Equivalent Circuit Parameters for Large Brushless Doubly Fed Machines (BDFMs)

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Abstract—This paper presents analytical methods to calculate the equivalent circuit parameters for large-scale Brushless Doubly-Fed Machines (BDFMs) with magnetic wedges utilized for closing stator open slots. The use of magnetic wedges reduces the magnetizing currents in the machine, reflected in the inductances, but also increases leakage fluxes affecting the value of series inductances in the equivalent circuit. Though such effects can be modelled by numerical models, the proposed analytical methods are particularly helpful in optimizing machine design, inverter rating, reactive power management and grid low-voltage ride-through performance. The conventional analytical methods cannot be readily applied to the BDFM due to its complex magnetic field distribution; this paper presents analytical methods to calculate the magnetizing and leakage inductances for the BDFM with magnetic wedges used in the stator slots. The proposed methods are assessed by experimentally-verified FE models for a 250 kW BDFM.


I. INTRODUCTION

The BDFM is an alternative to the well-established Doubly-Fed Induction Generator (DFIG) for use in wind turbines as it retains the benefit of utilizing a fractional-size converter, but it also offers higher reliability and lower maintenance costs than the DFIG due to absence of brush gear and slip-rings [1]. In addition, the BDFM is intrinsically a medium-speed machine, enabling the use of a simplified one or two-stage gearbox, hence reducing the cost of the overall drivetrain and giving further reliability improvement.

The modern BDFM as a variable speed drive or generator comprises two electrically separate stator windings, one connected directly to the grid, called the power winding (PW), and the other supplied from a variable voltage and frequency converter, called the control winding (CW). The pole numbers are selected in a way to avoid direct transformer coupling between the stator windings and the coupling between the windings is through the rotor [2]. The rotor is specially designed to couple to the two air-gap fields associated with the two stator windings, the nested-loop design being commonly used [3]. The normal mode of BDFM operation is as a synchronous machine with the rotor rotating at a speed determined by the winding pole numbers and the mains and converter frequencies.

The equivalent circuit is a simple method of representing the steady-state performance of the BDFM, allowing the rapid calculation of operating conditions of the BDFM and its associated converter and grid connection. The equivalent circuit is also helpful for the design and optimization of the machine. The equivalent circuit parameters can be calculated from the machine geometry using the analytical methods described in [4]. They can also be extracted from BDFM steady-state operation, provided from numerical models or experimental tests, by applying curve-fitting methods [2]. Extracted parameters are likely to be more accurate as it is hard to obtain analytically precise values of certain quantities such as the leakage reactances of end-windings.

To date, there have been several attempts to manufacture large BDFMs, for example in Brazil with a 75 kW machine [5], China with the design of a 200 kW machine [6] and the 250 kW BDFM reported by the authors [7]. The latter was built in a frame size D400 as a stepping-stone towards a megawatt-scale BDFM wind turbine and it involved construction and winding techniques appropriate to large machines including magnetic wedges. These are often used in large machines with open slots [8] to reduce the effective air gap length, thereby resulting in lower magnetizing currents, which is advantageous in BDFM design. Wedges, typically comprising 75% iron powder, 7% glass mat, and 18% epoxy resin [9], also reduce core losses, and hence machine temperature rise [8], [10], [11]. On the other hand, magnetic wedges will increase the slot leakage by providing easier paths for fluxes not crossing the air gap [12].

Though the effects of magnetic wedges on the performance of induction and permanent magnet machines have been studied by others, for example in [11], [13], the methods of analysis proposed in those papers cannot be readily applied to the BDFM due to its complex design and magnetic field distribution [14]. This paper investigates the effects of magnetic wedges on the performance of the BDFM by analysing their impacts on the equivalent circuit parameters. Analytical methods are presented to calculate the magnetizing and leakage inductances for a 250 kW BDFM with magnetic wedges and Finite Element models are used to verify the accuracy of the proposed methods.

II. BDFM EQUIVALENT CIRCUIT MODEL

A simplified equivalent circuit for the BDFM is shown in Fig. 1 where all parameters are referred to the power winding side and iron losses are neglected [2]. The circuit is valid for all modes of operation, including the induction, cascade and synchronous modes and can be utilized for the analysis.
Two of the models utilized are based on the coupled-circuit equivalent circuit parameters, which are described in Table I. The referred rotor inductance, $L'_r$ in the simplified circuit shown in Fig. 1 represents the series inductances in the full equivalent circuit [2], including the stator PW and CW and rotor leakage inductances. Hence, the effects of stator slot magnetic wedges on the flux leakage, will be reflected in the value of the $L'_r$ in the simplified equivalent circuit.

As described, the equivalent circuit parameters can be calculated from the machine geometry, during the design stage, using the method described in [15]. The procedure involves deriving the machine’s coupled-circuit model, followed by performing a series of transformations to obtain the d-q, sequence components and equivalent circuit parameters, respectively.

