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Analysis of impact of uncertainty in global production networks' parameters

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Highlights

- The impact of risks on a global production networks GPNs is considered
- Epistemic uncertainties are modelled using fuzzy numbers
- Ambiguity is used as a measure of uncertainties
- Sensitivity of GPN performance to parameter values is examined
- Sensitivity to uncertainty in GPN's parameters is analysed
- Implications for decision making under uncertainty are discussed

ABSTRACT

As production networks grow globally, their complexity and susceptibility to risk are increasing as well. Due to internal and external factors, risks affect individual network nodes and their impact propagates through the network to affect other nodes. A Fuzzy Dynamic Inoperability Input/output Model (FDIIM) is developed to facilitate and analyse the risk and its propagation in global production networks (GPN), at the strategic level. This method applies fuzzy arithmetic to track and operate with uncertainty in GPN parameters and to estimate the confidence in the results obtained. The expert provides a judgement on relevant risk parameters' values in the form of linguistic values, where relevant statistical data is absent. We used the measure of ambiguity to measure uncertainty in the GPN parameters. Two types of analyses are carried out: (1) to examine the sensitivity of the FDIIM to changes in input parameter values, including interdependencies between GPN nodes, resilience of the GPN, intended revenues and impact of disruptions, and (2) to examine sensitivity to uncertainty in the GPN's input parameters. A generic GPN example and different risk scenarios are defined to illustrate these analyses. The analyses provide an insight into the importance of different GPN's parameters in the risk analysis. Furthermore, we demonstrated how to identify GPN parameters which are important to specify with less uncertainty.

Keywords: Global Production Network, Risk Management, Sensitivity Analysis, Measure of Uncertainty, Fuzzy Arithmetic

1 INTRODUCTION

Global Production Networks (GPNs) are now the reality of manufacturing, where manufacturers, suppliers, customers and consumers can be located at various locations across the globe. These global arrangements can provide many benefits such as cutting costs, increasing revenue and increasing flexibility and agility. However, they could also lead to an increase in exposure to risks. Due to differences in a social, economic and political profile of different locations, GPNs could be affected unexpectedly. Also, individual network nodes can behave differently amid cultural and legal differences. As a result, it is more important than ever to have a correct understanding and proper risk management of GPNs. Risk consideration needs to be carried out at the strategic level, when decisions about GPN configuration and selection of GPN nodes are going to be made.

A novel Fuzzy Dynamic Inoperability Input/output Model (FDIIM) for risk analysis in GPNs have been proposed (Niknejad & Petrovic, 2016; Niknejad, Petrovic, Popplewell, Jaekel, & Pajkovska-Goceva, 2016). The method allows for utilising experts' knowledge to construct a risk profile that can be used to evaluate the impact of risk on different GPN configurations at the strategic level. It may be convenient to express this knowledge in the form of linguistic values, such as "high interdependency between production facility and supplier", "small resilience of the production facility", or "medium disruption of the supplier". They are modelled using fuzzy sets. It has been demonstrated that the fuzzy sets provide a convenient framework for treating uncertainty in production networks which is expressed using natural language terms (Niknejad & Petrovic, 2014; Petrovic, Roy, & Petrovic, 1999).

The use of experts' knowledge and subjective estimates are particularly advantageous as most of the relevant information about the risks is not readily available at the strategic decision making level and a significant effort and time are required to obtain more accurate data. However, this could also lead to a high level of uncertainty in the results obtained, which can hinder the decision making. Hence, it is important to understand the relationships between GPN parameters relevant to risk, their uncertainty and the results obtained. Furthermore, the parameters that have contributed the most to the uncertainty of the results can be identified and then their accuracy should be improved in a bid to reduce the overall uncertainty in the results obtained.

In this paper, we will analyse (1) the sensitivity of the results, obtained by using the FDIIM, to different GPN parameters values and (2) the impact of uncertainty of the GPN's parameters to the GPN performance. We are proposing a novel framework for management of uncertainty in GPN decision making. Based on the two analyses carried out, GPN parameters whose values and uncertainty in these values have the highest impact on GPN performance in the presence of risk can be identified. We are investigating different measures of uncertainty including fuzziness, specificity and ambiguity to quantify uncertainty in the GPN parameters. In the proposed framework, we are applying the measure of ambiguity. We use a generic GPN structure with two suppliers, a production facility and a customer and consider two different uncertain disruptions, one on the suppliers and one on the production facility to observe differences in their propagation and their impact on the GPN performance.

This paper is arranged as follows. In Section 2, a brief review of the literature on GPN risk analysis, inoperability modelling, uncertainty management and analysis of sensitivity to uncertainty are introduced. Section 3 describes concepts of fuzzy arithmetic relevant to the FDIIM, uncertainty measures and the process of uncertainty management using the FDIIM. The FDIIM is discussed in Section 4. Following on, the general GPN structure and its parameters which are used for the sensitivity analysis are introduced in Section 5. Sensitivity of the results to the parameters' values is investigated in Section 6, while, Section 7 examines the sensitivity of the results to the uncertainty in the input parameters. Finally, a summary of the paper and a few main concluding remarks are provided in Section 8.

2 LITERATURE REVIEW

Considering the importance of GPN risk management, a considerable amount of research has been carried out in this area in the last decade or so (Ho, Zheng, Yildiz, & Talluri, 2015). Some of the main research topics include identification of relevant risk factors and drivers (Prinz & Bauernhansl, 2013; Wagner & Neshat, 2010), risk analysis and assessment (Hofman, 2011; Wei, Dong, & Sun, 2010), risk control and optimisation (Aqlan & Lam, 2016; Shin, Shin, Kwon, & Kang, 2012) and network design under risk and uncertainty (Gong, Mitchell, Krishnamurthy, & Wallace, 2014; Mizgier, Wagner, & Holyst, 2012; Yang, Liu, & Yang, 2015).

Multiple categories of risks of GPNs have been identified. One popular approach to categorise risks is through the zone of its impact; it can be: (1) organizational or internal risks such as issues related to individual nodes in the networks, including insolvencies, machine breakdowns or accidents, (2) environmental or external, such as disruptions due to the regional and external factors, including economic risks, legal uncertainties and floods, and, (3) network-related risks caused by interdependencies between the nodes of the network such as supplier risks and demand risks (Jüttner, Peck, & Christopher, 2003). Bogataj and Bogataj (2007) classify risks based on GPN processes, such as supply, production, distribution, demand, control and environmental.

A number of quantitative approaches have been proposed for the assessment and analysis of GPN risks, including simulation methods (Hua, Sun, & Xu, 2011), mathematical optimisation (Gong, Mitchell, Krishnamurthy, & Wallace, 2014), multi-criteria decision making methods such as Analytical Hierarchical Processes (Ganguly, & Guin, 2013), Bayesian Belief Networks (Shin, Shin, Kwon, & Kang, 2012) and Inoperability Input/output Model (Wei, Dong, & Sun, 2010).

Inoperability Input/output Model (IIM) has been initially proposed to analyse disruptions of different economic sectors (Haimes & Jiang, 2001). Statistical data has been utilised to estimate the interdependencies between the sectors (Haimes, Horowitz, Lambert, Santos, Lian, & Crowther, 2005), while Panzieri and Setola (2008) used fuzzy sets to model uncertainties in interdependent infrastructures. IIM based methods have been developed for supply chain risk as well (Niknejad & Petrovic, 2016; Wei et al., 2010).

It might be interesting to mention the NASA Langley Uncertainty Quantification Challenge (Crespo, Kenny, & Giesy, 2014) that identified a number of challenges related to reducing epistemic uncertainty and sensitivity of input parameters in a Twin-Jet aircraft model. Both epistemic and aleatory uncertainties in the inputs that propagate to the outputs have been considered, where, epistemic uncertainty refers to the incompleteness of knowledge about the subject, which can be reduced, and, aleatory uncertainty refers to irreducible uncertainties that exist due to the stochastic character of events. A number of approaches to the problem have been proposed, for example, using Simulation, Genetic Algorithm and Artificial Neural Networks (Pedroni, & Zio, 2015). This challenge, although it is in a different application area, served as a starting point to our analysis of the impact of uncertainty on the GPN performance.

