# Dual connectivity in backhaul-limited heterogeneous networks with massive-MIMO: User association and power allocation

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# Dual Connectivity in Backhaul-limited Massive-MIMO HetNets: User Association and Power Allocation

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***Abstract*—With dual connectivity, a mobile user can be served by a macro base station (MBS) and a pico base station (PBS) simultaneously. In this paper, we address the problem of optimizing user-PBS association and power allocation in the uplink such that the network can serve the users' demand at the minimum cost, where the PBSs are subject to backhaul capacity limitations and minimum rate requirements of users. We show that this non-convex problem can be formulated as a signomial geometric programming (SGP) whose solution can be found by solving a series of geometric programming (GP) problems. Simulation results are provided to demonstrate traffic offloading trend to PBSs for different cost and backhaul capacity settings, confirming the effectiveness of the proposed iterative algorithm. They also show that the output of the proposed algorithm closely matches the global optimal solution with affordable complexity.**

## I. Introduction

Network densification has been identified as one of the dominant themes for future fifth-generation (5G) networks, which could facilitate meeting the growing mobile traffic demand [1]. Two of the key enabling trends for network densification are heterogeneous networks and massive-MIMO. With proper resource allocation and interference management techniques, the deployment of heterogeneous networks by introducing pico-cells along with macro-cells can improve network throughput, coverage and energy efficiency. Furthermore, implementing a large number of antennas at macro base stations (MBSs) can help to further enhance the spectral efficiency by minimizing the inter and intra-cell interference.

Integration of pico-cells and macro-cells can be realized in different forms. One of these options is dual connectivity which has been introduced in Release 12 of Third Generation Partnership Project (3GPP) specifications [2]. Dual connectivity allows users to be simultaneously served by an MBS and a pico base station (PBS), which are operating at non-overlapping frequency carriers and are interconnected with X2-based backhaul links. It has been demonstrated that dual connectivity can significantly increase the user throughput [2–4] and at the same time improve the robustness against mobility [2].

Installing a massive number of antennas on MBSs can significantly boost the spectral efficiency. On the other hand, operating a large number of antennas consumes non-trivial amount of energy [5], and such cost must be taken care of responsibly to minimize the impact to the environment. With dual connectivity, part of the traffic can be off-loaded to PBSs, which typically consume much less power than MBSs [6].

Since PBSs are typically deployed in hotspot areas with high data-rate demands and their backhaul links are usually capacity limited, data offloading to PBSs needs to be optimally and dynamically adjusted such that the cost of serving the users can be minimized. The dense deployment of PBSs can further complicate the user-PBS association of the users which are located at the edges of several pico-cells. This then brings more difficulty to the decision of how data is to be split between an MBS and the PBSs.

Previous works [3, 4, 7, 8] address the traffic splitting optimization with dual connectivity without jointly considering the user-PBS association optimization and backhaul limitations. In this work, we focus on the optimization of traffic splitting and user-PBS association in the uplink by taking into account the backhaul capacity constraints of the single-antenna PBSs, where the MBS is equipped with a large number of antennas. Aiming to minimize the cost of serving the users' data rate requirements, an optimization problem is formulated to determine to which PBS each user should connect and moreover how a user should balance its power budget between the MBS and the associated PBS. The formulated problem is non-convex. To solve the problem with affordable complexity, we first transform it into a signomial geometric programming (SGP), and then solve the SGP through a series of geometric programming (GP) problems which can be solved efficiently. Numerical studies illustrate that the proposed iterative algorithm performs closely to the exhaustive search with affordable complexity. Also, the variations of the achieved data rate at the base station (BSs) are demonstrated for different cost and backhaul capacity settings, demonstrating the effectiveness of the proposed iterative algorithm.

The rest of this paper is organized as follows. Section II presents the system model and the formulation of the joint user association and power allocation problem. Section III develops an iterative algorithm to solve the problem formulated in Section II. Section IV illustrates simulation results to evaluate the performance of the proposed iterative algorithm. Finally,

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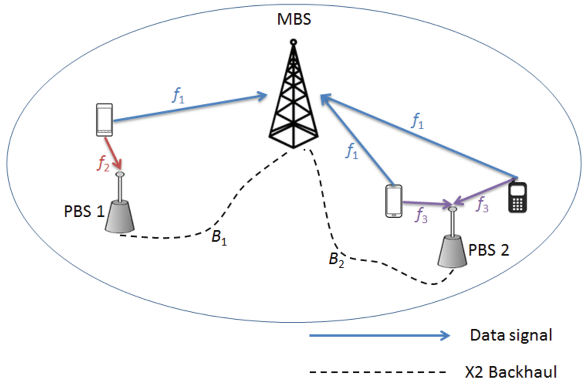



Fig. 1. A schematic diagram of the system model. The MBS operates on frequency band $f_1$. The two PBSs operate on frequency bands $f_2$ and $f_3$, respectively. At the same time, each user can only be connected to one PBS.

Section V draws the conclusion.

