Ergodic Capacity Analysis of Wireless Transmission over Generalized Multipath/Shadowing Channels

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Abstract—Novel composite models were recently proposed based on inverse gamma distributed shadowing conditions. These models were extensively shown to provide remarkable modeling of the simultaneous occurrence of multipath fading and shadowing phenomena in emerging wireless scenarios such as cellular, off-body and vehicular-to-vehicular communications. Furthermore, the algebraic representation of these models is rather tractable, which renders them convenient to handle both analytically and numerically. Based on this, the present contribution analyzes the ergodic capacity over the recently proposed $\kappa - \mu$ / inverse gamma composite fading channels, which were shown to characterize excellently multipath fading and shadowing in line-of-sight communication scenarios, including realistic vehicular-to-vehicular communications. Novel analytic expressions are derived which are subsequently used in the analysis of the corresponding system performance. In this context, the offered results are compared with respective results from cases assuming conventional fading conditions, which leads to the development of numerous insights on the effect of the multipath fading and shadowing severity on the achieved capacity levels. It is expected that these results will be useful in the design of timely and demanding wireless technologies such as wearable, cellular and vehicular communications.

I. INTRODUCTION

Accurate characterization and modeling of fading channels constitutes a core topic in wireless communications as fading phenomena affect considerably the performance of conventional and emerging communication systems. As a result, numerous fading models that provide adequate modeling accuracy to specific types of fading conditions have been proposed during the past years [1]–[5] and the references therein. In this context, it has been extensively shown that generalized fading models are capable of providing accurate characterization of multipath fading [6]–[10]. Yet, it has been also shown that multipath fading and shadowing phenomena practically occur simultaneously and can be modeled with the aid of composite fading distributions [11]–[19]. However, the existing composite fading models in the open technical literature do not typically provide holistic accurate modeling of fading phenomena, while they often have a complicated mathematical form, which renders them analytically intractable in numerous applications of interest. Motivated by this, the authors in [1]–[3] proposed two novel distributions, namely the $\kappa - \mu$ / inverse gamma and the $\eta - \mu$ / inverse gamma that constitute effective a composite fading models. The high modeling capability of these models has been validated by accurate fitting to results from extensive measurement campaigns. These campaigns also included communication scenarios in the context of wearable, cellular and vehicular communications, which constitute emerging and timely topics of interest. In addition, a distinct characteristic of the proposed models is their relatively convenient algebraic representation, which renders them tractable both analytically and numerically. Based on this, they overall constitute the most adequate balance between modeling accuracy and algebraic tractability compared to the existing composite fading models in the open technical literature.

It is recalled that fading distributions have been extensively used in the analysis and evaluation of wireless communications since they typically allow the derivation of explicit expressions for critical performance measures of interest. However, this task becomes considerably more challenging, if not impossible, in the case of generalized and/or composite fading conditions [6], [20]. Based on this, the authors in [21]–[23] analyzed the capacity over generalized fading channels under different adaptation policies. This topic was also addressed in [24] for the case of $K_G$ fading channels, in [13] and [25] for the case of $G$ fading channels and in [26] and [27] for the case of $\eta - \mu$ / gamma and $\kappa - \mu$ shadowed fading channels, respectively. In the same context, the outage probability (OP) over different generalized interference-limited scenarios was investigated in [28], whereas an analytical framework for the case of device-to-device communications in cellular networks was proposed in [29]. Finally, the outage capacity (OC) of orthogonal space-
time block codes over multi-cluster scattering multi-antenna systems along with the coverage capacity 5G millimeter wave cellular systems were addressed in [30] and [31], respectively.

Motivated by the above, the present work analyzes the channel capacity of digital communications over $\kappa - \mu$ / inverse gamma fading channels. To this end, we derive an explicit analytic expression for the ergodic capacity under these composite fading conditions in the form of a simple and convergent infinite series. An elegant upper bound for the corresponding truncation error is also derived in closed-form, allowing the precise determination of the number of terms required at given accuracy levels. Particularly in the considered case of the ergodic capacity, it is shown that only few terms are required in order to achieve a 1% accuracy, which is practically sufficient for channel capacity relating measures. Based on this, the derived expressions are utilized in quantifying the effects of different fading conditions on the corresponding system performance. This leads to the development of useful insights that are expected to be useful in the design of demanding emerging wireless technologies such as wearable, cellular and vehicular communications.

The remainder of this paper is organized as follows: Section II revisits the basic properties of the recently proposed $\kappa - \mu$ / inverse gamma fading model. Capitalizing on this, Section III is devoted to the analysis of the ergodic capacity under these fading conditions, followed by the corresponding numerical results and related discussions in Section IV. Finally, closing remarks are provided in Section V.

