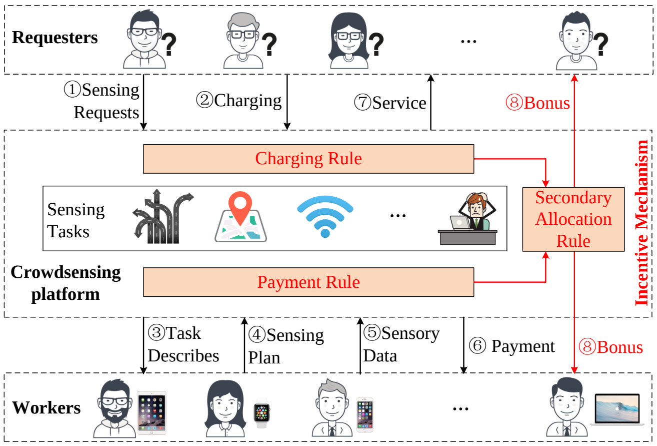

Fig. 1. The process of MTA in crowdsensing.

Table II
SUMMARY OF NOTATIONS IN THIS PAPER

Variable	Description
τ_j, $\mathcal{T}$	jth task, $\mathcal{T} = \{\tau_1, \cdots, \tau_m\}$.
r_j, $\mathcal{R}$	jth requester, $\mathcal{R} = \{r_1, \cdots, r_m\}$.
w_i, $\mathcal{W}$, $\mathcal{W}_j$	ith worker, $\mathcal{W} = \{w_1, \cdots, w_n\}$, a set of workers participated in τ_j.
κ_j, $\mathcal{K}$	unit value of τ_j, $\mathcal{K} = \{\kappa_1, \cdots, \kappa_m\}$
c_i, $\mathcal{C}$	unit cost of w_i, $\mathcal{C} = \{c_1, \cdots, c_n\}$.
t_i, t_{-i}, t_{ij}	sensing time of w_i, sensing time profile excluding t_i, sensing time of w_i when she undertakes τ_j.
s_i, s_{-i}	task selection of w_i, task selection profile excluding s_i.
π_i, π_{-i}, Π	$\pi_i = (s_i, t_i)$, strategy profile excluding π_i, $\Pi = \{\pi_1, \cdots, \pi_n\}$.
γ_j, γ_j^w, Γ	service fee for τ_j, reward for workers in $\mathcal{W}_j$, $\Gamma = \{\gamma_1, \cdots, \gamma_m\}$.
δ,	discount factor.
ε_i, ε_j^r	bonuses allocated to worker w_i, and requester r_j.
u_i, u_{ij}	utility function of w_i, w_i's utility when she undertakes τ_j.
v_j, $\mathcal{U}$	utility function of r_j, the sum of utilities of all users.

multiple workers together. Taking into account the practical factors (*e.g.*, time, location, effort, etc.), a worker is usually assumed to be able to perform one sensing task at one time, but this paper is not limited to this assumption. When a worker chooses to participate in multiple tasks, we simply treat her as multiple workers. Similarly, when a requester publicizes multiple sensing tasks, we simply treat her as multiple requesters. Conveniently, Table II lists frequently used notations in this paper.

According to Fig. 1 and Example 1, the typical process of a MTA system can be summarized as follows: First, requesters publish multiple tasks (e.g., τ_1 and τ_2) and send them to the crowdsensing platform (step ①). Next, the platform requires that the requester needs to pay the service fee (e.g., \$20 and \$10) according to the charging rules (step ②). After getting the description of sensing tasks from the platform (step ③), each worker determines her sensing plan (step ④, e.g., $\tau_1 \rightarrow \{w_1, w_2\}$, $\tau_2 \rightarrow \{w_3, w_4\}$, as well as the sensing time determined by these four workers), uploads the sensory data after completing the task (step ⑤), and receives the corresponding payment from the platform (step ⑥, the rewards received by workers minus their sensing costs are their utilities, such as u_1, u_2, u_3 and u_4 calculated in Table 1). Then, the platform collects the sensory data from the chosen workers, provides crowdsensing services to the requester based on aggregated sensory data (step ⑦). Finally, according to the required incentive, the platform distributes the bonus to both requesters and workers according to the secondary allocation rule, which will be addressed through our proposed fairness-aware incentive mechanism (step ⑧).

In the process of MTA, the decision-making behaviors of user are sequential: the decision of charging amount by requesters is made first, followed by the decision of sensing plan by workers. Therefore, we model this interaction as a multi-leader multi-follower Stackelberg game. In the first stage, each requester $r_j \in \mathcal{R}$ as a leader publicizes a task τ_i and its corresponding reward $\gamma_j > 0$ to the platform. While in the second stage, each worker w_i, as a follower, strategically selects task s_i and determines her sensing time t_i, which constitutes her sensing plan $\pi_i = (s_i, t_i)$. Similarly, let $\pi_{-i} = (s_{-i}, t_{-i})$ be the strategy profile excluding π_i. On the one hand, w_i will not participate in τ_j unless she can get enough reward $p_i^w(\gamma_j^w, t_{ij}, t_{-ij})$ to compensate for her cost $c_i t_{ij}$, where c_i is w_i's unit cost, γ_j^w is the sum of rewards paid to workers participating in τ_j, t_{ij} is w_i's sensing time when she undertakes τ_j, and t_{-ij} is the strategy profile excluding t_{ij}. Consequently, worker w_i's utility u_i can be formulated as:

$$u_i = \begin{cases} p_i^w(\gamma_j^w, t_{ij}, t_{-ij}) - c_i t_{ij}, & \text{if } s_i = \tau_j, \\ 0, & \text{else if } s_i \notin \mathcal{T}. \end{cases} \quad (1)$$

On the other hand, r_j's strategy is the reward γ_j she is willing to pay. As long as the set $\mathcal{W}_j$ of workers participating in task τ_j is not empty, r_j will obtain a service benefit $b_j^r(\kappa_j, \Pi_j)$. Otherwise, she will refuse to pay, and her utility will be zero. Thus, requester r_j's utility v_j can be formulated as:

$$v_j = \begin{cases} b_j^r(\kappa_j, \Pi_j) - \gamma_j, & \text{if } \mathcal{W}_j \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases} \quad (2)$$

where κ_j is the unit value of task τ_j, γ_j^r is the reward the requester pays to the platform, and Π_j is the strategy profile for a set $\mathcal{W}_j$ of workers.

At the crowdsensing platform's side, it charges a service fee γ_j from requester r_j, and pays reward γ_j^w to workers in $\mathcal{W}_j$. Here, we assume that the crowdsensing platform is non-profitable, i.e., it will distribute $\gamma_j - \gamma_j^w$ to the right requesters and workers as a bonus according to a secondary allocation rule as depicted in Definition 3. And hence, the social utility is defined as the sum of the utilities of all users, *i.e.*, $\mathcal{U} = \sum_{i:w_i \in \mathcal{W}} u_i + \sum_{j:r_j \in \mathcal{R}} v_j$.

B. Fairness-Aware Multi-Task Allocation Problem

In addition to the maximization of social utility, we also consider a classic fair solution called PF, which optimizes the total utility of users. That is, given a PF solution σ_{PF}, there does not exist any other solution σ that give a total relative improvement for a subset of users which is larger than the total relative loss inflicted on the other users. Referring to [12, 17], we give a formal definition of PF for crowdsensing.

Definition 1. *(PF) A solution $\sigma_{\mathrm{PF}} \in \Sigma$ is PF if, for any other Pareto optimal solution σ, it holds that*

$$\sum_{i:w_i\in\mathcal{W}} \frac{u_i^w(\sigma)-u_i^w(\sigma_{\mathrm{PF}})}{u_i^w(\sigma_{\mathrm{PF}})} + \sum_{j:r_j\in\mathcal{R}} \frac{u_j^r(\sigma)-u_j^r(\sigma_{\mathrm{PF}})}{u_j^r(\sigma_{\mathrm{PF}})} \le 0 \tag{3}$$

where $u_i^w(\sigma_{\mathrm{PF}}) > 0, \forall w_i \in \mathcal{W}$ and $u_j^r(\sigma_{\mathrm{PF}}) > 0, \forall r_j \in \mathcal{R}$.

Continuing with Example 1, none of the six solutions is a PF solution. Moreover, we cannot find any solution that satisfies Eq. (3) in Example 1. Obviously, the constraints of PF are too strict to strike a balance between fairness and efficiency. In general, as proved by Nicosiat *et al.* [17], the PF solution may not exist, if it exists it must be unique, and a Pareto-dominated solution can never be PF. Consequently, we need a relatively weak definition of PF as follows.

Definition 2. *(ξ-PF) Given $\xi \ge 0$, a solution σ_ξ is ξ-PF if, for any other Pareto optimal solution σ, it holds that*

$$\sum_{i:w_i\in\mathcal{W}} \frac{u_i(\sigma)-u_i(\sigma_\xi)}{u_i(\sigma_\xi)} + \sum_{j:r_j\in\mathcal{R}} \frac{v_j(\sigma)-v_j(\sigma_\xi)}{v_j(\sigma_\xi)} \le \xi, \tag{4}$$

where $u_i(\sigma_\xi) > 0, \forall w_i \in \mathcal{W}$ and $v_j(\sigma_\xi) > 0, \forall r_j \in \mathcal{R}$.

It is easy to find that the strictness of ξ-PF monotonically decreases with fairness threshold ξ. As long as ξ is sufficient large, ξ-PF solutions always exist. When $\xi = 0$, PF and ξ-PF are equivalent.

Definition 3. *(FAMTA) The Fairness-Aware Multi-Task Allocation problem with ξ-PF requirement in crowdsensing is formulated as:*

$$\begin{cases} \max\limits_{\sigma_\xi} \; \mathcal{U}(\sigma_\xi) \triangleq \sum\limits_{i:w_i\in\mathcal{W}} u_i(\sigma_\xi) + \sum\limits_{j:r_j\in\mathcal{R}} v_j(\sigma_\xi), \\ \text{s.t. Eq.}(4). \end{cases} \tag{5}$$

It is obvious that ξ-PF is a necessary but not sufficient condition to address the FAMTA problem. This is because due to the rational and selfish nature of users, neither requesters nor workers will follow a ξ-PF solution based social norm unless it is in their self-interest. Therefore, finding a ξ-PF solution is a necessary condition for addressing Eq. (5), while maximizing the social welfare is a sufficient condition, and thus a fundamental problem arises, *i.e.*, "How to apply the optimal ξ-PF solution for allocating multi-task to multi-worker in crowdsensing?".

