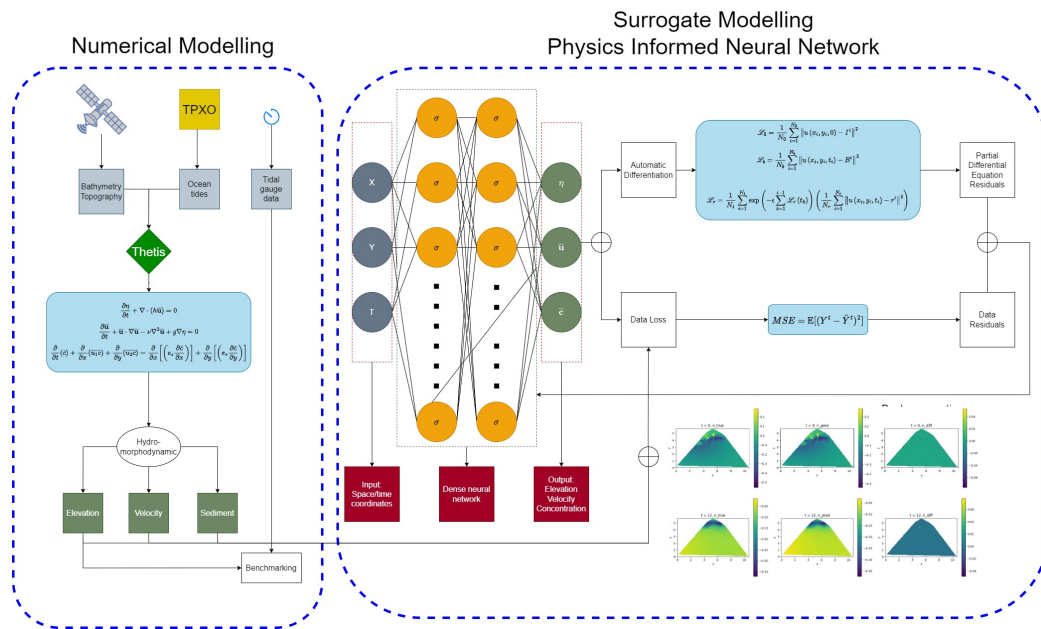
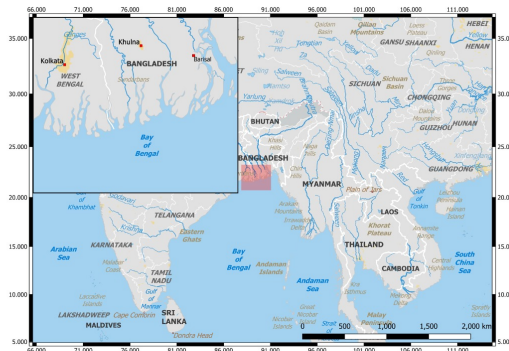


1 Graphical Abstract

2 Leveraging Physics-Informed Neural Networks for Efficient Modelling of Coastal Ecosystems Dynamics: A Case Study of Sundarbans Mangrove Forest

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5 Majdi Fanous, Alireza Daneshkhah, Jonathan M. Eden, Juntao Yang, Simon  
6 See, Vasile Palade



7 Highlights

8 **Leveraging Physics-Informed Neural Networks for Efficient Mod-**  
9 **elling of Coastal Ecosystems Dynamics: A Case Study of Sundar-**  
10 **bans Mangrove Forest**

11 Majdi Fanous, Alireza Daneshkhah, Jonathan M. Eden, Juntao Yang, Simon  
12 See, Vasile Palade

- 13 • Introduces hybrid PINNs, merging physics-informed modeling and neu-  
14 ral networks for advanced mangrove dynamics simulation.
- 15 • Utilises diverse datasets, including satellite imagery, simulations, and  
16 physics equations, ensuring comprehensive mangrove representation.
- 17 • Employs PINNs on complex PDE model, with complex boundary con-  
18 ditions and over an irregular and very large spatial domain.
- 19 • Significant time and computational improvements compared to tradi-  
20 tional numerical model.



47 mangrove forest, situated in a climatically vulnerable region of South Asia.  
48 The study’s findings revealed that PINNs significantly outperformed the nu-  
49 merical model exhibiting a five-fold decrease in computational cost, enabling  
50 near real-time predictions of mangrove dynamics. This improvement in com-  
51 putational efficiency is crucial for situations requiring rapid responses, such  
52 as evaluating the resilience of mangroves against extreme climate events like  
53 tropical cyclones. Furthermore, the accuracy of PINNs was found to be com-  
54 parable, if not superior, to the traditional model enabling accurate capturing  
55 of the dynamics around the mangrove environments.

56 *Keywords:* Coastal erosion, Hybrid modelling, Hydro-morphodynamic  
57 modelling, Mangrove Environments, Physics-informed neural networks

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## 58 1. Introduction

59 Considering the prevailing patterns of global warming, the projected esca-  
60 lation in sea levels entails the potential to trigger catastrophic consequences  
61 for coastal ecosystems, neighbouring communities, and interconnected ma-  
62 rine ecosystems. Thus, developing proper climate mitigation and adaptation  
63 strategies is crucial for a better understanding of the resilience of such ecosys-  
64 tems against climate change impacts. One solution is the use of artificial  
65 barriers as defense against rising sea levels Losada et al. (2019), but this is  
66 often cost prohibitive. The United Nations, in its Intergovernmental Panel  
67 on Climate Change (IPCC) Sixth Assessment Report (AR6), therefore en-  
68 couraged the use of natural defenses, known as ecosystem-based adaptation  
69 solutions, as an alternative to mitigate climate change impacts Cooley et al.  
70 (2022). Such defenses have huge potential to be an inexpensive, yet reliable,  
71 solution with the additional benefit of preserving natural ecosystems.

72 Among the most important natural defenses are mangrove ecosystems,  
73 which play a vital role in safeguarding coastal regions from the detrimental  
74 impacts of climate change, such as sea-level rise and land erosion Fanous  
75 et al. (2023b). Accurately modelling the complex dynamics of mangrove en-  
76 vironments is crucial for implementing effective protection and restoration  
77 strategies. Traditionally, numerical modelling techniques such as finite dif-  
78 ference, volume, or element methods have been employed for this purpose.  
79 However, these methods often face significant challenges related to time and  
80 computational complexities, which can hinder the success rates of proposed  
81 projects Fanous et al. (2023b). This is due to the fact that these models re-

82 quire high-resolution inputs, that account for spatial and temporal variations,  
83 to produce accurate solutions as well as solving complex physics equations,  
84 such as Navier–Stokes, that govern the hydro-morphodynamics of this region.  
85 Consequently, executing such models becomes time-consuming and imprac-  
86 tical, especially when real-time or near real-time predictions are necessary  
87 for effective coastal adaptation decision-making.

88 An alternative approach to traditional numerical modelling is using ma-  
89 chine learning models as surrogates, which can efficiently replace the existing  
90 solvers by learning the dynamics entirely from the data Pinto et al. (2021);  
91 Partee et al. (2022); Weber et al. (2020). The utilisation of progressively  
92 larger models and datasets in deep learning has resulted in significant ad-  
93 vancements across various scientific disciplines, which is particularly evident  
94 in fields such as computer vision and natural language processing Liu et al.  
95 (2023); Høye et al. (2021). Such models require very large datasets in order  
96 to be properly trained, i.e. minimising the loss function that represents the  
97 misfit of the data. However, the acquisition of data for many scientific and  
98 engineering problems is accompanied by considerable costs. This is particu-  
99 larly true in the field of climate modelling where direct numerical simulation  
100 is utilised Sivarajah et al. (2017); Kochkov et al. (2021). Consequently, there  
101 arises a pressing need for machine learning models that are capable of gen-  
102 eralising effectively within the confines of limited data.

103 One approach could be using Gaussian process (GP) surrogate models,  
104 which are particularly useful in cases with limited data or noisy observations  
105 Knudde et al. (2020); Donnelly et al. (2022). Unlike other machine learning  
106 methods, that assume a specific functional form, GPs provide a flexible and  
107 adaptable way to model complex relationships in data. The main issue,  
108 nonetheless, with GPs, and generally most other deep learning models, is  
109 that they are considered black box models where the underlying process used  
110 to provide the output is not fully understandable or explainable Vakili et al.  
111 (2021); Wang et al. (2019); Chatrabgoun et al. (2022). This can be a concern  
112 in applications where understanding the reasoning or factors influencing the  
113 predictions is crucial such as climate modelling. In this case, for example,  
114 such models could provide physically impossible outputs, whilst considering  
115 it a plausible solution, which could lead to dangerous consequences if relied  
116 upon in developing mitigation strategies.

117 In recent years, the emergence of physics-informed machine learning (PIML)  
118 has provided a promising avenue for simulating complex dynamics while ad-  
119 hering to the fundamental laws governing physical systems Karniadakis et al.

120 (2021); Kumar et al. (2021); Mahjoubi et al. (2022). PIML represents an inter-  
121 disciplinary approach that combines concepts from physics and machine  
122 learning by leveraging the representation and approximation capabilities of  
123 neural networks and integrating domain knowledge and governing physics  
124 equations into the learning process. This concept tackles both data scarcity  
125 and model explainability, which are shortcomings of deep learning and GP  
126 models.

127 Physics-informed neural networks (PINNs) are a specific type of PIML  
128 models that has gained significant attention as they use the physics equations  
129 as regularisation terms in the loss function thus constraining the outputs to  
130 physically consistent solutions Raissi et al. (2019). By introducing a physics  
131 loss term in the loss function, PINNs would tend to minimise both unrealistic  
132 solutions and data fitting errors. The physics loss is determined by calculat-  
133 ing the residuals associated with the model and the physics equations. This  
134 can be done easily using Automatic Differentiation (AD) to calculate the  
135 partial derivatives of the outputs for the corresponding inputs Raissi et al.  
136 (2019).

137 Previously, PINN models would either heavily rely on data (e.g., as com-  
138 putational fluid dynamics (CFD) solutions at selected input configurations)  
139 or be trained solely based on the underlying physics equations. Both ap-  
140 proaches may be challenged by large and complex applications, such as mod-  
141 elling the hydro-morphodynamics around mangrove environments. The for-  
142 mer requires input data that is computationally expensive to generate, while  
143 the latter may fail to capture the full complexities of the region (e.g., the  
144 interactions of the mangroves with tidal waves). In this paper, we propose  
145 a novel hybrid PINNs model, which partially uses data from CFD simula-  
146 tions and also partially uses physics equations to constrain the predicted  
147 solutions, in order to model different fields such as elevations, velocities, and  
148 sediment dynamics around mangrove environments. By incorporating both  
149 data-driven insights and physics-based constraints, PINNs offer a promising  
150 approach to efficiently and effectively address the limitations of traditional  
151 numerical and machine learning modelling methods in capturing the complex  
152 interactions and non-linearities, present in modelling mangrove environments  
153 dynamic.