II. THE $\kappa$-$\mu$ / INVERSE GAMMA FADING MODEL

The $\kappa - \mu$ / inverse gamma fading model assumes that the mean power of both the dominant and scattered signal components is subject to shadowing, which is weighted by an inverse gamma random variable (RV). This model was shown to provide remarkable accuracy in line of sight (LOS) communication scenarios and its envelope probability density function (PDF), $R$, is expressed as [1, 3]

$$f_R(r) = \frac{2\mu^\mu(1 + \kappa)^\mu m_s^{m_s\Omega}e^{-\mu r}r^{2\mu - 1}}{B(m_s, \mu)\mu(1 + \kappa)^{r} + m_s\Omega^{m_s + \mu}} \times 1_F \left( m_s + \mu; \mu; \frac{\mu^2(1 + \kappa)r^r + m_s\Omega}{\mu(1 + \kappa)^r + m_s\Omega} \right)$$

(1)

where $\kappa$ denotes the ratio of the total power of the dominant components to the total power of the scattered waves, $\mu$ is related to the number of multipath clusters, $m_s$ is the shadowing parameter and $\Omega$ is the the mean signal power. Furthermore, $B(\cdot, \cdot)$ and $1_F(\cdot; \cdot; \cdot; \cdot)$ denote the Beta function and the Kummer hypergeometric function, respectively [32].

Based on (1), the signal-to-noise ratio (SNR) PDF of the $\kappa - \mu$ / inverse gamma fading model is given by

$$f_r(\gamma) = \frac{\mu^\mu(1 + \kappa)^\mu m_s^{m_s\Omega}\gamma e^{-\mu \gamma}r^{2\mu - 1}}{B(m_s, \mu)\mu(1 + \kappa)^{\gamma} + m_s\Omega^{m_s + \mu}} \times 1_F \left( m_s + \mu; \mu; \frac{\mu^2(1 + \kappa)\gamma + m_s\Omega}{\mu(1 + \kappa)^\gamma + m_s\Omega} \right)$$

(2)

where $\gamma = E[\gamma]$ is the corresponding average SNR, with $E[\cdot]$ denoting statistical expectation.

It is evident that the algebraic representation of the PDF of the $\kappa - \mu$ / inverse gamma fading model is relatively convenient both analytically and numerically. In what follows, we capitalize on the above statistical results to derive a useful analytic expression for the ergodic capacity under these composite fading conditions.

III. ERGODIC CAPACITY OVER $\kappa - \mu$ / INVERSE GAMMA FADING CHANNELS

A. Ergodic Capacity

**Theorem 1.** For $\kappa, \gamma, B \in R^+$ and $m_s \in N$, the following analytic expressions hold for the ergodic capacity in $\kappa - \mu$ / inverse gamma fading channels

$$C_E = \frac{m_s - 1}{B} \sum_{l=0}^{m_s - 1} \sum_{i=0}^{\infty} \left( \frac{m_s - 1}{l} \right) \left( \frac{1 - \mu^l(1 + \kappa)^\mu\kappa r e^{-\mu \kappa}}{B(m_s, \mu)\mu(1 + \kappa)^l + m_s\Omega} \right) \times H_{l+i+\mu} + \ln(\eta m_s) - \ln(\mu + \ln(1 + \kappa))$$

$$+ \sum_{l=0}^{m_s - 1} \sum_{i=0}^{\infty} \left( \frac{m_s - 1}{l} \right) \left( \frac{1 - \mu^l(1 + \kappa)^\mu\kappa r e^{-\mu \kappa}}{B(m_s, \mu)\mu(1 + \kappa)^l + m_s\Omega} \right) \times \ln(\eta m_s)\Gamma(i + l + \mu)$$

$$\times 2F_1 \left( 1; 1 + l + i + 1; \mu + l + i + 2; 1 - \frac{m_s\gamma}{\mu(1 + \kappa)} \right)$$

(3)

which is valid when $\mu \in R^+$, with $H_n$ denoting the $n^{th}$ harmonic number, and

$$C_E = \frac{m_s - 1}{B} \sum_{l=0}^{m_s - 1} \sum_{i=0}^{\infty} \left( \frac{m_s - 1}{l} \right) \left( \frac{1 - \mu^l(1 + \kappa)^\mu\kappa r e^{-\mu \kappa}}{B(m_s, \mu)\mu(1 + \kappa)^l + m_s\Omega} \right) \times \frac{1}{l+i+\mu}$$

$$+ \sum_{l=0}^{m_s - 1} \sum_{i=0}^{\infty} \left( \frac{m_s - 1}{l} \right) \left( \frac{1 - \mu^l(1 + \kappa)^\mu\kappa r e^{-\mu \kappa}}{B(m_s, \mu)\mu(1 + \kappa)^l + m_s\Omega} \right) \times \ln(\eta m_s)\Gamma(i + l + \mu)$$

$$\times 2F_1 \left( 1; 1 + l + i + 1; \mu + l + i + 2; 1 - \frac{m_s\gamma}{\mu(1 + \kappa)} \right)$$

