Multi-period data envelopment analysis based on Chebyshev inequality bounds

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Abstract

Data envelopment analysis is a cross-sectional approach to evaluate the relative efficiency of a set of homogeneous units in a single time point; nonetheless, organizational units have been performing continuously over a period of time; hence, their performances are considered within this period. Cumulating inputs and outputs over the time periods provide an unnecessary compensating impact, making the efficiency appraisal unrealistic. To avoid this negative impact of data accumulation, a two-stage approach on the basis of Chebyshev inequality bounds is proposed to find interval efficiency of decision making units (henceforth DMUs). The proposed method is applied in a real case encompassing 115 bank branches over 6 periods of time. This application indicated the significant cautious approach of the proposed method in multi-period data envelopment analysis (hereafter DEA).

Keywords: Data envelopment analysis; Multi-period Efficiency; Chebyshev inequality bounds; Two-stage efficiency approximations.

1. Introduction

Data envelopment analysis, initially introduced by Charnes, Cooper, and Rhodes (1978) is an accepted and widely employed framework to analyze the relative efficiency of a set of DMUs, using m inputs to produce a set of s outputs. DEA is extended based on economic foundations of production possibility sets and seeks a production frontier to measure the relative efficiency of DMUs (Charnes, Cooper, Lewin, & Seiford, 1994; Førsund, Kittelsen, & Krivonozhko, 2009). Recent investigations by Emrouznejad, Parker and Tavares (2008) and Liu, Lu, Lu and Lin (2013) have shown a large variety of applications using DEA for measuring and improving the efficiency.

Classical DEA models can be considered as cross-sectional analysis. Admittedly, the performance of DMUs is compared with a particular point of time. Contrariwise, comparing the performance of DMUs over several periods of time is considerable, knowing as longitude or time series analysis (Charnes, Cooper, Lewin, & Seiford, 1994; Ramanathan, 2003). This problem is generally called multi-period DEA. Some implications of multi-period DEA can be found at universities (Kao, & Liu, 2014) and international airports (Kao, & Hwang, 2014).

In a nutshell, multi-period DEA deals with inputs and outputs fluctuation among DMUs. Classically, stochastic DEA models can be applied for handling this fluctuation, where inputs and outputs are assumed to follow certain statistical distributions. Cooper, Huang, and Susan (2011), Wei, Chen, and Wang (2014), and Branda (2015) are some of the latest researches regarding stochastic DEA; nevertheless, a different perspective is followed in multi-period DEA models in which, data are observed in different time points and are captured in the form of time series. A conventional approach for dealing with multiple periods is to aggregate the data of different periods in a single data point and to ignore the specific situation of each period (Charnes, Clarck, Cooper, & Golany, 1985).

To avoid this simplification mode, several methods are proposed for time series DEA problems. One of the first approaches in DEA analysis of multiple time periods is window analysis; where a moving average pattern of analysis is applied (Caves, Christensen, & Diewert, 1982). Actually, the performance of a DMU is compared with its performance in other periods, and with other DMUs' performance in the same period (Ramanathan, 2003). However, as mentioned by Charnes, Cooper, Lewin, and Seiford (1994), choosing the number of time periods in the window is really a controversial issue. Alongside with, another classic approach is Malmquist-type indexes of productivity (Färe, & Grosskopf, 1996). Beyond their usefulness, Kao and Liu (2014) pointed that these methods "do not take into account an aggregated measure of efficiency for multiple-period production systems".

Classes of dynamic DEA models are also extended for multi-period problems. Their main advantage is the ability to account the effect of carry-over activities between two consecutive terms. Dynamic DEA models were initially introduced by Färe and Grosskopf (1996), subsequently were developed by Nemoto and Goto (1999, 2003), Sueyoshi and Sekitani (2005), as well as Bogetoft, Färe, and Grosskopf (2008). On the other hand, it is worth noting here that, as a weakness these models need a perfect foresight regard to input costs, while Thompson, Langemeier, Lee, Lee, and Thrall (1990), besides Thompson, Dharmapala, and Thrall (1995) believed that exact input cost is not determined even in a given period.