IV. Fairness-Aware Incentive Mechanism

In this section, we introduce PF into the development of a novel fairness-aware incentive mechanism to address the FAMTA problem as shown in Eq. (5). Specifically, we adopt the proportional sharing method to design the charging rule and payment rule [27], and design a secondary allocation rule inspired by the idea of transferable utility [41].

Definition 4. *(FAIM) A Fairness-Aware Incentive Mechanism is represented as a 3-tuple $(\mathbb{C}, \mathbb{P}, \mathbb{A})$, i.e., a charging rule $\mathbb{C}$, a payment rule $\mathbb{P}$, and a secondary allocation rule $\mathbb{A}$.*

- *$\mathbb{C} : R^+ \cup \{0\} \to R^+ \cup \{0\}$ determines how much reward γ_j requester r_j should pay to the platform, i.e.,*

$$\gamma_j \leftarrow \arg\max_{\gamma_j \ge 0} v_j. \tag{6}$$

- *$\mathbb{P} : N^n \to R^+ \cup \{0\}$ represents the amount of reward worker w_i should receive, which is proportional to t_{ij}, i.e.,*

$$p_i^w(\gamma_j^w, t_{ij}, t_{-ij}) = \frac{t_{ij}}{\sum_{x:w_x\in\mathcal{W}_j} t_{xj}} \gamma_j^w, \tag{7}$$

where $\gamma_j^w = \delta\gamma_j$, and the discount factor $\delta \in (0, 1]$.

- *$\mathbb{A} : R^+ \cup \{0\} \to \{R^+ \cup \{0\}\}^{m+n}$ decides how much bonuses ε_i^w and ε_j^r should be allocated to worker w_i and requester r_j, respectively, i.e.,*

$$\sum_{i:w_i\in\mathcal{W}} \varepsilon_i^w + \sum_{j:r_j\in\mathcal{R}} \varepsilon_j^r = \sum_{j:r_j\in\mathcal{R}} (1-\delta)\gamma_j. \tag{8}$$

FAIM can be regarded as a set of rules that a crowdsensing platform uses to regulate the behaviors of its users. Combining Eq. (1) and Eq. (7), the utility maximization for each worker $w_i \in \mathcal{W}$ can be rewritten as:

$$\begin{cases} u_i^w \triangleq \max\limits_{j \le [1,m]} u_{ij}, \\ \text{s.t.} \begin{cases} u_{ij} = \frac{t_{ij}}{\sum_{x:w_x\in\mathcal{W}_j} t_{xj}} \gamma_j^w - c_i t_{ij}, \\ t_{ij} \ge 0, \forall i : w_i \in \mathcal{W}, \forall j : r_j \in \mathcal{R}. \end{cases} \end{cases} \tag{9}$$

According to the ubiquitous phenomenon of diminishing marginal utility in economics [42], we define the valuation function of requester r_j to the set $\mathcal{W}_j$ of workers' sensing time as a submodular function, and materialize it as $\kappa_j \log_\alpha(\sum_{x:w_x\in\mathcal{W}_j} t_{xj} + 1)$, where the range of $\alpha > 1$ makes v_j a strictly concave function in $\sum_{x:w_x\in\mathcal{W}_j} t_{xj}$. Accordingly, combining Eq. (2) and Eq. (6), the utility maximization for each requester $r_j \in \mathcal{R}$ can be rewritten as:

$$\begin{cases} v_j \triangleq \max\limits_{\gamma_j} \kappa_j \log_\alpha(\sum\limits_{x:w_x\in\mathcal{W}_j} t_{xj} + 1) - \gamma_j, \\ \text{s.t. } \gamma_j \ge 0, \forall j : r_j \in \mathcal{R}. \end{cases} \tag{10}$$

According to FAIM, the reward received by a worker is a function of her sensing plan $\pi_i = (s_i, t_i)$, and hence, it is in the self-interest of each worker to actively undertake sensing tasks and take the incentive to contribute a high level effort. Meanwhile, the rewards paid by requesters are directly related to the number of workers they can attract and the total sensing time. Therefore, FAIM can play a positive motivating role not only for workers but also for requesters. According to Eq. (9) and Eq. (10), it can be seen that FAIM always achieves a higher social utility than a non-incentive one. Therefore, the FAMTA problem can be equivalently transformed into the FAIM design problem.

Definition 5. *The FAIM design problem is formulated as*

$$\begin{cases} \max\limits_{(\mathbb{C},\mathbb{P},\mathbb{A})} \; \mathcal{U} \triangleq \sum\limits_{i:w_i\in\mathcal{W}} u_i + \sum\limits_{j:r_j\in\mathcal{R}} v_j, \\ \text{s.t.} \begin{cases} \sum\limits_{i:w_i\in\mathcal{W}} \frac{\widehat{u_i}-u_i}{u_i} + \sum\limits_{j:r_j\in\mathcal{R}} \frac{\widehat{v_j}-v_j}{v_j} \le \xi, \\ u_i = \max\limits_{j:r_j\in\mathcal{R}, t_i \ge 0} u_{ij}, \; \forall i : w_i \in \mathcal{W}, \\ v_j = \max\limits_{\gamma_j \ge 0} v_j, \; \forall j : r_j \in \mathcal{R}, \end{cases} \end{cases} \tag{11}$$

where $\hat{u_i}$ and $\hat{v_j}$ are obtained in Pareto optimal solution σ.

V. Optimal Design of FAIM

In this section, we first analyze whether there is a feasible solution to the FAIM design problem and how to find it, and then we design a secondary allocation rule to further increase the social utility.

A. A Feasible Solution to the FAIM Design Problem

Our goal in designing FAIM is to establish an ideal Stackelberg Equilibrium (SE) under which the social utility can be maximized. Note that the MTA Stackelberg game consists of two subgames, i.e., the Sensing Plan (SP) game and the Reward Declaration (RD) game. Specifically, in the RD game, the decision of charging amount by requesters is made first, followed by the decision of sensing plan by workers in the SP game. The Nash Equilibrium (NE) of the SP game and the RD game may together form an SE, which is provided as follows.

Definition 6. *(SE) Let $\Gamma^* = (\gamma_1^*, \cdots, \gamma_m^*)$ and $\Pi^* = \{\pi_1^*, \cdots, \pi_n^*\}$ be Nash Equilibria of the RD and SP game, respectively, (Γ^*, Π^*) is an SE for the MTA Stackelberg game if $\forall(\Gamma, \Pi)|\Gamma \neq \Gamma^* \vee \Pi \neq \Pi^*$,*

$$\begin{cases} u_i(\Gamma^*, \pi_i^*, \pi_{-i}^*) \geq u_i(\Gamma^*, \pi_i, \pi_{-i}^*), \forall i: w_i \in \mathcal{W}, \\ v_j(\gamma_j^*, \gamma_{-j}^*, \Pi^*) \geq v_j(\gamma_j, \gamma_{-j}^*, \Pi^*), \forall j: r_j \in \mathcal{R}. \end{cases} \tag{12}$$

Finding an SE is the prerequisite for addressing Eq. (11). An SE, if it exists, can be obtained by employing backward induction, that is, the SP game is solved first and then the RD game is solved. In the SP game, we are trying to determine whether there is a unique NE in the SP game with given $\Gamma = \{\gamma_1^w, \cdots, \gamma_m^w\}$. We first suppose that worker w_i has already selected task s_i, and her competitors who selected the same task as her are also determined. Worker w_i will naturally choose her optimal sensing time strategy, denoted $\bar{t_{ij}}$, that maximizes her own benefit.

Definition 7. *Given γ_j^w, $\mathcal{W}_j$, and t_{-ij}, $u_{ij}(\bar{t_{ij}}, t_{-ij}) \geq u_{ij}(t_{ij}, t_{-ij})$ over all $\forall t_{ij} \neq \bar{t_{ij}}$.*

Second, each strategic worker will also choose her optimal task selection strategy, denoted $\bar{s_i}$, i.e., the more the task reward and the weaker the competitor, the better.

Definition 8. *Given Γ, $\mathcal{W}$, and π_{-i}, $u_i(\bar{s_i}, \bar{t_i}, \pi_{-i}) \geq u_i(s_i, \bar{t_i}, \pi_{-i})$ over all $s_i \neq \bar{s_i}$.*

Obviously, each strategic worker will prefer $\bar{t_i}$ and $\bar{s_i}$ in an NE. The following theorem indicates that both of $\bar{t_i}$ and $\bar{s_i}$ are exist and unique.

Theorem 1. *Given Γ, there is a unique NE in the SP game.*

Proof. See Appendix A. □

In the following, we design Algorithm 1 to compute the unique NE of the SP game. Its time complexity is O($mn \log n$), where m and n are the number of tasks and workers, respectively. Detailed analysis and justification are shown in Corollary 1.

Algorithm 1: Computation of the NE for the SP game

Input: $\Gamma = \{\gamma_1^w, \cdots, \gamma_m^w\}$, $\mathcal{C} = \{c_1, \cdots, c_n\}$
Output: $\Pi^* = \{\pi_1^*, \cdots, \pi_n^*\}$

reorder elements in Γ and $\mathcal{W}$ so that $\gamma_1^w \geq \cdots \geq \gamma_m^w$ and $c_1 \leq \cdots \leq c_n$;
initialize $\bar{s_1} = \tau_1$ and $\mathcal{W}_1 = \{w_1\}$;
for $i = 2 : n$ **do**
  $k = \arg\max_{j \in [1,m]} \{\mathcal{W}_k \neq \varnothing\}$;
  for $j = 1 : k+1$ **do**
    $\mathcal{W}_j = \mathcal{W}_j \cup \{w_i\}$;
    compute u_{ij} according to Eq. (20);
    $\mathcal{W}_j = \mathcal{W}_j \backslash \{w_i\}$;
  $l = \arg\max_{l \in [1,j]} u_{il}$;
  if $\bar{u_{il}} > 0$ **then**
    $\mathcal{W}_v = \mathcal{W}_v \cup \{w_i\}$;
  else
    $\bar{s_i} = 0$;
    break;
foreach $j \in [1, m]$ **do**
  foreach $w_x \in \mathcal{W}_j$ **do**
    compute $t_x^* = \bar{t_{xj}}$ according to Eq. (16) ;
    $s_x^* = j$;

Corollary 1. *The output Π^* of Algorithm 1 is a unique NE of the SP game.*

Proof. See Appendix B. □

In the RD game, each requester declares her reward value corresponding to her requested task. In order to maximize her own utility by manipulating the system, each strategic requester will carefully choose the most favorable strategy $\gamma_j, \forall j : r_j \in \mathcal{R}$, which is define as the optimal reward declaration strategy $\bar{\gamma_j}$.

