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Reduction of structural weight, costs and complexity of control system in active vibration reduction of flexible structures

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This paper concerns active vibration reduction of a flexible structure with discrete piezoelectric sensors and actuators in collocated pairs bonded to its surface. In this study, a new fitness and objective function are proposed to determine the optimal number of actuators based on variations in average closed loop dB gain margin reduction for all optimal piezoelectric pairs and the modes which required to be attenuated using the optimal linear quadratic control scheme. The aim of this study is to find the minimum number of optimally located sensor/actuator pairs, which can achieve the same vibration reduction as a greater number, in order to reduce the cost, complexity and power requirement of the control system. This optimisation was done using a genetic algorithm.

The technique is may be applied to any lightly damped structure, and is demonstrated here by attenuating the first six vibration modes of a flat cantilever plate. It is shown that two sensor/actuator pairs located and controlled optimally give almost the same vibration reduction as ten pairs. These results are validated by comparing the open and closed loop time responses and actuator feedback voltages for various numbers of piezoelectric pairs using the ANSYS finite element package and a proportional differential control scheme.

1. Introduction

Active vibration control of smart structures is achieved by measuring the vibration at a number of points and applying dynamic forces by actuators, normally collocated with the sensors to avoid spill over. The efficiency with which the vibration reduction is obtained depends on the number of sensor/actuator pairs, their location about the structure and the settings of the controller. This paper describes the optimisation of these parameters, leading to cost and weight reduction as well as reduction in control system complexity.

A few studies have investigated the optimal size and number of sensors and actuators. The optimal placement and size (length) of piezoelectric sensor/actuator pairs [1, 2] and feedback gain [3] were investigated for a beam based on the minimization of the optimal linear quadratic index and the maximization of the controllability index and energy dissipation as objective functions respectively. The optimal number and placement of piezoelectric actuators has been investigated for the active vibration control of trusses [4-6] and plates [7]. Here, the eigenvalue distribution of the energy correlative matrix of control input force was presented to determine the required number of actuators, and a genetic algorithm found their optimal placement based on active vibration control effects as an objective function [4, 7]. The optimal location and number of discrete actuators has been determined using a genetic algorithm with a fitness function based on energy degree of controllability and arbitrarily weighting factors [5]. Li et al proposed the use of a genetic algorithm for optimal placement and number of actuators in multi-storey buildings using the minimisation of the maximum top floor displacement of as an objective function [6].

Previous studies [8-12] have investigated the active vibration reduction of a cantilever plate by various numbers of optimally placed sensors and actuators: six sensors and two actuators [8], two sensor/actuator pairs [9], six sensor/actuator pairs [10] and ten sensor/actuator pairs [11, 12]. In this paper, fitness and objective functions are proposed to optimize the placement, feedback gain and number of piezoelectric sensor/actuator pairs for flexible structures using the genetic algorithm. These are presented as applied to a cantilever plate and compared for effectiveness with previously published results.

2. Modeling

Finite element and Hamilton's principle have used to model a plate bonded with discrete piezoelectric sensor/actuator pairs using four nodes isoparametric element and equilibrium dynamic equations was written in state space form as [13]:

$$\dot{X} = AX + B\phi_a \tag{1}$$

$$\phi_a = -KX \quad , \quad \phi_s = C_i X_i \tag{2}$$

$$\dot{X} = (A - BK)X \tag{3}$$

$$A_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & -2\xi_i\omega_i \end{bmatrix} \quad , \quad B_i = \begin{bmatrix} 0 \\ -\varphi^T K_{u\emptyset}^a \end{bmatrix} \tag{4}$$

$$C_i = \begin{bmatrix} -\varphi^T \omega_i^{-1} \, {K_{\emptyset\emptyset}^s}^{-1} K_{u\emptyset}^s & 0 \end{bmatrix} \quad , \quad X_i = \{\omega_i\eta_i \quad \dot{\eta}_i\}^T \tag{5}$$

where A_i , B_i and C_i are individual modal state, input actuator and output sensor matrices respectively and the subscript (i) refers to mode number. Matrix φ is an open-loop mass-normalised modal matrix obtained by solving the eigenvalue free damped problem and η is a single vector of the modal coordinates. The matrices $K_{\emptyset\emptyset}$ and $K_{u\emptyset}$ are piezoelectric permittivity and electromechanical coupling matrices[13].

3. Control law

In this work, optimal linear quadratic control was implemented to attenuate vibration, which is based on minimisation of the performance index J. Minimisation of this index was used to optimise the locations of sensors, actuators and feedback gain.

$$J = \int_0^\infty (X^T QX + \phi_a^T R\phi_a)dt \tag{6}$$

The weighted matrices Q of dimensions $2n_m \times 2n_m$ and R of dimensions $r_a \times r_a$ are positive definite or semi-definite Hermitian or real symmetric matrices where n_m and r_a represent the number of modes and actuators respectively. These matrices are managed the relative importance of error and controller energy, with high values of Q giving high vibration suppression and controller energy.

Ogata has shown that it is possible to follow this derivation to design a linear quadratic controller, which leads to the minimum quadratic performance index equation (7) and Riccati equations (8) and (9) [14]:

$$J = X(0)^T P X(0) \tag{7}$$

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{8}$$

$$K = R^{-1}B^T P\,, \qquad \phi_a = -KX \tag{9}$$

Solution of the reduced Riccati equation (8) gives the value of the Riccati matrix P; if matrix P is positive definite then the system is stable or the closed loop matrix $A - BK$ is stable. This means that the real part of the closed loop poles of $A - BK$ are negative and the poles located on the left hand side of the s-plane. Optimal feedback control gain can be obtained after substitution of the matrix P in equation (9).

