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Integration of a geotechnical model within a morphodynamic model to investigate river meandering processes

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ABSTRACT: Despite significant progress made in the research conducted to understand the morphodynamics of meandering rivers using computer models, a number of challenges and limitations remain with respect to simulating lateral river channel adjustments. In particular, some biophysical processes critical to bank erosion (e.g. related to soil and vegetation) are often neglected or oversimplified, proxy variables such as flow velocity are used to predict lateral migration rates, non-physical assumptions are frequently made to simulate channel cut offs, and channel and floodplain processes are commonly studied separately. The objective of this paper is not to address all of these issues, but to present a new geotechnical model that was integrated into a numerical morphodynamic model to include lateral erosion due to mass wasting. The model accounts for floodplain morphology and river bank hydrology, without compromising computational efficiency. The integrated geotechnical component includes a set of physics-based rules to quantify slope stability across the simulation domain. It is managed by a fully configurable universal genetic algorithm with tournament selection to efficiently calculate the spatial extent of block slumps whose slip surface profile is allowed to be planar, circular or irregular. This module is compatible with any type of mesh structure, making it suitable for the investigation of the dynamics of single- and multi-threaded river channels. Following bank failure, the fine material is assumed to be immediately entrained by the flow, whereas the coarse fraction is deposited along the formally unstable slope at the friction angle of the bank material. By keeping track of floodplain topography, and not solely of channel morphology, the model allows for preferential pathways to develop on the valley floor, which may affect both the direction and rate of channel migration.

1 INTRODUCTION

1.1 *Meander dynamics*

The characteristics of the flow field and sediment transport patterns in meandering river channels are fairly well understood due to the observations made over decades in flume, field and numerical experiments. For instance, instrumental measurements (e.g. Bathurst et al., 1979; Thorne & Rais, 1984; Thorne et al., 1985; Frothingham & Rhoads 2003) and numerical simulations (e.g. Morvan et al., 2002) both demonstrated the existence of a helical flow in meander bends. The dynamics of planform development and evolution, however, remains less clear.

The irregularity of meander planform observed in nature is puzzling (Güneralp & Marston, 2012) and corresponds to the knowledge gaps on river meandering processes. Processes and features deemed complex have historically been left out of the equation in studies pertaining to meander morphodynamics, at best assuming uniformity in the environmental conditions. The effects of heterogeneity and spatiotemporal variability in the floodplain condi-

tions on planform evolution and on biophysical feedbacks are largely ignored (Güneralp et al., 2012; Güneralp & Marston, 2012). But, more generally, there is a notable paucity of knowledge related to the feedbacks between channel morphodynamics and floodplain patterns and processes (Pittaluga & Seminara, 2011; Güneralp & Marston, 2012).

1.2 *Morphodynamic modelling*

River meandering morphodynamics has been studied using numerical models for many years (e.g. Ikeda et al., 1981). While certain features typical of this type of river planform were reproduced to a certain extent, some issues remain. Firstly, the idealized meander bends developed through numerical simulations depart from those observed in nature, the latter being fairly irregular (Shen, 1984). This may be partly explained by the fact that many numerical models of meandering rivers focused on the initiation of channel meandering in the virtual environment (e.g. Duan et al., 2001; Asahi et al., 2013). It may also indicate that the available models are not

holistic enough to take account of all the important features of the natural environment in which meandering rivers develop and evolve.

Secondly, the physics and river geometry are often greatly simplified to allow for simulations to run for longer temporal scales, with the consequence that important processes are not taken into account. As an example, although process simplification is required to study the long-term evolution of meandering rivers, the HIPS formulation (see Ikeda et al., 1981; Johannesson & Parker, 1989; Zolezzi & Seminara, 2001) lumps the effects of hydraulic entrainment and mechanical bank failures into a single erodibility coefficient without physically describing the geotechnical processes that are responsible for lateral channel migration (Camporeale et al., 2005). The role of riparian vegetation in modifying river bank erodibility is generally not included in morphodynamic models (Malkinson & Wittenberg, 2007). Other features that are challenging to deal with in a meandering model are the chute and neck cut offs, for which no analytical solution exists (Chen & Tang, 2012). Finally, as a consequence of the geometrical constraints imposed by model design choices, a flat floodplain deprived from paleochannels is often assumed as the environment on which the single-threaded meandering channel will migrate. This prevents the creation of preferential pathways within an alluvial valley.

