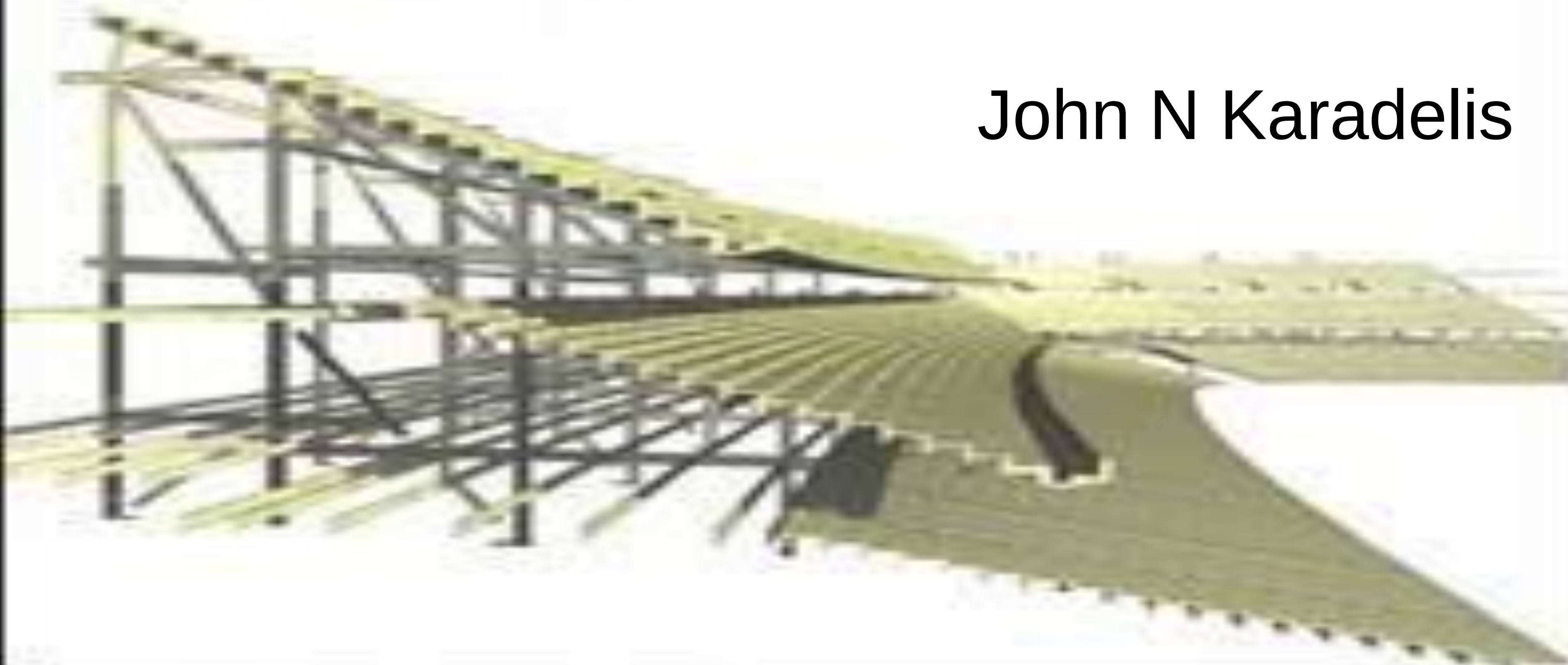
Grandstand Terraces.
Experimental and Computational Modal Analysis.

John N Karadelis



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INTRODUCTION

- Structural vibrations caused by human activities are not known to be particularly damaging or catastrophic.

- They may be the cause of **concern, discomfort and distress** to occupants or users of the structure.

- Researchers agree: it's a **SLS** problem.

- Research at Coventry University, aims to provide a rigorous interpretation of the behaviour of these structures under dynamic actions, by means of numerical modelling.

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Main Objectives

- Use FE method to accurately **simulate** a group of grandstand terraces.

- **Compare** these models with carefully conducted lab. and/or full scale tests. Highlight inherent uncertainties in simulation and (why not?) experimentation.

- **Alert** on the effect of steel reinforcement on the dynamic properties of these structures and identify any likely patterns related to the above.

- **Investigate and quantify** the effect of support conditions on these properties.

- Use these models for **further**, numerical analysis work.

BACKGROUND

Modelling real behaviour of RC structures, is not straight forward because of,

- the inherent **non-linear** behaviour of the material,
- the fact that there is no, so far, globally accepted **constitutive law**.

Nevertheless, a respectful number of models have been developed in the past.

- Comprehensive survey of computational ‘anthology’ by Hofstetter & Mang (1995)

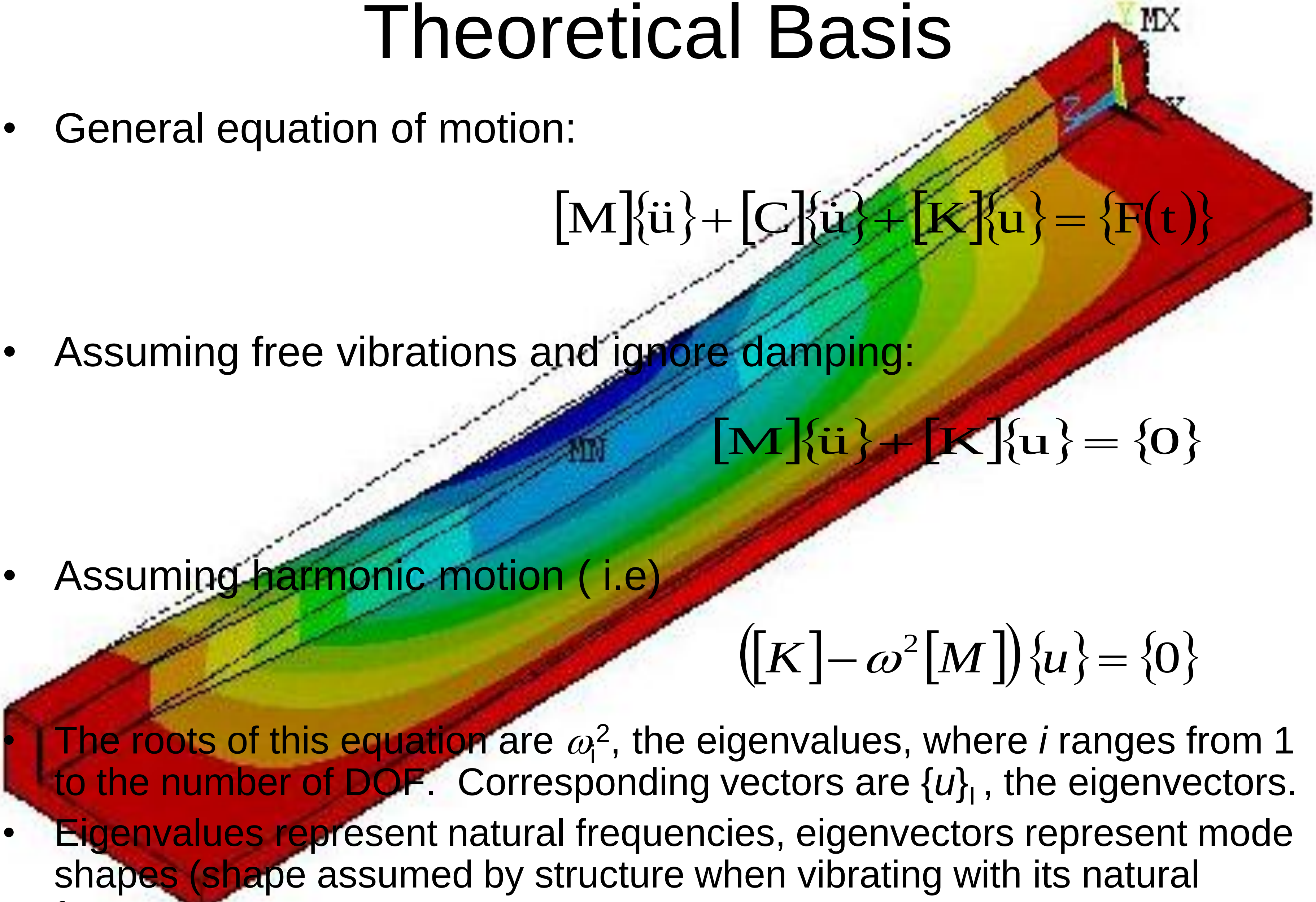
Several researchers have made successful (and not so successful) attempts in the past.