The equivalent circuit parameters can also be extracted from steady-state measures, such as torque, speed, voltages and currents, obtained from BDFM’s operation in the induction and cascade modes [2]. The steady-state data can be obtained from numerical models or experimental tests. The stator winding resistances, $R_1$ and $R_2$ are either calculated from the machine geometry at a certain operating temperature or obtained from DC measurements. The magnetizing inductances $L_{m1}$ and $L_{m2}$ are obtained from the magnetizing tests where a single stator winding is supplied in turn whilst the other winding is left open and the rotor is driven at the synchronous speed to eliminate rotor currents. Finally, the rotor parameters $L_r$ and $R_r$ are obtained from applying a curve fitting method to the data from cascade tests, assuming the stator resistance and magnetizing parameters are fixed [2].

### III. Models Considered in This Study

Five models have been utilized in this study to obtain the equivalent circuit parameters, which are described in Table I. Two of the models utilized are based on the coupled-circuit approach: one neglecting the effects of magnetic wedges (CC-LN-NW) and the other incorporating the authors’ proposed approach to model the effects of wedges (CC-LN-W). The coupled-circuit model comprises the machine inductance and resistance matrices and the effects of iron saturation are neglected. It has been shown to give close agreement with experimental tests on a D180 size machine, for example in [17]. The inductance and resistance matrices are calculated from the machine geometry and can be transformed into the equivalent circuit parameters by performing a series of transformations [16].

The other three models are based on the 2D Finite Element (FE) approach. One assumes a linear iron circuit and no magnetic wedges (FE-LN-NW), the second one assumes a linear iron circuit and takes into account the magnetic wedges with a constant relative permeability (FE-LN-W), and the third model includes the nonlinear properties of the iron circuit and magnetic wedges (FE-NLN-N-W). Clearly, the latter has a significantly more computational time than the linear models, but will be shown to give the closest predictions to experimental results.

### IV. Prototype 250 kW D400 BDFM Specifications

#### A. Specifications

The specifications of the 250 kW D400 BDFM are shown in Table II. The D400 BDFM was constructed as a frame size 400 machine with the stack length of 820 mm. The stator windings were form wound from copper strips. The power winding was rated at 690 V, 178 A, at 50 Hz and the control winding was designed for 620 V at 18 Hz and rated at 73 A. Both stator windings were connected in delta. The rotor comprises six sets of nests each consisting of a number of concentric loops [18], the conductors being solid bars with one common end ring [7]. The magnetic properties for the iron and magnetic wedges were provided by the machine manufacturer.

#### B. Test rig and instrumentation details

Fig. 2 shows the BDFM on test bed, which was set up at the manufacturer’s facility. The BDFM was driven via a torque transducer by a 355 kW 8-pole induction motor fed from a 400 kW ACS800 inverter. The PW was connected to the mains and supplied at 690 V, 50 Hz and the CW was supplied from the converter. Speed and position signals are obtained from an incremental encoder with a resolution of 10,000 pulses per revolution. The voltages and currents of each stator phase are measured by LEM AV100-750 and LEM LA 205/305-s transducers respectively. The accuracy of voltage, current, torque and speed measurements are shown in Table III, which are calculated from the specifications provided by the manufacturers and the accuracies of the conditioning circuitries.

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**Fig. 1:** Simplified equivalent circuit

**TABLE I:** Different approaches utilized for obtaining the equivalent circuit parameters

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Model</th>
<th>Iron circuit</th>
<th>Magnetic wedges</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC-LN-NW</td>
<td>Coupled-Circuit</td>
<td>Linear</td>
<td>Not included</td>
</tr>
<tr>
<td>CC-LN-W</td>
<td>Coupled-Circuit</td>
<td>Linear</td>
<td>Included</td>
</tr>
<tr>
<td>FE-LN-NW</td>
<td>Finite Element</td>
<td>Linear</td>
<td>Not included</td>
</tr>
<tr>
<td>FE-LN-W</td>
<td>Finite Element</td>
<td>Linear</td>
<td>Included</td>
</tr>
<tr>
<td>FE-NLN-W</td>
<td>Finite Element</td>
<td>Non-linear</td>
<td>Included</td>
</tr>
</tbody>
</table>

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TABLE II: Specifications of the 250 kW D400 BDFM

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame size</td>
<td>400</td>
</tr>
<tr>
<td>PW pole number</td>
<td>4</td>
</tr>
<tr>
<td>Speed range</td>
<td>500 rpm ±36%</td>
</tr>
<tr>
<td>PW rated voltage</td>
<td>690V at 50 Hz (delta)</td>
</tr>
<tr>
<td>Rated torque</td>
<td>3670 Nm</td>
</tr>
<tr>
<td>PW rated current</td>
<td>178 A (line)</td>
</tr>
<tr>
<td>Rated power</td>
<td>250 kW at 680 rpm</td>
</tr>
<tr>
<td>CW pole number</td>
<td>8</td>
</tr>
<tr>
<td>Stack length</td>
<td>0.82 m</td>
</tr>
<tr>
<td>CW rated voltage</td>
<td>620 V at 18 Hz (delta)</td>
</tr>
<tr>
<td>Efficiency (at full load)</td>
<td>&gt; 95%</td>
</tr>
<tr>
<td>CW rated current</td>
<td>73 A (line)</td>
</tr>
</tbody>
</table>

