Multicriteria group decision making with ELECTRE III method based on interval-valued intuitionistic fuzzy information

Hashemi, S. S., Hajiagha, S. H. R., Zavadskas, E. K. & Amoozad Mahdiraji, H.

DOI 10.1016/j.apm.2015.08.011
ISSN 0307-904X

Publisher: Elsevier

Multicriteria group decision making with ELECTRE III method by interval-valued intuitionistic fuzzy information

Abstract

Many real world problems can be associated with multicriteria decision making. These problems are often characterized by a high degree of uncertainty. Interval-valued intuitionistic fuzzy sets are a generalized form of an ordinal fuzzy set to deal with this natural uncertainty. In this paper, an enhanced version of ELECTRE method, called ELECTRE III, is extended under the interval-valued intuitionistic fuzzy environment. The advantages and strengths of ELECTRE III as a decision aid technique and interval-valued intuitionistic fuzzy sets as an uncertain framework make the proposed method a suitable choice in solving practical problems. The application of the proposed method is illustrated by the solution of an investment project selection problem.

Keywords: Multiple Criteria Decision Making; ELECTRE III; Interval-Valued Intuitionistic Fuzzy Information; Investment Project Selection.

1. Introduction

The history of operations research science in its structured and exhaustive form shows that this field is a response to the needs of managers, decision makers, and resource owners for having criteria to judge their decisions. In fact, decision makers always seek a criterion to evaluate the favourite of their decisions. Decision making plays an important role for managers in different organizational levels. They should make the best decisions to assign resources fairly and gain benefit for organizations and employees. Consequently, they should solve many problems and choose among the conflicting alternatives. Therefore, many effective methods of selecting the appropriate options have been developed for decision-making. The multiple-criteria decision making (MCDM) provides an effective framework for comparison, based on the evaluation of multiple conflict criteria. MCDM has been one of the fastest growing areas of operational research because it is often found that many concrete problems can be represented by several conflicting criteria. Since, usually, there are numerous and antithetic criteria in actual decision making problems, the MCDM methods have become an important branch of operations research in the last decades [1, 2, 3]. Decision makers should select the alternatives by comparing them, based on several conflicting criteria and, using multicriteria decision making methods. This comparison is not completely definitive because of unreliable and imprecise available information about the considered alternatives. Therefore, it is very difficult to numerically express their estimates. Hence, uncertainty contexts are widely applied to MCDM problems.

A series of MCDM models use the so-called ‘outranking relations’ to rank a set of alternatives. The method of elimination and choice translating reality (ELECTRE) and its derivatives play a prominent role in this group. The ELECTRE approach was first introduced in 1966 [4]. The origin of ELECTRE methods goes back to 1965 to the European consultancy company SEMA. At that time, a research team from SEMA worked on a concrete, multiple criteria and real-world problem, regarding decisions on the development of new activities in the firms [1]. The main idea of this method is based on outranking relations and concordance and discordance concepts [5]. This method uses concordance and discordance indices to analyse the outranking relations [6]. Soon after the introduction of the first version known as ELECTRE I [7], this approach evolved into a number of other variants. Today, the most widely used versions are ELECTRE II [8], ELECTRE III [9], ELECTRE IV [10], ELECTRE IS [11] and ELECTRE TRI [12].

Under many conditions, exact data are inadequate to model the real-life situations. These situations are called uncertainty, and many researchers developed some structures, such as bounded data, ordinal data, fuzzy data, and grey numbers in response to these situations. Liu and Lin [13] believe that uncertainty frameworks can be classified into three distinct fields: probability and statistics, grey system theory and fuzzy set theory. The evolution of approaches to decision making problems is evident. As a generalization of the fuzzy set theory, Attanasov [14] developed intuitionistic fuzzy sets, where a non-membership degree is also associated with each element, in addition to ordinal membership degree in classic fuzzy sets. Recent years have witnessed a growing interest in the study of decision making problems with intuitionistic fuzzy sets/numbers.

Hung and Chen [15] applied intuitionistic fuzzy sets to a new fuzzy TOPSIS decision making model, using the entropy weight for dealing with multicriteria decision making problems under intuitionistic fuzzy environment. Ye [16] and Park et al. [17] developed different frameworks for TOPSIS method under IVIF data. Li [18] applied the triangular intuitionistic fuzzy numbers to choose between three alternatives in a MADM problem. Dymova and Sevastjanov [19] presented an analysis of the basic definition of the theory of intuitionistic fuzzy sets and surveyed its application to MCDM problems. Daneshvar Rouyendegh [20] used Intuitionistic fuzzy TOPSIS in a model of group decision making for project selection. Yu et al. [21] proposed some IVIF aggregation operators and investigated their application to group decision making under IVIF environment. Chen et al. [22] also proposed a new multicriteria decision making method under IVIF environment.

In the context of ELECTRE III method, as the method considered in this paper, Leyva Lopez [23] applied the ELECTRE III for student selecting problems. Papadopoulos and Karagiannidis [24] used this method for the optimization of decentralized energy systems. Li and Wang [25] used ELECTRE III method, based on fuzzy numbers, for ranking the alternatives. Amiri et al [26] proposed a new method of ranking alternatives, based on interval grey data by ELECTRE method. Vahdani et al. [27] also extended another approach for applying ELECTRE method by interval weights and data. Atici and Ulucan [28] and Banias et al. [29] used this method for the optimization of energy systems. Radziszewska-Zielina [30] applied the ELECTREIII for selecting the best partner construction enterprise in terms of partnering relations. Zavadskas et al. [31] used the considered method for evaluating commercial construction projects for investment purposes. Marzouk [32], Thiel [33], Ulubeyli and Kazaz [34] used this method in value engineering applications, while Vahdani et al. [35] offered the extension of ELECTRE method to fuzzy environment in flexible manufacturing.

The remaining part of the paper is organized as follows: In Section 2, the ELECTRE III method is briefly described. Then, in Section 3, a concept and basic calculation of triangular intuitionistic fuzzy is shortly reviewed (algebraic operations). In Section 4, following the introduction of MCDM problems with triangular intuitionistic fuzzy weights and data, an algorithm is presented to extend the ELECTRE III method, to deal with triangular intuitionistic fuzzy weights and data. In Section 5, the proposed algorithmic method is illustrated by a case study. Section 6 presents the conclusions and outlines the aims of the future work.