4. Fitness and objective function: first stage

Minimisation of the linear quadratic index was implemented by Kondoh as an objective function to optimise actuator location and feedback gain for a flexible beam [15]. The performance index depends on actuator location and initial state conditions according to equation (7). The effect of initial state conditions was reduced by taking the average cost function [11, 16-18], giving:

$$J(x,y) = Trace\big(\mathrm{P}(x,y)\big) \qquad 10$$

$$J_{opt}(x,y) = \min(Trace\big(\mathrm{P}(x,y)\big), \mathrm{K}) \qquad 11$$

where $x, y \in$ structure dimensions

It can be seen from the Riccati equation (8) that the Riccati solution matrix P is a function of the actuator location represented by matrix B, while the matrices R, Q and A are constant for a particular control system. A range of high and low values of P can be gained by changing the location of actuators in matrix B. However, the optimisation of controller gain K and the locations of the actuators require a lower value of P, and it is difficult to find the correct the actuator matrix B using trial and error or simple optimisation methods especially when the optimisation problem has a very large number of candidate solutions. In this context, a genetic algorithm was implemented, which is a powerful optimisation method depending on fitness and objective functions in finding an optimal solution.

5. Fitness and objective function: second stage

A second stage of optimisation is based on the development of new fitness and objective functions in order to find the optimal number of actuators by measuring the variation of closed loop average dB gain reduction for all optimal sensor/actuator pairs and for all the required modes to be attenuated using the optimal linear quadratic index. These fitness and objective functions lead to a system with low weight, cost and system complexity. The second stage of optimisation is based on the following assumptions:

1. The optimal locations of sensor/actuator pairs n_{sa} , n_{sa+1} , , , n_{sa+n} and optimal feedback gain are determined based on the previous section using equation (11) and genetic algorithms placement strategy as explained in the next section .

2. A disturbance sinusoidal voltage of unit amplitude is applied at the first optimal actuator or a specified actuator throughout the optimisation as a reference for comparison.
3. The variation of average closed loop gain reduction for the first number of piezoelectric pairs in the optimal location (fitness value) equals 100% compared with the open loop gain reduction (or no sensor/actuator).

Take the Laplace transform of equations (1) and (2):

$$sX(s) = AX(s) + B\phi_a(s) \quad , \quad \phi_a(s) = B^{-1}(sI - A)X(s)$$
$$\phi_s(s) = CX(s) \tag{12}$$

The open loop transfer function of a system in the frequency domain is:

$$GO(\omega) = \frac{\phi_s(s)}{\phi_a(s)} = C(j\omega I - A)^{-1}B \tag{13}$$

$$GO(\omega) = C(j\omega I - A)^{-1}B \tag{14}$$

The open loop dB gain for the sensor j as a result of the unit voltage applied to the first actuator at the specified natural mode i is:

$$GO_{j,i} = 20\log_{10}(|C_i(j\omega_i I - A_i)^{-1}B_i|) \tag{15}$$

The average open loop dB gain for all sensors and modes due to applying the sinusoidal unit voltage at a specified single actuator is:

$$MGO = \frac{1}{n_m n_{sa}} \sum_{i=1}^{n_m} \sum_{j=1}^{n_{sa}} GO_{j,i} \tag{16}$$

where MGO refers to the average open loop dB gain for all sensors n_{sa} as a result of the applied sinusoidal unit voltage at the first actuator at all natural frequencies n_m to be suppressed. The closed loop state matrix is:

$$Ac_i = A_i - B_i K_i \tag{17}$$

The closed loop dB gain for a single sensor *j* as a result of applying the unit sinusoidal voltage disturbance at a specified single actuator and for a single mode *i* is:

$$GC_{j,i} = 20\log_{10}(|C_i(j\omega_i I - Ac_i)^{-1}B_i|) \tag{18}$$

The mean average closed loop dB gain for all sensor/actuator pairs and modes as a result of applying the unit voltage disturbance at a specified single actuator for different values of linear quadratic weighted matrices is:

$$MGC = \frac{1}{3n_m n_{sa}} \sum_{k=1}^{3} \sum_{i=1}^{n_m} \sum_{j=1}^{n_{sa}} GC_{j,i} \Bigg|_{at\ R=1, Q_k=10^6, 10^7, 10^8} \tag{19}$$

where MGC refers to the average closed loop dB gain for all sensor/actuator pairs n_{sa} as a result of applied the unit voltage at a specified actuator at all natural frequencies n_m for three values of linear quadratic weighted matrix settings. These weighted matrices select one, two or three setting values shown in equation (9) depending on a designer in order to get more accurate results and system behaviour. The average closed loop dB gain reduction *MGR* for an arbitrary number of piezoelectric pairs located at optimal locations is equal to the absolute difference in value between the open and closed loop dB gain:

$$MGR = |MGC - MGO| \tag{20}$$

$$J(n_{sa+1}) = MGV = \frac{MGR_{n_{sa}+1} - MGR_{n_{sa}}}{MGR_{n_{sa}+1}} \times\ 100\% \qquad 21$$

$$J_{opt}(n_{sa}) = MGV|_{n_{sa}+1} \leq 20\% \qquad 22$$

The developed fitness function MGV is a percentage of the variation of the closed loop dB gain margin of a number of sensor/actuator pairs n_{sa} and n_{sa+1} bonded at optimal locations on a flexible structure. The fitness function MGV mathematically represents by the ratio of the difference in average closed loop dB gain reduction for a structure bonded with numbers of optimal piezoelectric pairs equal to n_{sa} and n_{sa+1}divided by the average closed loop dB gain reduction for an optimal piezoelectric pair equal to (n_{sa+1}).

The fitness function *MGV* represents the percentage of closed loop gain marginal improvement over a given number of optimally located sensor/actuator pairs n_{sa} by the addition of one more pair. The optimum is taken as the smallest number of pairs for which this improvement is less effective when MGV less than 20%".

According to the objective function equation (22), the optimal number of piezoelectric actuators n_{sa} is selected at fitness value $J(n_{sa+1})$ less or equal to 20%, which could be varied by the designer depending on the requirements of the system.

The expected relationship between the average closed loop dB gain reduction *MGV* and the number of sensor/actuator pairs n_{sa} is represented in figure (1). It can be seen that the curve is divided into three zones ***a***, ***b*** and ***c***. The first zone **a** is considered to be the active zone limited to between 100% and 20% of *MGV*. The active zone is considered as a highly progressive zone which gives a sensibly increasing closed loop vibration reduction with increasing numbers of optimally placed sensor/actuator pairs. The second zone ***b*** is an important transition zone between the active zone ***a*** and the passive zone ***c***, limited to between 20% and 0% *MGV*. In this zone, the variation in closed loop vibration reduction is low. The final and biggest zone ***c*** is a passive zone with no variation in closed loop vibration reduction irrespective of the number of sensor/actuator pairs. The passive zone ***c*** involves merely drawback effects, by increasing the weight of the structure, the cost of materials, and the complexity of the control system.