Sensitivity analysis has been predominantly carried out to investigate the impact of uncertainty modelled by probability distributions. Probability Bounding Analysis (PBA) as a method of measuring the sensitivity of system's output to its input uncertainty was considered by Ferson & Troy Tucker (2006) and Aughenbaugh & Paredis (2007). PBA combines probability distributions with interval calculus to determine the boundaries of variations in system's output and its application to modelling both aleatory and epistemic uncertainties. Hall (2006) examined the individual or combined contribution of inputs to the output's variance by using sensitivity indices. Furthermore, Oberguggenberger, King, & Schmelzer (2009) explored a case-study from aerospace engineering and examined various methods of sensitivity analysis based on probabilities, fuzzy sets, intervals and sensitivity indices.

The FDIIM for the analysis of strategic risks in GPNs and an illustrative example from the pump industry have been introduced in Niknejad & Petrovic, 2016 and Niknejad et al., 2016. In this paper, we will investigate different measures of uncertainty which is modelled by fuzzy numbers. We will apply an appropriate uncertainty measure to analyse the sensitivity of the proposed GPN risk model to different parameter values and their uncertainty, and its effect on the uncertainty of the results obtained. We will focus on epistemic uncertainties, modelled using fuzzy numbers, and analyse how to efficiently reduce the impact of uncertainties. A generic GPN will be used as an illustrative example.

3 UNCERTAINTY MANAGEMENT USING FUZZY ARITHMETIC

Fuzzy set theory is an extension of the classic set theory that allows for partial membership of elements to a set (Zadeh, 1965). While in a classical set, an element either belongs to the set or not, with membership 1 or 0, respectively, in the case of fuzzy sets, it is possible to have a degree of membership. Fuzzy set theory is suitable for modelling epistemic uncertainties, where the lack of knowledge about the subject leads to uncertainty. Fuzzy sets have been commonly used to capture experts' knowledge using linguistic values where the level of human knowledge is explicitly quantified.

In this section, basic concepts of fuzzy numbers and arithmetic are introduced. Three uncertainty measures including fuzziness, specificity and ambiguity are examined for the purpose of evaluating the level of uncertainty in values, arguing that the ambiguity measure is suitable for measuring uncertainty in GPN parameters and output values. Furthermore, the cyclical process of uncertainty management is introduced, which uses fuzzy arithmetic and uncertainty measurement to manage and control the epistemic uncertainties in decision making.

3.1 FUZZY ARITHMETIC

Fuzzy numbers are used to represent uncertain quantities. They map a real domain to the interval [0, 1] of membership degrees. Their membership function is required to be non-decreasing from 0 until reaching its maximum value 1, and then, non-decreasing until it reaches zero again. Very often, real-world problems deal with uncertain data specified to be *around* a certain value, *close* to a certain value or *approximately equal* to a certain value. A triangular fuzzy number (TFN) is one of the most used and simple forms of fuzzy numbers which can be used to represent the above-mentioned linguistic terms. It consists of a 3-tuple $\tilde{X} = (X_1, X_2, X_3)$. The 3-tuple includes the modal value of the number - X_2, which corresponds to a real number that is the most likely value to belong to the fuzzy number, the lowest possible value - X_1 and a highest possible value - X_3, where $X_1 \leq X_2 \leq X_3$. The modal value X_2 has a membership degree of 1, while X_1 and X_3 have a membership degree of 0. The memberships of all other values are determined linearly. Alternatively, bell-shaped membership functions, such as Beta, Gaussian or PI membership functions can be used as well (Cox, 1994).

One way to converts a fuzzy number into a crisp interval is to determine its α-cuts. It is the interval within the real domain consisting of elements of the fuzzy number with the membership degree higher or equal to α. Using α-cuts, it is possible to estimate fuzzy number calculations by carrying out interval calculations on its various α-cuts.

One method of representing membership function of fuzzy numbers is through LR functions (Delgado, Vila, & Voxman, 1998b). This representation uses two functions to define the left hand side $L_{\tilde{X}}(\alpha)$ and the right hand side $R_{\tilde{X}}(\alpha)$ of an α-cut of the fuzzy number, respectively. The numbers that have the membership value α on the left and the right side of the LR fuzzy number are determined. For a triangular fuzzy number $\widetilde{X} = (X_1, X_2, X_3)$, functions $L_{\tilde{X}}$ and $R_{\tilde{X}}$ are as follows:

$$L_{\tilde{X}}(\alpha) = \alpha X_2 + (1 - \alpha)X_1 \tag{1}$$

$$R_{\tilde{X}}(\alpha) = \alpha X_2 + (1 - \alpha)X_3 \tag{2}$$

In the proposed model, input parameters of the FDIIM are modelled using triangular fuzzy numbers, while the generated output is in the form of fuzzy LR-numbers. However, the model can operate with generic fuzzy input parameters in the LR form.

3.2 UNCERTAINTY MEASURES

A number of measures have been proposed to measure uncertainty in fuzzy sets and fuzzy numbers, such as fuzziness, specificity and ambiguity.

Fuzziness signifies the difference between the membership function of a fuzzy set and its complement set (Delgado, Vila, & Voxman, 1998a). The further that these two sets are, the main fuzzy set is considered to be less uncertain. For crisp sets, as the membership degree can be either 0 or 1, the difference is always 1, which represents the minimum fuzziness, while for fuzzy sets it is higher. The following formula can be used for measuring the fuzziness of a fuzzy number:

$$F(X) = \int_0^{\frac{1}{2}} [R_X(\alpha) - L_X(\alpha)]d\alpha + \int_{\frac{1}{2}}^1 [L_X(\alpha) - R_X(\alpha)]d\alpha \tag{3}$$

where $F(X)$ is the fuzziness of fuzzy number X, and, $R_X(\alpha)$ and $L_X(\alpha)$ are the right-side and left-side of the α-cut of X, respectively.

Specificity is a width-based measure of uncertainty that is concerned with the width of the α-cuts of a fuzzy set compared with the support of the fuzzy set (Yager, 2008), where the support represents the interval of all elements with the membership degree higher than 0. It equally considers all degrees of membership and aggregates the ratio of the width of the α-cut intervals and width of the fuzzy set support. It is worth noting that, unlike fuzziness, specificity has an inverse relationship with the uncertainty of the fuzzy set; the more uncertainty in the fuzzy set, the smaller its specificity. We can use the following formula for the measure of specificity:

$$SP(X) = 1 - \frac{1}{R_X(0) - L_X(0)} \int_0^1 [R_X(\alpha) - L_X(\alpha)]d\alpha \tag{4}$$

where $SP(X)$ is the specificity of fuzzy number X.

However, our focus will be on the ambiguity measure. The ambiguity measure is concerned with the divergence of the number from its modal value (Delgado, Vila, & Voxman, 1998b). It is similar to the specificity measure as it takes the α-cut intervals into account; however, it puts more emphasis on the α-cut intervals with the higher α values. Also, its value is absolute and is not normalised with respect to the support of fuzzy set, unlike the specificity measure. The ambiguity measure is formulated as follows:

$$A(X) = \int_0^1 \alpha[R_X(\alpha) - L_X(\alpha)]d\alpha \tag{5}$$

where $A(X)$ is the ambiguity of fuzzy number X. Formulas for calculating fuzziness, specificity and ambiguity of triangular fuzzy numbers are given in Appendix 1.