2. The ELECTRE III

The ELECTRE family in MCDM problems uses the concept of 'outranking relationship'. The outranking relationship of $A_k \rightarrow A_l$ means that, even though two alternatives *k* and *l* do not dominate each other mathematically, the DM accepts the risk of regarding A_k as almost surely better than A_l. This method includes pairwise comparison of alternatives, based on the degree, to which evaluations of the alternatives and the preference weights confirm or contradict the pairwise dominance relationship between the alternatives [36]. This method has been developed during a certain period of time and became more widely applicable. The versions, mentioned above, are based on the same foundation, but they differ slightly. Specifically, ELECTRE I is designed for selection problems, ELECTRE TRI for assignment problems and ELECTRE II, III and IV for ranking problems. ELECTRE III is used, when it is possible and desirable to quantify the relative importance of criteria, while ELECTRE IV is used in the cases, when this quantification is not possible. This paper is focused on the analysis and application of ELECTRE III method.

Let $A = (a, b, c, \ldots, n)$ be a set of alternatives and $(g_1, g_2, \ldots, g_m)$ a set of criteria for an MCDM problem; $g_j(a_j)$ represents the performance or the evaluation of the alternative $a \in A$ on criterion g_j. Depending on whether the target is to maximize or to minimize the criterion $g_j(a_j)$, the higher or lower it is, the better the alternative meets the criterion in question. Consequently, multicriteria evaluation of an alternative $a \in A$ will be represented by the vector $g(a) = (g_1(a), g_2(a), \ldots, g_m(a))$.

The evaluation procedures of the ELECTRE III model encompass the establishment of a threshold function, disclosure of concordance and discordance indices, determination of credibility degree, and the ranking of the alternatives. Let $q(g)$ and $p(g)$ represent the indifference and preference thresholds, respectively [37].

If $g(a) \geq g(b)$, then,

$$g(a) \succ g(b) + p(g(b)) \Leftrightarrow aPb \tag{1}$$

$$g(b) + q(g(b)) \prec g(a) \prec g(b) + p(g(b)) \Leftrightarrow aQb \tag{2}$$

$$g(b) \prec g(a) \prec g(b) + q(g(b)) \Leftrightarrow aIb, \tag{3}$$

where *P* denotes strong preference, *Q* denotes weak preference, *I* denotes indifference, and $g(a)$ is the criterion value of the alternative *a*.

The steps of ELECTRE III and calculations are presented below.

Step 1. The concordance index $C(a,b)$ is computed for each pair of alternatives:

$$C(a,b) = \frac{\sum_{i=1}^{m} w_i C_i(a,b)}{\sum_{i=1}^{m} w_i}, \tag{4}$$

where $C_i(a,b)$ is the outranking degree of the alternative *a* and the alternative *b* under criterion *i*, and

$$C_i(a,b) = \begin{cases} 0 \quad if \quad g_i(b) - g_i(a) \succ p_i(g_i(a)) \\ 1 \quad if \quad g_i(b) - g_i(a) \leq q_i(g_i(a)) \\ \dfrac{p_i + g_i(a) - g_i(b)}{p_i - q_i} \quad otherwise \end{cases} \tag{5}$$

Thus, $0 \leq c_i(a,b) \leq 1$.

The veto threshold $v_i(g_i(b))$ is defined for each criterion *i* as follows:

$$v_i(g_i(a)) = \alpha_v + \beta_v g_i(a) \tag{6}$$

The veto threshold, *v*i, allows for the possibility of a**S**b to be refused totally if, for any one criterion j, $g_i(b) \succ g_i(a) + v_i$.

Step 2. The discordance index $d(a,b)$ for each criterion is then defined as follows:

$$d_i(a,b) = \begin{cases} 0 \ \ \text{if} \ \ g_i(b) - g_i(a) \le p_i(g_i(a)) \\ 1 \ \ \text{if} \ \ g_i(b) - g_i(a) \succ v_i(g_i(a)) \\ \dfrac{g_i(b) - g_i(a) - p_i}{v_i - p_i} \ \ \text{otherwise} \end{cases} \tag{7}$$

Thus, $0 \le d_i(a,b) \le 1$

Step 3. Finally, the degree of outranking is defined by $S(a,b)$:

$$S(a,b) = \begin{cases} c(a,b) & \text{if} \ \ d_j(a,b) \le c(a,b) \ \ \forall j \in J \\ c(a,b) \times \prod_{j \in J(a,b)} \dfrac{1 - d_j(a,b)}{1 - c(a,b)} & \text{otherwise} \end{cases}, \tag{8}$$

where $J(a,b)$ is the set of criteria for which $d_j(a,b) \succ c(a,b)$ [37].

Step 4. To obtain the complete ranking of the alternatives, the normal ranking method of ELECTRE III uses a structured algorithm via two intermediate ranking procedures: one is descending, where the alternatives are classified from the best to the worst (descending distillation), while the other is based on the ascending order from the worst to the best alternative (ascending distillation). However, according to Li and Wang [25], a new ranking method based on the introduction of three concepts, including the concordance credibility degree, the discordance credibility degree and the net credibility degree, is applied.

i) The concordance credibility degree is defined by

$$\phi^{+}(x_i)=\sum_{x_j\in X} S(x_i,x_j), \quad \forall x_i \in X \tag{9}$$

The concordance credibility degree is a measure of the outranking character of x_i (showing how x_i dominates all the other alternatives of *X*).

ii) The discordance credibility degree is defined by

$$\phi^{-}(x_i)=\sum_{x_j\in X} S(x_j,x_i), \quad \forall x_i \in X \tag{10}$$

The discordance credibility degree describes the outranked x_j (showing how x_j is dominated by all the other alternatives of *X*).

iii) The net credibility degree is defined by

$$\phi(x_i)=\phi^{+}(x_i)-\phi^{-}(x_i), \quad \forall x_i \in X \tag{11}$$

The net credibility degree represents a value function, where a higher value reflects higher attractiveness of the alternative x_i. Next, all the alternatives can be completely ranked by the net credibility degree.

In addition, to solve practical problems of fuzzy multiattribute decision making (FMADM), evaluators and decision makers must be provided for and they should necessarily include various stakeholders and interest groups. The different backgrounds and positions of the members of these groups result in greatly varying subjective judgments. For example, the above thresholds (concordance, discordance, and veto) may be presented in fuzzy data. This shows that ELECTRE III and IV are more appropriate for the evaluation of real-world problems [37].

