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SIMPLIFIED SECOND-ORDER GENERALIZED INTEGRATOR — FREQUENCY-LOCKED LOOP

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Abstract. Second-Order Generalized Integrator — Frequency-Locked Loop (SOGI-FLL) is a popular technique available in the grid synchronization literature. This technique uses gain normalization in the frequency locked-loop. This increases the computational complexity. In this paper, we propose an alternative implementation to reduce the computational complexity of the SOGI-FLL. The proposed implementation modifies mainly the frequency locked-loop part and requires normalized voltage measurement. dSPACE 1104 board-based hardware implementation shows that the proposed implementation executes 20% faster than the standard implementation. This could be very beneficial for high switching frequency application e.g. ≥ 1 MHz. In addition to the nominal frequency case, multi-resonant implementation is also proposed to tackle grid harmonics using a simpler harmonic decoupling network. Small signal dynamical modeling and tuning are performed for both implementations. Dynamical equivalence is also established between the two implementations. Experimental comparative analysis demonstrates similar or better performance (depending on test scenarios) with respect to the standard implementation of the SOGI-FLL.

Keywords


1. Introduction

To reduce the worst impacts of climate change, Renewable Energy Sources (RES) can play a major role. In this regard, International Energy Agency (IEA) set up an ambitious target of 17.6 Gt energy-related carbon emission by 2040 (as opposed to 33.1 Gt in 2018). RES have the potential to contribute in this regard. RES are generally connected through inverters to the traditional power grid. Maximum power can only be transferred to the grid from RES, if the instantaneous phase of both RES and the grid are the same. This process is commonly known as grid synchronization. In addition, many other applications require the phase and frequency information of the traditional power grid [1], [2], [3], [4], [5] and [6]. As phase and frequency information have huge potential implications, this has attracted the attention of wider engineering communities, resulting to an increasing research effort.

In the grid synchronization literature, Phase-Locked Loops (PLL) are widely used. PLL works by synchronizing with the input signal. As such PLLs are sensitive to phase angle jump. To overcome this limitation of PLL, researchers have paid their attention to frequency which very rarely suffers from sudden change in the actual power system. As a result Frequency-Locked Loop (FLL) has been proposed which works by synchronizing itself with the frequency of the input signal.

In the literature of FLL, Quadratic Signal Generators (QSG) are widely used. There are many different QSG techniques available in the literature, e.g. Phase Locked-Loop [7], Enhanced Phased Locked-Loop (EPLL) [8], Second Order Generalized Integrator
The rest of this paper is organized as follows: Sec. 2 describes the proposed approach. Modeling and tuning are studied in Sec. 3. Extension to the case of harmonics is described in Sec. 4. Experimental results are given in Sec. 5. Finally, Sec. 6 concludes this paper.

2. Alternative Implementation of SOGI-FLL

In this work, we propose an alternative way to calculate the second state variable of the SOGI-FLL and propose a modification in the FLL equation. This along with the use of normalized or per unit voltage measurement, help to eliminate the need of gain normalization which reduces the computational complexity. Moreover, in the case of harmonics, the proposed approach has a simple harmonic decoupling method. One question that naturally arise is that are the two implementations dynamically equivalent. We address this question through small-signal modeling.

In this paper, an alternative implementation of the standard SOGI-FLL has been proposed. The proposed implementation is computationally simpler and has a simple harmonic decoupling network. Moreover, subject to parameter tuning, SOGI-FLL and the proposed alternative implementation are dynamically equivalent. This alternative implementation is the main contribution of this paper.


\[ y_0 = \hat{A}_0 \] is the estimation of DC bias \( A_0 \) and \( \hat{\theta} \) is the estimated value of \( \theta \). An advantage of the proposed modification is that it preserves all the transfer functions of the SOGI-FLL [9]. As such they are avoided here for the purpose of brevity. In the steady-state, solutions of \( x \) and \( y \) are given as:

\[
\begin{align*}
x &= -\hat{A} \cos(\hat{\theta}), \\
y &= \hat{A} \sin(\hat{\theta}).
\end{align*}
\]

It is to be noted here that amplitude normalization (i.e. measurement in per unit) is necessary for the proposed ASOGI-FLL. Since embedded hardwares always work in the low-voltage e.g. 0–5 V (micro-controller), ±10 V (dSPACE), normalized measurements are a reality in power system control design. As such no additional computational burden is required for the implementation of the proposed technique.

