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Fast Estimation of Phase and Frequency for Single Phase Grid Signal

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Abstract

Accurate and fast estimation of single phase grid voltage phase and frequency has the potential to improve the performances of various control and monitoring techniques used in electric power system. This letter applies an adaptive sliding mode observer for frequency and phase estimation. The observer is simple, easy to tune and suitable for real-time implementation. The proposed adaptive observer can be considered as an alternative to phase-locked loop (PLL) with better performance. dSPACE®based experimental results are given to show the effectiveness and performance improvement of the proposed approach with respect to two other advanced PLL techniques namely pseudo linear - enhanced PLL (PL-EPLL) and Second order generalized integrator- frequency locked loop (SOGI-FLL).

Index Terms

Phase estimation, Frequency Estimation, Adaptive observer.

I. Introduction

PHASE and frequency play a very important role in various power system applications, e.g. grid synchronisation, islanding detection [1]–[3]. To estimate the phase and frequency, various effective methods have been reported in the literature. They use a variety of techniques such as - discrete Fourier transform, Kalman filter, adaptive notch filter [4], weighted least-square [5], [6], phase locked loop (PLL) [7]–[12], passivity [13] etc.

As mentioned in the first paragraph, there are several widely accepted class of techniques available in the literature [4], [5], [8]–[10]. Least squares (LS) and its variants like weighted or recursive LS (WLS/RLS) [5] are one such. LS or RLS/WLS has higher computational cost (see Table II in [5]) and matrix inversion are required. Kalman filter (KF) is another effective tool for parameter estimation purpose. However, prior knowledge of the covariance matrix is required. Moreover, online matrix inversion is also required. According to [14, page 287], For a $n \times n$ matrix, the matrix inversion requires n^3 multiplications/divisions and $n^3 - 2n^2 + n$ additions/subtractions. This might be a big issue for low-cost computing hardwares.

Out of various techniques, PLL received wide spread attention due to its excellent performances yet having a simple structure [15]. Basic PLL performs well in the case of normal grid condition but fast dynamic response comes at a cost of disturbance rejection capability. Moreover, in the presence of uncertainties and disturbances like phase or frequency jump, the performance deteriorates. To overcome these limitations, numerous modifications have been proposed in the literature. These modifications are mainly done in the phase-detector (PD) part of the PLL. In [11], enhanced PLL (E-PLL) has been proposed. Adaptive filter and simple sinusoid multiplier has been used in the phase detector part. However, adaptive filtering introduces delay which in turn provide slow dynamic performance. Second order generalized integrator PLL (SOGI-PLL) [16] may improve the transient period of the E-PLL by using two adaptive weight. However, this introduces added computational burden. SOGI-PLL uses Park Transform ($\alpha\beta \to dq$). Inverse Park Transform-PLL (IPT-PLL) uses the other way around i.e. $dq \to \alpha\beta$. However, it uses a low-pass filter inside the PD part. As a result, slow dynamic response is inevitable. Apart from the mentioned

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references, there are numerous other works available in the literature regarding various modifications of PLL. For a comparative analysis of several other PLLs, please consult the surveys [12], [17] for an overview.

PLL system in general uses low-pass filter (LPF)'s. Presence of LPF slow down the dynamic response of PLL. To overcome this limitation of PLL, in this letter we present a very simple adaptive sliding mode observer motivated by [18]. The proposed observer uses both linear and nonlinear output injection terms. The linear part ensures stability while the nonlinear part enhances robustness with respect to parametric uncertainty and measurement noises as shown in [18]. The nonlinear injection also helps to improve the convergence time. As a result, the proposed observer can estimate accurately and fast the phase and frequency of the single phase grid voltage signal. The application of the sliding mode observer with frequency adaptation in the context of single-phase grid voltage parameters estimation is the main contribution of this letter. The proposed algorithm can be considered as an alternative to PLL. The stability analysis of the proposed observer is done based on Lyapunov stability theory. Comparative experimental validations are performed to show the effectiveness of the proposed approach with respect to two adaptive techniques, namely pseudo linear - enhanced PLL (PL-EPLL) and SOGI - frequency locked loop (SOGI-FLL).