Definition 9. *Given γ_{-j}, $v_j(\bar{\gamma_j}, \gamma_{-j}) \geq v_j(\gamma_j, \gamma_{-j})$ over all $\gamma_j \neq \bar{\gamma_j}$.*

In the NE of the RD game, $r_j, \forall j \in [0, m]$ will obviously choose $\overline{\gamma_j}$ in an NE. Given γ_{-j} and $\mathcal{C}$, we have

$$\overline{\gamma_j} = \arg\max_{\gamma_j \geq 0} \kappa_j \log_\alpha\Big(\sum_{x: w_x \in \mathcal{W}_j} t_{xj} + 1\Big) - \gamma_j. \tag{13}$$

In the following, we first prove that there exists a fixed point equivalent to an NE of the RD game, and then prove that such an NE is unique in the RD game.

Theorem 2. *There is a unique NE in the RD game.*

Proof. See Appendix C. □

Next, we design Algorithm 2 and prove that it can obtain the unique NE of the RD game, as shown Corollary 2. In particular, Algorithm 2 employs a two-step procedure in an

Algorithm 2: Computation of the NE for the RD game

Input: $\mathcal{K} = \{\kappa_1, \cdots, \kappa_m\}$, $\mathcal{C} = \{c_1, \cdots, c_n\}$
Output: $\Gamma^* = \{\gamma_1^*, \cdots, \gamma_m^*\}$

1 reorder elements in $\mathcal{K}$ so that $\kappa_1 \geq \cdots \geq \kappa_m$;
2 initialize $\overline{\gamma_j^0} = 0, \forall j \in [1, m]$ and $\theta = 1$;
3 **do**
4   **for** $j = m : 1$ **do**
5   obtain $\overline{\gamma_j^\theta}$ by solving Eq. (13) with given $\overline{\gamma_k^\theta}, \forall k \in (j, m]$ and $\overline{\gamma_v^{\theta-1}}, \forall l \in [1, j)$;
6   **if** $v_j(\overline{\gamma_j^\theta}) - v_j(\overline{\gamma_j^{\theta-1}}) \leq \epsilon$ **then**
7       $\overline{\gamma_j^\theta} = \overline{\gamma_j^{\theta-1}}$;
8     $\theta = \theta + 1$;
9 **while** $\{\overline{\gamma_1^\theta}, \cdots, \overline{\gamma_m^\theta}\} \neq \{\overline{\gamma_1^{\theta-1}}, \cdots, \overline{\gamma_m^{\theta-1}}\}$;

alternate manner. It has low-complexity computation, which is dominated by the loop (i.e., ϵ).

Corollary 2. *The output Γ^* of Algorithm 2 is a unique NE of the RD game.*

Proof. See Appendix D. □

According to Definition 5, Theorem 1, and Theorem 2, we know that (Γ^*, Π^*) is a unique SE for the MTA Stackelberg game.

B. Optimal Design of Secondary Allocation Rule

Although given any fixed $\delta \in (0, 1]$, we can obtain a unique SE (Γ^*, Π^*). However, it is may not be an optimal solution to Eq. (11). To further improve the social utility, we design a secondary allocation rule $\mathbb{A}$ based on the idea of transferable utility, our goal is to maximize the social utility while maintaining ξ-PF. Given δ, Eq. (11) can be rewritten as:

$$\begin{cases} \max\limits_{\delta \in (0,1]} \mathcal{U} \triangleq \sum\limits_{i:w_i \in \mathcal{W}} u_i(\delta) + \sum\limits_{j:r_j \in \mathcal{R}} v_j(\delta), \\ \text{s.t.} \begin{cases} \sum\limits_{i:w_i \in \mathcal{W}} \frac{\widehat{u_i} - u_i(\delta)'}{u_i(\delta)'} + \sum\limits_{j:r_j \in \mathcal{R}} \frac{\widehat{v_j} - v_j(\delta)'}{v_j(\delta)'} \leq \xi, \\ v_i(\delta)' = v_i(\delta) + \varepsilon_i^w, \forall i : w_i \in \mathcal{W}, \\ v_j(\delta)' = v_j(\delta) + \varepsilon_j^r, \forall j : r_j \in \mathcal{R}. \end{cases} \end{cases} \tag{14}$$

The key to solving the problem depicted in Eq. (14) is to find the optimal value of δ. We now design Algorithm 3, and prove that its output is an optimal solution to Eq. (14), since it maximizes the social utility and allocates the minimal bonuses to both workers and requesters to maintain the ξ-PF at a minimum cost.

Theorem 3. *The output of δ^* and $\{\varepsilon_k^* | \forall k \in [1, m+n]\}$ by Algorithm 3 is an optimal solution to Eq. (14).*

Proof. See Appendix E. □

VI. Performance Evaluation

In this section, we provide numerical results to evaluate the performance of our proposed FAIM designed for the MTA problem in crowdsensing with both synthetic and real-world datasets.

Algorithm 3: Optimal design of $\mathbb{A}$

Input: $\mathcal{K} = \{\kappa_1, \cdots, \kappa_m\}$, $\mathcal{C} = \{c_1, \cdots, c_n\}$
Output: δ^*, $\{\varepsilon_1^w, \cdots, \varepsilon_n^w, \varepsilon_1^r, \cdots, \varepsilon_m^r\}$

1 compute $\Gamma^* = \{\gamma_1^*, \cdots, \gamma_m^*\}$ according to Algorithm 2;
2 initialize $x_i = u_i, \forall i : w_i \in \mathcal{W}$, $x_{n+j} = v_j, \forall j : r_j \in \mathcal{R}$, and $\mathcal{U} = \sum_{k \in [1, m+n]} x_k$;
3 **for** $\delta = 1; \ell > 0; \delta = \delta - \ell$ **do**
4   let $y_i = u_i, \forall i : w_i \in \mathcal{W}$, $y_{n+j} = v_j, \forall j : r_j \in \mathcal{R}$;
5   **if** $\mathcal{U}' = \sum_{k=1}^{m+n} y_k + \sum_{j=1}^{m} (1 - \delta)\gamma_j > \mathcal{U}$ **then**
6   reorder elements in $\{\frac{x_k}{y_k} | \forall k \in [1, m+n]\}$ so that $\frac{x_1}{(y_1)^2} \geq \cdots \geq \frac{x_{m+n}}{(y_{m+n})^2}$;
7   **for** $\omega = 2 : m + n$ **do**
8   **if** *there exists a solution for*

$$\begin{cases} \frac{x_k}{(y_k + \varepsilon_k)^2} = \frac{x_\omega}{(y_\omega + \varepsilon_\omega)^2}, \forall k \in [1, \omega) \\ \sum_{k \leq \omega} \frac{x_k}{y_k + \varepsilon_k} + \sum_{k \in (\omega, m+n]} \frac{x_k}{y_k} \\ \quad = m + n + \xi \end{cases} \tag{15}$$

      and $\sum_{k \in [1, \omega]} \varepsilon_k \leq \sum_{j \in [1, m]} (1 - \delta)\gamma_j$ **then**
9         $\mathcal{U} = \mathcal{U}'$;
10         $\delta^* = \delta$;
11         $\varepsilon_k = 0, \forall k \in (\omega, m + n]$;
12         $\varepsilon_k^* = \varepsilon_k + [(1 - \delta) \sum_{j \in [1, m]} \gamma_j - \sum_{k \in [1, \omega]} \varepsilon_k] \frac{\frac{x_k}{y_k}}{\sum_{l \in [1, m+n]} \frac{x_v}{y_v}}, \forall k \in [1, m + n]$;
13   go to line 3;

A. Experiment Setup

1) Datasets: Two types of datasets are used in the experiment.

- **A Synthetic Dataset**. Without loss of generality, assume that the value of $\kappa_j, \forall j \in [1, m]$ is subject to a Gaussian distribution $\kappa_j \sim N(\mu_1, \sigma_1^2)$. Here, we fix $\mu_1 = 5$, and define its value boundary as (0,40). Similarly, we assume that the value of $c_i, \forall i \in [1, n]$ is also subject to a Gaussian distribution $c_i \sim N(\mu_2, \sigma_2^2)$. Here, we fix $\mu_2 = 3$, and define its value boundary as (0,10). Without further notification, we fix $\sigma_1^2 = 10$, $\sigma_2^2 = 1$, and $\xi = 1$ by default.
- **Four Real-world Datasets**. Four standard real-world datasets are utilized to make performance evaluation which have also been widespread used into much related research work in federated learning. Actually, federated learning is a novel paradigm of crowdsensing that enables collaborative training global models across multiple data silos without uploading raw data [44].
  - **MNIST**, which contains gray-scale images of 70000 handwritten digits, where 60000 for training and 10000 for testing [45].
  - **FMNIST**, which is replacement for MNIST database

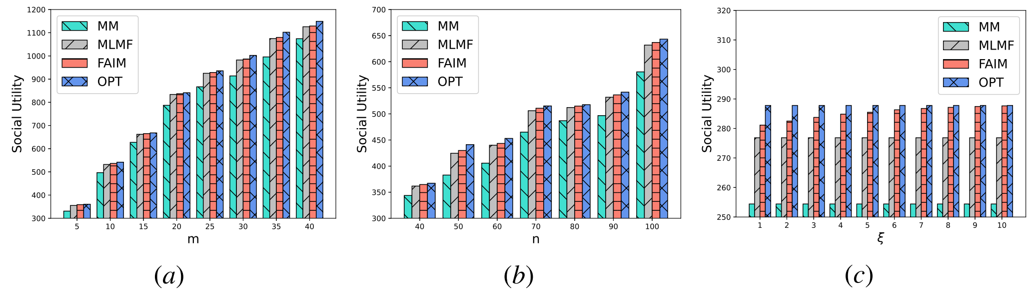

Fig. 2. The social utilities of the compared mechanisms against (a) m, (b) n, and (c) ξ.