- In 1998, Reynolds *et al*, developed a FE model of a post tensioned concrete floor to match their modal test results. However, they did not report on any specific findings resulting from the comparison of their final FE and experimental work.
- Same authors, a year later (1999), attempted the stepped modelling of a multi-storey car park including columns, as opposed to simple pin supports, and claimed better correlation. Again, no limitations or drawbacks were reported. Connections/interfaces were not mentioned anywhere.
- Slender steel columns and their connections can be very “tuneful” and highly “*emotional”.* Hence, require accurate computer representation.

- In 2006, Michel and Cunha independently, experimented with ambient type (output only) vibration, obtaining acceptable results for the first two modes.
- The drawback with the ambient vibration is that, methods such as the Fourier based spectral analysis is not appropriate to analyze ambient data because of low SNR (signal-to-noise ratio).

- Ambient, as opposed to input (hammer, shaker, or earthquake) records, have the advantages of being of ‘infinitely’ <u>long duration</u>, ‘<u>stationary</u>’, and in most cases <u>linear</u>. However, they contain an excessively high level of noise making their utilization problematic, or even prohibitive.

Theoretical Basis



- General equation of motion:

$$[\mathrm{M}]\{\ddot{\mathrm{u}}\}+[\mathrm{C}]\{\dot{\mathrm{u}}\}+[\mathrm{K}]\{\mathrm{u}\}=\{\mathrm{F}(\mathrm{t})\}$$

- Assuming free vibrations and ignore damping:

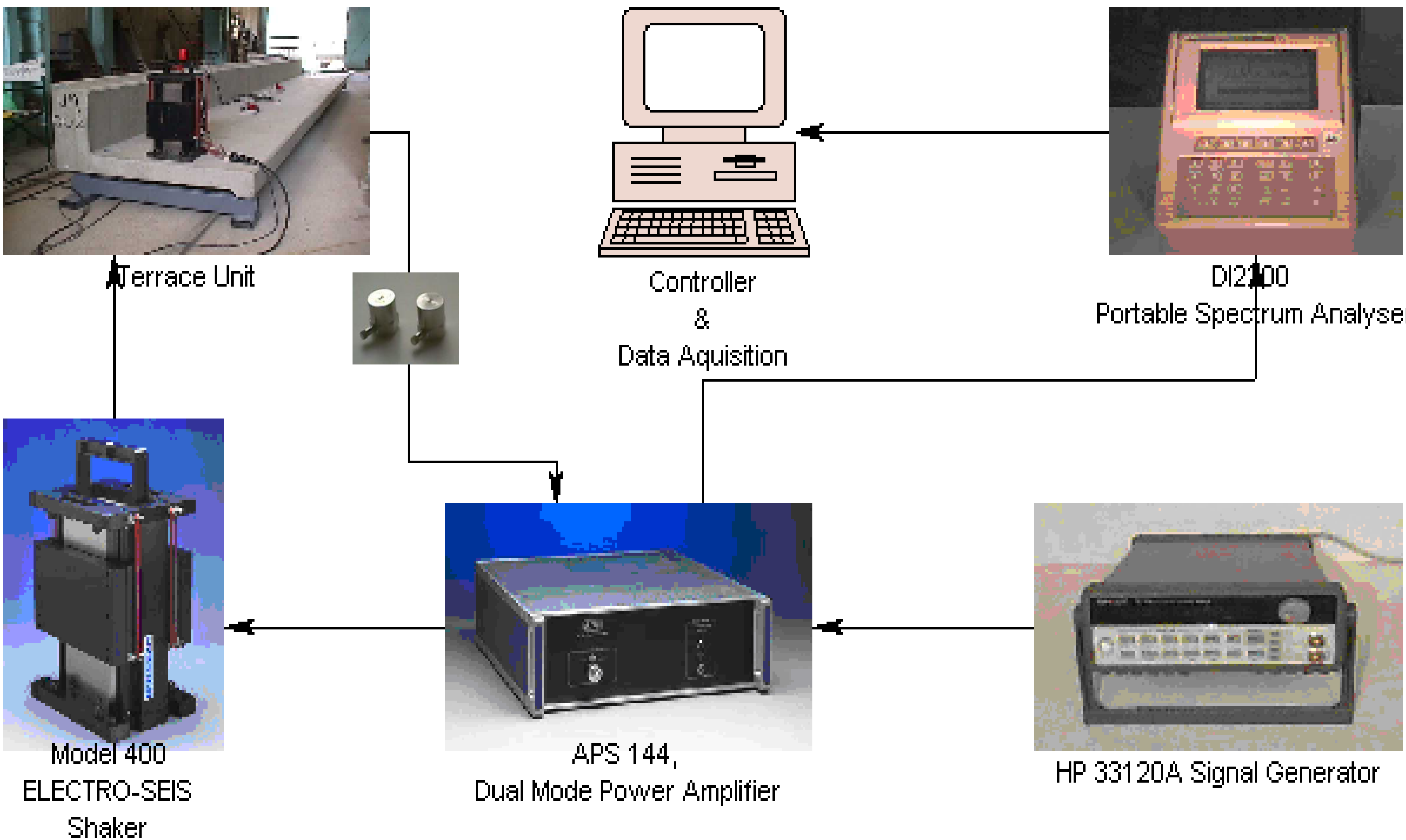
$$[\mathrm{M}]\{\ddot{\mathrm{u}}\}+[\mathrm{K}]\{\mathrm{u}\}=\{0\}$$

- Assuming harmonic motion (i.e)

$$\left([K]-\omega^2[M]\right)\{u\}=\{0\}$$

- The roots of this equation are ω_i^2, the eigenvalues, where *i* ranges from 1 to the number of DOF. Corresponding vectors are $\{u\}_i$, the eigenvectors.
- Eigenvalues represent natural frequencies, eigenvectors represent mode shapes (shape assumed by structure when vibrating with its natural frequency.