3. Interval-valued intuitionistic fuzzy sets

Intuitionistic fuzzy sets theory was introduced by Atanassov [14]. Let a set *E* be fixed. An intuitionistic fuzzy set *A* in *E* is defined as an object of the following form:

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in E\} \quad , (12)$$

where $\mu_A : E \rightarrow [0,1]$ indicates the degree of membership and $v_A : E \rightarrow [0,1]$ indicates the degree of non-membership of *x* in *A*.

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \tag{13}$$

Atanassov and Gargov [38] generalized the IFS to interval-valued intuitionistic fuzzy sets (IVIFS) as follows:

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. Let $X(\neq \Phi)$ be a given set. The IVIFS in X is an expression given by $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle | x \in X\}$, where $\mu_{\tilde{A}} : X \rightarrow D[0,1]$, $v_{\tilde{A}} : X \rightarrow D[0,1]$ with the condition $0 \prec sup_x\, \mu_{\tilde{A}}(x) + sup_x\, v_{\tilde{A}}(x) \leq 1$.

For each $x \in X$, $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ are closed intervals, whose lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $v_{AL}(x)$ and $v_{AU}(x)$. The IVIFS *A* is denoted by

$$A = \{\langle x, [\mu_{AL}(x), \mu_{AU}(x)], [v_{AL}(x), v_{AU}(x)]\rangle | x \in X\} \quad , (14)$$

where $0 \prec \mu_{AU}(x) + v_{AU}(x) \leq 1$, $\mu_{AL}(x), v_{AL}(x) \geq 0$. For convenience, the IVIFS value is denoted by $\tilde{A} = ([a,b],[c,d])$, referred to as an interval-valued intuitionistic fuzzy number (IVIFN).

Moreover, let $\tilde{A}_1 = ([a_1, b_1],[c_1, d_1])$ and $\tilde{A}_2 = ([a_2, b_2],[c_2, d_2])$ IVIFNs. Their operational laws are defined as follows [39]:

$$\tilde{A}_1 + \tilde{A}_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2],[c_1 c_2, d_1 d_2]) \tag{15}$$

$$\tilde{A}_1 \cdot \tilde{A}_2 = ([a_1 a_2, b_1 b_2],[c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2]) \tag{16}$$

$$\lambda \tilde{A}_1 = ([1-(1-a_1)^{\lambda}, 1-(1-b_1)^{\lambda}],[c_1^{\lambda}, d_1^{\lambda}]), \lambda \geq 0 \tag{17}$$

If $\tilde{A} = ([a,b],[c,d])$ is an IVIFN, then,

$$s(\tilde{A}) = \frac{1}{2}(a - c + b - d) \quad (18)$$

is the score function of $\tilde{A}$, where $s(\tilde{A}) \in [-1,1]$ and

$$h(\tilde{A}) = \frac{1}{2}(a + c + b + d) \quad (19)$$

is the accuracy function of *A*, where $h(\tilde{A}) \in [0,1]$ [39].

To compare two IVIFNs let us consider $\tilde{A}_a$ and $\tilde{A}_b$ as two IVIFNs. Therefore,

1. If $s(\tilde{A}_a) \prec s(\tilde{A}_b)$, then, $\tilde{A}_a$ is smaller than $\tilde{A}_b$, $\tilde{A}_a \prec \tilde{A}_b$.
2. If $s(\tilde{A}_a) = s(\tilde{A}_b)$, then,
   2-1) If $h(\tilde{A}_a) = h(\tilde{A}_b)$, then, $\tilde{A}_a = \tilde{A}_b$.
   2-2) If $h(\tilde{A}_a) \prec h(\tilde{A}_b)$, then, $\tilde{A}_a$ is smaller than $\tilde{A}_b$, $\tilde{A}_a \prec \tilde{A}_b$ [39].

Then, let $\tilde{A}_j = ([a_j, b_j], [c_j, d_j]), j = 1,2,\ldots,n$ be a collection of IVIFNs. The generalized interval intuitionistic fuzzy weighted average $GIIFWA_w(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)$ is defined as follows:

$$GIIFWA_w(A_1, A_2, \ldots, A_n) = \left(w_1\tilde{A}_1^{\lambda} + w_2\tilde{A}_2^{\lambda} + \ldots + w_n\tilde{A}_n^{\lambda}\right)^{1/\lambda} \quad , (20)$$

where $\lambda \succ 0$, and $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector with $w_j \geq 0, j = 1,2,\ldots,n$, and $\sum_{j=1}^{n} w_j = 1$. It can be shown that GIIFWA is also an IVIFN and can be calculated as follows [40]:

$$GIIFWA_w(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \left(\left[\left(1 - \prod_{j=1}^{n} (1 - a_j^{\lambda})^{w_j}\right)^{1/\lambda}, \left(1 - \prod_{j=1}^{n} (1 - b_j^{\lambda})^{w_j}\right)^{1/\lambda} \right], \right.$$
$$\left. \left[1 - \left(1 - \prod_{j=1}^{n} \left(1 - (1 - c_j)^{\lambda}\right)^{w_j}\right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^{n} \left(1 - (1 - d_j)^{\lambda}\right)^{w_j}\right)^{1/\lambda} \right] \right) \quad (21)$$

If $\lambda = 1$, then, GIIFWA is turned into the interval intuitionistic fuzzy weighted average (IIFWA).

4. The ELECTRE III with the interval-valued intuitionistic fuzzy number

In this section, the ELECTRE III method is extended to include the IVIFNs data. First, let us consider a decision making problem consisting of *m* alternatives $A = (A_1, A_2, \ldots, A_m)$, which will be evaluated based on *n* criteria $(g_1, g_2, \ldots, g_m)$, while $\tilde{x}_{ij}$ is the value of *i*-th alternative in *j*-th criterion, which is expressed as an IVIFN, and a group of *K* decision makers will assign their scores. In this case, the process of group decision making by ELECTRE III - IVIFN method is developed in the following steps:

Step 1. Determine the importance of each decision maker. First, the importance of each decision maker in the final decision should be determined. Suppose, that $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)$ is the decision makers' importance vector, where $\lambda_k \geq 0, k = 1, 2, \ldots, K$ is the importance of *k*-th decision maker and $\sum_{k=1}^{K} \lambda_k = 1$. Note, that if decision makers have the same importance, then, $\lambda_1 = \lambda_2 = \ldots = \lambda_K = 1/K$.