## II. System Model and Problem Formulation

We consider a heterogeneous cellular network which consists of one MBS and $N$ PBSs. The network serves $K$ single-antenna users on the uplink. We assume that dual connectivity is supported[1] in the system, so that each user can be served simultaneously by an MBS and a PBS. Each BS, i.e., an MBS or a PBS, uses a dedicated frequency band which is not used by any other BS. We assume that the MBS is equipped with a large number of antennas $M \gg 1$, while a single antenna is installed on each PBS. It is assumed that the MBS has the perfect knowledge of the channels between each user and each MBS/PBS. Fig. 1 shows an example of the system model.

Let $\mathcal{K}$ and $\mathcal{N}$ be the set of users and the PBSs, respectively. Also, let $\mathbf{P} \triangleq [\mathbf{P}_0, \mathbf{P}_1, ..., \mathbf{P}_N]$, $\mathbf{P}_0 \triangleq [P_{1,0}, P_{2,0}, ..., P_{K,0}]$ and $\mathbf{P}_n \triangleq \{P_{1,n}, P_{2,n}, ..., P_{K,n}\}$, $\forall n \in \mathcal{N}$, where $P_{k,0}$ denotes the transmission power of user $k$ towards the MBS and $P_{k,n}$ represents the transmission power of user $k$ towards PBS $n$. In this setup, to comply with dual connectivity regulations, we restrict the access of each user by the following constraint

$$\mathbf{C1:} \;\; P_{k,n} \cdot \Big( \sum_{n' \in \mathcal{N} \setminus \{n\}} P_{k,n'} \Big) = 0, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}. \quad (1)$$

The constraint $\mathbf{C1}$ ensures that each user can only be connected to one of the PBSs. More specifically, if user $k$ is associated with PBS $n$ (i.e., non-zero $P_{k,n} > 0$), then no power would be allocated toward any other PBS (i.e., $\sum_{n' \in \mathcal{N} \setminus \{n\}} P_{k,n'} = 0$).

We also consider finite power budget of each user as

$$\mathbf{C2:} \;\; P_{k,0} + \sum\nolimits_{n \in \mathcal{N}} P_{k,n} \le P_k^{\max}, \; \forall k \in \mathcal{K}, \quad (2)$$

where $P_k^{\max}$ is the maximum power budget of user $k$.

Let $h_{k,n}$ represent the channel gain between user $k$ and PBS $n$, where the attenuation due to both the large-scale fading and the small-scale fading are included in $h_{k,n}$. The achievable uplink rate of user $k$ at PBS $n$ can be expressed as

$$R_{k,n} = \log \Big( 1 + \frac{P_{k,n}|h_{k,n}|^2}{\sigma_n^2 + \sum_{j \neq k} P_{j,n}|h_{j,n}|^2} \Big),$$
$$\Rightarrow e^{R_{k,n}} = 1 + \frac{P_{k,n}|h_{k,n}|^2}{\sigma_n^2 + \sum\limits_{j \neq k} P_{j,n}|h_{j,n}|^2}, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}, \quad (3)$$

where $\sigma_n^2$ denotes the noise power at the PBS $n$ and $\sum_{j \neq k} P_{j,n}|h_{j,n}|^2$ represents the intra-picocell interference.

Furthermore, denote $d_k$ as the large-scale fading between user $k$ and the MBS. In the perfect channel state information (CSI) case, the achievable uplink rate of user $k$ at the MBS becomes (See [10])

$$R_{k,0} = \log \Big( 1 + \frac{P_{k,0} M d_k}{\sigma_0^2} \Big) \Rightarrow e^{R_{k,0}} = 1 + \frac{P_{k,0} M d_k}{\sigma_0^2}, \quad (4)$$

where $\sigma_0^2$ denotes the noise power at the MBS.

We assume that user $k$ needs a minimum rate requirement of $R_k^{\text{rsv}}$ through the transmission towards both the MBS and a PBS, which can be expressed as

$$\mathbf{C3:} \;\; R_{k,0} + \sum\nolimits_{n \in \mathcal{N}} R_{k,n} \ge R_k^{\text{rsv}}, \; \forall k \in \mathcal{K}. \quad (5)$$

Moreover, from a user-plane perspective, we consider a dual connectivity solution standardized by 3GPP, in which user-plane data is split in the MBS, such that the data being sent to the PBSs by the users need to be routed to the MBS via the backhaul connections between the PBSs and the MBS. Since the PBSs are interconnected with MBS through non-ideal X2 backhauls, we consider a maximum capacity on the backhaul between PBS $n$ and MBS as

$$\mathbf{C4:} \;\; \sum\nolimits_{k \in \mathcal{K}} R_{k,n} \le B_n, \forall n \in \mathcal{N} \quad (6)$$

to avoid an excessive load on one PBS and its backhaul link to MBS, where $B_n$ gives the backhaul capacity of PBS $n$. The capacity of the backhaul of the MBS is assumed to be sufficiently large.