Sengupta (1995; 1999) developed different types of dynamic DEA models, via which various possible scenarios of aggregating input costs were considered over the time. In these models, an optimal level is determined for inputs and the overall efficiency is defined as the ratio of actual used inputs over optimal expected inputs. Sengupta (1995) model assumed that inputs' future prices are determined exactly, while Sengupta (1999) extended his initial model to incorporate the uncertainty of inputs' future prices for measuring overall efficiency. As previously declared, the main restriction of dynamic DEA models is their dependency on knowledge about input prices, especially in the future, imposing an additional uncertainty to the models.

The multi-period DEA problems can be imagined in the context of network DEA models, whilst classic network DEA models (Kao, 2008, 2014a, b) considered DMUs internal structures and the relations among the subunits of DMUs, the multi-period DEA model can be considered as a network of time frames where a DMU performed continually in a time horizon whereas the aim of the model is to evaluate the relative efficiency of DMUs in this time-based network. A similar conceptualization of multi-period DEA in the form of network DEA is considered by Kao and Liu (2014).

Park and Park (1995) presented a multi-period data envelopment analysis (MDEA) model upon the concept of Debreu–Farrell's technical efficiency. The MDEA model relies on finding the efficiency of DMUs in different periods whereas a DMU is called full efficient if it gains full efficiency in all periods.

Amirteimoori and Kordrostami (2010) defined the aggregated efficiency of a DMU as a convex combination of individual period efficiencies and developed a model to find total and period efficiencies of DMUs. Kao and Liu (2014) proposed a model for multi-period efficiency evaluation, through which the overall and period efficiencies of a DMU are calculated simultaneously, while the overall efficiency is defined as the weighted average of the period efficiencies. Kao and Hwang (2014) applied the idea of overall efficiency as the weighted average of period efficiencies in two-stage network production systems.

Whilst different methods presented valuable views toward finding multi-period efficiency of a set of DMUs over a period of time, the main drawback to these models is ignoring the individual input and output variances during the time. Since the performance of DMUs is evaluated in multiple periods, the inputs and outputs of these DMUs are treated differently over the period of time, remark that these marked differences should be considered in their efficiency evaluation. Different methods usually accumulate inputs and outputs of considered periods to evaluate relative efficiency of units with cumulative data. This accumulation results in an unnecessary compensating effect ignoring the impact of inputs and outputs fluctuation on efficiency. As a case in point, if one of the outputs of a given DMU is increased in a specific time period, while this output is decreased dramatically in another time period, without considering the variance of this output engenders an unrealistic approximation of efficiency. The aim of this paper is to extend a DEA model incorporating the variability of input and output measures directly in the model.

The reminder of paper is organized as follows. Section 2 describes the perceived problem of evaluating bank branches efficiency during a period of 12 months. The proposed algorithm is explained in section 3. Numerical results are presented in section 4. Finally, the paper is concluded in section 5.

2. Problem description

Banks play an important role in the economic system of countries. This importance makes them an interesting subject of DEA applications. As Emrouznejad, Parker and Tavares (2008) and Liu, Lu, Lu and Lin (2013) surveys' illustrated, financial institutions including banks are the most implicational field of DEA.

The noted problem in this paper deals with evaluating the efficiencies of 115 branches of HNI, a corporate bank in Iran. A set of 5 inputs and 4 outputs are identified to appraise the branches efficiency. These inputs and outputs are specified upon Berger and Humphrey (1997) and Luo and Liang (2012). Table 1 depicts the input and output measures. Among the inputs, the ratio of non-current to total receivables is an undesirable output. Different methods are proposed encompassing undesirable outputs. Färe, Grosskopf, Lovell, and Pasurka (1989), Färe, Grosskopf, and Tyteca (1996), as well as Tyteca (1997) modeled and developed the concept of hyperbolic output efficiency measure to deal with undesirable outputs in terms of an equiproportionate increase in desirable outputs and decrease in undesirable ones. Seiford and Zhu (2002) introduced a linear monotone decreasing transformation for undesirable outputs. The transformed outputs are treated as desirable outputs. Considering undesirable outputs as inputs is applied in different DEA studies (Matthews, 2013). The main strength of this approach is its easiness of use. In conjunction with the main advantage this approach doesn't rely on any assumption about data structure and shape of linear transformation.

