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Original citation & hyperlink:
Razavi Hajiagha, Seyed Hossein, Hannan Amoozad Mahdiraji and Madjid Tavana. "A new bi-level data envelopment analysis model for efficiency measurement and target setting". Measurement. 2019. 147.
https://dx.doi.org/10.1016/j.measurement.2019.106877

DOI 10.1016/j.measurement.2019.106877
ISSN 0263-2241

Publisher: Elsevier

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First Revision
MEAS- D-19-01264.R1

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ACKNOWLEDGEMENT

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

All additions and changes to the original manuscript are highlighted in this revision.

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A New Bi-Level Data Envelopment Analysis Model for Efficiency Measurement and Target Setting

Abstract

Data envelopment analysis (DEA) is a well-known and widely used method for performance evaluation in a set of homogeneous units. We propose a new bi-level DEA model for efficiency measurement and target setting. The fundamental novelty of the proposed model is threefold. We: (1) set both efficiency and profit concurrently as targets; (2) limit the amount of changes in the inputs and outputs to prevent unachievable targets; and (3) predict some targets for efficient units beyond the inefficient ones. We present a case study in the banking industry to demonstrate the efficacy of efficiency measurement and target setting in the proposed models.

Keywords: Data envelopment analysis; Bi-level programming; efficiency planning; target setting; benchmarking.

1. Introduction

A large volume of literature has concerned itself with the study of efficiency. Considering efficiency as "how well an organization uses inputs to produce outputs" (Cochran, 2003), perhaps no manager or resource owner denies the importance of efficiency. A brief glance at the literature shows a wide range of discussions about how to measure and improve the efficiency.

Generally, efficiency evaluation methods are classified into two categories: non-parametric (Battese and Coelli 1995, 1998; Ajibefun et al. 2002) and parametric (Fare et al. 1993; Coelli and Perelman 1996; Forsund and Hjalmarsson 1997; Bardason and Vassdal 1998) methods. As a non-parametric efficiency evaluation method, data envelopment analysis (DEA) employs a linear programming based technique to estimate a piecewise linear production frontier and applies the estimated frontier to approximate efficiency (Charnes et al. 1978; Collier et al. 2011). Since its initial introduction, DEA has been applied and investigated in thousands of papers and practical projects. Liu et al. (2013) and later, Emrouznejad et al. (2017) surveyed the applications of DEA and represented a wide variety of applications. Originally, the main application of DEA is for efficiency evaluation. Classic DEA models were extended upon the assumption of certainty, where they assume that a set of decision making units (DMUs) act and use a set of known inputs and produce a set of determined outputs. Liu and Lin (2006) have classified uncertainty into three categories: (1) encompassing probability and statistics, (2) fuzzy set theory, and (3) grey systems theory. Accordingly, DEA models under uncertainty are generally are extended with probabilities, fuzzy logic or grey systems theory.

As highlighted by Olesen and Petersen (2016), stochastic DEA models have evolved in two different directions. In one direction, that was initiated by Banker (1993) and followed by Korostelev et al. (1995a), Korostelev et al. (1995b), Kneip et al. (1998), Simar and Wilson (1998, 2007), some statistical axioms are included that define a statistical model and a sampling process into a DEA framework. In the other direction, originated by Land et al. (1993) and proceeded by Olesen and Petersen (1995), Cooper et al. (1998) and Olesen (2006), the observed inputs and outputs are replaced with DMU-specific distributions. Furthermore, Olesen and Petersen (2016) presented a comprehensive review of stochastic DEA models. Jradi and Ruggiero (2019) considered a stochastic DEA model and emphasized on the most common stochastic frontier model. They used simulated data and compared the model to the econometric stochastic frontier model. Kao and Liu (2019) warned against using deterministic DEA models for measuring the relative efficiency in the real world stochastic problems for the sake of simplicity. They proposed a stochastic DEA model and used the correlation between the input and output factors to determine the distribution of the stochastic efficiency.

Fuzzy DEA has also been studied extensively. In this field of research, Hatami-Marbini et al. (2011) classified the proposed fuzzy DEA approaches into different categories: (1) the tolerance approach, (2) the α-level based approach, (3) the possibility approach, and (4) other approaches. Since 2011, Zerafat Angiz L. et al. (2012), Razavi Hajiagha et al. (2013), Hatami-Marbini et al. (2017) and Razavi Hajiagha et al. (2018) are examples of some other researches that added significant results to the field of fuzzy DEA. For instance, Zhang et al (2019) proposed a two-stage DEA model for resource allocation based on zero and fixed sum gain. Furthermore, Hatami-Marbini (2019) used network DEA as a non-parametric tool for benchmarking in supply chains. They considered human judgements as fuzzy numbers to formulate impreciseness and ambiguity in their model. It is worth noting here that, more improved versions of the fuzzy approach in DEA including intuitionistic fuzzy DEA and hesitant fuzzy DEA were presented by Puri Yadav (2015), Otay et al. (2017) and Zhou et al. (2018).

Moreover, DEA with grey or interval data is studied by some scholars. Wang et al. (2005) introduced a method of handling interval data in DEA. Consequently, Kao (2006) proposed a two-level mathematical programming based approach for solving DEA problems with imprecise data including ordinal and interval data. Figure 1 illustrates a possible classification of certain and uncertain DEA approaches.

Insert Figure 1 Here

Beyond the type of uncertainty in DEA, i.e. stochastic, fuzzy, or interval, all the aforementioned methods deal with the problem of evaluation that is a post-event analysis. While evaluation might be an important problem for managers, a more challenging issue is how to plan and target the performance of organizations. Analyzing related literature, a wide range of researches is performed in the field of performance evaluation using DEA; however, the category of planning based on DEA is limited. De facto, the stochastic DEA considered as a tool for forecasting efficiency has been ignored. Considering this fact, Gui-sheng (2009) proposed an extension of interval DEA to forecast the efficiency. Similarly, Klimberg et al. (2009) proposed a regression analysis for forecasting the performance of banks, when DEA relative efficiency is used as a variable. Furthermore, Kiani Mavi et al. (2010) applied the analytic hierarchy process along with DEA for forecasting efficiency. Moreover, Ebrahimnejad Tavana (2014) proposed an interactive multi objective linear programming model for identifying target units in output-oriented DEA. Shabanpour et al. (2017) proposed a combination of artificial neural networks and dynamic DEA to forecast the efficiency of green suppliers. Their approach initially approximated the inputs and outputs using artificial neural networks and then applied these estimated values in dynamic DEA to forecast the efficiency. Aparicio et al. (2017) proposed a bi-level model to determine the closest target based on the least distance criterion. The first level objective is to maximize the Russell output measure and the second level is to minimize the input slacks. The second level model is replaced with its equivalent Karush-Kuhn-Tucker optimality condition.

The aim of this research is to propose a method of planning based on efficiency. Literally, considering the performance of a DMU in the current period, an approach has been developed to indicate how it should perform in the upcoming period to improve its efficiency, by considering his/ her profit performance. For example, if DMU_j uses input vector $\left(x_{1j}^{t}, x_{2j}^{t}, \ldots, x_{mj}^{t}\right)$ to produce output vector $\left(y_{1j}^{t}\ {}^{t}\ ,\ \ {}^{t}\ \right)$ in period *t*, the proposed method seeks to justify the next period inputs (i.e., ${}^{t\ 1}\ {}^{t\ 1}\ \ {}^{t\ 1}$), and targets outputs (i.e., ${}^{t\ 1}\ {}^{t\ 1}\ \ {}^{t\ 1}$) to improve its efficiency in the forthcoming period with respect to inputs and outputs. Considering profit along with technical efficiency is one of the main contributions of the proposed model.

To determine the values of $x_{ij}^{t+1}, i = 1,2, \ldots, m; j = 1,2, \ldots, n$ and $y_{rj}^{t+1}, r = 1,2, \ldots, s; j = 1,2, \ldots, n$, a bi-level programming model is proposed in this paper that at the first level, efficiency of any DMU is maximized, while at the second level the combination of inputs and outputs based on their costs and profits are optimized. Bi-level programming (hereafter BLP) is a problem consisting of a two-level hierarchy. In a BLP, two decision makers at two different positions independently control a set of decision variables (Arora and Gupta, 2009). Candler and Townsley (1982) suggested that energy policy, drug abuse and economic development are some of the application areas of BLP. Ben-Ayed (1993) formulated the general structure of a BLP problem as follows:

$$\max_{x} f_1(x, y) = C_1 x + d_1 y$$

where y solves: (1)

$$\max_{y} f_2(x, y) = C_2 x + d_2 y$$

s.t.

$Ax + By \leq b$

$x, y \geq 0$

where x and y are vectors of the first and second level decision problems; c_1, c_2, d_1, d_2, and b are constant vectors; A and B are constant matrices.

Several solving approaches are proposed to solve BLP problems, among them can refer to Candler and Townsley (1982), Arora and Gupta (2009), and Baky (2009). In this paper, the formulated problem of input/output targeting by forecasting efficiency will be solved using the concept of BLP.

The paper is organized as follows. The bi-level efficiency model proposed in this study is formulated in Section 2. Next, the proposed solution method is presented in Section 3. A case study in the banking industry is presented in Section 4. Section 5 presents the results for the case study and the paper is concluded in Section 6.

2. Problem formulation

2.1. Notations

To formulate the considered problem in previous section, the following notations are used.

