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Reactive Power & Voltage Control in Grid-Connected Wind Farms: An Online Optimization Based Fast Model Predictive Control Approach

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***Abstract*—This paper presents the application of an online optimization based fast model predictive control scheme to grid-connected wind farms for reactive power and voltage control. A linear prediction model of the network was used to predict the behavior of the system for a certain prediction horizon while a modified quadratic programming problem was used for the optimization process. The proposed controller was tested in a 5-bus test system hosting 3 sub wind farms of total 36 MW active power production capacity connected in series to the external network. The controller performed its control action by changing the reactive power output of the sub wind farms and voltage set-points of an Online Load Tap Changer (OLTC) transformer to respect the safety limit imposed on the bus voltages and desired reactive power exchange.**

***Index Terms*—Model Predictive Control, Reactive Power Control, Voltage Control, Wind Farms, Grid Integration**

I. Introduction

The global energy market is currently experiencing a renewable energy revolution. Governmental policy, reduced carbon emission, fiscal benefit, pressure from various NGO's etc. are contributing to this revolution. Every day, more & more renewable energy sources are being added to the global energy network. Their contribution is also increasing steadily to the global energy-mix. Out of various renewable energy sources, wind power has been able to establish itself as one of the key player among the others. It is considered as one of the most promising & important renewable energy sources. The wind energy technology has progressed a lot in the last few decades. Because of this technological advancement, the growth of wind energy installation is also very high. As a result, it has started to become an influential factor in the overall power system operation. This rapid integration of wind energy/distributed generation into the grid has brought many operational challenges in the distribution network as most of them are directly connected to the distribution network instead of the transmission network [13], [2], [45], [36]. This operational challenge comes with the benefit of rapid extension of the distribution system. Although, they have started to create impact on the overall power system operation but until recent times they were not obliged to provide any support to the distribution/transmission system operation. Because of their impact, Distribution System Operator's (DSO) have started to oblige Distributed Generators (DG) like Wind farm to contribute to the power system operation. Reactive Power and Voltage supports are now sought from DG's [47].

The challenge here is to optimally manage the reactive power support while maintaining the acceptable grid voltages [44]. In order to provide the aforementioned supports, Wind farm must have to supply/absorb reactive power along with the active power supply. To do this, flexible and coordinated control actions are needed, which will take into account the problem of reactive power & voltage control (RPVC). Reactive power plays an important role in improving the voltage profile in the overall power system operation. Reactive power flow can be controlled through the generator voltages, transformer tap and also through the reactive power capability of the generator [12]. The problem of RPVC has been well studied in the literature. Various types of evolutionary optimization algorithm like Genetic Algorithm(GA) [18], [28], [35], shuffled frog-leaping algorithm [41], Particle Swarm Optimization (PSO) [11], various variants of Genetic algorithms [38], PSO [1], [24], [42], [48], [29], Ant colony optimization [21], [43], Differential Evolution (DE) [40], [20], Cuckoo Search (CS) [4], Harmony Search (HS) [3], [16], [9] etc. have been used. Classical methods like Genetic Algorithm suffers from various problems like sticking around the local minima instead of the global minima, high computation time etc.

However, the main problem associated with the abovementioned techniques is that the optimization problem is always formulated as a single step optimization problem. Single step problems are open-loop hence cannot be robust to various uncertainties or change in system properties. Instead of single-step optimization, multi-step optimization based Model Predictive Control can be used. Model Predictive Control (MPC) has recently gained much popularity among the research community [15], [52]. For example, in the area of electrical machine & power electronics, it has attracted a lot of attention recently [17], [33], [32], [31], [23], [22]. Although the application of MPC is very new in power system domain but still a number of researches have already been done and got wide acceptance. In [51], [50], [49], classical MPC have been used to control the voltages of active distribution network. MPC based control scheme for load management in power network with renewable energy sources have been discussed in [55]. MPC based control strategy has been developed in [25] to