3. Modeling and Tuning

In this section, small signal modeling and tuning of the two different implementations are performed. Small signal modeling will also help to establish the dynamical equivalence between the two implementations.

3.1. Modeling of SOGI-FLL

SOG-FLL shown in Fig. 1(b) can be mathematically described by the following equations:

\[
\begin{align*}
\dot{x} &= y, \\
\dot{y} &= \alpha (\nu - y - y_0) \hat{\omega} - x\omega^2, \\
\dot{\omega} &= -\frac{\alpha \beta \omega^2}{x^2 + y^2} (\nu - y - y_0), \\
\dot{y}_0 &= \gamma (\nu - y - y_0), \\
\dot{\hat{\theta}} &= \arctan \left( \frac{y}{-x\omega} \right),
\end{align*}
\]

where \( \alpha, \beta, \gamma > 0 \) are tuning gains and the rest of the variables retain the same meaning as in Sec. 2. For further analysis, it is assumed that the SOGI-FLL is near the equilibrium state, i.e. \( \theta = \theta_0 \), \( \omega = \hat{\omega} \), \( \hat{A} = \hat{A}_0 \) and \( A_0 = \hat{A}_0 \).

Using Eq. (1) and Eq. (3), the frequency estimation dynamics can be written as (using similar approaches as presented in [21]):

\[
\begin{align*}
\dot{\hat{\omega}} &= \frac{\alpha \beta \omega}{\hat{A}^2} \left\{ A \sin(\theta) - \hat{A} \sin(\hat{\theta}) + A_0 - \hat{A}_0 \right\} \hat{\omega} \\
&= \frac{\alpha \beta \omega}{2} \left\{ \sin(\theta - \hat{\theta}) + \sin(\theta + \hat{\theta}) - \sin(2\hat{\theta}) \right\} + \frac{\alpha \beta \omega}{\hat{A}} (A_0 - \hat{A}_0).
\end{align*}
\]

Near the equilibrium point or in the quasi-locked state one can safely assume that \( A_0 - \hat{A}_0 \to 0 \), \( \sin(\theta + \hat{\theta}) \to 0 \) and \( \hat{\omega} = \omega_n \). From the small-angle approximation formula, one can write that \( \sin(\theta - \hat{\theta}) \approx \theta - \hat{\theta} \). By applying all these simplifications, Eq. (4e) can be written as:

\[
\dot{\hat{\omega}} = \frac{\alpha \beta \omega}{2} (\theta - \hat{\theta}).
\]

To obtain the dynamics of the estimated phase-angle, let us consider the time derivative of Eq. (4e) given as:

\[
\dot{\hat{\theta}} = \frac{\dot{\hat{\omega}}}{\alpha \beta \omega} = \frac{\dot{\hat{\omega}}}{\hat{A}} = \frac{\omega_n}{\hat{A}}.
\]

Using Eq. (3) in Eq. (4c), it can be found that:

\[
-\alpha (\nu - y - y_0) \omega^2 x + \hat{A}^2 \dot{\hat{\omega}} = \omega + -\alpha (\nu - y - y_0) \omega^2 x = \dot{\hat{\omega}} \hat{A}^2 \beta^{-1}.
\]

Using Eq. (8) in Eq. (7), the instantaneous phase angle can be written as:

\[
\dot{\hat{\theta}} = \omega_n + \frac{\dot{\hat{\omega}}}{\beta}.
\]

Finally the dynamics of the DC bias estimation can be calculated as:

\[
\hat{A}_0 = \gamma \left[ A_0 + \hat{A} \sin(\theta) - \hat{A} \sin(\hat{\theta}) - \hat{A}_0 \right] \hat{\omega} = \gamma (\theta - \theta_0 + A_0 - \hat{A}_0) \hat{\omega}.
\]

In the quasi-locked state, it can be assumed that \( \theta - \hat{\theta} \to 0 \). Then the Eq. (10) can be simplified as:

\[
\hat{A}_0 = \gamma (A_0 - \hat{A}_0).
\]

