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A Conceptual Analysis and Framework

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Abstract— **The paper presents a conceptual interpretation of amalgamating inquiry-based learning and the flipped classroom model with gamification / game processes as means to strengthen and improve the way Mathematics learning and teaching content, strategies and approaches are enacted in primary, secondary and tertiary education. By introducing a Personal Learning Environment (PLE) where students and teachers can create, (re)use and repurpose their own learning content (e.g. inquiry-based games feedback, assessment, micro-blogs), mathematics modules can be accessed, created, remixed, learned and reviewed in-class and out-class through rapid, personalised and meaningful playful feedback. Learning activities and modules at any scale may be coupled with the GamifyMaths framework to offer engaging personalized learning experiences and promote ownership, accomplishment and creativity. Learners and teachers are finally enabled to discern the connection of mathematics with other disciplines highlighting its inter-disciplinary application.**

Keywords: mathematics education, inquiry-based learning, games, gamification.

I. Introduction

Mathematics is an intellectual inquiry to augment knowledge, resolve doubt and solve problems in topics such as quantity, structure, space and change. Resolution of doubt is based on the concept of mathematical proof as a rigorous type of reasoning that allows for the construction of new knowledge from given facts in a way that is indisputable.

The wide applicability of Mathematics in all sciences and technological domains is due to its foundational relation to pedagogy (Mathematics stems from the Greek word "mathesis" which means learning) and the generic nature of its study topics (quantity, structure, space, change). Consequently, from a learner's standpoint, mathematics goes beyond a utilitarian view as a tool. The pedagogical value of mathematics is much deeper and its relation to inquiry-based pedagogies is foundational. We therefore perceive mathematics as an inquiry-based intervention (resolve doubts by proof) employing learning prompts (mathematical problems) to learn universal truths (augment knowledge) in topics such as quantity (numbers), structure, space and change.

As Alan Kay [1] put it: *"Mathematics is really 'careful thinking about how representations of ideas could imply other representations of ideas', and the most important process in helping anyone learn how to do mathematical thinking is to put them in many situations in which they can use how they think right now in a more careful way"*.

To make mathematics accessible to a learner, one has to provide carefully designed/selected situations that afford personalized explorations during which the learner can develop mathematical thinking given his/her current knowledge and skills. This development takes place through thoughtful exploration of representations or prompts and their logical relationships. It is important for this process to take place in a learning environment that reverses the typical lecture (information provision) and homework (learner activities) elements following a flipped classroom model [2]. In the flipped classroom model, the teacher provides digital learning content (e.g. games, infographics, virtual experiments, gamified exercises, quizzes, videos) explaining mathematical prompts and visualising mathematical complexity for students to explore before entering the classroom. In-class time is devoted to collaborative investigations, projects and discussions.

The rest of this paper is organized as follows: Section 1 sets the paper into context by discussing the transformational impact of ICT in mathematical teaching and the core concepts related to inquiry-based learning in mathematics. Section 2 presents the elements that enable the transformation from current teaching practices in Mathematics to the framework set out by GamifyMaths. Section 3 provides indicative scenarios of use, the details of the framework and its implications. Section 4 summarizes and discusses future research.

II. Pedagogical Backround

GamifyMaths addresses ICT as a transformational force when appropriate pedagogies are employed. To see this potential of ICT in mathematics education, one could consider Lagrange [3] and its five dimensions for ICT in mathematical teaching:

1. **Epistemological and semiotic dimension:** Influence of ICT on (a) the mathematical knowledge and practices; (b) the representatives used in this activity.
2. **Cognitive dimension:** This includes (a) the cognitive frame (constructivist, socio-cultural, etc.); (b) the concepts used (schemes, webbing, etc.); and (d) the

cognitive role of ICT (visualisation, expression, connection, etc.).

3. **Instrumental dimension:** The tools' possibilities / constraints and instrumentation processes.
4. **Teacher dimension:** This refers to (a) the teacher's beliefs and representations of mathematics and of ICT; (b) new teaching situations; and (c) the influence of research and pre- and in-service programs.
5. **Institutional dimension:** Lagrange further points out that the integration of ICT into the classrooms seems to be more difficult than expected. The relationship between ICT and mathematical knowledge (epistemological and semiotic dimension) and the influence of ICT in conceptualisation (cognitive dimension) is underlined and the focus is given on the teaching dimension concluding that "the integration of technology [in the mathematics classroom] requires a deep transformation of the "habitus", of the classroom management and of the representations of mathematics that cannot be done in a single year."

Considering the above arguments, one could actually observe that a more holistic approach is actively investigated, though only marginally used in actual classrooms. This holistic approach is related to conceptualising students as designers and creators of their own digital artefacts (such as games, narratives, simulations, rich media presentations etc.). As a special case of this holistic approach a student, including those in tertiary education, could design their own software (presentations, quizzes, simulations, games, etc.) and engage in making decisions about their design and mechanics addressing also issues related to learning outcomes, activities, assessment. This is the foundational ground of GamifyMaths.