Step 2. Assign the scores. In this step, decision makers are asked to individually express their *opinions* and evaluate the alternatives based on criteria and their weights. Suppose, that $\tilde{x}_{ij}^k, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ is the *k*-th expert's evaluation rating of the alternative A_i on criterion *j*, which is expressed by the IVIFN $\tilde{x}_{ij}^k = ([\mu_{Lij}^k, \mu_{Uij}^k], [v_{Lij}^k, v_{Uij}^k])$. Then, the expert *k*'s decision matrix is constructed as follows:

$$\tilde{X}^k = \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \cdots & \tilde{x}_{2n}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1}^k & \tilde{x}_{m2}^k & \cdots & \tilde{x}_{mn}^k \end{bmatrix} \tag{22}$$

Then, aggregate the expert's matrices as follows:

$$\tilde{x}_{ij} = GIIFWA_{\lambda}\left(x_{ij}^1, x_{ij}^2, \ldots, x_{ij}^K\right) = \left(\left[\left(1 - \prod_{k=1}^{K}\left(1-\mu_{Lij}^k\right)^{\lambda_k}\right), \left(1 - \prod_{k=1}^{K}\left(1-\mu_{Uij}^k\right)^{\lambda_k}\right)\right], \left[\prod_{k=1}^{K}\left(v_{Lij}^k\right)^{\lambda_k}, \prod_{k=1}^{K}\left(v_{Uij}^k\right)^{\lambda_k}\right]\right) \quad (23)$$

Step 3. Determine the weights of criteria. Synchronously, expert k expresses his or her judgments with regard to the weights of criteria. Suppose, that $\tilde{w}_j^k = \left(\left[\mu_{Lj}^k, \mu_{Uj}^k\right], \left[v_{Lj}^k, v_{Uj}^k\right]\right)$ is the k-th expert's judgment on the importance of the j-th criterion. In this step, the aggregated weights of criteria are determined by calculating the GIIFWA operator, Eq. (21) and the criteria's weights, determined by decision makers. If $\tilde{w}_j^k, k = 1, 2, \ldots, K$ are the weights of the criterion j assigned by decision makers, then, the aggregated weight of the criterion j, $\tilde{w}_j = \left(\left[\mu_{Lj}, \mu_{Uj}\right], \left[v_{Lj}, v_{Uj}\right]\right)$, will be calculated as described below:

$$\tilde{w}_j = GIIFWA_{\lambda}\left(\tilde{w}_j^1, \tilde{w}_j^2, \ldots, \tilde{w}_j^K\right) = \left(\left[\left(1 - \prod_{k=1}^{K}\left(1-\mu_{Lj}^k\right)^{\lambda_k}\right), \left(1 - \prod_{k=1}^{K}\left(1-\mu_{Uj}^k\right)^{\lambda_k}\right)\right], \left[\prod_{k=1}^{K}\left(v_{Lj}^k\right)^{\lambda_k}, \prod_{k=1}^{K}\left(v_{Uj}^k\right)^{\lambda_k}\right]\right) \quad (24)$$

Step 4. Construct the concordance matrix. For this purpose, we should first determine the thresholds, where P denotes strong preference, Q denotes weak preference, and I denotes indifference.

The alternatives' performance can usually be determined with 'certain accuracy', and the imperfect knowledge about the evaluations can be taken into account, when defining the thresholds for the model.

There are several techniques, which can be used in this case. Some of them are directly associated with the definition of a threshold, while others are based on the concept of a dispersion threshold. A dispersion threshold allows us to take into account the concept of probable value and the notion of optimistic and pessimistic values. It translates the plausible difference, due to over- or under-estimation, which affects the evaluation of the consequence or performance level. It should be noted that there are no true values for thresholds. Therefore, the values chosen for thresholds are the most convenient (the best adapted) for expressing the imperfect character of the knowledge [1].

Then, a pair of alternatives should be compared and the concordance matrix is calculated as follows:

If $s(\widetilde{A}_a) \prec s(\widetilde{A}_b)$, then, $\widetilde{A}_a \prec \widetilde{A}_b$,
where $s(\widetilde{A}_i)$ is calculated by Eq. (18).

Now, $C(a,b)$ is calculated by Eq. (25),

$$C(a,b) = \frac{\sum_{i=1}^{m} w_i C_i(a,b)}{\sum_{i=1}^{m} w_i}, \tag{25}$$

where

$$C_i(a,b) = \begin{cases} 0 \quad if \quad s_i(b) - s_i(a) \succ s(\widetilde{p}_i) \\ 1 \quad if \quad s_i(b) - s_i(a) \le s(\widetilde{q}_i) \\ \dfrac{s(\widetilde{p}_i) + s(a) - s(b)}{s(\widetilde{p}_i) - s(\widetilde{q}_i)} \quad otherwise \end{cases} \tag{26}$$

Step 5. Determine the discordance index. The value of $d(a,b)$ for each criterion is defined as follows:

$$d_i(a,b) = \begin{cases} 0 \quad if \quad s(b) - s(a) \le s(\widetilde{p}_i) \\ 1 \quad if \quad s(\widetilde{A}_b) - s(\widetilde{A}_a) \succ v_i(\widetilde{A}_a) \\ \dfrac{s(\widetilde{A}_b) - s(\widetilde{A}_a) - s(\widetilde{p}_i)}{s(\widetilde{v}_i) - s(\widetilde{p}_i)}, \quad otherwise \end{cases} \tag{27}$$

Thus, $0 \le d_i(a,b) \le 1$.

Step 6. Finally, the degree of outranking is defined by $S(a,b)$:

$$S(a,b) = \begin{cases} c(a,b) \qquad if \quad d_j(a,b) \le c(a,b) \quad \forall j \in J \\ c(a,b) \times \prod_{j \in J(a,b)} \dfrac{1 - d_j(a,b)}{1 - c(a,b)} \quad otherwise \end{cases} \tag{28}$$

Step 7. According to Li and Wang [25], the alternatives could be ranked based on concordance credibility degree, the discordance credibility degree and the net credibility degree by Eqs. (9), (10), and (11).

The algorithmic scheme of the proposed model is shown in Figure 1.