Based on Eq. (6), Eq. (9) and Eq. (11), the transfer function of the linearized frequency, phase and DC offset dynamics of SOGI-FLL are given as:

\[
\begin{align*}
\frac{\hat{\omega}(s)}{\omega(s)} &= \frac{\alpha \beta \omega}{s^2 + \gamma^2 + \frac{\alpha \beta \omega}{\hat{A}^2} + \frac{\alpha \beta \omega}{\hat{A}}}, \\
\frac{\hat{\theta}(s)}{\theta(s)} &= \frac{\frac{\alpha \beta \omega}{s} + \frac{\alpha \beta \omega}{\hat{A}}}{\frac{\alpha \beta \omega}{s^2} + \frac{\alpha \beta \omega}{\hat{A}^2} + \frac{\alpha \beta \omega}{\hat{A}}}, \\
\frac{\hat{A}_0(s)}{A_0(s)} &= \frac{\gamma \omega_n}{s + \gamma \omega_n}.
\end{align*}
\]

3.2. Gain Tuning of SOGI-FLL

To tune the gains of SOGI-FLL, let us consider the standard second-order transfer function denominator polynomial given as:

\[
\begin{align*}
\omega(s) &= \frac{\alpha \beta \omega}{s^2 + \frac{\alpha \beta \omega}{\hat{A}^2} + \frac{\alpha \beta \omega}{\hat{A}}} = \frac{\alpha \beta \omega}{s^2 + \gamma^2} + \frac{\alpha \beta \omega}{\hat{A}}, \\
\theta(s) &= \frac{\frac{\alpha \beta \omega}{s} + \frac{\alpha \beta \omega}{\hat{A}}}{\frac{\alpha \beta \omega}{s^2} + \frac{\alpha \beta \omega}{\hat{A}^2} + \frac{\alpha \beta \omega}{\hat{A}}}, \\
A_0(s) &= \frac{\gamma \omega_n}{s + \gamma \omega_n}.
\end{align*}
\]
\[ s^2 + 2\zeta \omega_n s + (\omega_n')^2 = 0, \]

where \( \zeta \) and \( \omega_n' \) are the damping-ratio and natural frequency respectively. We assume that \( \omega_n' = \omega_n \). Next, by comparing the Eq. (13) with the denominator polynomials of Eq. (12a) and Eq. (12b), it can be found that:

\[
\alpha \beta \omega_n = 2(\omega_n')^2 \\
\omega_n' = \frac{\alpha \omega_n}{4\zeta} 
\]

Equation (14) can be used to calculate \( \beta \) from \( \alpha \) or vice-versa. However, it requires the value of the damping ratio, \( \zeta \). Using integral of time weighted absolute error (ITAE) criteria, Subsec. 5.8 in [22] has suggested \( \zeta = 1/\sqrt{2} \) as an optimal value for second-order system. This value will be considered here as well. With the selected value of \( \zeta \), \( \alpha = 1 \) corresponds to \( \beta = 78.5 \). To select the DC loop tuning gain \( \gamma \), we consider the first-order transfer function Eq. (12c). The settling time of this system is given by:

\[
t_{s(DC)} \approx \frac{3.9}{\gamma \omega_n}, \quad (15)
\]

A settling time of 50 ms (2.5 cycles) corresponds to \( \gamma \approx 0.25 \) in Eq. (15). This completes the tuning of the SOGI-FLL.

### 3.3. Modeling of ASOGI-FLL

Similar to the ideas presented in Subsec. 3.1, linear modeling of the ASOGI-FLL has also been performed. Details are avoided here for the purpose of brevity. The transfer function of the linearized frequency, phase and DC offset dynamics of SOGI-FLL are given as:

\[ \frac{\dot{\omega}(s)}{\omega(s)} = \frac{\rho \omega_n}{s^2 + \frac{2 \alpha \omega_n}{2} s + \frac{\omega_n^2}{2}}, \quad (16a) \]

\[ \frac{\dot{\theta}(s)}{\theta(s)} = \frac{\rho \omega_n}{s^2 + \frac{2 \alpha \omega_n}{2} s + \frac{\omega_n^2}{2}}, \quad (16b) \]

\[ \frac{\dot{A}_0(s)}{A_0(s)} = \frac{\mu}{s + \mu}, \quad (16c) \]

By comparing Eq. (12) and Eq. (16), it can be concluded that SOGI-FLL and ASOGI-FLL are mathematically very similar. Moreover, depending on the parameter values, the two technique coincides e.g. \( \kappa = \alpha = 1 \), \( \mu = \gamma \omega_n \) and \( \rho = \beta \). This shows that the two techniques are dynamically equivalent.

