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# Robust Synchronization of Master-Slave Chaotic Systems Using Approximate Model: An Experimental Study

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**Abstract**

Robust synchronization of master slave chaotic systems are considered in this work. First an approximate model of the error system is obtained using the ultra-local model concept. Then a Continuous Singular Terminal Sliding-Mode (CSTSM) Controller is designed for the purpose of synchronization. The proposed approach is output feedback-based and uses fixed-time higher order sliding-mode (HOSM) differentiator for state estimation. Numerical simulation and experimental results are given to show the effectiveness of the proposed technique.

***Index terms***— Robust Synchronization, Chaotic Systems, Sliding-Mode, Master-Slave Synchronization, Model-Free Control

## 1 Introduction

Over the last decades, the synchronization of chaotic systems has attracted a lot of attention of researchers from multidisciplinary research communities [1, 2]. Synchronization of chaotic systems has several potential applications. It can be used for secure communication [3, 4], electronic locking device [5], chemical and biological systems [6], neural network [7], signal processing [8] *etc.*

In the context of chaotic system synchronization, one important problem is the synchronization of master-slave systems. A collection of master-slave chaotic systems can be consulted from [9]. In this problem, the slave system needs to follow the trajectory of the master system. This problem has been

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studied very widely in the literature and as such various results are available. Various control approaches have been applied or can be applied for master-slave synchronization of chaotic systems, for example, linear output-feedback [10], linear state-feedback [11], state feedback linearization [12], sliding-mode [13], adaptive sliding-mode [14], proportional-derivative controller [15], nonlinear $\mathcal{H}_\infty$ controller [16], active disturbance rejection control [17] *etc.* Most of these controllers provide relatively good performances.

However, in order to design the previously mentioned robust controllers, good knowledge of the system dynamics are required. Obtaining good models are often not so easy due to various practical considerations like uncertainties, and external disturbances. To overcome the limitation of model based control, an alternative solution can be to use approximate model based control [18, 19].The main idea here is to approximate the system dynamics by a local model described by an appropriate input-output relationship.

Based on the previously mentioned works, this article proposes an approximate model based sliding-mode control strategy to achieve robust synchronization of master-slave chaotic systems. The novelty of this paper is to combine approximate-model and sliding-mode control techniques in order to control uncertain nonlinear systems like chaotic oscillators. Moreover, experimental verification is provided as well.

*Main contributions:* Firstly, unlike the existing model based approaches available in the literature e.g.,[15], an approximate-model based master-slave synchronization is proposed here. This is a big advantage over the existing results. Secondly, the dimension of the observer being used for synchronization is lower than the disturbance estimation based control schemes [20]. Finally, experimental validation is another contribution of this work.

In this work, the synchronization error system is approximated by a ultra-local model. Using this approximate model, a sliding-mode controller is designed so that the slave systems can track the master system. The proposed controller is output-feedback based and uses fixed-time Higher Order Sliding-Mode (HOSM) differentiator (see , [21] for an introduction of HOSM differentiator and [22, 23, 24, 25] for various applications) to estimate the states and perturbations. The proposed controller produces less chattering than the conventional sliding-mode controller. Moreover it does not require the estimate of the disturbance like [20, 17].

The rest of the article is organized as follows: Problem statement is given in Section 2, details of the proposed control strategy can be found in Section 3. Simulation results are given in Section 4 while experimental study can be found in Section 5 . Finally Section 6 concludes this article.

## 2 Problem statement

Consider the following master system

$$\mathbf{\Sigma}_M : \begin{cases} \dot{x}_M &= f_M\left(x_M\right) \\ y_M &= h_M\left(x_M\right) \end{cases} \tag{1}$$

where $x_M = \begin{bmatrix} x_{1M} & x_{2M} & \dots & x_{nM} \end{bmatrix}^T \in \mathbb{R}^n$ is the state vector and $y_M \in \mathbb{R}$ is the output of the master system. $f_M$ and $h_M$ are smooth vector fields. Consider a slave system described by

$$\mathbf{\Sigma}_S : \begin{cases} \dot{x}_S &= f_S\left(x_S\right) + g_S\left(x_S\right) u \\ y_S &= h_S\left(x_S\right) \end{cases} \tag{2}$$

where $x_S = \begin{bmatrix} x_{1S} & x_{2S} & \dots & x_{nS} \end{bmatrix}^T \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ ($u : \mathbb{R}_+ \to \mathbb{R}$ is locally essentially bounded and measurable signal) is the control and $y_S \in \mathbb{R}$ is the output of the slave system. $f_S$, $g_S$ and $h_S$ are smooth vector fields. Now, the synchronization problem considered can be established as follows.