Most of the aforementioned issues could be addressed by more sophisticated models of bank retreat (Camporeale et al., 2005; Motta et al., 2012). In addition, developing and employing hybrid models that employ both reductionist and holistic approaches could help identifying patterns in the co-evolution of the river channel and floodplain (Güneralp et al., 2012). Finally, designing models in which the key features of a meandering river's environment can develop would certainly unlock the potential for morphodynamic models to tackle research questions related to the long-term evolution of these rivers (Crosato et al., 2012).

1.3 Challenges and opportunities

The morphodynamic simulations achieved within computational fluid dynamics (CFD) models are computationally intensive and commonly require a significant amount of time to complete due to the non-simplified set of equations governing the flow and to the finer spatial and temporal scales generally considered. An additional set of equations describes spatially-varied fluvial erosion. The consideration of lateral migration from a physically-based perspective (e.g. using the principles of limit equilibrium to evaluate slope stability) within a CFD model adds to the computational burden and discourages the use of such an integrated modelling solution to study the

evolution of a river channel over long temporal scales.

There are nevertheless multiple reasons motivating the choice of a CFD model to examine river morphodynamics. Firstly, more powerful computers and more efficient models are now readily accessible and provide new opportunities to tackle complex river dynamics questions. Secondly, the current state of technology already makes it possible to devise a number of experiments aiming to better understand morphodynamic processes in a model that minimizes geometrical restrictions, is able to represent the 3D helical flow, and can deal with both in-channel and overbank flow (Howard, 1996; Pittaluga & Seminara, 2011; Güneralp & Marston, 2012). This, in turn, would help developing hypotheses on the controls on river meandering (e.g. sedimentology, hydrological regime) and on the biophysical interaction of the river channel and floodplain (e.g. channel morphology vs. effects of bank and floodplain vegetation on the hydraulic field and mechanical soil reinforcement).

A bank migration module was designed and implemented within a CFD model by Duan et al. (2001) and Darby et al. (2002), to simulate bank advance and retreat along alluvial channels in a more realistic manner. Similarly, by developing a model that can simulate bank erosion, bank failures, point bar accretion and channel cutoffs, Asahi et al. (2013) explained the evolution in sinuosity and width of a meandering river, and examined the effects of flow magnitudes on the physiological characteristic of the developed river planform and channel bars. The fact that a greater number of modelling tools now integrate bank erosion as a physically-based process is likely to increase the number of studies using CFD to investigate morphodynamic problems.

1.4 Research objectives

This paper describes a physics-based, deterministic morphodynamic model that was developed to examine some of the questions and hypotheses related to river morphodynamics, which are difficult to tackle using existing meandering models due to the structural restrictions they impose on the simulation domain. In particular, this new model allows to simulate lateral river channel adjustments that can lead to the development of a meandering river planform geometry.

2 MODEL DESCRIPTION

This section describes the geotechnical module (GEOTECH) that was developed and integrated into the hydraulic solver suite TELEMAC-MASCARET. This new module is divided into five components

(Figure 1). A *landscape analysis* algorithm (section 2.1) generates a network of transects along which slope stability assessments are performed during a morphodynamic simulation. This algorithm detects slopes anywhere across the simulation domain, and not strictly along the external river bank of meander bends. A *genetic algorithm* (section 2.2) searches for the geometry of the most likely failure profile along each transect. Another algorithm performs *slope stability assessment* (section 2.3) to obtain the safety factor associated with any potential failure profile. The geotechnical module also includes a *river bank hydrology manager* (section 2.4) that computes water table elevation in the floodplain, near the river bank. Finally, a *slump block analyzer removes* the unstable slump blocks, *deposits* the material downslope and *updates* the computational mesh (section 2.5). These components work together to assess the geotechnical stability of a river channel and floodplain described by a triangular irregular network (TIN), subject to specific river flow conditions (free surface elevation), to include sediment transport through mass movement in addition to transport by fluvial processes.