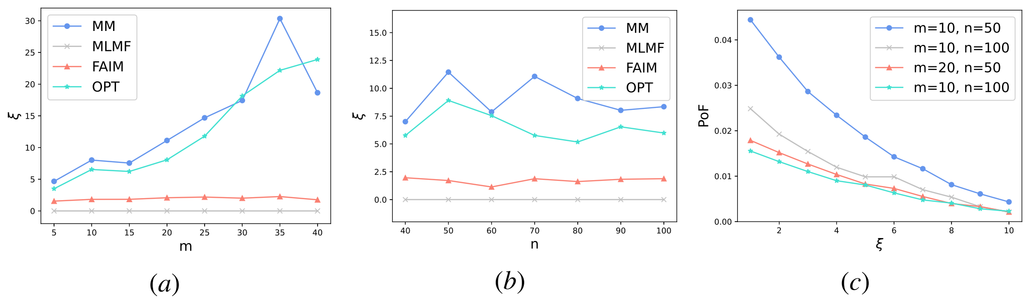

Fig. 3. The fairness threshold ξ is affected by intrinsic parameters (a) m, (b) n, and (c) how ξ affects the PoF.

containing 80000 images of fashion apparels with 10 distinct classes [46].

– **EMNIST**, which includes six different splits and the biggest one is EMNIST ByClass containing 814255 characters and 62 unbalanced classes [47].
– **CIFAR10**, which is a subset of the tiny images dataset and consists of 60000 32×32 color images with 50000 training images and 10000 test images [48].

2) Baselines: We mainly compare our proposed mechanism with the following three typical mechanisms:

- **MLMF**, which expands the single-leader multi-follower Stackelberg game [26–29, 49] to a multi-leader multi-follower Stackelberg game [30, 31]. It guarantees PF, but does not employ the secondary allocation rule to further optimize the social utility.
- **MM**, which looks for a solution in which even the least happy agent users as much as possible, i.e., the user who obtains the lowest utility, still receives the highest possible utility [50].
- **OPT**, which adopts the secondary allocation rule to exhaustively maximizes the social utility without guaranteeing ξ-PF.

3) Metrics: We use two metrics to evaluate the compared mechanisms with the synthetic dataset: (i) **Social utility**, which is the main metric to evaluate the performance of our FAIM. (ii) **Fairness**, which measures how the fairness threshold ξ is affected by intrinsic parameters and how it affects the PoF, where PoF is denoted as $\text{PoF} = \sup_{I \in \mathcal{I}} \min_{\sigma_\xi \in \sum_{\mathcal{F}}} \frac{\mathcal{U}(\sigma^*(I)) - \mathcal{U}(\sigma_\xi(I))}{\mathcal{U}(\sigma^*(I))}$, where instance $I \in \mathcal{I}$, $\sigma^*(I)$ is a system optimum, and $\sum_{\mathcal{F}}$ is the set of fair solution for I [12, 51].

We use two other metrics to evaluate the compared mechanisms with the four real-world datasets: (i) **Prediction accuracy**, which is expressed as the correlation between the model prediction and the actual score. Accuracy of 1 indicates a perfect learner, whereas the accuracy of 0 indicates a largest error. (ii) **Training loss**, which is a metric used to assess the error of the model on the training set.

B. Results on Synthetic Dataset

1) Social Utility: Fig. 2 plots the social utilities of the four compared mechanisms against (a) the number of available requesters m, (b) the number of available workers n, and (c) the fairness threshold ξ, respectively. Fig. 2(a) and 2(b) show that the social utilities of the four compared mechanisms monotonically increase with m and n. This is because the social utility depends on the number of tasks and workers, and more tasks and workers means more social utility is created. From the two figures, we sort the social utility values from large to small, OPT is the largest, FAIM is the second, MLMF is the third, and MM is the last. The fundamental reason for this phenomenon is that, on the one hand, OPT abandons PF and unilaterally pursues the maximization of social utility. On the other hand, although MLMF guarantees PF, it does not further optimize the social utility. OPT and MLMF can be regarded as the upper and lower bound of social utility obtained by FAIM, respectively. In contrast, MM achieves the maximin fairness by sacrificing the profit, so the profit is the lowest. Since ξ only affects FAIM, a larger ξ means a weaker constraint on ξ-PF. Consequently, the performance gap between FAIM and OPT monotonically decreases with ξ as shown in Fig. 2(c).

2) Fairness: Fig. 3 and Fig. 2 are from the same set of experiments, the major difference is that Fig. 2 focuses on social utility, while Fig. 3 focuses on social fairness. Fig. 3(a) and 3(b) plot the fairness threshold ξ is affected by the number of available requesters m and the number of available workers

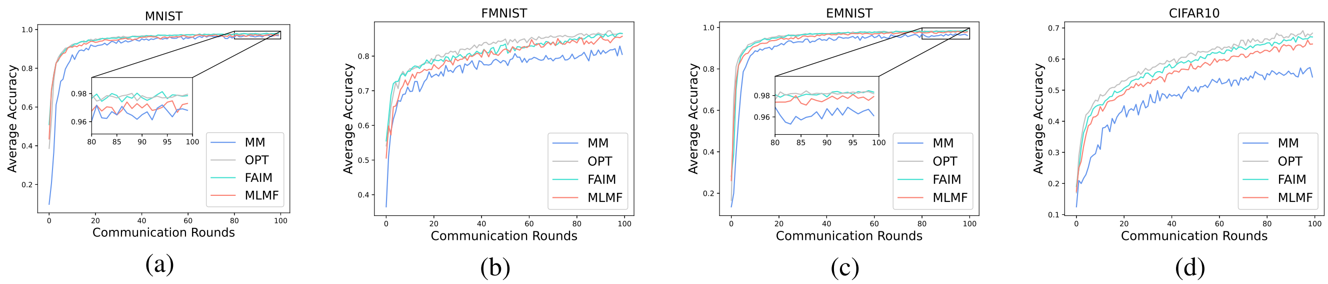

Fig. 4. The prediction accuracy: (a) MNIST, (b) FMNIST, (c) EMNIST, and (d) CIFAR10.

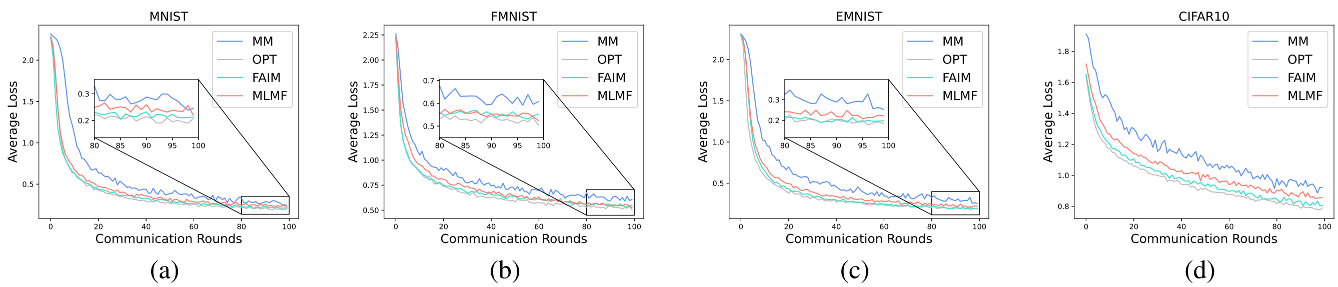

Fig. 5. The training loss: (a) MNIST, (b) FMNIST, (c) EMNIST, and (d) CIFAR10.

n, respectively. From the results in the two figures, MLMF has the lowest fairness threshold, which reflects that it maintains the highest fairness. FAIM achieves the second highest social utility with the second lowest fairness threshold. With the growth of m, both MM and OPT need to increase the fairness threshold ξ, that is, reduce the fairness requirement, in order to seek feasible solutions. Fig. 3(b) is similar to the results in Fig. 3(a), the difference is that ξ corresponding to MM and OPT do not show an obvious increasing trend, because the increase of m can aggravate the competition between users in the system more than the increase of n. The overall ξ value of OPT is lower than that of MM, because OPT employs the secondary allocation rule. In Fig. 3(c), an interesting phenomenon can be found in Fig. 3(d), that is, the values of PoF for all four cases monotonically decrease with ξ. Because as ξ increases, Algorithm 3 will have a better chance of being executed, resulting in higher social utility. When ξ is large enough, the judgment condition in line 8 of Algorithm 3 is always true, so that the fair solution is also the optimal solution, that is, PoF=0. Furthermore, the smaller n and m, the smaller the PoF. This is because the smaller the quantity of tasks and workers, the easier it is for the performance of FAIM to approach OPT.

C. Results on Real-World Dataset

1) Prediction Accuracy: Fig. 4 plots the prediction accuracy of the four compared mechanisms using four real-world datasets, i.e., MNIST, FMNIST, EMNIST, and CIFAR10. Although the experimental results vary across different datasets, we can observe that with the increasing of training rounds, the prediction accuracy grows very fast in the beginning, and then slows down until it converges. It is obvious that OPT attains higher accuracy than both FAIM and MLMF, while FAIM is very close to OPT, and MM has the worst performance across Fig. 4(a) to Fig. 4(b). The main reason behind this phenomenon is essentially the same as that in Fig. 2, i.e., both FAIM and OPT employ the secondary allocation rule to compensate workers with strong ability (i.e., low unit cost corresponding to the synthetic datasets, and high data quality corresponding to the four real-world datasets), so that they can obtain higher social utility and better system performance. Although the experimental results in the four datasets are different, the trends and performance rankings of the four curves representing the four compared mechanisms are basically consistent.

2) Training Loss: Fig. 5 plots the training loss of the four compared mechanisms using the same four real-world datasets as Fig. 4. It is obvious that all four mechanisms converge rapidly with fewer communication rounds. Consistent with Fig. 4, the training loss of FAIM is lower than that of MM and MLMF, and slightly higher than OPT, for the same reason. Looking closely at the training loss curve, it is clear that the difference between the curves of FAIM and OPT is very small, and the curve fluctuations are also smaller due to the secondary allocation rule. Conversely, MM's curve fluctuates the most. Besides, the main difference between the four figures is that different datasets are associated with distinct stable points due to their data disparity. In Fig. 5(d), the overall performance of the four compared mechanisms is worse than that of Fig. 5(a)-(c) because of the larger storage capacity of the dataset and the higher computational overhead.