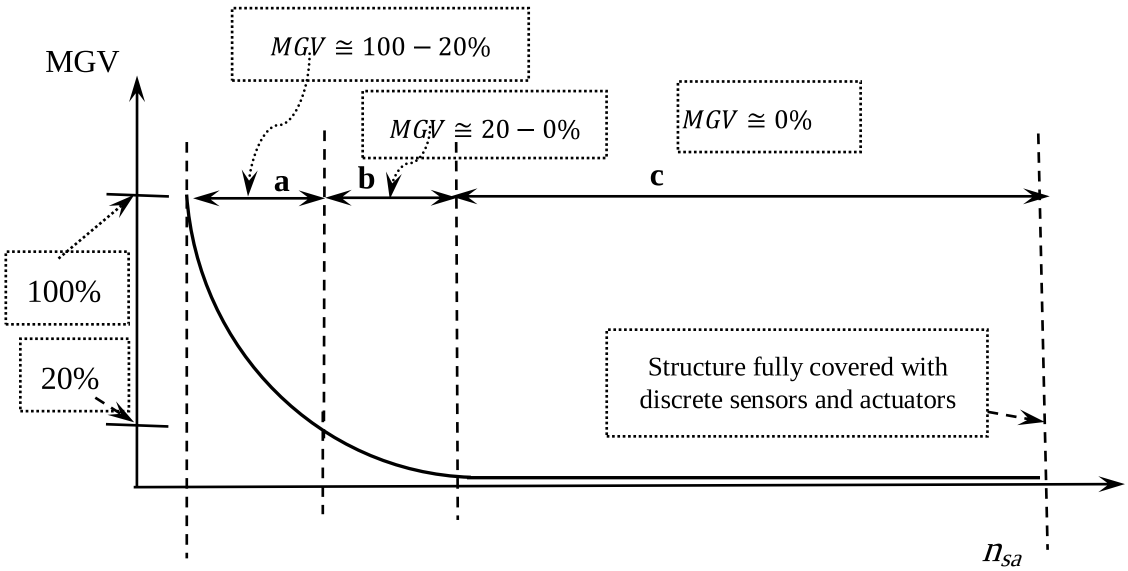


Figure 1. Expected relation between fitness function and number of sensor/actuator pairs

6. Genetic algorithm

In 1975, Holland invented the genetic algorithm, which is a superior guided random method based on the principle of survival of the fittest or natural evolution theory used to find optimal solutions. It has been continuously improved and is now a powerful method for searching for optimal solutions. In this work, the optimisation technique used is composed of two stages. The authors have proposed a placement strategy to optimise the location of discrete actuators and the feedback gain matrix using genetic algorithms [13], and this is used as a first stage of this study and summarised by the following points.

1. Suitable values of $Q = 10 \times 10^{11} \times I_{2n_m, 2n_m}$ and $R = 1 \times I_{r_a, r_a}$ are set by the user. The weighted matrix Q controls the level of vibration suppression of flexible structures. Increasing the value of the weighted matrix Q gives the optimal locations of sensor/actuator pairs on a structure in order to achieve higher vibration suppression, and this may require higher external energy and vice versa.
2. The state matrix A of dimension $(2n_m, 2n_m)$ is prepared for the first six modes of vibration according to equation (4).
3. One hundred chromosomes are chosen randomly from the search space to form the initial population.
4. The input (actuators) B matrix is calculated for each chromosome and for the first six modes of vibration according to equation (4).
5. A fitness value is calculated for each member of the population based on the fitness function, according to equation (10), and stored in the chromosome string to be saved for future recalculation.
6. The chromosomes are sorted according to their fitness value and the 50 chromosomes (less than or equal to the initial population depending on the problem size) with the lowest fitness values (i.e. the most fit) are selected to form the breeding population. These are called parents. The remaining, less fit, chromosomes are discarded.
7. The members of the breeding population are paired up in order of fitness and 50% crossover is applied to each pair; the crossover point being selected randomly and is different for each parent. This gives two new offspring (child) chromosomes with new properties.
8. A mutation rate of 5% is used on the child chromosomes.

9. The new chromosomes are filtered for repeated genes. It is a physical requirement of this work that there be a number of sensor/actuator pairs, so more than one gene for a particular location would be meaningless and disrupts the path to the optimal solution.
10. The input (actuators) matrix is calculated for each child chromosome according to equation (4) and thereafter the process is repeated from step 5 for a preset number of generations.

The placement strategy of the first stage is now improved to include a second stage which optimises the number of piezoelectric actuators based on the proposed objective function explained in the previous section. The genetic algorithm written in MATLAB m-code, described above, is improved to include the second stage with the following additional features:

1. The location of one piezoelectric actuator ($n_{sa} = 1$) is optimised according to the first stage as explained in the above listed ten points.
2. The linear quadratic control scheme is used with three settings of values of the weighting matrices values for $R = 1$ and $Q = 10^6, 10^7$ and 10^8. Low setting value of Q matrix achieves low vibration reduction with low energy consumed and vice versa at high setting. This range of setting is chosen to test the effectiveness of developed fitness and objective functions and the designer may choose the most appropriate. The average closed loop vibration level reduction is calculated for all optimal piezoelectric pairs, all vibration modes and all three weighted matrix values according to equation (20). The results are inserted into the optimal chromosome string for recalculation.
3. The area of the first optimal sensor/actuator pair is divided into five segments as a percentage from 4% to 100% in order to achieve more precise results as shown in the first part of table (4).

4. The fitness function is calculated according to equation (22).
5. If the fitness value is less than or equal to 20%, then the process is halted and the previous chromosome is taken to represent an optimal number and location of piezoelectric pairs according to the objective function in equation (22). Otherwise, the first step is repeated with a greater number of actuators $n_{sa}+1$ until the condition of the objective function is achieved.

