Structural use of bamboo: Part 4: Element design equations

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Technical Note Series: Structural Use of Bamboo

Technical Note 4: Element design equations

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Synopsis

Bamboo is a strong, fast growing and very sustainable material, having been used structurally for thousands of years in many parts of the world. In modern times, it has the potential to be an aesthetically pleasing and cost-competitive alternative to more conventional materials, such as timber, as demonstrated by some visually impressive recent structures.

This five-part technical series, aimed at both developed- and developing-world contexts, will bring together current knowledge and best practice on the structural use of bamboo, covering:

- an introduction to bamboo (part 1)
- durability and preservation (part 2)
- design values (part 3)
- element design equations (part 4)
- connections (part 5)

This fourth article proposes element design equations for typical loading conditions on a bamboo culm: flexure, shear, axial tension, axial compression and local bearing. The work draws on existing published bamboo and timber design codes, and sound engineering judgement.

Introduction

Element design in bamboo is relatively similar to timber and must always follow elastic stress distributions since bamboo is a brittle material. The methods proposed are based on a Limit State approach and have been developed based on ISO 22156: Bamboo – Structural Design[1], ISO 22157: Bamboo – Determination of Physical and Mechanical Properties[2,3], NSR-10 G-12: Colombian Code for Seismically-Resistant Construction: Structures of Timber and Guadua Bamboo[4], BS5268-2: Structural use of timber[5] and EN 1995-1-1: Eurocode 5: Design of Timber Structures[6]. Design strengths should be taken from Structural use of bamboo: Part 3: Design values[7].

When determining loads in bamboo elements, an approach similar to that used in timber structures should be used – i.e. a view should be taken on how the load path might vary depending on the possible variations in stiffness of the connections and elements. This is particularly important because most failure modes in bamboo elements and bolted connections are brittle. This is best done by a sensitivity analysis of the structural load path, using good engineering judgement.

For all elements and all structures, it is important to use bamboo that has been dried to a moisture content close to the moisture content of equilibrium in the location where the structure will be built. Failure to do so may result in splits in elements and joints that may compromise the strength of the structure.

It is recommended that for safety critical elements, such as primary beams and columns, designers consider bundling culms together. Although composite action flexurally is unlikely to be able to be justified and therefore cannot be exploited, bundling has the potential to make the overall element less sensitive to splitting of individual culms, hence they are more robust. Similarly, consideration should be given in design to the possibility of replacing individual elements in service should these exhibit severe splitting – bundling may aid this process due to redundancy in the element.

Nomenclature

Nomenclature used throughout this Note:

bundle = several culms bundled together forming an element, generally sharing load but not acting compositely
culm = stem or stalk of bamboo
element = a part of the structure (beam, column etc.), formed from one or more culms
fibres = the cellulose fibres of the bamboo which run longitudinally along the length of the culm
internode = the culm region between nodes
lignin = a weak matrix which holds the longitudinal fibres of the culm together
node = a solid band at regular intervals along the culm, where the fibres acts as a diaphragm and close off the section
splits = a longitudinal crack along the length of the culm

b = length of loaded area for local crushing (mm)
d_i = distance between the centroid of the combined section and the centroid of each individual culm (mm)
k_{cr} = crack factor
$X_{i,d}$ = design strength (N/mm^2)[7]
$X_{c,0,d}$ = design compressive strength parallel to fibre (N/mm^2)[7]
$X_{m,d}$ = design flexural strength (N/mm^2)[7]
$X_{t,0,d}$ = design tensile strength parallel to fibre (N/mm^2)[7]

$X_{v,d}$ = design shear strength about primary axis about either axis (N/mm^2)[7]
k = coefficient of effective length
l_e = effective length of element (mm)
r = radius of gyration of section/composite section
t = average wall thickness of culm, based on the average of the eight dimensions shown in Figure 1 (four at each end) (mm)
γ_E = material factor of safety for modulus of elasticity
λ = slenderness of element/composite element

A = net area of section (less any holes) (mm^2)
A_i = net section area (less any holes) of each individual culm (mm^2)
A_{tot} = sum of the net section areas (less any holes) of the individual culms that make up the compressive element (mm^2)
C_k = boundary coefficient between intermediate and long slender elements for axial compression
D_e = average external diameter of culm (mm), based on the average of the four diameters shown in Figure 1 (two at each end)
D_{out} = external diameter of second culm when checking local crushing (mm)
$E_{0.05}$ = modulus of elasticity, 5th percentile (N/mm^2)
F_i = design capacity of element (N)
F_c = design compressive capacity of element (N)
M_m = design flexural capacity of element (Nmm)
F_t = design tensile capacity of element (N)
F_v = design shear capacity of element (N)
I = second moment of area of combined/composite section (mm^4)
I_i = second moment of area of each individual culm forming the section (mm^4)
L = length between points at which the element is restrained against buckling in that plane (mm)
M = moment applied to element (Nmm)
N_c = axial compressive force applied to element (N)
N_e = Euler critical load for axial compression (N)
N_t = axial tensile force applied to element (N)
$S_{elastic}$ = elastic section modulus (mm^3)
V = shear force applied to element (N)