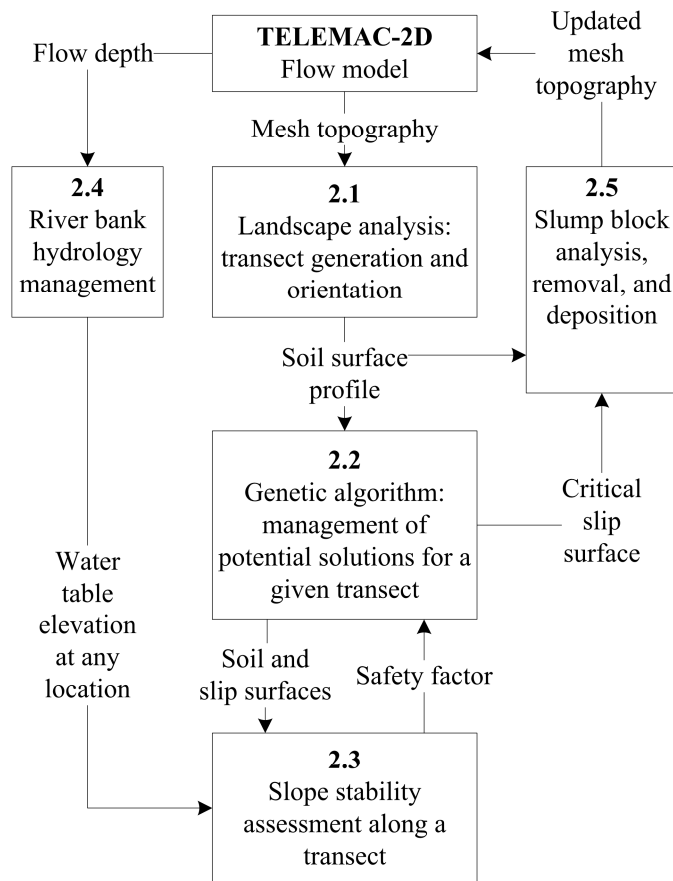


Figure 1. The five components of GEOTECH and coupling with the flow model TELEMAC-2D.

Information related to the flow and sediment transport modules of TELEMAC is available from the project website (<http://www.opentelemac.org>). The GEOTECH module was designed to be used with the depth-averaged 2D version of the TELEMAC flow model, but it should in theory also work with the 3D version. Furthermore, the new module is compatible with any computational mesh structure, and thus is compatible with the finite-element discretization scheme implemented in TELEMAC.

2.1 Landscape analysis

The geotechnical module presented in this paper was developed and coupled with TELEMAC to enable the examination of single- and multi-threaded channel dynamics within a CFD model as well as to provide a more holistic modeling tool for investigating the interactions between channel and floodplain processes.

The selected design, in particular the independence of the landscape stability assessment algorithm in respect to the computational mesh, allows geotechnical failures to occur at any location across the landscape. This fundamental feature of the module contrasts with the strategy implemented in most bank erosion models, where a body-fitted coordinate system is used to describe the bathymetry of a single-threaded channel, whilst the floodplain lacking topography and not considering the impact of previous erosion and deposition events (i.e. ignorance of paleo- and ephemeral channels). In these models, a stability assessment is achieved at each cross-section (corresponding to the longitudinal axis), which greatly simplifies stability assessment, but imposes constraints on the mesh structure, which renders the inclusion of complex floodplain processes challenging.