Parameters:

t The current time period;

j The index used for determining DMUs, $j = 1,2, \ldots, n$;

i The index used for determining inputs, $i = 1,2, \ldots, m$;

r The index used for determining outputs, $r = 1,2, \ldots, s$;

x_{ij}^t The value of the ith input for the jth DMU in period t;

y_{rj}^t The value of the rth output for the jth DMU in period t;

θ_j^t The efficiency of the jth DMU in period t;

c_{ij} The cost of using or obtaining a unit of the ith input by the jth DMU;

p_{rj} The profit of producing a unit of the rth output by the jth DMU;

ID_{ij}^{t+1} The maximum allowable decrease in the ith input of the jth DMU in period $t+1$;

OI_{rj}^{t+1} The minimum allowable increase in the rth output of the jth DMU in period $t+1$;

Variables:

ε_j The minimum amount of growth in the efficiency of the jth DMU;

α_{ij}^{t+1} The amount of decrease in the ith input of the jth DMU in period $t+1$;

β_{rj}^{t+1} The amount of increase in the rth output of the jth DMU in period $t+1$;

v_{ij}^{t+1} The associated weight of the ith input in efficiency appraisal of the jth DMU in period $t+1$;

u_{rj}^{t+1} The associated weight of the rth output in efficiency appraisal of the jth DMU in period $t+1$;

2.2. Modeling the first level

Consider the DMU_0 at the current time period t, that uses the input vector $\{x_{10}^t, x_{20}^t, \dots, x_{m0}^t\}$ to produce the output vector $\{y_{10}^t, y_{20}^t, \dots, y_{s0}^t\}$. Using the CCR input-oriented model, the efficiency of this unit at period t, i.e. θ_0^t, can be determined by solving the following model:

$$\begin{aligned} &\theta_0^t: max \sum_{r=1}^{s} u_{r0}^t y_{r0}^t \\ &\sum_{i=1}^{m} v_{i0}^t x_{i0}^t = 1 \\ &\sum_{r=1}^{s} u_{r0}^t y_{rj}^t - \sum_{i=1}^{m} v_{i0}^t x_{ij}^t \leq 1, \qquad j = 1,2,\dots,n \\ &u_{10}^t, u_{20}^t, \dots, u_{s0}^t, v_{10}^t, v_{20}^t, \dots, v_{m0}^t \geq 0 \end{aligned} \tag{2}$$

The above model will be solved for all DMUs and an initial vector of efficiencies, i.e. $\theta^t = \{\theta_1^t, \theta_2^t, \dots, \theta_n^t\}$, is obtained:

$$\theta^- = \mathrm{Min}\{\theta_1^t, \theta_2^t, \dots, \theta_n^t\} \tag{3}$$

At the next stage, the decision maker aims to determine the target values of the input and output measures for DMUs at the upcoming time period. In this case, the model seeks to determine the unknown values $\alpha_{i0}^{t+1}, i = 1,2,\dots,m$ and $\beta_{r0}^{t+1}, r = 1,2,\dots,s$ such that the targeted input vector $\{x_{10}^t - \alpha_{10}^{t+1}, x_{20}^t - \alpha_{20}^{t+1}, \dots, x_{m0}^t - \alpha_{m0}^{t+1}\}$ and output vector $\{y_{10}^t + \beta_{10}^{t+1}, y_{20}^t + \beta_{20}^{t+1}, \dots, y_{s0}^t + \beta_{s0}^{t+1}\}$ reaches an efficiency level at least as great as θ_0^t, i.e.,

$$\frac{\sum_{r=1}^{s} u_{r0}^{t+1}(y_{r0}^t + \beta_{r0}^{t+1})}{\sum_{i=1}^{m} v_{i0}^{t+1}(x_{i0}^t - \alpha_{i0}^{t+1})} \geq \theta_0^t \tag{4}$$

Therefore, the firs level objective function of the considered problem is formulated as:

$$Max \frac{\sum_{r=1}^{s} u_{r0}^{t+1}(y_{r0}^t + \beta_{r0}^{t+1})}{\sum_{i=1}^{m} v_{i0}^{t+1}(x_{i0}^t - \alpha_{i0}^{t+1})} \tag{5}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_{r0}^{t+1}(y_{r0}^{t}+\beta_{r0}^{t+1})}{\sum_{i=1}^{m} v_{i0}^{t+1}(x_{i0}^{t}-\alpha_{i0}^{t+1})} \geq \theta_0^t$$

$$\frac{\sum_{r=1}^{s} u_{r0}^{t+1}(y_{rj}^{t}+\beta_{rj}^{t+1})}{\sum_{i=1}^{m} v_{i0}^{t+1}(x_{ij}^{t}-\alpha_{ij}^{t+1})} \leq 1, j=1,2,\dots,m$$

Using the Charnes–Cooper (1962) transformation, the above fractional programming problem is transformed into the following non-fractional non-linear programming model:

$$Max \sum_{r=1}^{s} u_{r0}^{t+1}(y_{r0}^{t}+\beta_{r0}^{t+1})$$

s.t.

$$\sum_{i=1}^{m} v_{i0}^{t+1}(x_{i0}^{t}-\alpha_{i0}^{t+1}) = 1 \tag{6}$$

$$\sum_{r=1}^{s} u_{r0}^{t+1}(y_{r0}^{t}+\beta_{r0}^{t+1}) - \theta_0^t \sum_{i=1}^{m} v_{i0}^{t+1}(x_{i0}^{t}-\alpha_{i0}^{t+1}) \geq 0$$

$$\sum_{r=1}^{s} u_{r0}^{t+1}(y_{rj}^{t}+\beta_{rj}^{t+1}) - \sum_{i=1}^{m} v_{i0}^{t+1}(x_{ij}^{t}-\alpha_{ij}^{t+1}) \leq 1, j=1,2,\dots,m$$

2.3. Modeling the second level

In the second level, each unit of output, e.g. y_{r0}^{t+1}, will have a profit p_{r0} for DMU$_0$, while each unit of e.g. x_{i0}^{t+1}, will bring a cost c_{i0} to the DMU$_0$. Thus, DMU$_0$ seeks to maximize its profit earned from producing outputs while simultaneously tries to minimize its cost of using inputs. The profit maximization objective is formulated as:

$$Max \sum_{r=1}^{s} p_{r0}(y_{r0}^{t}+\beta_{r0}^{t+1}) \tag{7}$$

And the cost minimization function is formed as

$$Min \sum_{i=1}^{m} c_{i0}(x_{ij}^{t}-\alpha_{i0}^{t+1}) \tag{8}$$

Therefore, the expected profit of next period is formulated as:

$$Max \sum_{r=1}^{s} p_{r0}(y_{r0}^{t}+\beta_{r0}^{t+1}) - \sum_{i=1}^{m} c_{i0}(x_{ij}^{t}-\alpha_{i0}^{t+1}) \tag{9}$$

The lower level of the decision making hierarchy may have some regulatory or operational limitations on the values of $\beta_{r0}^{t+1}, r=1,2,\dots,s$ and $\alpha_{i0}^{t+1}, i=1,2,\dots,m$. Suppose that this set of limitations are specified as Ω.

Therefore, joining all the above equations, the second-level problem is constructed as:

$$Max \sum_{r=1}^{s} p_{r0}(y_{r0}^{t} + \beta_{r0}^{t+1}) - \sum_{i=1}^{m} c_{i0}\left(x_{ij}^{t} - \alpha_{i0}^{t+1}\right)$$

s.t. (10)

$$(\beta_{10}^{t+1}, \beta_{10}^{t+1}, \ldots, \beta_{s0}^{t+1}, \alpha_{10}^{t+1}, \alpha_{20}^{t+1}, \ldots, \alpha_{m0}^{t+1}) \in \Omega$$

Combining Eqs. (6) and (10), the bi-level model of planning based on efficiency is formulated as Eq. (11).

$$Max \sum_{r=1}^{s} u_{r0}^{t+1}(y_{r0}^{t} + \beta_{r0}^{t+1})$$

s.t.

$$\sum_{i=1}^{m} v_{i0}^{t+1}(x_{i0}^{t} - \alpha_{i0}^{t+1}) = 1$$

$$\sum_{r=1}^{s} u_{r0}^{t+1}(y_{r0}^{t} + \beta_{r0}^{t+1}) - \theta_{0}^{t} \sum_{i=1}^{m} v_{i0}^{t+1}(x_{i0}^{t} - \alpha_{i0}^{t+1}) \geq 0$$

$$\sum_{r=1}^{s} u_{r0}^{t+1}\left(y_{rj}^{t} + \beta_{rj}^{t+1}\right) - \sum_{i=1}^{m} v_{i0}^{t+1}\left(x_{ij}^{t} - \alpha_{ij}^{t+1}\right) \leq 1, j = 1,2, \ldots, m \quad (11)$$

$$\max_{\alpha,\beta} \sum_{r=1}^{s} p_{r0}(y_{r0}^{t} + \beta_{r0}^{t+1}) - \sum_{i=1}^{m} c_{i0}\left(x_{ij}^{t} - \alpha_{i0}^{t+1}\right)$$

s.t.

$$(\beta_{10}^{t+1}, \beta_{10}^{t+1}, \ldots, \beta_{s0}^{t+1}, \alpha_{10}^{t+1}, \alpha_{20}^{t+1}, \ldots, \alpha_{m0}^{t+1}) \in \Omega$$

It can be assumed that the last constraint of Eq. (11), i.e. $(\beta_{10}^{t+1}, \beta_{10}^{t+1}, \ldots, \beta_{s0}^{t+1}, \alpha_{10}^{t+1}, \alpha_{20}^{t+1}, \ldots, \alpha_{m0}^{t+1}) \in \Omega$ can be shown in the form of a series of convex functions g and h.