Serving the uplink users comes at a cost at the MBS and the PBSs. Assume that the price per unit of data at the MBS is $\pi_0 \ge 0$ and the price per unit of data at PBS $n$ is $\pi_n \ge 0$. Then, we adopt the widely used linear cost model [11], such that the cost for MBS and PBS $n$ will respectively be $\pi_0 \sum_{k \in \mathcal{K}} R_{k,0}$ and $\pi_n \sum_{k \in \mathcal{K}} R_{k,n}$. For ease of referencing, we define

$$\psi(\mathbf{P}) \triangleq \pi_0 \sum\nolimits_{k \in \mathcal{K}} R_{k,0} + \pi_n \sum\nolimits_{k \in \mathcal{K}} R_{k,n} \quad (7)$$

as the total cost of all the BSs.

In this work, we aim to jointly optimize user-PBS association as well as power control between MBS and the associated PBS in a dual-connectivity heterogeneous network. The goal is to minimize the cost experienced by the network, while satisfying the minimum data rate requirements of the users and backhaul capacity limits of the PBSs. The joint user association and power control optimization problem can be

[1]Dual connectivity in the uplink is now being considered for future extension of the 3GPP standardization [9].

formulated as

$$\texttt{MinCost}\text{: } \underset{\mathbf{P}}{\text{minimize}}\ \psi,\ \text{subject to: } \mathbf{C1}-\mathbf{C4}.$$

The objective function and constraints ($\mathbf{C1}, \mathbf{C3},$ and $\mathbf{C4}$) in `MinCost` are non-convex due to intra-cell interference in pico-cells and user association restrictions. Next, we develop an efficient algorithm to solve `MinCost`.

## III. An Iterative Algorithm for Joint User Association and Power Control

In this session, we develop an iterative algorithm to solve the optimization problem (8a). We show that the problem can be transformed into a SGP. Then, to solve the SGP with low complexity, we use successive convex approximation (SCA) [12] in which a series of standard GPs are solved to find a solution of the SGP, where a GP can be efficiently solved by optimization toolboxes such as CVX [13].

### *A. Preliminary on SGP*

A SGP in standard form can be written as [14]

$$\underset{\mathbf{x}}{\text{minimize}}\ x_0, \tag{8a}$$

$$\text{subject to } \frac{f_k(\mathbf{x})}{f'_k(\mathbf{x})} \le 1,\ k \in \mathcal{K}_1, \tag{8b}$$

$$\frac{f_k(\mathbf{x})}{f'_k(\mathbf{x})} = 1,\ k \in \mathcal{K}_2, \tag{8c}$$

where $\mathbf{x} \triangleq [x_0, x_1, ..., x_N]$ is the vector of optimization variables, $f_k(\mathbf{x})$ and $f'_k(\mathbf{x})$ are posynomials, and $\mathcal{K}_1$ and $\mathcal{K}_2$ are the index sets such that $\mathcal{K}_1 \cap \mathcal{K}_2 = \emptyset$.

To solve a SGP with an affordable complexity, an iterative algorithm based on SCA is proposed with guaranteed convergence to an optimal point of the SGP [15]. More specifically, in each iteration, a two-step transformation is performed to formulate a GP which is an approximation of the original SGP. An optimal solution can then be found by solving a series of GPs, where a GP can be solved efficiently by a convex optimization toolbox [16].

We now describe the details of the two-step transformations to reach the GP formulation that approximates the original SGP. Denote $\mathbf{x}(t)$ as the variables at the $t$-th iteration of the iterative algorithm which solves (8). Also, define $\{s_k(t)|k \in \mathcal{K}_2\}$ as the set of auxiliary variables, and define $\{w_k(t)|k \in \mathcal{K}_2\}$ as the non-negative weighting factors on the respective auxiliary variables. The problem (8) is first transformed into

$$\underset{\mathbf{x}, \{s_k(t)|k\in\mathcal{K}_2\}}{\text{minimize}}\ x_0(t) + \sum\nolimits_{k\in\mathcal{K}_2} w_k(t)s_k(t), \tag{9a}$$

$$\text{subject to } \frac{f_k(\mathbf{x}(t))}{f'_k(\mathbf{x}(t))} \le 1,\ k \in \mathcal{K}_1 \cup \mathcal{K}_2, \tag{9b}$$

$$\frac{s_k(t)^{-1} f'_k(\mathbf{x}(t))}{f_k(\mathbf{x}(t))} \le 1,\ k \in \mathcal{K}_2, \tag{9c}$$

$$s_k(t) \ge 1,\ k \in \mathcal{K}_2, \tag{9d}$$

so that the equality constraints on ratios between posynomials become inequality constraints. Note that (9) is equivalent to (8) when all the constraints in (9d) are active [15]. To ensure the equivalence, $\{w_k(t)|k \in \mathcal{K}_2\}$ can be configured as an increasing function of $t$ [15].

In the second step of the transformation, arithmetic-geometric mean approximation (AGMA) is applied to the denominators of the left hand sides (LHSs) of the constraints in (9b) and (9c), so that the posynomials in the denominators are approximated by monomials. After the AGMA, the constraints in (9b) and (9c) become upper bound constraints on posynomials, so that (9) becomes a GP [16].