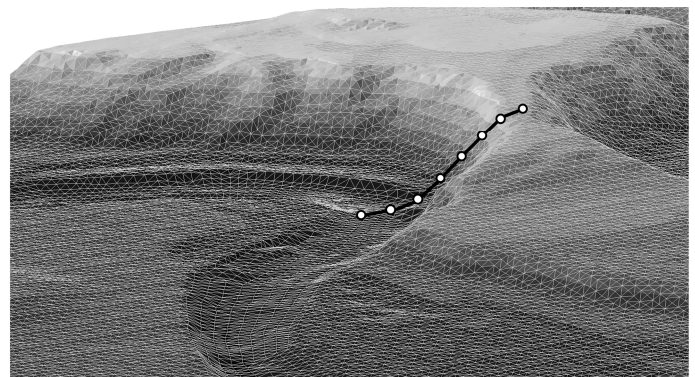


Figure 2. Bird's-eye view of the computational mesh, featuring a transect that consists of height nodes placed along a river bank for which a geotechnical analysis is to be performed.

At each iteration of the geotechnical module, horizontal transects are generated, and spaced evenly along the x- and y-axes of the computational mesh. The length of transects, their spacing, and the num-

ber of points forming each one are specified by the user, albeit the module automatically adjusting transect length according to the location of domain boundaries. Following this, each transect is rotated until pointing in the direction of steepest ascent. This operation is necessary to avoid underestimating the gradient of hillslopes, and thus not properly detecting geotechnical instabilities. Finally, the length of each transect is adjusted by removing or adding nodes at both extremities until the computational

mesh profile associated with the transect is monotonically increasing or decreasing, e.g. rising from channel bed to bank top (Figure 2).

To improve computational efficiency, the geotechnical model can be configured to consider only those transects that are entirely submerged, entirely dry, or partially dry, i.e. located at the edge of the flow. This option is relevant to the investigation of landscapes in which the dry areas are assumed to be geotechnically stable. An example illustrating the