TABLE III: Measurement Accuracy

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Total accuracy of measured value</th>
<th>DC Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>2.1%</td>
<td>2.3(V)</td>
</tr>
<tr>
<td>PW current</td>
<td>1.4%</td>
<td>0.5(A)</td>
</tr>
<tr>
<td>CW current</td>
<td>1.4%</td>
<td>0.3(A)</td>
</tr>
<tr>
<td>Torque</td>
<td>0.15%</td>
<td>0.1(Nm)</td>
</tr>
<tr>
<td>Speed</td>
<td>0.0024%</td>
<td>–</td>
</tr>
</tbody>
</table>

rotor speed is set to the synchronous speed using an external load machine, only the field due to the excited winding will exist in the machine and hence the equivalent circuit of Fig. 1 will reduce to only include the supplied winding’s resistance and magnetizing inductance. The magnetizing inductances of PW and CW, $L_{m1}$ and $L_{m2}$, can be derived from these tests.

The magnetizing characteristic of the D400 BDFM is shown in Fig. 4 for the two stator windings. Results from experiments and FE-LN-NW and FE-NLN-W models are shown. The supply frequency was 20 and 10 Hz for the PW and CW respectively.

There are two horizontal lines shown in Fig. 4: the solid line shows the rated voltage of the corresponding winding at the excited frequency and the dashed line shows the voltage at which the air gap flux is rated in the absence of the other stator field. As can be seen from Fig. 4, the effect of iron saturation is not significant below the air gap rated flux. The effect of stator slot magnetic wedges in reducing magnetizing currents is evident. The magnetizing inductances, $L_{m1}$ and $L_{m2}$ can be obtained from the slope of the linear region below the rated voltages.

B. Cascade tests

The BDFM can be operated as a self-cascaded induction machine by exciting the PW or CW and shorting the other winding. A cascade induction machine formed from $p_1$ and $p_2$ pole pair induction machines has characteristics which resemble an induction machine with $p_1 + p_2$ pole pairs.

The rotor parameters, $L_r$ and $R_r$ and turns ratio $N_1/N_2$ may be obtained by applying a curve fitting method to results from BDFM cascade operation. The stator currents (both supplied and shorted windings) and torque are used for extracting the parameters. The fitting algorithm assumes fixed values for stator winding resistances and magnetizing inductances and finds values for $L_r$, $R_r$ and $N_1/N_2$ which give the best fit. Fig. 5 shows the results from cascade tests obtained from experiments and FE-LN-NW and FE-NLN-W models. The cascade tests have been carried out at reduced voltages, i.e. PW and CW were supplied at 70 V, 20 Hz and 240 V, 20 Hz respectively, due to restrictions on stator currents, hence the extracted parameters do not represent the effect of iron saturation.

VI. COMPARISON OF EXISTING MODELS

The equivalent circuit inductance parameters $L_{m1}$, $L_{m2}$ and $L_r$, obtained from different modelling methods and experimen-
(a) Magnetizing characteristic of the stator PW.

(b) Magnetizing characteristic of the stator CW.

Fig. 4: Magnetizing characteristic of the D400 BDFM

(a) Torque in PW cascade test

(b) Torque in PW cascade test

(c) $I_{pw}$ in PW cascade test

(d) $I_{pw}$ in CW cascade test

(e) $I_{cw}$ in PW cascade test

(f) $I_{cw}$ in CW cascade test

Fig. 5: Cascade results and fitted curves for Experimental data, FE-LN-NW and FE-NLN-W models
nal tests are shown in Tables IV and V. The parameters shown in Table IV are for the machine without magnetic wedges in stator slots. The two linear models are in close agreement, as expected, which validates the assumptions used to obtain the coupled-circuit model and its transformation to equivalent circuit model proposed by Roberts in [15]. Nevertheless, neither of the models are able to take into account the effect of magnetic wedges.

Table V shows the inductance parameters obtained from FE models and experiment when magnetic wedges are considered. There is acceptable agreement between the parameters obtained from FE-NLN-W model and experiments, considering the measurement accuracies and limitations in obtaining geometrical data. There is about 15% difference in the rotor inductance $L_r$, mainly due to the fact that $L_r$ represents the stator and rotor leakage inductances and the modelling of the leakage effects, for example in the case of winding overhang, is difficult. The agreement between FE-NLN-W and FE-LN-W, both using identical geometrical data, is close which shows that the non-linearity of the iron circuit can be neglected due to reduced voltage levels in the tests. It also confirms that assuming the linear magnetic property for the magnetic wedges, i.e. a constant permeability, gives acceptable accuracy.

The differences between the parameters obtained assuming no magnetic wedge in stator slots, shown in Table IV, and the ones obtained when magnetic wedges are present, shown in Table V, are significant: both magnetising and leakage inductances are larger in the latter table. Though the nonlinearity of iron circuit may also contribute in the differences, since the tests were undertaken at reduced voltage levels, such effects can be neglected.