To illustrate the AGMA, let

$$\tilde{f}_{j,k}(\mathbf{x}(t)) \triangleq \varphi_{j,k} \prod\nolimits_i x_i(t)^{\delta_{i,j,k}} \tag{10}$$

be a monomial, where $\varphi_{j,k} > 0$ and $\delta_{i,j,k}$ is a real number. Without loss of generality, the posynomial in the denominator of the LHS of the $k$-th constraint in (9c) can be written as

$$f_k(\mathbf{x}(t)) \triangleq \sum\nolimits_j \tilde{f}_{j,k}(\mathbf{x}(t)). \tag{11}$$

From the AGMA, we have

$$\frac{1}{f_k(\mathbf{x}(t))} = \frac{1}{\sum_j \tilde{f}_{k,j}(\mathbf{x}(t))} \le \prod\nolimits_j \left(\frac{\tilde{f}_{k,j}(\mathbf{x}(t))}{b_{k,j}(t)}\right)^{-b_{k,j}(t)}, \tag{12}$$

where $b_{k,j}(t) \triangleq \frac{\tilde{f}_{k,j}(\mathbf{x}(t-1))}{f_k(\mathbf{x}(t-1))}$ and $\mathbf{x}(t-1)$ is obtained from the last iteration [15]. Then, using the right hand side of (12) to approximate $\frac{1}{f_k(\mathbf{x}(t))}$, the constraints in (9c) can be approximated as

$$s_k(t)^{-1} f'_k(\mathbf{x}(t)) \prod\nolimits_j \left(\frac{\tilde{f}_{k,j}(\mathbf{x}(t))}{b_{k,j}(t)}\right)^{-b_{k,j}(t)} \le 1, k \in \mathcal{K}_2. \tag{13}$$

Observe that the constraints in (13) are tighter than those in (9c). As a result, the solution obtained from the iterative algorithm is always feasible to the original SGP. Similar approximations can be applied to the constraints in (9b), and the details are omitted for simplicity.

In the following, we first reformulate the problem in (8a) as a SGP, and then we apply the iterative GP approximation method to solve (8a) in SGP form.

### *B. Reformulation of `MinCost` as a SGP*

As a first step, we treat $R_{k,0}$ and $R_{k,n}$ as new variables, so that (4) and (3) become the new constraints of `MinCost`. For formulating the new problem as a SGP, we need to rewrite the constraints in (4) and (3) as equality constraints on posynomials or ratios between posynomials. To this end, we apply the Taylor series approximation on the exponential functions, such that

$$e^{R_{k,j}} = \sum\nolimits_{i=0}^{+\infty} \frac{R_{k,j}^i}{i!} \approx 1 + \sum\nolimits_{i=1}^{U} \frac{R_{k,j}^i}{i!},\ j \in \{0\} \cup \mathcal{N}, \tag{14}$$

where $U$ is a sufficiently large positive integer to ensure the accuracy of the approximation. Then, the constraints in (3) and (4) can respectively be rewritten as

$$\textbf{C5:}\ \sum_{i=1}^{U}\frac{R_{k,0}^{i}\sigma_0^2}{i!\cdot P_{k,0}Md_k}{=}1,\ \forall k{\in}\mathcal{K} \tag{15}$$

$$\textbf{C6:}\sum_{i=1}^{U}\frac{R_{k,n}^{i}\cdot\left(\sigma_n^2{+}\sum_{j\neq k}P_{j,n}|h_{j,n}|^2\right)}{i!\cdot P_{k,n}|h_{k,n}|^2}{=}1,\forall k,\forall n. \tag{16}$$

The constraints in **C2**, **C4**, **C5**, and **C6** are readily admissible to a SGP. The constraints in **C1** and **C3** are equivalent to

$$\hat{\textbf{C}}\textbf{1}:1+P_{k,n}\cdot\left(\sum_{n'\in\mathcal{N}\setminus\{n\}}P_{k,n'}\right)=1,\forall k,\forall n \tag{17}$$

and

$$\hat{\textbf{C}}\textbf{3}:\ R_k^{\text{rsv}}\cdot\left(R_{k,0}+\sum_{n\in\mathcal{N}}R_{k,n}\right)^{-1}\leq 1,\ \forall k\in\mathcal{K}, \tag{18}$$

respectively, where $\hat{\textbf{C}}\textbf{1}$ and $\hat{\textbf{C}}\textbf{3}$ can be accepted in a standard SGP formulation. Then, `MinCost` can be rewritten as the following SGP

$$\texttt{MinCost-SGP}:$$

$$\underset{\mathbf{P},\mathbf{R}}{\text{minimize}}\ \psi,\ \text{subject to: } \hat{\textbf{C}}\textbf{1},\ \textbf{C2},\ \hat{\textbf{C}}\textbf{3},\ \textbf{C4}-\textbf{C6},$$

where $\mathbf{R}\triangleq\{\mathbf{R}_0,\mathbf{R}_1,...,\mathbf{R}_N\}$ gives the achieved rate of all users at all the BSs, and $\{\mathbf{R}_j\triangleq\{R_{1,j},R_{2,j},...,R_{K,j}\}|j=0,1,...,N\}$.