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correct the transmission voltages. A two-stage MPC controller has been proposed in [27], [26] to prevent voltage collapse. In the first stage, a static load shedding algorithm was used to obtain stable MPC prediction simulations. While in the second stage, a linear programming problem was used to optimize the control action. In [34], the authors have proposed a coordinated system protection scheme (SPS) against voltage collapse based on MPC and heuristics tree search. In [10], authors have proposed a MPC based voltage stabilization using the state automata description of the primary voltage controller from the hybrid system (System having both continuous and discrete dynamics, Example-Bouncing Ball) point of view. The hybrid system arises from the interaction of the continuous dynamics of the nonlinear load to the discrete dynamics of the controller. This paper proposes a centralized MPC based control scheme for grid connected wind farms to address the problem of reactive power and voltage control. Instead of using traditional MPC, this paper uses a recently developed algorithm named as online optimization based fast model predictive control [53]. In this algorithm, the structure of the quadratic programming problem is exploited to speed up the optimization process. This approach makes the computation of control action faster in the order of maximum 100 times than a method that uses generic optimizer for some specific classes of problems. This faster computation time makes it a suitable candidate for real-time operation. There also exists some other fast computation method for solving Quadratic programming problems [37], [39], [19]. Some them are not suitable for all class of system. For example, in [19], primal-dual interior-point methods have been proposed. Again, some are suitable for special pursoses like embedded optimization [37], [19]. Commercial efficient QP solvers like [39] are very costly. But the method being used in this paper is applicable to large class of MPC problems while freely available in public domain.

The paper is organized as follows: Section II discusses the control objective while section III discusses about model predictive control based problem formulation. The speedup of the optimization process i.e. fast model predictive control is discussed in section IV. The test system is discussed in section V. Section VI is devoted for simulation results while the last section i.e. section VII concludes the present work.

II. Control Objective

Typically, wind farms are located in distant geographical location far from the city where the wind profile is very high and also the population density is low. Onshore wind farms are also very popular. In fact, most of the large capacity wind farms are located on the shore. As wind is an intermittent renewable energy, the active power generation of the wind farms is very unpredictable. It may vary from zero to the rated capacity depending on the wind profile. Each wind generator has its own reactive power capability which depends on the type of generator being used and also on the power electronics setup. This reactive power capability can be derived from the P-Q diagram of the respected wind generator [5]. For example, a general rule of thumb for DFIG based wind turbine's reactive power capability is, $\pm Q = 0.33 * P$. When these wind farms are connected to the grid, they create challenges like voltage problem in the grid [46].

The objective of the present work is to address the problem of reactive power and voltage control. In case of voltage control, the objective is to maintain the voltages of different buses around predefined safe limits. These safe limits are generally defined by the system operators. Regarding reactive power support, the wind farms must respond to the requirements of the system operators. In this case, the objective is to optimally manage the reactive power resources of the wind farms to respond to the operator's request. The proposed controller is based on the idea that once we have achieved our goal/target, then control variables will not change their values instead will work on the previously computed values. This idea will help to reduce the introduction of any additional disturbance in the power system which rises from the continuous control effort. For example, we are in a case that all our bus voltages are around our desired limit and also we are successfully meeting the demand of reactive power support. In this case, the controller will stop working and the system will run on the previously computed control variable. The controller will come into action once it detects that the system is outside the limit. This approach will be illustrated through simulation results in section VI. The controlled variables in the case of our problem are the reactive power output of wind farms and voltage set-points of the OLTC transformer.

III. MPC Based Control Problem Formulation

Model Predictive Control (MPC) also referred to as receding (or moving) – horizon control is a feedback strategy that attempts to solve at every decision instant an open-loop optimal control problem over some prediction horizon, to apply the first input in the optimal control sequence and to repeat the process at the next decision instant [7]. The main idea is, at time instant k, with the help of latest available measurements data, a cost minimizing control strategy is computed for a time horizon in the future $[k, k+p]$, so that the optimal change of control variable helps to reach the target at the end of the prediction horizon, p. However, only the first step of the predicted control sequence is applied and the optimization process repeats. Keeping this idea in mind, the MPC based reactive power & voltage control problem is formulated. The controller calculates the change of reactive power output $(\mathbf{Q_g})$ of wind farms and the voltage set-point (V_{tap}) of the transformer to maintain the voltages at different buses while providing the requested reactive power support at the end of the prediction horizon. The change of the control variables at time instant k can be written as,

$$\Delta\mathbf{u}(k) = [\Delta\mathbf{Q}_g(k)^T, \Delta V_{tap}(k)]^T \tag{1}$$

Then the objectives of our problem become to minimize the change of the control variables while keeping constraints like bus voltages around a safe limit. This objective can be translated as the following standard Quadratic Programming (QP) problem:

$$minimize \quad \sum_{i=0}^{p-1} ||\Delta \mathbf{u}(k+i)||^2_{\mathbf{R}} \tag{2}$$

subject to,

$$\begin{aligned} \mathbf{u}^{min} &\leq \mathbf{u}(k+i) \leq \mathbf{u}^{max} \\ \Delta\mathbf{u}^{min} &\leq \Delta\mathbf{u}(k+i) \leq \Delta\mathbf{u}^{max} \\ \mathbf{x}^{min} &\leq \mathbf{x}(k+i) \leq \mathbf{x}^{max} \end{aligned}$$

Where, $\mathbf{x}(k)$ is the state of the system, $\mathbf{R}$ is the weight matrices to penalize the controlled variables. The states of the system $\mathbf{x}(k+i|k)$ are the voltages at different buses and reactive power exchange with the grid. The prediction model for the state can be expressed as,

$$\mathbf{x}(k+i|k) = \mathbf{x}(k+i-1|k) + \frac{\delta \mathbf{x}}{\delta \mathbf{u}} \Delta\mathbf{u}(k+i-1) \tag{3}$$

Here, $\frac{\delta \mathbf{x}}{\delta \mathbf{u}}$ is the sensitivity matrix of the state with respect to change in control variables and has the following form:

$$\frac{\delta \mathbf{x}}{\delta \mathbf{u}} = \begin{bmatrix} \frac{\delta x_1}{\delta u_1} & \frac{\delta x_1}{\delta u_2} & \cdots & \frac{\delta x_1}{\delta u_m} \\ \frac{\delta x_2}{\delta u_1} & \frac{\delta x_2}{\delta u_2} & \cdots & \frac{\delta x_2}{\delta u_m} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\delta x_n}{\delta u_1} & \frac{\delta x_n}{\delta u_2} & \cdots & \frac{\delta x_n}{\delta u_m} \end{bmatrix}$$

The controller updates itself with the data $\mathbf{x}(k|k)$ and previous control inputs $\mathbf{u}(k-1)$ to compute the next optimal control sequence [50]. The calculation of sensitivity matrix was done with power-flow analysis. The value of the weight matrices were assigned according to the priority. In our case, we assumed that the transformer voltage set-point is costlier than the reactive power output of the sub wind farms. It's because the no. of change of voltage set-points has an effect on the lifetime of OLTC transformer. So, the controller will always try not to use the voltage set-points of the OLTC as this will minimize the objective function.

IV. Fast Model Predictive Control

MPC relies on a dynamic model of the system being controlled which is very often a linear model. The state dynamics of a basic linear time invariant system can be written as,

$$x(t+1) = ax(t) + bu(t) + \omega(t), \quad t = 0, 1, \cdots\cdots\cdots \tag{4}$$

Where time is denoted by t, $x(t) \in \mathbb{R}^n$is the state, $u(t) \in \mathbb{R}^m$ is denoting the output and the disturbance is denoted by $\omega(t) \in \mathbb{R}^n$. The state matrix $A \in \mathbb{R}^{nxn}$ and the input matrix $B \in \mathbb{R}^{nxm}$ are known data. It is assumed that for different values of t, $\omega(t)$ values are independent identically distributed (IID) with known distribution. Then let, $\varpi = E\omega(t)$ is the mean of $\omega(t)$ (which is independent of t) [53].

Then the current input $u(t)$ must have to be determined from the previous & current states $x(0), \ldots, x(t)$ by the control policy. The role of the Model Predictive Control (MPC) is to find this control policy by solving an optimization problem. The control input $u(t)$ in MPC can be found at each step by solving the following Quadratic Programming (QP) problem

$$minimize \quad \frac{1}{T} \sum_{\tau=t}^{\tau=t+T-1} s(x(\tau), u(\tau)) \tag{5}$$

subject to,

$$\begin{gathered} F_1 x(\tau) + F_2 u(\tau) \leq f, \quad \tau = t, \ldots, t+T-1 \\ x(\tau+1) = Ax(\tau) + B(\tau) + \varpi, \\ \tau = t, \ldots, t+T-1 \\ 0 = Ax(t+T-1) + Bu(t+T-1) + \varpi \end{gathered}$$

with variables $x(t+1), \ldots, x(t+T-1), u(t+1), \ldots, u(t+T-1)$, and with problem data $x(t), A, B, Q, S, R, F_1, F_2, f, \varpi$. Here, T is known as the time horizon. If we assume that $u^*(t), \ldots, u^*(t+T-1), x^*(t), \ldots, x^*(t+T-1)$ is optimal for the QP (5), then at each time instant t, the MPC controller will take the policy $u(t) = u^*(t)$.