3. Proposed solution model

The concept of bi-level programming has been previously used in DEA. Zhou et al. (2018) considered a hierarchical structure to evaluate the cost efficiencies based on the Stackelberg relationships. They first minimize the total cost of leader and follower, and then, minimize the independent inputs and outputs of the followers. The proposed model is then solved by replacing each follower's model with its associated Kuhn-Tucker condition. Omrani et al. (2018) proposed a bi-level DEA model that considered profit efficiencies of DMUs in the first level and operational efficiencies in the second level. The model is then solved using a fuzzy goal programming approach.

The model in Eq. (11) is a bi-level non-linear programming (hereafter BNLP) problem. Different methods are proposed to solve BNLP problems. Global optimization of BNLP problems were discussed by several authors among them Amouzegar (1999), Gümüş and Floudas (2001), and Tuy et al. (2007). In this paper, the global optimization approach of Gümüş and Floudas (2001) is employed to solve the BNLP problem of Eq. (11). Consider the following BNLP problem:

$$\min_x F(x,y)$$
$$\text{s.t.}$$
$$G(x,y) \leq 0$$
$$H(x,y) = 0$$
$$\tag{12}$$
$$\min_y f(x,y)$$
$$\text{s.t.}$$
$$g_j(x,y) \leq 0, j = 1,2,\dots,J$$
$$h_i(x,y) = 0, i = 1,2,\dots,I$$

where, each of the functions F, G, H, f, g, or h can be nonlinear. If the lower level problem is a convex one, i.e. its objective and constraints are convex; the lower level problem is replaced with its equivalent KKT condition. Therefore, the problem in Eq. (11) is replaced with the single level programming problem of the following form:

$$\min_x F(x,y)$$
$$\text{s.t.}$$
$$G(x,y) \leq 0$$
$$H(x,y) = 0$$
$$h(x,y) = 0$$
$$\frac{\partial f(x,y)}{\partial y} + \sum_{j=1}^{J} \lambda_j \frac{\partial g_j}{\partial y} + \sum_{i=1}^{I} \mu_j \frac{\partial h_i}{\partial y} = 0 \tag{13}$$
$$g_j(x,y) + s_j = 0, j \in J$$
$$\lambda_j - U y_j \leq 0, j \in J$$
$$s_j + U y_j \leq U, j \in J$$
$$\lambda_j, s_j \geq 0, j \in J$$

Since the lower level problem of Eq. (11) is a convex problem, therefore its single level equivalence is formulated based on Eq. (13). Now, consider the constraint (v) in Eq. (13) and suppose that the functions g and h are specified. Then, this inequality can be shown as the following inequalities:

$$-p_{r0}+\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \beta_{r0}^{t+1}}+\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \beta_{r0}^{t+1}}\le 0, r=1,2,\dots,s$$

$$p_{r0}-\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \beta_{r0}^{t+1}}-\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \beta_{r0}^{t+1}}\ge 0, r=1,2,\dots,s \tag{14}$$

$$-c_{i0}+\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \alpha_{i0}^{t+1}}+\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \alpha_{i0}^{t+1}}\ge 0, i\ \ 1,2,\dots,m$$

$$c_{i0}-\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \alpha_{i0}^{t+1}}-\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \alpha_{i0}^{t+1}}\le 0, i\ \ 1,2,\dots,m$$

Hence, the final single level model is formulated as:

$$Max\sum_{r=1}^{s}u_{r0}^{t+1}(y_{r0}^{t}+\beta_{r0}^{t+1})$$

s.t.

$$\sum_{i=1}^{m}v_{i0}^{t+1}(x_{i0}^{t}-\alpha_{i0}^{t+1})=1$$

$$\sum_{r=1}^{s}u_{r0}^{t+1}(y_{r0}^{t}+\beta_{r0}^{t+1})-\theta_0^t\sum_{i=1}^{m}v_{i0}^{t+1}(x_{i0}^{t}-\alpha_{i0}^{t+1})\ge 0$$

$$\sum_{r=1}^{s}u_{r0}^{t+1}\left(y_{rj}^{t}+\beta_{rj}^{t+1}\right)-\sum_{i=1}^{m}v_{i0}^{t+1}\left(x_{ij}^{t}-\alpha_{ij}^{t+1}\right)\le 1, j=1,2,\dots,m$$

$$-p_{r0}+\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \beta_{r0}^{t+1}}+\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \beta_{r0}^{t+1}}\le 0, r=1,2,\dots,s \tag{15}$$

$$p_{r0}-\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \beta_{r0}^{t+1}}-\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \beta_{r0}^{t+1}}\le 0, r=1,2,\dots,s$$

$$-c_{i0}+\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \alpha_{i0}^{t+1}}+\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \alpha_{i0}^{t+1}}\le 0, i\ \ 1,2,\dots,m$$

$$c_{i0}-\sum_{j=1}^{J}\lambda_j\frac{\partial g_j}{\partial \alpha_{i0}^{t+1}}-\sum_{i=1}^{I}\mu_j\frac{\partial h_i}{\partial \alpha_{i0}^{t+1}}\le 0, i\ \ 1,2,\dots,m$$

$$\lambda_j-Uy_j\le 0, j\in J$$

$$s_j+Uy_j\le U, j\in J$$

$$\lambda_j, s_j\ge 0, j\in J$$

In case managers have some limitations or requirements on the amount of input decreasing or output increasing, these limitations can be worked into the model. As a case in point, if managers determined a minimum (maximum) percent a of decreasing over input i, i.e. $\alpha_{i0}^{t+1} \leq a x_{i0}^{t}$ ($\alpha_{i0}^{t+1} \geq a x_{i0}^{t}$), or similarly a minimum (maximum) percent b of increasing in output r, i.e. $\beta_{r0}^{t+1} \leq b y_{r0}^{t}$ ($\beta_{r0}^{t+1} \geq b y_{r0}^{t}$), these constraints can be added to the first level problem. However, assuming that these additional constraints make the problem infeasible, they must be relaxed. The significance and value of the proposed method can be seen based upon the presentation of a real world case study described in the next section.

4. Case study

4.1. Background

In this section, we present a real-life case study in the banking industry to demonstrate the applicability and exhibit the efficacy of the proposed method. DEA has been widely used in banking (Emrouznejad et al., 2008). The data presented here are taken from Peoples Bank[1], a private bank, in Iran with over 45 branches nationwide. The bank headquarters in Tehran annually announces the goals for the upcoming year in the following areas:

a) Deposits;

b) Loans;

c) Incomes; and

d) Cost.

In the following section, we explain all the variables used as inputs and outputs in this study:

- **Inputs:**
  - Non-interest expenses (x_1): Non-interest expenses include a wide range of operating and overhead expenses such as salaries, benefits, rent, utilities, insurance, information technology expenses, legal expenses, and other fixed assets.
  - Interest expenses (x_2): Interest expenses are incurred from various deposits, short-term loans, long-term loans, and trading account liabilities.
- **Outputs:**
  - Checking deposits (y_1): A deposit account against which checks or drafts may be

[1] The name is changed to protect the anonymity of the bank.

written for checking, savings and money market accounts.

- Savings deposits (y_2): Interest paying accounts, typically at below-market interest rates that do not have a specific maturity and can be withdrawn upon demand.
- Money market deposits (y_3): A money market deposit account often has limited check-writing privileges and pays an interest rate that is typically higher than a regular savings account but lower than a certificate of deposit or other time deposits.
- Certificates of deposits (CD) (y_4): A CD is a promissory note issued by a bank. It is a time deposit that restricts account holders from withdrawing funds on demand.
- Individual retirement account (IRA) deposits (y_5): An IRA is an account that allows an individual to save for retirement with tax-free growth or on a tax-deferred basis, depending on the type of IRA.
- Fixed rate loans (y_6): A fixed interest rate loan is a loan where the interest rate doesn t fluctuate during the fixed rate period of the loan.
- Variable rate loans (y_7): Variable interest rate loan is a loan in which the interest rate charged on the outstanding balance varies as market interest rates change.
- Installment loans (y_8): An installment loan is a loan that is repaid over time with a set number of scheduled payments; normally at least two payments are made towards the loan.
- Secured loans (y_9): A secured loan is a loan in which the borrower pledges some asset as collateral for the loan, which then becomes a secured debt owed to the creditor who gives the loan.
- Unsecured loans (y_{10}): An unsecured loan is a loan that is issued and supported only by the borrower s creditworthiness, rather than by any type of collateral.
- Convertible rate loans (y_{11}): Debt financing is when an investor loans funds to a business with a set interest rate.
- Personal account fees (y_{12}): Bank fees are nominal fees for a variety of account set-up and maintenance, and minor transactional services for personal customers.
- Business account fees (y_{13}): Bank fees are nominal fees for a variety of account set-up and maintenance, and minor transactional services for retail and business customers.
- Miscellaneous income (y_{14}): This income may be gained in property, money or other assets from royalties, rents, prizes, dividends, interest, or any other income otherwise

unreported (Investopedia; Wikipedia).

Table 1 presents a list of the two inputs and 14 outputs used in this study.