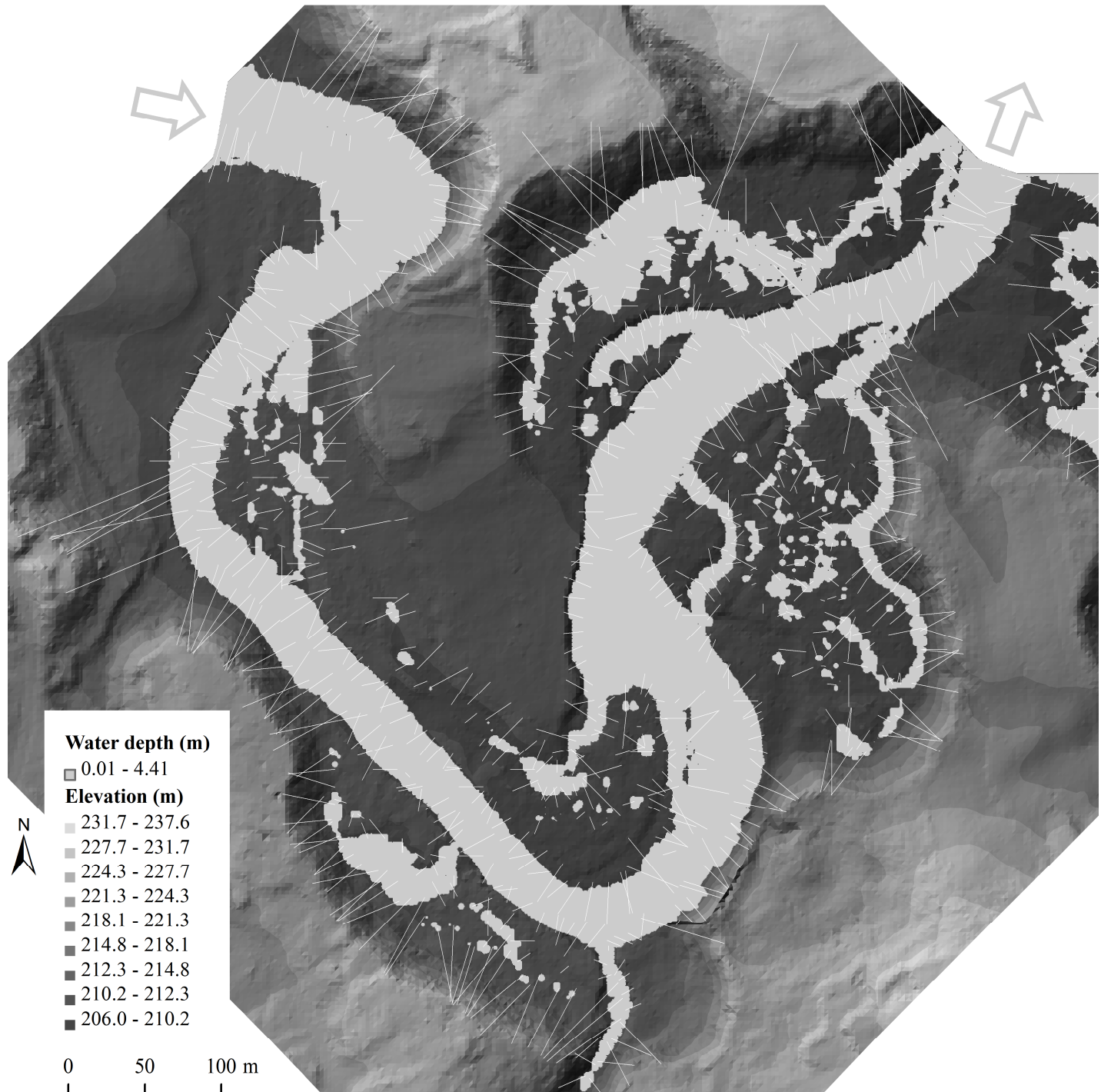


Figure 3. Outcome of the transect generation and orientation procedure in GEOTECH, performed for a short reach comprised in the valley of the semi-alluvial Medway Creek, London, Canada. The topographic dataset is a combination of data acquired using LiDAR (floodplain) and centimeter-level differential GPS (main river channel) technologies. Flow depth is predicted with TELEMAC-2D and corresponds to a discharge of $70\text{m}^3/\text{s}$ (recurrence interval of 4.2 years). In this reach, mean channel width is 20m. The arrows indicate the location of the inlet and outlet, and flow direction. A horizontal spacing of 0.71 times the transect length (16m) is employed, which ensures a complete coverage of the landscape in the stability analysis. Only the partially submerged transects are shown.

outcome of the transect generation and orientation procedure for an entire simulation domain is given in Figure 3.

2.2 Genetic algorithm

A critical slip surface, i.e. the interface between a block slump and the underlying more stable soil material, is commonly located using a grid search strategy. This procedure searches for the slip surface producing the lowest safety factor within a slope, considering a set of trial surfaces obtained by gradually varying geometrical characteristics. The genetic algorithm with tournament selection described in Li et al. (2010) was selected and implemented to maximize the efficiency of the slip search process. The aim was to design a single algorithm that would consider different slip surface shapes, namely planar, circular and non-circular, and which thus includes the mechanisms of mass movement resulting in the translation and rotation of the soil material. The current algorithm, however, imposes a monotonic slope to the critical slip surface, a constraint that will be removed in a subsequent version of the module.

The genetic algorithm is based on the theory of natural selection, whereby an optimal solution is allowed to 'evolve' out of a set of randomly chosen initial population of solutions. Each potential solution has a genetic makeup consisting of the points defining its geometry, such that a solution S with identifier id is represented by the vector:

$$\vec{S}_{id} = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_{n-1}, \vec{v}_n\} \quad (1)$$

A set of user-configurable rules dictate partner matching during cross-breeding. These are designed to optimize diversity, and thus reduce the time required to perform a stability assessment. The first rule ensures that two partners are not family relatives. A family policy then limits the permitted number of children per couple. Finally, an option allows partner exclusivity to be enabled or disabled.