A. Primal Barrier Interior-Point Method

In this section, a basic primal barrier interior-point method will be described for the solution of the QP problem mentioned in equation (5). Let us introduce an overall optimization variable $z = (u(t), x(t+1), u(t+1), \ldots, x(t+T-1), u(t+T-1))$, where $z \in \mathbb{R}^{Tm+T(n-1)}$. Then the QP problem of (5) can be expressed as

$$minimize \quad z^T Hz + g^T z \tag{6}$$

subject to,

$$Pz \leq h, \ Cz = b$$

Where,

$$H = \begin{bmatrix} R & 0 & 0 & \ldots & 0 & 0 \\ 0 & Q & S & \ldots & 0 & 0 \\ 0 & S^T & R & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & Q & S \\ 0 & 0 & 0 & \ldots & S^T & R \end{bmatrix}, g = \begin{bmatrix} 2S^T x(t) \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} F_1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & F_1 & F_2 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & F_1 & F_2 \end{bmatrix},$$

$$C = \begin{bmatrix} -B & I & 0 & 0 & \ldots & 0 & 0 \\ 0 & -A & -B & I & \ldots & 0 & 0 \\ 0 & 0 & 0 & -A & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & I & 0 \\ 0 & 0 & 0 & 0 & \ldots & -A & -B \end{bmatrix},$$

$$b = \begin{bmatrix} Ax(t) + \varpi \\ \varpi \\ \varpi \\ \vdots \\ \varpi \\ \varpi \end{bmatrix}, h = \begin{bmatrix} f - F_1 x(t) \\ f \\ \vdots \\ f \end{bmatrix}$$

To solve the QP problem (6), an infeasible start primal barrier method will be used. In this method, the inequality constraint of the QP problem of (6) will be replaced by a barrier term in the objective function. So, the approximate problem becomes [53]

$$minimize \quad z^T H z + g^T z + k\phi(z) \tag{7}$$

subject to

$$Cz = b,$$

where the barrier parameter $\kappa > 0$ and the log barrier ϕ associated with the inequality constraints can be defined as,

$$\phi(z) = \sum_{i=1}^{lT} -log(h_i - p_i^T z)$$

where, $p_i^T, \ldots, p_{lT}^T$ are the rows of P defined in (3). The equality constrained QP problem of equation (4) is a convex optimization problem with smooth objective function. This can be solved using any Newton's method. Infeasible start Newton method can be a solution.

B. Infeasible Start Newton Method

According to [14], let us associate a dual variable $v \in \mathbb{R}^{Tn}$ with the equality constraint $Cz = b$. Then the optimality conditions become,

$$\begin{aligned} r_d &= 2Hz + g + \kappa P^T d + C^T v = 0, \\ r_p &= Cz - b = 0 \end{aligned} \tag{8}$$

where, $d_i = 1/(h_i - p_i^T z)$ and p_i^T denotes the i-th row of P. The term $\kappa P^T d$ is the gradient of $\kappa\phi(z)$. There also an implicit constraint exist here which is $Pz < h$. r_p and r_d are called as primal residual and dual residual respectively. They constitute the residual vector r denoted as, $r = [r_d^T \, r_p^T]^T$. Then the optimality condition for (8) becomes $r = 0$.

In this algorithm i.e. Infeasible Start Newton Method, the initial point z^0 strictly satisfies the implicit inequality constraints ($Pz^0 < h$) but may not satisfy the equality constraints $Cz = b$. So, the initial point z^0 can be infeasible. The name of the algorithm as infeasible starts comes from this infeasible point. For the dual variable v, any initial point v^0 can be chosen to start with.