II. Estimation of Phase and Frequency

Single phase grid voltage signal can be described as

$$V = A\sin(\omega t + \phi) = A\sin(\psi) \tag{1}$$

where the amplitude of the signal is denoted by A, ω denotes the angular frequency and $\psi \in [0, 2\pi)$ denotes the instantaneous phase. The objective of the estimator is to estimate ω and ψ in the presence of various disturbances like phase, frequency or amplitude jumps. To do that, let us consider the dynamics of the grid signal (1) in state-space form given as,

$$\begin{aligned} \dot{\chi} &= Q\chi \\ y &= R\chi \end{aligned} \tag{2}$$

where

$$\begin{aligned} \chi &= \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = \begin{bmatrix} A\sin(\omega t + \phi) \\ A\omega\cos(\omega t + \phi) \end{bmatrix}, \\ Q &= \begin{bmatrix} 0 & 1 \\ -\nu\omega_n^2 & 0 \end{bmatrix} \text{and } R = \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$

In Q, ω_n denotes the nominal angular frequency of the grid and $\nu = \omega^2/\omega_n^2$. System (2) is observable as such the design of a Luenburger type state observer is pretty straight forward in the nominal case. However, in real life the actual frequency is unknown and time varying. As such to design the observer, the knowledge of ν is required. In other words, adaptation is necessary to find the value of ν. To do that, let us introduce the following state transformation

$$\xi = \begin{bmatrix} \xi_1 & \xi_2 \end{bmatrix}^T = M\chi$$

where $M = \begin{bmatrix} \omega_n^{-2} & \omega_n^{-3} \\ \nu\omega_n^{-1} & \omega_n^{-2} \end{bmatrix} / (1+\nu)$. In ξ, the system (2) can be written as

$$\begin{aligned} \dot{\xi} &= A\zeta \\ y &= C\zeta \end{aligned} \tag{3}$$

where $A = MQM^{-1} = Q$ and $C = RM^{-1} = \left[\begin{array}{cc} \omega_n^2 & \omega_n \end{array} \right]$. For system (3), motivated by the main idea presented in [18], the following sliding mode observer is designed

$$\dot{\hat{\zeta}} = \hat{A}\hat{\zeta} + L\left(y - C\zeta\right) + K\text{sgn}\left(y - C\zeta\right) \tag{4}$$

to estimate the state vector ζ, where $\hat{\zeta}$ is the estimate of ζ, $\hat{\nu}$ is the estimation of ν, L and K are observer gain matrix and

$$\hat{A} = \left[\begin{array}{cc} 0 & 1 \\ -\hat{\nu}\omega_n^2 & 0 \end{array} \right].$$

The performance of observer (4) depends on the value of unknown ν. As a result, an updating law is required to estimate ν. For technical simplicity, we will assume that the grid voltage is an unknown constant. To this end, let us consider the observer output error given as

$$\begin{aligned} e &= y - C\hat{\zeta} \\ \dot{e} &= \frac{d}{dt}\left(C\zeta - C\hat{\zeta}\right) \\ &= C\left(A - LC\right)C^Te - C\left(\hat{A} - A\right)\hat{\zeta} - CK\text{sgn}\left(e\right) \end{aligned} \tag{5}$$

The parameter estimation error can be defined as $e_\nu = \hat{\nu} - \nu$. With estimation error e and e_ν, the following Lyapunov function candidate is considered:

$$V\left(e, e_\nu\right) = e^Te + \frac{e_\nu^2}{\mu}, \mu > 0 \tag{6}$$

$$\begin{aligned} \dot{V}\left(e, e_\nu\right) &= e^T\dot{e} + \dot{e}^Te + 2\frac{e_\nu}{\mu}\dot{\hat{\nu}} \\ &= e^T\left[\left\{C\left(A - LC\right)C^T\right\}^T + \left\{C\left(A - LC\right)C^T\right\}\right]e \\ &- K^TC^Te\,\text{sgn}\left(e\right) - e^TCK\text{sgn}\left(e\right) + 2\frac{e_\nu}{\mu}\dot{\hat{\nu}} \\ &- \hat{\zeta}^T\left(\hat{A} - A\right)^TC^Te - e^TC\left(\hat{A} - A\right)\hat{\zeta} \end{aligned} \tag{7}$$