1.0 Dimensions for structural design

Culm dimensions and basic properties can be determined as follows and from Figure 1:

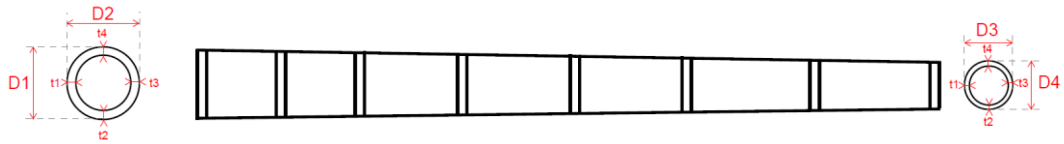

Figure 1: Locations of external diameters and wall thicknesses to measure (refer to nomenclature).

The net area of section, A (less any bolt holes), can be determined by the following equation:

$$A = \left[\frac{\pi}{4}\left({D_e}^2 - (D_e - 2t)^2\right)\right] - \sum area\ of\ any\ holes \qquad \text{Eq 1}$$

2.0 Element failure modes

2.1 Flexure

The capacity of a bamboo culm in flexure is limited by the transverse strain capacity of the lignin[8] (which is very roughly 1.1×10^{-3} – see Technical Note 1[9]), hence flexural failure generally manifests itself as a collapse by separation of the fibres (similar to bursting or local buckling) in the compressive zone[10].

The design flexural capacity of an element, M_m, can be determined assuming an elastic stress distribution and based on the average wall thicknesses and diameters determined in Section **Error! Reference source not found.**. Local buckling of walls cannot be ruled out. For species where the D_e/t ratio is larger than 11, this possibility needs to be examined either through analysis or experimentation.

$$M_m = X_{m,d} S_{elastic} \qquad \text{Eq 2}$$

where:

$$S_{elastic} = \frac{\pi\left(D_e{}^4 - [D_e - 2t]^4\right)}{32 D_e} \qquad \text{Eq 3}$$

Note that to date a connection system that allows composite action between culms in bending has not been developed – for further details refer to Trujillo and Archila[11].

Deflection of beams in bending often governs above flexure and should therefore normally be checked – see Section 4.0.

2.2 Shear

The capacity of a bamboo culm in shear is limited by the longitudinal shear capacity of the lignin between the fibres.

The design shear capacity of an element, F_v, can be determined assuming an elastic (approximately sinusoidal~~ly~~) stress distribution utilising the entire section of the culm, and based on the average wall thickness and diameter.

$$F_v = X_{v,d} k_{cr} \frac{3\pi t\left(D_e{}^4 - (D_e - 2t)^4\right)}{8\left(D_e{}^3 - (D_e - 2t)^3\right)} \qquad \text{Eq 4}$$

Longitudinal splits are common in bamboo (Figure 2) and are difficult to avoid, therefore it is recommended to apply a crack reduction factor, k_{cr}, similar to that used in EN 1995-1-1[6]. The recommended value for k_{cr} is given as follows:

$$k_{cr} = 0.5$$

This factor accounts for the risk of a single split through the culm wall.