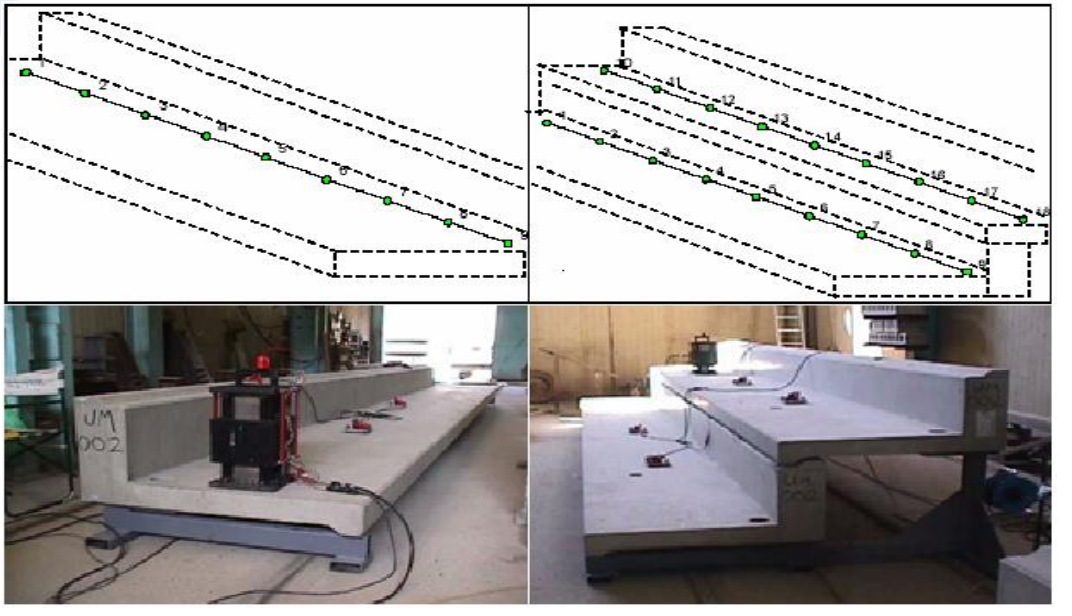
Data Acquisition System



Test Programme & Methodology

<u>Objective</u>: Was to take measured data, which relate to the *response properties* (eg.: frequency) of a structure and from these to extract the *modal properties* (natural frequencies, etc.)

- **<u>Method</u>:**
- Divide each unit into a set of lumped masses "connected" to a set of *'spring/damper'* elements.
- Masses were used as data collection test points (TPs).
- Properties of structure were determined by measuring the FRF between each of the TPs.



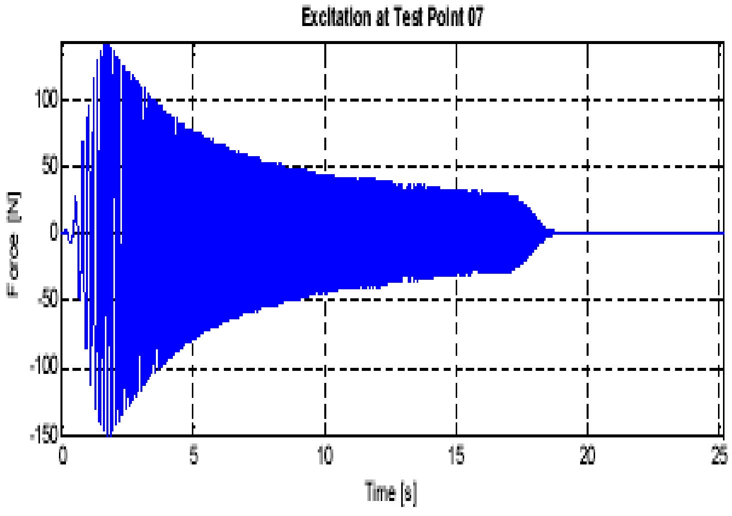
- **<u>Procedure</u>:** By shaking, say, TP_1 and measure acceleration at all other mass positions in the structure, the $FRF_{1,1}$, $FRF_{1,2}$, …,$FRF_{1,n}$ were obtained. Natural Freq: determined from the peaks of (FRF) v ω, or ϕ v ω.
- Mode Shapes: by assuming displacement @ TP_1 = 1, and then use displ't transfer functions to determine how all the other TPs move, relative to TP_1.

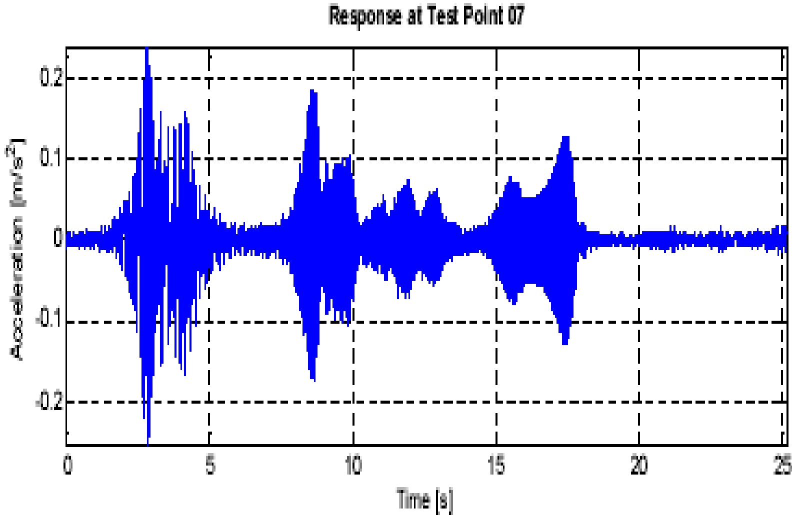
- A *chirp* excitation (short bursts of sine sweeps) was selected for the FRF measurements. Chirp signals allow for a series of averages to be taken with a spectrum analyser over its analysis range. Hence, more representative.

- Chirp excitation swipes from 1 Hz to 70 Hz, while the unit passes through a number of resonance frequencies.

- *Coherence* was used to validate testing (cause and effect signals were collected and compared; accepted when ratio: cause/effect ≃1).