(4)

which is valid when $\mu \in N$, where in both cases $2F_1(\cdot; \cdot; \cdot; \cdot)$ is the Gaussian hypergeometric function [32].
Proof. By recalling that
\[ C_e \triangleq B \int_0^\infty \log_2(1 + \gamma)p_\gamma(\gamma) \, d\gamma \quad (5) \]
and substituting [3, Eq. (4)] yields (6), at the top of the next page. Based on this and setting
\[ u = \mu(1 + \kappa)\gamma + m_s\tau \]
it immediately follows that
\[ C_e = \frac{e^{-\mu}}{B(m_s, \mu) \ln(2)} \times \int_{m_s\tau} \frac{(u - m_s\tau)^{\mu-1}}{u^{m_s+\mu}} \ln \left(1 + \frac{u - m_s\tau}{\mu(1 + \kappa)}\right) \times \frac{1}{\Gamma(1)} \left( m_s + \mu; \mu; \mu\kappa - \frac{m_s\mu\kappa\tau}{u} \right) \, du \quad (7) \]
which upon setting
\[ t = 1 - m_s\tau/u \]
and after some algebraic manipulations yields
\[ C_e = \frac{e^{-\mu}}{B(m_s, \mu) \ln(2)} \times \left\{ \int_0^1 \frac{t^{\mu-1}}{1 - t}^{m_s-1} \ln \left(1 + \frac{(m_s\tau - (1 + \kappa))t}{\mu(1 + \kappa)}\right) \times \frac{1}{\Gamma(1)} \left( m_s + \mu; \mu; \mu\kappa - \frac{m_s\mu\kappa\tau}{u} \right) \, dt \right\} \quad (8) \]
By applying [32, Eq. (1.111)] in (8) and expanding the involved hypergeometric functions along with straightforward algebraic manipulations one obtains (3), which is valid for \( \mu \in \mathbb{R}^+ \). To this effect and by recalling that
\[ H_{i+l+\mu} \triangleq \sum_{j=1}^{i+l+\mu} j^{-1} \quad (9) \]
which holds for \( \mu \in \mathbb{N} \), equation (4) is deduced, which completes the proof.

It is noted that the series representation in (4) is fully convergent and it is evident that it has a relatively convenient algebraic form that renders it tractable both analytically and numerically. Furthermore, only a few number of terms are required to achieve adequate truncation accuracy. Yet, a simple upper bound that determines the involved truncation error in an accurate manner is essential, particularly in analyses relating to emerging wireless communication scenarios, such as those encountered in vehicular communications. To this end, a simple and tight closed-form bound to (4) is derived in the following proposition.

B. A closed-form upper bound for the truncation error of (4)

Proposition 1. For \( \kappa, \tau, B \in \mathbb{R}^+ \) and \( m_s \in \mathbb{N} \), the following closed-form upper bound is valid for the truncation error of the infinite series in (3):
\[ \mathcal{T} \leq \frac{e^{-\mu}}{B(m_s, \mu) \ln(2)} \times \left\{ \sum_{i=0}^{m_s-1} \frac{(m_s - 1)}{l} \log(\tau m_s/\mu) - \log(1 + \kappa) + \sum_{i=0}^{m_s-1} \frac{(m_s - 1)}{l} (\mu(1 + \kappa) - \tau m_s/\mu) \times \frac{1}{\Gamma(1)} \left( m_s + \mu; \mu; \mu\kappa - \frac{m_s\mu\kappa\tau}{u} \right) \, du \right\} \quad (10) \]
where \( _pF_q(\cdot) \) denotes the generalized hypergeometric function for the specific case \( p = q = 2 \). [32]