In this study, data are gathered from the second half of the financial year 2013, from 1 July 2013 to 1 January 2014. Accordingly, there are a set of 6 input/output matrices of the size of 115×9. According to these matrices, a set of 9 time series data, each for one of the inputs or outputs is constructed. The aim of the problem is to assess the efficiency of bank branches in aforementioned time horizon.

<Please insert table 1 here>

The attended data in this problem are observed values of inputs and outputs in discrete time points forming different time series for every input and output of each DMU.

3. Problem formulation

Generally, suppose that there are a set of n DMUs which received an m-dimensional input vector X to produce an s-dimensional output vector Y. The efficiency of these DMUs will be evaluated in a time horizon of T periods. Let

- $x_j^t = (x_{1j}^t, x_{2j}^t, \dots, x_{mj}^t)$ be the m-dimensional input vector of DMU_j, $j = 1,2,\dots,n$;
- $y_j^t = (y_{1j}^t, y_{2j}^t, \dots, y_{sj}^t)$ be the s- dimensional output vector DMU_j, $j = 1,2,\dots,n$;
- $u = (u_1, u_2, \dots, u_s)$ be the s- dimensional output vector prices;
- $v = (v_1, v_2, \dots, v_m)$ be the m- dimensional input vector costs;

Referring to DEA fundamentals, efficiency of a DMU is defined as dividing its weighted sum of outputs by weighted sum of inputs, e.g.:

$$E_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \tag{1}$$

Where $E_j \leq 1$ for all $j = 1,2,\dots,n$ and $u_r \geq 0, r = 1,2,\dots,s$ and $v_i \geq 0, i = 1,2,\dots,m$. In cross sectional DEA problems, the efficiency of DMUs is obtained easily by solving a problem of maximizing individual efficiencies subjected to the above constraints. De facto, to evaluate the relative efficiency of a given DMU$_0$, a linear programming problem of the form $\{Maximize\ E_0, \text{Subject To } E_j \leq 1, j = 1,2,\dots,n\}$ is solved.

Note that, there are a set of $m+s$ time series associated with each inputs and outputs. Suppose that $\mu_{ij} = \sum_{t=1}^{T} x_{ij}^t / T$ and $s_{ij} = \sqrt{\sum_{t=1}^{T} (x_{ij}^t - \mu_{ij}) / (T-1)}$ are the mean and standard deviation of ith input of jth DMU, and $\delta_{rj} = \sum_{t=1}^{T} y_{ij}^t / T$ and $d_{rj} = \sqrt{\sum_{t=1}^{T} (y_{rj}^t - \delta_{ij}) / (T-1)}$ are the mean and standard deviation of rth output of jth DMU, respectively; thus, the input mean and standard deviation matrices are constructed as:

$$\mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1m} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2m} \\ \vdots & \vdots & & \vdots \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{nm} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix} \tag{2}$$

And

$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1m} \\ s_{21} & s_{22} & \cdots & s_{2m} \\ \vdots & \vdots & & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nm} \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} \tag{3}$$

Similarly, for output variables:

$$\delta = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1s} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2s} \\ \vdots & \vdots & & \vdots \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{ns} \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_1 \\ \vdots \\ \delta n \end{bmatrix} \tag{4}$$

And

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1s} \\ d_{21} & d_{22} & \cdots & d_{2s} \\ \vdots & \vdots & & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{ns} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} \tag{5}$$

