# On a stochastic model for the spin-down of solar-type stars

Leprovost, N. & Kim, E. J.

# Published PDF deposited in Coventry University's Repository

# **Original citation:**

Leprovost, N & Kim, EJ 2010, 'On a stochastic model for the spin-down of solar-type stars' Astrophysical Journal, vol. 719, no. 1, pp. 287-298. https://dx.doi.org/10.1088/0004-637X/719/1/287

DOI 10.1088/0004-637X/719/1/287 ISSN 0004-637X ESSN 1538-4357

Publisher: American Astronomical Society

Copyright © and Moral Rights are retained by the author(s) and/ or other copyright owners. A copy can be downloaded for personal non-commercial research or study, without prior permission or charge. This item cannot be reproduced or quoted extensively from without first obtaining permission in writing from the copyright holder(s). The content must not be changed in any way or sold commercially in any format or medium without the formal permission of the copyright holders.

# ON A STOCHASTIC MODEL FOR THE SPIN-DOWN OF SOLAR-TYPE STARS

NICOLAS LEPROVOST AND EUN-JIN KIM

Department of Applied Mathematics, University of Sheffield, Sheffield S3 7RH, UK Received 2009 July 7; accepted 2010 June 16; published 2010 July 19

#### ABSTRACT

Modeling the rotation history of solar-type stars is an outstanding problem in modern astrophysics. One of the main challenges is to explain the dispersion in the distribution of stellar rotation rate for young stars. Previous works have advocated diverse mechanisms to explain the presence of fast rotators (FRs) and also of slow rotators (SRs). For instance, dynamo saturation can limit the stellar spin-down and explain the presence of FRs but does not produce enough SRs. Here, we present a new model that can account for the presence of both types of rotators by incorporating fluctuations in the angular momentum loss. This renders the spin-down problem probabilistic in nature, some stars experiencing more braking on average than others. We show that random fluctuations in the loss of angular momentum enhance the population of both FR and SR compared to the deterministic cases (with a linear dynamo prescription or with dynamo saturation). The stochastic angular momentum loss thus provides an alternative physical mechanism to that of a saturated dynamo, with an even better agreement with observations.

Key words: stars: evolution - stars: interiors - stars: rotation

Online-only material: color figures

# 1. INTRODUCTION

To first approximation, the present Sun is rotating as a solid body, the average rotation of the core and the envelope being roughly the same (see Thompson et al. 2003, for instance). However, this has not always been the case, as suggested by stellar evolution theory (Tassoul 2000): during the pre-main sequence (PMS) phase of their evolution, stars experience a contraction accompanied by changes in their internal structure. As a consequence, they spin-up and develop a radiative core which rotates faster than the envelope. If the coupling between the core and the envelope is sufficiently strong, the core can in turn accelerate the envelope. Toward the end of the PMS, the loss of angular momentum through stellar wind gradually halts the spin-up of the convective envelope and leads to a fast spindown of the convective envelope on the main sequence (MS), as the wind braking timescale becomes the shortest scale (Kepens et al. 1995; Herbst & Mundt 2005). Note however that there are still small changes in the structure at the beginning of the MS.

Modeling the rotation history of solar-type stars is a formidable task due to the number of unknown physical processes to be understood (Schrijver et al. 2003). The two main ingredients to be modeled are the following.

- 1. The loss of angular momentum through stellar wind. Idealistically, this requires a three-dimensional model of the solar wind.
- 2. The transfer of angular momentum between the convective zone of the star and its core. This could be achieved through transfer of mass or viscous-like transport (such as transport by waves or turbulence).

Once these two processes are properly modeled and appropriate initial conditions for the rotation rates of the core and envelope are chosen, it is possible to trace a star rotation history in order to explain observations. The main observations to account for are those of equatorial rotational velocity (v) distributions (or rotational rate) of solar-type star clusters of different ages, for instance  $\alpha$ Persei, the Pleiades, or the Hyades. Using data from Soderblom et al. (1993), we construct Figure 1

which shows the percentages of the population of stars having  $v \sin i$  in a given velocity band for the three clusters. The first feature to be noticed is that  $\alpha$ Persei (age  $\sim 50$  Myr) exhibits a bimodal distribution. Twenty percent of stars are very rapid rotators ( $v \sin i > 140 \text{ km s}^{-1}$ ) and the remaining 80% of stars are rotating with speed  $v \sin i < 100 \text{ km s}^{-1}$ ; furthermore, a large proportion of them are very slow rotators (SRs) ( $v \sin i < v$ 20 km s<sup>-1</sup>). At later time, in the Pleiades cluster (age  $\sim$ 70 Myr). the distribution is fairly less bimodal with only 10% of the stars in the tail of fast rotators (FRs;  $v \sin i > 70 \text{ km s}^{-1}$ ). Note also that there is a very substantial spin-down as the fastest rotators have speed  $v \sin i \sim 80-90 \text{ km} \text{ s}^{-1}$  and there is a large proportion of SRs: 80% of stars are rotating with speed less than 20 km s<sup>-1</sup> (and 50% with  $v \sin i < 10$  km s<sup>-1</sup>). At the Hyades ages (~600 Myr), most stars are SRs ( $v \sin i < 10 \text{ km s}^{-1}$ ). Spin-down is also observed for times later than 600 Myr, the rate of slowing-down being given by the Skumanich relation (Skumanich 1972):  $v \sin i \propto t^{-1/2}$ , where t is the age of the star.

Different mechanisms have been proposed to explain the spindown of solar-type stars. As the loss of angular momentum is achieved through the magnetized stellar wind, an important ingredient is the prescription of the dependence of the magnetic field of a star on its rotation rate (the so-called dynamo relation). When this relation is linear (as suggested by an  $\alpha$ - $\Omega$  dynamo model), Kepens et al. (1995) and Barnes & Sofia (1996) have shown that the decrease in rotation rate is too rapid during the early evolution and cannot thus explain the large tail of rapid rotators at the age of  $\alpha$  Persei. Consequently, a number of models have invoked the saturation of the loss of angular momentum for rapidly rotating stars as a way to reduce the spin-down experienced by the fast rotators (FRs). In particular, Kepens et al. (1995) and Barnes & Sofia (1996) have assumed that the dynamo mechanism saturates for a rotation rate above a prescribed threshold. As a consequence (see for instance Equation (4)), the loss of angular momentum is limited for rapidly rotating stars. Using this model, Kepens et al. (1995) obtained rotation distributions that agree reasonably well with observations for the value of threshold of  $\Omega_t \sim 20 \,\Omega_{\odot}$ . The saturation of the dynamo mechanism for  $\Omega > 20 \,\Omega_{\odot}$  is supported by the



Figure 1. Percentages of the population of stars having a projected velocity  $v \sin i$  inferred from the data in Soderblom et al. (1993).

observation of chromospheric activity which is linked to the magnetic activity (Stauffer et al. 1994; Pizzolato et al. 2003). However, observations of star-spot coverage (O'Dell et al. 1995) seem to indicate a saturation for a somewhat higher value of the rotation rate (typically  $\Omega_t \sim 60-100 \,\Omega_{\odot}$ ). It thus led Solanki et al. (1997) to suggest that the saturation of the angular momentum loss is due to a polar localization of the magnetic activity rather than the saturation of a dynamo process. Indeed, the localization of open magnetic field lines at higher latitude, which is observed for rapidly rotating stars, was shown to reduce the transport of angular momentum and thus limit the spin-down of fast-rotating stars (Buzasi 1997). Note however that there is a very large uncertainty both in parameter values and modeling. In particular, an accurate dynamo prescription is limited by the uncertainty in the details of magnetic dynamo processes, magnetic flux transport to the stellar surface, the geometry (for instance, polar localization), and strength of surface magnetic fields (Jardine & Donati 2008).

