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# The $\eta - \mu$ / Inverse Gamma Composite Fading Model

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**Abstract**—In this paper we propose a new composite fading model which assumes that the mean signal power of  $\eta - \mu$  signal envelope follows an inverse gamma distribution. This inverse gamma distribution has a simple relationship with the gamma distribution and can be used to model shadowed fading due to its semi heavy-tailed characteristics. To demonstrate the utility of the new  $\eta - \mu$  / inverse gamma composite fading model, we investigate the characteristics of the shadowed fading behavior observed in body centric communications channels which are known to be susceptible to the shadowing, particularly generated by the human body. It is shown that the  $\eta - \mu$  / inverse gamma composite fading model provided an adequate fit to the measurement data. Moreover, using Kullback-Leibler divergence, the  $\eta - \mu$  / inverse gamma composite fading model was found to provide a slightly better fit to the measured data than the  $\kappa - \mu$  / inverse gamma composite fading model, in certain communication scenarios.

## I. INTRODUCTION

In wireless communications channels, the random fluctuations of the received signal envelope can be attributed to multipath fading and shadowing fading [1]. The multipath fading is caused by the constructive and destructive interference between two or more versions of the transmitted signal over short distances and is generally modeled using the Rayleigh, Rice, Nakagami- $m$  and Weibull distributions [2]–[5]. Moreover, the generalized fading models, namely the  $\alpha - \mu$  [6], the  $\kappa - \mu$  and the  $\eta - \mu$  distribution models [7], have also been recently proposed. These models have proved their flexibility and versatility by including the majority of the aforementioned fading models as special cases. On the contrary, the shadowed fading is introduced by the topographical elements and the presence of obstructions in propagation path and is commonly modeled using the lognormal distribution [8]. The shadowing can be further divided into two types: the *line-of-sight (LOS)* shadowing and the *multiplicative* shadowing. The former, is caused by complete or partial obstacles between transmitter and receiver and refers to random variation of the amplitude of the LOS component. The latter renders the total power of

the multipath components including both LOS and scattered components a random variable [9].

In reality, both multipath fading and shadowing exist simultaneously and affect the wireless communication channels concurrently, reducing the quality of the radio link and degrading the overall performance. Therefore, it is inevitable to take into account the combined effect of both multipath fading and shadowing. To this end, composite fading models, which are also called shadowed fading models, have been proposed for different communications channels such as land mobile satellite communications channels [9]–[11] and body centric communications channels [12], [13]. Traditionally, composite fading channels have considered Rayleigh or Nakagami- $m$  signal envelopes in which the mean signal power is assumed to follow either a lognormal or gamma distribution. More recently, the  $\kappa - \mu$  signal envelope has also been studied. The  $\kappa - \mu$  fading model is an extremely versatile fading model which contains as special cases other important distributions such as the one-sided Gaussian, Rice (Nakagami- $n$ ), Nakagami- $m$  and Rayleigh distributions [7]. However, the  $\kappa - \mu$  fading model is mainly used to represent the multipath fading in a LOS conditions. For example in traditional cellular communications, it is difficult to achieve a LOS condition all the time. Therefore, in this paper, we consider the  $\eta - \mu$  fading model, which is used to represent multipath fading under NLOS conditions [7].

Motivated by this, there has been some published research on the  $\eta - \mu$  composite fading models [14], [15]. Specifically, the authors of [14], [15] have obtained a closed-form expression for the probability density function (PDF) of the  $\eta - \mu$  / gamma composite fading model which, however, is only valid for integer values of  $\mu$ . To generalize this, the authors of [15] provided an approximation of the PDF of the  $\eta - \mu$  / gamma composite fading model using an infinite series expansion. In this paper, we explore the use of the closely related inverse gamma distribution to represent the shadowing in  $\eta - \mu$  fading

channels. It is also noted that similar to the gamma PDF, the inverse gamma PDF can exhibit adequate semi heavy-tailed behavior. Furthermore, as it is subsequently shown, the proposed model allows us to derive a useful and relatively simple closed-form solution for the  $\eta - \mu /$  inverse gamma composite PDF without the need for infinite series expansion.

The remainder of this paper is organized as follows. Section II introduces the  $\eta - \mu /$  inverse gamma composite fading model and also illustrates the versatility of this model by highlighting a number of well-known distributions which exist as special cases. Section III describes the measurement set-up, environment and scenarios. In Section IV, the shadowed fading parameter estimates are presented alongside some example of model fitting. Furthermore, a useful comparison with the  $\kappa - \mu /$  inverse gamma composite fading model is provided in Section IV. Finally, Section V concludes the paper with some closing remarks.