Next,

$$\begin{gathered} -\hat{\zeta}^T\left(\hat{A} - A\right)^TC^Te - e^TC\left(\hat{A} - A\right)\hat{\zeta} \\ = -2\hat{\zeta}^T\left(\hat{A} - A\right)^TC^Te \\ = -2\left[\begin{array}{cc} \zeta_1 & \zeta_2 \end{array} \right]\left[\begin{array}{cc} 0 & \nu\omega_n^2 - \hat{\nu}\omega_n^2 \\ 0 & 0 \end{array} \right]\left[\begin{array}{c} e\omega_n^2 \\ e\omega_n \end{array} \right] \\ = -2e\omega_n^3\zeta_1\left(\hat{\nu} - \nu\right) \\ = -2e\omega_n^3\zeta_1e_\nu \end{gathered}$$

As a result, by selecting the following parameter update law

$$\dot{\hat{\nu}} = -\mu\hat{\zeta}_1\omega_n^3e \tag{8}$$

eq. (7) becomes

$$\begin{aligned} \dot{V}\left(e, e_\nu\right) &= e^T\left[\left\{C\left(A - LC\right)C^T\right\}^T + \left\{C\left(A - LC\right)C^T\right\}\right]e \\ &- K^TC^Te\text{sgn}\left(e\right) - e^TCK\text{sgn}\left(e\right) \end{aligned} \tag{9}$$

For any $K > 0$ and any L that satisfies $\text{Re}\left\{\lambda\left(A - LC\right)\right\} < 0$, the right hand side of equation (9) is always less than zero except at the origin, which is the equilibrium. As a result,

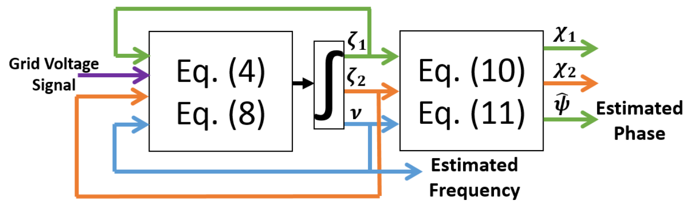


Figure 1. Implementation overview of the proposed estimation technique.

Table I
COMPARATIVE TIME DOMAIN PERFORMANCES FOR VARIOUS TESTS.

Characteristics	Test I			Test II			Test III		
	AO	PL-EPLL	SOGI-FLL	AO	PL-EPLL	SOGI-FLL	AO	PL-EPLL	SOGI-FLL
Freq. Settling Time ($\pm 2\%$ in Cycles)	1	2.5	3.3	3	3.12	3.9	1.44	2.1	2.82
Max. Peak Freq. Error (Hz)	2	2.1	2.5	5.87	3.37	6.26	0.85	0.95	2.3
Phase Settling Time ($\pm 3^\circ$ in Cycles)	0	0.5	3	0.06	1.14	1.32	0.24	0.42	0.6
Max. Peak Phase Error	1.2°	3.3°	4.6°	20°	20°	20°	3.4°	4.22°	3.5°

$$\dot{V}(e, e_\nu) < 0, (e, e_\nu) \neq 0.$$

This proves the global stability of the closed loop system (observer (4) coupled with the adaptation law (8)). Using $\hat{\nu}$ and $\hat{\zeta}$, the state vector χ and the instantaneous phase can be calculated as:

$$\hat{\chi} = M^{-1}\hat{\zeta} = \begin{bmatrix} \omega_n^2 & \omega_n \\ -\hat{\nu}\omega_n^3 & \omega_n^2 \end{bmatrix} \hat{\zeta} \tag{10}$$

$$\hat{\psi} = \arctan\left(\hat{\omega}\hat{\chi}_1/\hat{\chi}_2\right) \tag{11}$$

To implement the proposed technique, Eq. (4), (8), (10) and (11) are required. Implementation overview of the proposed technique is given in Fig. 1.