The purpose of this paper is to investigate the effects of intermittent angular momentum loss on the spin-down process and propose a new stochastic model that can explain the presence of both FR and SR in the early MS evolution. Specifically, we adopt the one-dimensional dynamical model of Kepens et al. (1995) for the coupled spin-down of the stellar envelope and core (see Equation (1)) where the angular momentum loss is prescribed by a spin-down time  $\tau_w$  which depends on the state of the star (essentially, its rotation velocity and magnetic field). The main novelty of our approach is to allow  $\tau_w$  to take a range of values for a given rotation. Our stochastic model is motivated by a growing number of recent observations indicating that solar/stellar magnetic fields exhibit much more intermittent behavior than has previously been thought (Jardine 2005). For instance, in addition to the 22 year activity cycle, the solar magnetic field shows variations on a broad range of temporal and spatial scales (Goldstein et al. 1995; Javaraiah 2003). Periodicity on temporal scales as long as 2400 years have been reported by Hood & Jirikowic (1990). Furthermore, solar activity exhibits intermittency all the way back to the

17th century (Nandy 2004). Therefore, it seems reasonable to consider that solar/stellar magnetic fields vary on various timescales up to evolution timescales. We thus explore the effect of different correlation times in the angular momentum loss on the probability distribution of stellar rotations.

The remainder of the paper is organized as follows. In Section 2, we present our model for the stochastic angular momentum loss and we use this model to compute the evolution of a single star and an initial distribution in Sections 3 and 4, respectively. In Section 5, we compared the results of different model to available observations and we discuss the results in Section 6.

# 2. MODEL

To study stellar spin-down, we assume that both the core and the envelope rotate rigidly and that the coupling between the two is achieved through viscous-like transport mechanism (for details, see Kepens et al. 1995, and references therein). The evolution of the angular momentum of the radiative core  $J_c = I_c \Omega_c$  and envelopes  $J_e = I_e \Omega_e$  can be written as

$$\frac{dJ_c}{dt} = -\dot{J}_v + \dot{J}_m,$$
(1)
$$\frac{dJ_e}{dt} = \dot{J}_v - \dot{J}_m - \dot{J}_w.$$

Here  $J_c$ ,  $J_e$ , and  $\Omega_c$ ,  $\Omega_e$  are the angular momentum and angular rotation of the radiative core and envelope, respectively. In the following, all of our calculations are performed by using parameter values typical of the Sun with moments of inertia  $I_c = 59.87 \times 10^{52} \text{ g cm}^2$  and  $I_e = 3.398 \times 10^{52} \text{ g cm}^2$ .  $J_m$  is the transfer of angular momentum associated with the mass exchange between the core and the envelope which is important primarily during the PMS phase. In the following, we focus on the evolution on the zero-age main sequence (ZAMS) and consequently assume that  $J_m$  is negligible compared with the exchange of angular momentum by the visco-magnetic coupling mechanism between the core and the envelope  $J_v$ . The latter is given by

$$\dot{J}_{v} = \frac{J_{c}I_{e} - J_{e}I_{c}}{\tau_{c}(I_{c} + I_{e})} = \frac{I_{c}I_{e}}{\tau_{c}(I_{c} + I_{e})}(\Omega_{c} - \Omega_{e}),$$
(2)

where  $\tau_c$  is the coupling time between the radiative core and convection zone. In the following, we assume, as is the case for most previous studies, that the coupling timescale is constant during the evolution of a star with  $\tau_c \sim 20$  Myr (Kepens et al. 1995). Note that this is unlikely to be the case as a strong differential rotation between the core and envelope could trigger an instability and increase the transport. The effect of fluctuations in  $\tau_c$  is left for future contributions.

The braking due to the solar wind is assumed to be of the viscous type:

$$\dot{J}_w = \frac{J_e}{\tau_w},\tag{3}$$

where  $\tau_w$  is the braking time. To compute the braking time, we use the Weber–Davis model (Weber & Davis 1967) which exhibits a transition between slow magnetic rotator (SMR) and a fast magnetic rotator (FMR) regimes (Belcher & MacGregor 1976). In this paper, we do not solve the Weber–Davis model for any value of the rotation and magnetic field but use asymptotic expression for the braking term given by MacGregor & Brenner (1991):  $\tau_w^{-1} \propto \Omega_e^{-2/3} B^{4/3}$  and  $\tau_w^{-1} \propto B^2$  in the FMR and SMR, respectively. We therefore write the braking time as

$$\frac{1}{\tau_w} = \frac{1}{\tau_{w\odot}} \times \begin{cases} (\Omega_* B_* / \Omega_e)^{2/3} B^{4/3} & \text{if } \Omega_e > \Omega_* \\ B^2 & \text{otherwise} \end{cases}.$$
 (4)

Here,  $\tau_{w\odot}$  is the spin-down time of the present Sun and  $\Omega_*$ (corresponding to a magnetic field  $B_*$ ) is the threshold at which the transition between the SMR and the FMR regimes occurs. Equation (4) is chosen to ensure continuity of  $\tau_w$  at the transition point and to match the present spin-down time of the solar rotation. Note that in the FMR regime, the braking time associated with the loss of angular momentum directly depends on the rotation rate and magnetic field, whereas in the SMR regime, the braking time depends only on the magnitude of the magnetic field. In all the calculations presented in the paper, we use the following values for the parameters:  $\tau_{w\odot} = 300$  Myr and  $\Omega_* = 3.5 \,\Omega_{\odot} = 1.05 \times 10^{-5} \,\mathrm{s}^{-1}$ . The value for the spin-down time of the present Sun is taken from Figure 4 of MacGregor & Brenner (1991) where the braking time can be seen to be 300 Myr at age of the Sun and the threshold value  $\Omega_*$  is found from Figure 10 of Kepens et al. (1995). Note that the value for the braking time  $\tau_{w\odot} = 300$  Myr may be an underestimate since a spin-down time up to 100 times larger has been suggested from observations (Li 1999). Different values for the parameter  $\tau_{w\odot}$ are considered in Appendix C. Note also that alternate formulae to Equation (4) have been used (e.g., Kawaler 1988).