154 To demonstrate the effectiveness of the proposed PINNs in modelling  
155 mangrove dynamics, we conducted a case study in the Sundarbans, the  
156 world’s largest mangrove forest situated between India and Bangladesh. The  
157 Sundarbans face significant climate change impacts, including sea-level rise

158 and land erosion, making it an ideal location to investigate the potential of  
159 the PINNs in supporting ecosystem-based adaptation solutions Mukul et al.  
160 (2019). In our study, we compared the performance of PINNs against a tradi-  
161 tional FE model that was developed to simulate the hydro-morphodynamics  
162 of mangrove environments. We focused on achieving two key aspects: de-  
163 creasing computational cost and increasing accuracy.

164 This paper is organised as follows. In Section 2, we introduce the PINNs  
165 architecture and underlying equations. In Section 3, we introduce the region  
166 of study with its characteristics and modelling conditions to demonstrate  
167 the performance of PINNs in modelling complex dynamics around mangrove  
168 environments in real-world settings. We also compare the performance of  
169 PINNs with a traditional numerical FE model at the same region, which is  
170 recently developed in Fanous et al. (2023a). Finally, we conclude our work  
171 and discuss present limitations and possible solutions in Section 4.

## 172 **2. Physics-informed neural networks**

173 In this section, we first provide the methodology illustrated in Figure 1,  
174 where panel a) shows the numerical model used to solve Navier Stokes equa-  
175 tions to model the hydro-morphodynamics of mangrove environments, which  
176 is briefly discussed in Section 2.1 (the full details can be found in Fanous  
177 et al. (2023a)). This numerical model was used to generate simulations over  
178 the region of interest and is required to construct the novel surrogate model  
179 proposed in this paper. Panel b) illustrates the the PINNs model including  
180 its equations, architecture, and evaluation metrics, which is developed for  
181 this complex real-world problem, over a large irregular spatial domain with  
182 complex boundary conditions. The details of this model will be discussed in  
183 Section 2.2.

### 184 *2.1. Hydro-morphodynamic modelling of mangrove environments*

185 Simulating the hydro-morphodynamics at a region of interest requires  
186 solving the Navier–Stokes (NS) equations that encompass the continuity in  
187 addition to the momentum equations Fanous et al. (2023a). Having a depth  
188 scale much smaller than the horizontal scale, it is possible to then use the  
189 depth-averaged NS, also known as the Shallow Water Equations (SWEs), as  
190 it saves on some unnecessary computational complexities.

191 The hydrodynamic equations of the 2D model are derived by depth-  
192 averaging from the bed,  $z_b$ , to the water surface,  $\eta$ . The model incorporates

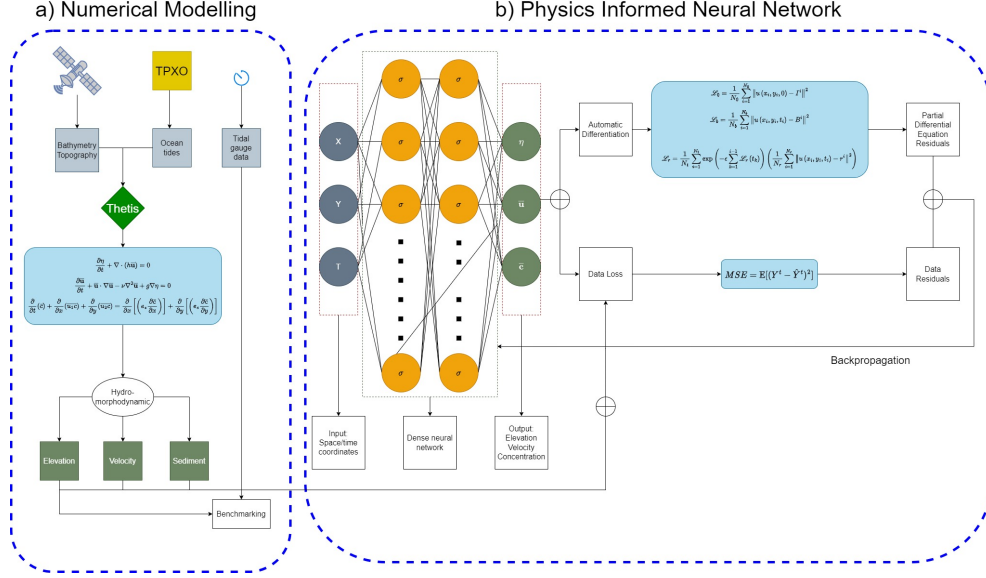


Figure 1: Proposed methodology outline

193 a kinematic boundary condition for the water surface, treating it as a free-  
 194 moving boundary while assuming the impermeability of the bed, i.e., water  
 195 does not pass through it. Therefore, the nonlinear SWEs used in this model  
 196 can be expressed as follows:

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (h\bar{\mathbf{u}}) = 0, \quad (1)$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} - \nu \nabla^2 \bar{\mathbf{u}} + g \nabla \eta = 0, \quad (2)$$

197 where,  $h = \eta - z_b$  represents the depth,  $\nu$  denotes the turbulent kinematic  
 198 eddy viscosity, and  $\bar{\mathbf{u}}$  is the depth-averaged velocity vector, and its  
 199 components,  $\bar{u}_1$  and  $\bar{u}_2$ , correspond to the flow in the  $x$  and  $y$  directions,  
 200 respectively (refer to Fanous et al. (2023a) for further details).

201 Regarding the morphodynamics, we adopt an Eulerian approach, which  
 202 considers the concentration of sediment particles and determines sediment  
 203 dynamics using an advection-diffusion equation. By combining diffusion and  
 204 dispersion effects over a prolonged sedimentation process, the depth-averaged  
 205 sediment concentration is given by:



$$\frac{\partial}{\partial t}(\bar{c}) + \frac{\partial}{\partial x}(\overline{u_1 c}) + \frac{\partial}{\partial y}(\overline{u_2 c}) = \frac{\partial}{\partial x} \left[ \left( e_s \frac{\partial \bar{c}}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ \left( e_s \frac{\partial \bar{c}}{\partial y} \right) \right] \quad (3)$$

206 Here,  $\bar{c}$  represents the sediment concentration,  $e_s$  denotes the sediment  
 207 turbulent diffusivity coefficient, given by  $e_s = v_s^h / \sigma_s$ , where  $v_s^h$  represents the  
 208 horizontal viscosity, and  $\sigma_s$  stands for the turbulent Schmidt number.

209 In order to solve the above equations, we developed a two-dimensional  
 210 coupled hydro-morphodynamic model within *Thetis*, a finite element model  
 211 for simulating coastal and estuarine flows. The main advantage of using  
 212 *Thetis* is that it uses a discontinuous Galerkin (DG)-based finite element  
 213 discretisation, which is proven to be robust for solving NS problems Fehn  
 214 et al. (2018). DG employs an unstructured mesh composed of triangular  
 215 elements for tessellation, upon which a FE space is established. The repre-  
 216 sentation of variables within a discontinuous space involves solving for the  
 217 variables along element edges. Due to its suitability for advection-dominated  
 218 problems and its ability to handle unstructured meshes, which are particu-  
 219 larly important for irregular geometries in coastal areas, this approach has  
 220 gained significant interest in hydro-morphodynamic applications Weinberg  
 221 and Wieners (2021).

222 A semi-implicit Crank-Nicolson time-stepping method is employed to en-  
 223 sure second-order accuracy and computational efficiency. This approach  
 224 requires only one non-linear solution per time-step, contributing to faster  
 225 execution. Furthermore, this particular time integration scheme minimises  
 226 excessive dissipation of tidal waves, preserving the solutions' characteristics  
 227 without excessively smoothing them, unlike the fully implicit backward Euler  
 228 method Fanous et al. (2023a).

229 In order to avoid some instabilities in the numerical model, as a result  
 230 of not reaching a stable flow state at the beginning of the simulation, which  
 231 may result in unrealistic sediment changes, we initialise the model first by  
 232 spinning the hydrodynamics. Once the elevation and velocity fields have  
 233 reached a steady state, we introduce then the morphodynamic equations  
 234 (see Fanous et al. (2023a) for further details).

235 Using an unstructured mesh and a semi-implicit time-stepping method,  
 236 we provide the numerical model with appropriate time and space-varying  
 237 conditions to describe the settings at the region of interest so that the model  
 238 can accurately simulate the hydro-morphodynamics.

239 *2.2. PINNs model*

240 The solutions to partial differential equations (PDEs) can be generally  
 241 represented in the following form:

$$u_t + \mathcal{N}[u; \lambda] = 0, \quad \mathbf{x} \in \Omega, \quad t \in [0, T] \quad (4)$$

242 Here,  $u(t, \mathbf{x})$  represents the solution, and  $\mathcal{N}[:, \lambda]$  is a general linear or  
 243 nonlinear operator with system parameters  $\lambda$ . The variables  $t$  and  $\mathbf{x}$  corre-  
 244 spond to the time and spatial inputs of the system, respectively. The spatial  
 245 domain  $\Omega$  can be bounded based on prior knowledge of the dynamical sys-  
 246 tem, and  $[0, T]$  denotes the time interval over which the system evolves. To  
 247 properly define the problem and solve Equation (4), it is typically necessary  
 248 to specify initial conditions  $u(\mathbf{x}, 0)$  and/or boundary conditions  $u(\mathbf{x}^0, t^0)$ .

249 This general form encompasses a wide range of problems, where  $\mathcal{N}$  can  
 250 be parabolic, hyperbolic, or elliptic, representing fluid dynamics, heat con-  
 251 duction, or steady-state diffusion, respectively.

252 For a two-dimensional problem, following Raissi et al. (2019), the func-  
 253 tion  $u(x, y, t)$  is approximated using a fully connected network denoted as  
 254  $f(x, y, t)$ . This network takes the coordinates  $(x, y, t)$  as inputs and provides  
 255 the corresponding outputs  $u_{\mathcal{N}}(x, y, t)$ . Then, by using AD, we can back-  
 256 propagate from the outputs to the inputs to calculate the partial derivatives  
 257 in terms of both time and space coordinates, i.e.  $\frac{\partial u}{\partial x}$  or  $\frac{\partial u}{\partial t}$ . A residual is  
 258 calculated between the calculated partials of  $f(x, y, t)$  and equation partials  
 259  $u(x, y, t)$ , which will be added as an equation loss term in the loss function.  
 260 Hence,  $f(x, y, t)$  and  $u(x, y, t)$  have shared parameters, but with different  
 261 activation functions, which is attributed to the inclusion of the differential  
 262 operator  $\mathcal{N}$ . The main advantage of using AD is its ability to calculate the  
 263 exact derivatives, thus eliminating the discretisation error.