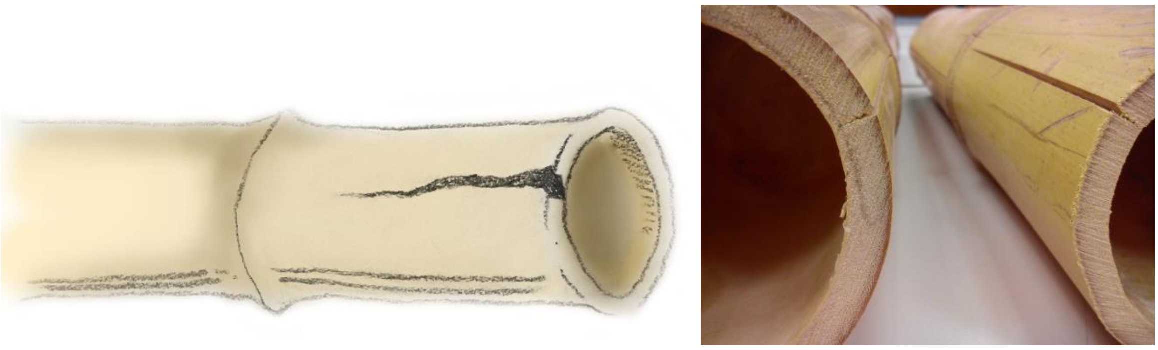

Figure 2: Splits in bamboo

2.3 Axial tension

The capacity of a bamboo culm in axial tension is limited by the tensile capacity of the fibres, however, this rarely governs since it is very difficult to mobilise the full axial tension capacity of a culm because the connections will tend to fail first in other mechanisms such as shear, local crushing or tension perpendicular to fibre.

The design allowable tensile capacity of an element, F_t, can be determined utilising the net area of the section (less any holes), and based on the average wall thickness and diameter.

$$F_T = X_{t,0,d}A \qquad \text{Eq 5}$$

2.4 Axial compression

The capacity of a bamboo culm in axial compression is limited by: local crushing of the fibres in short elements (~L<2D_e), bursting of the culm wall and separation of the fibres (due to failure of the lignin holding the fibres together) in medium length elements (~L>2D_e), and global Euler buckling of the section in long elements (~L>10D_e). The design allowable axial compressive capacity of an element, F_c, can be determined as per the following section. The approach uses equations which follow a Perry-Robertson curve with an initial imperfection of 3%, which assumes a maximum out-of-straightness of 2% (i.e., no part of the culm should deviate more than 20mm per metre length from a straight line drawn between the ends of a culm); the other 1% of the imperfection is a notional allowance for variations in the geometry of the cross section and the material properties. ~~The approach uses equations which follow a Perry-Robertson curve with an initial imperfection of 3%, which assumes a maximum out-of-straightness of 1% (i.e., drawing a straight line between the ends of the culms, no part of the culm should deviate more than 10mm per metre length outside this).~~ This out-of-straightness limit would need to be part of any grading regime. For some species of bamboo this may be difficult to achieve: where the out-of-straightness exceeds ~~1~~2%, the additional moment induced by this PΔ effect with the axial load should be considered together with the axial load.

Local buckling of walls cannot be ruled out. For species where the D_e/t ratio is larger than 11, this possibility needs to be examined either through analysis or experimentation.

2.4.1 Effective length, l_e

$$l_e = kL \qquad \text{Eq 6}$$

where k is taken from Figure 2.

	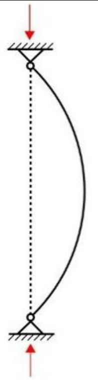	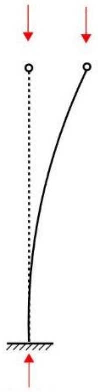
Support conditions	Non-sway mode, effectively held in position at both ends, not restrained in direction at either end	Sway mode, effectively held in position and restrained in direction at one end, not held in position or restrained in direction at other end
k	**1.0**	**2.1**

Figure 2: Coefficient of effective length, *k*

Note that other typical coefficients of effective length can theoretically be used (based on achieving a distance between points of contraflexure). However, due to the difficulty in actively restraining bamboo culms against rotation, the limited number of fasteners that can be used in connections and the brittle nature of connections, consideration should be given to whether connections can provide adequate restraint.

2.4.2 Slenderness, λ

$$\lambda = \frac{l_e}{r} \qquad \text{Eq 7}$$

2.4.3 Net area of section, A_{tot}

$$A_{tot} = \sum A_i \qquad \text{Eq 8}$$

2.4.4 Radius of gyration, r

$$r = \sqrt{\frac{0.9I}{A_{tot}}} \qquad \text{Eq 9}$$

The second moment of area has been reduced by 10% to account for the effect of the taper of bamboo[1].

2.4.5 Second moment of area, I

2.4.5.1 Elements formed from a single culm

For an element formed from a single culm:

$$I = \pi \frac{\left({D_e}^4 - [D_e - 2t]^4\right)}{64} \qquad \text{Eq 10}$$

2.4.5.2 Elements formed from multiple culms

It may be possible to achieve composite action in columns formed by multiple culms bundled together, however, to the authors' knowledge this has not been demonstrated experimentally[12].

Therefore whenever an element is formed from multiple culms, and composite action is not demonstrable, the second moment of area (or moment of inertia) for the element would be calculated thus:

$I = \sum I_i$ Eq 11

An element formed from multiple culms can be considered to act compositely in axial compression if full scale experimentation demonstrates that the experimentally measured second moment resembles that of Eq 12.