The master-slave synchronization objective: Given two chaotic system of same order, find a control to force the states of the slave system (2) to be synchronized with the states of the master system (1). In order to attain this objective, let us define the synchronization error, $\varepsilon := x_M - x_S$. Then, master-slave synchronization is defined as follows:

**Definition 1.** A slave system (2) exhibits master-slave synchronization with the master system (1), if

$$\lim_{t \to \infty} \varepsilon = 0 \tag{3}$$

for all $t \geq 0$ and any initial condition $\varepsilon(t_0) = x_M(t_0) - x_S(t_0)$.

## 3 Approximate Model Based Sliding-Mode Control

This section provides the detail of the proposed controller which will be used later for the purpose of master-slave synchronization. Consider a general nonlinear Single-input Single-Output (SISO) chaotic system represented in the general form as [26]

$$f\left(y, \dot{y}, \dots, y^{(a)}, u, \dot{u}, \dots, u^{(b)}, d\right) = 0, \tag{4}$$

where $y \in \mathbb{R}$ is the measurable output, $u \in \mathbb{R}$ is the input signal and $d \in \mathbb{R}$ is a bounded disturbance.

**Property 1.** The system (4) is BIBS (bounded input bounded state), and the derivative of its input is bounded.

Property 1 is not restrictive as it can be fulfilled by chaotic systems. It is not possible to have chaotic behavior without boundedness of the trajectory. However, it is to be noted here that for general nonlinear systems, Property 1 can be restrictive.

Roughly speaking, the main idea of approximate model based control is to replace complex "unknown" mathematical model by a simple ultra-local model which is only valid during a very short time interval. In this direction, model (4) can be approximated by the following ultra-local model

$$y^{(\nu)} = F + \alpha u, \tag{5}$$

where $\nu$ is the derivative of order $\nu \geq 1$ of $y$, $F$ is the compensation term, which carries the unknown and/or nonlinear dynamics of the system as well as the time-varying external disturbances and $\alpha \in \mathbb{R}$ is a "nonphysical" constant parameter for scaling. The compensation term $F$ can be estimated by the measurements of the system input and output.

**Assumption 2.** *The disturbance signal $F : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable for almost all $t \geq 0$, and there is a constant $0 < \kappa^+ < \infty$ such that ess $sup_{t\geq 0}|\dot{F}(t)| \leq \kappa^+$.*

The existing literature on the application of model-free control depends on the estimation of $F$ (*e.g.*, [18, 19]). In this work we will not consider this direction. In our case, $F$ will be considered as a disturbance and the objective is to design a robust controller in the presence of $F$ using differentiator based approach. In this paper, uniform finite-time convergent differentiator proposed by Cruz-Zavala et al. [27] will be used for the purpose of state estimation. This will be detailed later on in this Section. To design the tracking controller, the tracking errors can be defined as $e_1 = y - y_d$ and $\dot{e}_1 = e_2 = \dot{y} - \dot{y}_d$. Then the tracking error dynamics can be written as

$$\dot{e}_1 = e_2, \tag{6a}$$

$$\dot{e}_2 = F + \alpha u - \ddot{y}_d. \tag{6b}$$

The tracking problem for system (5) is essentially the stabilization of the error dynamical system (6). To stabilize the system (6), it is necessary to design a controller $u$ under the presence of external perturbations and/or parametric uncertainties which are included in the unknown function $F$. For this purpose, inspired by the ideas given in [28, 29], the following CSTSM controller is used in this work:

$$\sigma_L(e_1, e_2) = e_1 + \frac{\alpha}{L^{0.5}}\lceil e_2 \rfloor^{\frac{2}{3}}, \tag{7a}$$

$$u = \ddot{y}_d - k_1 L^{\frac{2}{3}} \lceil \sigma_L \rfloor^{\frac{1}{3}} + z, \tag{7b}$$