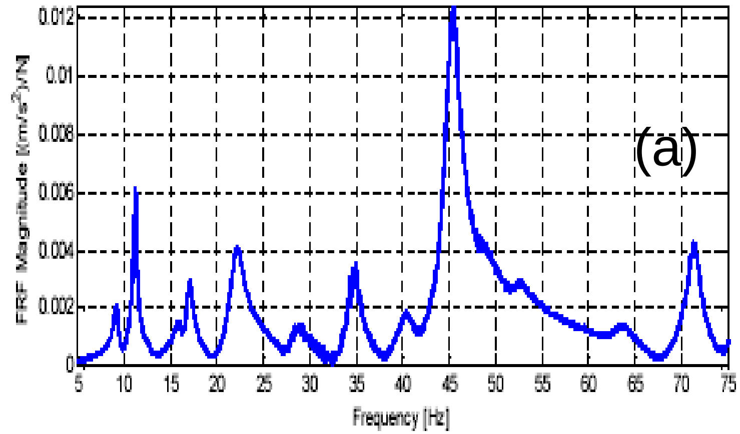
- As motion may be described in terms of displacement, velocity, or acceleration, FRF is called **compliance, mobility, accelerance.**

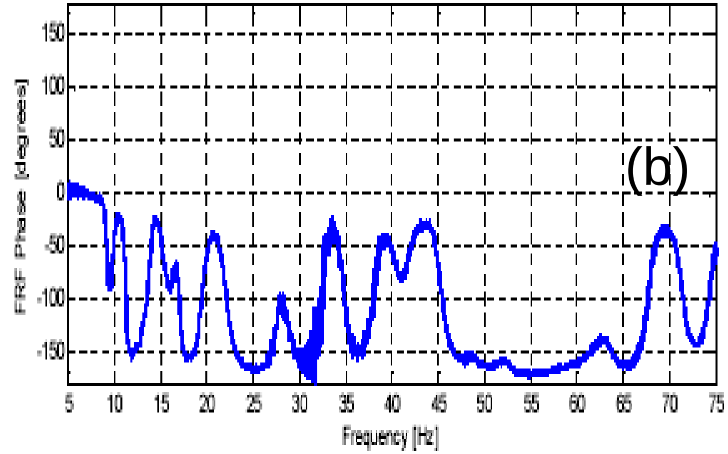




- The term ‘mobility measurement’ is used to describe any form of FRF.

Typical *point mobility* FRF, at TP7 after four frequency domain averages.
(a) FRF peaks and
(b) Characteristic phase changes at frequencies corresponding to the natural frequencies of certain, estimated modes of vibration.





FRF data were imported into *ICATS* for modal parameter estimation and for creating eigen files. The program allows the user to perform ‘what-if’ analysis introducing new masses, bracing, other forms of damping etc.

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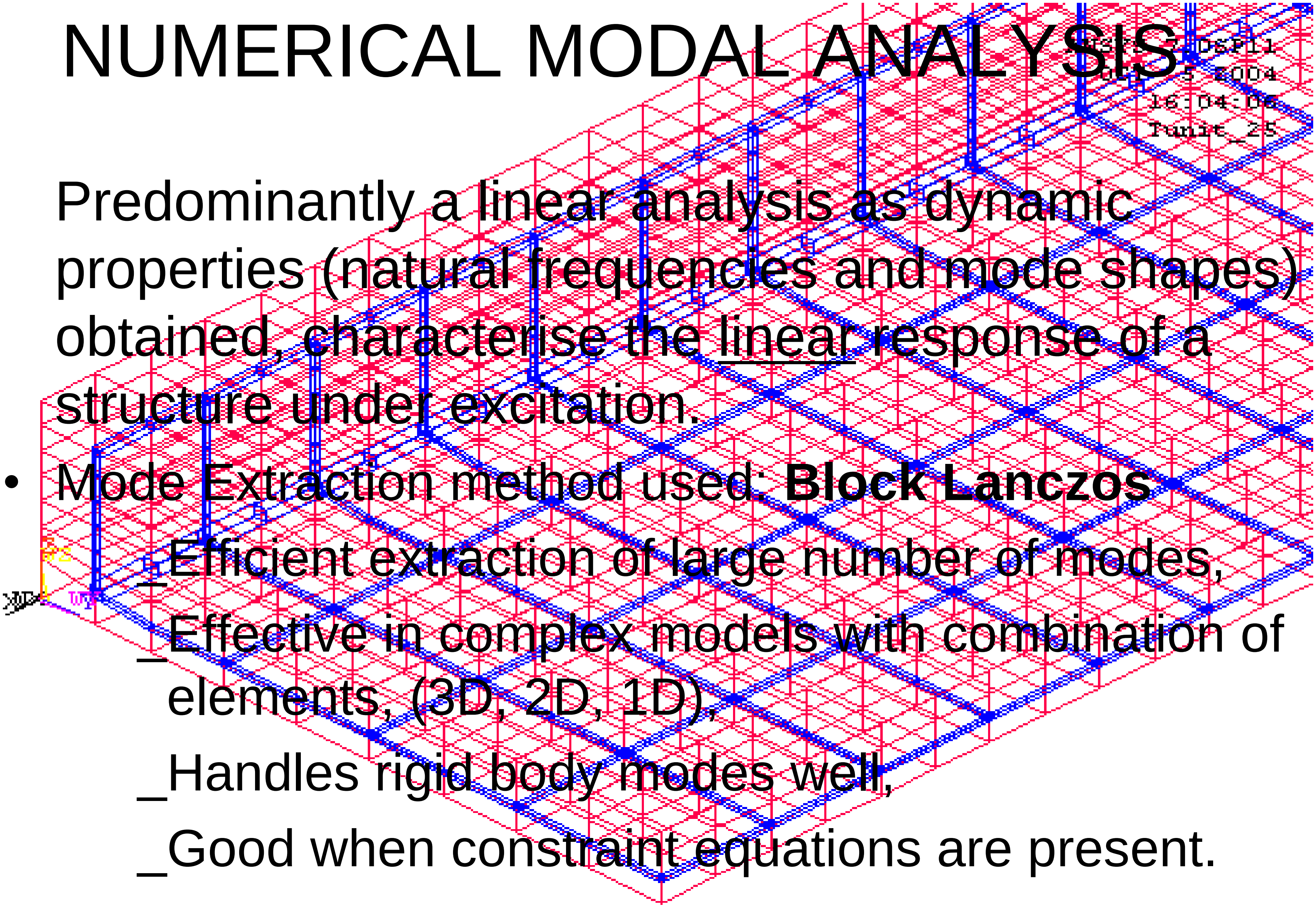
NUMERICAL MODAL ANALYSIS



Predominantly a linear analysis as dynamic properties (natural frequencies and mode shapes) obtained, characterise the linear response of a structure under excitation.

- Mode Extraction method used: **Block Lanczos**
  - _Efficient extraction of large number of modes,
  - _Effective in complex models with combination of elements, (3D, 2D, 1D),
  - _Handles rigid body modes well,
  - _Good when constraint equations are present.

The steel reinforcement

- Inserted in stages, to study its effect on natural frequencies of the unit, as there are conflicting ‘signals’ from a number of researchers!

  Similar studies at MSc level revealed no specific pattern. Demonstrated a tendency to change the dynamic behaviour of a SS-RC beam of rectangular section.

- Noticed (with caution) that considerable increase in the amount of reinforcement (no quantification attempted), is likely to increase certain modal frequencies and decrease others.

  Ie: introducing flexural reinforcement resulted in increasing the first two natural frequencies associated with bending modes. However, had no effect on the next two modes associated with predominantly ‘torsional’ vibrations.