VII. Conclusion

In this paper, we have developed a fairness-aware incentive mechanism to stimulate participation of users while maintaining PF for multi-task allocation in crowdsensing. First of all, the FAMTA problem has been transformed into a FAIM design problem via introducing the concept of PF to re-define a novel incentive mechanism. Secondly, the multi-task allocation has been modelled as a Stackelberg game consisting of multi-requester and multi-worker, and two novel algorithms have been presented to compute the unique Nash equilibria for the sensing plan game and reward declaration game respectively. And then, a secondary allocation rule has been designed to maximize the social utility while maintaining PF at a minimum cost by calculating the minimum bonus and optimal discount factor. Finally, experiment results using both synthetic and real-world datasets have further demonstrated that our proposed FAIM effectively balances system efficiency and fairness with a low PoF overall.

In the future research, we intend to extend this work in two directions. First, although seeking a PF solution is an critical issue in crowdsensing, seeking other solutions such as Max-min or Kalai-Smorodinsky is also a very interesting topic. Second, there are many practical factors in crowdsensing task allocation beyond our work such as the consideration of uncertain nature of worker participation, we will incorporate as many practical factors as possible into our problem formulation and solve them in the future.

Appendix

A. Proof of Theorem 1

The proof consists of two parts: we first assume that $\mathcal{W}_j = \{w_1, \cdots, w_v\}, \forall j \in [1, m]$ is given, and prove that each worker's optimal sensing time strategy is fixed. We then revoke this assumption and show that each worker's optimal task selection strategy is also fixed, relying solely on their unit costs and reward of all tasks.

First, given $\mathcal{W}_j = \{w_1, \cdots, w_v\}, \forall j \in [1, m]$, we will prove that w_i's $\bar{t_i}$ is

$$\bar{t_{ij}} = \begin{cases} \epsilon, & \text{if } v = 1 \\ 0, & \text{else if } i > z \\ \frac{\gamma_j^w (z-1)\left[\sum_{x \le z} c_x - c_i(z-1)\right]}{(\sum_{x \le z} c_x)^2}, & \text{otherwise} \end{cases} \tag{16}$$

where we assume that the workers in $\mathcal{W}_j$ are sorted as $c_1 \le \cdots \le c_v$, and let $z = \max\{x\colon 2 \le x \le k, c_x < \frac{\sum_{y=1}^{x} c_y}{x-1}\}$. When $v = 1$, the single worker w_i, can earn the total reward γ_j^w by free riding, while only having to contribute a sufficiently small amount of sensing time that its cost is approximately zero.

When $v \neq 1$, and according to Eq. (9), we can obtain that $t_{ij} \in [0, \frac{\gamma_j^w}{c_i}]$ as $u_{ij} \ge 0$, and have $\frac{\partial u_{ij}}{\partial t_{ij}} = \frac{\gamma_j^w (\sum_{x:w_x \in \mathcal{W}_j} t_{xj} - t_{ij})}{(\sum_{x:w_x \in \mathcal{W}_j} t_{xj})^2} - c_i$ and $\frac{\partial^2 u_{ij}}{\partial t_{ij}^2} = -\frac{2\gamma_j^w \sum_{x:w_x \in \mathcal{W}_j \setminus \{w_i\}} t_{xj}}{(\sum_{x:w_x \in \mathcal{W}_j} t_{xj})^3}$, respectively. Given any $\gamma_j^w > 0$, we know that $\frac{\partial^2 u_{ij}}{\partial t_{ij}^2} < 0$. To find the unique $\bar{t_{ij}}$, we let $\frac{\partial u_{ij}}{\partial t_{ij}}$=0, and obtain the following conclusion:

$$\gamma_j^w (\sum_{x:w_x \in \mathcal{W}_j} t_{xj} - t_{ij}) = c_i (\sum_{x:w_x \in \mathcal{W}_j} t_{xj})^2. \tag{17}$$

Let $\mathcal{W}_j^+ = \{w_i \in \mathcal{W}_j : t_{ij} > 0\}$, by taking all workers in $\mathcal{W}_j^+$ into Eq. (17) and adding them up, we can get

$$\sum_{x:w_x \in \mathcal{W}_j^+} t_{xj} = \frac{\gamma_j^w (|\mathcal{W}_j^+| - 1)}{\sum_{x:w_x \in \mathcal{W}_j^+} c_x}. \tag{18}$$

Since $t_{ij} = 0$ for each $w_i \in \mathcal{W}_j \setminus \mathcal{W}_j^+$, we have $\sum_{x:w_x \in \mathcal{W}_j^+} t_{xj} = \sum_{y:w_y \in \mathcal{W}_j} t_{yj}$. We substitute Eq. (18) into Eq. (17) to get

$$t_{ij} = \frac{\gamma_j^w (|\mathcal{W}_j^+| - 1)\left[\sum_{x:w_x \in \mathcal{W}_j^+} c_x - c_i(|\mathcal{W}_j^+| - 1)\right]}{(\sum_{x:w_x \in \mathcal{W}_j^+} c_x)^2}. \tag{19}$$

Next, we determine the set $\mathcal{W}_j^+$. According to Eq. (19), if $c_i < \frac{\sum_{x:w_x \in \mathcal{W}_j^+} c_x}{|\mathcal{W}_j^+| - 1}$, we know that $t_{ij} > 0 | w_i \in \mathcal{W}_j$, and $w_i \in \mathcal{W}_j^+$. Moreover, it is obvious that t_{ij} monotonically decreases with c_i. This means that workers with low unit costs are more motivated to contribute more perceived time.

Furthermore, it can be seen that t_{ij} monotonically decreases with c_i. Hence, a worker with smaller c-value has more incentive to devote more time. And hence $\mathcal{W}_j^+$ consists of a consecutive set of workers, namely $\mathcal{W}_j^+ = \{w_1, \cdots, w_s\}$ for some $s \in [2, k]$ (recall that workers are ordered such that $c_1 \le \cdots \le c_v$). Notice that if $c_x \ge \frac{\sum_{y \in [1,x]} c_y}{x-1}$, then $c_{x+1} \ge \frac{\sum_{y \in [1,x+1]} c_y}{x}$. Thus s must be the last index x satisfying $c_x < \frac{\sum_{y \in [1,x]} c_y}{x-1}$, that is, $s = z$ as defined in the statement of Eq. (16).

Second, we revoke the assumption that $\mathcal{W}_j = \{w_1, \cdots, w_v\}, \forall j \in [1, m]$ is given in the above proof, and show that each worker $w_i \in \mathcal{W}_j$ will choose $\bar{s_i}$ in the NE, depending on the unit costs $c_i | w_i \in \mathcal{W}_j$. Given $\Gamma = \{\gamma_1^w, \cdots, \gamma_m^w\}$ and $\{\mathcal{W}_1, \cdots, \mathcal{W}_m\}$, let w_α be a worker who has just joined the task with $\bar{s_\alpha} = \tau_k$ $(k \in [1, m])$, where $w_\alpha \notin \cup_{j \in [1,m]} \mathcal{W}_j$ and $c_\alpha \ge \max_{j \in [1,m]} \max_{i:w_i \in \mathcal{W}_j} c_{ij}$, then for any worker $w_\beta \in \cup_{j \in [1,m]} \mathcal{W}_j$, her optimal choice is to keep $\bar{s_\beta}$ unchanged. By substituting Eq. (16) into Eq. (9), u_{ij} can be obtained when she chooses $\bar{t_{ij}}$.

$$u_{ij} = \begin{cases} \gamma_j^w, & \text{if } v = 1 \\ 0, & \text{else if } i > z \\ \gamma_j^w [1 - \frac{c_i(z-1)}{\sum_{x:w_x \in \mathcal{W}_j} c_x}]^2, & \text{otherwise} \end{cases} \tag{20}$$

According to Eq. (16) and Eq. (20), we know that both $\bar{t_{ij}}$ and u_{ij} monotonically decrease with c_i.

According to Eq. (20), as w_α joins $\mathcal{W}_k$, the utility of other workers in that set does not increase. Thus, other workers not in $\mathcal{W}_k$ clearly have no incentive to deviate from her current choice and join $\mathcal{W}_k$ instead. Without loss of generality, we assume that $w_\beta \in \mathcal{W}_k$ changes her previous choice to participate in τ_v with $v \neq k$. Now, $u_{\beta v} = \gamma_v^w (1 - \frac{c_\beta z_v}{\sum_{q:w_q \in \mathcal{W}_v} c_q + c_\beta})^2$, where $z_j = |\mathcal{W}_j|, \forall j \in [1, m]$. If w_β does not deviate after worker w_α joins $\mathcal{W}_k$, then $u_{\beta k} = \gamma_k^w (1 - \frac{c_\beta z_k}{\sum_{p:w_p \in \mathcal{W}_k} c_p + c_\alpha})^2$. Since $c_\alpha \ge c_\beta$, we know that $\frac{c_\beta z_k}{\sum_{p:w_p \in \mathcal{W}_k} c_p + c_\alpha} \le \frac{c_\beta z_k}{\sum_{p:w_p \in \mathcal{W}_k} c_p + c_\beta}$, as well as $u_{\beta k} \ge {u_{\beta k}}' = \gamma_k^w (1 - \frac{c_\beta z_k}{\sum_{p:w_p \in \mathcal{W}_k} c_p + c_\beta})^2$. If and only if

$\bar{u_{\beta k}} \geq \bar{u_{\beta l}}$, w_β has incentive to choose τ_v instead of τ_k. We can show that $u_{\beta k}{}' \geq u_{\beta l}$ to confirm this. Let

$$f(x) = \begin{array}{l} \sqrt{\gamma_k^w}(1 - \frac{x z_k}{\sum_{p: w_p \in \mathcal{W}_k} c_p + x}) \\ -\sqrt{\gamma_v^w}(1 - \frac{x z_v}{\sum_{p: w_p \in \mathcal{W}_v} c_p + x}), \end{array} \tag{21}$$

and set $\rho_1 = \sqrt{\gamma_v^w}(z_v - 1) - \sqrt{\gamma_k^w}(z_k - 1)$, $\rho_2 = \sqrt{\gamma_v^w} z_v + \sqrt{\gamma_k^w} - \sqrt{\gamma_v^w}) \sum_{p: w_p \in \mathcal{W}_k} c_p - (\sqrt{\gamma_k^w} z_k + \sqrt{\gamma_v^w} - \sqrt{\gamma_k^w}) \sum_{q: w_q \in \mathcal{W}_v} c_q$, and $\rho_3 = (\sqrt{\gamma_k^w} - \sqrt{\gamma_v^w}) \sum_{p: w_p \in \mathcal{W}_k} \sum_{q: w_q \in \mathcal{W}_v} c_p c_q$, we get $f(x) = \rho_1 x^2 + \rho_2 x + \rho_3$. For w_α, it is obvious that $u_{tk} \geq u_{tl}$, i.e., $f(c_t) \geq 0$. To show that $f(x) \geq 0, \forall x \in (0, c_t]$, we divide the value interval of ρ_1, ρ_2, and ρ_3 into five subcases for verification.