As can be seen from Equation (4), the spin-down timescale  $\tau_w$  depends on the rotation rate of the star and the magnetic field. To express  $\tau_w$  solely in terms of rotation rate, one requires the dependence of the magnetic field on the rotation rate. This is the so-called dynamo prescription. To obtain this, most previous models assumed that the magnetic field depends only on the angular rotation of the convection zone, with the magnetic field varying linearly with the rotation rate as  $B = B_{\odot}\Omega_e/\Omega_{\odot}$ . Here,  $B_{\odot}$  is the present magnetic field of the Sun. This linear relation was however shown to lead to too rapid spin-down in early MS



Figure 2. Time history of the angular rotation of the core (in thin lines) and the envelope (in thick lines) of a solar-type star for different values of the saturation threshold.  $\Omega_{\odot}$  is the present solar rotation rate.

(A color version of this figure is available in the online journal.)

when the rotation rate of the stars is high (Kepens et al. 1995; Barnes & Sofia 1996) and cannot explain the heavy tail of FRs at the age of  $\alpha$  Persei as noted in the introduction. Consequently, other authors assumed that the dynamo and thus the angular momentum loss saturates at high rotation rate. For example, Kepens et al. (1995) and Barnes & Sofia (1996) assumed the following dependence of the magnetic field on the rotation rate:

$$B = B_{\odot} \times \begin{cases} \Omega_e / \Omega_{\odot} & \text{if } \Omega_e < Q_s \Omega_{\odot} \\ Q_s & \text{otherwise} \end{cases},$$
 (5)

with the value of the saturated magnetic field being  $Q_s B_{\odot}$ . To illustrate the effect of this dynamo saturation, we computed the evolution of a solar-type star with the dynamo prescription (Equation (5)) for different values of the saturation threshold  $Q_s$  starting from ZAMS by using initial condition:  $\Omega_e = 20 \Omega_{\odot}$  and  $\Omega_c = 35 \Omega_{\odot}$ . The rotation rates of the core and the envelope are shown in Figure 2.

It can be seen that as the saturation threshold  $Q_s$  is lowered, the spin-down of the envelope is delayed because the loss of angular momentum is reduced. For the lowest threshold considered (Qs = 5), we even observe a spin-up at the early stage of the evolution, as the loss of angular momentum is smaller than the acceleration provided by the fast-rotating core. It is important to note that the relation between stellar rotation and magnetic field is very uncertain due to the lack of understanding of the properties of stellar dynamos. For instance, the stellar rotation does not only affect the magnetic field generation, but also modifies the transport of magnetic flux to the surface (Schrijver et al. 2003).

#### 3. EFFECT OF STOCHASTIC FLUCTUATIONS

To model the effect of uncertainties in the modeling of stellar dynamo and temporal fluctuations in the properties of the wind,



**Figure 3.** Mean rotation rate (left panel) and standard deviation (right panel) of the core (in thin lines) and the envelope (in thick lines) of a solar-type star for different values of the parameter  $\sigma$ . The initial conditions are  $\Omega_e = 20 \,\Omega_{\odot}$  and  $\Omega_c = 35 \,\Omega_{\odot}$ . (A color version of this figure is available in the online journal.)

we consider a time dependent loss of angular momentum by replacing Equation (3) by the following:

$$\dot{J}_w = \frac{J_e}{\tau_w} \xi(t),\tag{6}$$

where  $\tau_w$  is given by the Weber–Davis model of Equation (4).  $\xi(t)$  in Equation (6) is a stochastic noise chosen to be multiplicative as a toy model for the intermittent loss of angular momentum. The statistics of this noise is chosen to be a  $\Gamma$  distribution so that  $\xi$  is defined only for positive values to ensure that the loss of angular momentum given by Equation (6) is always positive (i.e., momentum is extracted from the star by the solar wind). Specifically, the distribution of the noise is taken to be

$$P(\xi) = \frac{1}{N} \xi^{1/2} \exp\left[-\frac{3\xi}{2}\right].$$
 (7)

Here, N is a normalization factor. The distribution (Equation (7)) has been chosen such that the average loss of angular momentum is the same as the deterministic case since  $\langle \xi \rangle = 1$ . Here, the angular brackets denote the average over the statistics of the noise (see Appendix A). We have an additional parameter  $\sigma$ which characterizes the time correlation of the fluctuations: the larger the parameter  $\sigma$ , the shorter the time correlation (see Appendix A). Once the distribution of Equation (7) has been fixed, all the statistical properties of the noise are fixed. For instance, the standard deviation is  $\sqrt{\langle \xi^2 \rangle - \langle \xi \rangle^2} = \sqrt{2/3} \sim$ 0.81. This means that the intensity of the fluctuations are around 80% of the mean. This value seems a little high compared to that of the fluctuations reported in the literature (see figures in Bruno & Carbone 2005, for instance). Note that in order to study other values of the intensity of fluctuations, one has to consider other form of the noise. Indeed, this cannot be done in this paper, as distribution (Equation (7)) can only fix one between the mean and the standard deviation. This issue will be addressed in a future work.

To illustrate the effect of stochastic fluctuations, we start from a star with the initial condition  $\Omega_e = 20 \,\Omega_{\odot}$  and  $\Omega_c = 35 \,\Omega_{\odot}$ and evolve Equation (1) with a linear dynamo prescription (i.e., no saturation or  $Q_s = \infty$  in Equation (5)). As the model is probabilistic, it is not possible to predict with certainty the stellar rotation rate at any later time. We first examine the evolution of average rotation rate and standard deviation, which are shown in Figure 3. It is seen that as the parameter  $\sigma$  is decreased, the spin-down of both the core and the envelope is reduced on average. Since the mean rotation rate of the stars gives us only one measure of the rotational evolution on average, we compute the probability distribution function (PDF) of having a certain value of the angular rotation velocity of the envelope. Figure 4 shows this PDF at three different times t = 50, 100, and 600 Myr. The main notable feature is that there is a large dispersion in the distribution around the mean value. Another important point is that the tail of FRs is more pronounced than the tail of SRs. This is due to the fact that the noise is multiplicative. That is, in Equation (6), the noise  $\xi$  multiplies  $J_e$ , which is a function of  $\Omega_e$ . This makes the effect of the noise effectively stronger when  $\Omega_e$  (or equivalently  $J_e$ ) is larger.

The value of the parameter  $\sigma$  is related to the correlation time of the loss of angular momentum. To see this from the time evolution of a single star, we compute the average loss of angular momentum given by  $\langle J_w \rangle = \langle J_e / \tau_w \xi(t) \rangle$  which is a function of time. From this function, we can obtain the autocorrelation function of the angular momentum which is shown in Figure 5. From this autocorrelation function, we can obtain a time correlation for the loss of angular momentum which is 6.7, 1.6, 0.7, and 0.3 Myr for  $\sigma^2 = 0.1, 0.5, 1,$  and 5, respectively. Note that as shown in Appendix A, the variation of the parameter  $\sigma$  corresponds to a variation of the correlation time of the noise (see Figure 10).