Insert Table 1 Here

Let $\{x_{1j}^t, x_{2j}^t, x_{3j}^t, x_{4j}^t, y_{1j}^t, y_{2j}^t, y_{3j}^t, y_{4j}^t\}$ be the input-output vector of DMU_j at the current period. The planning section of bank wants to determine the values of $\{x_{1j}^{t+1}, x_{2j}^{t+1}, x_{3j}^{t+1}, x_{4j}^{t+1}, y_{1j}^{t+1}, y_{2j}^{t+1}, y_{3j}^{t+1}, y_{4j}^{t+1}\}$ for the next upcoming time period. At the upper level, bank management should determine the positive values $\{\beta_{1j}, \beta_{2j}, \beta_{3j}, \beta_{4j}\}$ for outputs such that $y_{rj}^{t+1} = y_{rj}^t + \beta_{rj}, r = 1,2,3,4$ and the positive values $\{\alpha_{1j}, \alpha_{2j}, \alpha_{3j}, \alpha_{4j}\}$ for inputs such that $x_{ij}^{t+1} = x_{ij}^t - \alpha_{ij}, i = 1,2,3,4$ in a way that the efficiency of DMUj at period $t + 1$ becomes larger than its efficiency at the current period *t*. On the other hand, at the second level of planning, each DMU wants to determine its levels of outputs for the next period while maximizing their profits and simultaneously determines its levels of inputs such that it minimizes their costs. The considered problem is named *targeting inputs/ outputs level based on efficiency forecasting*. The bank sets the following policies about its branches inputs/ outputs:

a) $y_7 + y_8 + y_9 + y_{10} + y_{11} \le 80\%(y_3 + y_4)$
b) $y_7 \le 30\%(y_7 + y_8 + y_9 + y_{10} + y_{11})$
c) $y_8 \le 30\%(y_7 + y_8 + y_9 + y_{10} + y_{11})$
d) $y_9 \le 15\%(y_7 + y_8 + y_9 + y_{10} + y_{11})$
e) $y_{10} \le 15\%(y_7 + y_8 + y_9 + y_{10} + y_{11})$
f) $y_{11} \le 5\%(y_7 + y_8 + y_9 + y_{10} + y_{11})$

Furthermore, according to the economic situation, the bank policy makers believe that the maximum amount of decrease in input variables should be lower than 30% of the current inputs, while it is expected that branches can increase their outputs in the range of 10% - 50%. Since it might possible that these constraints made the model infeasible along with the constraint of targeted efficiency being more than its current efficiency, if the model becomes infeasible, the efficiency constraint can be relaxed.

4.2. Results

To apply the proposed method in this case, first the current CCR efficiencies of the DMUs are computed using the MaxDEA software. Table 2 illustrates the obtained results. Moreover, the following regulations are implied over the identified inputs/outputs. Considering DMU 1001, its current inputs/outputs are illustrated in Table 3.

Insert Tables 2 and 3 Here

Using the information from the above table, the model in Eq. (11) can be formulated as follows:

$$
\begin{aligned}
&\min_{u,v} u^1_{1,1001}\left(105292334097 + \beta^1_{1.1001}\right) + u^1_{2,1001}\left(1955255295 + \beta^1_{2,1001}\right) + \cdots \\
&\qquad\qquad + u^1_{14,1001}\left(269602 + \beta^1_{14,1001}\right) \\
&\text{s.t.} \\
&v^1_{1,1001}\left(49960905 - \alpha^1_{1,1001}\right) + v^1_{2,1001}\left(1955255295 - \alpha^1_{2,1001}\right) \quad 1 \\
&u^1_{1,1001}\left(105292334097 + \beta^1_{1,1001}\right) + u^1_{2,1001}\left(1955255295 + \beta^1_{2,1001}\right) + \cdots + u^1_{14,1001}\left(269602\right. \\
&\qquad\qquad \left.+ \beta^1_{14,1001}\right) - v^1_{1,1001}\left(49960905 - \alpha^1_{1,1001}\right) - v^1_{2,1001}\left(1955255295 - \alpha^1_{2,1001}\right) \\
&\qquad\qquad \geq 0 \\
&u^1_{1,1001}\left(105292334097 + \beta^1_{1,1001}\right) + u^1_{2,1001}\left(1955255295 + \beta^1_{2,1001}\right) + \cdots + u^1_{14,1001}\left(269602\right. \\
&\qquad\qquad \left.+ \beta^1_{14,1001}\right) - v^1_{1,1001}\left(49960905 - \alpha^1_{1,1001}\right) - v^1_{2,1001}\left(1955255295 - \alpha^1_{2,1001}\right) \\
&\qquad\qquad \leq 0 \\
&u^1_{1,1001}\left(292087864495 + \beta^1_{1,1002}\right) + u^1_{2,1001}\left(1270011337 + \beta^1_{2,1002}\right) + \cdots + u^1_{14,1001}\left(103174\right. \\
&\qquad\qquad \left.+ \beta^1_{14,1002}\right) - v^1_{1,1001}\left(49960905 - \alpha^1_{1,1002}\right) + v^1_{2,1001}\left(1741020126 - \alpha^1_{2,1002}\right) \\
&\qquad\qquad \leq 0 \\
&\qquad\qquad\qquad\qquad \vdots \\
&u^1_{1,1001}\left(6119116105 + \beta^1_{1,1034}\right) + u^1_{2,1001}\left(450000 + \beta^1_{2,1034}\right) + \cdots + u^1_{14,1001}\left(355289\right. \\
&\qquad\qquad \left.+ \beta^1_{14,1034}\right) - v^1_{1,1001}\left(291173327 - \alpha^1_{1,1034}\right) - v^1_{2,1001}\left(1020785582 - \alpha^1_{2,1034}\right) \\
&\qquad\qquad \leq 0
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\min_{\alpha,\beta} -0.04\left(13691426672 + \beta^1_{6,1004}\right) - 0.175\left(894934898450 + \beta^1_{7,1004}\right) \\
&\qquad\qquad - 0.175\left(26384060006 + \beta^1_{8,1004}\right) - 0.18\left(55427596141 + \beta^1_{9,1004}\right) \\
&\qquad\qquad - 0.12\left(141375407693 + \beta^1_{10,1004}\right) - 0.05\left(9724164019 + \beta^1_{11,1004}\right) \\
&\qquad\qquad - 0.15\left(125659100000 + \beta^1_{3,1004}\right) - 0.10\left(1049767301275 + \beta^1_{4,1001}\right) \\
&\qquad\qquad + \left(49960905 - \alpha^1_{1,1001}\right) + \left(1955255295 - \alpha^1_{2,1001}\right)
\end{aligned}
$$

s.t.

$$
\begin{aligned}
&\left(894934898450+\beta^1_{7,1004}\right)+\left(26384060006+\beta^1_{8,1004}\right)+\left(55427596141+\beta^1_{9,1004}\right)\\
&\qquad+\left(141375407693+\beta^1_{10,1004}\right)+\left(9724164019+\beta^1_{11,1004}\right)\\
&\qquad-0.8\left[\left(125659100000+\beta^1_{3,1004}\right)+\left(1049767301275+\beta^1_{4,1001}\right)\right]\leq\\
&\left(894934898450+\beta^1_{7,1004}\right)\\
&\qquad\leq 0.3\left[\left(894934898450+\beta^1_{7,1004}\right)+\left(26384060006+\beta^1_{8,1004}\right)\right.\\
&\qquad+\left(55427596141+\beta^1_{9,1004}\right)+\left(141375407693+\beta^1_{10,1004}\right)\\
&\qquad\left.+\left(9724164019+\beta^1_{11,1004}\right)\right]\\
&\left(26384060006+\beta^1_{8,1004}\right)\\
&\qquad\leq 0.3\left[\left(894934898450+\beta^1_{7,1004}\right)+\left(26384060006+\beta^1_{8,1004}\right)\right.\\
&\qquad+\left(55427596141+\beta^1_{9,1004}\right)+\left(141375407693+\beta^1_{10,1004}\right)\\
&\qquad\left.+\left(9724164019+\beta^1_{11,1004}\right)\right]\\
&\left(55427596141+\beta^1_{9,1004}\right)\\
&\qquad\leq 0.15\left[\left(894934898450+\beta^1_{7,1004}\right)+\left(26384060006+\beta^1_{8,1004}\right)\right.\\
&\qquad+\left(55427596141+\beta^1_{9,1004}\right)+\left(141375407693+\beta^1_{10,1004}\right)\\
&\qquad\left.+\left(9724164019+\beta^1_{11,1004}\right)\right]\\
&\left(141375407693+\beta^1_{10,1004}\right)\\
&\qquad\leq 0.15\left[\left(894934898450+\beta^1_{7,1004}\right)+\left(26384060006+\beta^1_{8,1004}\right)\right.\\
&\qquad+\left(55427596141+\beta^1_{9,1004}\right)+\left(141375407693+\beta^1_{10,1004}\right)\\
&\qquad\left.+\left(9724164019+\beta^1_{11,1004}\right)\right]\\
&\left(9724164019+\beta^1_{11,1004}\right)\\
&\qquad\leq 0.05\left[\left(894934898450+\beta^1_{7,1004}\right)+\left(26384060006+\beta^1_{8,1004}\right)\right.\\
&\qquad+\left(55427596141+\beta^1_{9,1004}\right)+\left(141375407693+\beta^1_{10,1004}\right)\\
&\qquad\left.+\left(9724164019+\beta^1_{11,1004}\right)\right]
\end{aligned}
$$

Using the aforementioned method, the model in Eq. (16) is transformed into the equivalent single objective problem by substituting the lower level problem by its KKT condition. Using this transformation, the above model is transformed into the following form:

$$\min_{u,v} u_{1,1001}^{1}(105292334097+\beta_{1.1001}^{1})+u_{2,1001}^{1}\left(1955255295+\beta_{2,1001}^{1}\right)+\cdots$$
$$+u_{14,1001}^{1}\left(269602+\beta_{14,1001}^{1}\right)$$

s.t.

