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Parametric analysis of the potential of energy harvesting from commercial vehicle suspension system

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Abstract

An accurate estimation of the harvestable energy from a vehicle suspension under typical operating conditions is vital for design and implementation of efficient energy harvesters in vehicles. In this study, a generic three-dimensional model of a commercial vehicle is formulating by integrating nonlinear models of suspension components and tires to determine the harvestable power considering the effects of suspension parameters and road characteristics. The component characteristics of the suspension system and tires are obtained through the reported laboratory-measured data acquired under an extensive range of loadings conditions. The vehicle model is subsequently employed to investigate the harvestable energy potential considering variations in the driving speed, chassis load, road waviness and roughness, suspension and tire stiffness, compression mode damping ratio, and asymmetric suspension damping over the most possible ranges of running conditions. The results suggested significant influences of these parameters, while the driving speed, damping asymmetry factor, compression mode damping ratio and road condition revealed the most pronounced effect on the harvestable power. The results obtained in terms of RMS and PSD of harvestable power are also indicative that rough terrains yield incomparably larger magnitudes of energy dissipation than relatively smooth road classes defined in ISO 8608:1995, and thereby suggestive of the greater potential of energy recovery from commercial vehicles on off-road surfaces.

Keywords: sensitivity analysis; suspension damping; asymmetric damping; harvestable power; off-road terrain

1. Introduction

Vibratory energy is a prevalent source of energy in a broad range of physical phenomena and mechanical systems [1]. Consequently, vibratory energy harvesters have captured a lot of attention during the recent years [2]. The mechanical energy dissipated via heat waste through the suspension dampers has been reported as an interesting application for the vibratory energy harvesters. The implementation of an energy harvester in the vehicle suspension may be considered feasible provided that the suspension offers reasonable amount of vibratory energy under typical operating conditions. It has been estimated that recycling of 30% of the dissipated power of a vehicle suspension could yield about 0.8% reduction in the fuel consumption in cars. A recent study has suggested that implementation of regenerative suspensions in only 10% of the light cars can translate in 8.2 million liters reduction in consumption of gasoline and 43.2 ton reduction in pollutants in Canada [3]. Although scalability and miniaturization [2], dynamics of circuitry and device efficiency subject to parametric uncertainties are critical design factors, the key
challenging question arises - how much is the energy recovery potential of the suspension system under typical operating conditions? The answer to this pivotal question renders to discover the limitations, applicability and cost effectiveness of the energy harvester exploitation in a vehicular suspension system. In some circumstances, either the insufficient vibration amplitude [4] or the excitation frequencies [5] greatly limit the feasibility of an energy harvesting system. Considerable efforts have thus been made to establish a better understanding of the energy harvesting limits, potentials and feasibility [5, 6].

The reported studies show large discrepancies with regard to the harvestable power of vehicle suspensions, which report harvestable power ranging from a low value of 46 W to as high as 7500 W [6]. Such wide variations in the harvestable power can be attributed to the broad ranges of design and operating factors considered, including the vehicle type, test or operating conditions, and analysis approach. The reported studies have mostly focused on relations of the harvestable power with the driving speed and road roughness, ranging from class A to D, as defined in ISO-8606 [28]. Zuo and Zhang [7] showed that the potential of energy harvesting from a passenger car ranges from 100 to 400 W when operating on good and average roads at a constant forward speed of 96.5 km/h. Segel and Lu [8] investigated the effect of highway pavement roughness on energy loss due to tire and suspension damping and concluded that about 200 W of power is dissipated by dampers of a passenger car at speed of 48.2 km/h. Using a quarter-vehicle model, Sulton et al. [9] showed that peak and average power of 45 and 11.43 W can be recaptured from a vehicle operating on a class C road at a speed of 50 km/h. Xie and Wang [10] achieved maximum power recovery of up to 738 W from a quarter-car model with a piezoelectric beam based energy harvester. The study reported RMS power in the 17–40 W, 70–160 W and 280–660 W ranges, when operating on class B, C and D roads, respectively, at speeds ranging from 54 to 126 km/h. Another study reported average dissipated power of 150 W, 3 W and 613 W when operating on typical roads in Germany, ranging from a newly paved highway to a rugged country road [11].

