THE BEHAVIOUR OF REINFORCED CONCRETE AS DEPICTED IN FINITE ELEMENT ANALYSIS.

THE CASE OF A TERRACE UNIT.

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1. INTRODUCTION.

- Aim to replicate the behaviour of reinforced concrete in a **multi-scale** numerical model by developing a mathematical representation of physical and mechanical parameters (material and geometric properties, actions, displacements, etc)

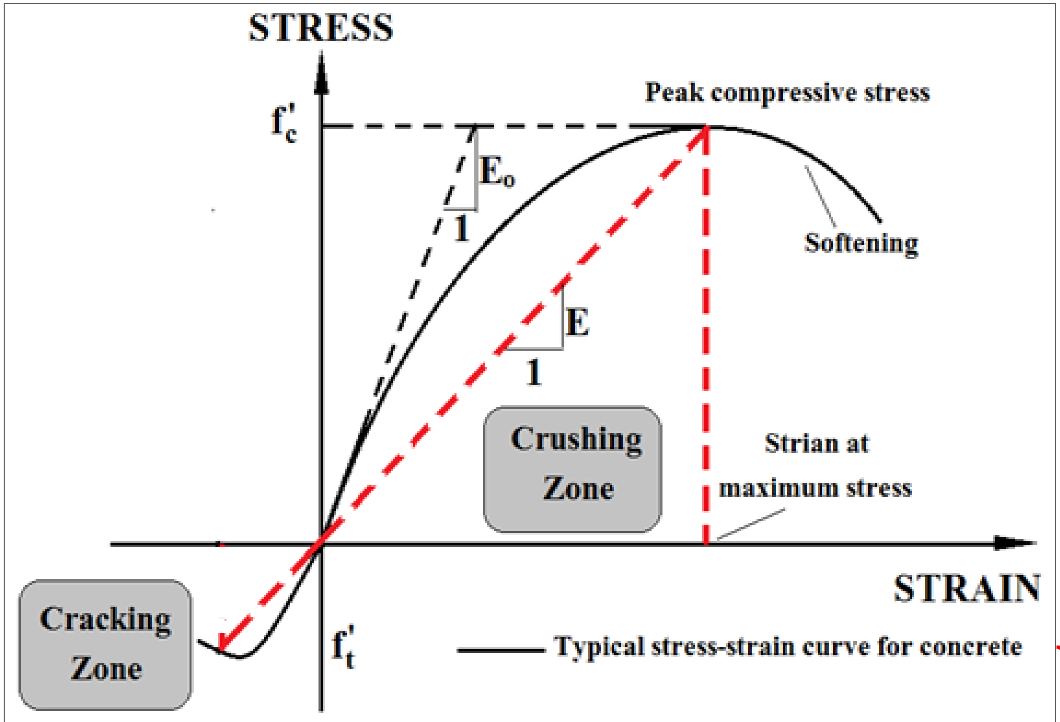
- Follow the way the latter affect structures, from the laboratory, to the computer.

Objectives:

- Review maths models representing RC, focusing on main areas: Plain concrete and steel reinforcement.
- Develop a flexible arrangement, capable of describing and predicting the structural performance of a specific study and later generalise.
- Incorporate ‘mechanism-of-action’, that is, make strategic observations at laboratory level, into computer models.

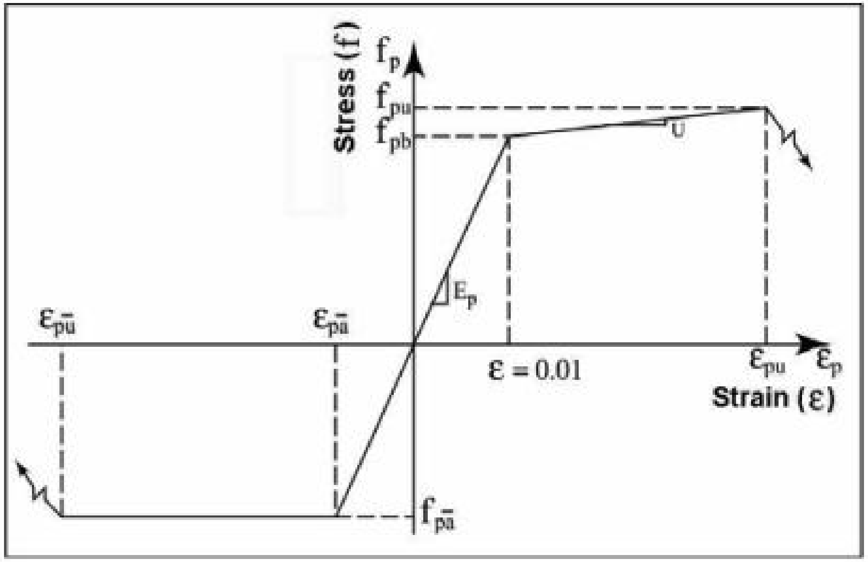
General Concrete Behaviour Under Load

- initial linear portion lasting 30% – 40% of ultimate load.
- after that curve becomes non-linear, with large strains registered for small increments of stress.
- non-linearity is credited to microcracks at the cement paste - aggregate interface.
- strain corresponding to ultimate stress is around 0.003 for normal strength concrete.



General Steel Behaviour Under Load

- <u>medium/high C steels</u> – no well defined yield point.
- hence, <u>0.2% proof stress</u> definition.
- <u>elastic region</u> – material will return to its original shape if load is removed.
- <u>plastic region</u> – some permanent deformation will remain, even if the load is removed.
- <u>failure point</u> - the body ruptures.