III. EXPERIMENTAL RESULTS

To verify the practical feasibility and performance of the proposed adaptive sliding mode observer (AO), experimental validations are considered in this Section. The nominal grid signal being considered is $110\sqrt{2}\sin(\omega_n t)$ with $\omega_n = 120\pi$. Parameter design guidelines: to obtain the observer gain, L, readers can start with the nominal value of A. The eigen-values of A are $0 \pm i\omega_n$. If we want to obtain good performance, then the closed loop poles (*i.e.* $A - LC$) should have only negative real parts without any imaginary part. To obtain that, one may start with desired closed loop pole locations as $\lambda_1 \approx -\omega_n$ and $\lambda_2 \approx -4\omega_n$. Assume that $L = [l_1; l_2]$. Then the values of l_1 and l_2 can be easily found by solving the equation $|sI - (A - LC)| = (s - \lambda_1)(s - \lambda_2)$. Once, L is obtained, K can be selected as $K = \gamma L$, with γ being a very small but positive positive constant. In this work, we have selected $L = [0.001; 40]$ and $K = 10^{-2}L$. The gain μ can be selected as 0.002 or lower for slow dynamic response and $\mu = 0.008$ or higher for fast dynamic response. The sign function of observer (4) has been approximated as Sigmoid function to avoid the chattering effect. dSPACE 1104 board has been selected as the rapid prototyping solution. The AO was implemented using Simulink. The solver was the Runge Kutta's method and the sampling frequency was 10KHz. For comparison purpose we have selected two well established adaptive PLL techniques namely SOGI-FLL [19] and PL-EPLL [11]. Parameters of the PLL techniques are selected according to the guidelines given in [11], [19]. The designed tests are as follows:

- Test I: Frequency jump from 60 to 62 Hz
- Test II: Phase jump from 0° to -20° (Fig. 4)

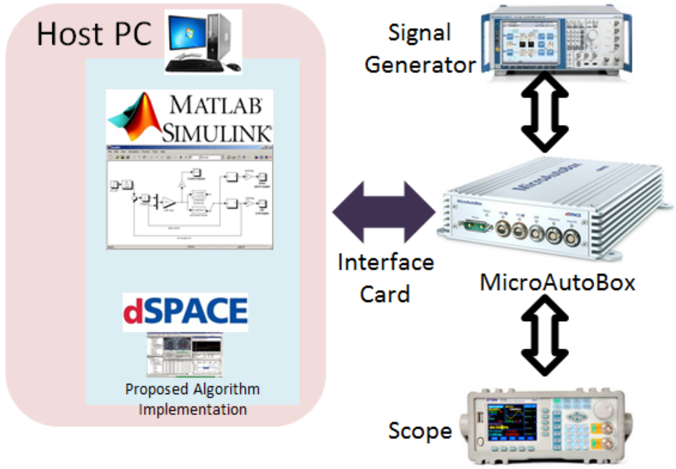
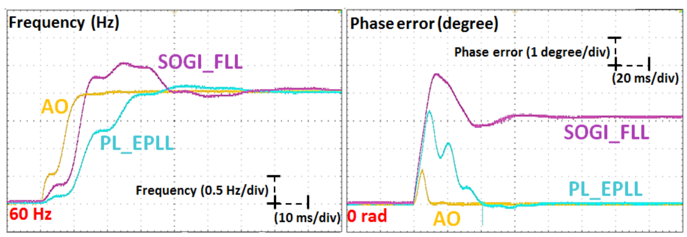


Figure 2. Experimental setup



(a) (b)

Figure 3. Experimental response for Test I. (a) Frequency estimation (0.5Hz/div, 10ms/div). (b) Phase estimation error (1°/div, 20ms/div).

- Test III: Amplitude jump from $110\sqrt{2}$ to $130\sqrt{2}$

The experimental setup can be seen in Fig. 2. The results are shown Fig. 3, 5 and 6 respectively. These results show that in most cases the AO outperform the selected PLL techniques in estimating the phase and frequency. The frequency estimation error convergence time of the AO is few times smaller than that of the PLLs in Test I. Similarly other characteristics like maximum peak overshoot of phase and frequency estimation error are also in favour of the proposed technique in most of the cases. Comparative time domain performance summary for the different tests can be found in Table I. These excellent performances show that the AO can be a good alternative of PLLs for various applications e.g., grid synchronisation, islanding detection [1].