7. Results and discussion

7.1 Research problem

The technique was applied to a flat plate of dimensions $500 \times 500 \times 1.9$mm constrained on one edge as a cantilever as shown in Figure (2). The plate is discretised into one hundred elements in a 10×10 matrix, sequentially numbered from left to right and down to up as shown in Figure (2). The properties of plate and piezoelectric materials are given in Table 1. The results of the first six natural frequencies using the ANSYS finite element package are shown in Table 2 and validated experimentally. These results are used to build the state space matrices of the control system based on plate discretisation of (10×10) elements according to the equations (4) and (5).

Table 1. Plate and piezoelectric properties

Properties	Steel plate	Piezoelectric PIC255
Modulus, GPa	210	----------
Density, Kg/m^3	7810	7180
Poisson’s ratio	0.3	---------
Thickness, mm	1.9	0.5
Length, width, mm	500	50
e_{31}, e_{32}, e_{33} , C/m^2	-------	-7.15, -7.15,13.7
$C_{11}^E, C_{12}^E, C_{13}^E, C_{55}^E$, GPa	-------	123,76.7,70.25, 22.226
μ_{33}^{σ} F/m	-------	1.5×10^{-8}

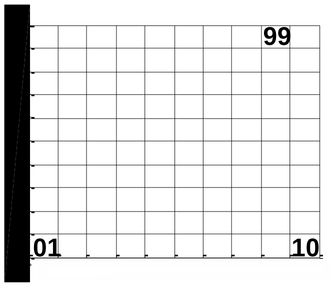


Figure 2. Cantilever plate mounted rigidly from the left hand edge descritised to one hundred elements sequentially numbered from left to right and down to up

Table 2 natural frequencies compared with MATLAB model and experiment for a cantilever plate

Element type	Mode (Hz)					
	1^{st}	2^{nd}	3^{td}	4^{th}	5^{th}	6^{th}
ANSYS shell63 (10×10)	6.59	16.19	40.49	51.66	59.01	103.34
Experimental Frequency	5.90	16.90	37.30	51.60	58.20	101.00
Damping ratio $\times 10^{-4}$	124.4	46.2	25.6	23.1	21.2	14.9

7.2 Optimisation of piezoelectric location

The genetic algorithm described in section 6 was used to find the optimal locations and feedback gain for different numbers of sensor/actuator pairs on the same 0.5m square cantilever plate. Optimal placements for up to ten sensor/actuator pairs are shown in Figure (3). It may be seen that in all cases the optimum locations for *n* sensor/actuators were also members of the optimal set for *n*+1, and that the improvement in attenuation for an added sensor/actuator quickly falls off with added numbers.

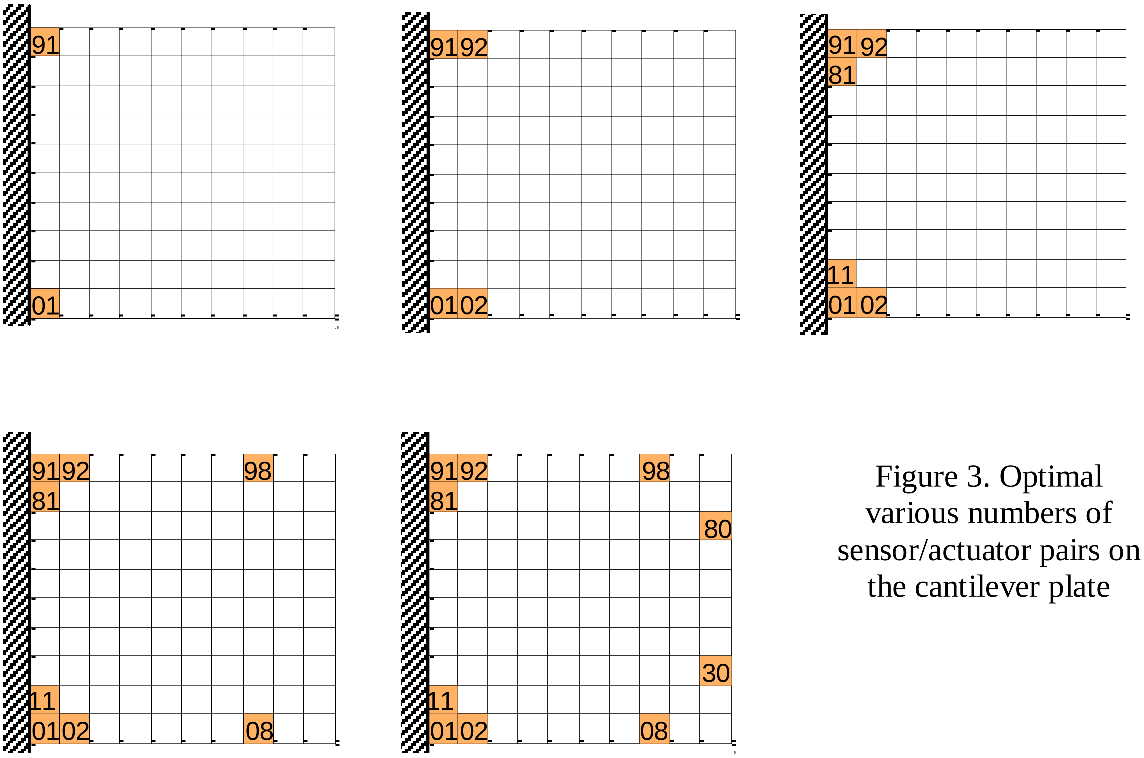


Figure 3. Optimal various numbers of sensor/actuator pairs on the cantilever plate

7.3 Optimisation of number of actuators

The second stage of the genetic algorithm is the determination of the optimal number of actuators based on measuring the variation in the average closed loop gain reduction for all optimal sensor/actuator pairs according to the fitness equation. A measure of the closed loop dB gain margin reduction over the whole plate is obtained by taking the average vibration reduction for all sensors and modes as described in Section 6.