There is therefore need for an analytical approach to model the effect of magnetic wedges in the coupled circuit model, so these effects can be considered during the design optimisation when computational cost is important. The following sections will present the authors’ proposed approach.

### TABLE IV: Equivalent circuit parameters without stator magnetic wedges

<table>
<thead>
<tr>
<th>Model</th>
<th>$L_{m1}(\text{mH})$</th>
<th>$L_{m2}(\text{mH})$</th>
<th>$L'_r(\text{mH})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC-LN-NW</td>
<td>82.8</td>
<td>282.8</td>
<td>9.67</td>
</tr>
<tr>
<td>FE-LN-NW</td>
<td>84.2</td>
<td>285</td>
<td>9.40</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>1.4%</td>
<td>0.8%</td>
<td>2.9%</td>
</tr>
</tbody>
</table>

### TABLE V: Equivalent circuit parameters with stator magnetic wedges

<table>
<thead>
<tr>
<th>Model</th>
<th>$L_{m1}(\text{mH})$</th>
<th>$L_{m2}(\text{mH})$</th>
<th>$L'_r(\text{mH})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>104</td>
<td>368</td>
<td>12.45</td>
</tr>
<tr>
<td>FE-NLN-W</td>
<td>97.1</td>
<td>341</td>
<td>14.3</td>
</tr>
<tr>
<td>FE-LN-W</td>
<td>96.8</td>
<td>348.5</td>
<td>15.2</td>
</tr>
</tbody>
</table>

### VII. Calculation of Magnetizing Inductances

The magnetizing inductances of electrical machines correspond to the magnetic flux that links the stator and rotor, passing the air gap twice. The effects of stator and rotor slotting in the magnetizing inductance are often modelled by Carter factors which effectively scale the air gap length. The effect of magnetic wedges in reducing magnetizing currents, hence increasing magnetizing inductances, can also be modelled by scaling the air gap length, similar to the concept of Carter factors [19]. Hence, modified Carter factors may be derived which can take into account the effect of both slotting and magnetic wedges.

The Carter factor is effectively the ratio of the air gap peak flux density, which would be the case in the absence of slotting, and its average value when slotting is considered [20]. When both rotor and stator have open slots, it is assumed that the resultant Carter factor is the product of the Carter factors for the stator and rotor [21]. Since magnetic wedges are only used in stator slots of the BDFM, the following presents the calculation of Carter factor for the stator assuming an un-slotted rotor using the magnetic equivalent circuit (MEC) method. The Carter factor for the rotor can be calculated using conventional methods described, for example, in [21]. The following assumptions are made:

- There is no saturation in the iron circuit and in magnetic wedges and hence infinite and constant permeabilities are assumed for iron and magnetic wedges, respectively.
- The magnetic wedges are assumed to have a rectangular shape.
- There are three magnetic paths between the stator tooth and un-slotted rotor, through which magnetic fluxes $\phi_x$, $\phi_y$ and $\phi_w$ enter the rotor as shown in Fig. 6. $\Theta$ is considered as the total MMF that produces these fluxes.
- The reluctance between the above mentioned magnetic paths is infinite and hence the interaction between their corresponding magnetic fluxes is negligible.

A typical variation of air gap flux density over a slot pitch is shown in Fig. 6 with presence of a magnetic wedge, where $B_{av}$, $B_{max}$ and $B_{min}$ are average, maximum and minimum air gap flux density, respectively.

The reluctances associated with magnetic paths corresponding to $\phi_x$ and $\phi_w$ include:

$$R_{ox} = \frac{x}{\mu_0 A_{ox}}, \quad R_{oy} = \frac{y}{\mu_0 A_{oy}}, \quad R_{og} = \frac{g}{\mu_0 A_{og}},$$

$$R_{wx} = \frac{x}{\mu_0 A_w A_{wx}}, \quad R_{wy} = \frac{y}{\mu_0 A_w A_{wy}}, \quad R_{wg} = \frac{g + d}{\mu_0 A_{wg}},$$

Where $g$ is the air gap length, $d$ is the depth of air-filled region above the magnetic wedge, $\mu_w$ is the relative permeability of the magnetic wedge and $A$ is the area through which the magnetic flux is passing. The flux entering the rotor consists of three fluxes: $\phi_x$, $\phi_y$ and $\phi_w$ shown in Fig. 6. $B_{wx}$, $B_{wy}$, $B_{ox}$ and $B_{oy}$ are the flux densities associated to reluctances $R_{wx}$, $R_{wy}$, $R_{ox}$ and $R_{oy}$ respectively. Assuming $\Theta$ is the MMF producing $\phi_x$, $\phi_y$ and $\phi_w$, and the MMF across $R_{wx}$, $R_{wy}$, $R_{wg}$ and $R_{ox}$, $R_{oy}$, $R_{og}$ are $\Theta_{wx}$, $\Theta_{wy}$, $\Theta_{wg}$ and $\Theta_{ox}$, $\Theta_{oy}$, $\Theta_{og}$ respectively, the following equations can be obtained:

$$\Theta = \Theta_{ox} + \Theta_{oy} + \Theta_{og} = \Theta_{wx} + \Theta_{wy} + \Theta_{wg}$$