#### 4. ROTATION HISTORY OF SOLAR-TYPE STARS

As previously noted, our model does not allow us to study stellar evolution starting with the PMS as structural changes are neglected. In order to study the evolution of a distribution of solar-type stars from ZAMS, we use as an initial condition the results from the model of Kepens et al. (1995). We consider both cases of dynamo saturation with no fluctuation and fluctuations with no saturation  $Q_s = \infty$  (i.e., linear dynamo). We also run one case without saturation or fluctuations (which is labeled the linear case). As our model cannot take into account structural changes  $J_m$  which are important only in PMS, we simulate the evolution on the MS for a given distribution of initial rotation rates (for the core and the envelope) at ZAMS by using the result of Kepens et al. (1995) to fix our initial distribution at ZAMS = 30 Myr as follows. The distribution of envelope velocity is chosen to fit their results (see Appendix B for details) while the ratio of core angular rotation to envelope rotation  $\Omega_c/\Omega_e$  is fixed to be constant with the value 1.75. After choosing the distribution (see Appendix B for details), we take 5000 initial conditions according to this distribution and evolve them in time



Figure 4. Probability distribution of the angular rotation of the envelope at different times. The initial condition is  $\Omega_e = 20 \,\Omega_{\odot}$  and  $\Omega_c = 35 \,\Omega_{\odot}$ . (A color version of this figure is available in the online journal.)

by solving Equation (1) both in the deterministic ( $\xi = 1$ ) and in the stochastic cases ( $\xi = \xi(t)$ ). Figure 6 shows the evolution of the distribution of rotation rate in the linear case without saturation or noise, the saturated case (for two different values of saturation threshold  $Q_s = 10$  and  $Q_s = 5$ ), and the stochastic case ( $\sigma^2 = 1$  and  $\sigma^2 = 0.1$ ). The top row shows that in the linear case without saturation or noise, the entire distribution simply shifts to the left, for smaller rotation rate, without dramatic modification in the shape of the distribution. In contrast, the next two rows show that, in the saturated case, the distribution develops a large tail of FRs for rapid rotators. It is because stars rotating faster than the rotation threshold experience a weaker spin-down (compared with the linear case) due to the saturation of magnetic field. The last two rows show the influence of the noise on the rotation distribution. In this case, the distribution develops a tail of FRs due to fluctuations in the spin-down. Recall that the tail of FRs consists of stars that have experienced less braking in their spin-down. Therefore, compared with the linear case, both saturated and stochastic cases enhance the population of higher rotation, leading to a heavier right tail. The main difference between the two is that the distribution in the stochastic case is a little shifted to the left compared with that in the saturated case, therefore accounting for a larger proportion of SRs. This is because a substantial fraction of stars experiences a stronger braking in the stochastic case compared to the linear case. In contrast, the dynamo saturation solely affects the tail of rapidly rotating stars and SRs are not affected.



**Figure 5.** Temporal autocorrelation of the loss of angular momentum for different values of the parameter  $\sigma$ .

In the following, we compare our results with the observation from  $\alpha$ Persei (age ~50 Myr), the Pleiades cluster (~70 Myr), and the Hyades Cluster (~600 Myr).

# 5. COMPARISON WITH OBSERVATIONS

To compare our simulations in Figure 6 with the observational data in Figure 1, we construct a histogram of the number of stars having an equatorial rotation  $V_{eq}$  between 0 and 200 km s<sup>-1</sup>. The results are shown both for the saturated and stochastic cases in Figure 7.



**Figure 6.** Evolution of the probability density with time for different values of the noise intensity and the saturation threshold. The top panel is for the case without saturation or noise, the second and third for two values of the saturation threshold and the last two for two values of the noise parameters. Recall that  $\sigma$  is inversely proportional to the correlation time of the noise. Note that the curves in the stochastic cases are smoother as each initial conditions is evolves many times and thus the number of points used to construct the histogram is greater in the stochastic simulation than in the linear or saturated one.

(A color version of this figure is available in the online journal.)



Figure 7. Percentages of stars having a certain angular velocity for different values of the saturation rate  $Q_s$  and the noise parameter  $\sigma$ . The other parameters have been fixed to  $\tau_w = 300$  Myr and  $\tau_c = 20$  Myr.



Figure 8. Corrected figure using formula (8) for the percentage of stars having a given equatorial velocity.

One striking feature of the results from Figure 7 is that the saturated case does not show any SRs ( $V_{eq} < 10$ ) for short times (t = 50-70 Myr). On the contrary, the stochastic case leads to a small, but non-zero, fraction of SRs. Furthermore, one can see that both the saturated and stochastic cases have a heavier tail of FRs than the linear case. When comparing with the observations summarized in Figure 1, it is easy to see that the saturated case cannot explain at all the existence of SRs for short times. The agreement is a little better for the stochastic case as it maintains both SR and FR for short times (t = 50-70 Myr). However, the percentage of SRs in the stochastic case is much smaller than that in the observations. Furthermore, none of the models can exactly reproduce the observations which are bimodal at 50 Myr and 70 Myr. This bimodal character of the distribution cannot be reproduced either through saturation or noise in our model and can be taken into account only by assuming the initial distribution to be bimodal. Other authors have invoked different mechanisms to justify a bimodal distribution at ZAMS. For instance, disk-star coupling during the PMS is often invoked (see Kepens et al. 1995, for instance): some stars are coupled to the accretion disk and therefore are prevented from spinning-up and therefore constitute the left side of the bimodal distribution. This disk-star coupling cannot be investigated within the framework of the present paper which ignores the structural changes in the star. It is important to note though that the bimodal distribution observed in the data might just be an artifact due to line-ofsight effect, as discussed in the following.

#### 5.1. Correction of the Data for the Projection Angle

The main problem when comparing our results in Figure 7 with the observational data in Figure 1 is that they do not concern the same quantities. Specifically, the former gives the rotation rate of the envelope of the star  $V_{eq}$  whereas the latter gives the projected rotational velocity  $V_{eq} \sin i$ , where *i* is the angle at which a star is observed. As a consequence, the observational data overestimate the number of SRs. That is as  $\sin i \leq 1$ , it gives only a lower bound on the equatorial velocity of stars. To

compensate for this sin *i* factor, we assume that the distribution of the angle *i* is uniform between i = 0 and  $i = \pi/2$ . This is equivalent to assuming that when stars form, they do not have any preferred orientation (i.e., the direction of their rotation axis is random). This random orientation can be captured by a probability distribution of the angle given by  $P(i) = 2/\pi$  for  $0 \le i \le \pi/2$ . The probability of having an equatorial velocity between  $v_1$  and  $v_2$  is then determined by

$$P(v_{1} \leqslant V_{eq} \leqslant v_{2}) = \int_{v_{1} \leqslant x/\sin i \leqslant v_{2}} P(x = v \sin i) P(i) \, dx \, di$$
  
$$= \int_{0}^{v_{2}} dx P(x) \int_{\arcsin(x/v_{2})}^{\min(\arcsin(x/v_{1}), \pi/2)} di \, P(i)$$
  
$$= \frac{2}{\pi} \int_{0}^{v_{2}} dx P(x) \left[\min(\arcsin(x/v_{1}), \pi/2) - \arcsin(x/v_{2})\right].$$
(8)

Using Equation (8), we construct Figure 8 which shows the probability of  $V_{eq}$  under the assumption that *i* is random. One striking feature is that the bimodal character of the distribution is much less pronounced than in Figure 1.