Table (3) shows the percentage variation of closed dB-gain margin reduction for each addition of sensor/actuators according to the fitness function equation (22). It can be observed that three sensor/actuator pairs gives only small improvements and further increases beyond eight actually reduces the attenuation.

Table 3 Percentage variation of closed loop gain reduction ($100\% * \Delta dB/dB$) for the cantilever plate with different numbers of piezoelectric patches in optimal locations

Number of piezoelectric pairs	Linear quadratic weighted matrices R=1 with three Q settings			
	$Q = 10^6$	$Q = 10^7$	$Q = 10^8$	Fitness
(1/25), 1×1cm	100	100	100	100
(4/25), 2×2cm	88.11	75.77	57.61	73.83
(9/25), 3×3 cm	55.86	38.81	27.44	40.71
(16/25), 4×4cm	33.79	22.83	15.86	24.16
(25/25), 5×5cm	22.24	15.20	10.55	16.00
2	19.64	11.07	4.69	11.80
3	4.34	3.21	1.94	3.17
4	3.44	2.55	2.27	2.75
5	4.91	2.33	1.69	2.98
6	3.54	1.62	-0.23	1.64
7	1.16	0.87	0.69	0.90
8	0.87	0.41	0.27	0.52
9	4.70	1.31	0.13	2.04
10	1.95	1.14	1.15	1.41

The results are represented in Figure (4) for percentage variation for different values of weighted matrices respectively. It can be seen that Figure (4) is divided into three zones: an active progressive zone, an objective less progressive zone, and a passive zone. This shows that the optimal number of piezoelectric sensor/actuator pairs lies between one and three, depending on the marginal improvement required.

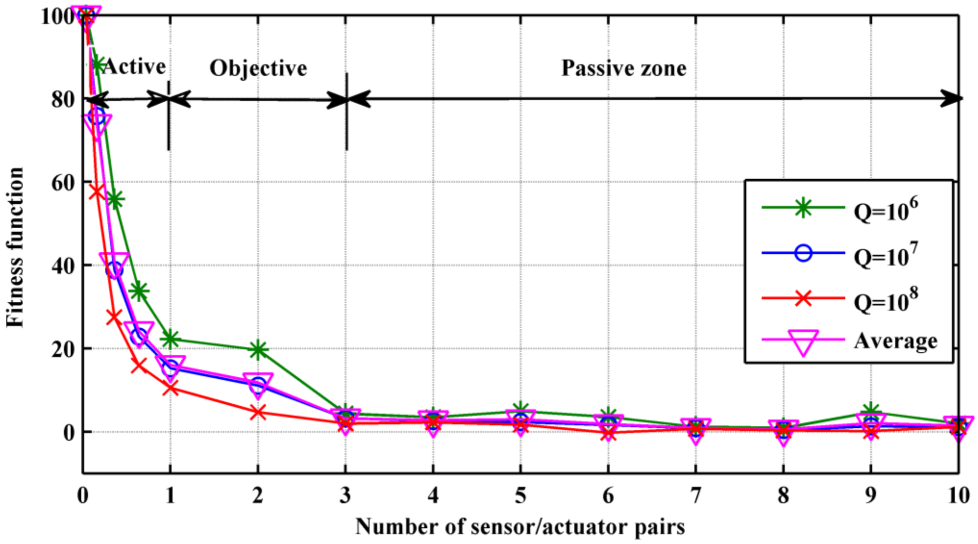


Figure 4. Variation of closed loop dB gain reduction for different values of weighted matrices against number of piezoelectric pairs

7.4 Validation of the optimal location and number of sensor/actuator pairs

The results in Section 7.3 are compared with previous published studies [11, 12]. These apply an alternative optimisation process to the same cantilever plate with sensor/actuator pairs of the same dimensions as in the current work and so are directly comparable. However, they used an arbitrary number of sensor/actuator pairs and optimised just the locations of ten pairs and feedback gain to suppress vibration. In this study, the result shows that the collective optimisation of number of sensor/actuator pairs in addition to their locations and feedback gain are given high reduction in material cost, structural weight and complexity of control system at the same level of vibration reduction as using more pairs. In this section the developed fitness and objective functions are tested and validated using the ANSYS finite element model embedded in an APDL program simulating a proportional differential (PD) control scheme. This was used to investigate the open and closed loop time responses against the number of sensor/actuator pairs in optimal locations.

To test the effectiveness of the vibration control the out of plane acceleration of one corner of the free end of the cantilever plate was taken as of the plate vibration because it has large displacement in all modes. The investigation involved exciting the plate in one of its resonant modes by driving a piezoelectric element close to a constrained corner (element 91) with a sinusoidal voltage of constant amplitude of 20V. The response at free end plate was determined with and without the active vibration control activated, using various numbers of optimally located sensor/actuator pairs. The results are shown in Figs (5) to (8) and summarised in tables (4) to (5). In each case the feedback gains are K_p = 24, K_d = 12.

Figure (5) shows the open and closed loop time responses for the first mode with the excitation is started at time t = 0. For the open loop (i.e. without active vibration control) the vibration level rises steadily and is still rising after 16s. In the closed loop with a single optimally located sensor/actuator pair there is a steady state vibration reduction of more than 89% (-19dB), and this steady state is reached in ten seconds. With two sensor/actuator pairs there is a significant further steady state vibration reduction to >95% (-26.8dB) which is reached within four seconds. Adding more sensor/actuator pairs has little additional effect on the steady state vibration amplitude, though it does progressively reduce the time to reach the steady state to less than one second for five pairs.