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2. Some Failure Criteria for Concrete

- Strength of concrete under multiaxial stresses conditions is a function of the state of stress. It cannot be predicted by considering T, C, V stresses independently of each other.

> <u>Example</u>: concrete with,
> f_c (uniaxial compressive strength) = 100 (units)
> v_c (shear strength) = $0.08f_c$ = 8
> would fail under comp. stress, $f_{fail} = 0.5f_c = 50$
> when shear stress increases to $0.2f_c = 20$

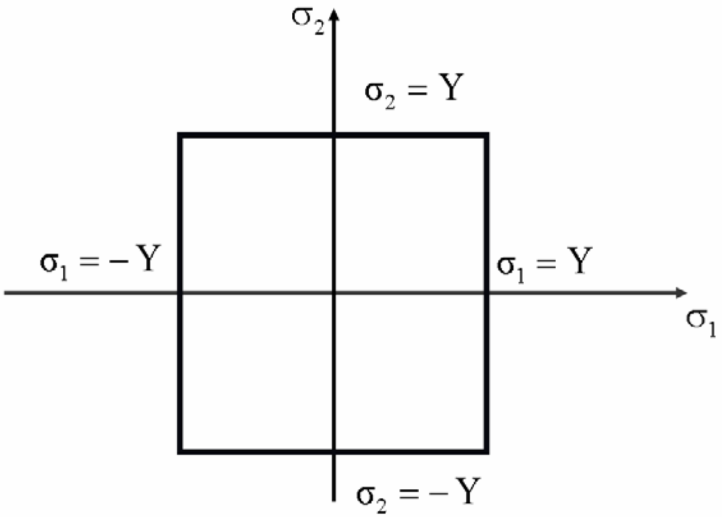
Hence, accurate strength determination is only possible by considering interaction of the various components of stress.

Maximum Principal Stress Criterion

William Rankine (1820-1872)

- One parameter model applicable to brittle materials
- failure occurs when a maximum principal stress equals either the *uniaxial* tension strength or the *uniaxial* compression strength.
- Graphically, the max. stress criterion requires that the two principal stresses lie within the square zone below.

$$f = max(|\sigma_1|, |\sigma_2|, |\sigma_3|) = Y$$



Maximum Principal Stress Yield Surface

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Mohr-Coulomb Criterion.

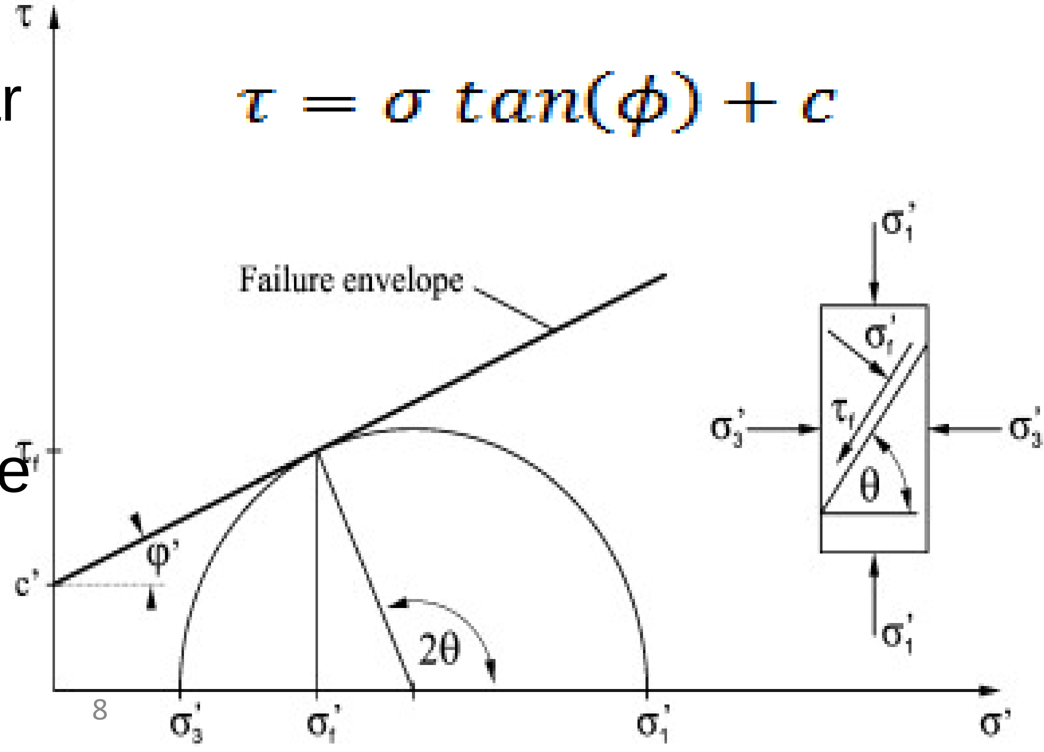
George Mohr (1640 – 1697), Charles. A. Coulomb (1736 – 1806)

- Two parameter model describing the response of brittle materials to shear and normal stresses.
- applies to materials for which the compressive strength far exceeds the tensile strength.

✓ Failure criterion is a linear envelope obtained by plots of τ v σ.