$I_{test} \approx \ I_{composite} = \sum(A_i \cdot d_i^2) + \sum I_i$ Eq. 12

2.4.6 Slenderness classification

Table 1 can be used to classify the element according to its slenderness from cl. 2.4.2 and Eq 7.

Table 1: Column slenderness classification (from NSR-10 G-12[4])

Column classification	Slenderness
Short (stocky)	$\lambda < 30$
Intermediate (slender)	$30 < \lambda < C_k$
Long (very slender)	$C_k < \lambda < 150$

where (from NSR-10 G-12[4]):

$C_k = \pi \sqrt{\frac{E_{0.05}}{\gamma_E X_{c,0,d}}}$ Eq 13

The γ_E is a material factor of safety, and its recommended value is given as follows:

$\gamma_E = 1.5$

In no circumstances should an element have a slenderness $\lambda > 150$.

2.4.7 Capacity

2.4.7.1 Short (stocky) elements, $\lambda < 30$

For stocky elements, the compressive capacity can be based on the squash load.

$F_c = X_{c,0,d} A_{tot}$ Eq 14

2.4.7.2 Intermediate (slender) elements, $30 < \lambda < C_k$

For intermediate slender elements, the compressive capacity can be a transition curve (from NSR-10 G-12[4]).

$F_c = A_{tot} X_{c,0,d} \left(1 - \frac{2}{5}\left[\frac{\lambda}{C_k}\right]^3\right)$ Eq 15

2.4.7.3 Long (very slender) elements, $C_k < \lambda < 150$

For long, very slender elements, the compressive capacity can be based on Euler.

$F_c = N_e = \frac{\pi^2 A_{tot} E_{0.05}}{\gamma_E \lambda^2}$ Eq 16

The γ_E is a material factor of safety, and its recommended value is given as follows:

$$\gamma_E = 1.5$$

2.5 Elements subject to combined bending and axial tension

For elements subject to combined bending and axial tension, the sum of the utilisations should be ≤ 1:

$$\frac{N_t}{F_t} + \frac{M}{MF_m} \leq 1 \qquad \text{Eq 17}$$

2.6 Elements subject to combined bending and axial compression

For elements subject to combined bending and axial compression, the sum of the utilisations should be ≤ 1:

$$\frac{N_c}{F_c} + \frac{M}{MF_m\left(1 - \frac{1.5N_c}{N_e}\right)} \leq 1 \qquad \text{Eq 18}$$

where

$$N_e = \frac{\pi^2 E_{0.05} I}{l_e^2} \qquad \text{Eq 19}$$

3.0 Local failure modes: bearing (crushing)

Bamboo is particularly susceptible to crushing under point loads and over supports. Without infilling the internode with mortar, crushing occurs by splitting of the sides of the culm outwards, and splitting of the top and bottom of the culm inwards. Where the bamboo is carrying a significant load (in practice in most cases), the internode should be infilled with an iuncompressible material such as cement mortar and the capacity should be checked as will be outlined in Article 5. Where the bamboo is not carrying a significant load (e.g., roof beams supporting a light roof with no live load) infilling the internode may be avoided, the procedure for checking the capacity will also be outlined in Article 5.

4.0 Deflection

Deflection checks of bamboo elements in bending under service loads often govern above flexural capacity. These can be checked using standard beam theory, using *I* as defined in Equation 10. Ranges of moduli of elasticity are suggested in Article 3[7], and should be selected based on the confidence required in the deflection calculation.

When checking deflections of a whole structure, consider accounting for slip in connections, especially if nailed or bolted.

Summary

This article proposes element design equations for typical loading conditions on a bamboo culm: flexure, shear, axial tension, axial compression and local bearing. Structural design of bamboo structures should follow a similar approach to that used in timber structures should be used – i.e. a view should be taken on how the load path might vary depending on the possible variations in stiffness of the connections and elements. Element design should always follow elastic stress distributions, and existing defects (such as cracks/splits), taper and out-of-straightness need to be considered – these can be controlled through a robust selection regime. It is important to use bamboo that has been dried to a moisture content close to the equilibrium moisture content in service ~~of equilibrium in the location where the structure will be built~~.

Areas of research that would be particularly beneficial with regards to element design are composite action between culms in flexure and composite action between culms working as a slender bunched column under axial compression – to-date these have been very difficult to achieve in the laboratory.

An update to the ISO bamboo standards[1,2,3] is currently underway and due for publication within the next few years. These standards should incorporate the state-of-the-art in bamboo research, and therefore are recommended to be used for bamboo design once available.

The next article in the series will cover connections.

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