## II. THE $\eta - \mu /$ INVERSE GAMMA COMPOSITE FADING MODEL

The PDF of the received signal envelope in an  $\eta - \mu /$  inverse gamma composite fading channel,  $R$ , is given as the integral of the conditional probability density of the  $\eta - \mu$  fading with respect to the random variation of the mean signal power  $\Omega$

$$f_R(r) = \int_0^\infty f_{R|\Omega}(r|\omega) f_\Omega(\omega) d\omega. \quad (1)$$

If we initially hold the mean signal power constant, then the PDF of the  $\eta - \mu$  composite fading channel proposed here can be expressed as

$$f_{R|\Omega}(r|\omega) = \frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^\mu r^{2\mu}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}} \left(\frac{1}{\sqrt{\omega}}\right)^{2\mu+1} \times \exp\left[-2\mu h\left(\frac{r}{\sqrt{\omega}}\right)^2\right] I_{\mu-\frac{1}{2}}\left[2\mu H\left(\frac{r}{\sqrt{\omega}}\right)^2\right] \quad (2)$$

which is practically the PDF of the  $\eta - \mu$  fading model given in [7] where

$$\mu = \frac{E^2(r^2)}{\text{Var}(r^2)} \left[1 + \left(\frac{H}{h}\right)^2\right] \quad (3)$$

or equivalently

$$\mu = \frac{1}{2\text{Var}(P^2)} \left[1 + \left(\frac{H}{h}\right)^2\right]. \quad (4)$$

Furthermore,  $I_\nu(\cdot)$  denotes the modified Bessel function of the first kind and order  $\nu$  whereas  $\eta$  is defined in two different formats according to two corresponding physical models as shown in Table I. Based on this, *Format 1* can be obtained from *Format 2* and vice versa by the relation

$$\eta_{\text{Format1}} = \frac{1 - \eta_{\text{Format2}}}{1 + \eta_{\text{Format2}}} \quad (5)$$

or, equivalently

$$\eta_{\text{Format2}} = \frac{1 - \eta_{\text{Format1}}}{1 + \eta_{\text{Format1}}} \quad (6)$$

where  $0 < \eta_{\text{Format1}} < \infty$  in *Format 1* and  $-1 < \eta_{\text{Format2}} < 1$  in *Format 2*.

Next, by also letting the corresponding mean signal power  $\Omega$  vary according to the inverse gamma distribution, the corresponding probability density function is given by

$$f_\Omega(\omega) = \frac{\beta^\alpha}{\Gamma(\alpha)} \omega^{-\alpha-1} \exp\left(-\frac{\beta}{\omega}\right) \quad (7)$$

where  $\Gamma(\cdot)$  is the gamma function,  $\alpha > 0$  is the shape parameter and  $\beta > 0$  is scale parameter. By substituting (2) and (7) into (1), the PDF of the received signal envelope in a  $\eta - \mu /$  inverse gamma composite fading channel,  $R$ , can be expressed as follows

$$f_R(r) = \frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^\mu\beta^\alpha r^{2\mu}}{\Gamma(\mu)\Gamma(\alpha)H^{\mu-\frac{1}{2}}} \times \int_0^\infty \left(\frac{1}{\sqrt{\omega}}\right)^{2\alpha+2\mu+3} \frac{I_{\mu-\frac{1}{2}}\left[2\mu H r^2 \left(\frac{1}{\sqrt{\omega}}\right)^2\right]}{\exp\left(\frac{2\mu h r^2}{\omega} + \frac{\beta}{\omega}\right)} d\omega \quad (8)$$

To this effect, by also setting  $y = 1/\omega$  and applying [16, eq. (4), pp. 270] along with some long but basic algebraic manipulation, it follows that

$$f_R(r) = \frac{4\sqrt{\pi}\mu^{2\mu}h^\mu\beta^\alpha r^{4\mu-1}}{(2\mu h r^2 + \beta)^{\alpha+2\mu}} \frac{\Gamma(\alpha + 2\mu)}{\Gamma(\mu)\Gamma(\alpha)\Gamma(\mu + \frac{1}{2})} \times {}_2F_1\left(\frac{\alpha + 2\mu}{2}, \frac{\alpha + 2\mu + 1}{2}; \mu + \frac{1}{2}; \left(\frac{2\mu H r^2}{2\mu h r^2 + \beta}\right)^2\right) \quad (9)$$

where  ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$  denotes the Gauss hypergeometric function.