264 The structure of the explained PINNs model is shown in Figure 2, com-  
 265 promising of a fully connected neural network with multiple hidden layers  
 266 with each hidden neuron containing a weight  $w_{i,j}$ , bias  $b_j$ , and a nonlin-  
 267 ear activation function  $\sigma$  such as hyperbolic tangents, ReLUs, leaky ReLUs,  
 268 ELUs or Swish Bihlo and Popovych (2022).

269 The neural network parameters are learned by minimising the mean  
 270 squared error (MSE) loss, which is defined as follows:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_b + \mathcal{L}_r, \quad (5)$$

271 where

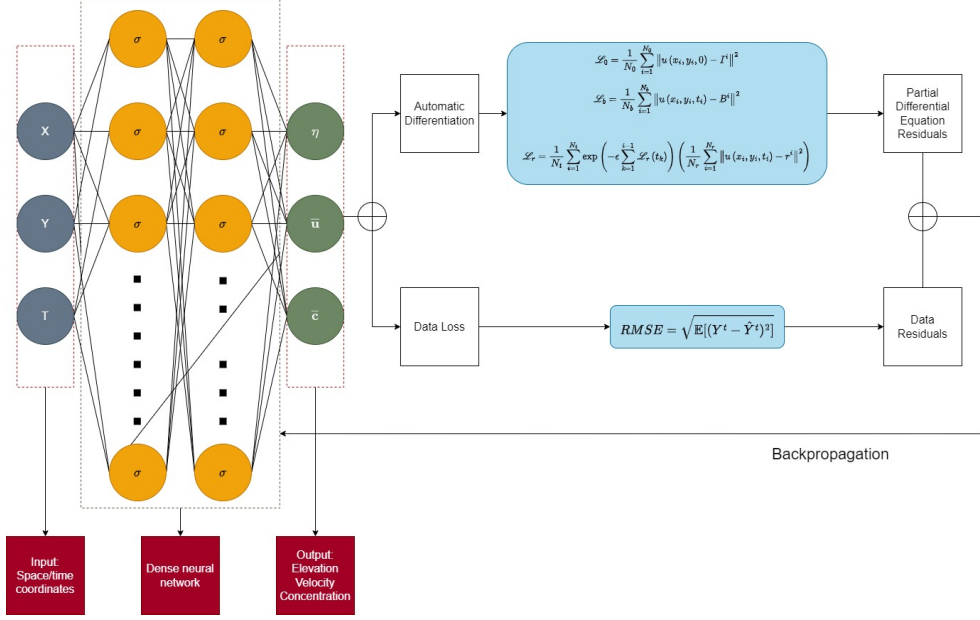


Figure 2: **Physics-informed neural network (PINN) architecture.** The inputs to the network are the time and space coordinates, which are passed through a deep fully connected neural network to obtain the desired quantities of interest such as water elevation ( $\eta$ ), velocity ( $\bar{\mathbf{u}}$ ) in both  $x$  and  $y$  directions, and sediment concentration ( $\bar{c}$ ). Then, gradients of the network's output with respect to its input are computed at these locations using automatic differentiation. Finally, the residual of the underlying differential equation is computed using these gradients and added as an extra term in the loss function in addition to the data loss.

$$\begin{aligned}
 \mathcal{L}_0 &= \frac{1}{N_0} \sum_{i=1}^{N_0} \|u(x_i, y_i, 0) - I^i\|^2 \\
 \mathcal{L}_b &= \frac{1}{N_b} \sum_{i=1}^{N_b} \|u(x_i, y_i, t_i) - B^i\|^2 \\
 \mathcal{L}_r &= \frac{1}{N_r} \sum_{i=1}^{N_r} \|u(x_i, y_i, t_i) - r^i\|^2
 \end{aligned} \tag{6}$$

272 In these equations,  $\mathcal{L}_0$ ,  $\mathcal{L}_b$ , and  $\mathcal{L}_r$  represent the initial loss, boundary  
 273 loss, and residuals of the governing equations, respectively. These losses are

274 computed using a finite set of collocation points. These points are sampled  
 275 uniformly, although there are different sampling strategies that could be  
 276 implemented as well, across the domain and constitute the location where  
 277 the solutions of the PINNs model is compared against the actual solutions of  
 278 the numerical model Raissi et al. (2019); Daw et al. (2022). Furthermore,  $I^i$ ,  
 279  $B^i$ , and  $r^i$  correspond to the initial, boundary, and domain solutions at these  
 280 collocation points, respectively. Finally,  $N_0, N_b, N_r$  are the number of points  
 281 at these domains. The obtained residuals are minimised by adjusting the  
 282 neural network parameters through optimisation algorithms such as Adam  
 283 or L-BFGS-B, which utilise gradient descent or quasi-Newton methods, re-  
 284 spectively Cuomo et al. (2022).

285 While PINNs provide promising alternatives for numerical models, the  
 286 standard formulation explained above fails to capture complex multi-scale  
 287 high nonlinear solutions Monaco and Apletti (2023). This is due to the  
 288 model minimising all losses  $\mathcal{L}$  simultaneously even if predictions at previous  
 289 time are inaccurate Wang et al. (2022). This would inevitably violate the  
 290 temporal causality, and thus lead to errors especially for time-dependent  
 291 PDEs. In order to avoid this issue, Wang et al. (2022) suggest reducing the  
 292 emphasis on subsequent time steps. In pursuit of this objective, the authors  
 293 reformulate the residual term as a weighted combination of residual losses  
 294 calculated at a fixed time step using the following equation:

$$\mathcal{L}_r = \frac{1}{N_t} \sum_{i=1}^{N_t} w_i \mathcal{L}_r(t_i) \quad (7)$$

295 where  $N_t$  is the temporal discretisation, and the weights  $w_i$  would have large  
 296 enough values to enable the minimisation of  $\mathcal{L}_r(t_i)$  upon the condition if all  
 297 previous residuals  $\{\mathcal{L}_r(t_k)\}_{k=1}^{i-1}$  prior to  $t_i$  are suitably minimised. To achieve  
 298 this, the weights  $w_i$  would be defined as the following:

$$w_i = \exp\left(-\epsilon \sum_{k=1}^{i-1} \mathcal{L}_r(t_k)\right) \quad (8)$$

299 where  $\epsilon$  is the temporal causality parameter, controlling the steepness of the  
 300 weights  $w_i$ . Consequently, the reformulated residual loss term can be written  
 301 as follows:

$$\mathcal{L}_r = \frac{1}{N_t} \sum_{i=1}^{N_t} \exp\left(-\epsilon \sum_{k=1}^{i-1} \mathcal{L}_r(t_k)\right) \mathcal{L}_r(t_i). \quad (9)$$

302 Defining  $w_i$  as an inversely exponential related to the magnitude of the  
 303 cumulative residual loss from previous time steps ensures that  $\mathcal{L}_r(t_i)$  will not  
 304 be minimised unless all previous residuals  $\{\mathcal{L}_r(t_k)\}_{k=1}^{i-1}$  decrease to a small  
 305 value such that  $w_i$  is sufficiently large.

306 In addition to introducing the causality weighted loss, we introduce a data  
 307 loss term  $\mathcal{L}_{data}$  which will account for the residual between the predicted and  
 308 actual output, where the latter would come from the CFD simulation. The  
 309 motivation behind the data loss term is that the interaction between the  
 310 mangrove dynamics and incoming tidal waves are not fully accounted for in  
 311 Equations (1) and (2). Thus, the modified loss equation would become:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_b + \mathcal{L}_r + \mathcal{L}_{data}, \quad (10)$$

312 To properly capture such interactions, it would require defining spatially  
 313 varying parameters such as Manning’s friction coefficient, kinematic viscos-  
 314 ity, and varying bed levels which could significantly increase the training  
 315 time for the PINNs model (see Fanous et al. (2023a) for details about these  
 316 parameters). Therefore, we utilise a small dataset from the numerical simu-  
 317 lation output to train the PINNs on the elevation and velocity fields, while  
 318 the sediment concentration is inferred purely from the physics equations,  
 319 i.e., from Eq.3. Consequently, this becomes a hybrid data and physics driven  
 320 PINNs model, which would result in faster training and convergence times  
 321 than regular PINNs.

### 322 2.3. Model setup

323 Our PINNs model is constructed using the NVIDIA Modulus framework  
 324 (see details about this Modulus, mod), which is a PyTorch-based neural  
 325 network framework designed for PINNs. The PINNs model we developed  
 326 consists of six fully connected multi-layer perceptron architectures, each com-  
 327 prising 256 neurons, and utilises the “swish” activation function. The swish  
 328 activation function, defined by the following equation

$$\phi(x) = \frac{x}{1 + e^{-x}}, \quad (11)$$

329 which is a smooth non-monotonic function that has demonstrated im-  
 330 proved performance over ReLU in deeper models Ramachandran et al. (2017).

331 To incorporate boundary and initial conditions into the PINNs, we sam-  
 332 pled points both on the domain boundary and in the interior. We employed  
 333 the *Adam* optimiser and utilised an exponentially decaying learning rate of

334 0.95 per 100,000 iterations for a total of 1,000,000 iterations and with a batch  
335 size of 512. An  $L_2$  regularisation (sum of squares error) was applied to mea-  
336 sure the approximation error of the neural network, which was minimised  
337 using the Adam optimiser Raissi et al. (2019).

338 To assess the performance and evaluate the accuracy of the constructed  
339 PINNs model, a comprehensive validation process was conducted by com-  
340 paring its predictions with the simulation data obtained from *Thetis*. The  
341 comparison was carried out at multiple time-steps, specifically at 12, 16, 20,  
342 and 24 hours, which show the spatial and temporal evolution of the model  
343 under different tidal stages (i.e., beginning of the tide, tidal peak, and end  
344 of tide).

345 The model simulation period was chosen from June 30, 2013, to July 1,  
346 2013, due to the availability of reliable tidal gauge records. Furthermore,  
347 the numerical model was validated against this tidal data to ensure that the  
348 model is accurate and can be used as validation against the output of the  
349 PINNs model Fanous et al. (2023a).

#### 350 2.4. Performance metrics

351 In order to assess the model’s predictive performance, the root mean  
352 squared error (RMSE) was employed as a performance metric to assess the  
353 predictive capability of the model for elevation,  $\bar{u}_1$  and  $\bar{u}_2$  velocities, and con-  
354 centration outputs at times 12, 16, 20, and 24 hours. It is calculated using  
355 the equation:

$$\text{RMSE} = \sqrt{\mathbb{E}[(\mathbf{y} - \hat{\mathbf{y}})^2]}, \quad (12)$$

356 where  $\mathbf{y}$  represents the actual output obtained from the numerical simu-  
357 lation, and  $\hat{\mathbf{y}}$  corresponds to the predicted output from the PINNs model.