$$v_{1,1001}^{1}\left(49960905-\alpha_{1,1001}^{1}\right)+v_{2,1001}^{1}\left(1955255295-\alpha_{2,1001}^{1}\right) \quad 1$$

$$u_{1,1001}^{1}\left(105292334097+\beta_{1,1001}^{1}\right)+u_{2,1001}^{1}\left(1955255295+\beta_{2,1001}^{1}\right)+\cdots+u_{14,1001}^{1}\left(269602+\beta_{14,1001}^{1}\right)-v_{1,1001}^{1}\left(49960905-\alpha_{1,1001}^{1}\right)-v_{2,1001}^{1}\left(1955255295-\alpha_{2,1001}^{1}\right)\geq 0$$

$$u_{1,1001}^{1}\left(105292334097+\beta_{1,1001}^{1}\right)+u_{2,1001}^{1}\left(1955255295+\beta_{2,1001}^{1}\right)+\cdots+u_{14,1001}^{1}\left(269602+\beta_{14,1001}^{1}\right)-v_{1,1001}^{1}\left(49960905-\alpha_{1,1001}^{1}\right)-v_{2,1001}^{1}\left(1955255295-\alpha_{2,1001}^{1}\right)\leq 0$$

$$u_{1,1001}^{1}\left(292087864495+\beta_{1,1002}^{1}\right)+u_{2,1001}^{1}\left(1270011337+\beta_{2,1002}^{1}\right)+\cdots+u_{14,1001}^{1}\left(103174+\beta_{14,1002}^{1}\right)-v_{1,1001}^{1}\left(49960905-\alpha_{1,1002}^{1}\right)+v_{2,1001}^{1}\left(1741020126-\alpha_{2,1002}^{1}\right)\leq 0$$

$$\vdots \tag{17}$$

$$u_{1,1001}^{1}\left(6119116105+\beta_{1,1034}^{1}\right)+u_{2,1001}^{1}\left(450000+\beta_{2,1034}^{1}\right)+\cdots+u_{14,1001}^{1}\left(355289+\beta_{14,1034}^{1}\right)-v_{1,1001}^{1}\left(291173327-\alpha_{1,1034}^{1}\right)-v_{2,1001}^{1}\left(1020785582-\alpha_{2,1034}^{1}\right)\leq 0$$

$$-0.8\lambda_1 \leq 0$$
$$0.8\lambda_1 \geq 0$$
$$0.15-0.8\lambda_2 \leq 0$$
$$-0.15+0.8\lambda_2 \geq 0$$
$$0.10-0.8\lambda_2 \leq 0$$
$$-0.10+0.8\lambda_2 \geq 0$$
$$0.04-\lambda_1 \leq 0$$
$$-0.04+\lambda_1 \geq 0$$
$$-0.175+\lambda_2+0.7\lambda_3-0.3\lambda_4-0.15\lambda_5-0.15\lambda_6-0.05\lambda_7 \geq 0$$
$$0.175-\lambda_2-0.7\lambda_3+0.3\lambda_4+0.15\lambda_5+0.15\lambda_6+0.05\lambda_7 \leq 0$$
$$-0.175+\lambda_2-0.3\lambda_3+0.7\lambda_4-0.15\lambda_5-0.15\lambda_6-0.05\lambda_7 \geq 0$$
$$0.175-\lambda_2+0.3\lambda_3-0.7\lambda_4+0.15\lambda_5+0.15\lambda_6+0.05\lambda_7 \leq 0$$

$-0.18 + \lambda_2 - 0.3\lambda_3 - 0.3\lambda_4 + 0.85\lambda_5 - 0.15\lambda_6 - 0.05\lambda_7 \geq 0$

$0.18 - \lambda_2 + 0.3\lambda_3 + 0.3\lambda_4 - 0.85\lambda_5 + 0.15\lambda_6 + 0.05\lambda_7 \leq 0$

$-0.12 + \lambda_2 - 0.3\lambda_3 - 0.3\lambda_4 - 0.15\lambda_5 + 0.85\lambda_6 - 0.05\lambda_7 \geq 0$

$0.12 - \lambda_2 + 0.3\lambda_3 + 0.3\lambda_4 + 0.15\lambda_5 - 0.85\lambda_6 + 0.05\lambda_7 \leq 0$

$-0.02 + \lambda_2 - 0.3\lambda_3 - 0.3\lambda_4 - 0.15\lambda_5 - 0.15\lambda_6 + 0.95\lambda_7 \geq 0$

$0.02 - \lambda_2 + 0.3\lambda_3 + 0.3\lambda_4 + 0.15\lambda_5 + 0.15\lambda_6 - 0.95\lambda_7 \leq 0$

$\lambda_j - Uy_j \leq 0, j = 1,2, \dots ,7$

$s_j + Uy_j \leq U, j = 1,2, \dots ,7$

$\lambda_j, s_j \geq 0, j = 1,2, \dots ,7$

$y_j \in \{0,1\}, j = 1,2, \dots ,7$

Since the constraints $0.15 - 0.8\lambda_2 \leq 0$ and $-0.15 + 0.8\lambda_2 \geq 0$ along with $0.10 - 0.8\lambda_2 \leq 0$ and $-0.10 + 0.8\lambda_2 \geq 0$ make the model infeasible, and since the two later inequalities are the subset of two first ones, to solve the model only the inequalities $0.10 - 0.8\lambda_2 \leq 0$ and $-0.10 + 0.8\lambda_2 \geq 0$ are considered.

At the first step, the model is solved without any limitation on the allowable amount of decreasing input or increasing output. At the obtained results, some of the targeted input values are degraded to zero for many DMUs. Therefore, managers decided to set the following limitations on the α and β values:

(I) Maximum allowable decrease in inputs must be lower than 30 percent, i.e. $\alpha_{i,\mathrm{j}}^1 \leq 0.3x_{ij}^t$.

(II) Outputs must be increased at least 10 percent and no DMU is required to increase none of its outputs more than 50 percent.

This model is solved using Lingo global optimization. The obtained results are shown in Table 4. The values under the last two columns, $\alpha_{1,j}^1$ and $\alpha_{2,j}^1$, illustrate the amount of decrease targeted on input variables while the first 14 columns illustrate the amount of increase targeted on output variables. It is noted that in DMUs 1005, 1008, 1009, 1010, 1012, 1013, 1015, 1024, 1028, and 1032 the efficiency constraint is relaxed.

Insert Table 4 Here

4.3. Comparison with the current state

To compare the performance of the proposed method, in this section the efficiency and profit of DMUs under proposed targets are compared with their current condition. Figure 2 illustrates the obtained efficiency scores of DMUs by running the CCR model on current and targeted inputs/ outputs. As is clear, the efficient units maintain their efficiency while inefficient units generally improve their efficiencies.

Insert Figure 2 Here

According to Figure 2 the efficiency of units appraised upon their target inputs/ outputs improved with regard to their current performance. This was the first level objective assumed by the proposed model. Figure 3 presents the percentage of increase in branches profit obtained as $(\text{target profit} - \text{current profit})/\text{current profit}$. This figure illustrates a 22.96% increase of profit on the average. This objective was assumed in the second level of the proposed model.

Insert Figure 3 Here

4.4. Comparison with the CCR projections

The CCR model proposes a projection for inefficient DMUs to become efficient. The profit obtained with the CCR projection compared with the target profit in Figure 4.

Insert Figure 4 Here

Since there is no clear evidence of the superiority of the CCR projections over the proposed target, a paired t-test was performed to test the equality of the obtained profit from the CCR projection and the proposed target. The p-value of 0.934 indicates that there is no significant difference between the CCR projection profit and the target profit. A comparison among range of change in inputs and outputs using dot-plots are illustrated in Figure 5.

Insert Figure 5 Here

As shown in Figure 5, there are some extraordinary changes proposed by the CCR projection while the proposed method maintains the amount of the aforementioned changes in an acceptable range provided by bank managers. For example, according to Fig 5(a), the CCR projection proposed some decrease in input x_1 as follow: DMU 1009: 81%, DMU 1016: 95%, DMU 1018: 80%, DMU 1024: 96%, and more than 90% for DMUs 1031, 1032, and 1033. In practice, the amount of this decrease in input resources is usually impossible. Considering the amount of decrease targeted by the proposed method, the maximum decrease is maintained at 30% determined by bank managers as a possible target.

Moreover, according to Fig 5(b) for output variable y_1, CCR proposed an increase of 273% for DMU 1008, 453% for DMU 1010, 284% for DMU 1023, more than 700% for DMU 1028, and an increase of 226% for DMU 1029. Note that reaching these objectives in real world situations would be essentially impossible. However, the maximum increase of output variable y_1 targeted by the proposed method is 50% which seems more achievable. A similar comparison is possible for other input/ output variables.

On the other hand, the CCR model doesn't propose any changes in inputs and outputs of = CCR efficient units and cannot act as a goal setting tool for these units. According to the proposed method, this limitation is alleviated by imposing targets of efficient units is shown in Table 4.