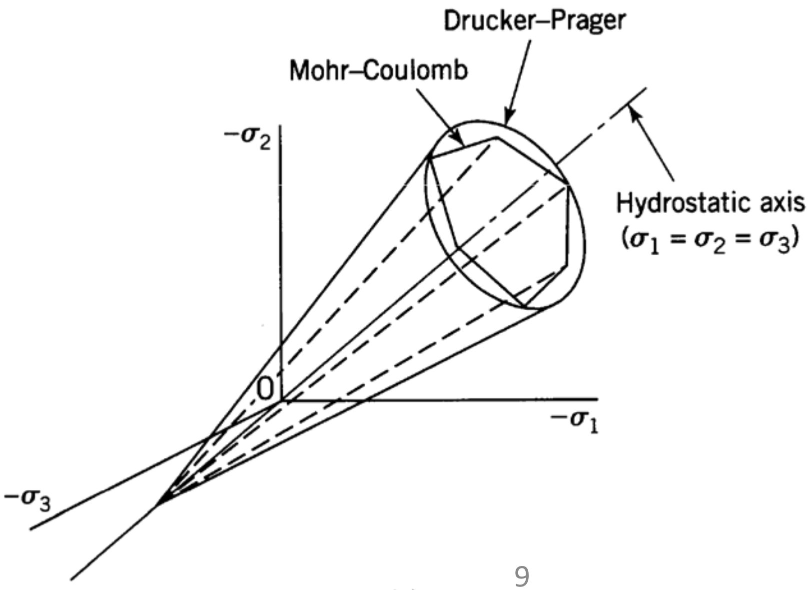
✓ Failure will occur for all stress states for which the largest of Mohr's circle is just tangential to the envelope.

$$\tau = \sigma \tan(\phi) + c$$

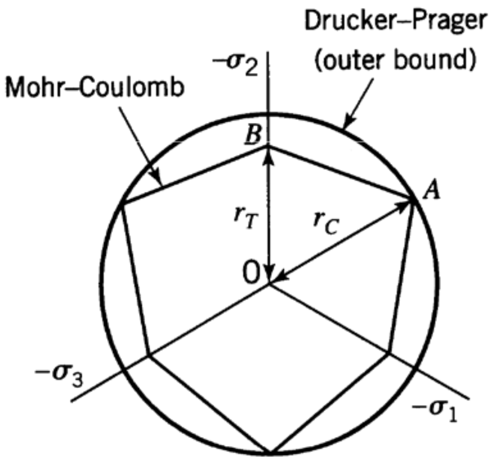


Drucker-Prager Criterion

- Two parameter model
- A modification of the well-known von Mises criterion, whereby the hydrostatic-dependent first invariant, $I_1 = \sigma_{ii}$, is introduced to Von Mises eqn.
- main difference between M-C & D-R is the shape of failure surface (because it accounts for hydrostatic pressure).



(a)



(b)

Agryris & Willam+Warnke Criterion

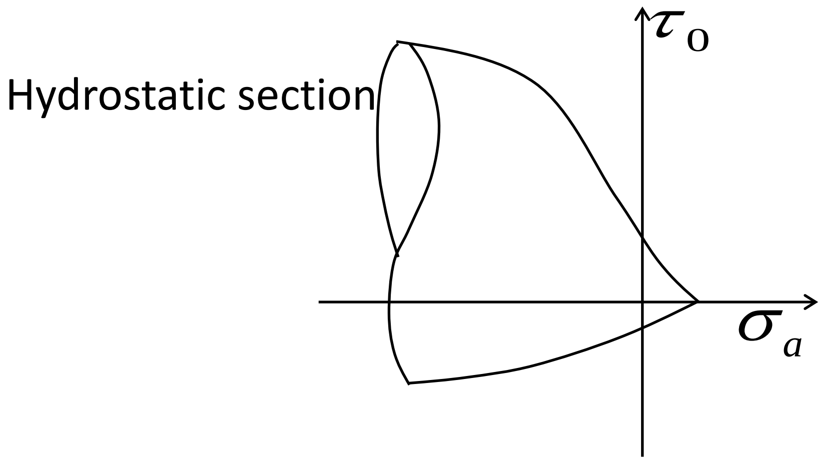
- First, Agryris suggested a three parameter failure criterion to cater for multi-axial stress states, involving both stress invariants.
- Willam+Warnke added two additional parameters to cater for low and high compression regions within struct. They developed a math. relationship for the failure curve in the deviatoric plane by modelling this curve as part of an ellipse. [deviator stresses: on-diagonal (direct) stresses minus the hydrostatic stress]
- Their five parameter model is characterised by a smooth surface and produces the main features of the triaxial failure surface of concrete.

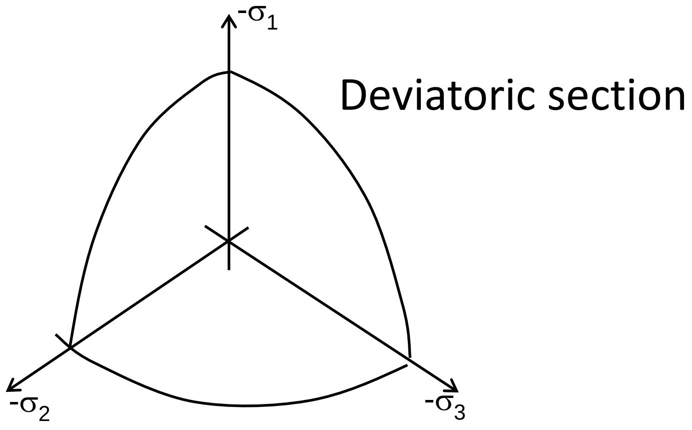
- Hence, assuming $\sigma_1 \geq \sigma_2 \geq \sigma_3$, the failure condition of concrete can be divided into four discrete domains:

1. when: $0 \geq \sigma_1 \geq \sigma_2 \geq \sigma_3$, (C – C – C), crushing occurs
2. when: $\sigma_1 \geq 0 \geq \sigma_2 \geq \sigma_3$, (T – C – C), cracking occurs
3. when: $\sigma_1 \geq \sigma_2 \geq 0 \geq \sigma_3$, (T – T – C), cracking occurs
4. when: $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$, (T – T – T), cracking occurs

- Five input strength parameters needed to define the failure surface, plus a hydrostatic stress state.