It is evident that the PDF given in (9) is a rather general representation as it inherits all of the generality of the  $\eta - \mu$  fading model. Therefore, as shown in Fig. 1, it includes as special cases the Nakagami- $m$ , the Nakagami- $q$  (Hoyt) and the Rayleigh distributions. More specifically, when there is no shadowing of the mean signal power ( $\alpha \rightarrow \infty$ ), the  $\eta - \mu /$  inverse gamma composite fading model coincides with the  $\eta - \mu$  fading model. Likewise, the Rayleigh distribution can be readily deduced by setting  $\mu = 1$  and  $\eta = 0$  for large  $\alpha$  ( $\alpha \rightarrow \infty$ ). Similarly, the Nakagami- $m$  distribution can be obtained by letting  $\eta = 0$  or  $\eta \rightarrow \infty$  in *Format 1* with the  $\mu$  parameter becoming equivalent to the  $m$  parameter of Nakagami- $m$  distribution. It should also be noted that as  $\alpha \rightarrow \infty$  (in reality,  $\alpha$  becomes large), the mean signal power becomes deterministic and in this case the PDF given in (5) coincides with the  $\eta - \mu$  PDF given in [7] with  $\Omega = \beta(\alpha - 1)$ .

TABLE I  
SUMMARY OF TWO DIFFERENT FORMATS FOR THE  $\eta - \mu$  DISTRIBUTION [7]

	<i>Format 1</i>	<i>Format 2</i>
Physical Model	In-phase and quadrature components of the fading signal within each multipath cluster are assumed to be independent from each other and to have different powers.	In-phase and quadrature components of the fading signal within each multipath cluster are assumed to have identical powers and to be correlated with each other.
$\eta$	$0 < \eta < \infty$ , the scattered-wave power ratio between the in-phase and quadrature components of each cluster of multipath	$-1 < \eta < 1$ , the correlation coefficient between the scattered-wave in-phase and quadrature components of each cluster of multipath
$h$	$h = \frac{2+\eta^{-1}+\eta}{4}$	$h = \frac{1}{1-\eta^2}$
$H$	$H = \frac{\eta^{-1}-\eta}{4}$	$H = \frac{\eta}{1-\eta^2}$

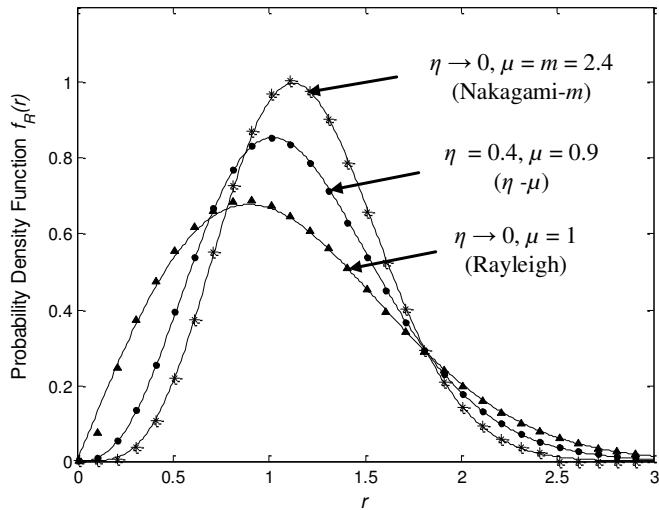


Fig. 1. PDFs of  $\eta - \mu /$  inverse gamma composite fading model (continuous lines) for special cases:  $\eta - \mu$  (circles), Nakagami- $m$  (asterisks) and Rayleigh (triangles) PDFs.

### III. FIELD MEASUREMENTS

This Section is devoted to the application of the proposed composite fading model in the context of the emerging field of body area networks (BAN). To this end, the measurements were performed at 5.8 GHz in the seminar room (7.92 m  $\times$  12.58 m) situated on the 1st floor of the Institute of Electronics, Communications and Information Technology (ECIT) at Queen's University Belfast in the United Kingdom as shown in Fig. 2(a). The ECIT building consisted of metal studded dry wall with a metal tiled floor covered with polypropylene-fiber, rubber backed carpet tiles, and metal ceiling with mineral fiber tiles and recessed louvered luminaries suspended 2.70 m above floor level. The seminar room contained a large number of chairs, desks, a projector and a white board. It also featured an external facing boundary constructed entirely from glass with some metallic supporting pillars. The seminar room was unoccupied for the duration of the experiments.