Apart from the road roughness, a few studies have investigated the suspension energy dissipation associated with different vibration modes of the vehicle. Khoshnoud et al. [12] used a vehicle model to determine the required power related to the bounce, pitch and roll modes of the vehicle, and presented model verification using the experimental data. The study reported maximum theoretical power of 1.1 kW, 0.88 kW and 0.97 kW related to bounce, pitch and roll oscillations of the vehicle, respectively, under a constant excitation frequency of 20 Hz. Such an excitation frequency, however, is substantially higher than the typical ride frequencies encountered on roads. Bazios et al. [13] explored the condition for self-powered operation of an active suspension system numerically considering a 7-degrees-of-freedom (DOF) vehicle model with a combination of PID, LQR and Fuzzy control algorithms. The study reported that the average power of a single wheel actuator could approach 8 MW in a limited frequency band, when operating on a class B road at a speed of 60 km/h. The reported power is well beyond the ranges of harvestable energy reported in the literature [6]. Ataei et al. [3] solved the optimization problem for coupled ride, road holding and energy harvesting by using a hybrid hydro-electromagnetic suspension system and reported the RMS power of 15–32 W, when operating on a class C road at a speed of 50 km/h. Huang et al. [14] developed a vehicle suspension setup with energy harvesting capability together with a theoretical model. An electronic circuit was employed to generate damping variations using an optimization rule and for applying limits on the forward speed when travelling on lower-quality roads. The study reported peak values of RMS harvestable power of 11, 25, 30 and 45 W at speeds of 120, 90, 50 and 30 km/h, respectively, when operating on road classes A to D. Mapelli et al. [15] employed a
linear permanent magnet alternator in a road vehicle suspension system and obtained mean dissipated power in the 3.15-5.58 W, 6.36-15.97 W and 12.75-37.98 W ranges for good, average and poor roads, respectively, with speeds ranging from 20 to 140 km/h depending on the road class. Singh and Satpute [16] simulated a quarter-car model with an electromagnetic hydraulic shock absorber and obtained peak and average power of 94 W and 60 W, respectively, on a smooth city road at a speed of 35 km/h.

The vast majority of the studies have focused on the effects of road roughness and driving speeds on the harvestable power of passenger cars, while the effects of vehicle parameters have been addressed in only a few studies. Zuo and Zhang [3] explored the effects of stiffness and damping properties of the suspension and tires, and the wheel and chassis masses on the harvestable power using a 2-DOF linear model of a passenger car. Huang et al. [14] evaluated the effects of suspension stiffness and damping coefficients on the harvestable power considering the 2-DOF linear car model. Wei et al. [17] studied the pitch and heave motions of the sprung mass, while the effects of suspension system parameters were not considered.

The reported studies have been mostly limited to simple models of road vehicles, particularly, the passenger cars, with linear suspension properties. These may not accurately describe the energy dissipation properties of the suspension, which invariably employ nonlinear dampers with asymmetric properties in compression and rebound. Moreover, these show widely different magnitudes of harvestable power. Further comprehensive analyses are thus essential to establish an understanding of the roles of vehicle design/tuning parameters on the amount of dissipated power. Additionally, energy harvesting potentials of commercial off-road and road vehicles have not yet been thoroughly explored. The efforts could be directed towards energy harvesting from the vehicles with greater mass, relatively soft suspension with high damping operating on irregular terrains such as commercial vehicles where larger magnitudes of energy can be recovered [18].

In the present study, the effects of different design and operating factors on the energy harvest potentials of commercial vehicles are investigated. These include the stiffness and damping properties of the suspension, tire stiffness, chassis load, driving speed and terrain roughness. A generic three-dimensional model of the vehicle integrating nonlinear damping characteristics of the suspension is used to study the effects of different design and operating parameters. The simulation results are analyzed to highlight energy harvest potentials considering different operating conditions, which would provide essential guidance on the design as well as feasibility of an energy harvesting suspension system.

2. Model development

The energy dissipation properties of a vehicle suspension are related to the bounce, pitch and roll motions of the vehicle, and closely affected by system dynamics and operating parameters, when subject to road excitations [19]. A generic three-dimensional model of a commercial vehicle is formulated for the analysis that deals with the substantial vehicle vibration modes, and the influences of different system parameters and operating factors on the dynamic responses such as driving speed, road roughness, tire inflation pressure, suspension properties and the vehicle load. Furthermore, a variable stiffness based tire model is also included to represent the effect of variations in tire inflation pressure. Additionally, road characteristics can potentially affect the vehicle dynamic response. Road waviness and roughness are the contributory factors in the intensity of the wheel hop, suspension deflection and thereby the amount of
dissipated energy. Furthermore, a wide range of smooth to rugged road surface conditions is considered to investigate the harvestable power potentials from a commercial vehicle suspension system.

2.1. Analytical vehicle model

A generic three-dimensional 7-DOF vehicle model is developed to consider the pitch and roll motions in addition to the heave motions of the sprung and unsprung masses by integrating the nonlinear suspension system components. The model includes the sprung and unsprung masses where the flexibility of the chassis structure is assumed negligible. The suspension component characteristics were experimentally derived from the laboratory tests through an extensive range of deflection and loading conditions [21]. The nonlinear suspension components are the asymmetric suspension damping properties during the compression and rebound and the nonlinear suspension spring stiffness with cubic hardening.

The typical suspension dampers generate high damping coefficients, \( C_{c1} \) in compression and \( C_{e1} \) in rebound, at low speeds due to bleed flows. The damping coefficients decrease considerably to \( C_{c2} \) in compression and \( C_{e2} \) in rebound at higher velocities due to the reduced flow resistance of the damping valves in order to achieve improved ride quality [20]. Moreover, the damping coefficients in compression and rebound are highly asymmetric. Such multi-stage force–velocity characteristics of the suspension damper can be defined by a generalized piecewise-linear model, shown in Fig. 1. A typical multi-stage asymmetric piecewise damping force of the suspension system is thus formulated as follows.

\[
F_c = \begin{cases} 
C_{c1}V ; & \text{for } 0 \leq V \leq V_e \\
C_{e1}[(V_e)^+ + \gamma_e (V_e - V)] ; & \text{for } V_e \leq V \\
pC_{c1}V ; & \text{for } V_e \leq V < 0 \\
pC_{e1}[(V_e)^+ + \gamma_e (V_e - V)] ; & \text{for } V \leq V_e 
\end{cases}
\]  