Figure 1. The scheme of the ELECTRE III IVIF algorithm

5. Application of ELECTRE III with the interval-valued intuitionistic fuzzy number

Case 1. In this section, the application of ELECTRE III, based on the interval-valued intuitionistic fuzzy numbers, is presented. A management team is going to select the best among the investment projects. For this purpose, they asked managers of 5 departments to evaluate four projects, based on different criteria. The economic and financial analysis of the project is based on the comparison of the cash flow of all costs and benefits, resulting from the project's activities. There are four common criteria for comparing the alternative investments [41, 42]:

1) Net present value (NPV),

2) Rate of Return (ROR),

3) Benefit-Cost analysis (CB),

4) Pay Back Period (PBP).

First, a management team determined their importance in terms of the weight vector $\lambda = (0.25, 0.25, 0.25, 0.25)^T$

Then, these managers were asked to express their opinion about the alternatives in the linguistic terms to show their preference with respect to each criterion and to weigh them according to Table 1.

Table 1. The IVIFN scale for rating the alternatives on criteria

Table 2. Linguistic preference matrix of an expert

The aggregation of the expert's preference matrices, based on Eq. (23), is shown in Table 3.

Table 3. The aggregated matrix of experts' preferences

Table 4 demonstrates the proposed scale to show the importance of each criterion, while the experts are asked to define their opinions as shown in Table 5.

Table 4. Linguistic terms used to define the importance of each criterion

Table 5. The weighting matrix of experts

Now, the aggregated weight of criteria determined by experts is calculated by Eq. (24):

$\tilde{w}_1 = ([0.7648, 0.8712], [0, 0.1768])$ $\tilde{w}_2 = ([0.7084, 0.8027], [0.1257, 0.2307])$

$\tilde{w}_3 = ([0.1549, 0.5838], [0, 0.3872])$ $\tilde{w}_4 = ([0.7434, 0.8458], [0, 0.3872])$

Then, according to Eq. (26), the alternatives are compared, using the thresholds for this purpose, as shown in Table 6:

Table 6. The matrix of the alternative thresholds

To construct the concordance matrix, it is necessary to calculate $s(\tilde{A}_j)$ and $s(\tilde{w}_i)$ first. The results of $s(\tilde{A}_j)$ are shown in Table 7.

Table 7. Deterministic format of the alternatives and thresholds

$s(\tilde{w}_1) = 0.729590194$ $s(\tilde{w}_2) = 0.577357189$

$s(\tilde{w}_3) = 0.175780762$ $s(\tilde{w}_4) = 0.601035037$

Now, the concordance matrix is constructed, based on the comparison of the alternatives, according to Eqs. (25), (26).

Table 8. Concordance matrix

Then, the discordance matrix is calculated and the results are shown in Table 9.

Table 9. Discordance matrix

In this step, the comparison between the concordance and discordance matrices is made, and $S(a,b)$ is determined by Eq. (28). The comparison results and $S(a,b)$ are presented in Tables 10 and 11, respectively.

Table 10. The comparison matrix

Table 11. The credibility matrix

In the final step, according to Li and Wang [25], the ranking is made and the results are shown in Table 12.

Table 12. The ranking matrix

This case is also solved by the method described by Ye [16]. The final result obtained by this method is as follows:

Table 13. The results yielded by Ye [16] method versus those obtained by the proposed method

Table 13 shows that the 1st and the 2nd ranks are the same, while the 3rd and the 4th places have changed.

Case 2. Here, to demonstrate the application of the suggested method to other fields and to validate the results more thoroughly, the problem of Park et al. [17] is solved by the proposed method. This problem is associated with determination of the type of air-conditioning systems to be installed in a library. The contractor offered four feasible alternatives A_1, A_2, A_3, A_4, which could be ranked, based on the following five attributes: (1) performance (C_1), (2) maintainability (C_2), (3) flexibility (C_3), (4) cost (C_4) and (5) safety (C_5). Table 14 shows the aggregated preference matrix of decision makers, while Table 15 illustrates the results yielded by the proposed method and the method described in the considered paper.

Table 14. The aggregated matrix of experts' preference

Table 15. Comparison of the results of the original research and the proposed method

Table 15 shows that the first and fourth alternatives are the same in both methods and only the 2nd and 3rd alternatives are different.

It should be noted that the partial differences between the proposed method and other considered methods can be caused by the impact of the threshold values on the ELECTRE III results. Therefore, the selection of suitable values for these thresholds may hide these differences.

6. Conclusion

In the present paper, the application of ELECTRE III with interval-valued intuitionistic fuzzy information is proposed for solving the MCDM problems. These types of decision making problems require managers to consider different aspects of the problem and the conflicting evaluation criteria. In this paper, the ELECTRE III is used as an appropriate method. The distinctive characteristic of ELECTRE III is its ability to handle a data set with a high degree of uncertainty. As another advantage of the ELECTRE III, the fact that it is completely compatible with the environmental applications, may be mentioned. Moreover, the ELECTRE III uses pairwise comparison of the alternatives. It means that one can select the best alternative according to predefined preference criteria. Each criterion is weighted to represent its relative importance according to the preference structure of the decision maker. Therefore, this ELECTRE version is more preferable than its previous versions. In order to treat the ambiguity and ill-defined data, the interval-valued intuitionistic fuzzy numbers are used, which consider a membership and non-membership interval for each element and give a more tangible picture of the inaccuracy of the real world. The application of the interval-valued intuitionistic fuzzy numbers is extended so that the ELECTRE III method could help managers to make decisions, based on nondeterministic data, and to decrease the ambiguity of information. Hence, this combination makes the proposed method more widely applicable and reliable. Finally, the proposed method is applied to two problems of the investment project and air-conditioning system selection. In both cases, the results yielded by the proposed method were compared with the data yielded by some other methods and only partial differences could be observed between them. This method can be considered to be a decision making system, which can be used in different fields of project management, construction decisions, supply chain decisions, etc.

References

[1] J. Figueira, S. Greco, M. Ehrgott, Multiple criteria decision analysis: state of the art surveys, Springer, New York, 2004.

[2] E. Triantaphyllou, Multi-criteria decision making methods: a comparative study, Kluwer Academic, London, 2000.

[3] E. K. Zavadskas, Z. Turskis, Multiple criteria decision making (MCDM) methods in economics: an overview, Technological and economic development of economy 17 (2) (2011) 397-427.