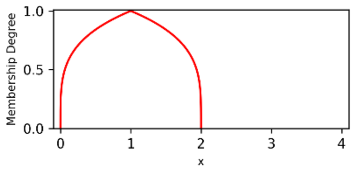
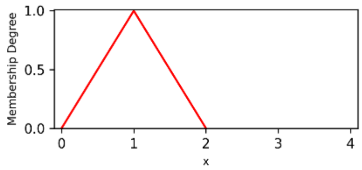
For general LR fuzzy numbers, we approximate the ambiguity by considering a discrete set of α-cuts. The integration can be approximated using the trapezoidal integration rule on $N + 1$ equally spaced α-cuts, as follows:

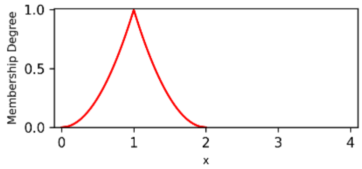
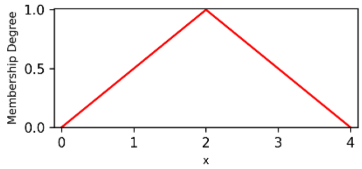
$$A(X) \cong \frac{1}{2N} \sum_{k=0}^{N-1} \left(\frac{k}{N} \left[R_X\left(\frac{k}{N}\right) - L_X\left(\frac{k}{N}\right) \right] + \frac{k+1}{N} \left[R_X\left(\frac{k+1}{N}\right) - L_X\left(\frac{k+1}{N}\right) \right] \right) \tag{6}$$

Similar discretisation can be done for the fuzziness and specificity measures.

In Table 1, a few examples of membership functions with the corresponding ambiguity, fuzziness and specificity measures are shown.

Table 1. Ambiguity, fuzziness and specificity measures of LR fuzzy numbers

#	L_X	R_X	Ambiguity A	Fuzziness F	Specificity SP	Diagram
1	α^4	$2-\alpha^4$	0.67	0.38	0.20	
2	α	$2-\alpha$	0.33	0.50	0.50	

#	L_X	R_X	Ambiguity A	Fuzziness F	Specificity SP	Diagram
3	$\sqrt{\alpha}$	$2-\sqrt{\alpha}$	0.20	0.39	0.67	
4	2α	$4-2\alpha$	0.67	1.00	0.50	

Examining Table 1, one can notice that the fuzziness measure, unlike ambiguity and specificity measures, does not differentiate between the first and third examples. Although, we consider the first example to be more uncertain than the third example, as it contains more uncertainty around the modal value. On the other hand, specificity measure is indifferent to the support of the fuzzy number, while ambiguity and fuzziness increase as the support expands, which is evident from the second and forth examples. We selected the ambiguity measure, as it is both sensitive to the uncertainty around the modal value, and also takes into account the size of fuzzy number support.

3.3 Uncertainty Management Process

Epistemic uncertainty is an inherent part of the real-world information and cannot be eliminated completely. This uncertainty affects the analysis we carry out using the information and, therefore, decisions that we make based on the analysis. Hence, it is important to understand and track the uncertainty which enables us to measure reliability of the results and whether they are conclusive enough to be used in decision making.

If the uncertainty is not tracked explicitly, it should be either ignored or eliminated pre-emptively. In the former case, we are making decisions based on unreliable data that can prove to be wrong and, as a result, damage our position. On the other hand, in the latter case, we have to aim to reduce uncertainty as much as is possible. This could prove to be costly and time consuming, and, if the uncertainty is not significant enough to impact the results, unnecessary. In this paper, we suggest an uncertainty management cycle that explicitly tracks the level of uncertainty in the parameters and results, and, use this information to make reliable decision without unnecessary data collection and refinement. This process is illustrated in Figure *1*.

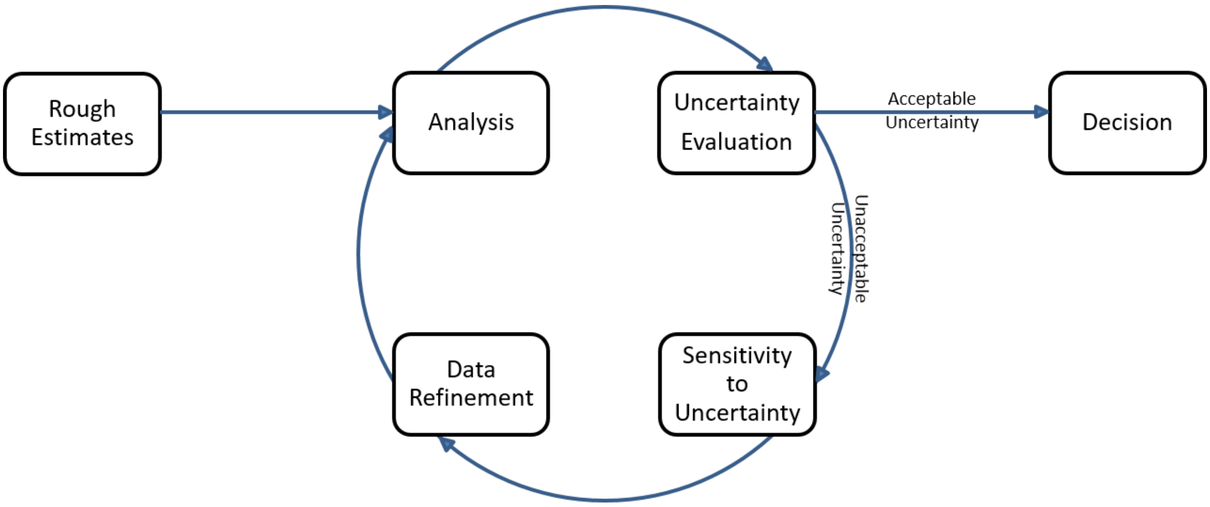


Figure 1. The framework for GPN uncertainty management

The uncertainty management process starts with rough estimates of the parameters which may carry high uncertainty levels. This information is based on what is readily available, such as experts' opinions or even pre-defined default values for parameters with high uncertainty. Such uncertain values can be immediately used to complete a first iteration of analysis.

Once the initial analysis is carried out, it is possible to evaluate and measure the uncertainty of results to determine if the uncertainty allows a conclusive and reliable decision to be made. If the uncertainty is not of an acceptable level, it is possible to utilise the results to extract information on where the uncertainty has a big impact. This is done in the step Sensitivity to Uncertainty, where the impact of the uncertainty of individual parameters on the uncertainty of results is estimated. This step will be examined in more details in Section 7.

Once the sensitivities to uncertainty are determined, the most influential parameters on the uncertainty in the results can be targeted for further data collection and refinement. In this way, resources can be focussed on parameters that are the most influential on uncertainty, while simultaneously avoiding making erroneous decisions based on unreliable analysis. Once this step is finished, the analysis can be repeated with the updated parameters. If the uncertainty is again unacceptable, this cycle will be repeated as necessary until a point where a reliable decision can be made based on the analysis with an acceptable level of uncertainty is reached.

4 FUZZY DYNAMIC INOPERABILITY INPUT/OUTPUT MODEL – FDIIM FOR GPNS

In order to analyse the propagation of risk within a GPN, a FDIIM is developed (Niknejad & Petrovic, 2016). The method is based on the assumption that the impact of the risk on a GPN's node can be measured by an inoperability value that determines the percentage of deviation of the node from its intended level of operation. The FDIIM considers external perturbations, resilience of the nodes and the interdependencies between them. Very often in practice, there are no historical data which can be used to determine the parameter values precisely. However, these parameters can be specified based on managerial subjective experience using imprecise linguistic terms. We can then translate these linguistic values into the corresponding fuzzy numbers.

A perturbation refers to a disruption that initially disturbs a GPN node and leads to the propagation of that disruption within the GPN. The impact of perturbation is the level of its influence on the GPN node, which is quantified as a fuzzy number for each time period of the disruption. It is defined on the interval [0,1], where 0 represents no impact of perturbation on the node and 1 represents a total disruption of the node. Resilience of GPN nodes refers to their speed of reaction to disruptions and how quickly they can recover from such events. It is modelled as a fuzzy number defined on the interval [0,1], where 0 models the slowest and 1 the fastest recovery. Furthermore, an interdependency between two nodes, namely a supporting and a dependency nodes is concerned with the strength of the relationship between these two nodes, and, how much the dependent node would be affected by an inoperability of the supporting node. Dependency between the two nodes depends on several factors, such as trade volume between the nodes, substitutability of the product supplied, lead time, distance, collaboration agreement, etc. It is defined using linguistic terms, such as *low*, *medium*, *high*, *fairly high* etc. and modelled using triangular fuzzy numbers defined on the interval [0,1].