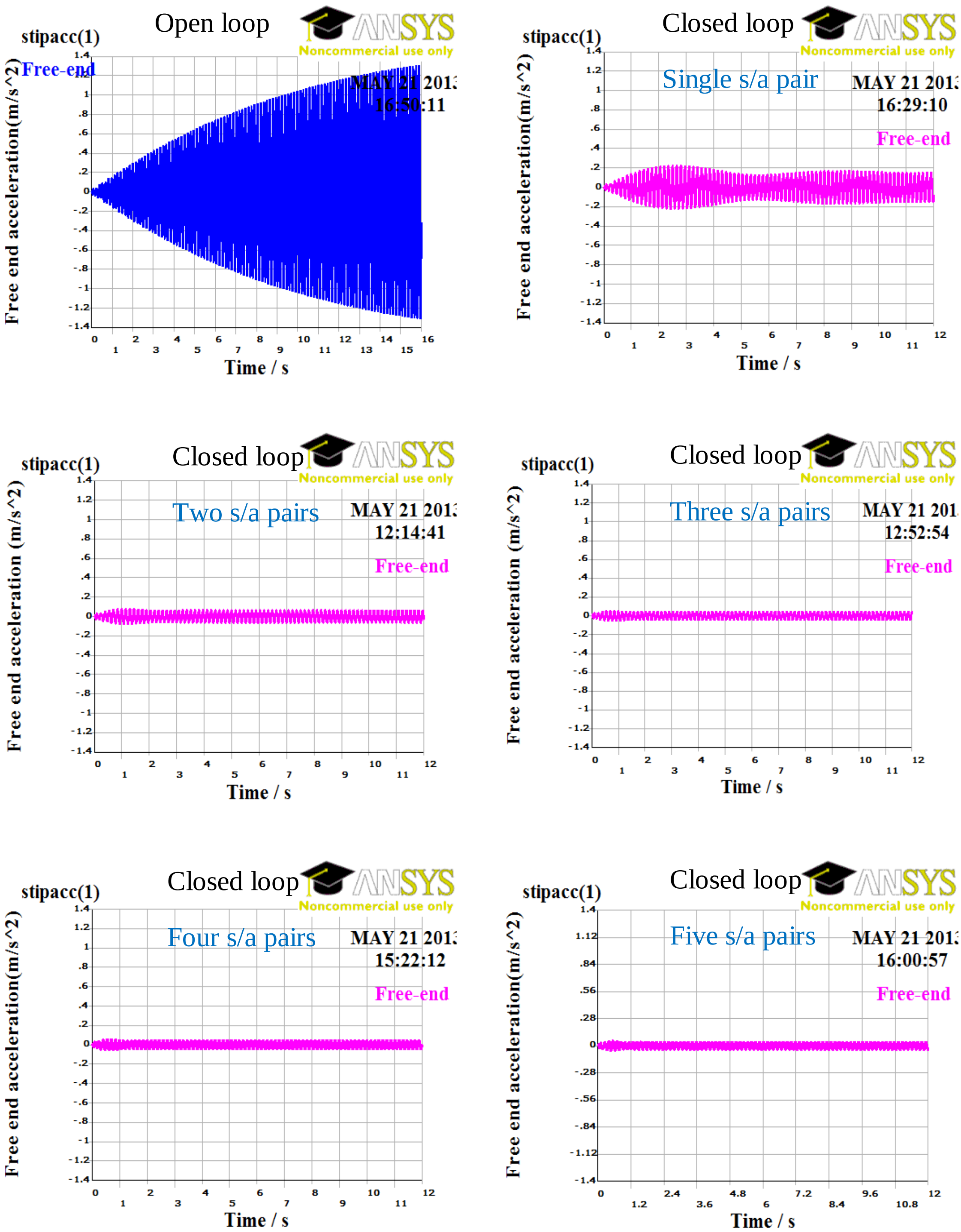


Figure 5. Open and closed loop displacement response with various number of sensor/actuator pairs. Plate driven at first mode resonant frequency.

Figure (6) shows the steady state closed loop actuator feedback voltage for increasing numbers of sensor/actuator pairs at the first mode. It can be seen that the increase in the number of sensor/actuator pairs gives a small reduction in the feedback voltage to actuator 01, but increases in the summation of the overall actuator feedback voltage and hence power in each case.

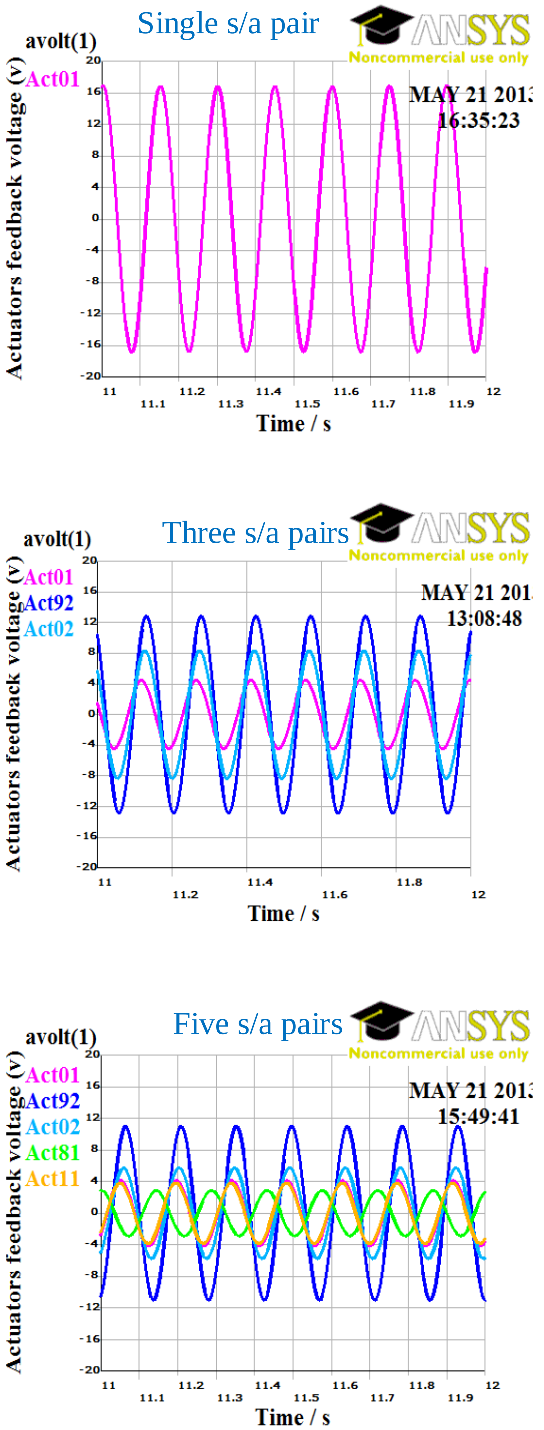


Figure 6. Closed loop time responses of the actuators feedback voltage at the first mode for the cantilever plate bonded to various number of sensor/actuator pairs in the optimal locations,

$K_p = 24,\ K_d = 12$

Figure (7) shows the same open and closed loop acceleration time responses at the same point on the cantilever plate when driven in the second mode. It can be seen that there is again a large attenuation using one sensor/actuator pair, but little further improvement using two or more pairs.