(1)

The respective damping coefficients for rebound and compression are represented by \( C_{c1}, C_{e1}, C_{c2}, C_{e2} \), with the corresponding transition velocities, \( V_e \), and \( V_c \), in rebound and compression modes. In the above model, \( p, \gamma_e \), and \( \gamma_c \) are dimensionless asymmetry and damping reduction factors, respectively, given by:

\[
p = \frac{C_{e1}}{C_{c1}}; \quad \gamma_c = \frac{C_{c2}}{C_{c1}}; \quad \gamma_e = \frac{C_{e2}}{C_{c1}}
\]  

(2)

The compression and rebound mode reduction factors as well as the asymmetric factor are used to obtain the rebound mode and the second stage damping coefficients with reference to the compression mode damping coefficients for the front and rear axles’ suspension dampers. The front and rear suspension damping is further represented by the respective compression-mode damping ratios of the front and rear suspensions, \( \xi_{f1} \) and \( \xi_{r1} \), defined as the ratio of \( C_{c1} \) to the uncoupled vertical mode critical damping coefficients of the sprung mass supported by the suspension[20], such that:

\[
\xi_{c\cdot f} = \frac{2C_{c1}}{2\sqrt{k_{sf}M_{sf}}}; \quad \xi_{c\cdot r} = \frac{4C_{c1}}{2\sqrt{k_{sr}M_{sr}}}
\]  

(3)

where \( k_{sf} \) and \( k_{sr} \) are the linearized equivalent vertical spring rates due to front and rear axles suspensions about the operating point, and \( M_{sf} \) and \( M_{sr} \) are the portions of the sprung mass supported by the front and
rear suspensions, respectively. Table 1 summarizes the optimal damping properties of the front and rear-axle suspension dampers for a commercial vehicle, as reported in [20]. The reported results were suggestive that the optimal damping design, based on 20% low-speed compression damping ratio, provides a considerable potential for improvement of driver- and road-friendliness of the commercial vehicle and thereby these damping parameters are adopted as nominal parameters for the generic vehicle model in the present study. The nonlinear forces due to tires, similar to the suspensions system components, are derived from the component models [22], where the force-displacement characteristic of tire is indicative of a progressively hardening stiffness of quadratic nature. The tire damping characteristic is excluded from the model due to the negligible magnitude of tire damping compared to the suspension damping. Additionally, the tire was considered as a contact-point tire model constrained to remain in ground contact without the wheel hop.

![Figure 1. A generic piecewise-linear representation of the multi-stage force-velocity characteristics of typical hydraulic suspension dampers](image)

**Table 1. Damping properties derived from objective optimal damper model [20].**

<table>
<thead>
<tr>
<th>Axle Suspension</th>
<th>Damping Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\xi_c$</td>
</tr>
<tr>
<td>Front Axle Damper</td>
<td>0.2</td>
</tr>
<tr>
<td>Rear Axle Damper</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The DOFs in developed model include: vertical ($z_v$), pitch ($\theta$) and roll ($\phi$) motions of the sprung mass, and vertical motions of the unsprung masses ($z_{ufr}$, $z_{ulf}$, $z_{urr}$, and $z_{url}$). The subscripts fr, fl, rr and rl refer to front-right, front-left, rear-right and rear-left suspensions, respectively. The 7-DOF full-vehicle model is schematically illustrated in Fig. 2. The developed equations of motion concerned with the vehicle model, which include the nonlinear force-deflection and force-velocity characteristics of the suspension components and tires, are summarized below:
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\begin{equation}
M_i \dddot{z}_i + \sum_{i,j}(F_{cij} + F_{kij}) = 0
\end{equation}

\begin{equation}
I_\phi \dddot{\phi} - \frac{w}{2} \left[ \sum_j (F_{cij} + F_{kij} - F_{cil} - F_{km}) \right] = 0
\end{equation}

\begin{equation}
I_\theta \dddot{\theta} - a \sum_j (F_{cij} + F_{kij}) + b \sum_j (F_{cij} + F_{kij}) = 0
\end{equation}

\begin{equation}
m_i \dddot{z}_{uij} - \sum_{i,j} (F_{cij} + F_{kij} - F_{tij}) = 0 \quad i = f, r; j = r, l
\end{equation}

where the first subscript \((f, r)\) refers to front- or rear-axle tire, and the last subscript \((l, r)\) refers to left- or right-hand tire. The body and chassis mass, front-axle and rear-axle assemblies are represented by \(M_s, m_{uf}\) and \(m_{ur}\), respectively, \(a\) and \(b\) denote the distances between \(C.G.\) of the chassis to the front-axle and rear-axle, respectively, and \(w\) shows the vehicle track width (Fig. 2). \(I_\phi\) and \(I_\theta\) are the roll and pitch mass moments of inertia of the sprung mass about the mass center, respectively. The forces developed by suspension dampers \((F_{cij}; i = f, r\) and \(j = r, l\)), suspension springs \((F_{kij}; i = f, r\) and \(j = r, l\)) and tires \((F_{tij}; i = f, r\) are presented in the Appendix. The inertial and geometric parameters of the vehicle are extracted from [19] and the optimal suspension parameters used in the simulation are those presented in Table 1 [20]. The vehicle speed range, 10 to 70 km/h, used in the modeling also complies with the range employed in the reported study [19].