Inoperability of nodes is calculated as follows:

$$\widetilde{\boldsymbol{q}}(\mathbf{t}+\mathbf{1}) = \widetilde{\boldsymbol{K}}\widetilde{\boldsymbol{A}^{*}}\widetilde{\boldsymbol{q}}(\boldsymbol{t}) + \widetilde{\boldsymbol{K}}\widetilde{\boldsymbol{c}^{*}}(\boldsymbol{t}) + \left(\boldsymbol{I}-\widetilde{\boldsymbol{K}}\right)\widetilde{\boldsymbol{q}}(\boldsymbol{t}) \tag{7}$$

where $\widetilde{\boldsymbol{q}}(\mathbf{t})$ is the fuzzy inoperability vector of all the GPN nodes at time period $\boldsymbol{t}$, $\widetilde{\boldsymbol{K}}$ is the diagonal matrix of resilience of all the nodes, $\widetilde{\boldsymbol{A}^{*}}$ is the interdependency matrix of the nodes, $\widetilde{\boldsymbol{c}^{*}}(\boldsymbol{t})$ is the external perturbation impact vector in time period $\boldsymbol{t}$, and $\boldsymbol{I}$ is the identity matrix. The fuzzy operations on these triangular fuzzy numbers are defined in Appendix 1.

The next step is to calculate a financial loss of risk that determines a financial impact of the inoperability of each node and on the whole network. The financial loss of risk is calculated as the product of inoperability and the intended revenue expected to be made by the nodes in the network. In our analysis, the intended revenue is based on the focal firm's perspective and, therefore, only nodes that have a value-added for the focal firm would be assigned an intended revenue.

The financial loss of risk is formulated as follows:

$$\widetilde{\boldsymbol{Q}} = \widetilde{\boldsymbol{x}^{\boldsymbol{T}}} \sum_{\boldsymbol{t}=\boldsymbol{1}}^{\boldsymbol{T}} \widetilde{\boldsymbol{q}}(\boldsymbol{t}) \tag{8}$$

where $\widetilde{\boldsymbol{Q}}$ denotes the fuzzy financial loss of risk of the network, $\widetilde{\boldsymbol{x}^{\boldsymbol{T}}}$ is the transposed vector of intended revenues of the nodes, $\boldsymbol{T}$ is the number of time periods in the time horizon and $\widetilde{\boldsymbol{q}}(\boldsymbol{t})$ is the vector of fuzzy inoperabilities of nodes in time period $\boldsymbol{t}$.

5 ILLUSTRATIVE EXAMPLE

We consider a generic illustrative example of a GPN for the analysis hereafter. The structure, parameters and disruption scenarios used in this example are described in the following sections.

5.1 GPN STRUCTURE

The network consists of two suppliers, Supplier A and Supplier B, a Production Facility and a node representing Customers. These nodes are interconnected as represented in Figure 2. The numbers on the arrows show the modal of the fuzzy interdependency values between the nodes.

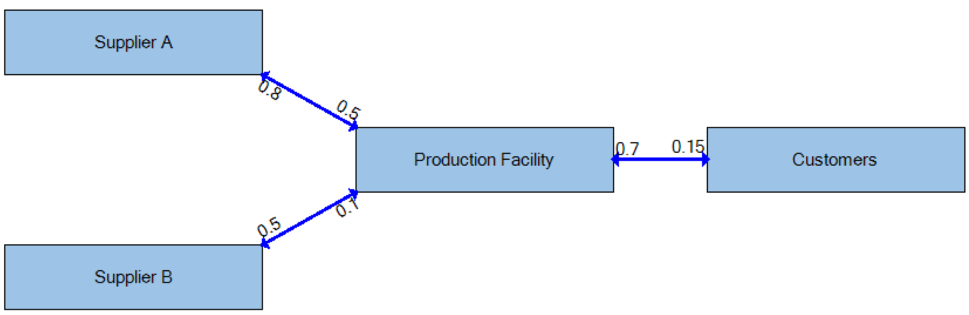


Figure 2. The structure of the GPN

Fuzzy interdependency values are shown in Table 2. These values represent the rate at which the operation at the dependent node will be affected by inoperability of the supporting node. The links can be bi-directional, as these dependencies could be caused by both the flow of materials, and, financial and order flows. For example, Production Facility can be dependent on Supplier A for delivering raw materials needed in the production process. However, Supplier A can also be dependent on Production Facility if the Production Facility is the main customer of Supplier A responsible for most of its income.

Additionally, nodes are assigned fuzzy resilience and intended revenue, as discussed in the previous section. These values are presented in
Table *3*.

Table 2. List of interdependencies between the nodes in the considered GPN

Supporting	Dependent	Interdependency
Production Facility	Supplier A	(0.6, **0.8**, 1)
Production Facility	Supplier B	(0.35, **0.5**, 0.65)
Supplier A	Production Facility	(0.4, **0.5**, 0.6)
Supplier B	Production Facility	(0.05, **0.1**, 0.15)
Customers	Production Facility	(0.5, **0.7**, 0.9)
Production Facility	Customers	(0.1, **0.15**, 0.2)

Table 3. Resilience and intended revenue of nodes per time period in the considered GPN

Name	Resilience	Intended Revenue
Supplier A	(0.7, **0.8**, 0.9)	(0, **0**, 0)
Supplier B	(1, **1**, 1)	(0, **0**, 0)
Production Facility	(0.7, **0.8**, 0.9)	(1,000, **1,250**, 1,500)
Customers	(0.5, **0.6**, 0.7)	(0, **0**, 0)

5.2 SCENARIOS OF DISRUPTION

Two scenarios of disruption in the GPN defined in Section 5.1 are considered: Scenario 1, where there is a disruption in Supplier A, and, Scenario 2, where there is a disruption in Production Facility. In Scenario 1, the network is affected in the upstream of the GPN, while, in Scenario 2 the centrally located production facility is impacted. In both scenarios, it is assumed that the node is affected by a single constant perturbation for 10 time periods with an impact of (0.5, **0.6**, 0.7). This effectively means

$$\widetilde{c_x^*}(t) = \begin{cases} (0.5, \mathbf{0.6}, 0.7) & if\ 1 \le t < 10 \\ (0, \mathbf{0}, 0) & otherwise \end{cases} \tag{9}$$

where $\widetilde{c_x^*}(t)$ is the perturbation impact of the node x at time period t, where node x is Supplier A in Scenario 1 and Production Facility in Scenario 2.

The results obtained by applying the FDIIM are provided in Table 4 and Table 5.

Table 4. Loss of risk in the two risk scenarios

Risk Scenario	Loss of risk
Scenario 1: Disruption in Supplier A	(£2,340, **£7,535**, £31,098)
Scenario 2: Disruption in Production Facility	(£6,072, **£13,856**, £39,079)

Table 5. Average inoperability of the nodes in the two scenarios

Node	Average Inoperability of time periods in Scenario 1	Average Inoperability of time periods in Scenario 2
Supplier A	(0.114, **0.204**, 0.497)	(0.070, **0.177**, 0.523)
Supplier B	(0.016, **0.060**, 0.268)	(0.043, **0.111**, 0.337)
Production Facility	(0.047, **0.121**, 0.415)	(0.121, **0.222**, 0.521)
Customers	(0.004, **0.018**, 0.083)	(0.011, **0.033**, 0.105)

Table 4 provides the financial loss during 50 time periods in the two risk scenarios, approximated and represented by triangular fuzzy numbers. It is clear that the disruption in production facility has a much higher overall impact on the company than the disruption in supplier A. Additionally, Table 5 shows the average value of inoperability of time periods of each of the individual nodes in the network over the time horizon of 50 time periods in the risk scenarios. It can be observed from Scenario 2 that, due to the higher interdependency of Supplier A on Production Facility (0.8) than that of Supplier B (0.5), the impact is higher on Supplier A. Also, as the Production Facility is directly affected in Scenario 2, both the Production Facility and Customers are affected considerably more than they are in Scenario 1.