Then an approximate z (satisfying $Pz < h$) and v are maintained at each step. If the primal residual r_p and dual residual r_d are small enough, the algorithm quits. Otherwise, the estimation is refined by linearizing the optimality condition given in (8). And computing the primal and dual steps $\Delta z, \Delta v$ for which$z + \Delta z$, $v + \Delta v$ give zero residuals in the linearized approximation.

The search steps Δz and Δv are found by solving the following linear equations,

$$\begin{bmatrix} 2H + \kappa P^T diag(d)^2 P & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta v \end{bmatrix} = - \begin{bmatrix} r_d \\ r_p \end{bmatrix} \tag{9}$$

Where, $2H + \kappa P^T diag(d)^2 P$ is the Hessian of $\kappa\phi(z)$. Once the search steps Δz and Δv are computed, a step size $s \in (0, 1]$ can be found by using a backtracking line search on the norm of the residual vector r while making sure that the implicit inequality constraint $Pz < h$ holds true for every updated point. Then the primal & dual variable are updated as $z := z + s\Delta z$ and $v := v + \Delta v$. As long as the norm of the residual vector r is above an acceptable value, the procedure keeps repeating.

If the problem (6) is strictly feasible then it can be shown that the primal feasibility $Cz = b$ will be achieved in a finite number of steps. Once the primal residual r_p becomes zero, it will be unchanged for the rest of the iterations. Also within a finite number of steps, the convergence of z and v for optimal point will also happen.

C. Fast Computation of the Newton Step

The solution of linear equations has a great role in terms of speed up of the optimization problem. Instead of exploiting the structure of the linear equations given in (6), if they are solved using a densed LDL^T factorization, the total cost will be $(1/3)T^3(2n + m)^3$ flops. This computational cost can be reduced by exploiting the structure of (9). One of the ways is to use the block elimination procedure. Let us start with denoting the Hessian $2H + \kappa P^T diag(d)^2 P$, of the barrier term $\kappa\phi(z)$ as, $\Phi = 2H + \kappa P^T diag(d)^2 P$ which is block diagonal. With the first block mxm, the last block nxn and the remaining $T - 1$ block $(n + m)x(n + m)$. Its inverse is also block diagonal. The inverse can be written as,

$$\Phi^{-1} = \begin{bmatrix} \tilde{R}_0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & \tilde{Q}_1 & \tilde{S}_1 & \ldots & 0 & 0 & 0 \\ 0 & \tilde{S}_1^T & \tilde{R}_1 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \tilde{Q}_{T-1} & \tilde{S}_{T-1} & 0 \\ 0 & 0 & 0 & \ldots & \tilde{S}_{T-1} & \tilde{R}_{T-1} & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & \tilde{Q}_T \end{bmatrix}$$

The algorithm for solving equation (9) by block elimination method can be summarized as [14]:

- 1) Form the Schur complement $Y = C\Phi^{-1}C^T$ and $\beta = -r_p + C\Phi^{-1}r_d$
- 2) Determine Δv by solving $Y\Delta v = -\beta$
- 3) Determine Δz by solving $\Phi\Delta z = -r_d - c^T v$

The Schur complement Y is a block diagonal matrix which has the form,

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & 0 & \ldots & 0 & 0 \\ Y_{21} & Y_{22} & Y_{23} & \ldots & 0 & 0 \\ 0 & Y_{32} & Y_{33} & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & Y_{T-1,T-1} & Y_{T-1,T} \\ 0 & 0 & 0 & \ldots & Y_{T,T-1} & Y_{T,T} \end{bmatrix}$$

Where,

$$\begin{aligned} Y_{11} &= B\tilde{R}_0 B^T + \tilde{Q}_1 \\ Y_{ii} &= A\tilde{Q}_{i-1}A^T + A\tilde{S}_{i-1}B^T + B\tilde{S}_{i-1}^T A^T + B\tilde{R}_{i-1}B^T + \tilde{Q}_i, \quad i \\ Y_{i,i+1} &= Y_{i+1,i}^T = -\tilde{Q}_i A^T - \tilde{S}_i B^T, \quad i = 1, \ldots, T-1 \end{aligned}$$