[4] R. Benayoun, B. Roy, N. Sussman, Manual de reference du programme electre, Note De Synthese et Formaton, no. 25, Direction Scientifique SEMA, Paris, 1966.

[5] B. Roy, D. Vanderpooten, An overview on The European school of MCDA: Emergence, basic features and current works, Journal of multi-criteria decision making 5 (1) (1996) 22-37.

[6] A. T. De Almeida, Multi-criteria decision model for outsourcing contracts selection based on utility function and ELECTRE method, Computers and Operations Research 34 (12) (2007) 3569-3574.

[7] T. Gal, T. Hanne, Multi-criteria decision making: advances in MCDM models, algorithms, theory, and applications, Kluwer Academic Publishers, New York, 1999.

[8] B. Roy, P. Bertier, La methode ELECTRE II: une method de classement en presence de critteres multiples, SEMA (Metra International), Direction Scientifique, Note de Travail No. 142, Paris, 1971.

[9] B. Roy, ELECTRE III: un algorithme de classements fonde sur une representation floue des preference en presence de criteres multiples, Cahiers de CERO, 20 (1978) 3-24.

[10] B. Roy, J. Hugonnard, Classement des prolongements de lignes de m´etro en banlieue parisienne (pr´esentation d'une m´ethode multicrit`ere originale), Cahiers du CERO, 24 (1982) 153-171.

[11] B. Roy, J. Skalka, ELECTRE IS: Aspects m´ethodologiques et guide d'utilisation, Document du LAMSADE 30, Universit´e Paris Dauphine, 1984.

[12] W. Yu, ELECTRE TRI: Aspects m´ethodologiques et manuel d'utilisation, Document du LAMSADE 74, Universit´e Paris-Dauphine, 1992.

[13] S. Liu, Y. Lin, On measures of information content of grey numbers, Kybernetes 35 (6) (2006) 899-904.

[14] K. T. Atanassov, Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1986) 87–96.

[15] C. C. Hung, L. H. Chen, A Fuzzy TOPSIS decision making model with Entropy weight under Intuitionistic Fuzzy environment proceedings of the International Multi-Conference of Engineers and Computer Scientists, Vol 1, IMECS, March 18 – 20 (2009) Hong Kong.

[16] F. Ye, An extended TOPSIS method with interval-valued intuitionistic fuzzy numbers for virtual enterprise partner selection. Expert Systems with Applications, 37 (10) (2010) 7050-7055.

[17] J. H. Park, I. Y. Park, Y. C. Kwun, X. Tan, Extension of the TOPSIS method for decision making problems under interval-valued intuitionistic fuzzy environment. Applied Mathematical Modelling, 35 (5) (2011) 2544-2556.

[18] C. Li, ELECTRE III based on ranking fuzzy numbers for deterministic and fuzzy maintenance strategy decision making problems, Automation and Logistics. IEEE International Conference, (2007) 309-312.

[19] L. Dymova, P. Sevastjanov, Operations on intuitionistic fuzzy values in multiple criteria decision making. Scientific Research of the Institute of Mathematics and Computer Science, 1(10) (2011) 41-48.

[20] B. Daneshvar Rouyendegh, Evaluating projects based on intuitionistic fuzzy group decision making, Hindawi Publishing corporation. Journal of applied mathematics. article ID 824265 (2012).

[21] D. Yu, Y. Wu, T. Lu, Interval-valued intuitionistic fuzzy prioritized operators and their application in group decision making. Knowledge-Based Systems, 30 (2012) 57-66.

[22] S. M. Chen, M. Y. Yang, S. W. Yang, T. W. Sheu, C. J. Liau, Multicriteria fuzzy decision making based on interval-valued intuitionistic fuzzy sets. Expert Systems with Applications, 39 (15) (2012) 12085-12091.

[23] J. C. Leyva Lopez, Multi criteria decision and application to a student selection problem. Pesquisa Operacional, 25 (1) (2005) 45-68.

[24] A. Papadopoulos, A. Karagiannidis, Application of the multi-criteria analysis method ELECTRE III for the optimization of decentralized energy systems. Omega, 36 (5) (2008) 766-776.

[25] H. F. Li, J. J. Wang, An improved ranking method for ELECTRE III. In: International conference on wireless communications. Networking and Mobile Computing, 21-25, (2007) 6659-6662.

[26] M. Nosratian, N. E. Amiri, A. Jamshidi, A. Kazemi, Developing a new ELECTRE method with interval data in Multiple Attribute Decision Making problems. Journal of Applied Sciences, 8 (22) (2008) 4017-4028.

[27] B, Vahdani, A. H. K. Jabbari, V. Roshanaei, M. Zandieh, Extension of the ELECTRE method for decision-making problems with interval weights and data. The International Journal of Advanced Manufacturing Technology, 50 (5-8) (2010) 793-800.

[28] K. B. Atici, A. Ulucan, A multiple criteria energy decision support system. Technological and economic development of economy, 17 (2) (2011) 219-245.

[29] G. Banias, C. Achillas, C. Vlachokostas, N. Moussiopoulos, S. Tarsenis, Assessing multiple criteria for the optimal location of a construction and demolition waste management facility. Building and Environment, 45 (10) (2010) 2317-2326.

[30] E. Radziszewska-Zielina, Methods for Selecting the Best Partner Construction Enterprise in Terms of Partnering Relations. Journal of civil engineering and management, 16 (4) (2010) 510-520.

[31] E. K. Zavadskas, L. Ustinovičius, A. Stasiulionis, Multi-criteria Valuation of Commercial Construction Projects for Investment Purposes. Journal of Civil Engineering and Management, 10 (2) (2004) 151–166.

[32] M. M. Marzouk, ELECTRE III model for value engineering applications. Automation in construction, 20 (5) (2011) 596-600.

[33] T. Thiel, Determination of the relative importance of criteria when the number of people judging is a small sample. Technological and economic development of economy, 14 (4) (2008) 566-577.

[34] S. Ulubeyli, A. Kazaz, A Multiple Criteria Decision- Making Approach to the Selection of Concrete Pumps. Journal of Civil Engineering and Management, 15 (4) (2009) 369–376.

[35] B. Vahdani, S. M. Mousavi, R. Tavakkoli-Moghaddam, A new design of the elimination and choice translating reality method for multi-criteria group decision making in an intuitionistic fuzzy environment. Applied mathematical modelling (article in press) (2012).