- It seems that, reinforcing and somehow increasing the ‘specific’ stiffness of a particular structural element, may result in “forcing” this element into a different mode of vibration, resulting in lower corresponding natural frequencies.

Modelling Support Conditions

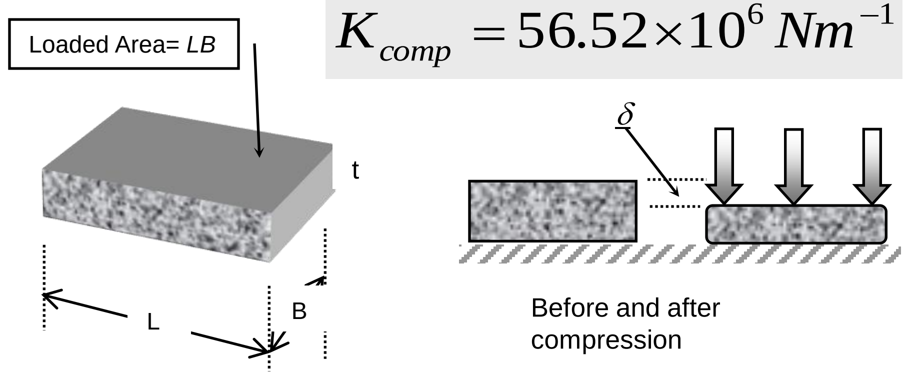
Objective: Seek better correlation between measured and predicted results. Achieved by accurate representation of support conditions.

MODE NUMBER	Measured (Hz)	Damping Ratio (%)	Predicted Plain (Hz)	Predicted Reinforced (Hz)
1	12.000	1.4	17.276	17.708
2	14.700	2.0	29.375	29.380
3	30.000	1.2	41.672	41.373
4	40.000	1.0	76.801	76.933
5	67.300	1.6	122.740	125.010
6			143.170	142.670
7			172.980	171.620
8			186.670	184.270

Precast concrete terrace units rest on elastomeric bearings (neoprene pads). They can act as natural vibration absorbers, influencing the vibration performance of the units.

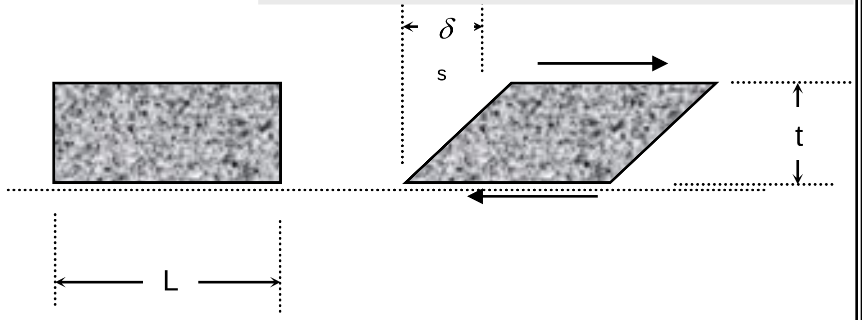
A lab. investigation was launched.

$$K_{comp} = 56.52 \times 10^{6}\, Nm^{-1}$$



Before and after compression

$$K_{shear} = 1.248 \times 10^{6}\, Nm^{-1}$$



RESULTS

ANSYS provides the user with a variety of elements with stiffness capabilities of which two were set apart:

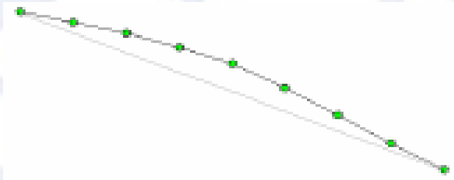
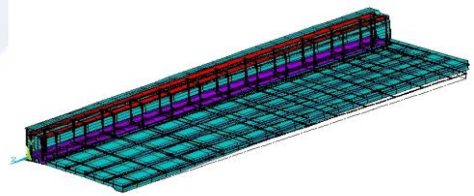
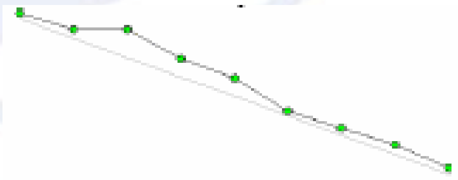
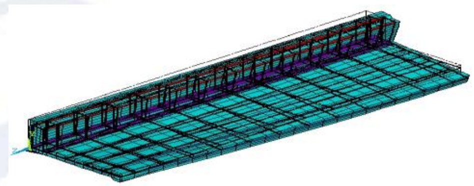
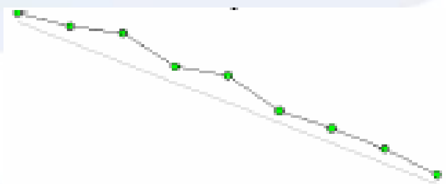
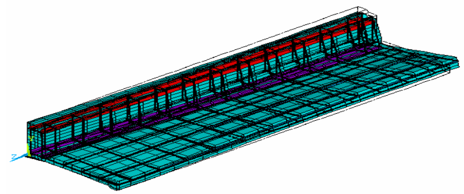
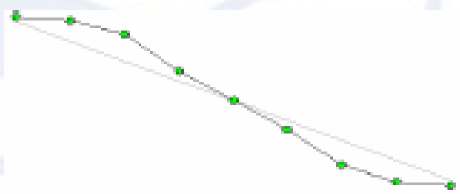
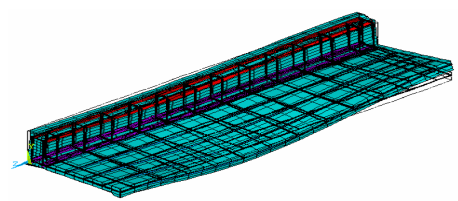
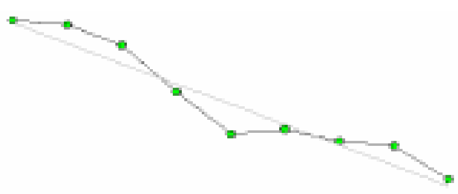
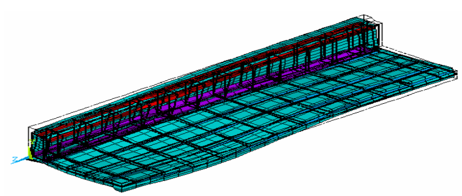
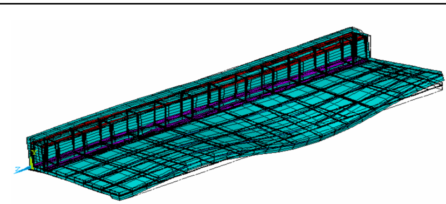
COMBIN14, element with a combination of spring and damper capabilities.