$$\Theta = R_{ox} A_{ox} B_{ox} + (R_{oy} A_{oy} + R_{og} A_{og})B_{oy}$$

$$\Theta = R_{wx} A_{wx} B_{wx} + (R_{wy} A_{wy} + R_{wg} A_{wg})B_{wy}$$

$$\Theta = \Theta_{wx} + \Theta_{wy} + \Theta_{wg}$$

$$\Theta = \Theta_{ox} + \Theta_{oy} + \Theta_{og}$$

$$\Theta = R_{ox} A_{ox} B_{ox} + (R_{oy} A_{oy} + R_{og} A_{og})B_{oy}$$

$$\Theta = R_{wx} A_{wx} B_{wx} + (R_{wy} A_{wy} + R_{wg} A_{wg})B_{wy}$$

$$\Theta = \Theta_{wx} + \Theta_{wy} + \Theta_{wg}$$
Along the path $R_{ox}$ and $R_{oy}$, the area through which $\phi_o$ is traveling can be assumed uniform, i.e. $A_{ox}=A_{oy}$, hence:

$$B_{ox} = B_{oy}$$  \hspace{1cm} (8)

Considering the path for $\phi_w$, the flux densities $B_{wx}$ and $B_{wy}$ are related as:

$$B_{wx} = B_{wy} \frac{w_o}{2h_w}$$  \hspace{1cm} (9)

From (6) to (9) and since $R_{oy}$ and $R_{wy}$ are negligibly small,

$$\Theta = B_{oy} \frac{x + g}{\mu_0} = B_{wy} \left( \frac{w_o x}{2w_o \mu_0 h_w} + \frac{g + d}{\mu_0} \right)$$  \hspace{1cm} (10)

Hence:

$$B_{oy} = \frac{\mu_0 \Theta}{x + g}$$  \hspace{1cm} (11)

$$B_{wy} = \frac{w_o}{2h_w} \frac{\Theta}{x + \frac{g + d}{\mu_0}}$$  \hspace{1cm} (12)

The magnetic fluxes $\phi_o$ and $\phi_w$ can be worked out from the flux densities $B_{ox}$ and $B_{wy}$ as:

$$\phi_o = \int_0^d B_{ox} \cdot dx = \mu_0 \Theta \ln \left( \frac{g + d}{g} \right)$$  \hspace{1cm} (13)

$$\phi_w = \int_0^{w_2} B_{wy} \cdot dx = \frac{2h_w \mu_0 \Theta}{w_o} \ln \left( \frac{w_0 w_2}{4h_w} \mu_0 (g + d) \right)$$  \hspace{1cm} (14)

Where $w_o$ and $w_2$ correspond to the stator slot opening width and magnetic wedge width respectively, as shown in Fig. 6. Considering $B_{\text{max-airgap}}$ corresponds to the magnetic flux $\phi_{\text{max}}$ passing through the air gap for an unslotted airgap, the airgap correction factor for taking magnetic wedges into account can be derived as:

$$K_{c-\text{stator}} = \frac{B_{\text{max-airgap}}}{B_{\text{mean-airgap}}} = \frac{\phi_{\text{max}}}{\phi_t + 2(\phi_o + \phi_w)}$$  \hspace{1cm} (15)

Where

$$\phi_{\text{max}} = \frac{\mu_0 y_s \Theta}{g}$$  \hspace{1cm} (16)

$$\phi_t = \frac{\mu_0 (y_s - w_o) \Theta}{g}$$  \hspace{1cm} (17)

Substituting Equations (13), (14), (16) and (17) in Equation (15) give

$$K_{c-\text{stator}} = \frac{y_s}{(y_s - w_2) + \frac{4h_w g}{w_o} \mu_0 \ln \left( \frac{w_0 w_2 (g + d)}{4h_w \mu_0 (g + d)} \right) + 2g \ln (\frac{g + d}{g})}$$  \hspace{1cm} (18)

The rotor Carter factor $k_{c-\text{rotor}}$ is calculated for the D400 BDFM using the method proposed by [21]:

$$k_{c-\text{rotor}} = \frac{y_r + 8g}{y_r - w_r + 8g}$$  \hspace{1cm} (19)

Where $y_r$ is the rotor slot pitch and $w_r$ is the rotor slot opening width. Equation 19 has empirical justification and is valid when $\frac{y_r}{g}$ is not sufficiently large (less than 12) [21]. The overall effective air gap $g_e$ can be calculated using the product of stator and rotor Carter factors [22]:

$$g_e = g \times k_{c-\text{stator}} \times k_{c-\text{rotor}}$$  \hspace{1cm} (20)

The effective air gap can be used in the calculation of magnetising inductances for the BDFM. The validity of the proposed method will be verified in Section IX using numerical analysis.

**VIII. Calculation of Leakage Inductances**

Inductance arising from leakage effects are inductances due to the magnetic flux not linking stator and rotor conductors. This definition is similar to that found in [23], except that for the purposes of this paper, the harmonic mutual inductance terms are not considered part of the leakage terms, rather as a separate term.