[36] C. L. Hwang, K. P. Yoon, Multiple Attribute Decision Making: An Introduction. Springer-Verlag, Berlin Heidelberg, 1995.

[37] G. H. Tzeng, J. J. Huang, Multiple attribute decision making, methods and applications. Boca Raton, Taylor & Francis group, 2011.

[38] K. Atanassov, G. Gargov, Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 31(3) (1989) 343-349.

[39] Z. S. Xu, Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. Control and Decision, 22 (2) (2007) 215-219.

[40] H. Zhao, Z. Xu, M. Ni, S. Liu, Generalized aggregation operators for intuitionistic fuzzy sets. International Journal of Intelligent Systems, 25 (1) (2010) 1-30.

[41] N. P. Archer, F. Ghashemzadeh, An integrated framework for project portfolio selection.

[42] C. Liang, Q. Li, Enterprise information system project selection with regard to BOCR. Internationa; Journal of Project Management, 26(8) (2008) 810-820.

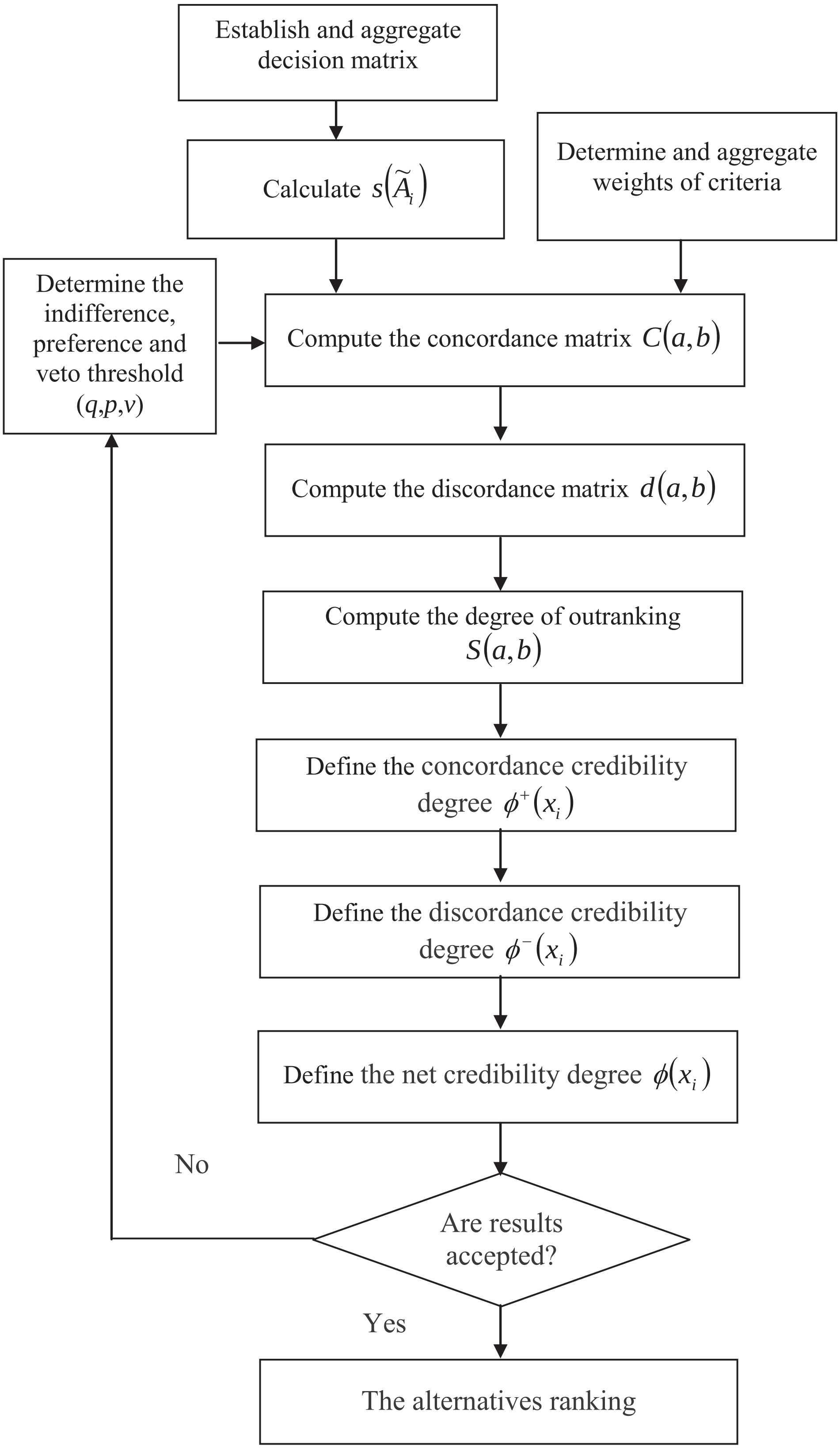

Figure 1. The scheme of the ELECTRE III IVIF algorithm

Table1. An IVIFN scale to rate alternatives on criteria

Linguistic term	IVIFNs
Extremely good (EG)/ extremely high (EH)	([1, 1], [0, 0])
Very very good (VVG)/ very very high (VVH)	([0.9, 0.9], [0.1, 0.1])
Very good (VG)/ very high (VH)	([0.7333, 0.825], [0, 0.125])
Good (G)/ high (H)	([0.6333, 0.725], [0.1, 0.225])
Medium good (MG)/ medium high (MH)	([0.5333, 0.625], [0.2, 0.325])
Fair (F)/ medium (M)	([0.4333, 0.525], [0.3, 0.425])
Medium bad (MB)/ medium low (ML)	([0.3333, 0.425], [0.4, 0.525])
Bad (B)/ low (L)	([0.15, 0.2875], [0.45, 0.6375])
Very bad (VB)/ very low (VL)	([0, 0.1375], [0.6, 0.7875])
Very very bad (VVL)/ very very low (VVL)	([0.1, 0.1], [0.9, 0.9])