The computation procedure for the first step of the algorithm just mentioned before can be stated as, first we have to compute the Cholesky factorization of the block diagonal matrix Φ. Then the formation of Y can be easily done by backward and forward substitution with columns taken from A and B, followed by multiplying the associated blocks in C. In order to compute Δv i.e. step-2, we have to compute the Cholesky factorization of Y, followed by backward and forward substitution. The Cholesky factorization of Y can be written as $Y = LL^T$ where L is the lower triangular and has the following lower bidiagonal block structure,

$$L = \begin{bmatrix} L_{11} & 0 & 0 & \dots & 0 & 0 \\ L_{21} & L_{22} & 0 & \dots & 0 & 0 \\ 0 & L_{32} & L_{33} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & L_{T-1,T-1} & 0 \\ 0 & 0 & 0 & \dots & L_{T,T-1} & L_{T,T} \end{bmatrix}$$

Where, L_{ii} are nxn lower triangular with positive diagonal entries and $L_{i+1,i}$ are general nxn matrices. From the Cholesky factorization of Y i.e. $Y = LL^T$, it can, be found that

$$\begin{aligned} L_{11}L_{11}^T &= Y11 \\ L_{ii}L_{i+1,i} &= Y_{i,i+1}, \quad i = 1, \dots, T-1 \\ L_{ii}L_{ii}^T &= Y_{ii} - L_{i,i-1}L_{i,i-1}^T, \; i = 2, \dots, T \end{aligned}$$

The computation speed and cost of step-3 i.e. computation of Δz depends heavily on the previous steps since this step requires the computation of the Cholesky factorization of Φwhich is already done in step-1.

D. Warm Start

If we solve an optimization problem, then the computational effort for solving another closely related optimization problem can be reduced by taking advantage of the information's gained while solving the original optimization problem. It means that the starting point for an optimization problem can be taken from a previously solve closely related optimization problem. This idea is known as 'Warm-start strategies' [30], [54]. As in MPC, we have to solve our QP problem (6) at each time step for a time horizon , the optimization problem is closely related to the problem of previous time steps. So, Warm start strategies can be a good solution to reduce the computational effort which in turn saves time. Which means, we can use the previously computed plan, suitably shifted in time, as a good starting point for the current plan [53]

Let us assume that, we have solved our QP problem (6) at time step $t-1$ with the trajectory $\tilde{z} = (\tilde{u}(t-1), \tilde{x}(t-1), \dots, \tilde{x}(t+T-2), \tilde{u}(t+T-2))$. Then the primal barrier method for time step t can be initialized with

$$z^{init} = (\tilde{u}(t), \tilde{x}(t-1), \dots, \tilde{x}(t+T-2), \tilde{u}(t+T-2), 0, 0)$$

z^{init} will satisfy the constraints if we assume that $\tilde{z}$ satisfies the equality and inequality constraints strictly. The assumption may not always work. So, there exists a possibility that z^{init} will not satisfy the constraints at the first step. In order to avoid constraints violation, the initial $u(t)$ can be modified in a way such that it satisfies, $F_1u(t) + F_2x(t) < f$. By this modification we can avoid violating the constraints initially.

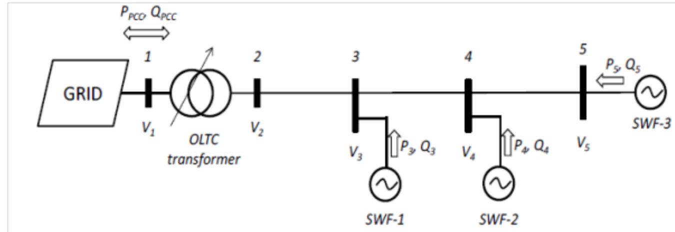

Figure 1. 5-bus test system

V. Test System

The proposed online optimization based fast MPC controller has been tested in a 5-bus 11 KV network. The network topology is shown in fig. 1 The network has three sub wind farms. The network is connected to the external grid through a 33/11 kV On Load Tap Changer (OLTC) transformer. The three sub wind farms are connected in series to the transformer. For simulation environment, Matlab/Simulink has been selected. The online optimization based fast MPC controller was developed in C using the fortran library LAPACK [8]. Later it was compiled through mex option of Matlab/Simulink to be useable in Matlab/Simulink[53].