Furthermore, the number of SRs is smaller than that in the original data of Figure 1, and thus agrees better with our prediction from the stochastic model in Figure 7.

#### 5.2. Effect of Saturation

Figure 7 shows that the saturated case  $Q_s = 20$  is very similar to the linear case without saturation or noise. This is simply because the initial distribution is chosen such that most of the stars have rotation rate smaller than the saturation value  $20 \Omega_{\odot}$ . However, as the saturation threshold is lowered, the number of rapidly rotating stars is increased whereas there is still no change in the number of SRs at 50 Myr and 70 Myr. Only at later time t = 600 Myr is the effect of the saturation on the SRs visible. For  $Q_s = 5$ , the number of SRs is significantly smaller than for the cases  $Q_s = 10$  and  $Q_s = 20$ . To get some qualitative results, we also computed the percentage of SRs,

Vol. 719

Table 1 Percentages of Slow Rotators (SR), Average Rotators (AR), Fast Rotators (FR), and Ultrafast Rotators (UFR) at Different Times from the Observations and the Simulation of the Saturated Dynamo

Simulation	Bin	$t = 50 \mathrm{Myr}$	$t = 70 \mathrm{Myr}$	$t = 600 \mathrm{Myr}$
Corrected observations	SR	35.3%	57.8%	57.5%
	AR	32.2%	26.1%	21.2%
	FR	15.4%	11.5%	19.3%
	UFR	17.1%	4.6%	2%
Linear case	SR	33.5%	39.0%	22.2%
	AR	66.5%	61.0%	74.6%
	FR	0.0%	0.0%	3.1%
	UFR	0.0%	0.0%	0.0%
$Q_s = 20$	SR	33.0%	38.3%	22.4%
	AR	66.7%	60.4%	74.4%
	FR	0.3%	1.2%	3.2%
	UFR	0.0%	0.0%	0.0%
$Q_s = 10$	SR	28.8%	38.0%	22.6%
	AR	69.0%	53.9%	70.4%
	FR	2.2%	8.1%	7.0%
	UFR	0.0%	0.0%	0.0%
$\overline{Q_s} = 5$	SR	1.9%	2.0%	7.7%
	AR	93.9%	75.1%	32.1%
	FR	4.1%	22.7%	60.1%
	UFR	0.1%	0.2%	0.0%

average rotators (ARs), FRs, and ultrafast rotators (UFRs) at different times. As the average rotation rate is decreasing with time, we choose the following definitions for the different cases.

- 1. t = 50 Myr: SR:  $V_{eq} < 20$ , AR:  $20 < V_{eq} < 100$ , FR:  $100 < V_{eq} < 200$ , and UFR:  $V_{eq} > 200$ . 2. t = 70 Myr: SR:  $V_{eq} < 20$ , AR:  $20 < V_{eq} < 60$ , FR:  $60 < V_{eq} < 200$ , and UFR:  $V_{eq} > 200$ . 3. t = 600 Myr: SR:  $V_{eq} < 10$ , AR:  $10 < V_{eq} < 20$ , FR:  $20 < V_{eq} < 200$ , and UFR= $V_{eq} > 200$ .

Note that different divisions into SR, AR, FR, and UFR are used at t = 50, 70, and 600 Myr. Table 1 shows the percentages for the corrected observations in Figure 8 and the results of the simulation for linear and saturated dynamos. As can be seen from this table, none of the cases can precisely reproduce the observations. This is because none of the scenarios can account for the existence of both FR and SR. For instance, the case  $Q_{\rm s} = 20$  has the correct percentage of SRs at t = 50 Myr but does not have any FRs. On the contrary, the case  $Q_s = 10$ has a satisfying percentage of FRs at t = 70 Myr but does not have enough SRs at later times (t = 600 Myr). This is because the saturated mechanism acts only on the tail of FRs of the distribution and cannot produce both FR and SR.

# 5.3. Effect of Noise

We now examine the effect of noise for different values of the parameter  $\sigma$ , which is related to the correlation time of loss of angular momentum (see Section 3). It can be seen from Figure 7 that the case with  $\sigma^2 = 5$  is very similar to the case without saturation or noise. When the parameter  $\sigma$  is decreased, corresponding to an increase in the correlation time of the loss of angular momentum (see Section 3), the proportion of FRs is increasing. Furthermore, the number of SRs is slightly decreasing as the effect of the noise becomes more important. Table 2 shows the percentage of SRs, ARs, FRs, and UFRs at different times for the stochastic case  $\sigma^2 = 0.1, 1, \text{ and } 5$  and

Table 2

Percentages of Slow Rotators (SR), Average Rotators (AR), Fast Rotators (FR), and Ultrafast Rotators (UFR) at Different Times for the Observations and the Simulation of the Case with Stochastic Loss of Angular Momentum

Simulation	Bin	$t = 50 \mathrm{Myr}$	$t = 70 \mathrm{Myr}$	$t = 600 \mathrm{Myr}$
Corrected observations	SR	35.3%	57.8%	57.5%
	AR	32.2%	26.1%	21.2%
	FR	15.4%	11.5%	19.3%
	UFR	17.1%	4.6%	2%
$\sigma^2 = 5$	SR	36.9%	40.7%	26.1%
	AR	63.1%	58.3%	67.4%
	FR	0.1%	0.9%	6.5%
	UFR	0.0%	0.0%	0.0%
$\sigma^2 = 1$	SR	32.1%	35.8%	21.8%
	AR	67.5%	60.8%	60.9%
	FR	0.4%	3.4%	17.4%
	UFR	0.0%	0.0%	0.0%
$\overline{\sigma^2} = 0.1$	SR	29.9%	32.5%	22.4%
	AR	69.4%	62.0%	49.7%
	FR	0.6%	5.5%	27.8%
	UFR	0.0%	0.0%	0.0%

the comparison with the corrected observations. As can be seen from this table, the agreement at small time (t = 50-70 Myr) is however rather poor as the number of FRs is always less than what inferred from observations. Furthermore, there is not much difference between the results from different values of  $\sigma$  for small times. However, when the effect of the noise is increased, the number of FRs is increased while the number of SRs remains almost constant, in qualitative agreement with the observations. In contrast, the behavior for long time (t = 600 Myr) depends very much on the parameter  $\sigma$  as the number of FRs is increasing significantly from 6.5% to 27.8% when the parameter  $\sigma$  is decreased (i.e., the correlation time of the angular momentum loss is increased). From the observations at t = 600 Myr, we see that the best fit for the number of FRs is obtained for  $\sigma^2 = 1$ , corresponding to a correlation time of the angular momentum loss of 0.3 Myr. In this case, the number of SRs at t = 600 Myr is small (22.4% instead of 57.5%).