![Figure 2. Three-dimensional representation of the 7-DOF vehicle model](image)

2.2. Terrain model

The model simulations are performed considering excitations arising from road as well as off-road terrain. The representation of off-road terrains, in-particular, is a difficult task due to the stochastic spatial variations in the terrain elevation and local changes in soil properties together with the number of contributory factors such as multipass effect and terrain compaction degree [23-25]. However, it is known that the road surface elevation features being spatially stationary and ergodic, and thereby can be
characterized by the spatial power spectral density (PSD) of the roughness. In order to analyze the vehicle response on off-road terrain, the surface is typically defined through an equivalent undeformable road [26-27]. International standards organization (ISO) has also defined the relationships between PSD of roughness of various classes of paved or undeformable surfaces as a function of spatial frequency [28]:

\[
S_g(\Omega) = \begin{cases} 
S_g(\Omega_0)(\Omega/\Omega_0)^{-N_1}; & \Omega \leq \Omega_0 \\
S_g(\Omega_0)(\Omega/\Omega_0)^{-N_2}; & \Omega > \Omega_0 
\end{cases} \tag{8}
\]

where \(S_g(\Omega)\) is PSD of road surface elevation, \(S_g(\Omega_0)\) is a roughness constant defined for different classes of roads which is related to the spatial frequency \(\Omega\), and \(N_1\) and \(N_2\) are waviness coefficients of the terrain profile and \(\Omega_0\) is a constant reference frequency. The spatial PSD of various undeformable surfaces from a smooth highway (class A) to a very rough road (class E) with \(\Omega_0=0.1\) cycles/m and constant waviness coefficients of 2.0 and 1.5 for \(N_1\) and \(N_2\), respectively, has been defined by ISO-8608 (1995) [28].

The roughness characteristics of different undeformable off-road surfaces have also been described through the power function as [31-32].

\[
S_g(\Omega) = C_{sp}\Omega^{-N}; \quad C_{sp} > 0 \text{ and } N > 0 \tag{9}
\]

where \(C_{sp}\) and \(N\) denote the roughness and waviness coefficients of the terrain, respectively, that are basically obtained from the measured terrain profiles. The above relation has also been used to describe roughness distributions of various road surfaces. Table 2 lists the reported values of \(C_{sp}\) and \(N\) for different road and off-road surfaces including the smooth and rough runways.

Power spectral density as a function of spatial frequency for various road types and runways is illustrated in Fig. 3. The results show greater roughness of the unpaved terrains compared to the paved roads, as it would be expected. The off-road terrains exhibit PSD of roughness very close to that of the grade E road defined in ISO-8608, 1995 [28]. The grade E spectrum may thus be considered representative of an off-rough terrain. The spatial PSD of the terrain roughness could also be described by a PSD function in temporal frequency and vehicle speed to synthesize the time domain signal of the terrain roughness using a random white noise signal of unity power [29-30].

<table>
<thead>
<tr>
<th>Road surfaces</th>
<th>(N)</th>
<th>(C_{sp})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth runway</td>
<td>3.8</td>
<td>4.3×10¹¹</td>
</tr>
<tr>
<td>Rough runway</td>
<td>2.1</td>
<td>8.1×10⁶</td>
</tr>
<tr>
<td>Smooth highway</td>
<td>2.1</td>
<td>4.8×10⁷</td>
</tr>
<tr>
<td>Highway with gravel</td>
<td>2.1</td>
<td>4.4×10⁶</td>
</tr>
<tr>
<td>Pasture</td>
<td>1.6</td>
<td>3×10⁻⁴</td>
</tr>
<tr>
<td>Plowed field</td>
<td>1.6</td>
<td>6.5×10⁻⁴</td>
</tr>
</tbody>
</table>
The differences in the elevations of two tracks of a road/terrain also impose roll excitation to the vehicle, which is known to be appreciable in case of off-road terrains. The reported terrain elevations, however, are considered to describe the mean profile of a terrain. The elevations of the two track profiles can be synthesized considering high correlation between the two at lower frequencies. The roughness of the two tracks of the terrain, $l(x)$ and $r(x)$, in this study are synthesized using two unity power band-limited white noise signals with zero mean together with the a frequency response function of the terrain and coherency function, as described in [27, 28, 30]. The coherency between the two tracks $\mu^2(\Omega)$ determines the roll excitations due to differences in elevations of the two tracks, which are defined as a function of the longitudinal coordinate $x$. The coherence function is defined as [28]:

$$\mu^2(\Omega) = \frac{G^2_{lr}}{G^2_{L(\Omega)}G^2_{R(\Omega)}}$$

where $G^2_{lr}$ is the cross-spectral density function of the left- and right-track elevations, $l(x)$ and $r(x)$, and $L(\Omega)$ and $R(\Omega)$ are the PSD spectra of $l(x)$ and $r(x)$, respectively. The frequency response functions $\sqrt{\mu^2(\Omega)}$ and $\sqrt{1-\mu^2(\Omega)}$ define the correlated and uncorrelated components of the left- and right-track profiles, respectively [27]. It has been reported that rough terrain tracks exhibit high coherency at low frequencies up to 0.3 cycles/m depending on the vehicle speed, while it diminishes in a nearly linear manner with increase in the frequency, such that [27,33];