$$\dot{z} = -k_2 L \lceil \sigma_L \rfloor^0, \tag{7c}$$

where $\sigma_L(e_1, e_2)$ is a continuously differentiable function of the state, $\ddot{y}_d$ is the second derivative of the reference trajectory to be tracked, $z$ is a dummy variable which extends the dynamics of the system so that the control signal become continuous, $k_1$, $k_2$ and $L$ are design parameters of the proposed control law and $\alpha > 0$ is a constant. Controller (7) requires all components of the state vector, which limits its implementation. However, in order to overcome this difficulty, observers/differentiators are proposed for solving the mentioned problem about state estimation.

## 3.1 Fixed-time differentiator for state estimation

According to Property 1, the system (4) is BIBS. Model (5) is a local approximation of model (4). So, similar assumptions are also applicable in this case. Lets consider that the upper bound for $F$ is $f^+$, *i.e.* $|\frac{dF}{dt}| \leq f^+$. To estimate the states of Model (5), the following HOSM differentiator can be applied based on [27]:

$$\dot{\zeta}_1 = -\eta_1 \left( \lceil \zeta_1 - y \rfloor^{\frac{1}{2}} + \gamma \lceil \zeta_1 - y \rfloor^{\frac{3}{2}} \right) + \zeta_2, \tag{8a}$$

$$\dot{\zeta}_2 = -\eta_2 \left( \frac{1}{2} \lceil \zeta_1 - y \rfloor^0 + 2\gamma (\zeta_1 - y) + \frac{3}{2}\gamma^2 \lceil \zeta_1 - y \rfloor^2 \right), \tag{8b}$$

where $\zeta = \begin{bmatrix} \zeta_1 & \zeta_2 \end{bmatrix}^T$ is the estimate of $Y = \begin{bmatrix} y & \dot{y} \end{bmatrix}^T$, $\eta_1, \eta_2$ are tuning parameters and $\gamma \geq 0$ is a scaler. When $\gamma = 0$, the standard robust exact differentiator proposed in [30] is recovered. The system (8) is discontinuous. As a result the solutions of the system (5) equipped by the differentiator (8) cannot be understood by the classical theory of differential equations, which assumes Lipschitz continuity to guarantee the existence of unique solutions. The solution has to be understood in the sense of Filippov [31]. The solution concept proposed by Filippov for differential equation with discontinuous right hand sides constructs a solution as the "average" of the solutions obtained from approaching the point of discontinuity from different directions. Then the following result can be provided:

**Theorem 3.** *[27] Let model (5) be BIBS. If the tuning parameters $\eta_1$ and $\eta_2$ are chosen from the following set $\eta$,*

$$\eta = \left\{ (\eta_1, \eta_2) \in \mathbb{R}^2 | 0 < \eta_1 < 2\sqrt{f^+}, \eta_2 > \frac{\eta_1^2}{4} + \frac{4\left(f^+\right)^2}{\eta_1^2} \right\} \cup \left\{ (\eta_1, \eta_2) \in \mathbb{R}^2 | \eta_1 > 2\sqrt{f^+}, \eta_2 > 2f^+ \right\},$$

*then the differentiator (8) is fixed-time convergent and the convergence time is independent of the initial differentiation error, i.e. there exists $\mathcal{T}_s > 0$ such that for the system in (5) and the differentiator in (8) for all $t \geq \mathcal{T}_s$, $\tilde{Y} = Y - \hat{Y} = 0$. In addition, the estimation error $\tilde{Y}$ stays bounded for all $t \geq 0$.*

With the estimated states, the estimated tracking errors can be defined as $\hat{e}_1 = \hat{y} - y_d$ and $\hat{e}_2 = \dot{\hat{y}} - \dot{y}_d$. Hence, the control law (7), in terms of estimated state variables, take the following form:

$$\sigma_L(\hat{e}_1, \hat{e}_2) = \hat{e}_1 + \frac{\alpha}{L^{0.5}} \lceil \hat{e}_2 \rfloor^{\frac{2}{3}}, \tag{9a}$$

$$u = \ddot{y}_d - k_1 L^{\frac{2}{3}} \lceil \sigma_L \rfloor^{\frac{1}{3}} + z, \tag{9b}$$

$$\dot{z} = -k_2 L \lceil \sigma_L \rfloor^0 \tag{9c}$$

Then the main result of this work is given below:

**Theorem 4.** *Let model (5) be BIBS, and tuning parameters of the HOSM differentiators be properly selected according to Proposition 3. Consider the system (5) with differentiator (8) and the controller (9). Then for some positive and sufficiently large values of $k_1$, $k_2$ and $L$, the output $y$ tracks the reference trajectory $y_d$ in a finite-time.*

*Proof.* The proof is straightforward. For $t_f \geq \mathcal{T}_s$, the trajectories of the differentiators converge to those of the system, and then $\hat{e}_1 = e_1$ and $\hat{e}_2 = e_2$. Therefore, substituting the control (9) into the error dynamics (6), one obtains:

$$\dot{e}_1 = e_2, \tag{10a}$$

$$\dot{e}_2 = e_3 - k_1 L^{\frac{2}{3}} \lceil \sigma_L \rfloor^{\frac{1}{3}} + F, \tag{10b}$$

$$\dot{e}_3 = -k_2 L \lceil \sigma_L \rfloor^0. \tag{10c}$$

Define $\bar{e}_3 = e_3 + F$. Then, eq. (10) can be written as follows:

$$\dot{e}_1 = e_2, \tag{11a}$$

$$\dot{e}_2 = \bar{e}_3 - k_1 L^{\frac{2}{3}} \lceil \sigma_L \rfloor^{\frac{1}{3}}, \tag{11b}$$

$$\dot{\bar{e}}_3 = -k_2 L \lceil \sigma_L \rfloor^0 + \dot{F}. \tag{11c}$$

Note that the error dynamics given in (11) mimics the one for the CSTSM controller presented in [28]. Using a Lyapunov function based argument, it has been shown there that for some positive and sufficiently large values of $k_1$, $k_2$ and $L$, the system (11) is UFTS and then

$$x_1(t) = x_d(t),\ x_2(t) = \dot{x}_d(t),\ e_3(t) = -F(t),\ \forall t \geq \mathcal{T}_s + \mathcal{T}_c,$$

where $\mathcal{T}_c$ is the convergence time of the controller. □

*Remark* 5. In this work, for the sake of simplicity, we have considered $\nu = 2$. If $\nu > 2$, then the controller (9) is not applicable. In that case, a recently proposed technique [32] can be easily applied for high relative degree system.

## 4 Implementation and Simulation Results

In this section, the implementation of the proposed controller will be demonstrated through numerical example. For this purpose, we consider the master-slave synchronization of the two identical chaotic duffing oscillator.

We take the following representation of the duffing oscillator

$$\dot{x}_1 = x_2, \tag{12a}$$

$$\dot{x}_2 = -p_1 x_1^3 - p_2 x_1 - p_3 x_2 + q\cos(\omega t) + u, \tag{12b}$$

where $p_1, p_2, p_3, q$ and $\omega$ are non-zero constant parameters and $u$ is the control. For the master oscillator, $u = 0$. It has been shown in [33] that if the parameters are set as: $p_1 = 1$, $p_2 = -1.1$, $p_3 = 0.4$, $q = 1$ and $\omega = 1.8$, then model (12) shows chaotic behavior. In order to design the master-slave synchronization controller, let us define the errors among the master and slave system as, $e_1 = x_{1m} - x_{1s}$ and $\dot{e}_1 = e_2 = x_{2m} - x_{2s}$ where $m$ stands for master system and $s$ for the slave system. Then the error dynamics can be written as

$$\dot{e}_1 = e_2, \tag{13a}$$

$$\dot{e}_2 = \phi_m(x_1, x_2, \omega) - \phi_s(x_1, x_2, \omega) - u, \tag{13b}$$

where $\phi_{m/s}\left(x_{1m/s}, x_{2m/s}, \omega\right) = -p_1 x_{1m/s}^3 - p_2 x_{1m/s} - p_3 x_{2m/s} + q\cos(\omega t)$. From the properties of $\phi_m$ and $\phi_s$, it can be substantiated that the difference of the two functions are bounded. Define $F := \phi_m - \phi_s$ with $|\frac{dF}{dt}| \leq f^+$ where $f^+$ being a known positive constant. Then eq. (13) can be written as