### C. Iterative GP approximations

Here, by following the two-step transformation in Section III-A, we formulate the GP at the $t$-th iteration of the iterative algorithm which approximate `MinCost-SGP`. For this reason, in the sequel, the objective function and the variables in `MinCost-SGP` will be appended with the index $t$.

First, define $\{s_{1,k,n}(t)|k\in\mathcal{K},n\in\mathcal{N}\}$, $\{s_{2,k}(t)|k\in\mathcal{K}\}$, and $\{s_{3,k,n}(t)|k\in\mathcal{K},n\in\mathcal{N}\}$ as the auxiliary variables for the equality constraints in $\hat{\textbf{C}}\textbf{1}$, **C5**, and **C6**, respectively. Also, define $\{w_{1,k,n}(t)|k\in\mathcal{K},n\in\mathcal{N}\}$, $\{w_{2,k}(t)|k\in\mathcal{K}\}$, and $\{w_{3,k,n}(t)|k\in\mathcal{K},n\in\mathcal{N}\}$ as the corresponding non-negative weighting factors for the auxiliary variables. Moreover, denote $\mathbf{s}(t)$ and $\mathbf{w}(t)$ as the vectors containing all the auxiliary variables and there corresponding weighting factors, respectively. The problem in `MinCost-SGP` can be transformed into

$$\begin{aligned}&\underset{\mathbf{P}(t),\mathbf{R}(t),\mathbf{s}(t)}{\text{minimize}}\ z(t),\ \text{subject to:}\\&\hat{\textbf{C}}\textbf{1}_1,\ \hat{\textbf{C}}\textbf{1}_2,\ \textbf{C2},\ \hat{\textbf{C}}\textbf{3},\ \textbf{C4},\ \textbf{C5}_1,\ \textbf{C5}_2,\ \textbf{C6}_1,\ \textbf{C6}_2,\\&\mathbf{s}(t)\geq 1,\end{aligned} \tag{19}$$

where

$$\begin{aligned}z(t)\triangleq&\psi(t)+\sum_{k\in\mathcal{K}}w_{2,k}(t)s_{2,k}(t)\\&+\sum_{j=1,3}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}}w_{j,k,n}(t)s_{j,k,n}(t),\end{aligned} \tag{20}$$

and the constraints $\hat{\textbf{C}}\textbf{1}_1$, $\hat{\textbf{C}}\textbf{1}_2$, $\textbf{C5}_1$, $\textbf{C5}_2$, $\textbf{C6}_1$, and $\textbf{C6}_2$ are defined as follows,

**Algorithm 1** Iterative GP Approximations

Initialize $\mathbf{P}(0)$. Initialize $\mathbf{R}(0)$ according to $\mathbf{P}(0)$.
Initialize $\mathbf{w}(0)$ to small positive values.
Set $t:=0$.
**while** Stopping criterion is not met **do**
  $t=t+1$.
  Solve `GP-Appx`$(t)$.
  Set $\mathbf{w}(t)$ such that $\mathbf{w}(t)>\mathbf{w}(t-1)$.
**end while**
**return** $\mathbf{P}(t)$.

$$\hat{\textbf{C}}\textbf{1}_1\textbf{:}\ 1{+}\sum_{n'\in\mathcal{N}\setminus\{n\}}P_{k,n}(t)P_{k,n'}(t){\leq}1,\forall k{\in}\mathcal{K},\forall n{\in}\mathcal{N},$$

$$\hat{\textbf{C}}\textbf{1}_2\textbf{:}\ s_{1,k,n}^{-1}(t)\cdot\Big[1{+}\sum_{n'\in\mathcal{N}\setminus\{n\}}P_{k,n}(t)P_{k,n'}(t)\Big]^{-1}{\leq}1,\forall k,\forall n,$$

$$\textbf{C5}_1\textbf{:}\ \sum_{i=1}^{U}\frac{R_{k,0}^{i}(t)\sigma_0^2}{i!\cdot P_{k,0}(t)Md_k}{\leq}1,\ \forall k{\in}\mathcal{K},$$

$$\textbf{C5}_2\textbf{:}\ s_{2,k}^{-1}(t)\cdot\Big(\sum_{i=1}^{U}\frac{R_{k,0}^{i}(t)\sigma_0^2}{i!\cdot P_{k,0}(t)Md_k}\Big)^{-1}{\leq}1,\ \forall k{\in}\mathcal{K},$$

$$\textbf{C6}_1\textbf{:}\ \sum_{i=1}^{U}\frac{R_{k,n}^{i}(t)\left(\sigma_n^2{+}\sum_{j\neq k}P_{j,n}(t)|h_{j,n}|^2\right)}{i!\cdot P_{k,n}(t)|h_{k,n}|^2}{\leq}1,\forall k,\forall n,$$

$$\begin{aligned}\textbf{C6}_2\textbf{:}\ & s_{3,k,n}^{-1}(t)\cdot\left[\sum_{i=1}^{U}\frac{R_{k,n}^{i}(t)\Big(\sigma_n^2{+}\sum\limits_{j\neq k}P_{j,n}(t)|h_{j,n}|^2\Big)}{i!\cdot P_{k,n}(t)|h_{k,n}|^2}\right]^{-1}{\leq}1,\\ & \forall k{\in}\mathcal{K},\ \forall n{\in}\mathcal{N}.\end{aligned}$$