5. Conclusion

DEA is a well-known and widely used method for performance evaluation of a set of homogeneous units. It usually performs as a posterior analysis tool. One of the side results of classic DEA models is their ability to determine some projections for inefficient units to become efficient. However, some limitations make the targeting result of classic DEA impractical. One of these limitations is that DEA doesn't propose any target for a DEA efficient unit, and it imposes the current status of inputs/outputs as the target one. On the other hand, sometimes DEA proposes some extraordinary targets for the DMU's=inputs and outputs. Furthermore, DEA doesn't=consider profit as an important performance measure in its targeting process as it just focuses on units' efficiency.=

The proposed method of targeting in this paper tries to address all of these limitations. First, considering a bi-level model for target setting, both efficiency and profit are considered in target determination. On the other hand, limiting the amount of changes in inputs and outputs prevented the model to propose unachievable targets for measures. Eventually, the proposed method predicted some targets for efficient units beyond the inefficient ones. By investigating the application of the proposed model in a real world case study, its advantages were highlighted. Briefly, the main advantages of the proposed method can be pointed as (1) considering profit as a targeting objective along with efficiency, (2) prohibiting inputs and outputs from unusual changes, and (3) proposing targets for efficient and inefficient units.

There are two limitations in the current study. First, the proposed model is bi-level and a multi-level formulation of the proposed model has many real-life applications. Enhancing the model in multi-period and network DEA settings and considering target setting for undesirable outputs is a potential for future research. Second, for qualitative and dubious variables, new uncertain approaches such as intuitionistic and hesitant sets for BLP or MLP DEA models are worthy of investigation.

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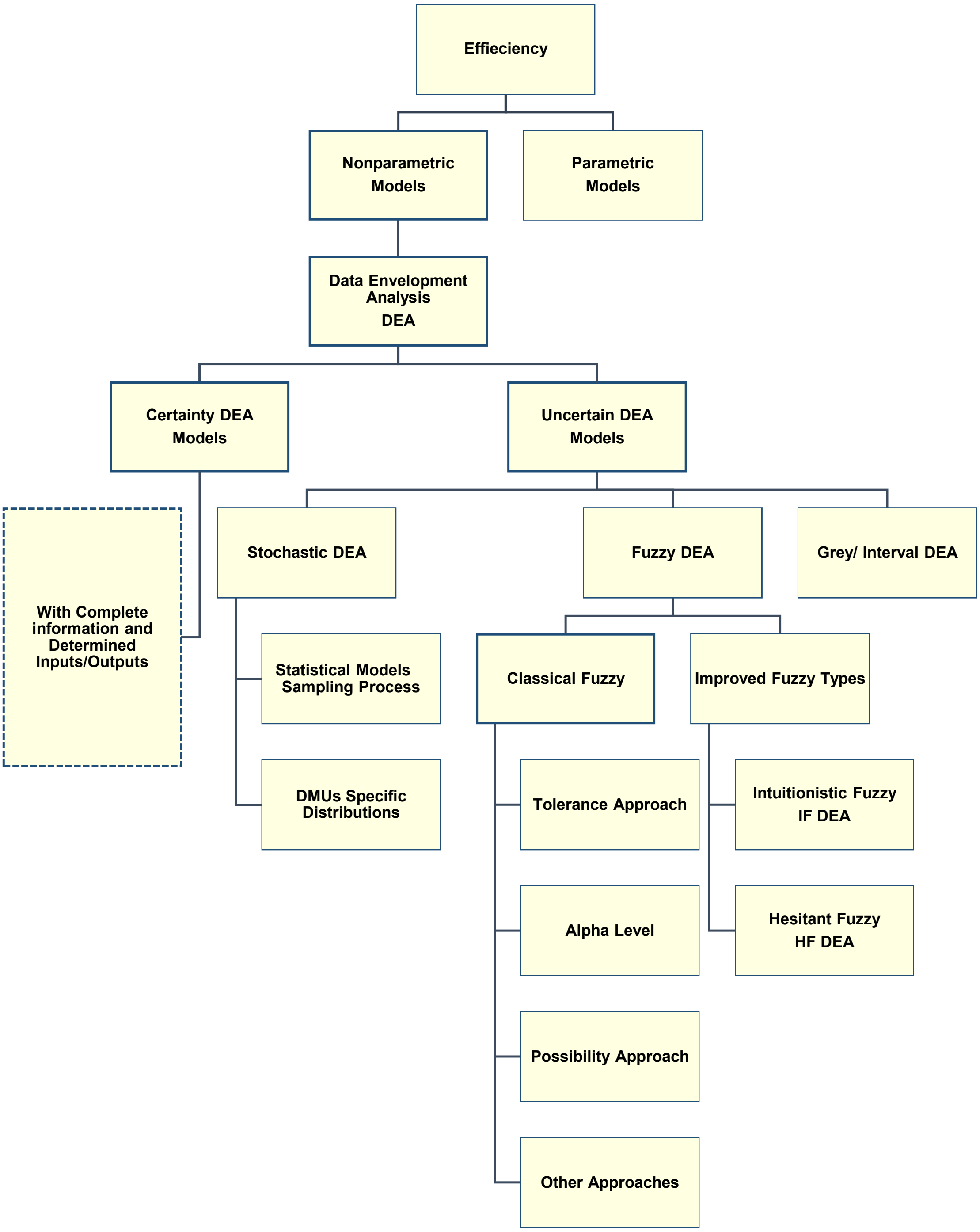


