

High Dynamic Precision Control System

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where

$$X_1 = [x_{11}, x_{12}, \dots, x_{1m}]^T,$$

$$X_2 = [x_{21}, x_{22}, \dots, x_{2v}]^T,$$

.....

$$X_n = [x_{n1}, x_{n2}, \dots, x_{nk}]^T.$$

X_1, X_2, \dots, X_n – are vectors of states of the plant,

$A_{11} = A_{11}(t), A_{12} = A_{12}(t), \dots, A_{mm} = A_{mm}(t)$ – are matrices of variable parameters of the plant,

U_1, U_2, \dots, U_n – are control vectors.

It is required to find such feedback laws that autonomous motion of plant's coordinates at each channel will be provided according to the following reference model:

$$\dot{X}_1^m = A_{11}^m X_1^m + G_1,$$

$$\dot{X}_2^m = A_{22}^m X_2^m + G_2,$$

.....

$$\dot{X}_n^m = A_{nn}^m X_n^m + G_n,$$

(2)

where

$$X_1^m = [x_{11}^m, x_{12}^m, \dots, x_{1m}^m]^T,$$

$$X_2^m = [x_{21}^m, x_{22}^m, \dots, x_{2v}^m]^T,$$

.....

$$X_n^m = [x_{n1}^m, x_{n2}^m, \dots, x_{nk}^m]^T.$$

$X_1^m, X_2^m, \dots, X_n^m$ – are vectors of states of the reference model,

$A_{11}^m, A_{22}^m, \dots, A_{nn}^m$ – are matrices of parameters of the reference model,

G_1, G_2, \dots, G_n – are inputs.

Synthesis of the Basic Loop's Structure

The feedback controls can be chosen as follows:

$$U_1 = G_1 + K_{11} X_1 + K_{12} X_2 + \dots + K_{1n} X_n,$$

$$U_2 = G_2 + K_{21} X_1 + K_{22} X_2 + \dots + K_{2n} X_n,$$

.....

$$U_n = G_n + K_{n1} X_1 + K_{n2} X_2 + \dots + K_{nn} X_n,$$

(3)

where

$K_{11} = K_{11}(t), K_{12} = K_{12}(t), \dots, K_{nn} = K_{nn}(t)$ – are matrices of the feedback variable gains.

According to the equations (1) and (3), the closed-loop system can be represented as follows:

$A_{ij}^0 = \text{const}$, ($i, j=1, \dots, n$), A_{ij}^0 – are the constant matrices whose coefficients correspond to the nominal operation mode of the plant,

$\Delta A_{ij} = \Delta A_{ij}(t)$, ΔA_{ij} – are deviation of the plant parameters from their nominal operation mode.

The controller parameters can be represented analogously:

$$K_{11} = K_{11}^0 + \Delta K_{11},$$

$$K_{12} = K_{12}^0 + \Delta K_{12},$$

.....

$$K_{nm} = K_{nm}^0 + \Delta K_{nm},$$

where

$K_{ij}^0 = \text{const}$, ($i, j=1, \dots, n$), K_{ij}^0 – are matrices of constant coefficients of the controller,

$\Delta K_{ij} = \Delta K_{ij}(t)$, ΔK_{ij} – are matrices of adjusting coefficients of the controller.

Thus, the motion of plant coordinates on a non-nominal operation mode can be represented as follows:

$$\dot{X}_i = \sum_{j=1}^n (A_{ij}^0 + \Delta A_{ij}(t)) X_j + U_i, \quad (i=1, \dots, n). \quad (8)$$

The feedback control is:

$$U_i = G_i + \sum_{j=1}^n (K_{ij}^0 + \Delta K_{ij}(t)) X_j, \quad (i=1, \dots, n). \quad (9)$$

Substituting the equation (9) into (8) we can obtain the equation of motion of the system "the plant + the controller" on each channel:

$$\dot{X}_i = \sum_{j=1}^n (A_{ij}^0 + \Delta A_{ij}(t)) X_j + \sum_{j=1}^n (K_{ij}^0 + \Delta K_{ij}(t)) X_j + G_i. \quad (10)$$

The model motion equation on each channel can be chosen as follows:

$$\dot{X}_i^m = A_{ii}^m X_i^m + \sum_{\substack{j=1 \\ j \neq i}}^n (A_{ij}^0 + K_{ij}^0) X_j + G_i. \quad (11)$$