$$\dot{e}_1 = e_2, \tag{14a}$$

$$\dot{e}_2 = F - u, \tag{14b}$$

If we consider $y = e_1$ as the output, then model (14) can be written as

$$y^{(2)} = F + \alpha u, \tag{15}$$

where $\alpha = -1$. Then the design of the synchronizing controller follows directly from Section 3. In this regard, the first step is to implement the observer (8) for model (14). The observer takes the following form:

$$\dot{\hat{e}}_1 = -\eta_1\left(\lceil \hat{e}_1 - e_1 \rfloor^{\frac{1}{2}} + \gamma \lceil \hat{e}_1 - e_1 \rfloor^{\frac{3}{2}}\right) + \hat{e}_2, \tag{16a}$$

$$\dot{\hat{e}}_2 = -\eta_2\left(\frac{1}{2}\lceil \hat{e}_1 - e_1 \rfloor^{0} + 2\gamma(\hat{e}_1 - e_1) + \frac{3}{2}\gamma^2 \lceil \hat{e}_1 - e_1 \rfloor^{2}\right) \tag{16b}$$

With the estimated state $\hat{e}_1$ and $\hat{e}_2$, the controller (9) for system (14) is given as:

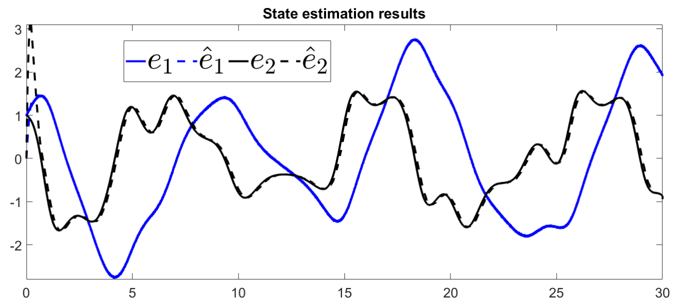


Figure 1: State estimation results

$$\sigma_L(\hat{e}_1, \hat{e}_2) = \hat{e}_1 + \frac{\alpha}{L^{0.5}} \lceil \hat{e}_2 \rfloor^{\frac{2}{3}}, \tag{17a}$$

$$u = -k_1 L^{\frac{2}{3}} \lceil \sigma_L \rfloor^{\frac{1}{3}} + z, \tag{17b}$$

$$\dot{z} = -k_2 L \lceil \sigma_L \rfloor^{0}. \tag{17c}$$

To illustrate the performance of the observer, digital simulations are performed using the following parameters of the fixed-time sliding-mode observer: $\gamma = 1$, $\eta_1 = 2\sqrt{5}$, $\eta_2 = 10$ and initial conditions $\hat{e}_1(0) = \hat{e}_2(0) = 0$. From Fig. 1, we can see that the estimation errors $\tilde{e}_1 = e_1 - \hat{e}_1$, and $\tilde{e}_2 = e_2 - \hat{e}_2$, converge to the origin in finite-time.

Parameters of the controller (17) are selected as: $k_1 = 16$, $k_2 = 7$, $\alpha = 1$ and $L = 1$. To compare the performance of the proposed technique, extended high-gain observer based output feedback controller is selected. Based on recent developments in the area of extended high gain observer [34, 20], the controller takes the following form,

$$u = -\hat{F} - k_1\hat{e}_1 - k_2\hat{e}_2 \tag{18}$$

where $\hat{e}_1, \hat{e}_2$ and $\hat{F}$ are the estimates of $e_1, e_2$and $F$ respectively. These estimates are generated by the following extended high gain observer,

$$\dot{\hat{e}}_1 = \hat{e}_2 + \frac{\zeta_1}{\epsilon}(e_1 - \hat{e}_1), \tag{19a}$$

$$\dot{\hat{e}}_2 = \hat{F} + u + \frac{\zeta_2}{\epsilon^2}(e_1 - \hat{e}_1), \tag{19b}$$

$$\dot{\hat{F}} = \frac{\zeta_3}{\epsilon^3}(e_1 - \hat{e}_1). \tag{19c}$$

Parameters of controller (18) are selected as: $k_1 = 5$ and $k_2 = 10$ while the parameters of EHG observer are selected as $\zeta_1 = \zeta_2 = 3, \zeta_3 = 1$ and $\epsilon = 0.01$. In the simulation, we assume that the noisy measurement of $x_{1,m}$ of the master system is available to the slave system. The measurement noise is added to make