358 The selection of the RMSE as the evaluation metric in this study is due  
359 to its ability to provide interpretable results by scaling the prediction errors  
360 back to the original unit of measurement, which in this case is expressed in  
361 meters [m].

362 In addition to providing RMSE values of the outputs (i.e., elevations at  
363 both directions and sediment concentrations) at the mentioned time-steps,  
364 we will illustrate the actual, predicted and their differences of these outputs  
365 at the time-steps over the entire spatial domain in Section 3.2. These images  
366 could be also used to examine the predictive performance of the proposed  
367 method in this paper.

368 **3. Case Study: PINNs for modelling Sundarbans mangroves**

369 In this section, we introduce the computational domain used to model the  
370 hydro-morphodynamics around mangrove environments at the Sundarbans.  
371 Then, we demonstrate the performance of the developed PINNs model and  
372 discuss its results and suitability as a surrogate when compared against the  
373 numerical solver.

374 *3.1. Computational domain*

375 The geographical extent of the model encompasses the complete shelf  
376 area of the Bay of Bengal, as well as the Sundarbans mangrove forest that  
377 straddles the border of India and Bangladesh. The spatial coverage of this  
378 region is illustrated in Figure 3.

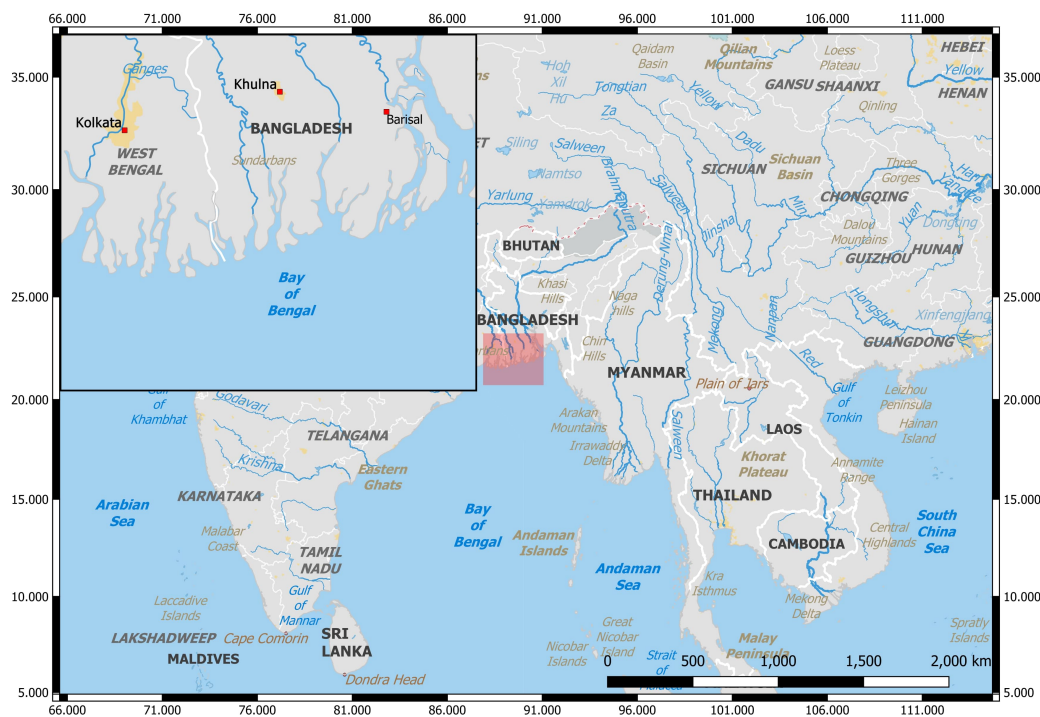


Figure 3: Geographical location of the Sundarbans

379 We developed a spatially varying mesh resolution to capture the dynamics  
380 of the tidal waves from the Indian Ocean up until the Sundarbans mangroves.  
381 The resolution of the domain varied from 8 km deep at the Ocean to 1.5 km at

382 the mangroves site. The generated mesh is shown in Figure 4. The resultant  
 383 mesh has over 125,000 cells with spatially varying resolutions.

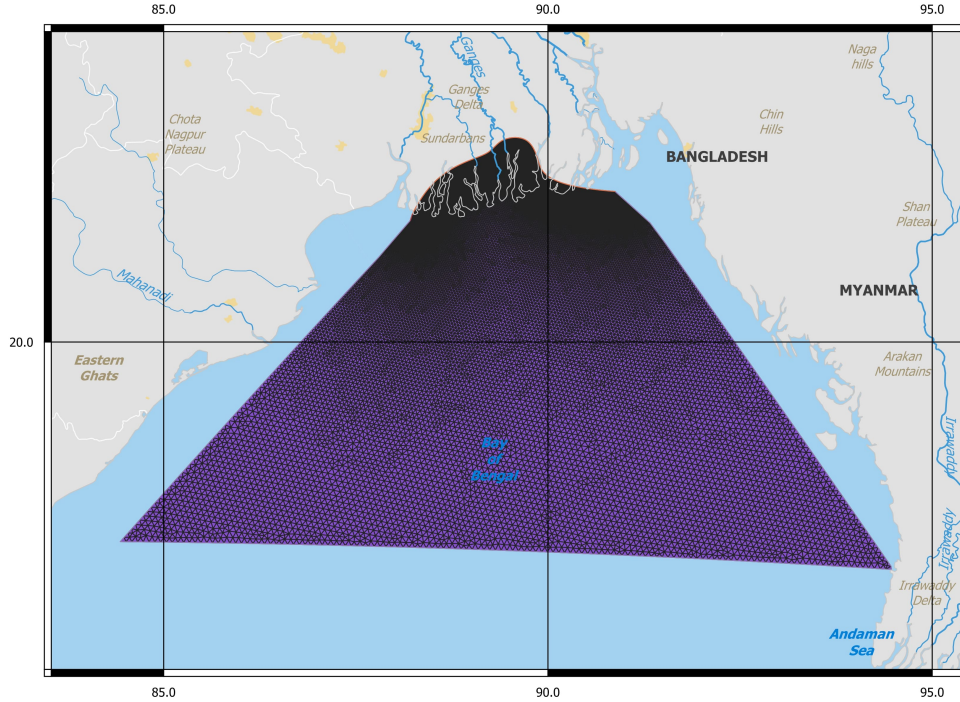


Figure 4: Mesh generated using Gmsh with varying resolution from 1.5 km to 8 km

384 In order to account for the absence of topography/bathymetry in the  
 385 PINNs model, we incorporated the effect of mangroves at the land border  
 386 by imposing a no-slip condition, where both horizontal velocities ( $\bar{u}_1$  and  $\bar{u}_2$ )  
 387 are set to zero. Similarly, to ensure comparability with the numerical solver,  
 388 we enforced the same boundary condition at the land boundary.

389 Furthermore, to simulate tidal waves, we implemented a periodic bound-  
 390 ary condition at the sea, which introduces a tidal elevation using the following  
 391 equation:

$$\text{Elevation} = A \sin\left(\frac{2\pi t_l}{T}\right), \quad (13)$$

392 where  $A$  represents the tidal amplitude,  $t_l$  is the simulation time, and  $T$   
 393 denotes the tidal period. For our model, we selected  $A$  to be 1 m and set  $T$



394 to 12 hours, which corresponds to a semi-diurnal tidal wave pattern observed  
395 in the Bay of Bengal.

396 Finally, in addition to the no-slip boundary conditions imposed at the  
397 land boundary, we imposed spatially varying Manning’s coefficient to simu-  
398 late the effect of mangrove environments. To explain, at the mangrove region,  
399 a Manning’s value of 0.15 was set that represents a dense forest Fanous et al.  
400 (2023a). However, moving towards the sea boundary, the value of Manning’s  
401 coefficient decreases reaching 0.001 as the friction is negligible deep in the  
402 Ocean.

### 403 *3.2. Results*

404 Running the PINNs model took approximately 24 hours in real time,  
405 which is significantly faster when compared to the numerical FE model that  
406 took approximately five days between hydrodynamic spin-up and full simu-  
407 lation. Such increase in the computational speed is critical, as discussed in  
408 Section 1, to increase the success rate of protection and mitigation projects  
409 when faced with different climate events.

410 With regards to the training loss of the model, Figure 5 shows the change  
411 of the log aggregated loss as the number of iterations increase. From Figure 5,  
412 it can be noticed that the loss decreases remarkably during the first 200,000  
413 iterations. From there, the loss almost stabilises with minimal decreases over  
414 the rest of the iterations. Furthermore, the loss appears to be decreasing in  
415 a stable manner, which proves the training stability of the PINNs model.

416 To quantify the model’s accuracy, we compared the results of the PINNs  
417 model with those of the numerical model, and Table 1 shows the RMSE  
418 scores of the elevation,  $\bar{u}_1$ -velocity and  $\bar{u}_2$ -velocity, and concentration at dif-  
419 ferent times of the simulation. From Table 1, it can be seen that the PINNs  
420 model performs very well over all fields and across different simulation times.  
421 This shows that the model is able to accurately capture the change in the  
422 hydrodynamics as well as the morphodynamics (sediment transport) across  
423 different stages of the tidal cycle.