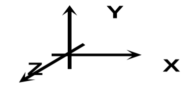
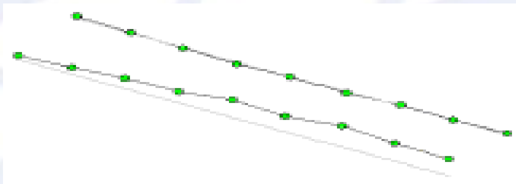
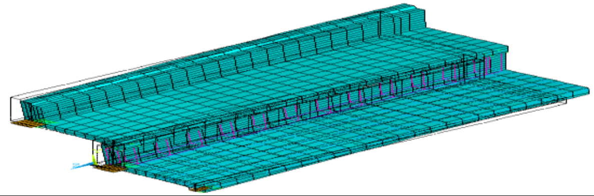
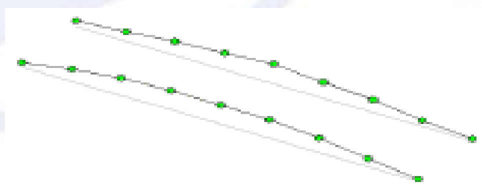
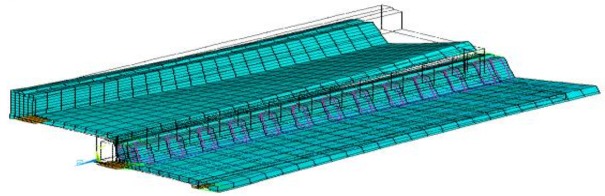
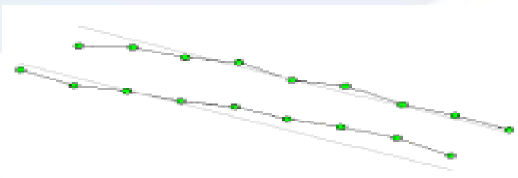
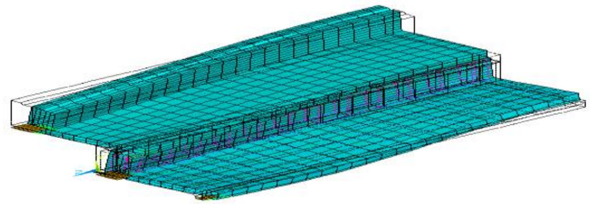
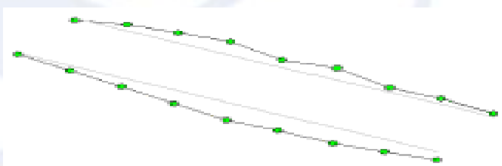
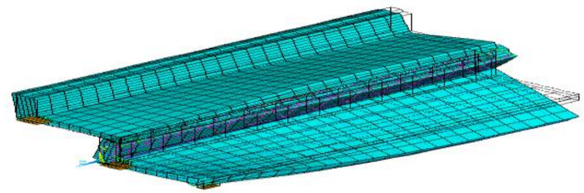
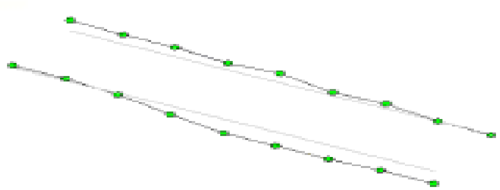
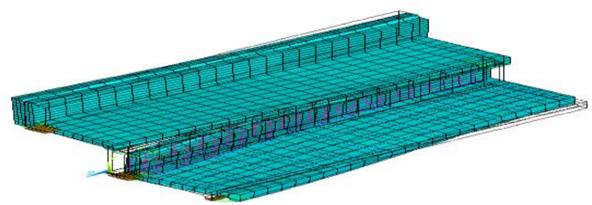
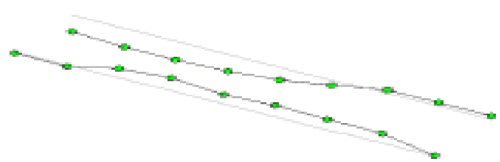
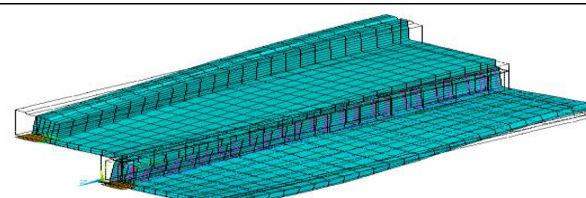
MATRIX27, element whose elastic kinematic response can be specified by stiffness, damping, or mass coefficients. Unique capability to relate two nodes (on structure and on some fixed medium) each with 6DOF, translations and rotations in and about X,Y,Z. Matrices generated by this element are 12x12. After several trials, MATRIX27 was adopted.

$$\begin{bmatrix} 1.25 & 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 & 0 & 0 \\ & 56.52 & 0 & 0 & 0 & 0 & 0 & -56.52 & 0 & 0 & 0 & 0 \\ & & 1.25 & 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & -1.25 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 56.52 & 0 & 0 & 0 & 0 \\ & & \mathit{symmetric} & & & & & & 1.25 & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & & 0 & 0 \\ & & & & & & & & & & & 0 \end{bmatrix}$$

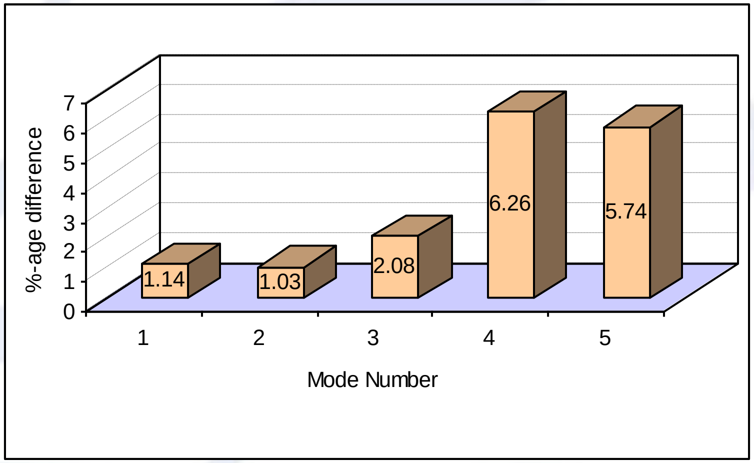
$\times 10^6$

Mode Number	Measured Damping Ratio (%)	Measured Freq. (Hz)	Predicted Freq. PC (Hz)	Predicted Freq. RC (Hz)	Predicted Freq. RC+NeoP (Hz)
1	1.4	12.000	17.276	17.708	12.243
2	2.0	14.700	29.375	29.380	14.549
3	1.2	30.000	41.672	41.373	30.872
4	1.0	40.000	76.801	76.933	44.560
5	1.6	67.300	122.740	125.010	71.473
6			143.170	142.670	
7			172.980	171.620	
8			186.670	184.270	