The wind farms are consisted of Doubly Fed Induction Generator (DFIG) driven by wind turbines. Each wind frams has a rated capacity of 12 MW. The centralized MPC controller works by requesting the change in reference of the reactive power output of each wind farms. The controller also requests the OLTC controller to change its voltage set points. The internal controller of both the wind farms & the OLTC transformer works according to the reference signal generated by the centralized controller. The data's are collected at every 0.5 seconds. So, the system has a sampling period of 0.5 seconds. The optimization should have to be done within this time. The system base is 40 MVA. The safe limit of bus voltages are $[0.95$ p.u. 1.05 p.u.$]$. From the test system shown in fig. 1, it can be said that the most important voltage buses are bus 2 and 5. If they stay inside the limit, then the other buses i.e. 3 and 4 are supposed to be inside the limit. So, we will consider only the voltages of Bus 2 and 5. The reactive power exchange with the grid will take place at bus 1 i.e. Point of Common Coupling (PCC).

VI. Simulation and Results

This section discusses about the simulation result. We have analyzed the performance of the controller with various prediction horizons. Prediction horizons influences also the optimization time. So, after considering all the factors, a prediction horizon of $p = 10$ was chosen. Also in simulation, load conventions have been used i.e. positive reactive power means absorption while negative reactive power means production.

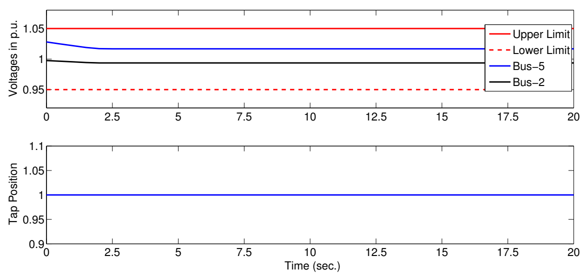

Figure 2. Bus voltages (top) and Tap position (bottom) scenario for case-1

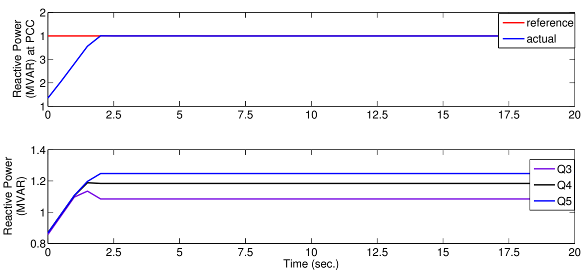

Figure 3. Reactive power exchange at PCC for case-1 (top) and each SWF's contribution (bottom)

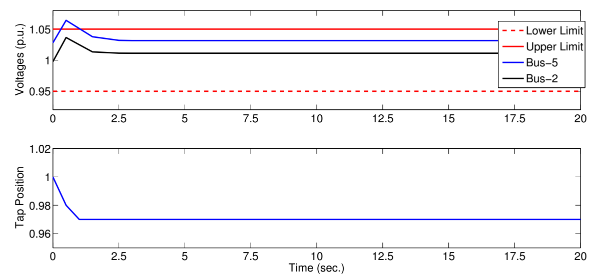

Figure 4. Bus voltages (top) and tap position (bottom) scenario for case-2

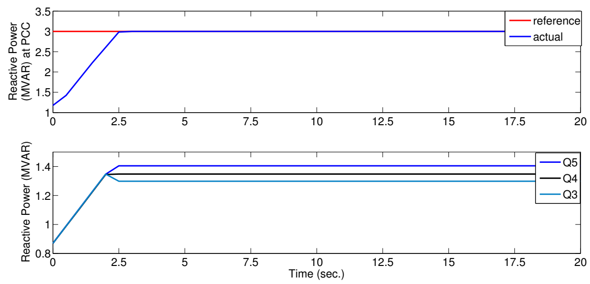

Figure 5. Reactive power exchange at PCC (top) and contribution of each SWF's (bottom) for case-2

To check the performance and effectiveness of the proposed controller, the controller was tested with three different various test scenarios. They are described below.

A. Case-1

In this case, initially the network is exchanging 1.2 MVAR (absorbing) reactive power with the grid and the voltage of the grid is 1 p.u. The bus voltages of the network are around the safety limit. At this time it received a request from the operator to increase the absorbed reactive power to 2.5 MVAR. The controller takes into account this request and starts to change the value of the control variable.