424 Finally, spatial illustrations in Figures 6 - 9 visualise the prediction power  
425 of PINNs when compared to that of the numerical model for all outputs at the  
426 same time snapshots of Table 1. From Figures 6 - 8, the ability of mangroves  
427 to attenuate incoming tidal waves is clearly visible. The mangrove reduced  
428 tidal heights and velocities almost entirely, and the region at the land border  
429 does not have any significant tidal heights or velocities left. Furthermore,  
430 the PINNs model was able to regenerate the complex tidal structure and

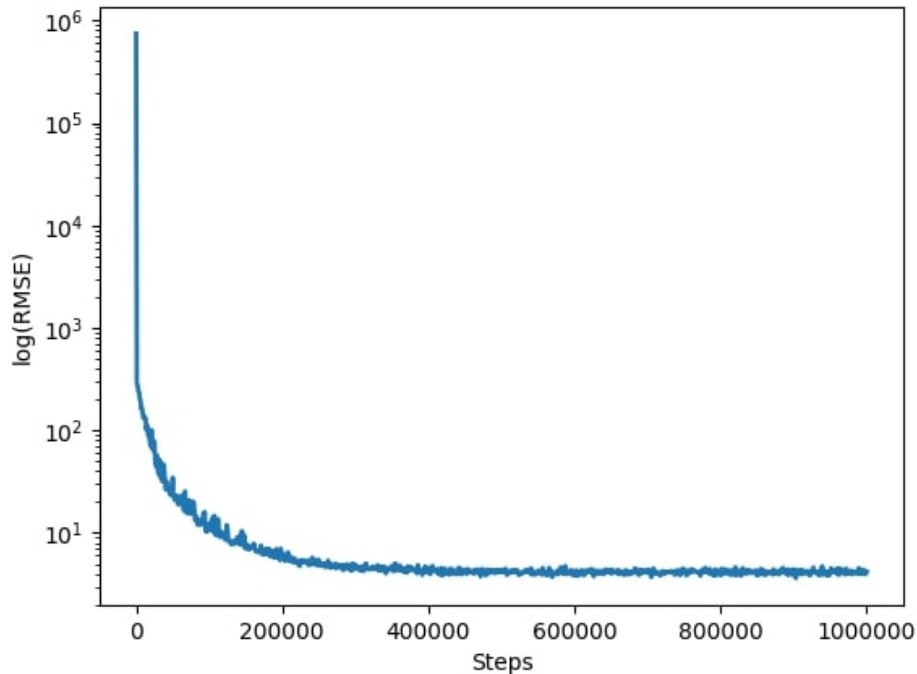


Figure 5: Training of PINNs model with  $y$ -axis showing the  $\log(\text{RMSE})$  loss and  $x$ -axis showing the training steps

431 the interaction between mangrove environments and the tides. This also  
 432 demonstrates the importance of adding the temporal causality. Without this  
 433 causality, the PINNs model would not have been properly trained for the  
 434 initial conditions at the region, which could result in significant errors, as  
 435 the initial structure is critical for determining the interaction for the rest of  
 436 the tidal cycle.

437 With respect to Figure 9, the output clearly shows the ability of man-  
 438 grove environments to prevent entirely any sediment erosion. The change in  
 439 sediment concentration over time is barely visible and just seen at the inter-  
 440 face of the incoming tides and mangrove region. It is important to note here  
 441 that the PINNs model did not predict the very small concentration change  
 442 at this interface. This is possibly due to the change being minimal, i.e. in  
 443 the order of  $e^{-6}$  and less, and the PINNs model focusing on higher losses in

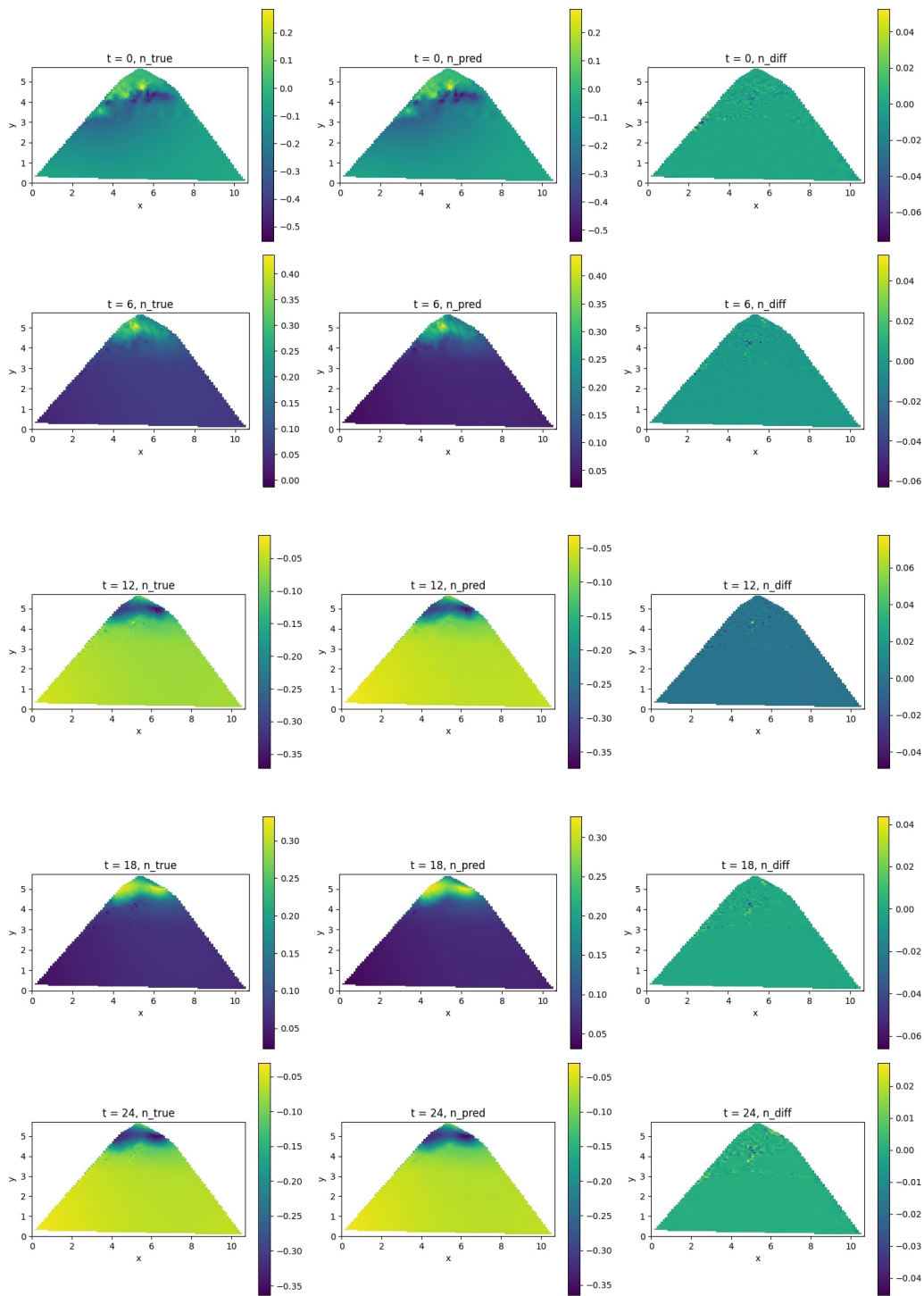


Figure 6: Actual vs predicted elevation at times 0, 6, 12, 18, and 24 hours.

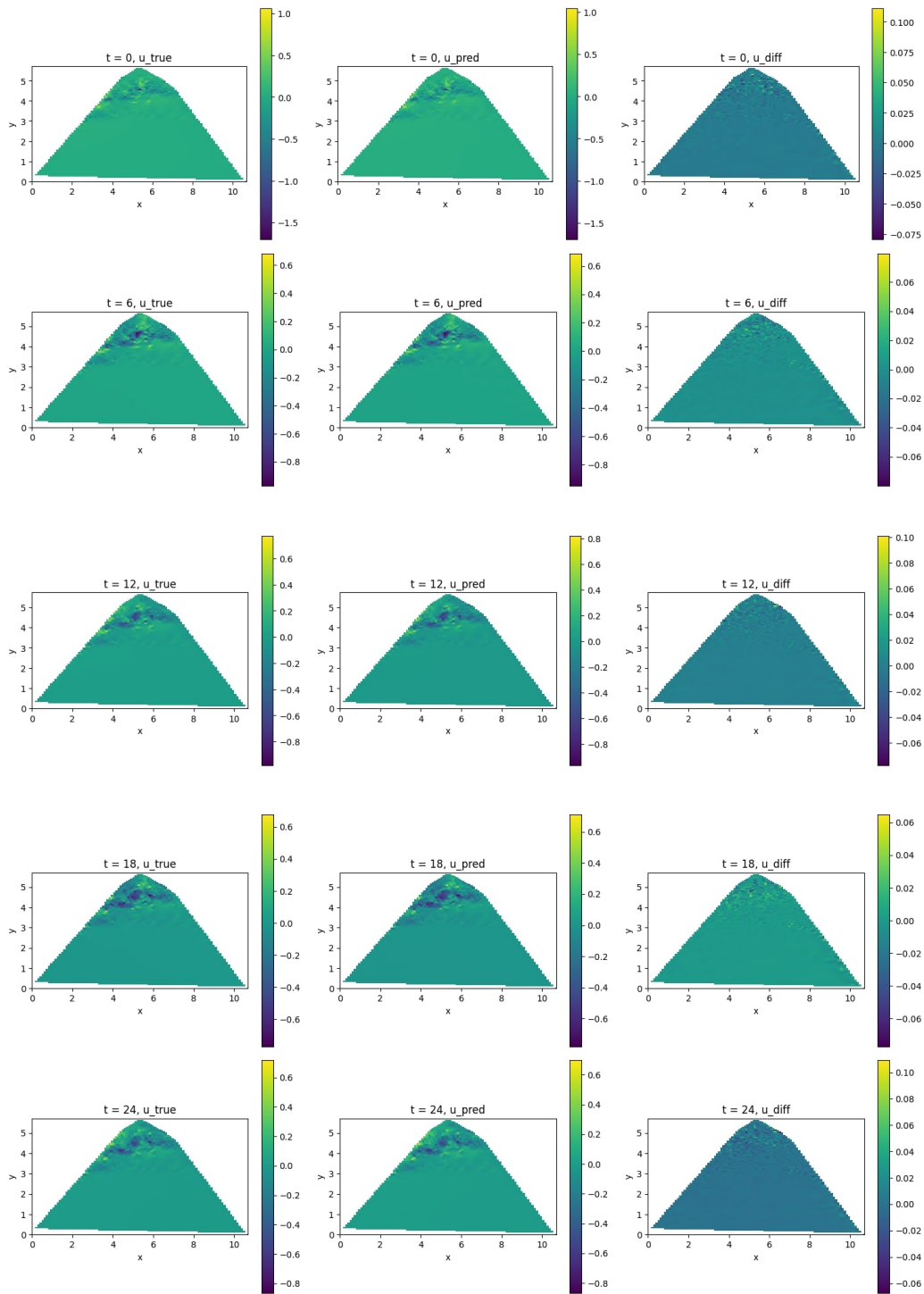


Figure 7: Actual vs predicted  $\bar{u}_1$ -velocity at times 0, 6, 12, 18, and 24 hours.