To recapitulate, despite a crude modeling of the distribution of the sin *i* factor, our stochastic model improves the agreement with observations compared to the linear and saturated dynamo cases. Therefore, whether or not the  $\sin i$  factor is taken into account, the stochastic case gives results in better qualitative agreement with the observations than the saturated model as it accounts for a larger proportion of SRs. Further improvements in the agreement would require fine-tuning of parameters, e.g.,

1. Calibrate the initial distribution such that the percentages at t = 50 Myr are closer to the observation. Recall that the main aim of this paper was to study the effect of stochastic momentum loss and compare with the effect of dynamo saturation studied in Kepens et al. (1995). We thus started from an initial distribution that matches theirs. However, when comparing the results with the corrected distribution of Figure 8, none of the cases starting from this initial distribution (neither based on saturation nor noise) can reproduce the small time behavior of the distribution. Therefore, one could instead start the simulation from t = 50 Myr with a synthetic distribution that matches the results of Table 2. This is done in Appendix D, with the main conclusion that the stochastic model agrees best with observations (see Appendix D for details).

No. 1, 2010

2. As noise tends to produce results in agreement with observations by widening the distribution of the stars' rotation rate, we can increase the efficiency of the noise by considering a distribution with fatter tails (corresponding to a larger number of stars experiencing weak or strong spin-down). This cannot be done in the present model as the shape of the distribution is chosen to be fixed while the correlation time of the loss of angular momentum loss is controlled by one free parameter.

These improvements are left for future contributions.

#### 6. CONCLUSION

In order to explain the presence of both FR and SR at the early stage of the rotational evolution of solar-type stars, we presented a new model of spin-down. After including the crucial effect of random fluctuations in the loss of angular momentum, we performed numerical simulations of the resulting stochastic system and showed that the distribution of star rotation rate has a wide dispersion severely skewed toward high rotation rates. Physically, it is because the effect of fluctuations is more pronounced for rapidly rotating stars as a stronger intermittency is expected for faster rotators. Mathematically, this follows from the property of multiplicative noise used here since the rotation rate multiplies the stochastic fluctuations, enhancing the effect of the noise for large rotation values and thus increasing the tail of FRs of the distribution.

We then compared our results from the stochastic model and from the deterministic model with a saturation in the dynamo prescription to observational data of Soderblom et al. (1993). While the best agreement was obtained for our stochastic model, none of these models can reproduce the bimodal character of the observational distribution. To account for the unknown angle at which stars are seen, we then made the crude assumption that this angle is randomly distributed and corrected the data to account for that factor. Interestingly, this gave corrected data with no bimodal character. By comparing these corrected observations to the simulation results of the models, we found that our stochastic model gave the best agreement with the corrected observations. While further improvements in the agreement with observations requires fine-tuning of the parameters which are not well understood (especially the initial distribution of star velocity at ZAMS), these results are encouraging, suggesting that the effect of stochastic angular momentum on the spindown process is as important as, or even more important than, saturation. These results thus imply that stochasticity can be crucial in determining the evolution of not only stellar rotation but also magnetic fields. Therefore, more attention should be paid to the effect of stochasticity and statistical analysis in terms of PDF in solar/stellar modeling.

Finally, there is considerable scope for the extension of our study. In particular, it is important to incorporate other physical processes and conditions contributing to stellar spindown. Some possibilities are as follows:

- 1. Different initial distributions. For instance, it has been proposed that the rotation distribution at ZAMS should be bimodal due to a number of stars being prevented from spinup during the PMS because of their coupling to companion accretion disk.
- 2. A combined effect of stochasticity and dynamo saturation.
- 3. Different values of the ratio of the core to envelope rotation rates at ZAMS.

- 4. Modeling of the momentum transfer between the core and the envelope by accounting for nonlinear dependence on the differential rotation. For instance, shear instability can occur for sufficiently large differential rotation, increasing the momentum transport between the two.
- 5. Different magnetic field configurations (e.g., dipolar) and distributions.
- These issues will be addressed in future contributions.

We thank K. B. MacGregor and M. J. Thompson for useful comments. This work was supported by U.K. STFC grant no. ST/F501796/1.

### APPENDIX A

# STOCHASTIC MODEL

To ensure that angular momentum is extracted from the star, we want the noise to take only positive values. One choice of noise distribution that has this property and which is used in this paper is a Gamma distribution

$$P(\xi) \propto \xi^{1/2} \exp[-c\xi]. \tag{A1}$$

To ensure that the average loss of angular momentum is the same as that in the deterministic case, we require the distribution (Equation (A1)) to have a mean value of 1, by fixing the value of *c* to be c = 3/2.

To generate a time series of noise  $\xi$  distributed according to Equation (A1), we numerically solve the following stochastic differential equation:

$$\dot{\xi} = a\xi - g\xi^2 + \xi\Gamma(t). \tag{A2}$$

Here,  $\Gamma$  is a Gaussian white noise with the correlation function:  $\langle \Gamma(t)\Gamma(t')\rangle = 2\sigma^2\delta(t-t')$ . We choose a random initial condition  $\xi(t_0) > 0$ ; as  $\xi = 0$  is an absorbing point of Equation (A2),  $\xi$  (and thus the loss of angular momentum) is always non-negative. Note that Equation (A2) is written assuming the Stratonovitch convention (Kloeden & Platen 1992). Using standard techniques, it is straightforward to show that the probability distribution  $P(\xi, t)$  satisfies the following Fokker–Planck equation:

$$\partial_t P = -\partial_{\xi} [(a\xi - g\xi^2)] + \sigma^2 \partial_{\xi} [\xi \partial_{\xi} (\xi P)].$$
(A3)

For sufficiently long time, the distribution P in Equation (A3) converges toward the stationary distribution  $P_s$ :

$$P_s(\xi) = \xi^{a/\sigma^2 - 1} \exp -\frac{g\xi}{\sigma^2} \quad . \tag{A4}$$

This stationary distribution can be chosen to match Equation (A1) with c = 3/2 by taking  $a/\sigma^2 = g/\sigma^2 = 3/2$ . We have three parameters  $(a, g, \sigma)$  and two relations, leaving only one free parameter. We choose to vary  $\sigma$  and fix the other two as  $a = g = 3/2\sigma^2$ . Equation (A3) is then integrated until the stationary distribution is reached; results are shown in Figure 9. It is interesting to note that regardless of the value of  $\sigma$ , the noise in the stationary regime is distributed according to Equation (A1). However, the autocorrelation function defined as

$$C(\tau) = \langle (\xi(0) - \langle \xi \rangle)(\xi(\tau) - \langle \xi \rangle) \rangle, \tag{A5}$$



Figure 9. Time series (left) of the noise used in the stochastic simulations and the corresponding probability distribution (right). The distribution (Equation (A1)) is plotted as a solid line in the right panel.