Case (*i*): $\rho_1 = 0$. Without loss of generality, we first assume that $\rho_3 < 0$ holds.From $\rho_1 = 0 \wedge \rho_3 < 0$, we can obtain that $\sqrt{\gamma_v^w}(z_v - 1) = \sqrt{\gamma_k^w}(z_k - 1) \wedge \gamma_1^w < \gamma_2^w$, and $\rho_2 = \sqrt{\gamma_k^w} z_k \sum_{w_p \in \mathcal{W}_k} c_p - \sqrt{\gamma_v^w} z_v \sum_{w_q \in \mathcal{W}_v} c_q < 0$. This contradicts $f(c_t) \geq 0$ and $\rho_3 < 0$. Consequently, $\rho_3 \geq 0$ must holds in case *i*, and get $f(x) \geq 0, \forall x \in (0, c_t]$.

Case (*ii*): $\rho_1 < 0 \wedge c_t \leq -\frac{\rho_2}{2\rho_1}$. Suppose, for the sake of contradiction, that $\rho_3 < 0$ is true. According to $\rho_1 < 0 \wedge c_t \leq -\frac{\rho_2}{2\rho_1} \wedge \rho_3 < 0$, we can obtain that $\sqrt{\gamma_v}(z_v - 1) < \sqrt{\gamma_k}(z_k - 1) \wedge c_t \leq -\frac{\rho_2}{2\rho_1} \wedge \gamma_k < \gamma_v \wedge z_v < z_k$. As $f(-\frac{\rho_2}{2\rho_1}) \geq f(c_t) \geq 0$, we can get $\rho_2^2 \leq 4\rho_1\rho_3$. However, it is easy to find a counterexample, e.g., $\gamma_k = \gamma_v$, $z_v = z_k - 1$ and $\sum_{w_p \in \mathcal{W}_k} c_p > \sum_{w_q \in \mathcal{W}_v} c_q$, which conflicts with $c < 0$. As a consequence, $f(x) \geq 0, \forall x \in (0, c_t]$ if $\rho_3 \geq 0$.

Case (*iii*): $\rho_1 < 0 \wedge c_t > -\frac{\rho_2}{2\rho_1}$. Similarly to Case (*ii*), it is easy to obtain that $f(x) \geq 0, \forall x \in (0, c_t]$ when $\rho_3 \geq 0$.

Case (*iv*): $\rho_1 > 0 \wedge \rho_2 > 0$. By contradiction, let's assume that $\rho_3 < 0$. According to $\rho_1 > 0 \wedge \rho_2 > 0 \wedge \rho_3 < 0$, we have $\sqrt{\gamma_v}(z_v - 1) > \sqrt{\gamma_k}(z_k - 1) \wedge (\sqrt{\gamma_v} z_v + \sqrt{\gamma_k} - \sqrt{\gamma_v}) \sum_{w_p \in \mathcal{W}_k} \wedge \quad - (\sqrt{\gamma_k} z_k + \sqrt{\gamma_v} - \sqrt{\gamma_k}) \sum_{w_q \in \mathcal{W}_v} > 0 \wedge \gamma_1 < \gamma_2$. There is at least one counterexample, such as $\gamma_k > \gamma_v$, $z_v = z_k$ and $\sum_{w_p \in \mathcal{W}_k} c_p = \sum_{w_q \in \mathcal{W}_v} c_q$. The above counterexample clearly contradicts with $\rho_3 < 0$. And hence, we can get $f(x) \geq 0, \forall x \in (0, c_t]$.

Case (*v*): $\rho_1 > 0 \wedge 0 < -\frac{\rho_2}{2\rho_2} \leq c_t$. Suppose, for the sake of contradiction, that $f(-\frac{\rho_2}{2\rho_1}) < 0 \wedge \rho_3 < 0$ is true. According to $\rho_1 > 0 \wedge f(-\frac{\rho_2}{2\rho_3}) < 0 \wedge \rho_3 < 0$, we get $\sqrt{\gamma_v}(z_v - 1) > \sqrt{\gamma_k}(z_k - 1) \wedge \gamma_1 > \gamma_2 \wedge 4\rho_1\rho_3 > \rho_2^2$. Similarly, we can construct a counterexample, e.g., $\gamma_k > \gamma_v$, $z_v = z_k$ and $\sum_{w_p \in \mathcal{W}_k} c_p = \sum_{w_q \in \mathcal{W}_v} c_q$. It is obviously contradicts with $f(-\frac{\rho_2}{2\rho_1}) < 0 \wedge \rho_3 < 0$. For the other subcases, $f(x) \geq 0, \forall x \in (0, c_t]$ clearly holds as well.

For the other cases not included In the abovementioned five cases, it is easy to show that $f(x) \geq 0, \forall x \in (0, c_t]$ holds. Therefore, this theorem is proved.

B. Proof of Corollary 1

According to Eq. (16) and Eq. (20), the competitiveness of workers monotonically decreases with their unit cost. For w_1, c_1 is the smallest element in set $\mathcal{C}$, so $\bar{s_1} = \tau_1$ (line 2). Moreover, according to Theorem 1, no matter any other worker chooses to participate in τ_1, w_1 has no incentive to deviate from current choice $\bar{s_1}$, because w_1 has the strongest competitiveness. By analogy, workers will choose tasks in order according to their competitiveness (line 3-14). Based on Eq. (16), we can obtain $\bar{t_i j}$ for each worker w_i when she has selected τ_j (line 15-18). According to the first part of the proof for Theorem 1, when a worker's unit cost is greater than the reward she gets from participating in any task, the optimal task selection strategy of the worker is not to participate in any task in order to avoid negative utility. Therefore, this corollary is proved.

C. Proof of Theorem 2

According to Eq. (10) in the main text, we show that $\Gamma \subset R$ (N-dimensional real number set) is a non-empty, compact convex set, and $v_j : \Gamma \to \Gamma$ is a continuous function from Γ to Γ. First, for each $j \in [1, m]$, we have $v_j(\gamma_j = 0) = 0$, *i.e.*, $0 \in \Gamma$. Second, by substituting Eq. (18) into Eq. (10), we can obtain that

$$v_j = \kappa_j \log_\alpha \Big(\frac{\delta \gamma_j (|\mathcal{W}_j^+| - 1)}{\sum_{x: w_x \in \mathcal{W}_j^+} c_x} + 1 \Big) - \gamma_j. \tag{22}$$

Let $\beta = \frac{\delta(|\mathcal{W}_j^+| - 1)}{\sum_{x: w_x \in \mathcal{W}_j^+} c_x} + 1$, Eq. (22) is rewritten as $u_j = \kappa_j \log_\alpha(\beta\gamma_j + 1) - \gamma_j$, and we can obtain that $\frac{\partial u_j}{\partial \gamma_j} = \frac{\kappa_j \beta}{\ln \alpha(\beta\gamma_j)} - 1$ and $\frac{\partial^2 u_j}{\partial \gamma_j^2} = -\frac{\kappa_j \beta^2}{\ln \alpha(\beta\gamma_j + 1)^2}$, respectively. It is easy to determine that $-\frac{\kappa_j \beta^2}{\ln \alpha(\beta\gamma_j + 1)^2} < 0$. We then know that $\overline{\gamma_j}$ exists and is unique, and the maximum value of u_j can be calculated by setting $\frac{\partial u_j}{\partial \gamma_j} = 0$. Without loss of generality, let the maximum value of u_j be $\bar{u_j}$, we can get that the function of u_j is bounded, *i.e.*, $u_j \in [0, \bar{u_j}]$. Third, it is not difficult to judge that $\frac{\partial u_j}{\partial \gamma_j}$ is differentiable on $\forall \gamma_j \in [0, \overline{\gamma_j}]$, which means that u_j must be continuous at this point. As a result, according to Brouwer fixed-point theorem, we know that $u_j(\cdot)$ has a fixed point, that is, $\exists \gamma_j \in \Gamma$, $\gamma_j = u_j(\gamma_j)$. This indicates that the RD game has at least one NE. Since the utilities of all requesters follow the increasing function until reaching a convergence, there is no further improvement from the NE solution, and hence such a NE is unique.

D. Proof of Corollary 2

This proof can be directly obtained from Theorem 2, and is omitted here.