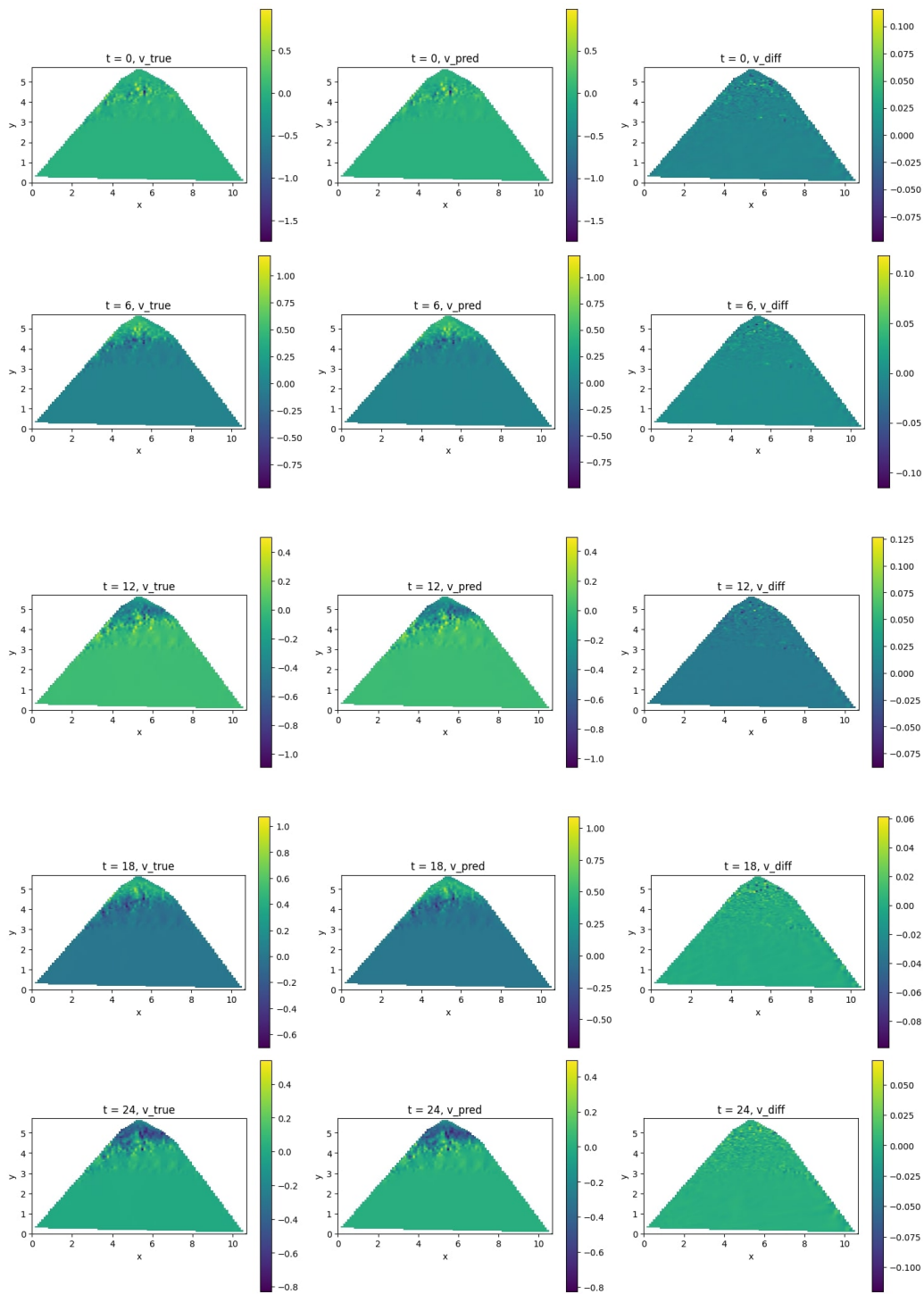


Figure 8: Actual vs predicted  $\bar{u}_2$ -velocity at times 0, 6, 12, 18, and 24 hours.

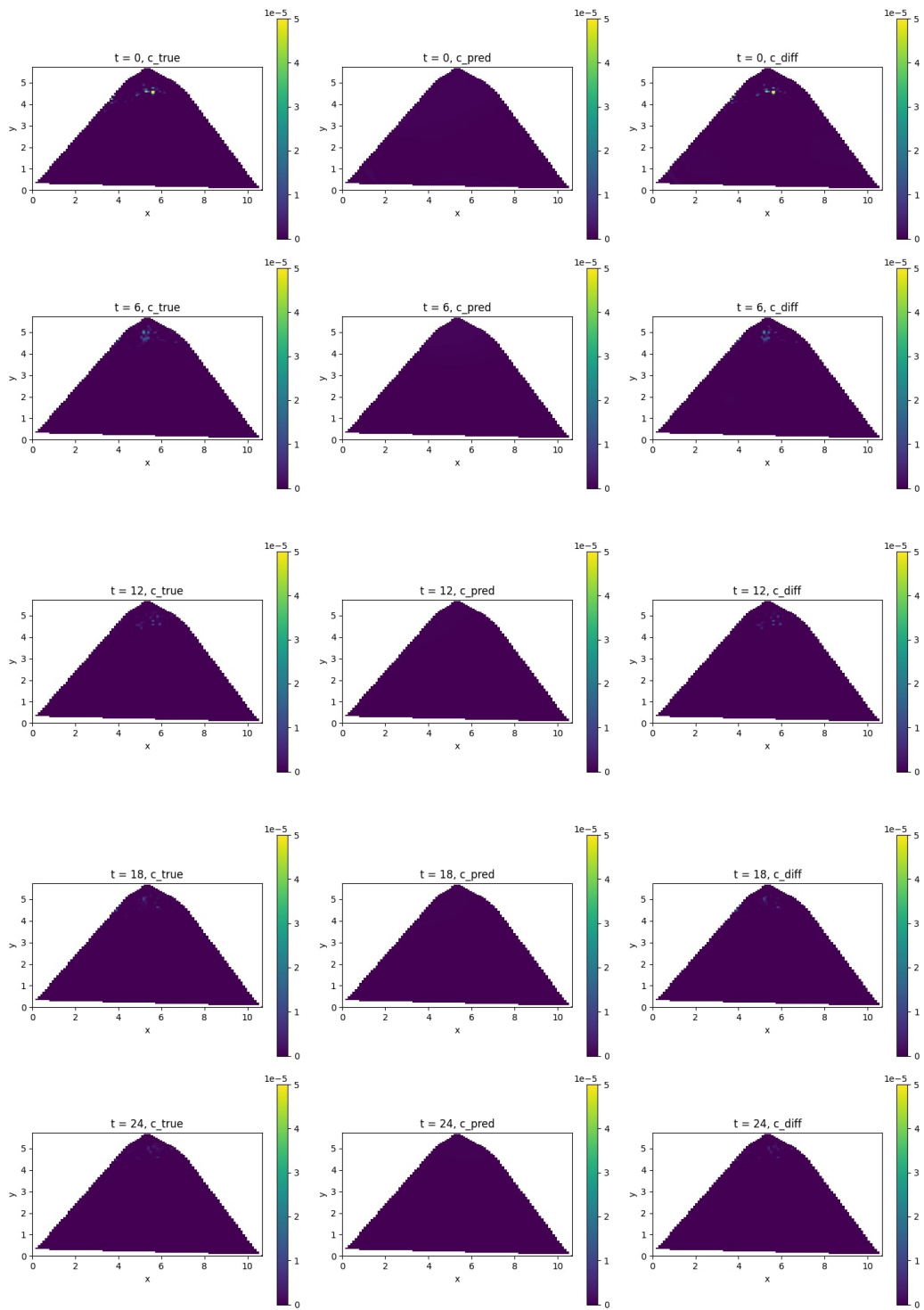


Figure 9: Actual vs predicted concentration at times 0, 6, 12, 18, and 24 hours.

Table 1: RMSE values for elevation,  $\bar{u}_1$ -velocity,  $\bar{u}_2$ -velocity, and concentration of PINNs against FE model/Thetis Fanous et al. (2023b)

<b>Time (hours)</b>	<b>Elevation</b>	<b><math>\bar{u}_1</math>-velocity</b>	<b><math>\bar{u}_2</math>-velocity</b>	<b>Concentration</b>
0	0.008	0.023	0.023	6.23e-06
6	0.007	0.017	0.02	2.12e-06
12	0.007	0.016	0.019	1.11e-06
18	0.006	0.015	0.018	1.25e-06
24	0.007	0.016	0.02	7.51e-07

444 the elevation and velocities. Nevertheless, the outputs still demonstrate the  
 445 model’s ability in predicting that almost no sediment change is happening at  
 446 the domain.

447 Thus, the figures illustrate the remarkable accuracy of the PINNs model  
 448 and its ability to handle complex interactions in large domains with complex  
 449 boundary conditions. The PINNs model was able to replicate with very  
 450 high accuracy the hydro-morphodynamic outputs simulated by the numerical  
 451 model without needing to solve the complex physics equations. In terms of  
 452 computational efficiency, the PINNs model, as discussed at the beginning of  
 453 the section, was considerably faster than the numerical model when training  
 454 the latter for initial and boundary conditions in addition to some simulation  
 455 data taken from the numerical model. Moreover, for providing inference at  
 456 unseen data-points, the model was substantially faster as it took less than 15  
 457 seconds to provide its predictions for the outputs fields. This is not possible  
 458 in the numerical model as it has to rerun the whole simulation again, which is  
 459 impractical in real-world cases where the speed of getting the information is  
 460 crucial for effective and quick decision making by the local and governmental  
 461 entities at the region.

462 It could be argued that the PINNs model used some numerical simula-  
 463 tion data for training, thus there is an inevitable computational demand for  
 464 running the numerical model first. However, as with any machine learning,  
 465 once the model is properly trained, i.e. converged to an acceptable loss, the  
 466 model does not have to be retrained again. In fact, just providing the new  
 467 conditions, the PINNs model would then be able to immediately predict the  
 468 output as long as the conditions were not significantly changed to the ones  
 469 used in training. This concept is present in all machine learning models,  
 470 where if the testing data is quite different to the data trained on, the model’s

471 accuracy and performance would deteriorate. In our case, the model would  
472 be able to perform well on similar tidal and mangrove boundary conditions,  
473 which are normally present in this region. Nonetheless, if one wants to in-  
474 corporate not only daily changes but also seasonal and annual dynamics, it  
475 would be possible to achieve performance comparable to the current model  
476 by training the PINNs model on appropriate time scales. Since the model is  
477 meshless, it does not require solving these equations in any spatial or time  
478 domains.

#### 479 **4. Conclusion**

480 Quantifying the role of mangrove environments in attenuating waves, pre-  
481 venting erosion, and providing ecosystem-based adaptation solutions is cru-  
482 cial for effective risk assessment, informed decision-making, and mitigating  
483 climate change impacts. Understanding the extent of these services enables  
484 scientists, policymakers, and stakeholders to make informed choices regarding  
485 coastal development, land-use planning, and infrastructure design, while also  
486 protecting natural ecosystems, biodiversity, and fragile habitats. Recognising  
487 the importance of mangroves in coastal areas is key to fostering sustainabil-  
488 ity, environmental management, and ensuring a sustainable future for both  
489 coastal ecosystems and neighbouring communities.