(A color version of this figure is available in the online journal.)



**Figure 10.** Left: temporal autocorrelation (Equation A5) of the noise for different values of the parameter  $\sigma$ . Right: correlation time of the noise as defined in Equation (A6) as a function of  $\sigma^2$ .

(A color version of this figure is available in the online journal.)

is different for different values of the parameter  $\sigma$  as shown in the left panel of Figure 10. The right-hand panel shows the correlation time computed as

$$\tau_c = \int_0^\infty \frac{C(\tau)}{C(0)} d\tau, \qquad (A6)$$

which is a decreasing function of  $\sigma$  as shown in the left panel of Figure 10: larger values of  $\sigma$  give shorter correlation times for the noise. For instance, the correlation time is 89, 21, 8, and 0.8 Myr for  $\sigma^2 = 0.1, 0.5, 1$ , and 5, respectively.

To simplify the formulation of the stochastic model, we use the following non-dimensionalized variables:

$$x_1 = \frac{J_c}{(I_c + I_e)\Omega_{\odot}},$$
  

$$x_2 = \frac{J_e}{(I_c + I_e)\Omega_{\odot}},$$
(A7)

where  $\Omega_{\odot} = 3 \times 10^{-6} \, \text{s}^{-1}$  is the present solar angular rotation. The two shell model (Equation (1)) coupled with the stochastic model (A2) can then be rewritten as

$$\begin{aligned} \frac{dx_1}{dt} &= -\alpha x_1 + \beta x_2, \\ \frac{dx_2}{dt} &= \alpha x_1 - \beta x_2 - \gamma(x_2, B) x_2 \xi(t), \\ \frac{d\xi}{dt} &= \frac{3\sigma^2}{2} (\xi - \xi^2) + \xi \Gamma(t). \end{aligned}$$
(A8)

Here,

$$\alpha_0 = \frac{I_e}{I_c + I_e}, \qquad \alpha = \frac{\alpha_0}{\tau_c}, \qquad \beta = \frac{1 - \alpha_0}{\tau_c} \quad \text{and}$$

$$\gamma(x_2, B) = \frac{1}{\tau_w(x_2, B)}. \quad (A9)$$

In terms of our notations, the braking due to the solar wind is given by

$$\gamma = \gamma_{\odot} \times \frac{(J_* B_* / x_2)^{2/3} B^{4/3}}{B^2} \text{ if } x_2 > J_*}{B^2},$$
 (A10)

where  $J_* = \alpha_0 \Omega_* / \Omega_{\odot}$  and  $\gamma_{\odot} = 1/\tau_{w\odot}$ . The magnetic field can then be taken to be linearly proportional to the angular rotation or to saturate above threshold. Results in this paper are obtained by integrating Equation (A8) using Heun's method (Kloeden & Platen 1992).

#### APPENDIX B

## INITIAL GAMMA DISTRIBUTION

To compare our results with those of Kepens et al. (1995), we choose a family of initial distribution characterized by two parameters n and m:

$$P(x) = \frac{A}{x^n} \exp \left(\frac{-1}{(mx)^2}\right)$$
 (B1)



**Figure 11.** Choice of the values of *n* and *m* when the percentage of rapidly rotating stars is fixed to 20%. (A color version of this figure is available in the online journal.)



**Figure 12.** Comparison of the initial distribution calculated by Kepens et al. (1995) and approximated by distribution (Equation (B1)). (A color version of this figure is available in the online journal.)

The results of Kepens et al. (1995) suggest that in the linear dynamo case (without saturation), the populations of 30%, 50%, and 20% of stars are observed for  $V_{eq}$  lying in the intervals 0–30 km s<sup>-1</sup>, 30–50 km s<sup>-1</sup>, and >50 km s<sup>-1</sup>, respectively. Figure 11 shows the evolution of the percentages with the parameters  $\alpha$  and *m* when the percentage of stars with velocity over 50 km s<sup>-1</sup> is fixed at 20%. The best fit to the initial distribution used by Kepens et al. (1995; 30%, 50%, and 20% for  $V_{eq}$  in the intervals 0–30 km s<sup>-1</sup>, 30–50 km s<sup>-1</sup>, and >50 km s<sup>-1</sup>, respectively) is obtained for n = 5.42 and m = 0.0425.

Figure 12 compares the initial distribution we use at 30 Myr for simulations and the one computed by Kepens et al. (1995).

# APPENDIX C

#### EFFECT OF THE PRESENT SUN SPIN-DOWN TIME

As noted in the main text, different values of the spin-down time  $\tau_{w\odot}$  have been proposed in the literature. For instance, the loss of angular momentum  $\dot{J}_w$  has been reported to be 2.4 ×  $10^{24}$  kg m<sup>2</sup>s<sup>-2</sup> and 2.1 ×  $10^{23}$  kg m<sup>2</sup>s<sup>-2</sup> by Kepens et al. (1995) and Li (1999), respectively. Using the angular momentum of the envelope  $I_e \Omega_e = 1.02 \times 10^{40}$  kg m<sup>2</sup>s<sup>-1</sup>, we obtain the spin-down time from Equation (3) with values 135 Myr and 1541 Myr, respectively. Note that the value 36,000 Myr quoted by Li (1999) is obtained using the total angular momentum of

 Table 3

 Percentages of Slow Rotators (SR), Average Rotators (AR), Fast Rotators (FR), and Ultrafast Rotators (UFR) at Different Times for the Simulation of the Case with Saturated Dynamo and Stochastic Loss of Angular Momentum

Linear case	SR AR	0.0%	0.0%	0.00
	AR	05.00		0.0%
	<b>FD</b>	97.2%	77.1%	0.2%
	FK	2.7%	22.9%	99.8%
	UFR	0.0%	0.0%	0.0%
$Q_s = 20$	SR	0.0%	0.0%	0.0%
	AR	94.9%	70.5%	0.2%
	FR	4.9%	29.0%	99.6%
	UFR	0.1%	0.4%	0.1%
$Q_{s} = 10$	SR	0.0%	0.0%	0.0%
	AR	94.0%	61.8%	0.2%
	FR	5.8%	37.6%	99.2%
	UFR	0.2%	0.6%	0.6%
$Q_s = 5$	SR	0.0%	0.0%	0.0%
	AR	93.7%	57.7%	0.0%
	FR	6.1%	41.7%	99.2%
	UFR	0.2%	0.6%	0.8%
$\sigma^2 = 5$	SR	0.0%	0.0%	0.0%
	AR	97.2%	77.6%	0.4%
	FR	2.8%	22.4%	99.6%
	UFR	0.0%	0.0%	0.0%
$\sigma^2 = 1$	SR	0.1%	0.0%	0.0%
	AR	96.7%	75.3%	0.5%
	FR	3.2%	24.6%	99.5%
	UFR	0.0%	0.0%	0.0%
$\overline{\sigma^2 = 0.1}$	SR	0.1%	0.1%	0.0%
	AR	96.3%	73.5%	1.4%
	FR	3.5%	26.3%	98.6%
	UFR	0.0%	0.1%	0.0%

Note. In that case, the present spin-down time is chose as  $\tau_{w\odot} = 3000 \text{ Myr}$ .

the Sun rather than the angular momentum of the envelope, which is different from the definition of  $\tau_{w\odot}$  in this paper and Kepens et al. (1995).