Then, by applying the AGMA in (12), the constraints in $\hat{\textbf{C}}\textbf{1}_2$, $\hat{\textbf{C}}\textbf{3}$, $\textbf{C5}_2$, and $\textbf{C6}_2$ can be approximated by the constraints in $\tilde{\textbf{C}}\textbf{1}_2$, $\tilde{\textbf{C}}\textbf{3}$, $\tilde{\textbf{C}}\textbf{5}_2$, and $\tilde{\textbf{C}}\textbf{6}_2$, respectively, where the details of $\tilde{\textbf{C}}\textbf{1}_2$, $\tilde{\textbf{C}}\textbf{3}$, $\tilde{\textbf{C}}\textbf{5}_2$, and $\tilde{\textbf{C}}\textbf{6}_2$ are given in Appendix A.

Finally, we obtain the GP approximation of `MinCost-SGP` at the $t$-th iteration as

$$\texttt{GP-Appx}(t):\ \underset{\mathbf{P}(t),\mathbf{R}(t),\mathbf{s}(t)}{\text{minimize}}\ z(t),\ \ \text{subject to:}$$

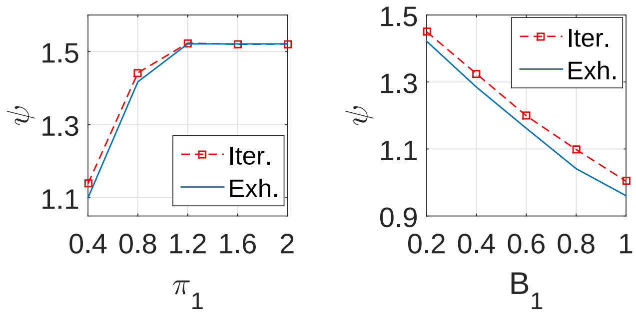
$$\hat{\textbf{C}}\textbf{1}_1,\ \tilde{\textbf{C}}\textbf{1}_2,\ \textbf{C2},\ \tilde{\textbf{C}}\textbf{3},\ \textbf{C4},\ \textbf{C5}_1,\ \tilde{\textbf{C}}\textbf{5}_2,\ \textbf{C6}_1,\ \tilde{\textbf{C}}\textbf{6}_2,\ \text{and (19)}.$$

Algorithm 1 outlines the steps of the proposed iterative algorithm that solves the problem in `MinCost-SGP`. To ensure that Algorithm 1 returns a solution that is close to the optimal solution of `MinCost-SGP`, the algorithm should stop at the $\tau$-th iteration when

1) The difference between $z(\tau)$ and $z(\tau-1)$ is sufficiently small, and
2) All elements in $\mathbf{s}(\tau)$ are close to 1's.

It has been shown that the SCA can converge to a Karush-Kuhn-Tucker (KKT) point of an SGP [15]. We state this convergence property of Algorithm 1 in the following proposition, where we refer interested readers to [15] for the proof.

**Proposition 1.** *Algorithm 1 converges to a KKT point of* `MinCost-SGP` *when* $s_{j,k,n}=1,\ s_{2,k}=1,\ \forall j,k,n.$




(a) $\psi$ vs $\pi_1$ (b) $\psi$ vs $B_1$

Fig. 2. Comparisons between Algorithm 1 and exhaustive search, where $N = K = 2$, $\pi_0 = 1$, $\pi_2 = 0.5$, $B_2 = 2$, and $R_1^{\text{rsv}} = R_2^{\text{rsv}} = 1$. (a) $B_1 = 2$. (b) $\pi_1 = 0.2$.

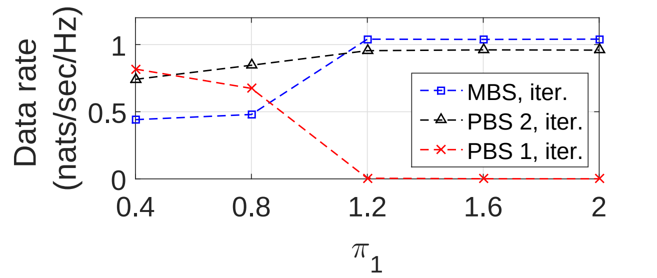
## IV. Simulation Studies

In this section, we present simulation results in a network with one MBS located at the origin and multiple PBSs and the users distributed around it. The channel gains between user $k$ and the MBS is modeled as $d_k = l_{k,0}^{-}$, where $l_{k,0}$ represents the normalized distance between user $k$ and the MBS, and $\delta = 2$ denotes the path-loss exponent. Note that the small-scale fading between a user and the MBS is ignored because the MBS is equipped with massive MIMO and therefore the channel gain due to small-scale fading converges to a deterministic value. On the other hand, the channel gain between user $k$ and the $n$-th PBS is modeled as $h_{k,n} = l_{k,n}^{-}\rho_{k,n}$, where $l_{k,n}$ represents the normalized distance between user $k$ and PBS $n$ and $\rho_{k,n}$ denotes the small scale fading component. The value of $\rho_{k,n}$ is randomly picked from an exponential distribution with unit mean.