490 The proposed machine learning model provides a fast and accurate alter-  
491 native to traditional numerical models, which could be critically important  
492 for real-time predictions and assessment of current climate conditions at the  
493 region of study. This paper uses novel machine learning methods to model the  
494 hydro-morphodynamics of mangrove environments for an expansive region  
495 with real complex boundary conditions. The approach involves developing a  
496 hybrid data and physics-informed neural network (PINNs) and a custom loss  
497 function that accounts for temporal causality when training the model. This  
498 would ensure accurate modelling of mangrove environments and their ability  
499 in attenuating waves and preventing erosion due to their complex root struc-  
500 ture. Furthermore, the temporal causality factor addresses the shortcomings  
501 of vanilla PINNs (consisting of a fully connected deep neural network and  
502 physics loss function) by forcing the model to converge on initial conditions  
503 before reducing the losses for upcoming temporal discretisations.

504 The developed model has several advantages over traditional deep learn-  
505 ing models. One key advantage is its ability to incorporate known physi-  
506 cal laws into the learning process by enforcing the governing equations and



507 boundary conditions as constraints during training. This leads to more ac-  
508 curate and physically consistent predictions over other deep learning models.  
509 Moreover, PINNs are data efficient as they require fewer training data points  
510 compared to traditional models since they effectively learn from limited data  
511 and extrapolate predictions to unseen scenarios. Finally, PINNs can han-  
512 dle irregular domains and complex geometries more effectively, making them  
513 suitable for a wide range of applications. To explain, traditional deep learn-  
514 ing models often require structured and evenly sampled data, which can be  
515 challenging when dealing with irregularly shaped domains or complex geome-  
516 tries. By incorporating the governing equations and boundary conditions as  
517 constraints during training, PINNs can capture the behaviour of the system  
518 even in regions with sparse or irregularly distributed data.

519 The results of the study demonstrate the suitability of the developed  
520 PINNs model as an effective surrogate when compared to a finite element  
521 numerical model. The RMSE of all output fields, including elevation, velocity  
522 in both  $x$  and  $y$  directions, and sediment concentration, were between  $e^{-2}$   
523 and  $e^{-3}$ . The model was also consistent for different stages of the tidal  
524 cycle, i.e. beginning of the tide, tidal peak, and end of tide as demonstrated  
525 in the figures in Section 3.2. This shows the robustness of the developed  
526 model and its flexibility in modelling different complex interactions between  
527 the incoming tidal waves and the mangrove environments. Furthermore,  
528 with regards to mangrove environments, the results showed their ability to  
529 attenuate most of the tidal heights and its velocity, in addition to preventing  
530 almost any sediment change where such changes were barely visible.

531 Computational efficiency was also a key element in showing the superi-  
532 ority of the PINNs model compared to the numerical model. The training  
533 time of the PINNs model was about 24 hours for 1,000,000 iterations al-  
534 though the model mostly converged after approximately 200,000 iterations.  
535 This is significantly faster than the numerical solver, which took approxi-  
536 mately 5 days to simulate the same period including hydrodynamic spin-up  
537 to prevent model instabilities.

538 Using a hybrid data and physics-driven approach for the PINNs model  
539 has some computational costs that we need to consider. Modelling the inter-  
540 actions of mangrove environments and the complex dynamics associated with  
541 them is non-trivial and requires extensive modifications to the Navier–Stokes  
542 equations. This makes the training procedure of the PINNs model more dif-  
543 ficult, demanding a significant increase in training time. Thus, utilising a  
544 hybrid approach, where some data from numerical simulations is used to aid

545 the PINNs model in modelling such complex processes, eases the computa-  
546 tional cost of the latter but inevitably requires some numerical simulation  
547 to run. Although this increases the overall cost in general, once sufficiently  
548 trained, i.e. the cumulative loss is reduced to an acceptable level, there is no  
549 further requirement for additional numerical simulation data for prediction  
550 and conducting inference at testing points, as the network parameters have  
551 been optimised. This makes the inference process at “unseen” data-points  
552 incredibly fast, taking just few seconds. The numerical model, in contrast,  
553 requires running the full simulation again, which is computationally very ex-  
554 pensive, particularly for the applications discussed in this study. As a result,  
555 the associated training cost for the PINNs model depends on the objective  
556 of the steady, i.e. determining daily, monthly, seasonal, or decadal trends,  
557 and in all cases prediction cost would be negligible.

558 While the developed PINNs model in this study focused on mangrove  
559 environments, the methodology described can be extended to model other  
560 coastal ecosystems (e.g., marshes) and even generalise to any problem involv-  
561 ing simulating hydrodynamics and morphodynamics. The only difference  
562 would be in the data-driven part of the PINNs model, which requires some  
563 simulations for the selected case. This demonstrates the potential of the de-  
564 veloped hybrid data and physics-driven PINNs, in addition to the modified  
565 temporal weighted loss, to be applied to a wide range of problems where  
566 the underlying physics of the domain can be described using Navier–Stokes  
567 equations.

568 Such flexibility of the proposed PINNs model can also be observed in  
569 training on different time scales. In this study, we trained the PINNs model  
570 to infer daily interactions of the mangrove environments and the tidal waves.  
571 Nonetheless, it is possible to expand the time scale to capture weekly, sea-  
572 sonal, and even yearly trends. This would mainly depend on the intended  
573 objective from building such models. Furthermore, this would mean some in-  
574 crease in the computational cost driven by the need of more simulation data  
575 to train the PINNs model. However, the inferencing time for such large time  
576 scale applications would be immeasurable compared to traditional numeri-  
577 cal modelling. Future works should consider modelling real extreme climate  
578 events such as tropical cyclones using PINNs. These events pose significant  
579 challenges due to their intricate dynamics and interactions with the envi-  
580 ronment posing significant threats to the coastal communities. Leveraging  
581 PINNs has the potential to provide huge advantages especially when real-  
582 time monitoring and evaluation is required in order to implement immediate

583 protection and mitigation strategies to save the lives of people living in those  
584 areas and preserve the natural ecosystems.

585 Furthermore, an important limitation of PINNs is the lack of a direct ap-  
586 proach to uncertainty quantification using a Bayesian paradigm. Quantifying  
587 uncertainty is crucial in many scientific and engineering applications, as it  
588 provides valuable insights into the reliability and confidence of the model’s  
589 predictions. While quantifying uncertainty could be done using sampling  
590 techniques, such as Monte Carlo, this limits the ability to propagate un-  
591 certainty from the inputs to the outputs, thus omitting valuable informa-  
592 tion when quantifying the confidence of the model in predicting the output  
593 fields. Future works could address this limitation by incorporating Gaussian  
594 Processes into the framework of PINNs, or as a standalone complementary  
595 procedure after training the PINNs model, would significantly enhance their  
596 practical utility and provide robust quantification of predictive uncertainty.

597

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### 603 **References**

604 , . Modulus — NVIDIA Developer. URL: [https://developer.nvidia.com/  
605 modulus](https://developer.nvidia.com/modulus).

606 Bihlo, A., Popovych, R.O., 2022. Physics-informed neural networks for  
607 the shallow-water equations on the sphere. *Journal of Computational  
608 Physics* 456, 111024. doi:<http://dx.doi.org/10.1016/J.JCP.2022.111024>,  
609 [arXiv:2104.00615](https://arxiv.org/abs/2104.00615).

610 Chatrabgoun, O., Esmaeilbeigi, M., Cheraghi, M., Daneshkhah, A., 2022.  
611 Stable likelihood computation for machine learning of linear differential  
612 operators with gaussian processes. *International Journal for Uncertainty  
613 Quantification* 12.

614 Cooley, S., Schoeman, D., Bopp, L., Boyd, P., Donner, S., Ghebrehiwet,  
615 D., Ito, S.I., Kiessling, W., Martinetto, P., Ojea, E., Racault, M.F.,

- 616 Rost, B. and, ., Skern-Mauritzen, M., 2022. Ocean and coastal ecosystems  
617 and their services, in: Pörtner, H.O., Roberts, D.C., Tignor, M.,  
618 Poloczanska, E.S., Mintenbeck, K., Alegría, A., Craig, M., Langsdorf, S.,  
619 Löschke, S., Möller, V., Okem, A., Rama, B. (Eds.), *Climate Change*  
620 *2022: Impacts, Adaptation and Vulnerability*. Contribution of Working  
621 Group II to the Sixth Assessment Report of the Intergovernmental Panel  
622 on Climate Change. Cambridge University Press, Cambridge, UK and  
623 New York, NY, USA. book section 3. URL: [https://www.ipcc.ch/report/  
624 ar6/wg2/downloads/report/IPCC\\_AR6\\_WGII\\_Chapter03.pdf](https://www.ipcc.ch/report/ar6/wg2/downloads/report/IPCC_AR6_WGII_Chapter03.pdf), doi:[http://  
625 dx.doi.org/10.1017/9781009325844.005](http://dx.doi.org/10.1017/9781009325844.005).
- 626 Cuomo, S., Di Cola, V.S., Giampaolo, F., Rozza, G., Raissi, M., Pic-  
627 cialli, F., 2022. Scientific Machine Learning Through Physics-Informed  
628 Neural Networks: Where we are and What’s Next. *Journal of Sci-*  
629 *entific Computing* 2022 92:3 92, 1–62. URL: [https://link.springer.  
630 com/article/10.1007/s10915-022-01939-z](https://link.springer.com/article/10.1007/s10915-022-01939-z), doi:[http://dx.doi.org/10.1007/  
S10915-022-01939-Z](http://dx.doi.org/10.1007/<br/>631 S10915-022-01939-Z), arXiv:2201.05624.
- 632 Daw, A., Bu, J., Wang, S., Perdikaris, P., Karpatne, A., 2022. Mit-  
633 igating Propagation Failures in Physics-informed Neural Networks us-  
634 ing Retain-Resample-Release (R3) Sampling URL: [https://arxiv.org/abs/  
635 2207.02338v3](https://arxiv.org/abs/2207.02338v3), arXiv:2207.02338.
- 636 Donnelly, J., Abolfathi, S., Pearson, J., Chatrabgoun, O., Daneshkhah, A.,  
637 2022. Gaussian process emulation of spatio-temporal outputs of a 2d inland  
638 flood model. *Water Research* 225, 119100.
- 639 Fanous, M., Daneshkhah, A., Eden, J.M., Remesan, R., Palade, V., 2023a.  
640 Hydro-morphodynamic modelling of mangroves imposed by tidal waves  
641 using finite element discontinuous Galerkin method. *Coastal Engineer-*  
642 *ing* 182, 104303. doi:[http://dx.doi.org/10.1016/J.COASTALENG.2023.  
643 104303](http://dx.doi.org/10.1016/J.COASTALENG.2023.104303).
- 644 Fanous, M., Eden, J.M., Remesan, R., Daneshkhah, A., 2023b. Challenges  
645 and prospects of climate change impact assessment on mangrove environ-  
646 ments through mathematical models. *Environmental Modelling Software*  
647 162, 105658. doi:<http://dx.doi.org/10.1016/J.ENVSOF.2023.105658>.
- 648 Fehn, N., Wall, W.A., Kronbichler, M., 2018. Robust and efficient dis-  
649 continuous Galerkin methods for under-resolved turbulent incompress-