Figure 1. A classification of certain and uncertain DEA approaches

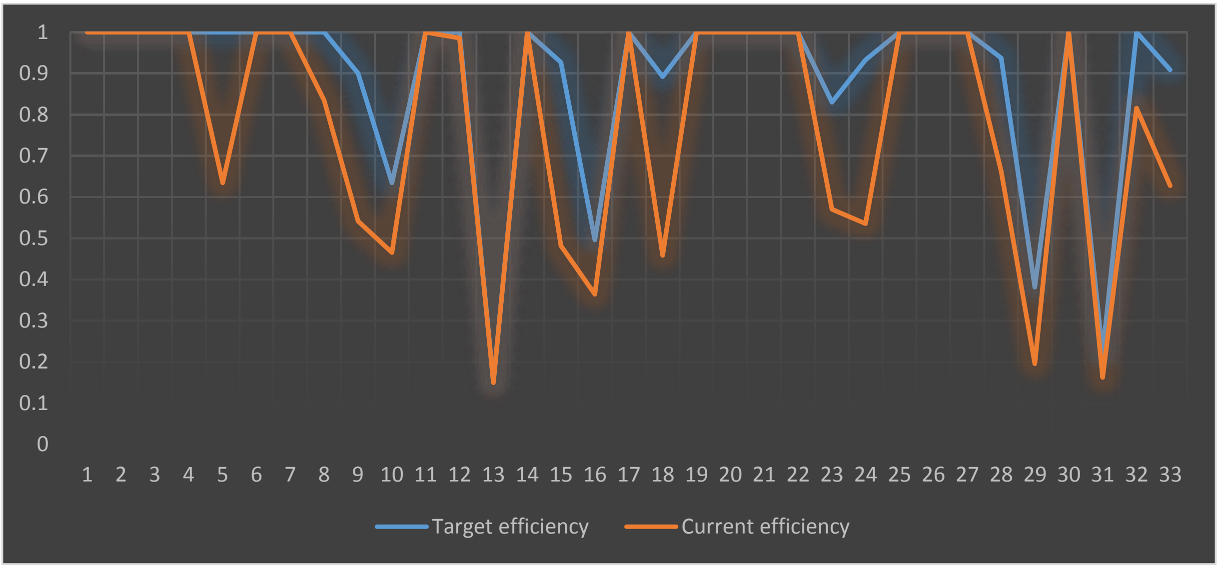


Figure 2. Comparison of the CCR efficiencies over the current state with targets

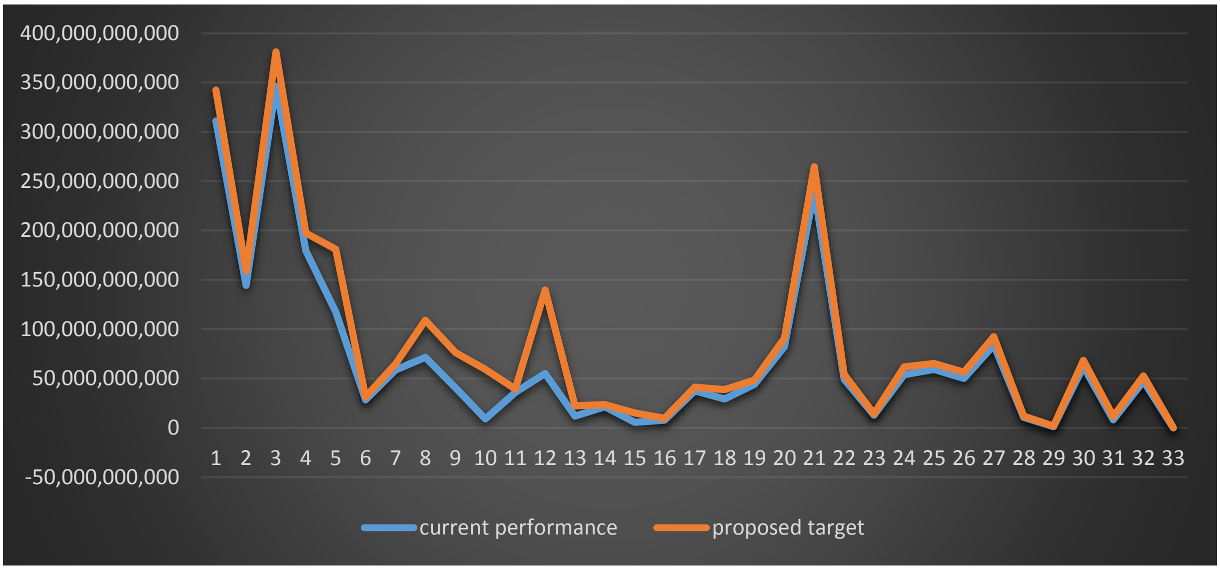


Figure 3. Comparison of profit between the current and target situation

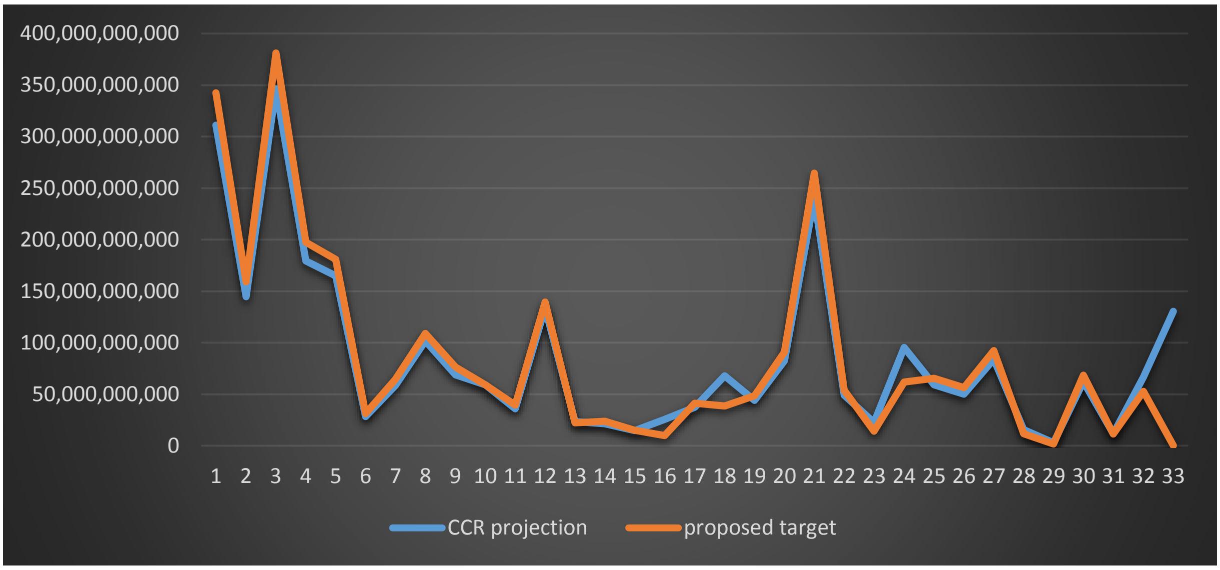
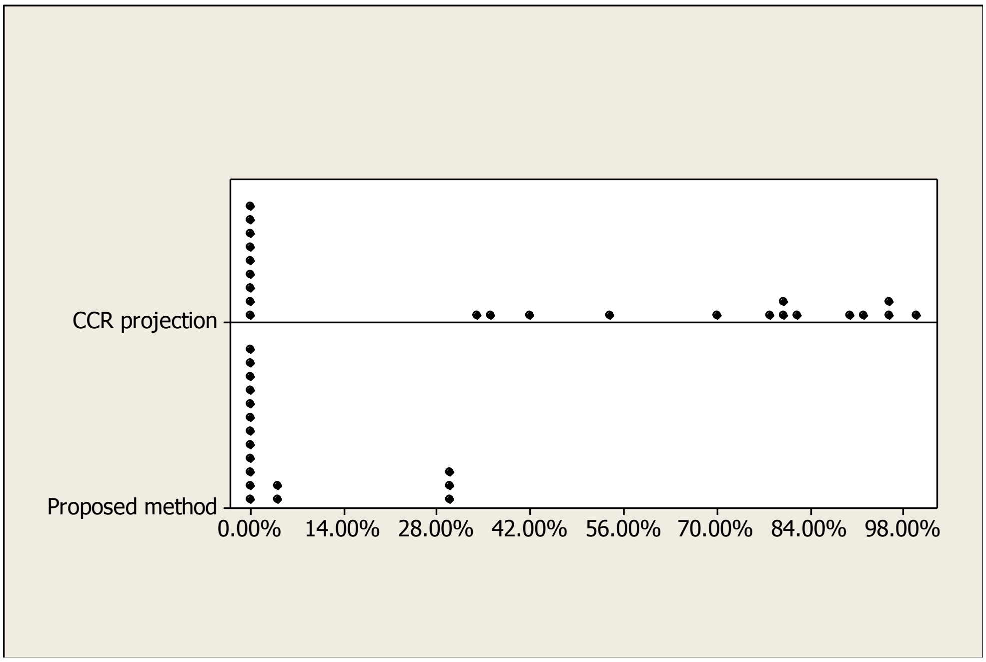
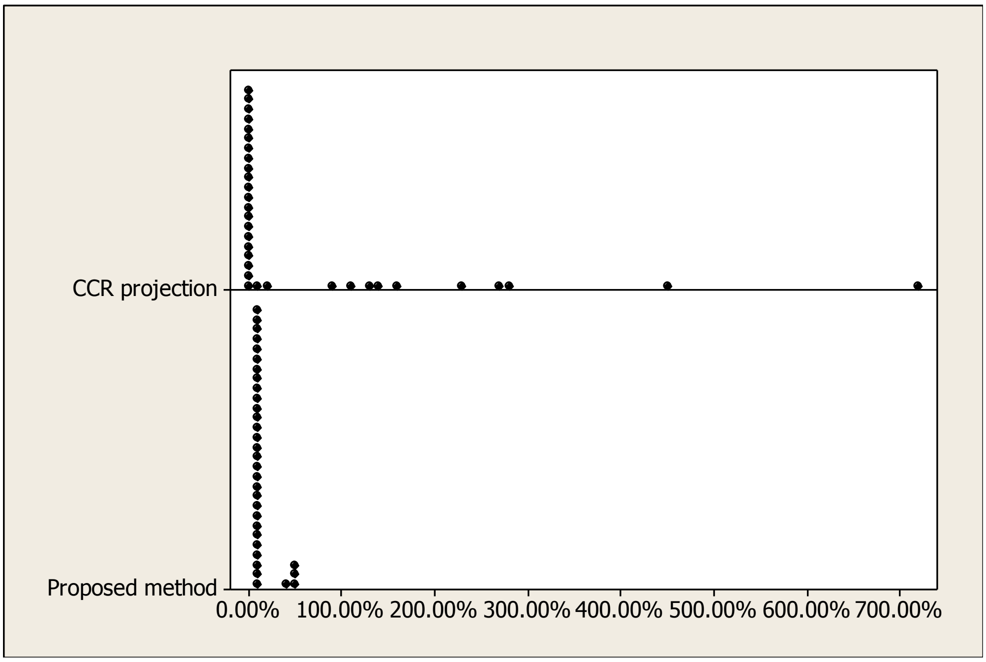


Figure 4. Comparison of profit between the CCR projection and target situation



ı) Percentage of decrease in input variable x_1 proposed by the CCR method and the proposed on



ı) Percentage of increase in output variable y_1 proposed by the CCR method and the proposed or

Figure 5. Dot-plots for percentage of change in inputs/ outputs with CCR projection and proposed method

Table 1. Input and output measures

Inputs	Outputs
• Non-interest expenses (x_1) • Interest expenses (x_2)	• Checking deposits (y_1) • Savings deposits (y_2) • Money market deposits (y_3) • Certificates of deposits (y_4) • Individual retirement account deposits (y_5) • Fixed rate loans (y_6) • Variable rate loans (y_7) • Installment loans (y_8) • Secured loans (y_9) • Unsecured loans (y_{10}) • Convertible rate loans (y_{11}) • Personal account fees (y_{12}) • Business account fees (y_{13}) • Miscellaneous income (y_{14})

Table 2. The CCR efficiencies

DMU	Efficiency (θ_0^1)
1001	1.000000
1002	1.000000
1003	1.000000
1004	1.000000
1005	0.634398
1006	1.000000
1007	1.000000
1008	0.834975
1009	0.541579
1010	0.465600
1011	1.000000
1012	0.985832
1013	0.150079
1014	1.000000
1015	0.482263
1016	0.364553
1017	1.000000
1018	0.458749
1019	1.000000
1020	1.000000
1021	1.000000
1022	1.000000
1023	0.570452
1024	0.535564
1025	1.000000
1026	1.000000
1027	1.000000
1028	0.662564
1029	0.195974
1030	1.000000
1031	0.162742
1032	0.816185
1033	0.628116

Table 3. Input/output vector of DMU 1001

Input/output	magnitude
$x_{1,1001}^{1}$	49960905
$x_{2,1001}^{1}$	1955255295
$y_{1,1001}^{1}$	105292334097
$y_{2,1001}^{1}$	4076266898
$y_{3,1001}^{1}$	125659100000
$y_{4,1001}^{1}$	1049767301275
$y_{5,1001}^{1}$	2685401958
$y_{6,1001}^{1}$	13691426672
$y_{7,1001}^{1}$	894934898450
$y_{8,1001}^{1}$	26384060006
$y_{9,1001}^{1}$	55427596141
$y_{10,1001}^{1}$	141375407693
$y_{11,1001}^{1}$	9724164019
$y_{12,1001}^{1}$	10412683233
$y_{13,1001}^{1}$	9675000
$y_{14,1001}^{1}$	269602