6 SENSITIVITY TO PARAMETERS

In this section, we analyse the sensitivity of the output (financial loss of risk) to the individual GPN uncertain parameters values. The values of input parameters are varying by -50%, -10%, -5%, 5%, 10% and 50%. As the input parameters' values are described by triangular membership functions, varying their values implies varying the corresponding values of the triangular membership function, i.e., shifting the membership function along the x-axes to the left or right, through multiplication by a suitable multiplier, when possible parameter values are decreased or increased, respectively, as illustrated in Figure 3.

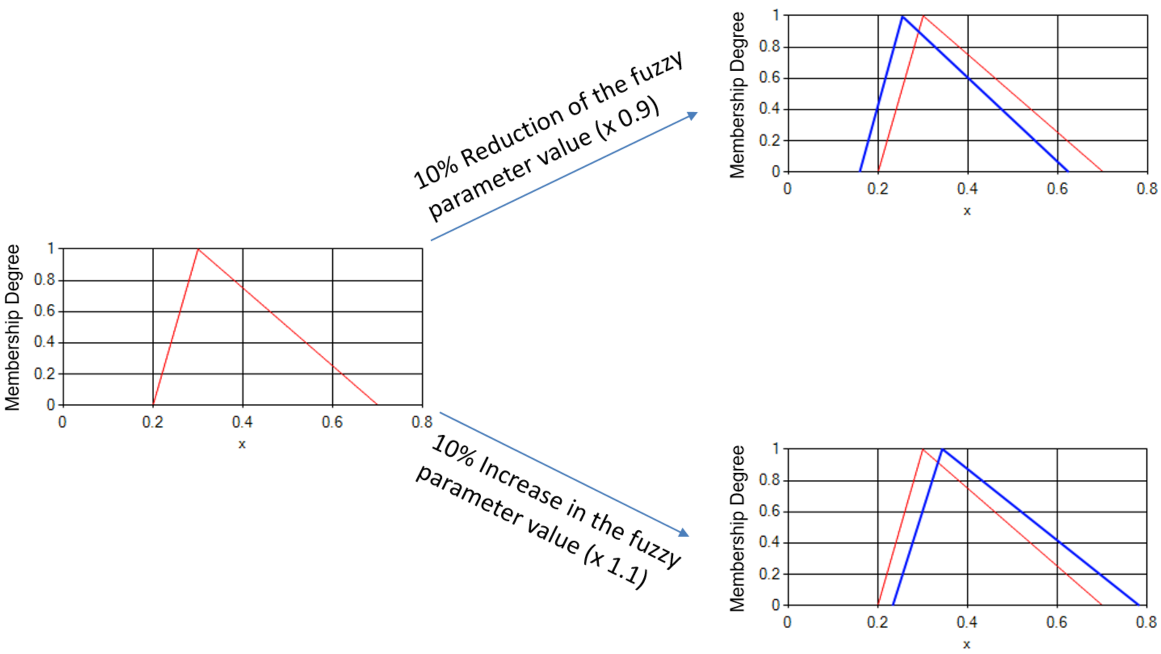


Figure 3. Example showing 10% reduction and 10% increase in the fuzzy parameter value

The objective is to understand how much the financial loss of risk – output would change as a result of a change in a GPN parameter, and also, how much the ambiguity of the output is changed as a result of this change. The results are analysed in the two scenarios defined previously.
It is worth noticing that by varying the parameters' values, the ambiguities of the parameters' values are also changed by the same factor, as proved in Appendix 2. This side-effect should be noted when interpreting the results reported in this section.
It is also worth mentioning that for some of the parameters, including interdependencies $\widetilde{\boldsymbol{A}}^{*}$, resilience $\widetilde{\boldsymbol{K}}$ and perturbation impacts $\widetilde{\boldsymbol{c}}^{*}$, their maximum value of 1 is considered, which means that if the increase leads to numbers higher than 1, they will be constrained to value 1.

6.1 SCENARIO 1: DISRUPTION IN SUPPLIER A

In this scenario, a disruption in supplier A, modelled as a fuzzy perturbation impact of (0.5, **0.6**, 0.7) for 10 time periods, is considered. Table 6 shows the result of changing individual GPN parameters by -50%, -10%, -5%, 5%, 10% and 50%. Two numbers are reported for each change: M - the percentage of change in the modal value of the loss of risk, and, A - the percentage of change in the ambiguity of the loss of risk.

Table 6. Scenario 1. Sensitivity of loss of risk to changes of -50%, -10%, -5%, 5%, 10% and 50% in the GPN parameters

	-50 %		-10 %		-5 %		5 %		10 %		50 %	
Parameter	M	A	M	A	M	A	M	A	M	A	M	A
Dependency of Production Facility on Supplier A	-31 %	-44%	-8 %	-12%	-4 %	-6%	3 %	2%	7 %	4%	19 %	9%
Dependency of Production Facility on Supplier B	-5 %	-13%	-1 %	-3%	-1 %	-1%	1 %	2%	1 %	3%	5 %	17%
Dependency of Supplier A on Production Facility	-65 %	-72%	-17 %	-21%	-8 %	-11%	9 %	13%	18 %	28%	122 %	283%
Dependency of Supplier B on Production Facility	-5 %	-13%	-1 %	-3%	-1 %	-1%	1 %	2%	1 %	3%	5 %	17%
Dependency of Customers on Production Facility	-10 %	-24%	-2 %	-6%	-1 %	-3%	1 %	3%	2 %	6%	10 %	15%
Dependency of Production Facility on Customers	-10 %	-24%	-2 %	-6%	-1 %	-3%	1 %	3%	2 %	6%	12 %	39%
Resilience of Supplier A	1 %	18%	0 %	2%	0 %	1%	0 %	-1%	0 %	-2%	-1 %	-8%
Resilience of Supplier B	0 %	1%	0 %	0%	0 %	0%	0 %	0%	0 %	0%	0 %	0%
Resilience of Production Facility	0 %	16%	0 %	3%	0 %	1%	0 %	-1%	0 %	-2%	-1 %	-9%
Resilience of Customers	0 %	2%	0 %	0%	0 %	0%	0 %	0%	0 %	0%	0 %	-2%
Intended revenue of Production Facility	-50 %	-50%	-10 %	-10%	-5 %	-5%	5 %	5%	10 %	10%	50 %	50%
Impact of Disruption in Supplier A	-50 %	-35%	-9 %	1%	-4 %	1%	3 %	-1%	5 %	-2%	13 %	-7%

As it is evident from Table 6, the loss of risk in Scenario 1 is sensitive to most of the parameters, albeit to different degrees. The ambiguity has a tendency to change in line with the change in the modal value. It can be observed from Table 6 that, for most reported changes in the GPN parameters, the direction and the range of change in the loss of risk and its ambiguity are similar. A mathematical explanation of this behaviour is provided in Appendix 2 where it is proved that the multiplication of a fuzzy number by a certain factor causes the corresponding change in the ambiguity of the multiplied number.

Sensitivity to changes in interdependency values is generally high and it is non-linear. Changes in interdependencies have a direct relationship with the loss of risk, i.e., decreases (increases) in interdependencies cause decreases (increases) in the loss of risk. As an example, the result shows high sensitivity to the Dependency of Supplier A on Production Facility, ranging from 65% reduction in the modal value of the loss of risk and 72% reduction in its ambiguity due to a 50% reduction in the parameter, to the increase of 122% in the modal value while ambiguity is increased by 283% as a result of 50% increase in the parameter. This is due to the high initial dependency between Supplier A and Production Facility (modal value of 0.8) and the fact that Supplier A is directly affected by the disruption.