### 3.4. Gain Tuning of ASOGI-FLL

Similar to Subsec. 3.2, by comparing the Eq. (13) with the denominator polynomials of Eq. (16a) and Eq. (16b), it can be found that

\[ \rho \omega_n = 2(\omega_n')^2 \\
\omega_n' = \frac{\kappa \omega_n}{4\zeta} \]

Equation (17) can be used to calculate \( \rho \) from \( \kappa \) or vice-versa. Similar to Subsec. 3.2, \( \zeta = 1/\sqrt{2} \) is selected here as well. With the selected value of \( \zeta \), \( \kappa = 1 \) corresponds to \( \rho = 78.5 \). To select the DC loop tuning gain \( \mu \), we consider the first-order transfer function Eq. (16c). The settling time of this system is given by:

\[ t_{s(DC)} \approx \frac{3.9}{\mu}. \quad (18) \]

A settling time of 50 ms (2.5 cycles) corresponds to \( \mu \approx 78.5 \) in Eq. (15). This completes the tuning of the ASOGI-FLL.

### 3.5. Remarks on the Similarity between SOGI-FLL and ASOGI-FLL

From the parameter tuning procedures described in Subsec. 3.2 and Subsec. 3.4, it is very clear that ASOGI-FLL and SOGI-FLL have the same linearized dynamics. The main difference is in the signal normalization procedure. SOGI-FLL does feedback linearization type signal normalization while ASOGI-FLL avoids this by using normalized measurement. One criticism of the proposed solution is that the frequency estimation dynamics become dependent on the amplitude changes. However, this is not the case if normalized measurements are used. Normalized or per unit measurements are widely used in the power system estimation literature. It is to be noted here that both techniques are nonlinear. As such having same linear dynamics does not necessarily mean that both techniques would react exactly the same way in the presence of disturbance or harmonics.

### 4. Multi-Frequency ASOGI-FLL

Harmonics polluted grid voltage signal is given by:

\[ \nu(t) = A_0 + \sum_{i=1,3}^{2n-1} A_i \sin (\omega_i t + \varphi_i), \quad (19) \]

where \( A_i \), \( \omega_i \), \( \varphi_i \) and \( \theta_i \) are the amplitude, angular frequency, phase and instantaneous phase of the individual harmonic components respectively. ASOGI-FLL as described in Sec. 2, considers only the
case of \(i = 1\). However, in the presence of harmonics ASOGI-FLL can longer provides zero steady-state error. To eliminate the steady-state error, following the ideas of [23], parallel operation of the multiple ASOGI block coupled with a frequency identification block can be one possible solution. For further development, we name this structure as parallel ASOGI-FLL (PASOGI-FLL). PASOGI-FLL can be mathematically represented as for \(n\) number of frequency components with \(i = 1, 2, \ldots, n\) as:

\[
\begin{align*}
\dot{y}_n &= -\omega_1 y_n + \alpha \omega_1 e_n, \\
\dot{x}_i &= y_i \omega_1, \\
\dot{z} &= -\beta x_1 e_1, \\
\dot{y}_0 &= \gamma e,
\end{align*}
\]

with \(\omega_i = \omega \cdot i\) and \(e = (v - v_0 - y_1 - y_2 - \ldots - y_n)\). Graphical representation of PASOGI-FLL is given in Fig. [2]. This figure shows that PASOGI-FLL has a very simple harmonic decoupling network. This can be considered an improvement w.r.t. the harmonic decoupling network of the standard SOGI-FLL given in Fig. 8 of [20].