$$\mu^2(\Omega) = \begin{cases} 1 - \frac{0.9\Omega}{\Omega_c}, & \Omega \leq \Omega_c \\ 0.1, & \Omega > \Omega_c \end{cases}$$

where $\Omega_c$ is the cut-off spatial frequency, which is taken as 0.3 cycles/m. Moreover, the terrain elevations are synthesized as a function of spatial frequency in the 0.02–5 cycles/m range, which corresponds to excitation frequencies in the 0.03–70 Hz range for forward speed of 50 km/h.

### 3. Analysis of Harvestable Power

The maximum harvestable energy is the energy consumed or dissipated by the suspension damper. The instantaneous damping force is directly related to relative velocity across the suspension damper, which is function of vertical, roll and pitch velocities of the vehicle masses. For instance, the relative velocity of

Figure 3. Comparisons of spatial power spectral density spectra of different terrains [25, 31]
The front suspension damper is obtained from: \( \ddot{z}_s - \dot{z}_{ufr} - a\dot{\theta} - \frac{w}{2}\dot{\phi} \) for front right suspension damper and the instantaneous power is subsequently obtained from the product of the damping force, Eq. (1), and relative velocity. The RMS value of the dissipated power \( P_{ms} \) can be obtained from:

\[
P_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} P(t) \, dt}
\]

where \( P(t) \) is the instantaneous power and \( T \) is the observed duration. Moreover, it is possible to present the PSD of harvestable power, \( S(f) \), based on the Wiener-Khinchin theorem considering frequency transformation of auto-correlation of the harvestable power, \( P_a(t) \), as follows:

\[
S(f) = \int_{-\infty}^{\infty} P_a(t) e^{-j2\pi ft} \, dt
\]

where the auto-correlation function is defined considering a time lag \( \tau \), such that:

\[
P_a(t) = \int_{-\infty}^{\infty} P(\tau)P(\tau - t) \, d\tau
\]

The Fourier transform of the above yields PSD of the power. The total harvestable power of the vehicle is subsequently obtained from summation of the RMS values of the power dissipated by all the suspension dampers considering 10s simulation duration. The nominal simulation parameters are defined in Section 4 where the sensitivity analysis of the parameters is also presented.

The energy harvest potential of a vehicle is strongly related to many factors such as speed, terrain properties, vehicle mass and suspension properties, in a highly coupled manner. A sensitivity analysis is performed to study the effects of main factors (speed, terrain roughness, tire stiffness, and suspension stiffness and damping) on the total harvestable power. Sensitivity analysis describes how the changes in the output of a model can be apportioned to different input parameter variations [35]. Mathematically, the sensitivity of the model output to definite parameters can be considered as the partial derivative of the output function against each of the model parameters. For simple functions a local technique is efficient where all derivatives are taken at a single point. However, this approach is not feasible for nonlinear models [36] as the local sensitivity analysis is a one-at-a-time (OAT) method to evaluate the influence of only one parameter on the model output. A drawback of OAT approach is that it neglects the influence of interactions among the parameters on the model output. Alternatively, the global sensitivity analysis method is employed using the Monte Carlo technique. Monte Carlo technique generates a global set of samples to investigate the design space and performs risk analysis by constructing the models of possible results through the substitution of a range of values considering a probability distribution for each model parameter. During a Monte Carlo simulation, values are sampled randomly from the input probability distributions that can take different forms including uniform, normal, logsig, triangular, etc. Each set of samples is known as iteration and the resulting outcome from that sample is stored during simulation runs. Monte Carlo simulation repeats the process several times, and the final result is a probability distribution of possible outcomes. The driving speed, road waviness, road roughness, damper asymmetry factor, compression mode damping ratio, damping reduction factor, sprung mass, and tire and suspension
spring hardening ratios are considered as the main factors for the sensitivity analysis, since these factors are known to effect the vehicle vibration response considerably.

4. Results and Discussions

Figure 4 illustrates comparisons of the RMS of harvestable power derived from the model considering five different road profiles, namely, smooth and rough runways, a smooth highway, a gravel road and the plowed field (Table 1). The RMS power is computed for different operating speeds in the 10 to 70 km/h range. It can be inferred that the RMS harvestable power increases with an increase in the driving speed in a nearly quadratic manner. This trend is more clearly evident for relative rough terrains. This tendency is also observed for all the roads considered, while the power lies in a relatively smaller range for smoother road surfaces. With increase in driving speed from 10 to 70 km/h yields nearly 4.6 times increase in harvestable power for the rough terrain, while the increase for the smooth roads is up to only 1.5 times the power obtained at the low speed of 10 km/h. A quadratic increase in harvestable power with respect to driving speed has also been reported in [12,17].