With regard to the changes in the resilience parameters, the sensitivity of the loss of risk is generally lower, non-linear and have an inverse relationship with the loss of risk as expected; the higher the resilience, the lower the loss of risk. The effect on the ambiguity seems to be higher than the effect on the modal value. Lower sensitivity can be explained by the fact that changes in the resilience parameters can either delay or fasten the impact of a disruption, but, it does not change the overall impact considerably.

As discussed previously, the intended revenue is considered for the Production Facility only, as we are assuming that other nodes do not bring a direct revenue to the focal firm. Using the formula for calculating the loss of risk, there is a linear and direct relationship between the loss of risk and the intended revenue of the Production Facility.

Finally, the disruption in supplier A directly, but non-linearly, affects the loss of risk. Interestingly, the loss of risk is more sensitive to decreases in this disruption than to its increases. This can be explained by the high initial disruption on Supplier A. A minor increase in the disruption leads to total inoperability of the GPN for the duration of the disruption, and, hence any more increase will not have any extra impact. Also, one can observe the reduction of ambiguity with a 50% increase in the impact of disruption in Supplier A, i.e., when there is 50% increase in the impact modal value. In this case, a 50% increase in the disruption leads to total inoperability of the GPN. As the inoperability is limited to 1, the distance between the upper bound and lower bound of the fuzzy number which represents the loss of risk decreases. As a result, the ambiguity of the loss of risk is reduced.

6.2 Scenario 2: Disruption in production facility

The sensitivity of loss of risk in the GPN when there is a disruption to the Production Facility is considered in this scenario. The disruption to the production facility is modelled by a fuzzy perturbation impact of (0.5, **0.6**, 0.7) for 10 time periods to this facility. The results are reported in Table 7.

Table 7. Scenario 2. Sensitivity of the loss of risk to changes of -50%, -10%, -5%, 5%, 10% and 50% in the GPN parameters

	-50 %		-10 %		-5 %		5 %		10 %		50 %	
Parameter	M	A	M	A	M	A	M	A	M	A	M	A
Dependency of Production Facility on Supplier A	-24 %	-36%	-5 %	-12%	-2 %	-7%	3 %	2%	6 %	4%	16 %	8%
Dependency of Production Facility on Supplier B	-3 %	-10%	-1 %	-2%	0 %	-1%	0 %	1%	1 %	2%	3 %	14%
Dependency of Supplier A on Production Facility	-24 %	-36%	-5 %	-12%	-2 %	-7%	3 %	9%	6 %	19%	43 %	210%
Dependency of Supplier B on Production Facility	-3 %	-10%	-1 %	-2%	0 %	-1%	0 %	1%	1 %	2%	3 %	14%
Dependency of Customers on Production Facility	-6 %	-18%	-1 %	-5%	-1 %	-2%	1 %	3%	1 %	5%	7 %	11%
Dependency of Production Facility on Customers	-6 %	-18%	-1 %	-5%	-1 %	-2%	1 %	3%	1 %	5%	8 %	33%
Resilience of Supplier A	6 %	23%	1 %	3%	0 %	1%	0 %	-1%	-1 %	-2%	-1 %	-7%
Resilience of Supplier B	1 %	3%	0 %	0%	0 %	0%	0 %	0%	0 %	0%	0 %	0%
Resilience of Production Facility	9 %	39%	1 %	4%	1 %	2%	0 %	-2%	-1 %	-4%	-2 %	-13%
Resilience of Customers	2 %	8%	0 %	1%	0 %	0%	0 %	0%	0 %	-1%	-1 %	-3%
Intended revenue of Production Facility	-50 %	-50%	-10 %	-10%	-5 %	-5%	5 %	5%	10 %	10%	50 %	50%
Impact of Disruption in Production Facility	-45 %	-8%	-4 %	6%	-2 %	3%	2 %	-2%	3 %	-4%	8 %	-11%

The changes in the loss of risk presented in Table 7 for Scenario 2 generally follow the same trend as the changes presented in Table 6 for Scenario 1. However, the sensitivity to the changes in the dependency parameters seems to be generally lower than in Scenario 1, while the sensitivity to the changes in the resilience parameters is higher. This is mostly due to the fact that the disruption is directly affecting the focal firm, and, the results are not as much sensitive to the nodes interdependencies, as it depends on the node resilience. However, as the GPN experiences higher inoperability values in Scenario 2 than Scenario 1, as shown in Table 7, the GPN is more reliant on the resilience to recover from the disruption.

7 SENSITIVITY TO UNCERTAINTY IN THE GPN PARAMETERS

In this experiment, we observe the ambiguity level of the financial loss of risk in the two risk scenarios. The objective is to understand how much each of the GPN parameters in the model contributes to the ambiguity of the output – the financial loss of risk. In order to measure this, we reduce the ambiguity of the input GPN parameters by various degrees, 100%, 50% and 10%, and measure the ambiguity of the financial loss of risk achieved. It is compared with the original ambiguity level of the financial loss of risk to understand the impact that the ambiguity of the parameter has on the results. Similar to the previous experiments, in these experiments, parameters are considered one at a time.

In order to reduce the ambiguity of the parameter value, the upper and the lower bounds of the corresponding triangular membership function (X_1, X_2, X_3) are moved closer to the modal value, by the specified degree, i.e.

$$(Y_1, Y_2, Y_3) = ((1-\lambda)X_1 + \lambda X_2, X_2, (1-\lambda)X_3 + \lambda X_2). \tag{10}$$

For $\lambda = 1$ (reduction of ambiguity of 100%), (Y_1, Y_2, Y_3) becomes a crisp number: (X_2, X_2, X_2).
For $\lambda = 0.5$ (reduction of ambiguity of 50%), (Y_1, Y_2, Y_3) becomes $\left(\frac{1}{2}X_1 + \frac{1}{2}X_2, X_2, \frac{1}{2}X_2 + \frac{1}{2}X_3\right)$.
For $\lambda = 0.1$ (reduction of ambiguity of 10%), (Y_1, Y_2, Y_3) becomes $\left(\frac{9}{10}X_1 + \frac{1}{10}X_2, X_2, \frac{1}{10}X_2 + \frac{9}{10}X_3\right)$.

This is illustrated in Figure 4.

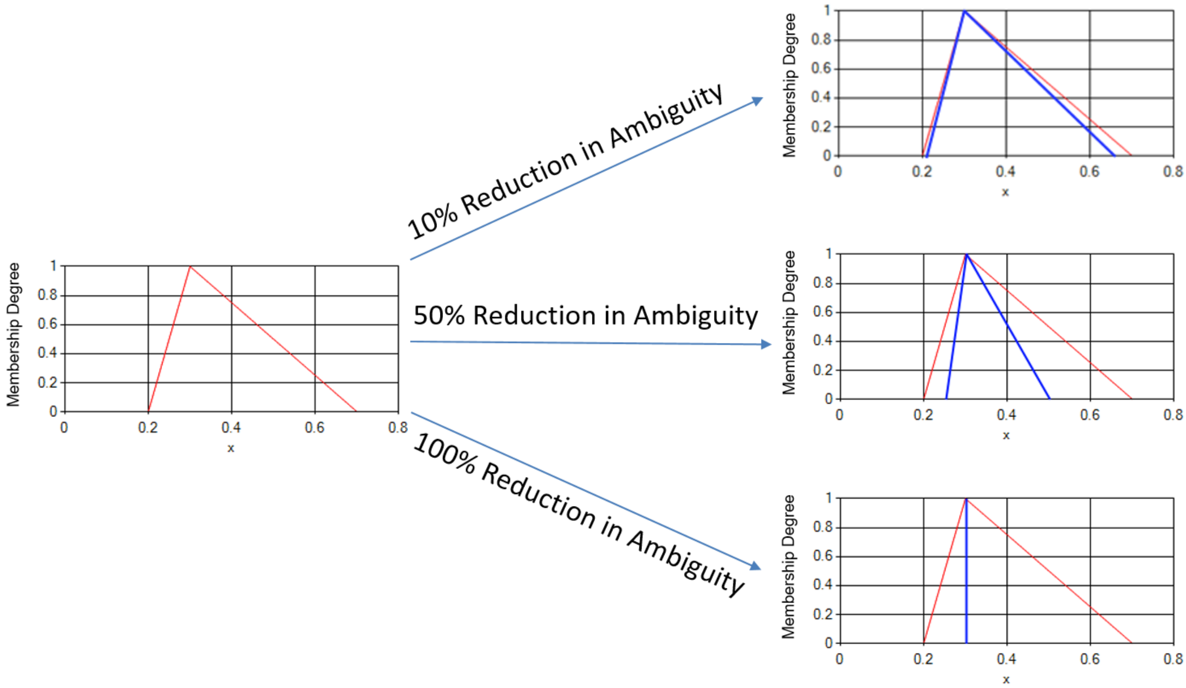


Figure 4. Example presenting the changes in ambiguity of a triangular fuzzy number by 100%, 50% and 10%

The proof that these changes in a triangular fuzzy number lead to the certain changes in the ambiguity is given in Appendix 3.