Parameter	Description
f_t	Ultimate uni-axial tensile strength
f_c	Ultimate uni-axial compressive strength
f_{cb}	Ultimate bi-axial compressive strength
σ_H	Hydrostatic stress (ambient)
f_1	Ult. comp. strength for state of biaxial comp. superimposed on σ_H
f_2	Ult. comp. strength for state of uniaxial comp. superimposed on σ_H





Failure Surface of Concrete Under Triaxial Conditions (Willam and Warnke 1974)

3. Cracking of Concrete

- Three types of crack models are used:
- Smeared model: Best choice, if overall behaviour of structure (with no great interest to local effects) is desired.
- Discrete model: If a well detailed behaviour is needed such as local stresses, length of crack, path, propagation, etc. Necessitates mesh adaptability techniques that can be complicated and inaccurate.
- Fracture model: For the special class of problems for which fracture mechanisms is of essence, a specialized fracture model may be more functional.
- The smeared crack model was used in the current study.

4. Shear Resistance

- SOLID65 can be used to model brittle materials, such as concrete, rock and ceramics.
- Both cracking and crushing failure modes are included.
- Initially, behaviour is assumed to be linear elastic.
- Plasticity and/or creep may be combined with concrete's own, to provide nonlinear behaviour prior to failure.
- This constitutive model is destined for materials with low tensile but high compressive strengths.
- A "smeared" reinforcement can be specified via *real constants* along the three 'elemental' directions.
- Discrete reinforcements can be separately added via LINK or COMBIN elements

- Smeared elements allow for reinf’t (residing at the centroid of element) to be modelled as smeared stiffness.
- 3-rebars may be defined. Can resist T & C but <u>not SHEAR (?)</u>
- Discrete Spar or Beam elements do not alleviate problem.
- Currently, either a new element (CONTAC?) is needed or… ... look out for the other option!

- By making use of the mechanism of shear transfer in a cracked concrete beam, the contribution of the main steel (modelled here with discrete truss elements) to shear resistance, could be attributed (*passed*) to the surrounding concrete. It can even be adjusted for either both, open-and-closed cracks or one case only.

- This procedure produced good results!

5. Softening of Concrete

- Concrete, unlike steel, shows a post yield, strain-softening behaviour, demonstrated by routine tests on specimens such as cubes, cylinders, prisms, etc.
- Traditional non-linear solution techniques like N-R, or mN-R cannot handle this. Even zero stiffness at the unstable region (top of the curve, where stiffness matrix, **K**, changes from +ve to -ve), possesses a problem for the Newton-Raphson method.
- The stiffness matrix becomes singular, inputted constraining equations become inadequate, the technique predicts an unbounded displacement increment and the model is declared unstable, often preventing further solution (*ill condition*).

- Solvers such as Riks’ and Crisfield’s arc-length methods promise sophisticated solution techniques but they are bounded with restrictions. Eg: only suitable for certain elements; when loading is strictly proportional; and when the problem is *“nice”*. Otherwise, they are not reported of producing good results.

- But attributing strain softening characteristics to the post-peak behaviour of concrete contradicts its brittle nature. Past studies have shown that: strain softening is merely attributed to interaction between specimen and loading platens of the apparatus (van Mier, Kotsovos, and others).

- In other words, if edge effects were eliminated, then concrete should be characterised by an almost immediate loss of load carrying capacity, after reaching its peak strength.

  Hence, the well known descending part of every concrete stress-strain laboratory test routine, is questionable (?), to say the least.