E. Proof of Theorem 3

Given a very small positive step number ℓ, we can find a δ to increase the social utility by looping (line 4-13). In order to satisfy the ξ-PF, we need to construct an optimal secondary allocation plan $\{\varepsilon_1^w, \cdots, \varepsilon_n^w, \varepsilon_1^r, \cdots, \varepsilon_m^r\} = \{\varepsilon_1, \cdots, \varepsilon_{m+n}\}$ with the minimization of utility which can not exceed the bonus that the platform can allocate to both requesters and workers, *i.e.*, $\sum_{k \in [1, m+n]} \varepsilon_k \leq \sum_{j \in [1, m]} (1 - \delta)\gamma_j$. We can obtain the optimal value of $\{\varepsilon_1^w, \cdots, \varepsilon_n^w, \varepsilon_1^r, \cdots, \varepsilon_m^r\}$ by solving the following problem, which has the same solution of ε^* as Eq. (15) in the main text.

$$\begin{cases} \min \sum\limits_{k \in [1, m+n]} \varepsilon_k, \\ \text{s.t.} \begin{cases} \sum_{k \in [1, m+n]} \frac{x_k}{y_k + \varepsilon_k} = m + n + \xi, \\ -\varepsilon_k \leq 0, \forall k \in [1, m+n]. \end{cases} \end{cases} \tag{23}$$

The above constraint optimization problem can be transformed into unconstraint optimization problem using Lagrange factors:

$$L(\{\varepsilon_k\}_{k=1}^{m+n}, \{\lambda_k\}_{k=1}^{m+n}, \mu) = \sum_{k=1}^{m+n} \varepsilon_k + - \sum_{k=1}^{m+n} \lambda_k \varepsilon_k + \mu\Big[\sum_{k=1}^{m+n} \frac{x_k}{y_k+\varepsilon_k} - (m+n+\xi)\Big]. \tag{24}$$

Using Karush-Kuhn-Tucker (KKT) conditions [43], $\{\varepsilon_k^*|\forall k \in [1, m+n]\}$ is an optimal solution iff

$$\begin{cases} \lambda_k + \mu \frac{x_k}{(y_k+\varepsilon_k^*)^2} = 1, \ k = 1, \cdots, m+n, \\ \sum_{k=1}^{m+n} \frac{x_k}{y_k+\varepsilon_k^*} = m+n+\xi, \\ \varepsilon_k(x) \geq 0, \ k = 1, \cdots, m+n, \\ \lambda_k \geq 0, \ k = 1, \cdots, m+n, \\ \lambda_k \varepsilon_k^* = 0, \ k = 1, \cdots, m+n. \end{cases} \tag{25}$$

According to the first subformula of Eq. (25), we know that $\lambda_k = 1 - \mu \frac{x_k}{(y_k+\varepsilon_k^*)^2}$. And according to the second subformula of Eq. (25), we can obtain that $\varepsilon_k^* = 0$ if $\lambda_k > 0$. Therefore, if $\frac{x_k}{y_k^2} < \frac{1}{\mu}$, then $\varepsilon_k^* = 0$. Hence, if we order indexes as $1, 2, \cdots, m+n$ such that $\frac{x_1}{(y_1)^2} \geq \cdots \geq \frac{x_{m+n}}{(y_{m+n})^2}$, then there exists index ω such that $\frac{x_\omega}{(y_\omega)^2} \geq \frac{1}{\mu}$ and $\frac{x_{\omega+1}}{(y_{\omega+1})^2} < \frac{1}{\mu}$, and thus $\varepsilon_1^*, \cdots, \varepsilon_\omega^* \geq 0$, $\varepsilon_{\omega+1}^*, \cdots, \varepsilon_{m+n}^* = 0$. We can construct $\{\varepsilon_k^*|\forall k \in [1, m+n]\}$ by solving Eq. (14) in the main text.

$$\begin{cases} \frac{1}{\mu} = \frac{x_k}{(y_k+\varepsilon_k^*)^2}, \ k = 1, \cdots, \omega, \\ \lambda_k = 0, \ k = 1, \cdots, \omega, \\ \sum_{k=1}^{\omega} \frac{x_k}{y_k+\varepsilon_k^*} + \sum_{k=\omega+1}^{m+n} \frac{x_k}{y_k} = m+n+\xi, \\ \varepsilon_k^* = 0, \ k = \omega+1, \cdots, m+n. \end{cases} \tag{26}$$

By removing the Lagrange factors $\{\lambda_k\}_{k=1}^{m+n}$ and μ, Eq. (26) can be further simplified as follows, which is equivalent to Eq. (14) in the main text together with line 11 in Algorithm 3.

$$\begin{cases} \frac{x_k}{(y_k+\varepsilon_k^*)^2} = \frac{x_\omega}{(y_\omega+\varepsilon_\omega^*)^2}, \ k = 1, \cdots, \omega, \\ \sum_{k=1}^{\omega} \frac{x_k}{y_k+\varepsilon_k^*} + \sum_{k=\omega+1}^{m+n} \frac{x_k}{y_k} = m+n+\xi, \\ \varepsilon_k^* = 0, \ k = \omega+1, \cdots, m+n. \end{cases} \tag{27}$$

As long as this system is feasible, then KKT conditions are satisfied. Index ω is the first index which makes this system feasible. Then the problem in Eq. (23) is solved and the theorem is also proved.

REFERENCES

[1] X. Li, and X. Zhang, “Multi-Task Allocation Under Time Constraints in Mobile Crowdsensing,” *IEEE Trans. Mob. Comput.*, vol. 20, no. 4, pp. 1494-1510, Apr. 2021.

[2] Z. Wang, J. Zhao, J. Hu, T. Zhu, Q. Wang, J. Ren, and C. Li, “Towards Personalized Task-Oriented Worker Recruitment in Mobile Crowdsensing,” *IEEE Trans. Mob. Comput.*, vol. 20, no. 5, pp. 2080-2093, May 2021.

[3] C. Dai, X. Wang, K. Liu, D. Qi, W. Lin, and P. Zhou, “Stable Task Assignment for Mobile Crowdsensing With Budget Constraint,” *IEEE Trans. Mob. Comput.*, vol. 20, no. 12, pp. 3439-3452, Dec. 2021.

[4] P. Sun, Z. Wang, L. Wu, Y. Feng, X. Pang, H. Qi, and Z. Wang, “Towards Personalized Privacy-Preserving Incentive for Truth Discovery in Mobile Crowdsensing Systemsm” *IEEE Trans. Mob. Comput.*, vol. 21, no. 1, pp. 352-365, Jan. 2022.

[5] Y. Deng, F. Lyu, J. Ren, Y. Chen, P. Yang, Y. Zhou, and Y. Zhang, “FAIR: Quality-Aware Federated Learning with Precise User Incentive and Model Aggregation,” in *Proc. of fortieth IEEE Conference on Computer Communications (INFOCOM)*, Vancouver, BC, Canada, May 10-13, 2021, pp. 1-10.

[6] J. Lu, Z. Zhang, J. Wang, R. Li, and S. Wan, “A Green Stackelberg-game Incentive Mechanism for Multi-service Exchange in Mobile Crowdsensing,” *ACM Trans. Internet Techn.*, vol. 22, no. 2, pp. 31:1-31:29, May 2022.

[7] F. Yucel, M. Yuksel, and E. Bulut, “Coverage-Aware Stable Task Assignment in Opportunistic Mobile Crowdsensing,” *IEEE Trans. Veh. Technol.*, vol. 70, no. 4, pp. 3831-3845, Apr. 2021.

[8] S. Bhattacharjee, N. Ghosh, V. K. Shah, and S. K. Das, “QnQ: Quality and Quantity Based Unified Approach for Secure and Trustworthy Mobile Crowdsensing,” *IEEE Trans. Mob. Comput.*, vol. 19, no. 1, pp. 200-216, Jan. 2020.

[9] G. Li, J. Cai, “An Online Incentive Mechanism for Collaborative Task Offloading in Mobile Edge Computing,” *IEEE Trans. Wirel. Commun.*, vol. 19, issue 1, pp. 624-636, Jan. 2020.

[10] D. Peng, F. Wu, and G. Chen, “Data Quality Guided Incentive Mechanism Design for Crowdsensing,” *IEEE Trans. Mob. Comput.*, vol. 17, no. 2, pp. 307-319, Feb. 2018.

[11] G. Zhang, S. Malekmohammadi, X. Chen, Y. Yu, “Equality Is Not Equity: Proportional Fairness in Federated Learning,” Available on arXiv:abs/2202.01666, Feb. 2022. https://arxiv.org/abs/2202.01666

[12] A. Agnetis, B. Chen, G. Nicosia, and A. Pacifici, “Price of fairness in two-agent single-machine scheduling problems,” *Eur. J. Oper. Res.*, vol. 276, no. 1, pp.79-87, Jun. 2019.

[13] Z. Wang, X. Fan, J. Qi, C. Wen, C. Wang, and R. Yu, “Federated Learning with Fair Averaging,” in *Proc. of Thirtieth International Joint Conference on Artificial Intelligence (IJCAI)*, Virtual Event / Montreal, Canada, 19-27 Aug. 2021, pp. 1615-1623.

[14] D. Halpern, and N. Shah, “Fair and Efficient Resource Allocation with Partial Information,” in *Proc. of Thirtieth International Joint Conference on Artificial Intelligence (IJCAI)*, Aug. 19-27, 2021, pp. 224-230.

[15] B. Fang, M. Jiang, P. Cheng, J. Shen, and Y. Fang, “Achieving Outcome Fairness in Machine Learning Models for Social Decision Problems,” in *Proc. of twenty-ninth International Joint Conference on Artificial Intelligence (IJCAI)*, July 2020, Yokohama, Japan, pp. 444-450.

[16] Z. Cai, J. Yuan, Y. Hua, T. Song, H. Wang, Z. Xue, N. Hu, J. Ding, R Ma, M. R. Haghighat. and H. Guan, “Themis: A Fair Evaluation Platform for Computer Vision Competitions,” in *Proc. thirtieth International Joint Conference on Artificial Intelligence (IJCAI)*, August 19-27, 2021, pp. 599–605.

[17] G. Nicosia, A. Pacifici, and U. Pferschy, “Price of fairness for allocating a bounded resource,” *Eur. J. Oper. Res.*, vol. 257, no. 3, pp. 933-943, 2017.

[18] X. Wang, et al., “Location-Aware Crowdsensing: Dynamic Task Assignment and Truth Inference”, *IEEE Trans. Mob. Comput.*, vol. 19, no. 2, pp. 362-375, Feb. 2020.

[19] D. Li, L. Yang, J. Liu, and H. Liu, “Considering Decoy Effect and Fairness Preference: An Incentive Mechanism for Crowdsensing,” *IEEE Internet Things J.*, vol. 6, no. 5, pp. 8835-8852, Oct. 2019.

[20] J. Ni, A. Zhang, X. Lin, and X. S. Shen, “Security, privacy, and fairness in fog-based vehicular crowdsensing,” *IEEE Commun. Mag.*, vol. 55, no. 6, pp. 146-152, Jul. 2017.

[21] X. Zhu, et al., “A Fair Incentive Mechanism for Crowdsourcing in Crowd Sensing,” *IEEE Internet of Things Journal*, vol. 3, no. 6, pp. 1364-1372, Dec. 2016.

[22] L. Carassus, and M. Rasonyi, “Risk-Neutral Pricing for Arbitrage Pricing Theory,” *J. Optim. Theory Appl.*, vol. 186, no. 1, pp. 248-263, Jun. 2020.