7.1 Scenario 1: Disruption in supplier A

In Scenario 1, the initial total ambiguity level of the financial loss of risk for 10 time periods under consideration is 3081. Table 8 presents the changes in the initial total ambiguity of loss of risk caused by reduction of ambiguity of the GPN parameters by 100%, 50% and 10%. The parameters are sorted in the ascending order in the case of reducing the parameter ambiguity by 100%.

Table 8. Scenario 1. Sensitivity of ambiguity of the loss of risk to changes in ambiguity of the GPN parameters

Parameter	-100%	-50%	-10%
Dependency of Supplier A on Production Facility	-33%	-17%	-4%
Dependency of Production Facility on Supplier A	-20%	-11%	-2%
Intended revenue of Production Facility	-18%	-9%	-2%
Dependency of Production Facility on Customers	-10%	-5%	-1%
Dependency of Customers on Production Facility	-9%	-5%	-1%

Parameter	-100%	-50%	-10%
Dependency of Supplier B on Production Facility	-7%	-4%	-1%
Impact of Disruption in Supplier A	-7%	-3%	-1%
Resilience of Production Facility	-5%	-3%	-1%
Dependency of Production Facility on Supplier B	-5%	-2%	0%
Resilience of Supplier A	-4%	-2%	0%
Resilience of Customers	-1%	-1%	0%
Resilience of Supplier B	0%	0%	0%

The four most influential parameters in Scenario 1 are identified to be Dependency of Supplier A on Production Facility, Dependency of Production Facility on Supplier A, Intended revenue of Production Facility and Dependency of Production Facility on Customers, respectively. One can see that the most influential parameters with respect to the impact of their uncertainty are those parameters whose values have the highest impact on the GPN financial loss of risk. The measure of ambiguity is used to quantify the impact of their uncertainty. Furthermore, the analysis shows that the uncertainties in these parameters are the most influential on the uncertainty in the loss of risk regardless of the level of reduction in their ambiguity. Also, the relationship between uncertainties in these parameters and uncertainty in the loss of risk seems to be nearly linear, e.g. on 100% reduction in ambiguity of Dependency of Supplier A on Production Facility, the 33% reduction in ambiguity of the loss of risk is recorded, while for 50% reduction, it is 17%, and, for 10% reduction it is 4%. Additionally, the model seems to be more sensitive to uncertainty in dependency parameters and intended revenue of the production facility than to the resilience parameters.

7.2 Scenario 2: Disruption in production facility

The initial total ambiguity level of financial loss of risk for 10 time periods under consideration in Scenario 2 is 3588. The result of sensitivity of the financial loss of risk in Scenario 2, in response to reduction of ambiguity in different GPN parameters, is reported in Table 9.

Table 9. Scenario 2. Sensitivity of ambiguity of the loss of risk to changes in ambiguity of the GPN parameters

Parameter	-100%	-50%	-10%
Intended revenue of Production Facility	-26%	-13%	-3%
Dependency of Production Facility on Supplier A	-25%	-13%	-3%
Dependency of Supplier A on Production Facility	-21%	-11%	-2%

Parameter	-100%	-50%	-10%
Dependency of Production Facility on Customers	-10%	-5%	-1%
Dependency of Customers on Production Facility	-9%	-5%	-1%
Impact of Disruption in Production Facility	-9%	-4%	-1%
Dependency of Supplier B on Production Facility	-7%	-4%	-1%
Resilience of Production Facility	-7%	-3%	-1%
Dependency of Production Facility on Supplier B	-4%	-2%	0%
Resilience of Supplier A	-3%	-1%	0%
Resilience of Customers	-1%	-1%	0%
Resilience of Supplier B	0%	0%	0%

The four most influential parameters on the ambiguity of the financial loss of risk in Scenario 2 are Intended revenue of Production Facility, Dependency of Production Facility on Supplier A, Dependency of Supplier A on Production Facility and Dependency of Production Facility on Customers. One can observe that ambiguities of the parameters related to the Production Facility are the most influential on the ambiguity of the loss of risk.

Also, ambiguity of the resilience parameters has a relatively small effect on the ambiguity of financial loss of risk, which is in line with the generally lower sensitivity of loss of risk to the resilience parameters observed in Section 6. Also, it can be concluded that the order of the most important parameters can change in different risk scenarios. Finally, changes in ambiguity of all parameters seem to have a linear (or near linear) relationship with the changes in the ambiguity of the loss of risk.

7.3 DOUBLE REDUCTIONS OF AMBIGUITY IN GPN PARAMETERS

In the previous sections, in each experiment we have reduced the ambiguity in one of the GPN parameters only. In this section, we will consider a simultaneous reduction of the ambiguities of two GPN parameters. The purpose of this experiment is to analyse if by reducing ambiguity of more than one parameter, we can achieve a higher reduction in the ambiguity of loss of risk, and how their combined influence is related to the loss of risk ambiguity. Table 10 and Table 11 provide the percentage of reductions in the ambiguity in loss of risk as a result of the 100% reduction in the ambiguity of two GPN parameters identified in the rows and columns, in Scenario 1 and Scenario 2, respectively. Only the five most influential parameters are included.

Table 10. Scenario 1. Reduction in the ambiguity of loss of risk when the ambiguity of two GPN parameters is reduced by 100% simultaneously

Parameters	Dependency of Supplier A on Production Facility	Dependency of Production Facility on Supplier A	Intended revenue of Production Facility	Dependency of Production Facility on Customers	Dependency of Customers on Production Facility
Dependency of Supplier A on Production Facility	-33%	-48%	-49%	-41%	-40%
Dependency of Production Facility on Supplier A		-20%	-36%	-28%	-27%
Intended revenue of Production Facility			-18%	-27%	-26%
Dependency of Production Facility on Customers				-10%	-17%
Dependency of Customers on Production Facility					-9%

Table 11. Scenario 2. Reduction in the ambiguity of loss of risk when the ambiguity of two GPN parameters is reduced by 100% simultaneously

Parameters	Intended revenue of Production Facility	Dependency of Production Facility on Supplier A	Dependency of Supplier A on Production Facility	Dependency of Production Facility on Customers	Dependency of Customers on Production Facility
Intended revenue of Production Facility	-26%	-50%	-45%	-35%	-34%
Dependency of Production Facility on Supplier A		-25%	-41%	-33%	-32%
Dependency of Supplier A on Production Facility			-21%	-29%	-28%
Dependency of Production Facility on Customers				-10%	-17%
Dependency of Customers on Production Facility					-9%

It can be seen that the reduction in the ambiguity of the loss of risk as a result of a reduction in the ambiguity of two parameters simultaneously provides slightly lower reduction in the ambiguity than when the ambiguity in the two parameters is reduced separately. This can be due to complex relationships between the GPN parameters and their impact on the loss of risk.