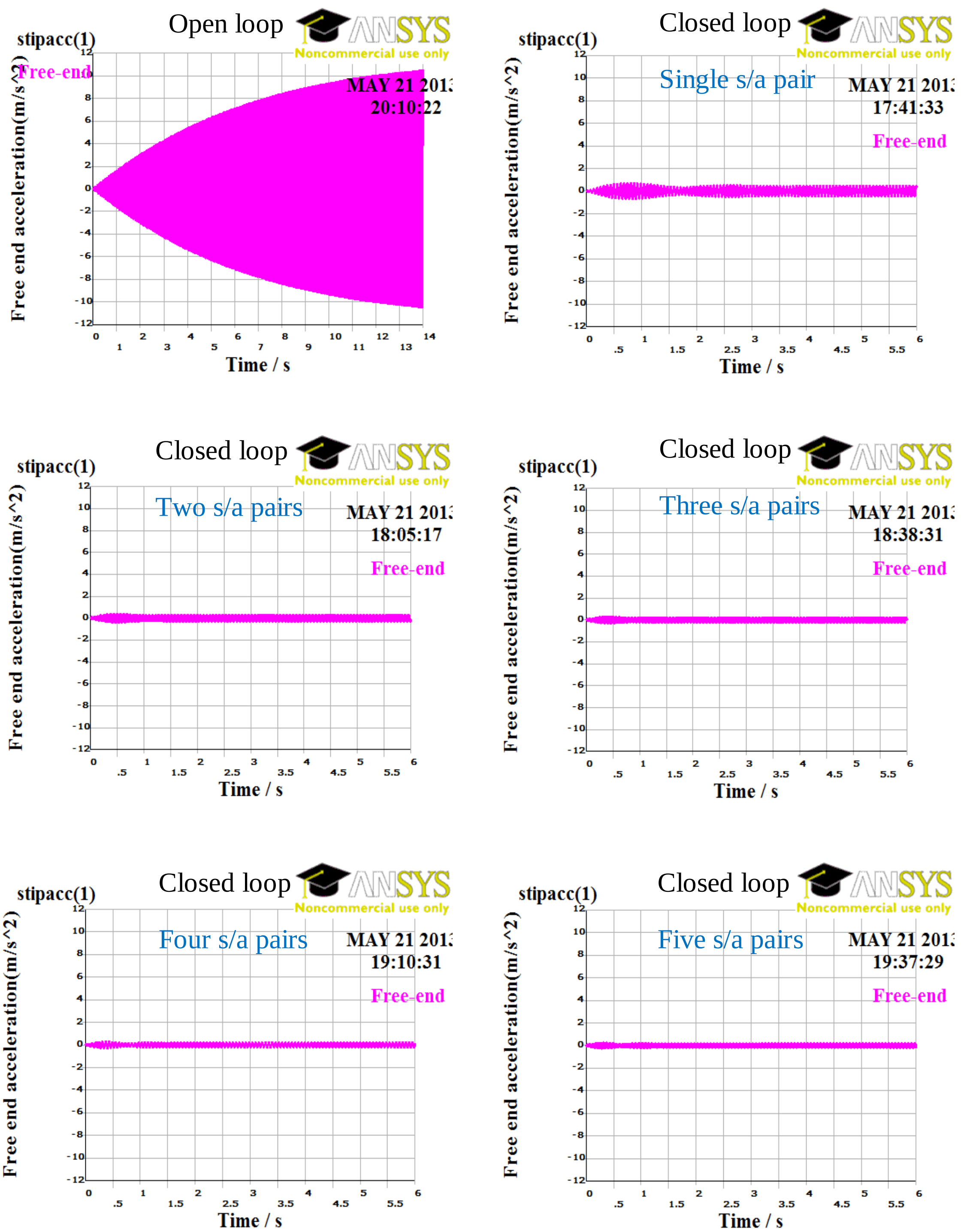
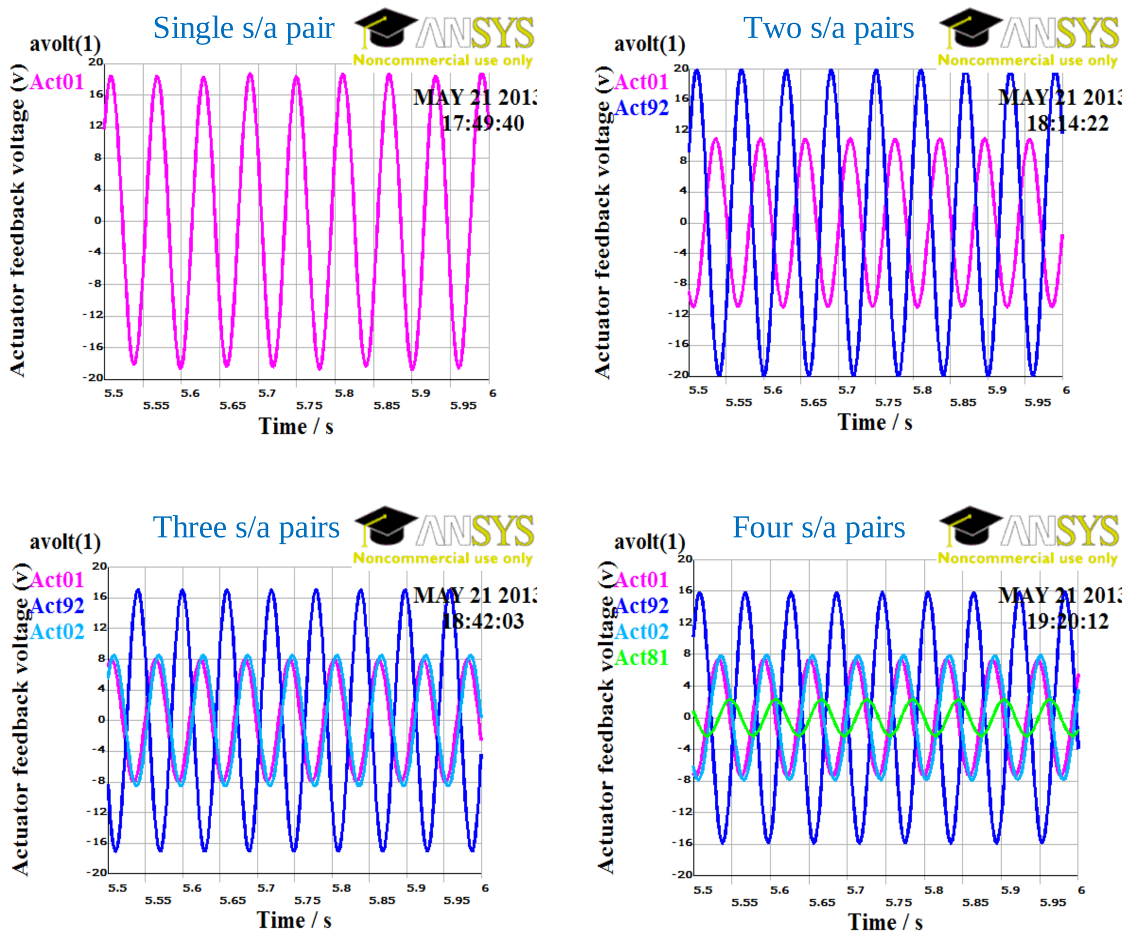