The calculation of stator and rotor winding leakage inductances is greatly simplified by the calculation of specific permeances which is defined as the self-inductance per unit length of coil per turn squared. Thus, to calculate the self-inductance of a coil due to leakage, the different permeance terms around the length of the coil should be summed and the result multiplied by the number of turns squared. Consequently, the per phase leakage inductances for stator PW and CW, $L_{s-pw}$ and $L_{s-cw}$ can be calculated:

$$L_{s-pw} = C_{pw} N_{pw}^2 \ell' \lambda_{s-pw}$$  \hspace{1cm} (21)

$$L_{s-cw} = C_{cw} N_{cw}^2 \ell' \lambda_{s-cw}$$  \hspace{1cm} (22)

Where $C_{pw}$ and $C_{cw}$ are the PW and CW number of coils per phase, $N_{pw}$ and $N_{cw}$ are the number of turns per coil, $\lambda_{s-pw}$ and $\lambda_{s-cw}$ are the specific permeances and $\ell'$ is the stator effective stack length. There are four major sources of leakage permeance in induction machines [22]-[24], which are also taken into account for the BDFM: slot permeance, tooth-top permeance, overhang permeance and zig-zag permeance. Among the leakage terms, the slot leakage in most cases has the largest contribution in the winding leakage.
As shown in Section VI, there is a large discrepancy in the value of \( L' \) when the effects of magnetic wedges are ignored. This is most likely due to easier paths for stator leakage fluxes in the stator slot openings enabled by the magnetic wedges. The other leakage terms as well as the rotor leakage are assumed to be largely unaffected by the stator wedges. The following presents the calculations of slot permeance when magnetic wedges are used in stator slot openings. The slot contains two stator windings, the PW and CW, and each winding is assumed to have a double-layer arrangement that is a common practice in large electrical machines. Derivations according to [23] are used for the other permeance terms.

Fig. 7 shows the stator slot. The slot leakage for the PW is calculated here; a similar approach can be used to derive the CW slot leakage. The slot permeance of the PW can be obtained by calculating the energy of the leakage magnetic field in a slot:

\[
W_o = \sum_{i=1}^{7} \frac{1}{2\mu_i} \int_{v_i} B_i^2(h)dh
\]

where

\[
B(h) = \begin{cases} 
B_1(h) = \mu_1 \frac{\Theta_{pw-A} h}{w}, & 0 < h < h_{pw}, \\
B_2(h) = \mu_2 \frac{\Theta_{pw-A} h}{w}, & h_{pw} < h < h_1, \\
B_3(h) = \mu_3 \frac{\Theta_{pw-A} + \Theta_{pw-B} h}{w}, & h_1 < h < h_2, \\
B_4(h) = \mu_4 \frac{\Theta_{pw-A} + \Theta_{pw-B} h}{w}, & h_2 < h < h_3, \\
B_5(h) = \mu_5 \frac{\Theta_{pw-A} + \Theta_{pw-B} h}{w}, & h_3 < h < h_4, \\
B_6(h) = \mu_6 \frac{\Theta_{pw-A} + \Theta_{pw-B} h}{w}, & h_4 < h < h_5, \\
B_7(h) = \mu_7 \frac{\Theta_{pw-A} + \Theta_{pw-B} h}{w}, & h_5 < h < h_6,
\end{cases}
\]

and

\[
dv_1 = dv_2 = dv_3 = dv_4 = \ell' w_1 dh \\
dv_5 = \ell' w_2 dh \\
dv_6 = \ell' \left[ w_2 + (w_0 - w_2) \frac{h}{w_1} \right] dh \\
dv_7 = \ell' w_6 dh
\]

\[
\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_7 = \mu_0 \\
\mu_5 = \mu_6 = \mu_w
\]

\( \ell' \) is the stator effective stack length and \( \Theta_{pw-A} \) and \( \Theta_{pw-B} \) are the total current linkages of the PW bottom and upper layers. All different heights and widths are specified in Fig. 7. The energy of the PW leakage magnetic field in the slot from (23) is:

\[
W_\theta = \frac{\ell' \mu_0}{2W_1} \left[ \Theta_{pw-A}^2 \int_{0}^{h_{pw}} \left( \frac{h}{h_{pw}} \right)^2 dh + \Theta_{pw-A}^2 \int_{h_{pw}}^{h_1} dh + \int_{h_1}^{h_2} \left( \Theta_{pw-A} + \Theta_{pw-B} \frac{h - h_1}{h_{pw}} \right)^2 dh + \right. \\
\left. (\Theta_{pw-A} + \Theta_{pw-B})^2 \int_{h_2}^{h_3} dh \right] + \\
\frac{\ell' \mu_0}{2W_2} (\Theta_{pw-A} + \Theta_{pw-B})^2 \int_{h_3}^{h_4} dh + \frac{\ell' \mu_0}{2W_0} \int_{h_4}^{h_5} dh + \\
\frac{\ell' \mu_w}{2} (\Theta_{pw-A} + \Theta_{pw-B})^2 \int_{h_5}^{h_6} \left[ w_2 + (w_0 - w_2) \frac{h-h_5}{h_1} \right] dh
\]