6. Numerical Modelling of a Reinforced Concrete Terrace Unit

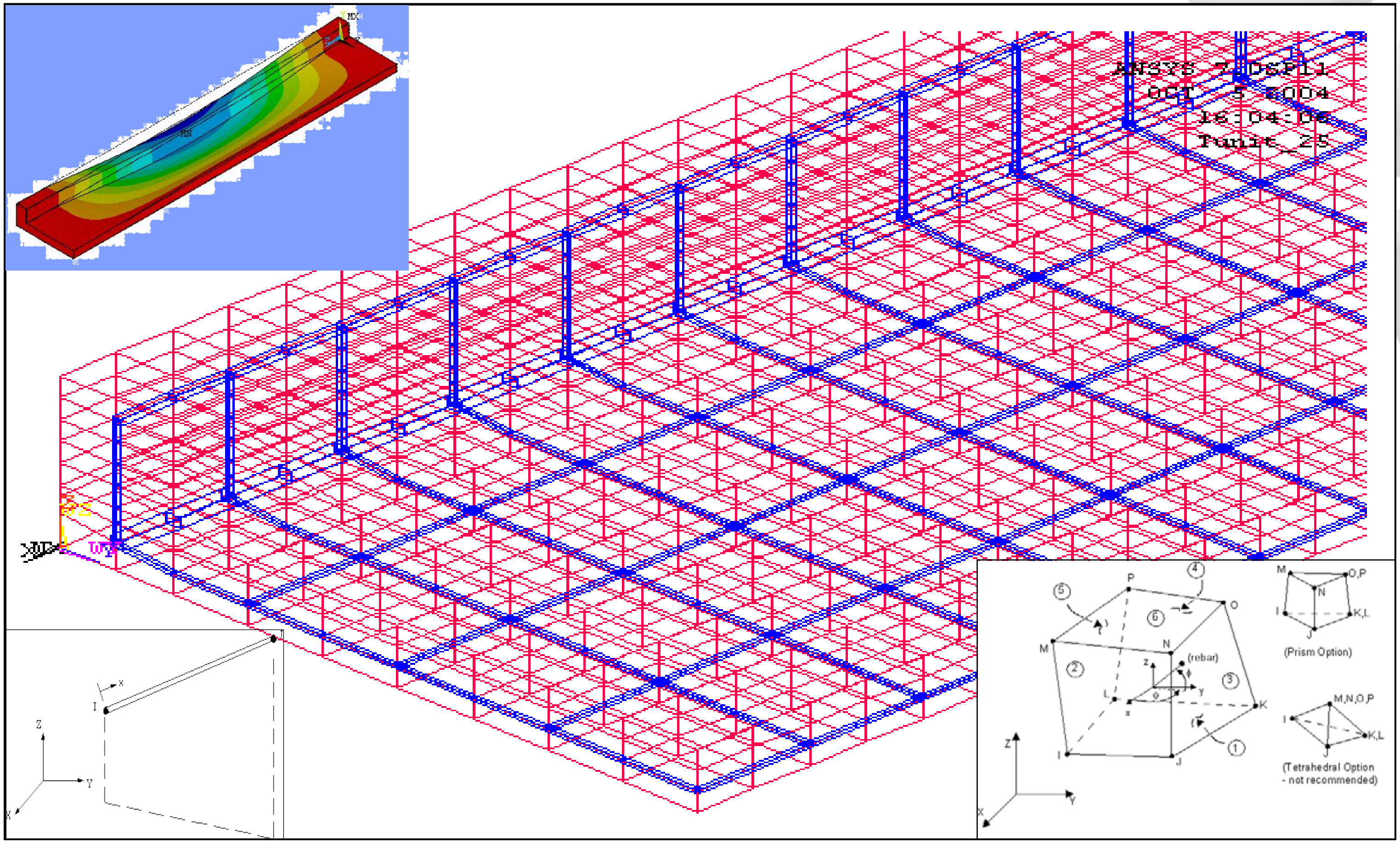
The algorithm at a glance.

START

1. Input Geometry.
2. Input initial Material Properties (next slide).
3. Input Non-Linear Material Properties & Failure Criteria (next slides).
4. Discretize Structure (meshing) .
5. Apply Constraints. [LH-support: UX, UY, UZ = 0, RH-support: UY, UX = 0, Front side: UZ = 0]
7. Apply Loads. [Incremental procedure. 1st incr. to cause T-steel to yield. Continue as per lab].
8. Solve.
9. Check Convergence [out-of-balance load]. NO: use denser mesh. YES: print results
10. Print Results.
11. Print nodal displacements, nodal and elemental strains and stresses. Show crack and crush location, size?, growth?.

STOP

The FE-model



- **SOLID65.** 3D, 8-node, 3DOF per node(translations), solid isoparametric (same shape function used to generate stiffness and mass matrices).
- **LINK8.** 3D, 2-node, 3DOF per node (translations), uni-axial tension-compression .

The Input data

Concrete		Steel re-bars	
E_{con}	30 $kNmm^{-2}$	E_s	198 $kNmm^{-2}$
f_{cu}	45 $kNmm^{-2}$	f_y	_
f_t	2.4 $kNmm^{-2}$	0.2%p	525 $kNmm^{-2}$
ν_{con}	0.15	ν_{steel}	0.3

Initial material properties derived from design, experimental investigation and routine laboratory tests.

Percentage of shear transfer and shear transfer coefficients attributed to concrete.

Shear Transf. Contribution. (Taylor)	**Closed Cracks (%)**	**Open Cracks (%)**	**ANSYS (input) Closed Cracks** (ShearTranCoef)	**ANSYS (input) Open Cracks** (ShearTranCoef)
Dowel Action	25	25	0.25*	0.25*
Aggr.Interlock	45	45	0.45+0.25*	0
Comp. Zone	30	30	0.3	N/A
			Total= 0.95	Total= 0.25

** Contribution of rebars (LINK elements) has been passed to surrounding concrete*

Failure criteria for concrete as inputted in the FE model

Failure criteria for Concrete. **Stress** ($kNmm^{-2}$)								
σ_x (t)	σ_x (c)	σ_y (t)	σ_y (c)	σ_z (t)	σ_z (c)	σ_{xy}	σ_{yz}	σ_{zx}
2.42	-45	2.42	-45	2.42	-45	0.45	0.45	0.45
Failure criteria for Concrete. **Strain**								
ε_x (t)	ε_x (c)	ε_y (t)	ε_y (c)	ε_z (t)	ε_z (c)	ε_{xy}	σ_{yz}	σ_{zx}
0.0001	-.0017	0.0001	-.0017	0.0001	-.0017	_	_	_