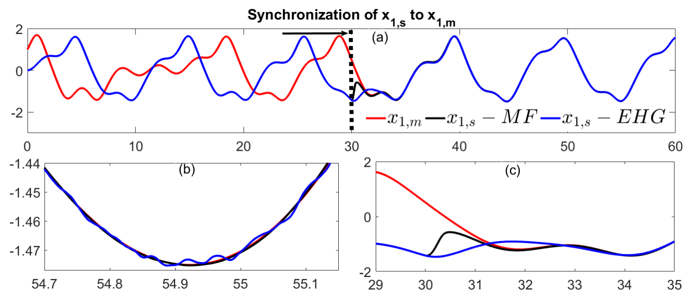


Figure 2: a) Synchronization of $x_{1,s}$ to $x_{1,m}$. MF-proposed control, EHG- eq. (18); b) zoomed view in steady-state; c) convergence of the slave system to master system.

the simulation realistic. Band-limited white noise block of Simulink is used to generate the noise. The result of the synchronization is given in Fig. 2 and 3. Fig. 2 and 3 show that master-slave synchronization is achieved after applying the control. Comparative simulation results show that the proposed controller perform better than the EHG based control scheme. The convergence time of the proposed controller is better than that EHG based controller. Steady-state error of the proposed controller is significantly small than that of EHG based controller. One problem of high-gain observer is that it amplifies the measurement noise. This is also evident here as shown in Fig. 3. These noise amplified estimated states deteriorates the performance of the controller.

From theoretical point of view, in the presence of noise, the asymptotic accuracy of differentiator deteriorates as the differentiation order increases. For example, in [21], it is shown that for the sliding-mode differentiator, the $n$-th order differentiation accuracy is of the order of $\epsilon^{\left(2^{-n}\right)}$, where $\epsilon$ is the maximum noise amplitude. So, higher the differentiation order, lower the accuracy. The proposed controller requires the first derivative of the measurement where as EHG based controller requires also the second derivative. Since the accuracy of the second derivative estimation is lower than the first derivative, this plays a role in the performance of the EHG based controller.

The comparative simulation results demonstrate the effectiveness of the proposed controller.

# 5 Experimental study

To verify practical feasibility and performance of the synchronization scheme developed in this study in Section 3, let us consider the synchronization of master-slave Van der Pol oscillators given by [35]:

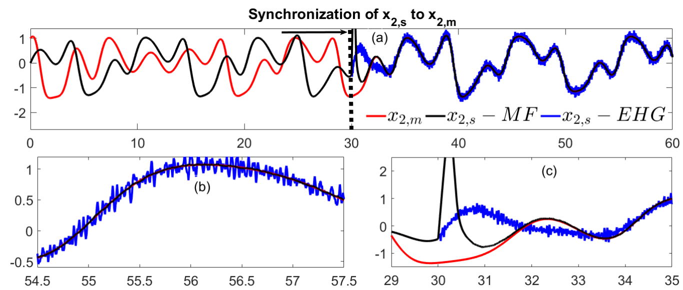


Figure 3: a) Synchronization of $x_{1,s}$ to $x_{1,m}$. MF-proposed control, EHG- eq. (18); b) zoomed view in steady-state; c) convergence of the slave system to master system.

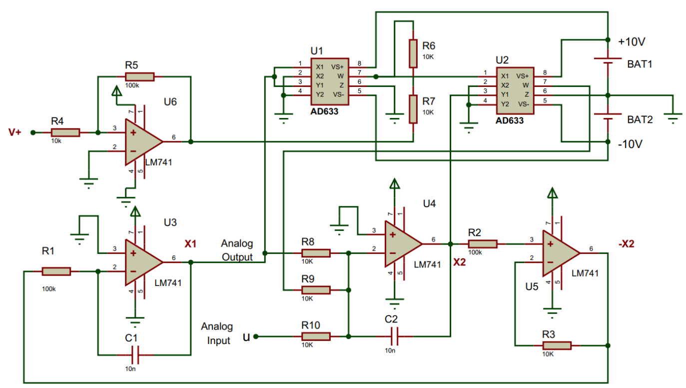


Figure 4: Circuit diagram of an autonomous Van der Pol oscillator.