The transmitter consisted of an ML5805 Frequency Shift Keyed (FSK) transceiver manufactured by RFMD and was configured to generate a continuous wave signal with an output power of +21 dBm. It was positioned on the central waist

TABLE II  
PARAMETER ESTIMATES FOR THE  $\eta - \mu /$  INVERSE GAMMA COMPOSITE FADING MODEL

	$\eta$	$\mu$	$\alpha$	$\beta$
<b>1 m</b>	0.09	0.43	1.04	1.91
<b>9 m</b>	2.52	0.36	6.20	15.00

region of an adult male of height 1.83 m and weight 73 kg, where it is a possible mounting point for a gateway node in body area network. For the receiver section, an antenna was positioned on a non-conductive pole at a height of 1.10 m above the floor level so that it was vertically polarized. It was then attached using a low-loss coaxial cable to port 1 of a Rohde & Schwarz ZVB-8 vector network analyzer (VNA). To eliminate the effects of power amplifier and cable loss, a pre-measurement calibration was conducted using a Rohde & Schwarz ZV-Z51 calibration unit. The VNA was configured to record the  $b_1$  wave quantity with a sample rate of 56 Hz. Both the transmitter and receiver utilized identical omnidirectional sleeve dipole antennas with +2.3 dBi gain (Mobile Mark model PSKN3-24/55S). The measured azimuthal radiation patterns for the sleeve dipole antenna for the central waist region was shown in Fig. 2(b). In this study we considered measurements of the off-body communication channel when the test subject moved randomly within a circle with a radius of 1 m at separation distances of 1 m and 9 m from the receiver.

### IV. NUMERICAL RESULTS

The parameter estimates for  $\eta$ ,  $\mu$ ,  $\alpha$  and  $\beta$  parameters of the  $\eta - \mu /$  inverse gamma fading model were obtained using a non-linear least squares routine programmed in MATLAB in order to fit (9) to the corresponding field measurements. To allow a direct comparison between the shadowed fading characteristics, the global mean signal power was removed from the measurement data for the random movement scenario.

As an example of the results of the model fitting, the PDFs of the  $\eta - \mu /$  inverse gamma composite fading model fitted to the measurement data are shown in Fig. 3. As we can see, the  $\eta - \mu /$  inverse gamma composite fading model provided a good fit to the measurement data. Also, the estimated shadowing of