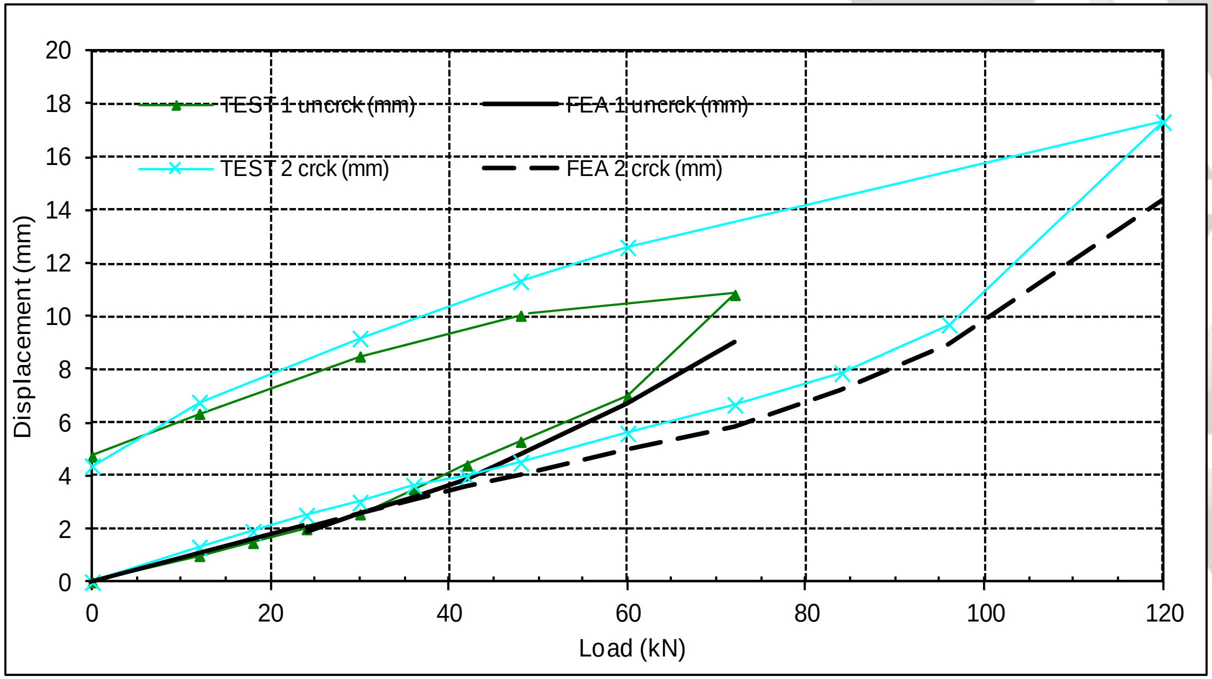
Failure criteria for steel as inputted in the FE model

Failure criteria for Steel. **Stress** ($kNmm^{-2}$)								
σ_x (t)	σ_x (c)	σ_y (t)	σ_y (c)	σ_z (t)	σ_z (c)	σ_{xy}	σ_{yz}	σ_{zx}
660	-660	660	-660	660	-660	_	_	_
Failure criteria for Steel. **Strain**								
ε_x (t)	ε_x (c)	ε_y (t)	ε_y (c)	ε_z (t)	ε_z (c)	ε_{xy}	σ_{yz}	σ_{zx}
0.09	-0.09	0.09	-0.09	0.09	-0.09	_	_	_

Results & Discussion. _Displacements

Predicted response is linear until the first crack has formed at apprx. 24 kN.

This compares very well with the exp. results.



Experience with FEA shows that "virtual structure" has tendency to have lower stiffness than the actual structure. Also, plastic properties of reinf't are such that converged solutions cannot be achieved beyond certain load step.

	Test 1(uncracked unit)	Test 2 (cracked unit)
Measured (W, δ) : (kN, mm)	(72, 10.7)	(120, 17.2)
Predicted (W, δ) : (kN, mm)	(73, 9.08)	126, 14.80)

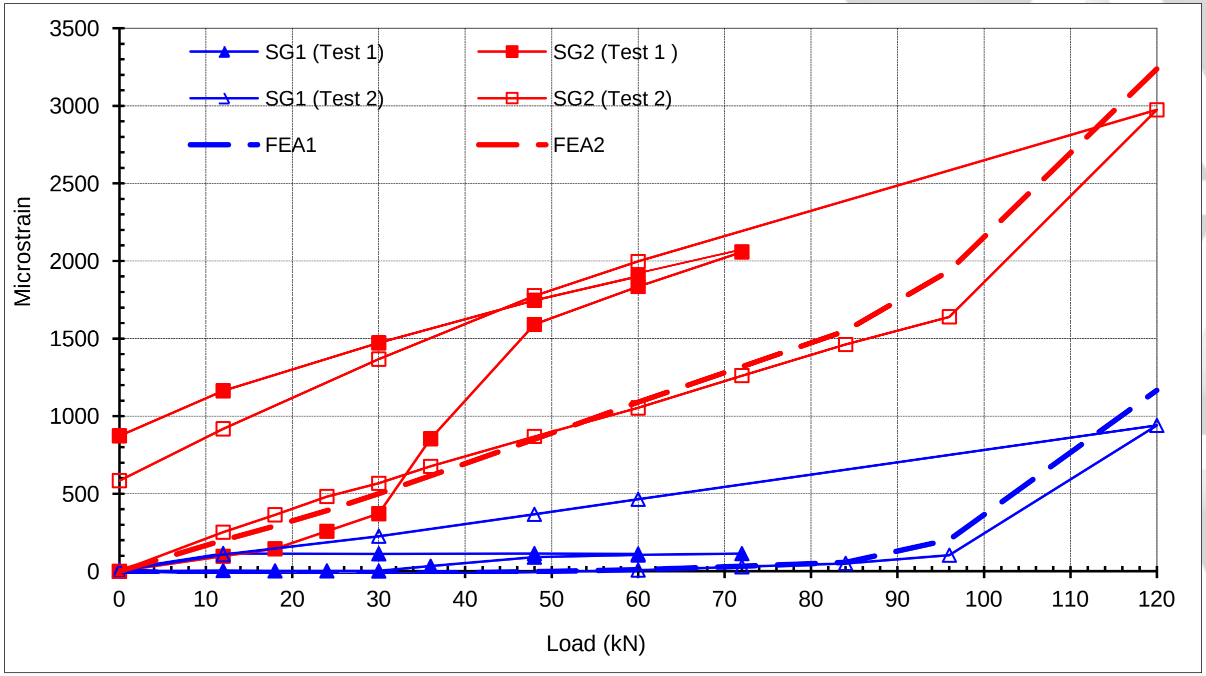
Measured and predicted ultimate values for mid-span displacement and corresponding load.