![Figure 4. Influence of variations in driving speed on the RMS harvestable power for different terrains](image)

The above results are indicative of distinctly different harvest potential of the vehicle for different terrain conditions. The results suggest greater feasibility of the energy harvesting systems for off-road vehicles than their road counterparts. It should be also noted that the simplified point-contact tire model using in the study may provide a relatively higher magnitude of the harvestable power when compared with the tire models involving enveloping of the tire over the terrain. The tire response in point contact tire models is quite sensitive to the road waviness and roughness especially to the high frequency components of the road input, which is generally filtered through finite contact patch tire enveloping models [37].
Apart from the terrain roughness, the energy harvesting potential of the vehicle is further related to waviness of the terrain, which directly relates to dominant frequencies of the suspension velocity response. Fig. 5 presents the influences of variations in waviness and roughness coefficients of the terrain on the harvestable power at a driving speed of 60 km/h, while all other parameters are held at their nominal values. The roughness coefficient axis is presented in logarithmic scale so as to illustrate the influence of road roughness more clearly. The road roughness coefficient is varied from $10^{-4}$ to $10^{-8}$ m$^3$/cycle, which would represent elevations of a broad range of terrains. A smooth road with very low roughness coefficient would yield negligible power, irrespective of the road waviness. The RMS power increases with increase in both the roughness coefficient and the waviness, and it approaches a peak value of 6531 W under a rugged terrain with high waviness. Roughness coefficient is directly related to the magnitude of road elevation, which directly affects suspension deflection and thereby the amount of stored energy of the suspension springs.

Figure 6. Variations in RMS harvestable power under the effect of: a) suspension stiffness hardening ratio and b) tire stiffness hardening ratio at different driving speeds of 25, 45 and 60 km/h (Terrain: Plowed field)

The harvestable power is also related to vibration properties of the vehicle, especially the resonant frequencies, which are directly related to vehicle mass, and suspension and tire stiffness. Suspension springs are invariably designed to yield low vertical and pitch mode natural frequencies of the vehicle. In this study, the progressively hardening property of each suspension spring is modeled using a linear and cubic function of the suspension deflection $\delta$, such that the spring force $F_k = k_1\delta + k_3\delta^3$. The effect of
changes in the suspension stiffness on the harvestable power is investigated by varying the suspension hardening ratio, while not altering the linear component. The hardening ratio is defined as the ratio of the cubic stiffness coefficient to the linear stiffness coefficient \( (k_3/k_1) \). Figure 6(a) illustrates the influence of variations in hardening ratio of the suspension spring on the RMS harvestable power at different forwards speeds, ranging from 25 to 60 km/h. The results are obtained for excitation arising from a plowed field. The results suggest important effect of the suspension hardening ratio on the harvestable power for all the speeds considered. Increasing the hardening ratio would yield higher oscillation frequencies of the vehicle and thereby higher suspension velocity and power, especially under relatively rough terrains. Figure 6(b) illustrates the effect of tire stiffness on the RMS harvestable power. The tire stiffness variation is also expressed by the ratio of quadratic stiffness constant to the linear tire stiffness constant \( k'_2/k'_1 \). The results show notable increase in the harvestable power with increase in the quadratic stiffness constant of the tire. This is likely due to higher wheel hop motion of the relatively stiff tire, which would result in greater suspension deflection and velocity. The results suggest that increasing inflation pressure of the tire can lead to higher harvestable suspension power. Higher tire inflation pressure, however, has been reported detrimental in view of road friendliness and ride comfort [34].

![Figure 6(a) Illustrates the Influence of Variations in Hardening Ratio of the Suspension Spring on the RMS Harvestable Power at Different Forwards Speeds](image)

**Figure 7.** Effect of variations in the vehicle sprung mass on RMS harvestable power at different driving speeds (Terrain: Plow field)

Commercial vehicles employed in the freight, resource and passenger transportation sectors encounter considerable variations in the sprung mass, which may lead to considerable changes in the suspension deflection and thus the harvestable power. For city buses, it has been reported that the maximum and minimum passenger load may vary during the off-rush hours and rush hours [18]. The corresponding variations in the sprung mass were reported from 10,575 kg to 16,231 kg. Such vehicles, however, employ air suspensions with controllable ride height, which yields only minimal changes in the vehicle resonant frequencies. Even greater variations in vehicle load have been reported for vehicles employed in the resource sector, which cause the notable variations in the natural frequencies [38]. The variations in harvestable power due to changes in the sprung mass at different driving speeds are evaluated considering sprung mass ranging from 10,000 kg to 22,000 kg, as seen in Fig. 7. The suspension stiffness is increased with increasing sprung mass so as to ensure a nearly constant vertical mode natural frequency. The results show increase in harvestable power with increase in the sprung mass, irrespective of the vehicle speed.
Fig. 8 illustrates the influences of asymmetry and damping reduction factors together with the compression mode damping ratio on the variation of the harvestable power at the nominal driving speed of 60 km/h. The range of asymmetry factor was selected between 2 to a maximum of 6 that includes most of the suspension damper properties presented in [20] for both the rear- and front-axle dampers. A broad range of damping reduction factor (0.3 to 0.6) is also analyzed and the results are presented in Fig. 8a. Due to the coupled relation between the asymmetry factor and the compression mode damping ratio, the effect of their interaction of harvestable power is also shown in Fig. 8. An increase in asymmetric factor coupled with an increase in the compression mode damping ratio yields the higher effective damping coefficients and consequently higher damping force and harvestable power.