$$\dot{x}_1 = x_2, \tag{20a}$$
$$\dot{x}_2 = -x_1 + \kappa\left(1 - x_1^2\right)x_2 + u, \tag{20b}$$
$$y = x_1 \tag{20c}$$

where $\kappa$ is the system parameter. In this work, the parameter $\kappa = 0.1$ is selected. The electronic circuit diagram can be seen in Fig. 4. The circuit parameters are: $Ri = 1\text{M}\Omega$, $i = \overline{1,6}$, $R7 = 130\Omega$, $R8 = 1.2K\Omega$ , $R9 = 100\Omega$, $R10 = 1.5K\Omega$, $C1 = C2 = 1\mu F$, $LM741$ is a general purpose operational amplifier and $AD633$ is a 4-quadrant multiplier operational amplifier. We used dSPACE 1106 board as rapid prototyping solution. The HOSM differentiator based CSTSM controller was implemented using Simulink. The solver was the Euler's method and the sampling frequency was 5KHz.

The objective here for the salve system is to follow the trajectory of the master system using the information of outputs only. As shown in Section 4,

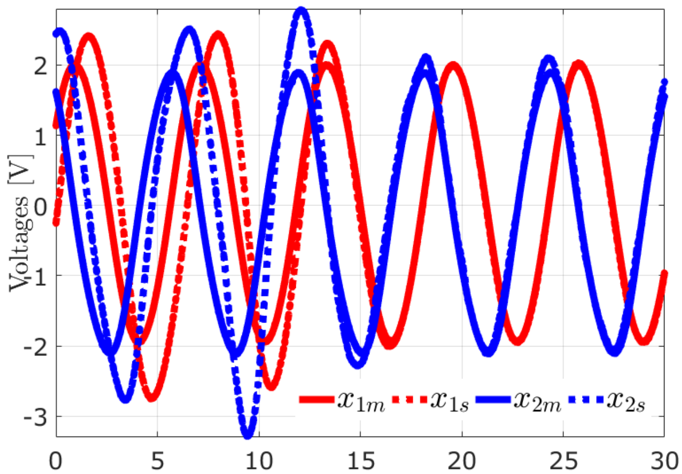


Figure 5: Experimental synchronization of master slave Van der Pol oscillator systems

the error dynamics in this case is also given by Eq. (14). Then the controller 17 can be similarly designed. The result of the experimental study can be seen in Fig. 5. From this figure, it can be seen that synchronization is achieved among the master-slave system. This shows the effectiveness of the controller used in this work. The estimation error of the fixed-time differentiator can be seen in Fig. 6.

# 6 Conclusion

In this work, an approximate model based master-slave synchronization of chaotic systems has been presented. The error dynamics of the master-slave systems is first approximated by a perturbed chain of integrators. Then the synchronization is achieved using control approaches based on sliding-mode control theory. The proposed controller is output-feedback-based and uses a fixed-time differentiator to estimate the unmeasurable states. Experimental study demonstrated the effectiveness of our method using master-slave Van der Pol oscillators.

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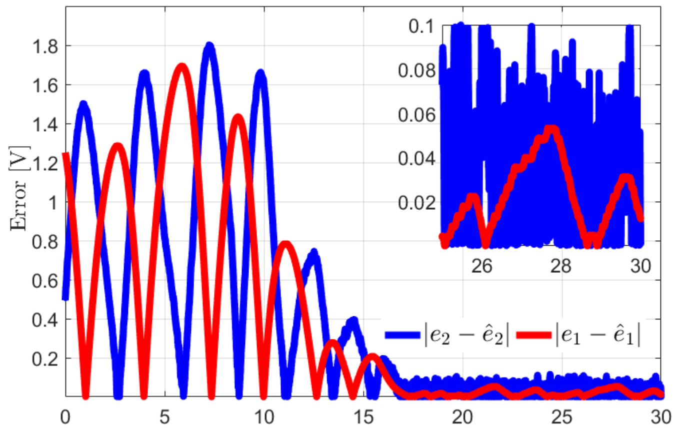


Figure 6: Experimental state estimation results


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