Table 4. The amount of decreasing input and increasing output

DMU	$\beta^1_{1,j}$	$\beta^1_{2,j}$	$\beta^1_{3,j}$	$\beta^1_{4,j}$	$\beta^1_{5,j}$	$\beta^1_{6,j}$	$\beta^1_{7,j}$	$\beta^1_{8,j}$	$\beta^1_{9,j}$	$\beta^1_{10,j}$	$\beta^1_{11,j}$	$\beta^1_{12,j}$	$\beta^1_{13,j}$	$\beta^1_{14,j}$	$\alpha^1_{1,j}$	$\alpha^1_{2,j}$
1	1.05E+10	4.08E+08	1.26E+10	1.05E+11	2.69E+08	1.37E+09	8.95E+10	2.64E+09	5.54E+09	1.41E+10	9.72E+08	1.04E+09	9.68E+05	2.70E+04	1.23E+00	1.23E+00
2	2.92E+10	1.27E+08	2.98E+09	3.00E+10	1.50E+09	6.55E+08	5.72E+10	5.88E+08	2.37E+09	5.03E+09	6.49E+08	8.26E+08	2.46E+07	1.03E+04	1.23E+00	1.23E+00
3	3.31E+09	1.49E+08	2.51E+09	1.10E+10	5.89E+09	1.30E+08	1.72E+11	1.53E+10	3.09E+06	5.36E+09	5.21E+07	5.67E+08	1.24E+07	6.24E+05	1.23E+00	1.23E+00
4	2.15E+10	8.71E+07	2.70E+09	3.56E+10	1.15E+09	1.91E+09	1.59E+08	2.61E+07	2.61E+07	1.17E+11	2.05E+07	8.55E+08	9.50E+04	3.91E+05	1.23E+00	1.23E+00
5	2.29E+09	3.78E+07	1.27E+09	2.49E+10	2.93E+07	4.59E+08	4.66E+10	1.10E+08	2.77E+10	8.54E+08	1.99E+08	7.63E+07	6.30E+05	2.45E+07	1.23E+00	1.23E+00
6	1.83E+09	1.53E+08	1.89E+09	1.39E+10	1.08E+07	6.37E+08	5.33E+09	4.75E+08	5.31E+07	1.73E+09	7.41E+08	5.19E+07	1.16E+06	8.31E+05	1.23E+00	1.23E+00
7	4.10E+09	7.30E+07	1.31E+09	1.68E+10	1.33E+09	1.48E+08	1.64E+10	6.31E+09	1.68E+07	1.48E+09	3.83E+08	1.72E+08	4.57E+06	8.79E+05	1.23E+00	1.23E+00
8	2.33E+09	7.10E+07	8.81E+09	2.27E+10	3.34E+08	2.79E+08	6.52E+09	1.10E+10	2.61E+07	5.00E+09	4.05E+09	1.15E+08	3.33E+07	3.63E+05	1.23E+00	1.23E+00
9	5.57E+08	4.54E+08	2.35E+10	7.77E+09	9.96E+07	2.11E+08	1.70E+09	1.83E+10	1.42E+08	1.95E+09	8.96E+08	4.01E+07	4.50E+05	1.13E+04	2.39E+08	4.75E+08
10	4.15E+08	5.82E+08	1.45E+09	4.97E+09	3.18E+06	2.97E+07	1.55E+09	8.10E+07	8.43E+06	5.66E+08	1.63E+08	2.84E+07	2.40E+06	2.95E+04	1.23E+00	1.23E+00
11	5.80E+08	9.44E+07	1.57E+10	6.57E+09	3.04E+07	1.63E+08	2.50E+09	2.37E+06	5.75E+07	1.77E+09	1.46E+08	2.42E+07	5.11E+05	1.02E+08	1.23E+00	1.23E+00
12	7.31E+08	2.64E+07	3.63E+08	1.18E+10	5.83E+08	8.08E+08	2.02E+09	2.19E+10	2.15E+08	1.01E+09	8.25E+08	5.24E+07	2.71E+05	6.95E+05	6.61E+06	3.96E+08
13	9.18E+08	5.97E+07	2.27E+09	4.90E+09	1.53E+08	1.16E+08	3.18E+09	4.26E+07	3.68E+06	7.28E+08	4.95E+07	2.61E+07	3.00E+04	3.93E+05	1.23E+00	1.23E+00
14	3.03E+08	6.81E+07	7.87E+07	9.42E+09	2.20E+06	2.07E+07	5.95E+09	3.82E+05	1.11E+08	1.84E+09	1.81E+07	1.11E+07	4.10E+05	9.00E+04	1.23E+00	1.23E+00
15	7.49E+08	4.08E+08	5.40E+08	2.13E+09	8.96E+07	2.47E+07	1.05E+09	1.23E+07	2.28E+08	1.08E+09	3.68E+08	1.51E+06	1.65E+05	9.01E+04	5.83E+06	2.17E+08
16	4.91E+09	4.94E+08	6.30E+09	5.19E+09	9.63E+07	2.47E+07	1.05E+09	1.23E+07	1.07E+09	1.08E+09	3.39E+08	2.63E+07	1.50E+05	1.01E+04	1.23E+00	1.23E+00
17	9.51E+08	3.30E+08	2.79E+09	1.35E+10	1.22E+07	2.17E+08	1.59E+09	9.01E+09	6.01E+05	1.67E+09	1.88E+09	1.22E+08	4.13E+05	1.34E+04	1.23E+00	1.23E+00
18	1.33E+10	4.84E+08	1.78E+10	1.14E+10	6.90E+07	2.16E+08	1.65E+09	2.70E+10	1.42E+07	1.20E+09	3.65E+08	3.56E+07	8.52E+05	1.05E+08	4.66E+06	4.04E+08
19	8.15E+08	6.92E+07	2.85E+09	2.22E+10	5.00E+06	1.51E+08	1.18E+09	8.49E+09	1.21E+07	1.21E+07	4.38E+08	2.21E+07	8.82E+05	3.58E+05	1.43E+05	2.90E+08
20	9.88E+08	1.28E+08	2.30E+10	2.30E+10	5.17E+07	2.13E+08	4.85E+09	9.02E+09	3.50E+07	1.70E+09	7.92E+08	3.91E+07	1.77E+06	8.02E+03	1.23E+00	1.23E+00
21	4.80E+09	2.59E+08	1.93E+09	3.35E+10	1.05E+09	2.53E+09	6.10E+10	4.36E+09	1.88E+10	4.58E+10	3.60E+09	4.72E+08	3.00E+04	5.91E+06	1.23E+00	1.23E+00
22	1.11E+10	1.06E+08	3.98E+09	2.18E+10	5.36E+07	6.80E+08	2.08E+09	8.47E+09	1.49E+06	2.64E+09	1.44E+09	6.70E+07	2.08E+06	4.85E+08	1.23E+00	1.23E+00
23	3.04E+08	4.14E+08	1.10E+09	7.13E+09	9.41E+06	4.87E+07	2.05E+09	2.44E+08	3.35E+05	9.74E+08	1.14E+08	1.39E+07	2.85E+05	1.35E+04	2.06E+06	1.23E+00
24	2.51E+10	7.31E+08	1.61E+09	3.10E+10	6.12E+07	5.64E+08	5.73E+09	1.39E+10	1.71E+07	2.41E+09	9.53E+08	9.81E+07	2.48E+06	1.36E+04	2.61E+08	5.44E+08
25	6.26E+09	2.67E+08	5.33E+08	2.75E+10	7.30E+08	1.57E+09	1.01E+10	2.56E+08	3.02E+09	6.85E+09	1.04E+09	2.46E+08	6.87E+06	8.05E+04	1.23E+00	1.23E+00
26	6.30E+09	2.78E+08	1.53E+09	2.33E+10	1.38E+09	9.66E+08	1.01E+10	1.19E+08	3.77E+08	1.48E+10	1.10E+09	1.79E+08	6.78E+05	8.58E+04	1.23E+00	1.23E+00
27	1.61E+09	1.39E+08	5.92E+09	1.08E+10	1.81E+08	1.48E+07	1.31E+10	2.05E+10	6.91E+05	9.27E+07	1.28E+10	7.61E+08	5.70E+06	5.33E+06	1.23E+00	1.23E+00
28	9.96E+07	4.50E+04	1.20E+09	6.59E+09	9.69E+07	8.68E+07	4.24E+08	1.30E+09	4.59E+06	1.28E+08	2.72E+08	9.89E+06	1.36E+05	2.67E+06	9.72E+05	1.23E+00
29	4.97E+07	4.50E+04	1.95E+07	5.68E+09	6.43E+07	5.03E+06	3.45E+08	1.13E+07	4.24E+07	8.93E+07	7.00E+08	3.85E+05	1.09E+05	1.23E+00	1.30E+06	1.23E+00
30	4.87E+08	4.50E+04	4.31E+09	1.72E+10	1.29E+07	2.21E+08	8.51E+08	2.15E+10	1.03E+06	1.75E+08	1.27E+07	1.22E+07	8.25E+04	1.64E+03	1.23E+00	1.23E+00
31	2.89E+09	4.50E+04	8.56E+09	2.99E+09	1.18E+07	2.63E+08	9.79E+09	8.49E+06	1.03E+06	5.77E+08	1.59E+09	1.52E+07	5.03E+05	3.55E+04	1.23E+00	1.23E+00
32	7.76E+08	4.50E+04	4.52E+09	4.34E+09	6.35E+07	7.88E+07	7.25E+08	2.15E+10	1.03E+06	7.74E+07	1.99E+07	8.95E+06	7.87E+05	3.55E+04	1.00E+07	3.19E+08
33	6.12E+08	4.50E+04	6.00E+06	4.45E+08	2.49E+09	7.88E+07	2.23E+08	8.49E+06	8.48E+06	6.44E+06	4.66E+08	3.31E+07	1.07E+06	3.55E+04	8.74E+07	3.06E+08