IV. Conclusion

This letter studies the phase and frequency estimation of the single phase grid voltage signal. Sliding mode observer with parameter update law has been proposed for that purpose. Closed loop stability analysis is also given for the adaptive observer. Experimental validations are provided to show the feasibility of the proposed observer in real-time. Experimental results showed that the algorithm provides excellent accuracy and fast convergence even in the presence of non smooth variations in phase, frequency and amplitude. The algorithm is easy to implement and simple tuning rules are also provided. The proposed algorithm can be considered as an alternative to PLL.

References

[1] N. Saxena, I. Hussain, B. Singh, and A. L. Vyas, "Implementation of grid integrated PV-battery system for residential and electrical vehicle applications," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 08, pp. 6592 – 6601, 2018.

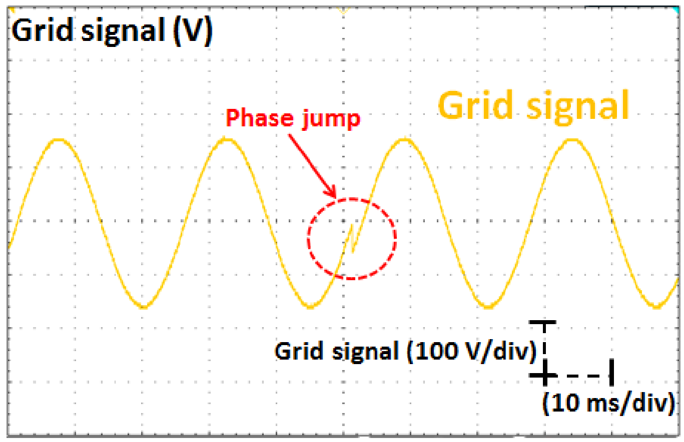
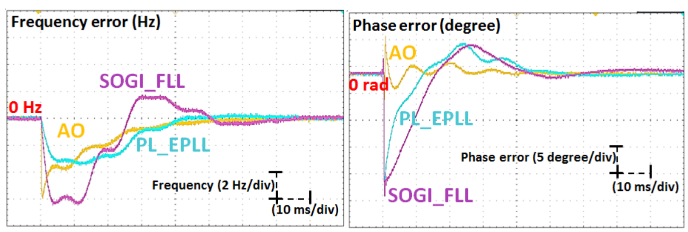


Figure 4. Input voltage waveform for Test II (100V/div, 10ms/div).



(a) (b)

Figure 5. Experimental response for Test II. (a) Frequency estimation error (2Hz/div, 10ms/div). (b) Phase estimation error (5°/div, 10ms/div).

[2] Q.-l. Zhao, X.-Q. Guo, and W.-y. Wu, "Research on control strategy for single-phase grid-connected inverter," in *Zhongguo Dianji Gongcheng Xuebao(Proceedings of the Chinese Society of Electrical Engineering)*, vol. 27, pp. 60–64, 2007.

[3] S. R. K. Hoseini, H. Givi, E. Farjah, and T. Ghanbari, "Phase locked loop-free sliding-mode-based power control for grid-connected voltage-sourced inverters under unbalanced network conditions," in *Power Electronics, Drive Systems & Technologies Conference (PEDSTC), 2017 8th*, (Mashhad, Iran), pp. 460–465, IEEE, 2017.

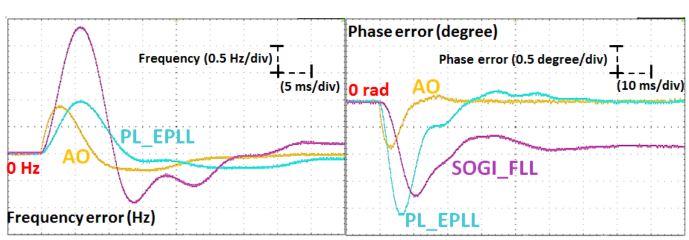
[4] K.-J. Lee, J.-P. Lee, D. Shin, D.-W. Yoo, and H.-J. Kim, "A novel grid synchronization PLL method based on adaptive low-pass notch filter for grid-connected PCS," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 1, pp. 292–301, 2014.