- 650 ible flows. *Journal of Computational Physics* 372, 667–693. doi:<http://dx.doi.org/10.1016/J.JCP.2018.06.037>, arXiv:1801.08103.
- 651
- 652 Høye, T.T., Ärje, J., Bjerger, K., Hansen, O.L., Iosifidis, A.,  
653 Leese, F., Mann, H.M., Meissner, K., Melvad, C., Raitoharju, J.,  
654 2021. Deep learning and computer vision will transform entomol-  
655 ogy. *Proceedings of the National Academy of Sciences of the United*  
656 *States of America* 118, e2002545117. URL: [https://www.pnas.org/](https://www.pnas.org/doi/abs/10.1073/pnas.2002545117)  
657 [doi/abs/10.1073/pnas.2002545117](https://www.pnas.org/doi/abs/10.1073/pnas.2002545117), doi:[http://dx.doi.org/10.1073/PNAS.](http://dx.doi.org/10.1073/PNAS.2002545117/SUPPL_FILE/PNAS.2002545117.SAPP.PDF)  
658 [2002545117/SUPPL\\_FILE/PNAS.2002545117.SAPP.PDF](http://dx.doi.org/10.1073/PNAS.2002545117/SUPPL_FILE/PNAS.2002545117.SAPP.PDF).
- 659 Karniadakis, G.E., Kevrekidis, I.G., Lu, L., Perdikaris, P., Wang, S.,  
660 Yang, L., 2021. Physics-informed machine learning. *Nature Reviews*  
661 *Physics* 2021 3:6 3, 422–440. URL: [https://www.nature.com/articles/](https://www.nature.com/articles/s42254-021-00314-5)  
662 [s42254-021-00314-5](https://www.nature.com/articles/s42254-021-00314-5), doi:<http://dx.doi.org/10.1038/s42254-021-00314-5>.
- 663 Knudde, N., Raes, W., De Bruycker, J., Dhaene, T., Stevens, N., 2020.  
664 Data-Efficient Gaussian Process Regression for Accurate Visible Light  
665 Positioning. *IEEE Communications Letters* 24, 1705–1709. doi:<http://dx.doi.org/10.1109/LCOMM.2020.2990950>.
- 666
- 667 Kochkov, D., Smith, J.A., Alieva, A., Wang, Q., Brenner,  
668 M.P., Hoyer, S., 2021. Machine learning–accelerated computa-  
669 tional fluid dynamics. *Proceedings of the National Academy*  
670 *of Sciences of the United States of America* 118, e2101784118.  
671 URL: <https://www.pnas.org/doi/abs/10.1073/pnas.2101784118>,  
672 doi:[http://dx.doi.org/10.1073/PNAS.2101784118/SUPPL\\_FILE/PNAS.](http://dx.doi.org/10.1073/PNAS.2101784118/SUPPL_FILE/PNAS.2101784118.SAPP.PDF)  
673 [2101784118.SAPP.PDF](http://dx.doi.org/10.1073/PNAS.2101784118/SUPPL_FILE/PNAS.2101784118.SAPP.PDF), arXiv:2102.01010.
- 674 Kumar, A., Ridha, S., Narahari, M., Ilyas, S.U., 2021. Physics-guided deep  
675 neural network to characterize non-Newtonian fluid flow for optimal use of  
676 energy resources. *Expert Systems with Applications* 183, 115409. doi:<http://dx.doi.org/10.1016/J.ESWA.2021.115409>.
- 677
- 678 Liu, P., Yuan, W., Fu, J., Jiang, Z., Hayashi, H., Neubig, G., 2023. Pre-train,  
679 Prompt, and Predict: A Systematic Survey of Prompting Methods in Nat-  
680 ural Language Processing. *ACM Computing Surveys* 55. URL: [https://](https://dl.acm.org/doi/10.1145/3560815)  
681 [dl.acm.org/doi/10.1145/3560815](https://dl.acm.org/doi/10.1145/3560815), doi:<http://dx.doi.org/10.1145/3560815>,  
682 arXiv:2107.13586.

- 683 Losada, I.J., Toimil, A., Muñoz, A., Garcia-Fletcher, A.P., Diaz-Simal, P.,  
684 2019. A planning strategy for the adaptation of coastal areas to climate  
685 change: The Spanish case. *Ocean Coastal Management* 182, 104983.  
686 doi:<http://dx.doi.org/10.1016/J.OCECOAMAN.2019.104983>.
- 687 Mahjoubi, S., Meng, W., Bao, Y., 2022. Logic-guided neural network for  
688 predicting steel-concrete interfacial behaviors. *Expert Systems with Appli-*  
689 *cations* 198, 116820. doi:<http://dx.doi.org/10.1016/J.ESWA.2022.116820>.
- 690 Monaco, S., Apiletti, D., 2023. Training physics-informed neural networks:  
691 One learning to rule them all? *Results in Engineering* 18, 101023. doi:<http://dx.doi.org/10.1016/J.RINENG.2023.101023>.
- 693 Mukul, S.A., Alamgir, M., Sohel, M.S.I., Pert, P.L., Herbohn, J., Turton,  
694 S.M., Khan, M.S.I., Munim, S.A., Reza, A.H., Laurance, W.F., 2019.  
695 Combined effects of climate change and sea-level rise project dramatic  
696 habitat loss of the globally endangered Bengal tiger in the Bangladesh  
697 Sundarbans. *Science of The Total Environment* 663, 830–840. doi:<http://dx.doi.org/10.1016/J.SCITOTENV.2019.01.383>.
- 699 Partee, S., Ellis, M., Rigazzi, A., Shao, A.E., Bachman, S., Marques, G.,  
700 Robbins, B., 2022. Using Machine Learning at scale in numerical simula-  
701 tions with SmartSim: An application to ocean climate modeling. *Journal*  
702 *of Computational Science* 62, 101707. doi:[http://dx.doi.org/10.1016/J.](http://dx.doi.org/10.1016/J.JOCS.2022.101707)  
703 [JOCS.2022.101707](http://dx.doi.org/10.1016/J.JOCS.2022.101707), [arXiv:2104.09355](https://arxiv.org/abs/2104.09355).
- 704 Pinto, G., Deltetto, D., Capozzoli, A., 2021. Data-driven district energy  
705 management with surrogate models and deep reinforcement learning. *Ap-*  
706 *plied Energy* 304, 117642. doi:[http://dx.doi.org/10.1016/J.APENERGY.](http://dx.doi.org/10.1016/J.APENERGY.2021.117642)  
707 [2021.117642](http://dx.doi.org/10.1016/J.APENERGY.2021.117642).
- 708 Raissi, M., Perdikaris, P., Karniadakis, G.E., 2019. Physics-informed neural  
709 networks: A deep learning framework for solving forward and inverse prob-  
710 lems involving nonlinear partial differential equations. *Journal of Compu-*  
711 *tational Physics* 378, 686–707. doi:[http://dx.doi.org/10.1016/J.JCP.2018.](http://dx.doi.org/10.1016/J.JCP.2018.10.045)  
712 [10.045](http://dx.doi.org/10.1016/J.JCP.2018.10.045).
- 713 Ramachandran, P., Zoph, B., Le Google Brain, Q.V., 2017. Searching for  
714 Activation Functions. 6th International Conference on Learning Repre-  
715 sentations, ICLR 2018 - Workshop Track Proceedings URL: [https://arxiv.](https://arxiv.org/abs/1702.03735)

- 716 [org/abs/1710.05941v2](https://arxiv.org/abs/1710.05941v2), doi:<http://dx.doi.org/10.48550/arxiv.1710.05941>,  
717 [arXiv:1710.05941](https://arxiv.org/abs/1710.05941).
- 718 Sivaraman, U., Kamal, M.M., Irani, Z., Weerakkody, V., 2017. Critical anal-  
719 ysis of Big Data challenges and analytical methods. *Journal of Business*  
720 *Research* 70, 263–286. doi:[http://dx.doi.org/10.1016/J.JBUSRES.2016.08.](http://dx.doi.org/10.1016/J.JBUSRES.2016.08.001)  
721 [001](http://dx.doi.org/10.1016/J.JBUSRES.2016.08.001).
- 722 Vakili, S., Moss, H., Artemev, A., Dutordoir, V., Picheny, V., 2021. Scalable  
723 Thompson Sampling using Sparse Gaussian Process Models. *Advances in*  
724 *Neural Information Processing Systems* 34, 5631–5643.
- 725 Wang, K.A., Pleiss, G., Gardner, J.R., Tyree, S., Weinberger, K.Q., Wilson,  
726 A.G., 2019. Exact Gaussian Processes on a Million Data Points. *Advances*  
727 *in Neural Information Processing Systems* 32.
- 728 Wang, S., Sankaran, S., Perdikaris, P., 2022. RESPECTING  
729 CAUSALITY IS ALL YOU NEED FOR TRAINING PHYSICS-  
730 INFORMED NEURAL NETWORKS URL: [https://arxiv.org/abs/2203.](https://arxiv.org/abs/2203.07404)  
731 [07404](https://arxiv.org/abs/2203.07404), [arXiv:2203.07404](https://arxiv.org/abs/2203.07404).
- 732 Weber, T., Corotan, A., Hutchinson, B., Kravitz, B., Link, R., 2020.  
733 Technical note: Deep learning for creating surrogate models of precip-  
734 itation in earth system models. *Atmospheric Chemistry and Physics*  
735 20, 2303–2317. URL: <https://acp.copernicus.org/articles/20/2303/2020/>,  
736 doi:<http://dx.doi.org/10.5194/acp-20-2303-2020>.
- 737 Weinberg, K., Wieners, C., 2021. Dynamic phase-field fracture with a first-  
738 order discontinuous Galerkin method for elastic waves. *Computer Meth-*  
739 *ods in Applied Mechanics and Engineering* 389. URL: [http://arxiv.org/](http://arxiv.org/abs/2106.15193)  
740 [abs/2106.15193](http://arxiv.org/abs/2106.15193)<http://dx.doi.org/10.1016/j.cma.2021.114330>, doi:[http://](http://dx.doi.org/10.1016/j.cma.2021.114330)  
741 [dx.doi.org/10.1016/j.cma.2021.114330](http://dx.doi.org/10.1016/j.cma.2021.114330), [arXiv:2106.15193v2](https://arxiv.org/abs/2106.15193v2).