Fig. 9 illustrates the correlation between the rms of harvestable power from vehicle suspension system and ride comfort terms of $W_k$-frequency-weighted rms acceleration of the sprung mass for assessing the human exposure to whole-body vibration [39] at different levels of damping properties $\rho = 2, 4.4$ and 6. It can be seen that an increase in the rms of harvestable power is achieved at higher magnitude of acceleration, a trend which is undesired according to the ride comfort measures. The results thereby are suggestive of practical limits on the magnitude of harvestable power and the significance of finding an optimal setting for suspension system parameters. A similar trend for the correlation between the energy harvesting and ride comfort exists in the literature [3] where the negative of regenerated power is illustrated against the ride comfort.

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As an example, Fig. 10 (a) compares the time-histories of instantaneous power computed for the four suspension dampers considering nominal vehicle parameters, speed = 60 km/h and \( M_s = 16,231 \) kg. The spectral components of the harvestable power are also illustrated in Fig. 10(b) for the right- and front left-dampers of the front wheels, as an example. Both the dampers exhibit comparable PSD magnitudes and the corresponding frequencies. The two dominant peaks in the spectra for both the dampers are indicative of the sprung mass and unsprung mass natural frequencies, while the fundamental peak near the sprung mass resonant frequency exhibits the dominant harvestable power.

![Graph](image)

(a)

![Graph](image)

(b)

Figure 10. (a) Time-history of moving rms envelope due to three dampers; and b) PSD of harvestable power for the front-left and front-right suspension dampers (Terrain: plowed field)

5. Sensitivity Analysis

In the present study, the normal probability distribution was used for generating the random values for the main factors, namely: speed; waviness and roughness coefficient; sprung mass; suspension and tire hardening ratio; and asymmetry factor, damping reduction factor and compression mode damping coefficient. In normal distribution, the mean or expected value and standard deviation are defined to
generate the variation about the mean value wherein the values close to the mean of parameter are most likely to occur. In the sampling method for the random parameter values, the Sobol’s method [40] was employed where the suggested sample size is adjusted in the range of 100.

The Sobol’s method was employed among the Monte-Carlo variance-based methods as a popular global sensitivity analysis method due of its effectiveness in determining the sensitivities and ranking parameters in spite of numerous parameters [36]. The Sobol-based sensitivity analysis technique is briefly described below considering a generalized model. Consider the following model relating the system output $Y$ and model parameters vector $U$, as:

$$Y = f(U) \quad (15)$$

The output $Y$ in this model refers to RMS harvestable power. The Sobol’s technique is on the basis of decomposition of the model output $Y$ variance, $\sigma^2$:

$$\sigma^2 = \sum_r \sigma_r^2 + \sum_{s>r} \sigma_{rs}^2 + \sigma_{1,2...n}^2 \quad (16)$$

where

$$\sigma_r^2 = \sigma^2 \left( E \left( Y \mid U_r = u_r^* \right) \right)$$

$$\sigma_{rs}^2 = \sigma^2 \left( E \left( Y \mid U_r = u_r^*, U_s = u_s^* \right) \right) - \sigma_r^2 - \sigma_s^2 \quad (17)$$

where $n$ denotes the number of model parameters and $r$ and $s$ represent the $r^{th}$ and $s^{th}$ model parameters, $s>r$, and $\sigma^2(.)$ and $E(.)$ show the variance and expectation operators, respectively. The normalized sensitivity indices ($S_r$, $S_{rs}$) can be defined using the variance, as:

$$S_r = \frac{\sigma_r^2}{\sigma^2}, \quad 1 \leq r \leq n$$

$$S_{rs} = \frac{\sigma_{rs}^2}{\sigma^2}, \quad 1 \leq r < s \leq n \quad (18)$$

The index $S_r$ denotes the average output variance reduction and $S_{rs}$ is used to compute the contribution due to interaction between $r^{th}$ and $s^{th}$ model parameters. In the adoption of variance-based method, two sensitivity indices are generally used, the first order index or main effect ($S_r$) and the total sensitivity index or total effect ($ST_r$), given by:

$$ST_r = S_r + \sum_{s>r} S_{rs} + \ldots$$

$$\quad (19)$$

The average output variance reduction $S_r$ can be achieved when the parameter $U_r$ is fixed, while $ST_r$ stands for the average output variance that would exist as long as $U_r$ remains unknown [40], that is, the total contribution of $U_r$ to the output variation.

As the Sobol’s method involves estimations of expectation $E(.)$ and variance $\sigma^2(.)$, it requires a large number of trials. For a sample size of $m$, $m \times (n + 2)$ simulations are necessary for computing ($S_r$, $ST_r$) for
a \( n \) parameter model \([41]\). A larger sample size would yield more precise sensitivity indices. It has been suggested that \( m \) should be in the range of 100 or higher \([42-43]\).

The variance-based method displays a number of attractive features including: (1) satisfactory performance when dealing with nonlinear and non-monotonic models, and for models involving interactions among the main factors; (2) the method can capture the effect of full range of variation of each parameter as well as the interaction effects, and (3); the method can treat sets of factors as a single factor \([36]\). However, the major drawback is that it requires a large number of model evaluations that may result in a considerable computational demand.