Table2. Expert's linguistic preference matrix

EX1	C_1	C_2	C_3	C_4
A_1	H	M	VH	H
A_2	M	ML	VL	M
A_3	M	VVL	M	VH
A_4	M	VH	H	M

EX2	C_1	C_2	C_3	C_4
A_1	M	M	M	ML
A_2	M	H	L	H
A_3	L	M	VVL	H
A_4	H	MH	M	VVH

EX3	C_1	C_2	C_3	C_4
A_1	H	H	M	VH
A_2	M	VH	M	H
A_3	VL	M	L	M
A_4	VH	H	VH	M

EX4	C_1	C_2	C_3	C_4
A_1	H	VH	H	ML
A_2	M	H	VL	M
A_3	VL	L	M	M
A_4	VH	M	MH	H

EX5	C_1	C_2	C_3	C_4
A_1	M	M	H	VH
A_2	H	VH	ML	VH
A_3	L	M	VL	M
A_4	M	H	H	VH

Table3. Aggregated matrix of experts' preference

EXs	C_1	C_2	C_3	C_4
A_1	([0.6452,0.7382], [0.0974,0.2129])	([0.6347,0.7320], [0,0.2155])	([0.6724,0.7662], [0,0.1838])	([0.6718,0.7703], [0,0.1764])
A_2	([0.5590,0.6560], [0.1687,0.2927])	([0.6997,0.7984], [0,0.1498])	([0.5148,0.6388], [0,0.2934])	([0.6724,0.7662], [0,0.1838])
A_3	([0.2000,0.3492], [0.3845,0.5721])	([0.3891,0.4880], [0.3233,0.4581])	([0.2959,0.4056], [0.3845,0.5345])	([0.6347,0.7320], [0,0.2155])
A_4	([0.6974,0.7912], [0,0.1587])	([0.6880,0.7796], [0,0.1719])	([0.6880,0.7796], [0,0.1719])	([0.7632,0.8184], [0,0.1501])

Table4. Linguistic terms to show the importance of each criterion

Linguistic term	IVIFNs
Very important (VI)	([0.9, 0.9], [0.1, 0.1])
Important (I)	([0.4, 0.7625], [0, 0.2115])
Medium (M)	([0.15, 0.5125], [0.25, 0.4625])
Unimportant (U)	([0, 0.3625], [0.4, 0.6125])
Very unimportant (VU)	([0.1, 0.1], [0.9, 0.9])

Table5. Experts weighting matrix

	C_1	C_2	C_3	C_4
EX1	VI	VI	M	VI
EX2	I	VI	I	M
EX3	I	M	U	VI
EX4	M	U	U	M
EX5	VI	M	U	I

Table6. Alternative's thresholds matrix

	C_1	C_2	C_3	C_4
Q	M	L	M	L
P	H	MH	MH	ML
V	VH	H	H	M

Table7. Deterministic format of alternatives and thresholds

EXs	C_1	C_2	C_3	C_4
A_1	0.5365	0.5756	0.6274	0.6328
A_2	0.3768	0.6742	0.4301	0.6274
A_3	-0.2037	0.0478	-0.1087	0.5756
A_4	0.6650	0.6478	0.6478	0.7157
q	0.1166	-0.325	0.1166	-0.325
P	0.5166	0.3166	0.3166	-0.0833
v	0.7166	0.5166	0.5166	0.1166

Table8. Concordance matrix

	A_1	A_1	A_1	A_1
A_1	1	0.5286	0.7115	0.5297
A_2	0.5420	1	0.6272	0.3898
A_3	0	0	1	0.0843
A_4	0.6024	0.5658	0.7793	1

Table9. Discordance matrix

D1				**D2**				**D3**				**D4**			
1	0	0	0	1	0	0	0	1	0	0	0	1	0.38	0.13	0.83
0	1	0	0	0	1	0	0	0	1	0	0	0.44	1	0.15	0.85
1	0.32	1	1	1	1	1	1	1	1	1	1	0.70	0.67	1	1.11
0	0	0	1	0	0	0	1	0	0	0	1	0.002	0	0	1

Table10. Comparison matrix

$c_{11} = d_{11}$	$c_{12} \succ d_{12}$	$c_{13} \succ d_{13}$	$c_{14} \prec d_{14}$
$c_{21} \prec d_{21}$	$c_{22} = d_{22}$	$c_{23} \succ d_{23}$	$c_{24} \prec d_{24}$
$c_{31} \prec d_{31}$	$c_{32} \succ d_{32}$	$c_{33} = d_{33}$	$c_{34} \prec d_{34}$
$c_{41} \succ d_{41}$	$c_{42} \succ d_{42}$	$c_{43} \succ d_{43}$	$c_{44} = d_{44}$

Table11. Credibility matrix

1	0.5286	0.7115	3.449402
0.542	1	0.6272	1.020628
0	0	1	0
0.6024	0.5658	0.7793	1

Table12. Ranking matrix

Alternatives	Scores	Ranks
A_1	3.545102	1
A_2	1.095428	2
A_3	-2.118	3
A_4	-2.52253	4

Table13. Results of Ye () method versus proposed method

	A_1	A_2	A_3	A_4
Proposed method's results	1	2	3	4
Ye ()	1	2	4	3

Table14. Aggregated matrix of experts' preference

	C_1	C_2	C_3	C_4	C_5
A_1	$([0.4385,0.6199],[0.1549,0.2848])$	$([0.3000,0.4573],[0.3404,0.4710])$	$([0.6116,0.7117],[0.1089,0.2083])$	$([0.5000,0.6395],[0.0980,0.2567])$	$([0.1323,0.3623],[0.3747,0.5482])$
A_2	$([0.3520,0.4797],[0.3114,0.4681])$	$([0.1138,0.3010],[0.2511,0.4773])$	$([0.3379,0.4387],[0.3872,0.4887])$	$([0.1758,0.3134],[0.5305,0.6496])$	$([0.6395,0.7521],[0.1089,0.2830])$
A_3	$([0.3516,0.4906],[0.2940,0.4214])$	$([0.6395,0.7711],[0.0980,0.2263])$	$([0.5213,0.7804],[0.0980,0.2083])$	$([0.4387,0.6252],[0.2263,0.3262])$	$([0.5452,0.6502],[0.1770,0.3205])$
A_4	$([0.3000,0.4170],[0.3114,0.4887])$	$([0.1000,0.2103],[0.6012,0.7678])$	$([0.1000,0.2366],[0.5577,0.7569])$	$([0.2103,0.3109],[0.4050,0.5613])$	$([0.1849,0.3121],[0.5031,0.6118])$

Table15.Comparison of original research and proposed method's results

	A_1	A_2	A_3	A_4
Proposed method's results	2	3	1	4
Park et al. (2011)	3	2	1	4