Where, δ and D Stand for output time series mean and variance matrices, respectively. It is well known that the variance of a sample mean is the ratio of variables' variance to the sample size. Therefore, input and output variables mean, Eqs. (2) and (4), have a sample variance of $S' = S/T$ and $D' = D/T$. Now, the Chebyshev inequality stated that for a random variable X with a mean of M and standard deviation of σ is remarkable:

$$\Pr(|X - M| \le K\sigma) \ge 1 - \frac{1}{K^2} \tag{6}$$

Or,

$$\Pr(M - K\sigma \le X \le M + K\sigma) \ge 1 - \frac{1}{K^2} \tag{7}$$

If $K = \sqrt{20}$, the above inequality is hold with a probability of at least 95%.

Applying the Chebyshev inequality, at least $100(1 - 1/K^2)\%$ significant confidence interval of input variables can be constructed as follows:

$$(\mu - KS', \mu + KS') \tag{8}$$

The corresponding inequality for outputs can be proclaimed as:

$$(\delta - KD', \delta + KD') \tag{9}$$

If the lower bound of inputs or outputs is a negative number, it is fixed at zero. The input oriented CCR model is formulated as the following linear programming problem:

$$
\begin{aligned}
&Max\sum_{r=1}^{s} u_r y_{r0} \\
&\text{S.T.} \\
&\sum_{i=1}^{m} v_i x_{i0} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \ j = 1,2,\ldots,n \\
&v_i \ge 0, i = 1,2,\ldots,m \\
&u_r \ge 0, r = 1,2,\ldots,s
\end{aligned}
\tag{10}
$$

It is worth noting here that the non-Archimedean element, ε, can be added to the model variables, if necessary (e.g. $v_i \ge \varepsilon, i \quad 1,2,\ldots,m$ and $u_r \ge \varepsilon, r \quad 1,2,\ldots,s$.). However, as pointed by Jahanshahloo and Khodabakhshi (2004), solving a DEA problem by simply choosing a value for ε can lead to erroneous results. Hence, the use of non-Archimedean elements is unusual in customary applications of linear programming and it is usually discarded (Arnold, Bardhan, Cooper, & Gallegos, 2008) as it is done in the proposed model.

The above problem can be easily presented with matrix notation:

$$
\begin{aligned}
&Max\, u^t y_0 \\
&\text{S.T.} \\
&v^t x_0 = 1 \\
&u^t y_j - v^t x_j \le 0, \ j = 1,2,\ldots,n \\
&v \ge 0, u \ge 0
\end{aligned}
\tag{11}
$$

Incorporating Eqs. (8) and (9) in Eq. (11), the Chebyshev-based DEA model is obtained:

$$
\begin{aligned}
&Max\, u^t \begin{pmatrix} \delta_0 - Kd'_0 \\ \delta_0 + Kd'_0 \end{pmatrix} \\
&\text{S.T.} \\
&v^t \begin{pmatrix} \mu_0 - Ks'_0 \\ \mu_0 + Ks'_0 \end{pmatrix} = 1 \\
&u^t \begin{pmatrix} \delta_j - Kd'_j \\ \delta_j + Kd'_j \end{pmatrix} - v^t \begin{pmatrix} \mu_j - Ks'_j \\ \mu_j + Ks'_j \end{pmatrix} \le 0, \ j = 1,2,\ldots,n \\
&v \ge 0, u \ge 0
\end{aligned}
\tag{12}
$$

Applying interval number operators (Moore, Kearfott, & Cloud, 2009) on Eq. (12), the equivalent formulation is garnered as:

$$Max\left(u^t\left(\delta_0 - Kd'_0\right), u^t\left(\delta_0 + Kd'_0\right)\right)$$

S.T.

$$X = \begin{cases} \left(v^t\left(\mu_0 - Ks'_0\right), v^t\left(\mu_0 + Ks'_0\right)\right) = 1 \\ \left(u^t\left(\delta_j - Kd'_j\right) - v^t\left(\mu_j + Ks'_j\right), u^t\left(\delta_j + Kd'_j\right) - v^t\left(\mu_j - Ks'_j\right)\right) \leq 0,\ j = 1,2,\ldots,n \\ v \geq 0, u \geq 0 \end{cases} \quad (13)$$

When the left hand side of the obtained confidence intervals becomes negative an important feature is noticeable. As it is described below, since the constraints of the problem are of the less than or equal type, the model manipulated their upper bound and central point's only; hence, the issue of negative data isn't considered here.