Results & Discussion. _Strains @ Reinf't

Comparison between measured and predicted strains as developed at the reinforcement.

Test 1: uncracked unit
Test 2: cracked unit
SG1= lateral reinf't
SG2= longit'l reinf't

Values are below the ultimate strain value of 3330μs.

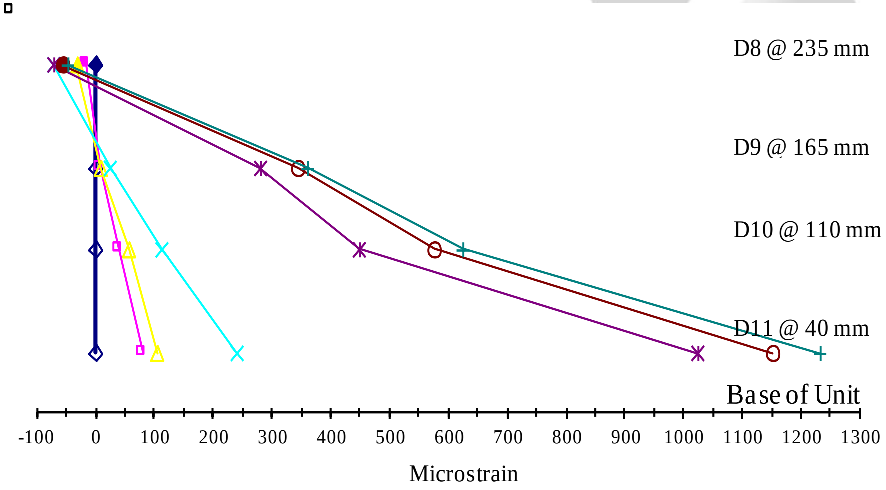


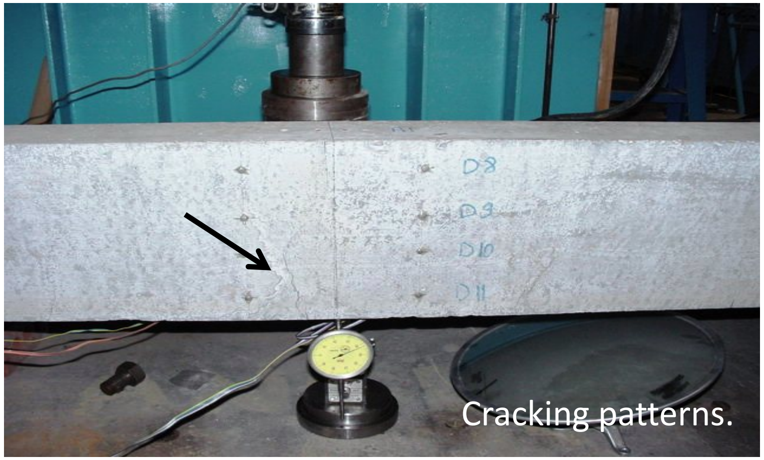
Lateral reinforcement		Longitudinal reinforcement	
Test No:	(kN, μs)	Test No:	(kN, μs)
1	(72, 114)	1	(72, 2058)
2	(120, 940)	2	(120, 2974)
FEA1	(126, 1167)	FEA2	(126, 3238)

Measured and predicted ultimate values of mid-span strains and the corresponding load.

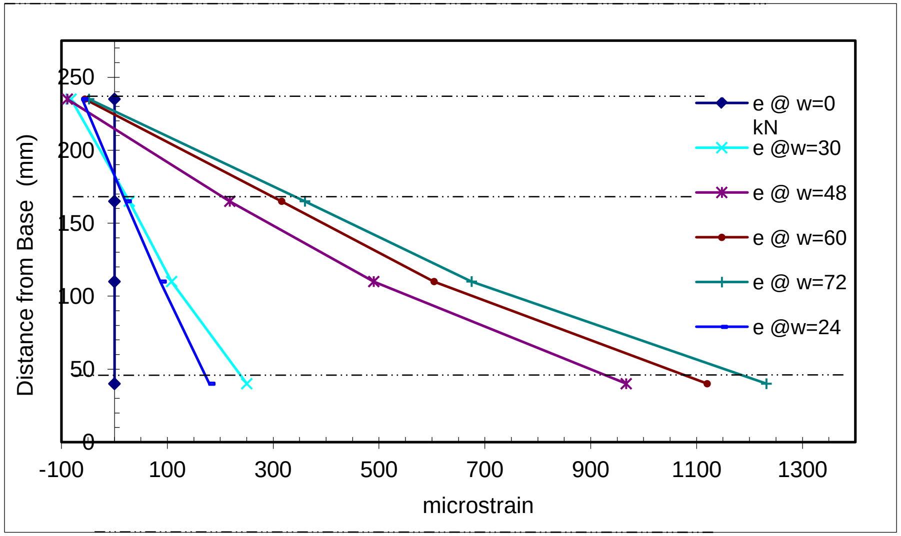
Results & Discussion. _Flexural Strains

Test 1.
Measured strain distribution across the depth of the riser.