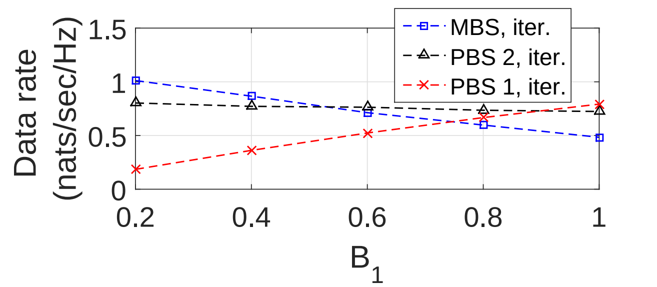
Fig. 2 compares the performance of Algorithm 1 with the exhaustive search, where $K = 2$ and $N = 2$. In this example, there are two PBSs located at $(5, 0.5)$ and $(5, -0.5)$ as well as two users located in between the two PBSs, creating a complicated situation for user association. More specifically, the locations of the two users are randomly generated within the square whose corners are $(5.1, 0.1)$, $(4.9, 0.1)$, $(4.9, -0.1)$, and $(5.1, -0.1)$. The results in Fig. 2 confirm the close-to-optimal performance of the proposed iterative algorithm, where the objective function of `MinCost` found by Algorithm 1 is sufficiently close to the optimal value found by the exhaustive search. Also, the maximum value of the auxiliary variables $s_{j,k,n}$ among all $j$, $k$, $n$ is 1.001, showing that Algorithm 1 converges to a KKT point of `MinCost-SGP`.

From Fig. 2(a), we can see that $\psi$ first increases and then saturates as $\pi_1$ increases, and the rate of increase of $\psi$ versus $\pi_1$ consistently reduces. This can be explained by Fig. 3(a), where PBS 1 serves less data as $\pi_1$ increases, and PBS 1 serves almost zero data when $\pi_1 > \pi_0$ and $\pi_1 > \pi_2$ given that PBS 1 has enough backhaul capacity.

Fig. 2(b) shows that when $\pi_1 < \pi_0$ and $\pi_1 < \pi_2$, $\psi$ decreases as $B_1$ increases. This observation is supported by Fig. 3(b), where the data rate served by PBS 1 increases as

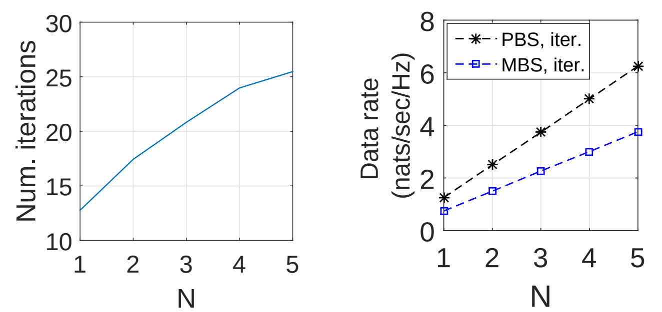



(a) Objective function of (8a) vs $B_1$




(b) Data rate at the BSs vs $B_1$

Fig. 3. Data rates at the BSs obtained from Algorithm 1, where $N = K = 2$, $\pi_0 = 1$, $\pi_2 = 0.5$, $B_1 = B_2 = 2$, and $R_1^{\text{rsv}} = R_2^{\text{rsv}} = 1$.




(a) (b)

Fig. 4. Evaluation of Algorithm 1, where $K/N = 2$, $\pi_0 = 1$, $\pi_n = 0.5$, and $B_n = 2$ for $n = 1, 2, ..., N$. Also, $R_k^{\text{rsv}} = 1$ for all $k$.

$B_1$ increases and the data rate served by the other two BSs decreases at the same time. Then, the total cost of the network reduces because the cost of PBS 1 is the lowest.

Fig. 4 examines the scenario where number of PBSs (i.e., $N$) increases in the network. In this example, the PBSs are randomly placed within the circle centered at the origin whose radius is five. Moreover, the closest normalized distance between two PBSs is 0.5, and the closest distance between the MBS and any PBS is 1. Two users are randomly placed in the vicinity of each PBS, where the normalized maximum and minimum distances between a user and a PBS are set to be 0.25 and 0.04, respectively. We can see that the number of iterations required by Algorithm 1 increases at a sub-linear rate with respect to $N$. In other words, let $t_{\text{conv}}$ be the number of iterations needed for Algorithm 1, then $t_{\text{conv}}$ is upper bounded by a multiple of $N$, or $t_{\text{conv}} = \mathcal{O}(N)$.