Note that:

\[
\Theta_{pw-A} = \Theta_{pw-B} = \frac{1}{2} \Theta_{pw}
\]

When a short pitched winding is used, the currents in the two CW coils in a stator slot are either in phase or have a 120° phase shift. In case of the latter, the term \( \Theta_{pw-A} \Theta_{pw-B} \) in (30) is multiplied by \( \cos \frac{2\pi}{3} \), representing the mutual influence of the bottom and upper currents [22]. Assuming \( q \) is the number of slots per phase per pole and \( a \) is the number of short pitched slots, the \( \cos \) terms can be averaged as:

\[
\cos \beta = \frac{1}{2q} \sum_{i=1}^{3q} \cos \gamma_i = \frac{1}{2q} \left[ 2a \cos \frac{2\pi}{3} + 2(q-a) \cos 0 \right]
\]

The term \( \Theta_{pw-A} \Theta_{pw-B} \) can hence be written as:

\[
\Theta_{pw-A} \Theta_{pw-B} = \Theta_{pw}^3 \cos \beta_{pw}
\]
Substituting (32) and (33) into (30), the PW leakage magnetic field energy in the slot is:

\[
W_{\phi} = \frac{1}{2} \ell' \Theta^2_{pw} \mu_0 \left( \frac{5}{3} + \cos \beta_{pw} \right) \frac{h_{pw}}{w_1} + \frac{h'}{w_1} + \frac{h_3 - h_2}{w_1} (2 + 2 \cos \beta_{pw}) + \frac{h_a}{w_2} (2 + 2 \cos \beta_{pw}) + \frac{1}{2} \ell' \Theta^2_{pw} \mu_0 \left[ \frac{h_{w_2}}{w_2} (2 + 2 \cos \beta_{pw}) + \frac{h_{w_1}}{w_2 - w_o} (2 + 2 \cos \beta_{pw}) \ln \left( \frac{w_3}{w_o} \right) \right]
\]

(34)

The total PW leakage magnetic energy stored in the slot is:

\[
W_{\phi} = \frac{1}{2} \ell'_{\text{slot-leakage}} \Theta^2_{pw} = \frac{1}{2} \ell' \Theta^2_{pw} \lambda_{s-pw}
\]

(35)

Where \( \lambda_{s-pw} \) is the PW slot leakage of one slot. From (34) and (35), \( \lambda_{s-pw} \) can be extracted by defining three terms: self-permeance for PW layer A \( \lambda_{pw-A} \), layer B \( \lambda_{pw-B} \), and mutual permeance between these two layers \( \lambda_{pw-M} \):

\[
\lambda_{pw-A} = \mu_0 \left( \frac{h_{pw}}{w_1} + \frac{h_3 - h_2}{w_1} + \frac{h_a}{w_o} \right) + \mu_w \left( \frac{h_{w_2}}{w_2} + \frac{h_{w_1}}{w_2 - w_o} \ln \left( \frac{w_2}{w_o} \right) \right)
\]

(36)

\[
\lambda_{pw-B} = \mu_0 \left( \frac{h_{pw}}{w_1} + \frac{h_3 - h_2}{w_1} + \frac{h_a}{w_o} \right) + \mu_w \left( \frac{h_{w_2}}{w_2} + \frac{h_{w_1}}{w_2 - w_o} \ln \left( \frac{w_2}{w_o} \right) \right)
\]

(37)

\[
\lambda_{pw-M} = \mu_0 \left( \frac{h_{pw}}{w_1} + \frac{h_3 - h_2}{w_1} + \frac{h_a}{w_o} \right) + \mu_w \left( \frac{h_{w_2}}{w_2} + \frac{h_{w_1}}{w_2 - w_o} \ln \left( \frac{w_2}{w_o} \right) \right)
\]

(38)

\( \lambda_{s-pw} \) is then:

\[
\lambda_{s-pw} = \lambda_{pw-A} + \lambda_{pw-B} + 2 \times \lambda_{pw-M} \times \cos \beta_{pw}
\]

(39)

The slot leakage for the CW can be derived using the same procedure as that of presented for the PW as:

\[
\lambda_{s-cw} = \lambda_{cw-A} + \lambda_{cw-B} + 2 \times \lambda_{cw-M} \times \cos \beta_{cw}
\]

(40)

The PW and CW leakage inductances can now be calculated from (21) and (22).

IX. Verification of Analytical Methods

In order to verify the accuracy of the proposed analytical methods in Sections VII and VIII, the magnetising and leakage inductances derived from the coupled circuit model embedding the new analytical methods are compared with parameters predicted by the numerical models, i.e. FE-LN-W.

Fig. 8 shows the area in the stator slot opening which is occupied by a magnetic wedge. Three cases have been considered to examine the accuracy of the analytical methods with respect to wedge dimensions and permeability, see Table VI:

Case A: this is the real case in the D400 BDFM. The magnetic wedge fills Area1 and Area2 and has a relative permeability of \( \mu_w = 20 \) which is an approximation of its non-linear magnetic characteristic provided by the manufacturer.