The genetic algorithm converges to a solution when a minimum number of generations have been created and if no new fittest solution was found during the course of a second user-defined number of generations.

2.3 Slope stability assessment along a transect

The safety factor (F_s) is used to quantify the stability of a slope, assuming a given geometry of slip surface, i.e. the potential solutions generated by the genetic algorithm. The two-dimensional Bishop's method of slices is employed to perform the geotechnical evaluation (Figure 4a). The following set of equations is solved iteratively:

$$F_s = \sum_{i=1}^n \frac{cb_i + \frac{[\tau_i - U_i b_i] \tan \phi}{m_i}}{\sigma_i} \quad (2)$$

$$\tau_i = W_{s,i} \cos \beta_i + F_{cp,i} \cos \delta_i \quad (3)$$

$$\sigma_i = W_{s,i} \sin \beta_i + F_{cp,i} \sin \delta_i \quad (4)$$

$$m_i = \cos \beta_i + \frac{\sin \beta_i \cdot \tan \phi}{F_s} \quad (5)$$

in which $W_{s,i}$ = weight of soil material in slice i ; U_i = pore water pressure at the base of slice; b_i = slice width; $F_{cp,i} = W_{w,i} \cos \beta_i$ = confined water pressure exerted by the flow; $W_{w,i}$ = weight of water content; β_i = slice base angle; α_i = slice top angle; δ_i = angle between the result of hydrostatic confining force and normal to failure plane; ϕ = friction angle; m = a term in Bishop formula; and n = number of slices. A slope is expected to fail if the safety factor associated with its critical slip surface is less than 1.0.

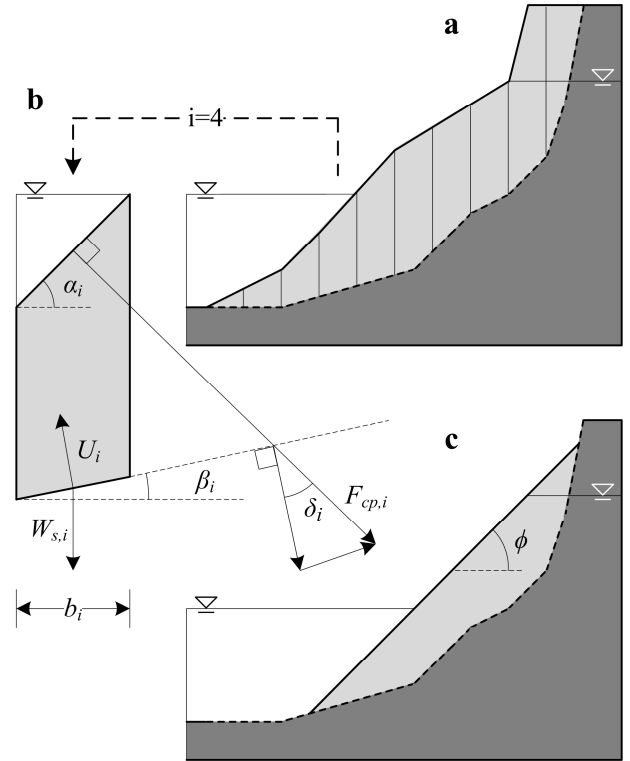


Figure 4. Example of slope stability assessment in 2D using Bishop's simplified method of slices. The light-shaded area represents the unstable portion of the river bank, whereas the dark-shaded portion is stable. The initially unstable river bank profile shown in 'a' is transformed into the profile shown in 'c' after disintegration of the slump block, entrainment of the finer particles by the flow, and deposition at the bank toe of the coarser sediment fraction at the friction angle ϕ . The forces acting on a slice i and the variables used in Equations 2-5 are shown in 'b'.