Eq. (13) is an interval linear programming problem. An algorithm is needed to solve the interval-DEA model (13). Different algorithms are proposed for solving interval linear programming problems (Chinneck, & Ramadan, 2000; Sengupta, Pal, & Chakraborty, 2001; Chen, Chen, Chen, & Wang, 2004; Razavi Hajiagha, Akrami, & Hashemi, 2012). The proposed algorithm is developed based on the idea of Kao and Liu (2000; 2011) on the basis of the relational ranking rules introduced by Ishibuchi and Tanaka (1990).

Considering $\widetilde{A} = [\underline{a}, \overline{a}]$ and $\widetilde{B} = [\underline{b}, \overline{b}]$, then $\widetilde{A} \leq \widetilde{B}$ if

$$\overline{a} \leq \overline{b} \quad (14)$$

And

$$\left(\frac{\underline{a} + \overline{a}}{2}\right) \leq \left(\frac{\underline{b} + \overline{b}}{2}\right) \quad (15)$$

Similarly, $\widetilde{A} \geq \widetilde{B}$ if:

$$\underline{a} \geq \underline{b} \quad (16)$$

And

$$\left(\frac{\underline{a} + \overline{a}}{2}\right) \geq \left(\frac{\underline{b} + \overline{b}}{2}\right) \quad (17)$$

These ordering relations are consistent with the possibility degree of Li, Yamaguchi, and Nagai (2007). Remained constraints are handled upon Ishibuchi and Tanaka (1990) ordering relations. For a constraint $\left(u^t\left(\delta_j - Kd'_j\right) - v^t\left(\mu_j + Ks'_j\right), u^t\left(\delta_j + Kd'_j\right) - v^t\left(\mu_j - Ks'_j\right)\right) \leq 0$, these ordering relations are implied:

$$\begin{cases} u^t\left(\delta_j + Kd'_j\right) - v^t\left(\mu_j - Ks'_j\right) \leq 0 \\ u^t\delta_j - v^t\mu_j \leq 0 \end{cases} \quad (18)$$

Applying Eq. (18), the Eq. (13) is now transformed into the model represented in Eq. (19):

$$
\begin{aligned}
&\max\left(u^t(\delta_0 - Kd'_0), u^t(\delta_0 + Kd'_0)\right)\\
&\text{S.T.}\\
&v^t\mu_0 = 1\\
&u^t\left(\delta_j + Kd'_j\right) - v^t\left(\mu_j - Ks'_j\right) \le 0,\ j = 1,2,\ldots,n\\
&u^t\delta_j - v^t\mu_j \le 0,\ j = 1,2,\ldots,n\\
&u^t \ge 0, v^t \ge 0
\end{aligned} \tag{19}
$$

Considering the second and third constraints of Eq. (19), we can easily recognize that the third constraint is a subset of the second one; thus, the third constraint can be eliminated. Solving Eq. (19) by eliminating its third constraint, an optimal weight vector u^* and v^* is obtained. Applying these weights, we can calculate the interval efficiency for DMU$_0$ as $\left(u^{t*}(\delta_0 - Kd'_0), u^{t*}(\delta_0 + Kd'_0)\right)$.