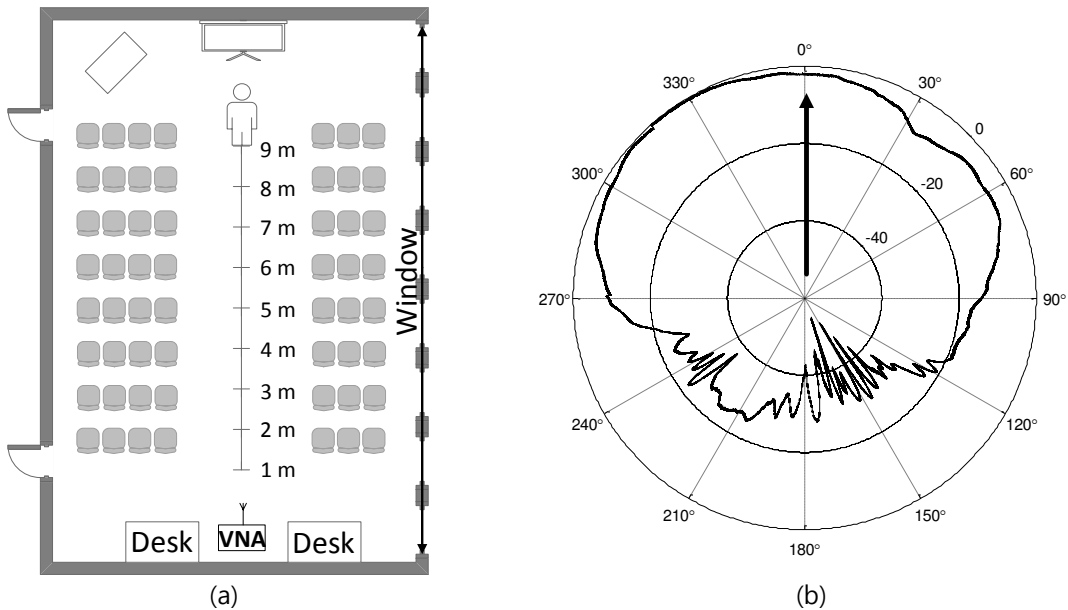


Fig. 2. (a) Indoor seminar room environment (99.63 m<sup>2</sup>) and (b) the measured azimuthal radiation pattern for the sleeve dipole antenna located on the central waist region. Please note that the black arrow in (b) denotes the direction that the test subject was facing.

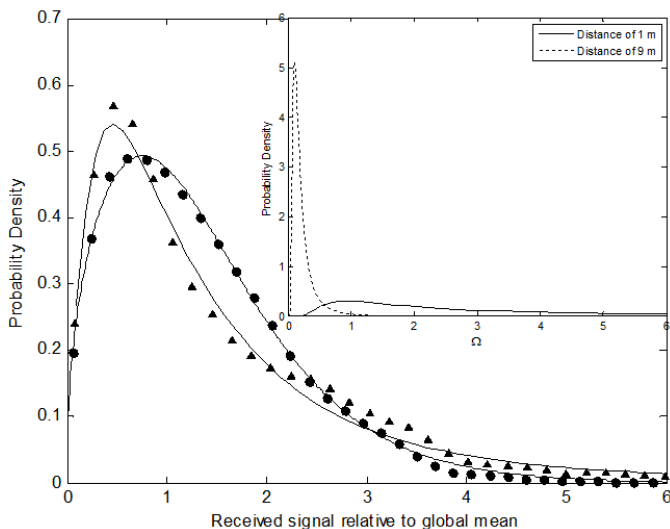


Fig. 3. Empirical (symbols) and theoretical probability densities for the random (continuous line) measurements at separation distances of 1 m (triangles) and 9 m (circles) from the receiver. Also shown inset is the estimated shadowing of the mean signal power. All parameter estimates are given in Table II.

the mean signal power is shown inset in Fig. 3. As expected, significant shadowed fading of the mean signal power was observed, because as the human body moves, it acts as a shadowing object in the channel. The channel between body worn node and receiver recorded the greater spread of the mean signal power at 1 m compared to that for 9 m. Moreover, as shown in Table II, the parameter estimate  $\alpha$  increased as the distance from the receiver increased from 1 m to 9 m, which

	1 m	9 m
$\eta - \mu$ / inverse gamma	0.2837	0.2037
$\kappa - \mu$ / inverse gamma	0.3158	0.2047

means there was less shadowing of the mean signal power.

To allow the reader to reproduce their own simulated data based on the presented empirical data, Table II provides the parameter estimates for all of the user movement scenarios. The Kullback-Leibler divergence [17] between empirical and their theoretical PDFs was also determined and is depicted in Table III for comparing the  $\eta - \mu$  / inverse gamma fading model with the  $\kappa - \mu$  / inverse gamma fading model [18]. It is evident that the  $\eta - \mu$  / inverse gamma composite fading model provided a better fit to the measurement data for the considered scenarios which is due to the fact that not only LOS conditions, but also NLOS conditions exist while the test subject moved in a random manner.

## V. CONCLUSION

A novel shadowed fading distribution was proposed, namely the  $\eta - \mu$  / inverse gamma composite fading model. In this fading model, it assumed that the mean signal power of an  $\eta - \mu$  signal envelope follows an inverse gamma distribution. A preliminary empirical validation of this model for use in body centric communication channels was also performed. To this end, real-time measurements were performed in an indoor

seminar room along with considered random movements at two different separation distances from the receiver. Excellent agreement was found between the  $\eta-\mu$  / inverse gamma model and the field measurements confirming its useful for shadowed body centric communication channels. Finally, when compared with the  $\kappa-\mu$  / inverse gamma composite fading model using Kullback-Leibler divergence, it has been shown that the  $\eta-\mu$  / inverse gamma composite fading model provided a better fit to the measurement data for the considered scenarios. To further validate this model, for use in the modeling of shadowed fading, future work will widen the experimental aspect of this study to consider a diverse range of fading channels such as those found in cellular device-to-device communications and vehicular communications.

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