Comparing (10) and (11) and taking into account that $A_{ii}^0 + K_{ii}^0 = A_{ii}^m$, we can obtain:

$$\dot{E}_i = A_{ii}^m E_i + (\Delta A_{ii}(t) + \Delta K_{ii}(t)) X_i + \left(\sum_{\substack{j=1 \\ j \neq i}}^n \Delta A_{ij}(t) + \sum_{\substack{j=1 \\ j \neq i}}^n \Delta K_{ij}(t) \right) X_j \quad (12)$$

where

$$E_i = X_i - X_i^m \quad (13)$$

Let us introduce the following notation:

$$\begin{aligned} Y_{ii} &= Y_{ii}(t) = \Delta A_{ii}(t) + \Delta K_{ii}(t), \\ Y_{ij} &= Y_{ij}(t) = \Delta A_{ij}(t) + \Delta K_{ij}(t), \quad (i \neq j). \end{aligned} \quad (14)$$

Thus, the error equation (12) can be represented as:

$$\dot{E}_i = A_{ii}^m E_i + Y_{ii} X_i + \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} X_j \quad (15)$$

Algorithms of adaptation can be specified in the following form:

$$\frac{d}{dt} \Delta K = \Psi \quad (16)$$

Admit, that parameters' deviation $\Delta A = \Delta A(t)$ is differentiable in time:

$$\frac{d}{dt} \Delta A(t) = R(t) \quad (17)$$

Therefore, according to (13) – (17) we obtain:

$$\begin{aligned} \dot{E} &= A^m E + YX, \\ \dot{Y} &= \Psi + R. \end{aligned} \quad (18)$$

where $R = R(t)$

The equations (18) with eq. (2) Describe the dynamic motion of the adaptive control system with the reference model.

Admit, that the matrix Ψ is a function of error deviation E and time t , and $\Psi(E, t) = 0$ at $t = 0$.

In this case the state and parametric error motion

$$E \equiv 0, \quad Y \equiv 0 \quad (19)$$

With the admission that $R(t) \equiv 0$ according to the hypothesis of quasi-stability, is the solution of the system (18).

It is suggested to use the second method of Lyapunov in order to obtain the adaptation algorithms Ψ from the condition of stability of the zero solution (19) of the system (18)) [3], [4], [5] and [6].

The quadratic function V can be chosen as follows:

$$V = \gamma E^T P E + tr(Y Y^T), \quad (20)$$

where

$\gamma = const$

P – is symmetric matrix.

Q – is negative definite matrix

A time derivative for the function V is obtained as follows:

$$\begin{aligned}\dot{V} &= \gamma \dot{E}^T P E + \gamma E^T P \dot{E} + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \gamma (A^m E + YX)^T P E + \gamma E^T P (A^m E + YX) + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \\ & \gamma [(A^m E)^T + (YX)^T] P E + \gamma E^T P (A^m E + YX) + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \\ & \gamma [E^T (A^m)^T + X^T Y^T] P E + \gamma E^T P A^m E + \gamma E^T P YX + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \\ & \gamma E^T (A^m)^T P E + \gamma X^T Y^T P E + \gamma E^T P A^m E + \gamma E^T P YX + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \\ & \gamma E^T ((A^m)^T P + P A^m) E + \gamma X^T Y^T P E + \gamma E^T P YX + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T)\end{aligned}\quad (21)$$

After some intermediate matrix manipulations we obtain that:

$$\Psi = -\gamma P E X^T \quad (22)$$

The derivative of the Lyapunov function is represented as:

$$\dot{V} = \gamma E^T Q E \quad (23)$$

Taking into account that A^T is Hurvitch matrix, we obtain:

$$\dot{V} \leq 0 \quad (24)$$

Therefore, according to (18), (16) and using conditions of quasi-stationarity of the system, it is straightforward to obtain the adaptation algorithms as:

$$\Delta \dot{K} = -\gamma P E X^T \quad (25)$$

It follows from the above that the motion of the system (16) with the algorithms (25) is stable and conditions (5) and (6) are satisfied.

CONCLUSIONS

The developed in this paper control algorithms allow one to improve the performance of a multivariable process or system due to the decoupling of channels of the process or system from the interaction via the process's or system's dynamics. The desirable performance of the process or system can be obtained using the relevant reference models for each channel. Adaptive control algorithms allow one to adjust the overall system's response in the case of parameters change. The stability of the overall system is guaranteed according to the Lyapunov stability criterion.

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