To find these weights, Kao and Liu (2000; 2011) proposed a two-stage model of finding interval efficiency of DMUs. Considering DMU$_0$, its lower bound efficiency, E_0^l, is found by setting its inputs in their upper bound and its outputs in their lower bound, while other DMUs inputs are fixed at their lower bound and their outputs at their upper bound. Therefore, the lower bound efficiency model is constructed as:

$$
\begin{aligned}
&E_0^l = \max u^t\left(\delta_0 - Kd'_0\right)\\
&\text{S.T.}\\
&v^t\left(\mu_0 + Ks'_0\right) = 1\\
&u^t\left(\delta_j + Kd'_j\right) - v^t\left(\mu_j - Ks'_j\right) \le 0,\ j = 1,2,\ldots,n,\ j \ne 0\\
&u^t \ge 0, v^t \ge 0
\end{aligned} \tag{20}
$$

On the other hand, the upper bound efficiency of DMU$_0$, E_0^u, is computed by setting its inputs at their lower bound and its outputs at their upper bound, while other DMUs inputs are fixed at their upper bound and their outputs at their lower bound. Hence, the upper bound efficiency model is constructed as:

$$
\begin{aligned}
&E_0^l = \max u^t\left(\delta_0 + Kd'_0\right)\\
&\text{S.T.}\\
&v^t\left(\mu_0 - Ks'_0\right) = 1\\
&u^t\left(\delta_j - Kd'_j\right) - v^t\left(\mu_j + Ks'_j\right) \le 0,\ j = 1,2,\ldots,n,\ j \ne 0\\
&u^t \ge 0, v^t \ge 0
\end{aligned} \tag{21}
$$

Solving two problems in Eqs. (20) and (21), the interval efficiency of DMUs is determined as $\widetilde{E}_j^* = \left[E_0^{l*}, E_0^{u*}\right]$. These interval efficiencies provide a lower bound and upper bound for efficiency score of DMUs. Subsequently, if ranking the efficiency of DMUs is considered, comparing relative interval efficiencies is necessary. For the sake of this comparison, a discrete representative of each DMU is required. As previously mentioned, Ishibuchi and Tanaka (1990) proposed a ranking method for interval numbers by computing their bounds and means; accordingly, the mean score of these intervals can be considered for ranking interval efficiencies:

$$\overline{E}_j = \frac{E_j^{l*} + E_j^{u*}}{2} \tag{22}$$

4. Case study

As described in section 2, the above algorithm is applied in a bank with 115 branches during a financial year. The data are gathered in 6 months; therefore, there are 6 input matrices of the size 115×5 and 6 output matrices of the size 115×4. The mean and variance matrices of input and output measures are calculated for different units and the Chebyshev-based confidence intervals are computed for different inputs and outputs of DMUs. Afterwards, the proposed model is applied for evaluating DMUs efficiency.

Following preparing the appropriate matrix, the model is solved for per DMU applying a MATLAB code. The obtained results are summarized in table 2. In this table, the problem is also solved by aggregated and network connected models of Kao and Liu (2014).

<Please insert table 2 here>

It is clear that there are marked differences between interval efficiencies lower bound and upper bound in the interval efficiency column of table 2. These differences are mainly due to large standard deviations of input and output measures in the original data. In addition, there is not a constant increasing trend in inputs and outputs of DMUs. As a case in point, the inputs and outputs trends of DMU 1002 are shown in Fig. 1.

<Please insert Fig. 1 here>

According to Fig. 1, there is a large increasing trend in inputs of DMU 1002, while the outputs trends do not indicate a similar trend. These large standard deviations cause a large difference between lower and upper bounds of efficiency scores. Coupled with, it is notable that penny difference of interval and mean efficiencies of the proposed method among many DMUs in this case are just due to their data similarity, which doesn't happen on a regular basis. The equality of upper bound efficiencies in DMUs is also due to a similar phenomenon, being not necessary in all problems.

Differently, Fig. 2 reveals the trends of interval efficiencies lower and upper bounds against the aggregated and network connected efficiencies in table 2.

<Please insert Fig. 2 here>

According to Fig. 2, the aggregated and network connected scores take their values among the lower and upper bounds efficiencies. A comparison is made between interval efficiency scores mean values and the aggregated and network connected scores. This comparison is shown in Fig. 3.