[23] D. Zhao, H. Ma, and X. Ji, “Generalized Lottery Trees: Budget-Balanced Incentive Tree Mechanisms for Crowdsourcing,” *IEEE Trans. Mob. Comput.* vol. 20, no. 7, pp. 2379-2394, Jul. 2021.

[24] S. Hao, and L. Duan, “Regulating Competition in Age of Information Under Network Externalities,” *IEEE J. Sel. Areas Commun.*, vol. 38, no. 4, pp. 697-710, Apr. 2020.

[25] J. Liu, Y. Yang, D. Li, X. Deng, S. Huang, and H. Liu, “An Incentive Mechanism Based on Behavioural Economics in Location-Based Crowdsensing Considering an Uneven Distribution of Participants,” *IEEE Trans. Mob. Comput.*, vol. 21, no. 1, pp. 44-62, Jan. 2022

[26] L. Xiao, et al., “A Secure Mobile Crowdsensing Game With Deep Reinforcement Learning”, *IEEE Transactions on Information Forensics and Security*, vo. 13, no. 1, pp. 35-47, Dec. 2018.

[27] W. Wu, W. Wang, M. Li, J. Wang, X. Fang, Y. Jiang, and J. Luo, “Incentive Mechanism Design to Meet Task Criteria in Crowdsourcing:

How to Determine Your Budget," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 2, pp. 502-516, Feb. 2017.

[28] Y. Jiao, P. Wang, D. Niyato, B. Lin, and D. I. Kim, "Mechanism Design for Wireless Powered Spatial Crowdsourcing Networks, " *IEEE Trans. Veh. Technol.*, vol. 69, issue 1, pp. 920-934, Jan. 2020.

[29] D. Yang, G. Xue, X. Fang, and J. Tang, "Incentive Mechanisms for Crowdsensing: Crowdsourcing With Smartphones," *IEEE/ACM Trans. Netw.*, vol. 24, no. 3, pp. 1732-1744, Aug. 2016.

[30] L. Chen, L. Libman, and J. Leneutre, "Conflicts and Incentives in Wireless Cooperative Relaying: A Distributed Market Pricing Framework," *IEEE Trans. Parallel Distributed Syst.*, vol. 22, issue 5, pp. 758-772, May 2011.

[31] Y. Zhan, C. H. Liu, Y. Zhao, J. Zhang, and J. Tang, "Free Market of Multi-Leader Multi-Follower Mobile Crowdsensing: An Incentive Mechanism Design by Deep Reinforcement Learning," *IEEE Trans. Mob. Comput.* , vol. 19, issue 10, pp. 2316-2329, Oct. 2020.

[32] K. Sooksatra1, R. Li, Y. Li, X. Guan, and W. Li, "Fairness-Aware Auction Mechanism for Sustainable Mobile Crowdsensing", in *Proc. of the 5th International Workshop on Automotive System/Software Architectures (WASA)*, Hamburg, Germany, 25 Mar. 2019, pp. 3103-321.

[33] Y. Liu, Y. Yang, E. Wang, W. Liu, D. Luan, X. Sun, and J. Wu, "A Fair Task Assignment Strategy for Minimizing Cost in Mobile Crowdsensing", in *Proc. of the Twenty-Sixth IEEE International Conference on Parallel and Distributed Systems (ICPADS)*, Hong Kong, China, December 2-4, 2020, pp. 44-53.

[34] N. Goel, and B. Faltings, "Deep Bayesian Trust: A Dominant and Fair Incentive Mechanism for Crowd," in *Proc. of the Thirty-Third Conference on Artificial Intelligence (AAAI)*, Honolulu, Hawaii, USA, January 27-February 1, 2019, pp. 1996-2003.

[35] Y. Wang, Z. Su, T. H. Luan, R. Li, and K. Zhang, "Federated Learning With Fair Incentives and Robust Aggregation for UAV-Aided Crowdsensing," *IEEE Trans. Netw. Sci. Eng.*, vol. 9, no. 5, pp. 3179-3196, Sep.-Oct. 2022.

[36] G. Sun, Y. Wang, X. Ding, and R. Hu, "Cost-Fair Task Allocation in Mobile Crowd Sensing With Probabilistic Users," *IEEE Trans. Mob. Comput.*, vol. 20, no. 2, pp. 403-415, Feb. 2021.

[37] W. Wang, Y. Yang, Z. Yin, K. Dev, X. Zhou, X. Li, N. M. F. Qureshi, and C. Su: BSIF, "Blockchain-Based Secure, Interactive, and Fair Mobile Crowdsensing," *IEEE J. Sel. Areas Commun.*, vol. 40, no. 12, pp. 3452-3469, Dec. 2022.

[38] Y. Li, F. Li, L. Zhu, H. Chen, T. Li, and Y. Wang, "Fair Incentive Mechanism with Imperfect Quality in Privacy-Preserving Crowdsensing," *IEEE Internet Things J.*, vol. 9, no.19, pp. 19188-19200, Oct. 2022.

[39] X. Song, E. Wang, W. Liu, Y. Liu, and Y. Dong, "Fairness task assignment strategy with distance constraint in Mobile CrowdSensing," *CCF Trans. Pervasive Comput. Interact.*, vol. 5, no. 2, pp. 184-205, Nov. 2023.

[40] F. Kelly, "Charging and rate control for elastic traffic," *Eur. Trans. Telecommun.*, vol. 8, no. 1, pp. 33-37, 1997.

[41] T. Demuynck, and T. Potoms, "Weakening transferable utility: The case of non-intersecting Pareto curves," *J. Econ. Theory*, vol. 188, pp. 105035, jul. 2020.

[42] Y. Miura, F. Toriumi, and T. Sugawara, "Evolutionary learning model of social networking services with diminishing marginal utility," in Proc. of Companion of the The Web Conference 2018 on The Web Conference 2018 (WWW), Lyon, France, April 23-27, 2018, pp. 1323-1329.

[43] L. Stefanini, and M. Arana-Jimenez, "Karush-Kuhn-Tucker conditions for interval and fuzzy optimization in several variables under total and directional generalized differentiability," *Fuzzy Sets Syst.*, vol. 362, pp. 1-34, May 2019.

[44] J. R. Zhao, R. Han, Y. Yang, B. Catterall, C. H. Liu, L. Y. Chen, R. Mortier, J. Crowcroft, and L. Wang, "Federated Learning With Heterogeneity-Aware Probabilistic Synchronous Parallel on Edge," *IEEE Trans. Serv. Comput.*, vol. 15, no. 2, pp. 614-626, Mar.-Apr. 2022.

[45] Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, "Gradient-based learning applied to document recognition," *Proc. IEEE*, vol. 86, no. 11, pp. 2278-2324, Nov. 1998.

[46] H. Xiao, K. Rasul, and R. Vollgraf, "Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms," 2017. [Online]. Available: https://doi.org/10.48550/arXiv.1708.07747.

[47] G. Cohen, S. Afshar, J. Tapson, and A. Schaik, "EMNIST: Extending MNIST to handwritten letters," in Proc. 2017 international joint conference on neural networks., May. 2017, pp. 2921-2926.

[48] A. Krizhevsky and G. Hinton, "Learning Multiple Layers of Features from Tiny Images," technical report, Univ. of Toronto, 2009.

[49] L. Duan, T. Kubo, K. Sugiyama, J. Huang, T. Hasegawa, and J. C. Walrand, "Motivating Smartphone Collaboration in Data Acquisition and Distributed Computing," *IEEE Trans. Mob. Comput.*, vol. 13, no. 10, pp. 2320-2333, Feb. 2014.

[50] X. Bei, S. Liu, X. Lu, H. Wang, "Maximin Fairness with Mixed Divisible and Indivisible Goods," in Proc. of Companion of Thirty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2021, Virtual Event, Feb. 2-9, 2021, pp. 5167-5175.

[51] I. Caragiannis, C. Kaklamanis, P. Kanellopoulos, and M. Kyropoulou, "The Efficiency of Fair Division," *Theory Comput. Syst.*, vol. 50, no. 4, pp. 589-610, 2012.

Jianfeng Lu received the Ph.D. degree in computer application technology from the Huazhong University of Science and Technology in 2010. He worked at Zhejiang Normal University from 2010 to 2021, served as a visiting researcher at University of Pittsburgh in 2013, and is currently a professor in the school of Computer Science and Technology at Wuhan University of Science and Technology, Wuhan, China. His research interests include algorithmic game theory and incentive mechanism with applications to crowdsensing.

Haibo Liu received the BE degree in Computer Science and Technology from Zhejiang Normal University, Jinhua, China, in 2022. He is currently a doctoral student of Computer Science and Technology at Shanghai Jiao Tong University, Shanghai, China. His research interests include federated learning, incentive mechanism, and game theory.

Riheng Jia received the B. E. degree in Electronics and Information Engineering from Huazhong University of Science and Technology, China, in 2012, and Ph.D. degree in Computer Science and Technology from Shanghai Jiao Tong University, Shanghai, China, in 2018. He is currently an Associate Professor in Department of Computer Science and Engineering of Zhejiang Normal University, China. His current research interests include wireless networks, energy harvesting networks and smart IoT.

Zhao Zhang received the Ph.D. from Xinjiang University in 2003, and received the Excellent Young Scientist Fundation of NSFC in 2012. She was with Xinjiang University from 1999 to 2014, and now is a professor in the Department of Computer Science and Engineering at Zhejiang Normal University, Jinhua, China. Her main interest is in combinatorial optimization, especially approximation algorithms for NP-hard problems which have their background in networks.

Xiong Wang received the Ph.D. degree in electronic engineering from Shanghai Jiao Tong University, Shanghai, China, in 2019. He was a Post-Doctoral Fellow with the Department of Computer Science and Engineering, The Chinese University of Hong Kong, Hong Kong, China, from 2019 to 2021. He is currently an Associate Professor with the School of Computer Science and Technology, Huazhong University of Science and Technology. His research interests include edge computing, crowdsourcing, resource allocation, and mobile computing.

Jiantao Wang received the PhD degree from Peking University, China, in 2015. He is currently an Associate Professor with Tenure in the Centre for Intelligent Healthcare, Coventry University, UK. Before that, he was an Assistant Professor with the School of Computer and Communications, Lancaster University, Lancaster, U.K., from 2019 to 2020. His research interests include pervasive and mobile computing, crowdsensing, digital health, Internet of Things, and smart cities.