Figure 7. Open and closed loop time responses for the free end plate displacement at the second mode for the cantilever plate bonded to various number of sensor/actuator pairs in the optimal locations, $K_p = 24$, $K_d = 12$



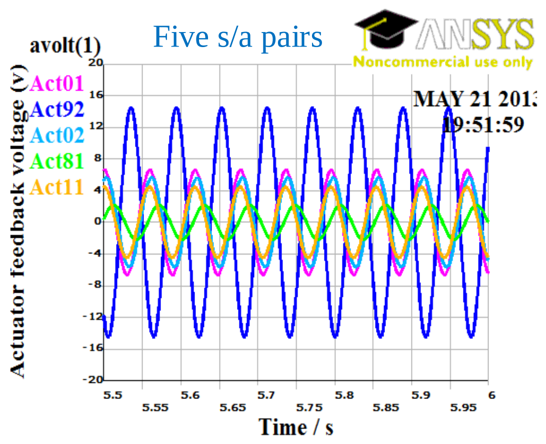


Figure 8. Closed loop time responses of the actuators feedback voltage at the second mode for the cantilever plate bonded to various number of sensor/actuator pairs in the optimal locations,

$$K_p = 24,\ K_d = 12$$

Figure (8) shows the steady state closed loop actuator feedback voltage time response at the second mode against the increase in the number of sensor/actuator pairs. It can be seen that the increasing in the number of sensor/actuator pairs gives little reduction in the feedback voltage to actuator 01, and required considerable voltage, and hence power, for the added actuators.

These ANSYS results shown in the Figs (5) to (8) are summarised in tables (4) and (5) for the first and second mode respectively. It can be seen from table (4) that there is a very large vibration reduction of 89.28% at the first mode using a single sensor/actuator pair, while a further improvement to 95.71% occurs using two pairs. However, negligible further reduction is achieved with more than two pairs. On the other hand, the overall actuator feedback voltage is increased by 64.7% at the first mode using five actuators compared with that using a single optimally placed sensor/actuator pair.

Table 4 Percentage vibration reduction against number of sensor/actuator pairs in the optimal locations for the first mode using ANSYS finite element package

Case	Number of sensor/actuator pairs in optimal locations				
	1 pair	2 pairs	3 pairs	4 pairs	5 pairs
Closed loop maximum Amplitude ($m.s^{-2}$)	0.15	0.06	0.043	0.045	0.038
Percentage reduction%	89.28	95.71	96.92	96.78	97.28
Total voltage consumption (V)	17	24.5	24.5	27	28
Percentage increased in feedback voltage %	-	44.1	44.1	58.8	64.7

Table (5) shows similar effects when controlling the second mode of vibration.

It can be concluded from this section that increasing in the number of sensor/actuator pairs gives little advantage provided they are correctly located. On the contrary, they add weight to a light weight structure, and increase the cost of the control system and its complexity. In addition, they are likely to have a significantly greater power consumption.

Table 5 Percentage vibration reduction against number of sensor/actuator pairs in the optimal locations for the second mode using ANSYS finite element package

Case	Number of sensor/actuator pairs in optimal locations				
	1 pair	2 pairs	3 pairs	4 pairs	5 pairs
Closed loop maximum amplitude($m.s^{-2}$)	0.48	0.28	0.228	0.208	0.17
Percentage reduction%	95.63	97.45	97.96	98.01	98.45
Total voltage consumption (V)	18	31	33	34.1	33.2
Percentage increased in feedback voltage %	-	72.2	83.3	89.4	84.4

8. Conclusion

In this study, new fitness and objective functions are proposed to optimise structural weight, costs and control system complexity in active vibration control of a smart structure by using a minimum number of sensors and actuators optimally placed by means of a genetic algorithm. The sensor/actuator placement and feedback gains are optimised by minimisation of the linear quadratic index as an objective function in the first stage of the genetic algorithm. The fitness and objective functions are developed to determine the optimal number of piezoelectric actuators based on variations in closed loop dB gain margin reduction with respect to the number of optimally located piezoelectric pairs taking the average effects of all optimal piezoelectric pairs and all modes required to be attenuated. It is shown that a few sensor/actuator pairs gives effective vibration reduction over the first six modes, where just two pairs in optimal locations give almost the same level of attenuation as up to five pairs. This leads to a potential saving in weight and cost of smart structures with active vibration control.

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