To illustrate the effect of varying the spin-down time, we performed some simulations with a larger spin-down time  $\tau_{w\odot} = 3000$  Myr and show the results in Table 3. As we can see, there is not enough spin-down to reproduce the observations: both for the stochastic and saturated models, there are no SRs for small and long times.

 Table 4

 Percentages of Slow Rotators (SR), Average Rotators (AR), Fast Rotators (FR), and Ultrafast Rotators (UFR) at Different Times for the Simulation of the Case with Saturated Dynamo and Stochastic Loss of Angular Momentum

Simulation	Bin	$t = 50 \mathrm{Myr}$	$t = 70 \mathrm{Myr}$	$t = 600 \mathrm{Myr}$
Linear case	SR	36.1%	44.5%	41.4%
	AR	32.1%	31.8%	23.4%
	FR	12.1%	23.7%	35.2%
	UFR	19.7%	0.0%	0.0%
$Q_s = 20$	SR	36.1%	45.2%	42.2%
	AR	32.1%	23.0%	22.5%
	FR	12.1%	13.5%	25.4%
	UFR	19.7%	18.3%	9.9%
$Q_{s} = 10$	SR	36.1%	44.3%	44.0%
	AR	32.1%	14.7%	16.5%
	FR	12.1%	18.3%	21.5%
	UFR	19.7%	22.7%	18.0%
$\overline{Q_s} = 5$	SR	36.1%	36.2%	39.2%
	AR	32.1%	18.6%	7.1%
	FR	12.1%	20.7%	30.1%
	UFR	19.7%	24.5%	23.7%
$\sigma^2 = 1$	SR	36.1%	42.9%	38.0%
	AR	32.1%	29.9%	22.1%
	FR	12.1%	22.4%	39.9%
	UFR	19.7%	4.8%	0.0%
$\sigma^2 = 0.5$	SR	36.1%	42.7%	37.0%
	AR	32.1%	29.2%	21.9%
	FR	12.1%	22.7%	41.0%
	UFR	19.7%	5.4%	0.0%
$\overline{\sigma^2} = 0.1$	SR	36.1%	42.5%	36.2%
	AR	32.1%	29.0%	21.2%
	FR	12.1%	22.7%	42.6%
	UFR	19.7%	5.8%	0.0%

**Note.** In that case, the distribution at 50 Myr is chosen to fit the modified distribution computed from the data in Section 5.

#### APPENDIX D

#### **EVOLUTION STARTING FROM 50 Myr**

For completeness, we also consider the case when the simulation is started at 50 Myr rather than 30 Myr. In order to compare the result of our simulations with the modified distribution that we compute in Section 5, we synthesize the initial distribution at 50 Myr to fit the percentage of slow and FRs given by the corrected distribution of Table 1. Note that we

fit the initial data with the distribution (Equation (B1)) as we did in Appendix B. Therefore, the percentages are not exactly matching that of the observations but are sufficiently similar to have a general trend.

The results of this simulation are given in Table 4. The first conclusion from Table 4 is that all cases are quite good without significant differences. The agreement with the corrected observations is also quite good. However, a closer inspection shows that the case with noise gives us the best agreement for small times (70 Myr) as it keeps a proportion of UFRs of around 5%. For large time (600 Myr), all the cases tend to overestimate the number of FRs (40% instead of 20%).

# REFERENCES

- Barnes, S., & Sofia, S. 1996, ApJ, 462, 746
- Belcher, J. W., & MacGregor, K. B. 1976, ApJ, 210, 498
- Bruno, R., & Carbone, V. 2005, Living Rev. Solar Phys., 2, 4
- Buzasi, D. L. 1997, ApJ, 484, 855
- Goldstein, M. L., Roberts, D. A., & Matthaeus, W. 1995, ARA&A, 33, 283
- Herbst, W., & Mundt, R. 2005, ApJ, 633, 967
- Hood, L. L., & Jirikowic, J. L. 1990, in Climate Impact of Solar Variability, ed. K. Schatten & A. Arking (Greenbelt, MD: NASA GSFC), 98
- Jardine, M. 2005, in IAU Symp. 226, in Coronal and Stellar Mass Ejections, ed. K. Dere, J. Wang, & Y. Yan (Cambridge: Cambridge Univ. Press), 481
- Jardine, M., & Donati, J.-F. 2008, in IAU Symp. 259, Cosmic Magnetic Fields: From Planets, to Stars and Galaxies, ed. K. G. Strassmeier, A. G. Kosovichev, & J. E. Beckman (Cambridge: Cambridge Univ. Press), 357
- Javaraiah, J. 2003, A&A, 401, L9
- Kawaler, S. D. 1988, ApJ, 333, 236
- Kepens, R., MacGregor, K. B., & Charbonneau, P. 1995, A&A, 294, 469
- Kloeden, P. E., & Platen, E. 1992, Numerical Solution of Stochastic Differential Equations (New York: Springer)
- Li, J. 1999, MNRAS, 302, 203
- MacGregor, K. B., & Brenner, M. 1991, ApJ, 376, 204
- Nandy, D. 2004, Sol. Phys., 224, 161
- O'Dell, M. A., Panagi, P., Hendry, M. A., & Collier Cameron, A. 1995, A&A, 294, 715
- Pizzolato, N., Maggio, A., Micela, G., Sciortino, S., & Ventura, P. 2003, A&A, 397, 147
- Schrijver, C., DeRosa, M. L., & Title, A. M. 2003, ApJ, 590, 493
- Skumanich, A. 1972, ApJ, 171, 565
- Soderblom, D. R., Stauffer, J. R., MacGregor, K. B., & Jones, B. F. 1993, ApJ, 409, 624
- Solanki, S. K., Motamen, S., & Keppens, R. 1997, A&A, 325, 1039
- Stauffer, J. R., Caillault, J.-P., Gagné, M., Prosser, C. F., & Hartmann, L. W. 1994, ApJS, 91, 625
- Tassoul, J.-L. 2000, Stellar Rotation (Cambridge: Cambridge Univ. Press)
- Thompson, M. J., Christensen-Dalsgaard, J., Miesch, M. S., & Toomre, J. 2003, ARA&A, 41, 599
- Weber, E. J., & Davis, L. J. 1967, ApJ, 148, 217