The simulation results are given in fig. 2 and fig. 3. From the figure it can be seen that the network gradually reaches to its target by changing the reactive power output of different subwind farms. As soon as, the target is reached, no change in the control variable can be seen. It's because the goal of the optimization is to minimize the change of control variable. When the target is achieved; the minimization of the objective function will be achieved if there are no changes in the controlled variable. Another thing to be noted that no change in transformer tap position can be seen. Its because the tap position change is penalized more in the objective function than the reactive power output.

B. Case-2

In this scenario, we assume that the system is absorbing 1.17 MVAR at a grid voltage of 1 p.u. Then it receives a request from the operator to absorb 3 MVAR however the grid voltage increased to 1.05 p.u. The simulation result for this case is shown in fig. 4 and 5.

From the fig. 4 and 5, we can see that because of sudden increase in the grid voltage the bus voltage of bus-5 went outside of the safety limit. However, playing with only reactive power output, the voltage will not enter inside the safe limit. So, the voltage set-points of OLTC transformer changed to keep the bus voltage inside the desired limit. Another point to be noted is that the voltage didn't enter into the desired limit instantaneously rather it entered slowly step by step. It is because of the ramp constraint introduced into the problem formulation in equation (2). This constraint was added to prevent sudden change in the control parameters which may introduce oscillation or unnecessary disturbances into the system.

C. Case-3

In this case, we will see the controller performance regarding reactive power production scenario. Initially the network is producing 1.4 MVAR at 1 p.u. grid voltage. However, it is requested to produce more to support the external network. The target is to produce 3 MVAR. The simulation result can be seen in fig. 6 and 7.

The figure is telling that the controller is well performing in this case also. It gradually reaches its target reactive power production. Bus voltages are increasing because of the reactive power production. However, as they are inside the limit, no change in voltage set-points of the transformer can be seen.

The simulation was done in a PC which is powered by an Intel Core 2 Duo Processor of 2.3 GHz & RAM of 2 GB. The PC was running on Linux operating system. The optimization process took around 7msec time. Further reduction in optimization time can be possible by using more high power processor like Core i-7. However, the optimization

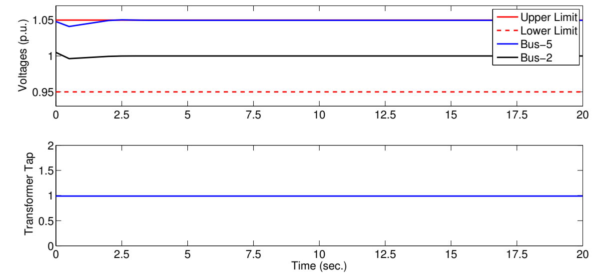

Figure 6. Bus voltages (top) and tap position (bottom) scenario for case-3

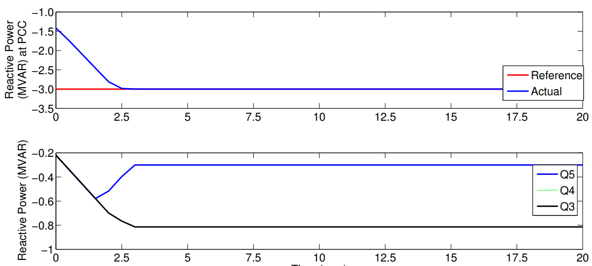

Figure 7. Reactive power exchange at PCC (top) and contribution of each SWF's (bottom) for case-3

time in our case was less than the sampling time which makes this optimization feasible for real-time implementation with physical networks.

VII. Conclusion

This paper presented the application of an online optimization based fast model predictive controller to the grid-connected wind farms. The controller coordinated the reactive power output and voltage set points of the OLTC transformer to keep the system states around their limit set by the distribution/transmission system operator. From the simulation results it can be said that the proposed controller is very useful in achieving its desires target i.e. reactive power & voltage control. The controller successfully performed in all the three test cases which include both reactive power production & absorption. Furthermore the optimization time it took was really small which opens up the possibility of real-time implementation.

Future works may include considering a non-linear prediction model of the network under consideration. This work was done for a 5-bus system which can be extended for higher bus system in future. Furthermore, the interaction of multi area system on the voltage & reactive power of the transmission network can be studied in future where a centralized/decentralized predictive controller might be considered. Another interesting thing could be applying Non-linear MPC. [6] Discussed real-time implementation of parameterized Non-linear MPC. This can also be considered.

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