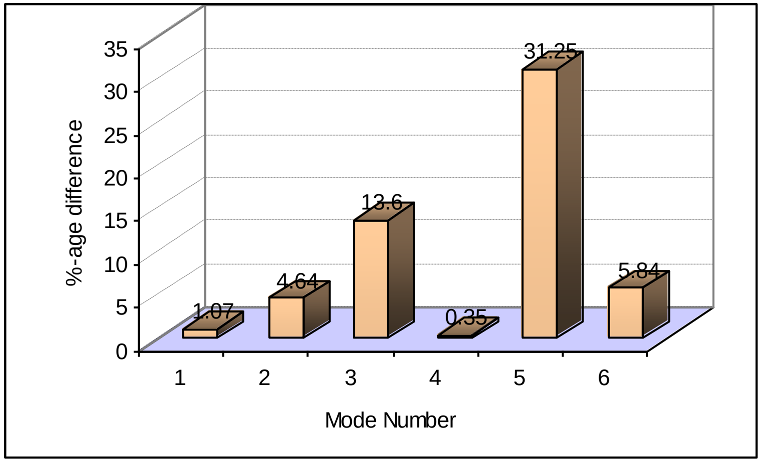
Mode No.	Experimental Modal Analysis Mode Shape	FE Modal Analysis Mode Shape	*f* (Hz) Ex p . Theo . ζ_{exp} (%)	Comments
1			12. 1 0 12. 24 1.4	The fundamental bending mode of vibration.
2			14.7 0 1 4.55 2.0	Predominantly torsional. May also be showing small amount of bending.
3			30.0 6 30 .7 0 1.2	Similar to Mode 2.
4			40. 0 1 4 4 .56 1.0	The second (flexural) mode of vibration.
5			67.3 7 1 .4 1.6	Near perfect flexural mode with a small amount of torsion.
6	?		103.00	Bending Mode

Mode No.	Experimental Modal Analysis Mode Shape	FE Modal Analysis Mode Shape	*f* (Hz) Exp. Theo. ζ_{exp} (%)	Comments 
1			9.30 9.40 2.2	Upper unit is static. Lower unit lifts corner over support (Rigid Body)
2			11.20 11.72 1.5	First bending & slightly twisting mode. "In phase" vibration
3			16.00 14.08 2.5	Coupled. Bending & twisting in unison about Z-axis. "In-phase" motion.
4			17.00 17.06 2.0	Mainly twisting mode about Z-axis "Out-of-phase" Vibration.
5			22.4 29.40 2.6	Upper unit: "Rigid Body" twisting about Y-axis. Lower unit: Mainly bending.
6			29.1 30.80 3.2	A complex, mainly torsional, and flexural vibration.

The following charts display the percentage difference between recorded and predicted frequencies per mode number for:



Single terrace unit



Double terrace unit

- No specific pattern emerged at this stage.
- Best agreement (at least for the first three modes of vibration) is achieved for the single unit, as expected.
- Certain findings from the single and double terrace units could be fed to the next stage: Modal Analysis of a (*mock*) Grandstand.

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Conclusions

- Overall, experimentally obtained natural frequencies were in good **agreement** with those predicted by the FE model.
- Experimental modal analysis alone may not be adequate, from the reliability point of view, unless a great deal of **experience** in interpreting results, combined with state-of-the-art **equipment** is available.
- FE analysis should serve as an **aid** and should supplement the former in an effort to explain more accurately the resulting mode shapes especially at higher modes of vibration. In fact, experimental and FE analyses should **interact** with each other for more accurate and successful results.
- Initial and short term tests and results suggest that the amount of reinforcement has only **very little** effect on the dynamic properties of the uncracked reinforced concrete terrace units.
- However, interim studies hinted towards the possibility that an increase in the amount of reinforcement is likely to force the structure into a different mode of vibration, hence **altering** the previously obtained dynamic properties.

- The dynamic properties of the terrace units were found to be very sensitive to **support conditions** (and/or other connection points, if any), Essentially, it was found that a gradual improvement of the predicted natural frequencies was evident by progressively improving simulation of the boundary conditions.

- Best correlation was achieved when the **behaviour** of neoprene pads was modelled by representing their stiffness characteristics with a matrix, MATRIX27.

- However, there seems to be a **limit**, above which further accuracy is actually **negligible,** in which case one should turn to the experimental methods/procedures to improve accuracy!

- The use of the appropriate number of **triaxial** accelerometers backed with the state-of-the-art **data logging** and **processing** equipment is recommended. However, such equipment is still very expensive and therefore not readily available.

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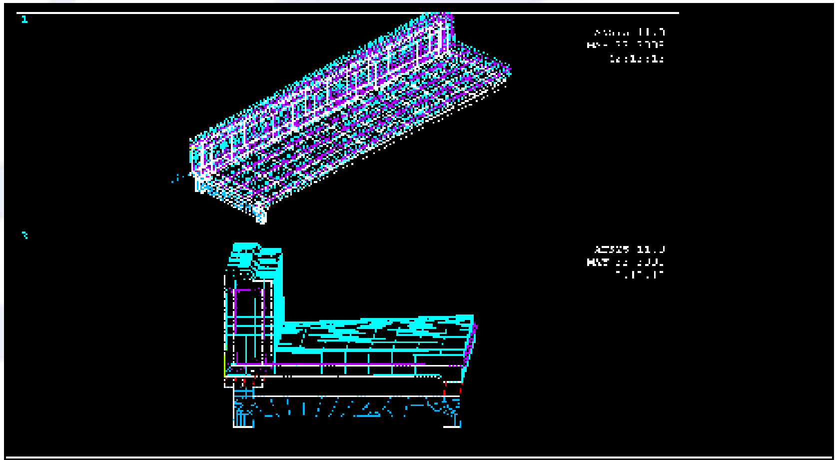
The mock grandstand,

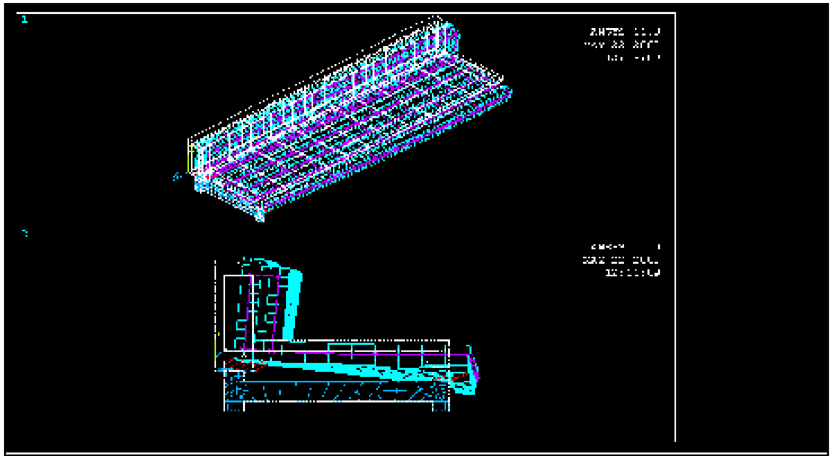
the next challenge

THANK YOU

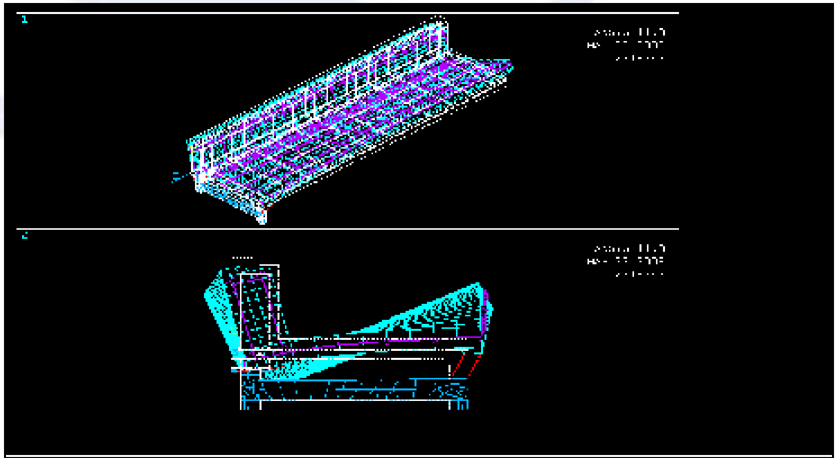
Single Terrace unit.
The first 4 Modes of Vibration