<Please insert Fig. 3 here>

Analyzing the differences between mean efficiency scores, aggregated and network connected scores indicates that mean efficiency scores of the proposed method is meaningfully smaller than both aggregated and network connected efficiencies with a confidence of 95%. In fact, the proposed method provides a warily approximation of DMUs efficiencies by applying their inputs and outputs variations directly in evaluating DMUs efficiency.

Considering the results emanate from proposed method in table 2, its main advantages can be summarized as follow. First of all, as previously mentioned, the proposed method directly applies the variance of inputs and outputs in evaluating DMU's efficiency over a time horizon consisting several time periods, providing a more realistic appraisal of DMUs by considering their fluctuations in inputs and outputs. Furthermore, as Liu and Lin (2006) proclaimed, a principal axiom of uncertain problems is nonexistence of exact result. As we clarified in table 2, since the outputs and inputs are fluctuated randomly over time horizon, the considered problem of multi period efficiency appraisal is an uncertain problem. Considering this axiom, the proposed method finds interval efficiencies for DMUs, while other methods determined exact efficiencies. Eventually, the proposed method revealed the highest discrimination power since just one DMU is classified as full efficient; nonetheless, other methods classified several DMUs as fully efficient declining their discrimination power.

Conclusion

Classic data envelopment analysis proposed a well-known and widely accepted method to evaluate the relative efficiency of a set of homogeneous units in a single period. However, organizational units perform in different periods of time, and it is interesting for their managers to have a comprehensive picture of the performance over multiple periods of time. In this paper, a method is proposed to measure the relative efficiency of DMUs in multiple periods, called time series DEA.

Aggregating the inputs and outputs of DMUs over different periods is one of the most popular approaches used in time series DEA. Cumulating inputs and outputs data in different periods in a single period provides a compensating impact for DMUs. Admittedly, the low values of outputs in some periods can be countervailed by high values of outputs in other periods. This compensating impact provides an easy-emptive efficiency evaluation framework for DMUs. The proposed approach in this paper, initially found a confidence interval for inputs and outputs of different DMUs over time periods based on Chebyshev inequality bounds. It is remarkable that if the time periods increased to more periods, exact parametric confidence intervals can be used to find confidence intervals of inputs and outputs. Following that a multi-objective model is solved to find the interval efficiencies of DMUs. It is a well-known axiom that when data are uncertain, it is impossible to obtain a crisp solution for the problem. This axiom implies that since input and output's data over a period of time follows a statistical distribution, the efficiency of DMUs cannot be an exact number. This fact is applied in the proposed method by finding interval efficiency for DMUs. In fact, since the inputs and outputs data are captured in the form of different time series, there isn't any deterministic and exact feasible space for the linear programming DEA model in a multi-period efficiency appraisal problem. This randomness is formulated with constructing interval constraints in the proposed method. Absence of an exact result for the uncertain multi-period DEA model is handled by interactively incorporating some confidence levels in finding inputs and outputs confidence intervals and finding efficiency scores under confidence levels. It is largely taken for granted that any change in this confidence level has certain impacts on objective function and feasible space of the proposed method and the results will be changed. By and large, if the problem is solved in a certain confidence level, the obtained efficiency results are only valid in its confidence level.

The proposed method is applied in a case of evaluating 115 bank branches in a six-period time horizon. The obtained results are compared with some previous presented methods of time series DEA. As shown in this case, results of the proposed method provide a cautious approximation of DMUs efficiencies. These approximations are more truly representing the real condition of DMUs efficiencies since data variations are implied in evaluations directly. This approach can increase the discrimination power of DEA model. As it can be seen from the results of case study, there is only one fully efficient DMU, 1001, while there are many full efficient DMUs in two other approaches.

Future researches could be directed toward extension of the proposed method when outputs and inputs of the DMUs are determined uncertainly by fuzzy sets. Besides, the proposed method is applicable in network and supply chain DEA. On the other hand, the problem of return to scale and its fluctuations is examinable in multi period DEA models.

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