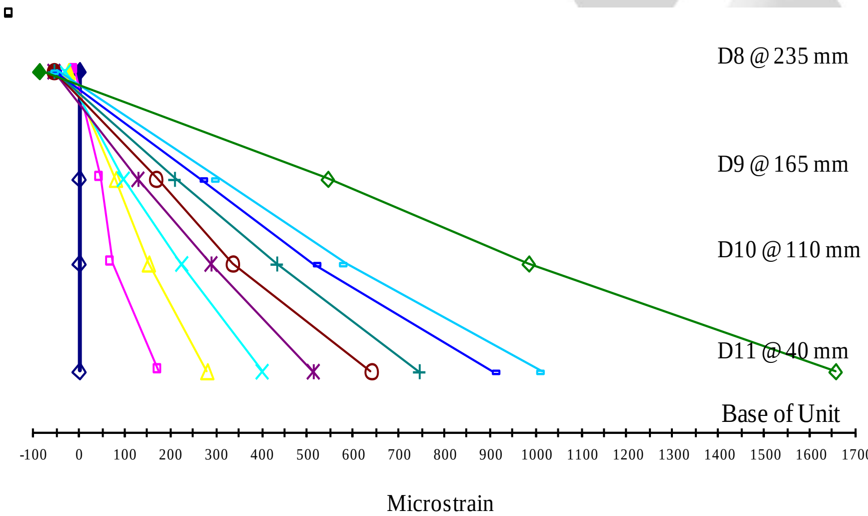


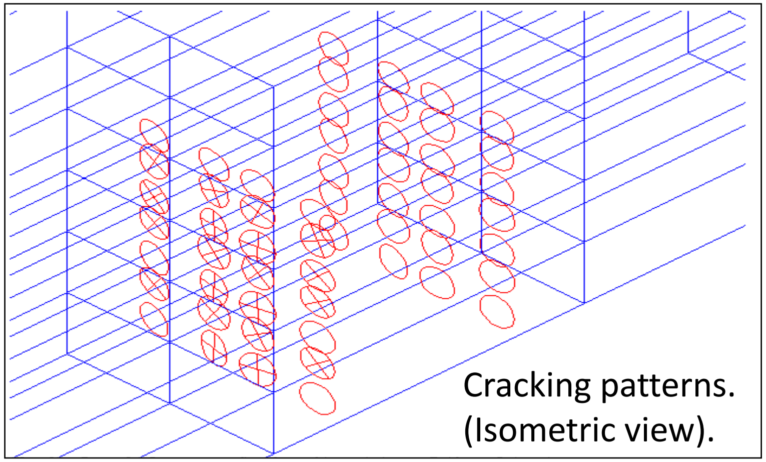
Test 1.
Predicted strain distribution across the depth of the riser.



Results & Discussion. _Flexural Strains

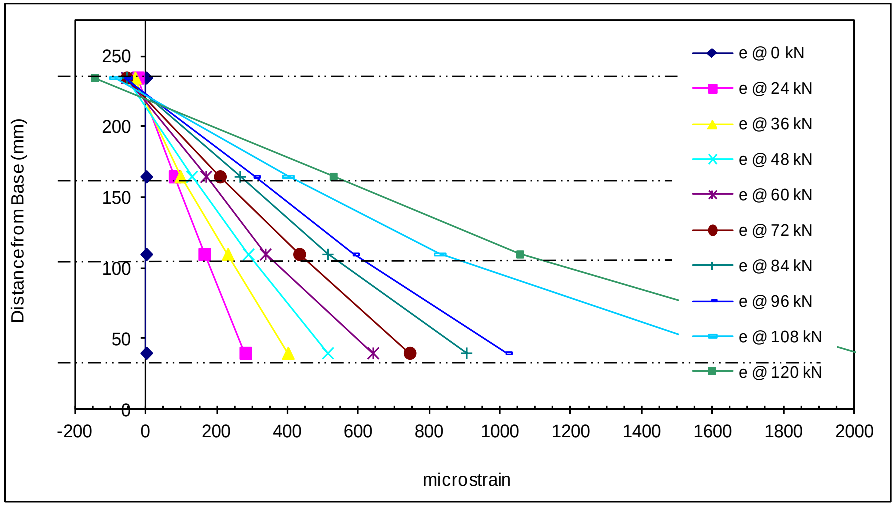
Test 2.
Measured strain distribution across the depth of the riser.





Cracking patterns.
(Isometric view).

Test 2.
Predicted strain distribution across the depth of the riser



7. Rounding up & Conclusions

- Some failure criteria for RC have been outlined, and a simple and yet accurate finite element model of a RC terrace unit was developed in ANSYS 7.0 environment.
- The FE-model employed data obtained from a parallel laboratory investigation. The general elasto-plastic constitutive approach with the cracking and crushing options has captured successfully the non-linear flexural behaviour of this composite unit to failure.
- The mode of failure predicted by the numerical model was of a flexural nature due to increasing plastic strains developing in the tension zone (reinforcement), consistent with the experimental response.
- The FE model depicted also: the 'bowl' at the centre, the 'region of inflexion', the lifting at the free ends, the rotation about the longitudinal axis.

Conclusions

- FE results were found to be rather sensitive to the Modulus of Elasticity assigned to concrete and the reinforcement. However, the various parameters controlling the non-linear performance of the model are numerous (materials, geometry and numerical techniques).
- in order to control the position of the reinforcement with accuracy and achieve better results, it is necessary to simulate the later in a discrete rather than smeared manner.
- The inability of smeared reinforcement to transfer shear stresses is a drawback. Hence, other options are recommended.
- FE-model was capable to predict the position of cracks but not the length (propagation).

Conclusions

- FEM is well suited in dealing with composite material models. One advantage of the theory of plasticity is the relatively simple and direct calibration of the state of stress. However, associated experimental data have been insufficient until now.
- Finally, the choice of a well established constitutive model in engineering research and practice is important as it affects accuracy. More experimental results and numerical models dealing with complex stress states are necessary for research and general engineering applications in the future.