Proof. Truncating the infinite series in (3) after \( p - 1 \) terms results to the following truncation error
\[ \mathcal{T} = \frac{e^{-\mu}}{B(m_s, \mu) \ln(2)} \times \left\{ 1 + \sum_{i=0}^{m_s-1} \frac{(m_s - 1)}{l} \times \log(\tau m_s/\mu) - \log(1 + \kappa) + \sum_{i=0}^{m_s-1} \frac{(m_s - 1)}{l} (\mu(1 + \kappa) - \tau m_s/\mu) \times \frac{1}{\Gamma(1)} \left( m_s + \mu; \mu; \mu\kappa - \frac{m_s\mu\kappa\tau}{u} \right) \, du \right\} \quad (11) \]
With the aid of the Pochhammer symbol identities and after some algebraic manipulations, the above representation can be upper bounded by the inequality in (12), at the top of the next page. Notably, the involved infinite series representations can be expressed in closed-form in terms of the generalized hypergeometric function, namely
\[ \sum_{i=0}^{m_s-1} \frac{(m_s + \mu)_i (\mu + l)_i \mu^{i\kappa}}{i! (\mu + l + 1)_i} = _2F_2 \left( \frac{m_s + \mu; \mu + 1}{\mu; \mu + 1 + l; \mu\kappa} \right) \quad (13) \]
and
\[ \sum_{i=0}^{m_s-1} \frac{(m_s + \mu)_i (l + \mu)_i \mu^{i\kappa}}{i! (l + \mu + 2)_i} = _2F_2 \left( \frac{m_s + \mu; l + \mu}{\mu; l + \mu + 2; \mu\kappa} \right) \quad (14) \]
where \(2 F_2\left(\frac{a,b}{c, d}; x\right) \triangleq 2 F_2\left(a, b; c, d; x\right)\). To this effect, by substituting (13) and (14) into (12) and performing the necessary change of variables along with some algebraic manipulations, equation (10) is deduced, which completes the proof.

**Remark 1.** It is noted that for the case of \(\mu \in \mathbb{N}\), a respective closed-form upper bound for the truncation error of (4) can be readily deduced by substituting (9) in (10), yielding

\[
T < \sum_{i=0}^{m_s - 1} \left( \frac{m_s - 1}{l} \right) e^{-\mu l} \frac{1}{\log(2)} \sum_{j=0}^{i+\mu} \frac{\mu(1+\kappa)(\mu l + j)}{\mu(1+\kappa)(\mu l + j + 1)}
\]

\[
\times 2 F_1 \left(1, l + p + 1; l + p + 2; 1 - \frac{\gamma_m}{\mu(1+\kappa)}\right) \sum_{i=0}^{m_s} \frac{(m_s + \mu + l + \mu + 2 l + \mu \mu \mu \mu)}{(l + \mu + 2 l i)!}.
\]

It is evident that both (10) and (15) have a tractable algebraic representation that allows their straightforward computation, since the involved functions are included as built-in functions in popular software packages such as MATLAB, MAPLE and MATHEMATICA.

**IV. Numerical Results**

This section employs the derived results in the previous sections in the quantification of the effects of composite multipath/shadowing conditions on the ergodic capacity, as this can occur in realistic communications scenarios undergoing fading effects, such as in wearable, cellular and vehicular communication scenarios. To this end, Fig. 1 illustrates the ergodic capacity over \(\kappa - \mu \) inverse gamma composite fading channels. It is evident that the joint effects of multipath fading and shadowing are considerable as significant deviations from the standard Rayleigh fading conditions are observed. For example, a 50% spectral efficiency increase is noted when \(\mu\) changes from \(\mu = 0.2\) to \(\mu = 2.0\), for light shadowing conditions at moderate SNR values and NLOS scenarios. Likewise, a nearly 55% spectral efficiency reduction is observed when shadowing changes from \(m_s = 0.2\) to \(m_s = 2.0\) for \(\mu = 0.2\) at \(\gamma = 20\) dB with \(\kappa = 1\). This corresponds to gains of several dBs for fixed spectral efficiencies, which is particularly advantageous in emerging applications of substantially increased quality of service requirements. It is noted that the offered results also verify that accurate characterization and modeling of multipath fading and shadowing are of paramount importance in the design of emerging communication systems; therefore, highly accurate composite fading models are highly essential in ensuring avoidance of unrealistic evaluation of conventional and emerging communication systems, such as wearable and vehicular communications.
V. Conclusion

This work was devoted to the ergodic capacity analysis of digital communications over $\kappa - \mu$ inverse gamma fading channels. Novel exact analytic expressions were derived for this measure which were subsequently employed in quantifying the effects of severity of multipath fading and shadowing fading conditions on the overall system performance. It was shown that the effect of different types of composite fading have a considerable effect across all SNR regimes. These effects are clearly beyond the range of the conventional Rayleigh distributed multipath fading effects and thus, they must be taken into account in the realistic design and deployment of emerging wireless systems that are characterized by their significantly increased quality of service requirements. Indicative examples include timely and critical topics of interest such as wearable, cellular and vehicular communications.

REFERENCES