Moreover, based on [16], the number of required iterations for solving a GP using the interior-point method is $t_{\text{GP}} = \lceil \frac{\log(\phi)-\log(\epsilon\tau^0)}{\log(\mu)} \rceil$, where $\phi$ is the number of constraints, and $\epsilon$, $\tau^0$, and $\mu$ are parameters related to the accuracy of the interior-point method. The number of constraints in $\texttt{GP-Appx}(t)$ is counted as $\phi = 6KN + 5K + N$. Thus, for fixed $\epsilon$, $\tau^0$, and $\mu$ as well as a fixed ratio between $N$ and $K$, $t_{\text{GP}} = \mathcal{O}(\log(N))$. Together with the observation that $t_{\text{conv}} = \mathcal{O}(N)$, the total number of required iterations for Algorithm 1 using the interior-point method in the considered network setup is $\mathcal{O}(N \log(N))$. In other words, the total number of required iterations only grows quasi-linearly with $N$.

## V. Conclusion

In this paper, we have studied the problem of joint user association and power allocation on the uplink of a heterogeneous network, where dual connectivity is enabled. We have formulated the cost minimization problem from the network perspective, where a user can be served simultaneously by the MBS and one of the PBSs on different frequency bands. To solve the non-convex problem, we have proposed an iterative algorithm which guarantees reaching an optimal solution (locally or globally) after convergence. Simulation studies have verified the close-to-optimal performance of the proposed algorithm with affordable complexity.

## Appendix A
## Approximations on $\hat{\mathbf{C1}}_2$, $\hat{\mathbf{C3}}$, $\mathbf{C5}_2$, and $\mathbf{C6}_2$

Define the following new variables, i.e.,

$$\alpha_{k,n,j}(t) \triangleq \begin{cases} P_{k,n}(t)P_{k,j}(t), \ j \neq n, \\ 1, \ j = n, \end{cases} \tag{21}$$

$$\beta_{k,i}(t) \triangleq \frac{R^i_{k,0}(t)\sigma_0^2}{i! \cdot P_{k,0}(t) M d_k}, \ i = 1, 2, ..., U, \tag{22}$$

$$\gamma_{k,n,i,j}(t) \triangleq \begin{cases} \frac{R^i_{k,n}(t)P_{j,n}(t)|h_{j,n}|^2}{i! \cdot P_{k,n}(t)|h_{k,n}|^2}, \ j \neq k, \\ \frac{R^i_{k,n}(t)\sigma_n^2}{i! \cdot P_{k,n}(t)|h_{k,n}|^2}, \ j = k, \end{cases} \tag{23}$$

Then, by applying the AGMA in (12), the constraints in $\hat{\mathbf{C1}}_2$, $\hat{\mathbf{C3}}$, $\mathbf{C5}_2$, and $\mathbf{C6}_2$ can be approximated as

$$\tilde{\mathbf{C1}}_2: \ s^{-1}_{1,k,n}(t) \cdot \prod_{j=1}^{N} \left( \frac{\alpha_{k,n,j}(t)\zeta_{k,n}(t)}{\alpha_{k,n,j}(t-1)} \right)^{-\frac{\alpha_{k,n,j}(t-1)}{\zeta_{k,n}(t)}} \leq 1, \\ \forall k \in \mathcal{K}, \ \forall n \in \mathcal{N},$$

$$\tilde{\mathbf{C3}}: R_k^{\text{rsv}} \cdot \prod_{j=0}^{N} \left( \frac{R_{k,j}(t) \cdot \eta_k(t)}{R_{k,j}(t-1)} \right)^{-\frac{R_{k,j}(t-1)}{\eta_k(t)}} \leq 1, \ \forall k \in \mathcal{K},$$

$$\tilde{\mathbf{C5}}_2: \ s^{-1}_{2,k}(t) \cdot \prod_{i=1}^{U} \left( \frac{\beta_{k,i}(t)\theta_k(t)}{\beta_{k,i}(t-1)} \right)^{-\frac{\beta_{k,i}(t-1)}{\theta_k(t)}} \leq 1, \ \forall k \in \mathcal{K},$$

and

$$\tilde{\mathbf{C6}}_2: \ s^{-1}_{3,k,n}(t) \cdot \prod_{i=1}^{U}\prod_{j=1}^{K} \left( \frac{\gamma_{k,n,i,j}(t)\vartheta_{k,n}(t)}{\gamma_{k,n,i,j}(t-1)} \right)^{-\frac{\gamma_{k,n,i,j}(t-1)}{\vartheta_{k,n}(t)}} \\ \leq 1, \ \forall k \in \mathcal{K}, \ \forall n \in \mathcal{N},$$

respectively, where

$$\zeta_{k,n}(t) \triangleq \sum_{j=1}^{N} \alpha_{k,n,j}(t-1), \ \eta_k(t) \triangleq \sum_{j=0}^{N} R_{k,j}(t-1),$$

$$\theta_k(t) \triangleq \sum_{i=1}^{U} \beta_{k,i}(t-1), \text{ and } \vartheta_{k,n}(t) \triangleq \sum_{i=1}^{U}\sum_{j=1}^{K} \gamma_{k,n,i,j}(t-1).$$

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