[5] A. Vidal, F. D. Freijedo, A. G. Yepes, P. Fernandez-Comesaña, J. Malvar, O. Lopez, and J. Doval-Gandoy, "A fast, accurate and robust algorithm to detect fundamental and harmonic sequences," in *Energy Conversion Congress and Exposition (ECCE), 2010 IEEE*, (Atlanta, GA, USA), pp. 1047–1052, IEEE, 2010.

[6] H.-S. Song and K. Nam, "Instantaneous phase-angle estimation algorithm under unbalanced voltage-sag conditions," *IEE Proceedings-Generation, Transmission and Distribution*, vol. 147, no. 6, pp. 409–415, 2000.

[7] M. Bierhoff, "A general PLL type algorithm for speed sensorless control of electrical drives," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 12, pp. 9253 – 9260, 2017.

[8] M. Ramezani, S. Golestan, S. Li, and J. M. Guerrero, "A simple approach to enhance the performance of complex-coefficient filter-based PLL in grid-connected applications," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 06, pp. 5081 – 5085, 2018.



(a) (b)

Figure 6. Experimental response for Test III. (a) Frequency estimation error (0.5Hz/div, 5ms/div). (b) Phase estimation error (0.5°/div, 10ms/div).

[9] S. Golestan, E. Ebrahimzadeh, J. M. Guerrero, J. C. Vasquez, and F. Blaabjerg, “An adaptive least-error squares filter-based phase-locked loop for synchronization and signal decomposition purposes,” *IEEE Transactions on Industrial Electronics*, vol. 64, no. 1, pp. 336–346, 2017.

[10] P. T. Lanza II, Y. B. Shtessel, and J. L. Stensby, “Improved acquisition in a phase-locked loop using sliding mode control techniques,” *Journal of the Franklin Institute*, vol. 352, no. 10, pp. 4188–4204, 2015.

[11] M. Karimi-Ghartemani, *Enhanced phase-locked loop structures for power and energy applications*. New York, USA: John Wiley & Sons, 2014.

[12] Y. Yang, L. Hadjidemetriou, F. Blaabjerg, and E. Kyriakides, “Benchmarking of phase locked loop based synchronization techniques for grid-connected inverter systems,” in *Power Electronics and ECCE Asia (ICPE-ECCE Asia), 2015 9th International Conference on*, (Seoul, South Korea), pp. 2167–2174, IEEE, 2015.

[13] Z. Dai, W. Lin, and H. Lin, “Estimation of single-phase grid voltage parameters with zero steady-state error,” *IEEE Transactions on Power Electronics*, vol. 31, no. 5, pp. 3867–3879, 2016.

[14] R. Burden, J. Faires, and A. Reynolds, *Numerical Analysis*. Boston, USA: Prindle, Weber and Schmidt, second ed., 1981.

[15] F. Blaabjerg, R. Teodorescu, M. Liserre, and A. V. Timbus, “Overview of control and grid synchronization for distributed power generation systems,” *IEEE Transactions on industrial electronics*, vol. 53, no. 5, pp. 1398–1409, 2006.

[16] M. Ciobotaru, R. Teodorescu, F. Blaabjerg, *et al.*, “A new single-phase PLL structure based on second order generalized integrator,” in *Power Electronics Specialists Conference*, (Jeju, South Korea), pp. 1–6, 2006.

[17] Y. Han, M. Luo, X. Zhao, J. M. Guerrero, and L. Xu, “Comparative performance evaluation of orthogonal-signal-generators-based single-phase PLL algorithms-a survey,” *IEEE Transactions on Power Electronics*, vol. 31, no. 5, pp. 3932–3944, 2016.

[18] J.-J. Slotine, J. K. Hedrick, and E. Misawa, “On sliding observers for nonlinear systems,” *Journal of dynamic systems, Measurement, and Control*, vol. 109, no. 3, pp. 245–252, 1987.

[19] P. Rodríguez, A. Luna, I. Candela, R. Mujal, R. Teodorescu, and F. Blaabjerg, “Multiresonant frequency-locked loop for grid synchronization of power converters under distorted grid conditions,” *IEEE Transactions on Industrial Electronics*, vol. 58, no. 1, pp. 127–138, 2011.