Case B: the magnetic wedge has identical dimensions as Case A, but its relative permeability is assumed to be \( \mu_w = 10 \).

Case C: the magnetic wedge only fills Area1 and hence Area2 is assumed unfilled i.e. is filled by air. The relative permeability of magnetic wedge is identical to Case A, i.e. \( \mu_w = 20 \).

<table>
<thead>
<tr>
<th>Magnetic Wedge</th>
<th>( \mu_w )</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>20</td>
<td>Area1 &amp; Area2</td>
</tr>
<tr>
<td>Case B</td>
<td>10</td>
<td>Area1 &amp; Area2</td>
</tr>
<tr>
<td>Case C</td>
<td>20</td>
<td>Area1</td>
</tr>
</tbody>
</table>

The magnetizing and leakage inductance parameters in the referred equivalent circuit shown in Fig. 1 are calculated using the coupled circuit and FE-LN-W models for Cases A, B and C and are shown in Tables VII, VIII and IX. As can be seen, there is generally close agreement between the parameters predicted by two models for all the three cases, hence validating the proposed analytical methods. The difference in the magnetizing inductances is below 2% in all cases which confirms that the proposed method to calculate the Carter factor used to obtain an effective air gap length leads to acceptable accuracy.

The difference in \( L_f \) which represents the stator and rotor leakage inductances is around 4 to 5%, well within the limitations of calculating leakage fluxes. The prediction of leakage inductance by the coupled circuit model in all three cases is always higher than the numerical model, showing that the analytical method overestimates the leakage inductances.

An important conclusion from the results shown in Tables VII, VIII and IX is that, the differences in the parameters calculated from analytical and numerical methods are small and, for all the three cases, are in a similar range; this confirms the accuracy of the analytical methods with respect to dimensions and magnetic property of magnetic wedges. Thus, the approach can be utilized during the BDFM design.
optimization to specify suitable magnetic wedges for stator slot openings in order to achieve an appropriate balance between the magnetization requirements, reactive power management and low-voltage ride-through performance of the machine.

TABLE VII: Equivalent circuit parameters Case A

<table>
<thead>
<tr>
<th></th>
<th>(L_{m1}(\text{mH}))</th>
<th>(L_{m2}(\text{mH}))</th>
<th>(L'_r(\text{mH}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE-LN-W</td>
<td>98.6</td>
<td>348.5</td>
<td>15.2</td>
</tr>
<tr>
<td>CC-LN-W</td>
<td>98.3</td>
<td>352</td>
<td>16</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>1.5%</td>
<td>1%</td>
<td>5.2%</td>
</tr>
</tbody>
</table>

TABLE VIII: Equivalent circuit parameters Case B

<table>
<thead>
<tr>
<th></th>
<th>(L_{m1}(\text{mH}))</th>
<th>(L_{m2}(\text{mH}))</th>
<th>(L'_r(\text{mH}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE-LN-W</td>
<td>93</td>
<td>329.2</td>
<td>12.3</td>
</tr>
<tr>
<td>CC-LN-W</td>
<td>94.6</td>
<td>330</td>
<td>12.8</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>1.7%</td>
<td>0.2%</td>
<td>4.1%</td>
</tr>
</tbody>
</table>

TABLE IX: Equivalent circuit parameters Case C

<table>
<thead>
<tr>
<th></th>
<th>(L_{m1}(\text{mH}))</th>
<th>(L_{m2}(\text{mH}))</th>
<th>(L'_r(\text{mH}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE-LN-W</td>
<td>90</td>
<td>319.4</td>
<td>12.8</td>
</tr>
<tr>
<td>CC-LN-W</td>
<td>91.7</td>
<td>323</td>
<td>13.4</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>1.9%</td>
<td>1.1%</td>
<td>4.7%</td>
</tr>
</tbody>
</table>

X. Conclusions

In this paper, the equivalent circuit parameters have been derived for a 250 kW D400 BDFM using several methods including analytical and numerical models and from experimental tests. One key feature of this BDFM as compared to the ones studied before is the use of magnetic wedges in stator slot openings, which affect the magnetising properties of the machine as well as reactive power management and performance during grid low voltage faults. The use of magnetic wedges in large electrical machines is common and is expected to be utilized in large BDFMs, and hence the approach of this paper is important.

It was shown that existing analytical methods are not able to give acceptable accuracy in calculating magnetizing and leakage inductances for the BDFM with magnetic wedges used in stator slots. Hence, analytical methods have been proposed to take into account the effects of magnetic wedges. A modified Carter factor has been derived which is used to calculate the effective air gap length to obtain the magnetizing inductances in the BDFM. In addition, a method to calculate slot leakages has been presented which calculates the increase in stator leakage inductances due to the presence of magnetic wedges. The analytical methods have been used to predict the parameter values for the D400 BDFM and two other design variations with magnetic wedges of different dimensions and magnetic property and results have been compared to predictions from numerical models. It has been shown that the analytical methods lead to good agreement with numerical models, hence have sufficient accuracy to be used in the BDFM design optimisation.

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References

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