For the sake of computational simplicity, let us consider that only the third harmonics is present. In this case, the grid frequency estimation transfer function is given by:

\[
y(s) = \frac{\alpha (\omega_1 + \omega_3)(s^2 + \omega_1 \omega_3)}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0},
\]

where \(a_3 = \alpha (\omega_1 + \omega_3)\), \(a_2 = (\omega_1^2 + \omega_3^2)\) and \(a_1 = \alpha \omega_1 \omega_2 (\omega_1 + \omega_2)\) and \(a_0 = \omega_1^2 \omega_2^2\). Bode magnitude plot of the transfer function is given in Fig. 3. Figure 3 demonstrates excellent filtering property of PASOGI.

5. Experimental Results

To evaluate the performance of the two techniques, dSPACE@1104 board based Hardware-In-the-Loop (HIL) experimental study has been performed. Voltage signal was acquired using the built-in 12-bit LTC1410 Analog-to-Digital Converter (ADC) from Analog Devices. The two techniques have been implemented in Matlab/Simulink with a sampling frequency of 10 kHz. All the continuous integrators are implemented using third-order discrete integrator [23]. The same parameter values are selected as given in Subsec. 3.2. and Subsec. 3.4. Block diagram representation of the experimental setup is given in Fig. 4.

To test the computational efficiency, dSPACE Profile has been used. According to the Profiler, the average execution time for ASOGI-FLL is approximately 2.35 ms while SOGI-FLL took approximately 2.95 ms. This counts for roughly 20 % time saving by the proposed implementation. It is to be noted here that dSPACE 1104 board is a powerful embedded computing hardware. For low-cost domestic applications, lower end micro controllers are more widely used. There is a possibility that the execution time saving could be even higher in those low-cost microcontrollers. However, this is yet to be tested.

To test the algorithms, we considered grid voltage signal with and without harmonics. For the harmonics case, we considered the grid voltage has 20 % Total Harmonic Distortion (THD) comprising of third, fifth and seventh order harmonics. In both cases, sudden jump of frequency, phase, amplitude and DC offset are considered. Comparative experimental results and test conditions are given in Fig. 5 and Fig. 6.

**Frequency jump test:** Both techniques have similar performances in terms of frequency estimation. How-
ever, the techniques slightly differed in terms of phase estimation error. ASOGI’s peak frequency estimation error is slightly higher than that of SOGI. Same goes for the case with harmonics.

*Phase jump test:* Similar behavior can be seen as that of frequency jump tests i.e. frequency estimation performance is very similar and slight difference for the phase estimation error. It is to be mentioned here that in the presence of harmonics, ASOGI converged faster than SOGI.

*Amplitude jump test:* In this case, significant performance difference can be observed between SOGI and ASOGI. With and without harmonics, both frequency and phase estimation error for ASOGI converged much faster than SOGI. In addition, the peak overshoot and peak estimation error were slightly smaller for ASOGI than that of SOGI.

*DC jump test:* This test demonstrate that ASOGI has much faster and smoother dynamics compared to SOGI. The frequency and phase estimation error converged at least 1 full cycle faster w.r.t. SOGI. Decaying oscillatory transient response can be seen for SOGI while no such phenomenon can be observed for ASOGI.

In addition to the above attractive features, in the case of multiple adaptive filters, the harmonic decoupling network is very simple for ASOGI-FLL whereas SOGI-FLL has a complicated harmonic decoupling network structure. SOGI-FLL uses FLL gain normalization whereas ASOGI-FLL does not use any gain normalization. Despite these differences, the performances of both techniques are similar. These results demonstrate the effectiveness of the proposed alternative implementation of the SOGI-FLL.

6. Conclusions

In this paper, an alternative implementation of the SOGI-FLL has been proposed for both nominal and distorted grid conditions. In the case of distorted grid, the proposed approach uses multiple filters in parallel using a simple harmonic decoupling network. The proposed modifications eliminate the need of FLL gain normalization. In this regard, it reduces the computational complexity. Extensive experimental results are presented. Experimental results demonstrated the suitability of the proposed alternative implementation. The proposed approach performed similar to SOGI-FLL when the grid voltage suffered from sudden jump in phase and frequency while performed better when the grid voltage suffered sudden jump in amplitude and DC offset.
References


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