The single objective function considered in the present study is the sum of RMS of harvestable power obtained from the four suspension dampers. The sensitivity analysis is performed to study the influences of variations in all the main factors, which are permitted to vary within \( \pm 25\% \) about the respective nominal value. Furthermore, the normal distribution of parameters was selected for generating the random parameter values via the random sampling method.

Table 3. Influence of variations in the main factors on the magnitude of harvestable power from vehicle suspension.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Harvestable power (W)</th>
<th>Nominal -25%</th>
<th>Nominal +25%</th>
<th>Maximum % change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving Speed (km/h)</td>
<td>60</td>
<td>2427</td>
<td>4987</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Road Waviness</td>
<td>3.5</td>
<td>2023</td>
<td>5191</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>Road Roughness</td>
<td>( 6 \times 10^{-4} )</td>
<td>2854</td>
<td>4844</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Asymmetric Factor</td>
<td>4.4</td>
<td>2312</td>
<td>4931</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Compression Damping Ratio</td>
<td>0.2</td>
<td>2564</td>
<td>4604</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Damping Reduction Factor</td>
<td>0.5</td>
<td>4281</td>
<td>2865</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Sprung Mass (kg)</td>
<td>16,231</td>
<td>2713</td>
<td>3843</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Tire Hardening Ratio</td>
<td>4.69</td>
<td>3035</td>
<td>4970</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Spring Hardening Ratio</td>
<td>71</td>
<td>3324</td>
<td>3892</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

The results obtained from sensitivity analysis performed in Matlab/Simulink environment are summarized in Table 3. The nominal values of model parameters (main factors) are also presented in the table, which resulted in total RMS harvestable power of 3566 W. The table presents the total harvestable power under the effect of \( \pm 25\% \) variations in given nominal values together with the maximum % change with respect to the nominal RMS power. The results clearly show that all the main factors considered in the study affect the RMS harvestable power significantly. The results obtained in terms of variations in the range of harvestable power are suggestive that the power is most strongly influenced by the driving speed, damper asymmetry factor, compression mode damping ratio, road roughness, and road waviness.

6. Concluding remarks

In this study, the energy harvestable potential of vehicles was explored considering a generic three-dimensional 7-DOF model of the vehicle with typical nonlinear suspension properties and a wide range of terrains. The results suggest notable harvestable power for commercial vehicles operating on relatively rough terrains, while the amount of energy available for vehicles operating on smooth highways may not justify the use of an energy harvesting system. The results thus suggest greater feasibility of energy
harvesting systems for off-road vehicles than their road counterparts. All of the main factors considered in the study, namely, driving speed, road roughness, chassis load, suspension spring stiffness, damping asymmetry factor, and tire stiffness showed significant effect on the total harvestable power. This was also evident from the sensitivity analyses. An increase in driving speed, tire stiffness, damping asymmetry factor, sprung mass, compression mode damping ratio, road roughness and waviness coefficients together with the decrease in damping reduction factor resulted in notable increase in the harvestable power. The results, however, are obtained using a simplified point-contact tire model. Further studies are needed to compare the obtained results for harvestable power with those obtained from a more comprehensive tire model involving enveloping of the terrain profile together with the effects of mechanical properties of the soils, especially for off-road vehicles. Although, the analytical method, presented in this study, could serve as a generic guidance for energy harvesting via passive suspension damping tuning, the effect of tuning on the ride performance and handling dynamics of the vehicle need to be considered.

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**References**


Appendix

The damping force at the shock absorbers together with nonlinear suspension spring and tire forces are defined as following.

\[
F_{ij} = \begin{cases} 
  C_{ij}(z_i - z_{w_i} + h_i\theta + g_j \phi) & \text{for } V_e \leq z_i - z_{w_i} + h_i\theta + g_j \phi < 0 \\
  C_{ij}(V_e) - C_{ij}(z_i - z_{w_i} + h_i\theta + g_j \phi - V_e) & \text{for } z_i - z_{w_i} + h_i\theta + g_j \phi \leq V_e \\
  C_{ij}(z_i - z_{w_i} + h_i\theta + g_j \phi) & \text{for } 0 \leq z_i - z_{w_i} + h_i\theta + g_j \phi \leq V_e \\
  C_{ij}(V_e) + C_{ij}(z_i - z_{w_i} + h_i\theta + g_j \phi - V_e) & \text{for } V_e \leq z_i - z_{w_i} + h_i\theta + g_j \phi 
\end{cases}
\]  
(A.1)

\[
F_{ij} = k_1(z_i - z_{w_i} + h_i\theta + g_j \phi) + k_3(z_i - z_{w_i} + h_i\theta + g_j \phi)^3 
\]  
(A.2)

where \(h_i = \begin{cases} 
  -a_i & i = f \\
  b_i & i = r 
\end{cases}\) and \(g_j = \begin{cases} 
  -\frac{w}{2} & j = r \\
  \frac{w}{2} & j = l 
\end{cases}\) and subscript index \(i\) refers to front- or rear-axle tire, and the subscript index \(j\) refers to left- or right-hand tire.

\[
F_{ij} = k_1'(z_{wij} - z_{wij}) + k_3'(z_{wij} - z_{wij})^2; \ i = f \ , \ j = l; \ (z_{wij} - z_{wij}) \geq 0
\]  
(A.3)