Interestingly, the best combination of parameters with respect to reducing the ambiguity of the loss of risk in Table 10 includes the first and the third most influential parameters Dependency of

Supplier A on Production Facility and Intended Revenue of Production Facility, respectively which led to 49% reduction in the ambiguity of the loss of risk. While the second parameter, Dependency of Production Facility on Supplier A, contributes to a higher individual reduction (20%) than the third parameter (18%), the first and the second most influential parameters provide only a 48% reduction in the ambiguity of the loss of risk.

Additionally, in Table 11, the individual contributions of the first and second most influential parameters, Intended revenue of Production Facility and Dependency of Production Facility on Supplier A, is -26% and -25%, respectively. The sum of these reductions is only slightly higher than the combined reduction of 50% . However, in Table 10, the combination of the first two most influential parameters provides 48% reduction, while the sum of their individual contributions to the reduction in ambiguity is much higher, 53% = 33% + 20%.

8 CONCLUSIONS

This paper proposes a new framework to support analyses of impact of parameters and their uncertainties on GPN's performance. It includes the FDIIM developed to model the risk propagation along a GPN and calculates a GPN performance as the financial loss of risk. The sensitivity of the FDIIM and financial loss of risk to the input parameters, including interdependencies, resilience, perturbation impact and intended revenues, and uncertainty in these input parameters is examined. In the absence of historical data and lack of precise method to obtain these data, they are specified based on managerial subjective judgement. Imprecise linguistic terms used are modelled by fuzzy numbers. Different measures of uncertainty including fuzziness, specificity and ambiguity are considered. Ambiguity is selected as an appropriate measure of uncertainty in fuzzy GPN parameters. Analyses are carried out considering a generic GPN structure with two suppliers, a production facility (belonging to the focal firm) and customer. Two disruption scenarios, affecting one of the suppliers and the production facility respectively, have been considered.

The analyses show that the model is sensitive to changes in input parameters, but sensitivity is not necessarily linear. The interdependencies among GPN nodes and the intended revenue are shown to have a higher influence on the value and ambiguity of loss of risk than resilience. Furthermore, the interdependencies and intended revenue have a direct influence and the resilience has an inverse influence on the value of loss of risk. Additionally, the ambiguity in the financial loss of risk is shown to be sensitive to the ambiguity in the input parameters. It is demonstrated how to rank the GPN parameters based on the impact that their ambiguity has on the ambiguity of the GPN financial loss of risk. The interdependency among some GPN's nodes and the intended revenue are identified to be among the most influential parameters. Furthermore, it is shown that, in the case when ambiguity of two parameters can be reduced, the selection of the two best parameters for ambiguity reduction does not necessarily include the two most influential parameters considered individually.

The uncertainty management framework proposed is of relevance to practical applications. The decision makers can carry out analysis to identify which parameters have the highest impact on GPN performance and analyse how much uncertainty in those parameters should be reduced in order to reduce the GPN's financial loss of risk. Furthermore, the analysis can be carried out to select parameters which ambiguity should be reduced in order to reduce the ambiguity of the obtained results in the most effective way.

One direction for future research is to develop a GPN optimisation model to minimise the cost in such a way as to reach a desirable level of financial loss of risk and its ambiguity, while considering the resources needed, including the cost of refinement of uncertain parameters and the cost of reducing interdependencies between GPN's nodes. The focus would be placed on those parameters which have the highest impact on financial loss of risk and its ambiguity.

APPENDIX 1. FUZZY OPERATIONS AND MEASURES OF UNCERTAINTY IN TRIANGULAR FUZZY NUMBERS

Fuzzy arithmetic defines necessary mathematical operations to allow calculations on fuzzy numbers. Fuzzy addition and scalar multiplication on triangular fuzzy numbers are determined as follows. Let $\tilde{X} = (X_1, X_2, X_3)$ and $\tilde{Y} = (Y_1, Y_2, Y_3)$ be triangular fuzzy numbers and λ be a non-negative scalar value. Then:

1. $\tilde{X} + \tilde{Y} = (X_1 + Y_1, X_2 + Y_2, X_3 + Y_3)$
2. $\lambda\tilde{X} = (\lambda X_1, \lambda X_2, \lambda X_3)$

Additionally, it is possible to approximate the fuzzy multiplication, assuming that $X_1, X_2, X_3, Y_1, Y_2, Y_3 \geq 0$, as follows (Chen, 1996):

$$\tilde{X} * \tilde{Y} = (X_1 Y_1, X_2 Y_2, X_3 Y_3)$$

Fuzziness (F), Specificity (SP) and Ambiguity (A) of triangular fuzzy number $\tilde{X} = (X_1, X_2, X_3)$ can be calculated as follows:

$$F(\tilde{X}) = \int_0^{\frac{1}{2}} [(\alpha X_2 + (1-\alpha)X_3) - (\alpha X_2 + (1-\alpha)X_1)]d\alpha$$

$$+ \int_{\frac{1}{2}}^{1} [(\alpha X_2 + (1-\alpha)X_1) - (\alpha X_2 + (1-\alpha)X_3)]d\alpha$$

$$= \int_0^{\frac{1}{2}} [(1-\alpha)X_3 - (1-\alpha)X_1]d\alpha + \int_{\frac{1}{2}}^{1} [(1-\alpha)X_1 - (1-\alpha)X_3]d\alpha = \frac{1}{4}(X_3 - X_1)$$

$$SP(\tilde{X}) = 1 - \frac{1}{X_3 - X_1}\int_0^1 [(\alpha X_2 + (1-\alpha)X_3) - (\alpha X_2 + (1-\alpha)X_1)]d\alpha$$

$$= 1 - \frac{1}{X_3 - X_1}\int_0^1 [(1-\alpha)X_3 - (1-\alpha)X_1]d\alpha = \frac{1}{2}$$

$$A(\tilde{X}) = \int_0^1 \alpha[(\alpha X_2 + (1-\alpha)X_3) - (\alpha X_2 + (1-\alpha)X_1)]d\alpha = \int_0^1 \alpha[(1-\alpha)X_3 - (1-\alpha)X_1]d\alpha$$

$$= [X_3 - X_1]\int_0^1 (\alpha - \alpha^2)d\alpha = [X_3 - X_1]\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}(X_3 - X_1)$$

APPENDIX 2. AMBIGUITY OF A TRIANGULAR FUZZY NUMBER MULTIPLIED BY A SCALAR

Assuming that the initial parameter is modelled by a triangular fuzzy number $\tilde{X} = (X_1, X_2, X_3)$ and its ambiguity identified by $A(\tilde{X})$, then:

$$A(\lambda\tilde{X}) = A(\lambda(X_1, X_2, X_3)) = A((\lambda X_1, \lambda X_2, \lambda X_3)) = \frac{1}{6}\lambda(X_3 - X_1) = \lambda A(\tilde{X})$$

Hence, multiplying the fuzzy parameter value by λ, the corresponding triangular fuzzy number is multiplied by λ, causing the ambiguity to be λ times higher.

APPENDIX 3. SENSITIVITY OF AMBIGUITY OF A PARAMETER MODELLED BY A TRIANGULAR FUZZY NUMBER

Let $\tilde{X} = (X_1, X_2, X_3)$ and its ambiguity be $A\big((X_1, X_2, X_3)\big)$. By moving its boundaries to (Y_1, Y_2, Y_3) = $((1-\lambda)X_1 + \lambda X_2, X_2, (1-\lambda)X_3 + \lambda X_2)$, the ambiguity of the fuzzy number (Y_1, Y_2, Y_3) is:

$$
\begin{aligned}
A\big((Y_1, Y_2, Y_3)\big) &= A\big(((1-\lambda)X_1 + \lambda X_2, X_2, (1-\lambda)X_3 + \lambda X_2)\big) \\
&= \frac{1}{6}((1-\lambda)X_3 + \lambda X_2 - (1-\lambda)X_1 - \lambda X_2) = \frac{1}{6}((1-\lambda)X_3 - (1-\lambda)X_1) \\
&= \frac{1}{6}(1-\lambda)(X_3 - X_1) = (1-\lambda)A\big((X_1, X_2, X_3)\big)
\end{aligned}
$$

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