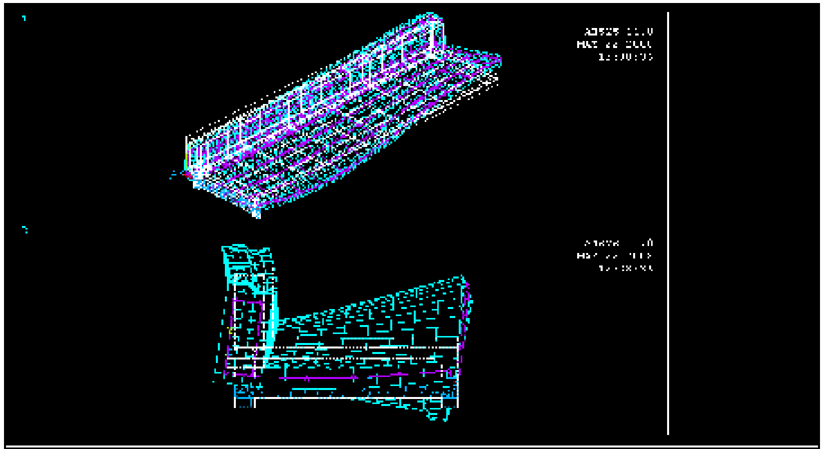
TUM003_M27_modeShape1



TUM003_M27_modeShape2

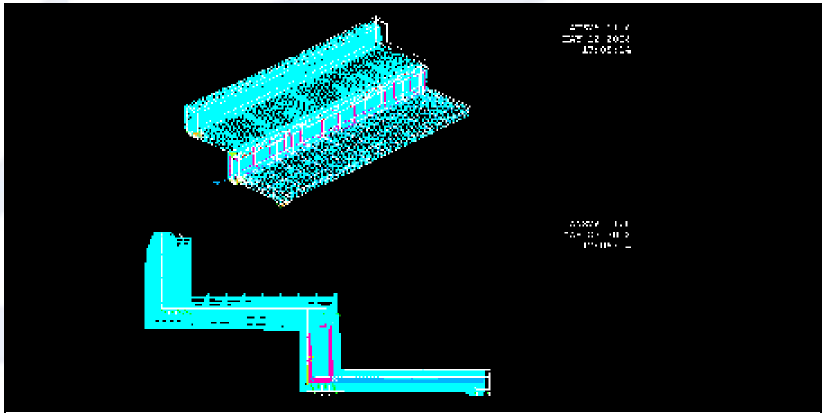


TUM003_M27_modeShape3

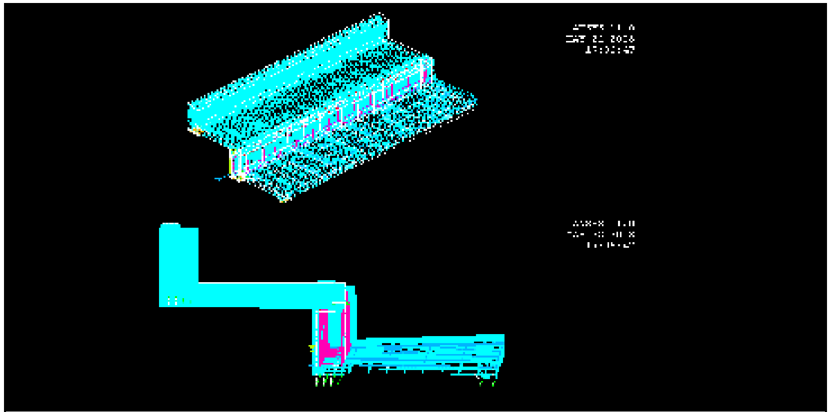


TUM003_M27_modeShape4

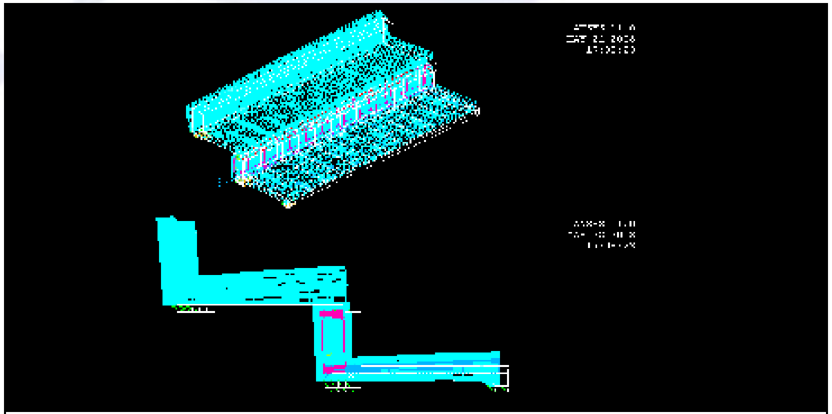
Double Terrace Unit
The First 4 Modes of Vibration



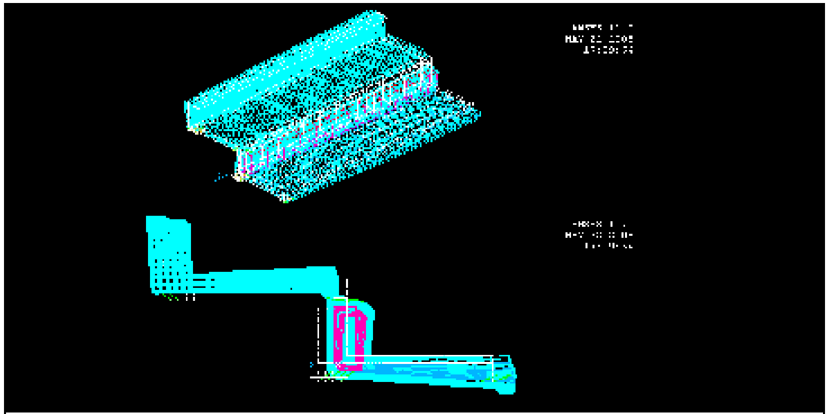
DU_3_ModeShape1



DU_3_ModeShape2



DU_3_ModeShape3



DU_3_ModeShape4