2.4 River bank hydrology management

The soil material comprised in a slice may be partially or entirely saturated with water if the elevation of its base is located below the water table, thereby increasing its weight (W_s in Equations 3-4) relative to that of its dry state. Thus, the elevation of the water table needs to be determined prior to calculating the weight of a slice. A river bank's water table responds slower to fluctuations in flow discharge than the river's water elevation. To account for the lag effect between flow and water table elevations, the following exponential function is used to calculate the elevation of the water table (z'_{wt}) at a time $t=t_0+\Delta t$:

$$z'_{wt} = z_{fs} - (z_{fs} - z_{wt})e^{-k\Delta t} \quad (6)$$

where z_{wt} and z'_{wt} = water table elevation at times t_0 and $t=t_0+\Delta t$; z_{fs} = flow surface elevation at time t_0 ; and k = rate of convergence of the water table elevation toward z_{fs} . Note that the value of the constant k must be adjusted to take account of the hydraulic conductivity of the simulated floodplain, and thus of the rapidity by which it adapts to a change in the river's free surface elevation. Since a different response time is expected between the rising and falling stages of the water table (faster response during the rising than the falling limb), two k -values must be provided per simulation.

2.5 Slump block removal and deposition

Two options are available in the geotechnical module to determine the fate of the soil material comprised in a failing slump block. The first option assumes that the material is immediately disintegrated and entirely entrained by the flow. With the second option, the finer material (clay, silt, and sand) is entrained by the flow while the coarser fraction (gravel) is deposited downslope at the friction angle of the bank material. The latter option is illustrated in Figure 4, assuming that the bank material consists of equal volumes of fine and coarse particles, a friction angle of 45° , and constant hillslope morphology along the river channel length. According to this scenario, the area of the slump block (zone in pale grey) is halved as a result of a mass wasting event (from Figure 4a to 4c).

An iterative procedure is employed to update the computational mesh following a slope failure event along an unstable transect. Note that the transects (used in the 2D geotechnical assessment) and the computational mesh (described in 3D) are comprised in two different layers. Hence, the location of a transect node generally does not coincide with that of a mesh node. In addition, mesh nodes are mobile vertically, but fixed laterally. Therefore, the nodes of the mesh elements overlaying the unstable transect nodes are displaced vertically until the difference in

volume between the pre- and post-failure computational meshes matches the desired volume of eroded material (according to the fraction of fines), and in a manner such that the gradient of the updated mesh elements (along the transect) form a slope at the friction angle (e.g. Figure 4c). A 3D solver is necessary to manage sediment deposition due to the morphological heterogeneity of landscapes (e.g. longitudinal variation in river bank morphology), and due to the option available in the GEOTECH that allows to determine the volume of material to be deposited downslope based on the fraction of coarse material in the slump block.

3 DISCUSSION AND CONCLUSION

The geotechnical module presented in this paper (GEOTECH) was developed and integrated into the computational fluid dynamics model (CFD) TELEMACH to provide a physically-based tool to study the dynamics of alluvial rivers. The set of algorithms provide a universal and efficient solution to describe lateral channel erosion in a range of river environments (e.g. single- or multi-threaded), geomorphic features and evolutionary phenomena, without the need to define context-based assumptions and rules. This is mainly attributed to the fact that the geotechnical module performs stability assessments independently of the structure of the computational mesh, and considering the morphology of a landscape rather than solely that of a channel bed, keeping track of paleo-channels and considering secondary channels. Therefore, the module is suitable to the examination of interactions between a river channel and its floodplain.

The inclusion of a slope stability analysis that takes into account the elevation of the water table (which varies with a lag as a function of river flow depth) and the confining pressure of the flow provides a modelling solution that is well suited for the examination of meander morphodynamics in floodplains with cohesive soils, but also for the study of alluvial rivers in general. Since the use of CFD models results in computationally intensive simulations, a genetic algorithm was implemented to converge more quickly to a solution during geotechnical assessments.

Future work will consist in validating the model against datasets from real rivers. Particular attention will be given to the predicted location, magnitude and timing of river bank failures during isolated flow events and over periods of several months.

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