To make this holistic approach more concrete and grounded to research on mathematics education, GamifyMaths adopts and adapts the principle of Inquiry-based Mathematics Education (IBME). IBME is a model of learning and teaching that takes to the foreground the development of questions for driving mathematical statements instantiated through prompts [4]. In inquiry for mathematics, students take responsibility for directing the lesson: They are co-designers of content, processes and assessment as means of creating personalised and memorable mathematics experiences.

IBME has common features with other inquiry-based approaches to learning as it encourages students to pose questions, create statements and test, connecting prior knowledge to new understandings, collecting and analysing data and solve problems. Mathematical quests may involve certain steps or phases that serve as a scaffold for designing the inquiry activities as well as for delivering the inquiry-based mathematical statements to students. For example, inquiry mathematics activities might start through posing an authentic question about a mathematical statement or problem that may be formulated by the teacher or by students themselves or their teachers. Tasks designed to provide a framework for IBME in terms of allowing students to access rigorous algebraic reasoning include problem or case scenarios, experiential learning and mathematical observations as well as mathematical research projects of various kinds. Students' mathematics inquiries may be small or large in scale, involving whole-cycle mathematical problems or only specific elements of a mathematical theorem (e.g. Pythagorean Theorem, algebraic structures, linear algebra etc.). Often working collaboratively, sometimes in partnership with teachers, students apply mathematical techniques and processes and they are encouraged to share the results of their investigations.

III. ENABLING ELEMENTS

GamifyMaths is addressing the twinning of gamified interventions with inquiry-based learning in mathematics. It represents the initiation of an applied movement or point of departure addressing the limited effectiveness of current teaching and learning processes in order to incentivise students to acquire fluency in mathematics and enhance their skills to effectively apply mathematical knowledge in real-life situations. To give a more vivid image of this movement from current teaching practices in mathematics education to the vision set out by GamifyMaths, we adopt the Aristotelian schema of the "four causes" [5] . The four Aristotelian causes that enable the vision of GamifyMaths are:

1. **Material cause - enabling technologies:** Adopting Personal Learning Environments (PLEs) as infrastructure to enable the set-up of flexible personalized learning spaces. PLEs range from modern web-based collaborative learning spaces to mobile context-aware application mixes. GamifyMaths exploits current devices and leverage the ICT skills of digital natives to further deepen their digital skills in a creative way and see the deep connections of mathematics to creative work and technology.
2. **Formal cause - subject domain:** Making mathematics concrete and grounded on learner's psyche and expectations, thus preparing the future citizens and knowledge workers.
3. **Moving cause - change agents:** Adoption of inquiry-based pedagogies and set-up of a community of practice with educators and future teachers able, after appropriate training and through continuous support, to bring inquiry-based practices in schools and universities.
4. **Final cause - gamified learning:** Injection of game mechanics in inquiry-based practices to provide motivation, immersion, and deep understanding as means of improving mathematical learning design and delivery across disciplines and in any scale (topics, lessons, or entire courses).

The rest of this section presents in more detail these four enabling elements.

A. *Enabling Technologies: Personal Learning Environments*

Personal Learning Environments (PLEs) offer an effective approach in assembling interoperating applications (including simulations, games, communication tools such as wikis and forums, reflection tools such as blogs etc.) in contextualized spaces that could be shared with other participants. These spaces are flexible and could be continuously adapted to reflect the gamified inquiry-based mathematical pathways designed by participants. Furthermore, these technologies provide an open infrastructure that could be continuously enriched with new functionalities/applications all conforming to certain standards that ensure their interoperation. The ultimate challenge is to set up a Mathland. As Seymour Papert [6] argues: *"Mathland is to Mathematics what France is to French. By using the computer as a mathematics speaking entity we can in fact make a Mathland, which is a place where mathematics can me learned not only effectively but honestly and respectfully."* (p. 71)

B. Enabling Domain: Mathematics

Alan Kay [1] argues that mathematics is careful thinking about how representations of ideas could imply other representations of ideas; and the most important process in helping anyone learn how to do mathematical thinking is to put them in many situations in which they can use how they think right now in a more careful way. Following this critical observation an insightful example is offered on how a mathematical concept should be presented to young students: *"The rule of thumb here is to find ideas and representations that allow beginners to act as intermediates, that is, for learners to immediately start doing the actual activity in some real form."*

Important conclusions could be drawn from this: (1) To make mathematics accessible to the learner, one has to provide carefully designed/selected situations that afford personalized explorations during which the learner can develop mathematical thinking given his/her current knowledge and skills. (2) This development takes place throughout exploration of representations and their logical relationships. (3) To function as facilitators in this process, teachers need the necessary training and support through appropriate tools and pedagogies. (4) To reverse the typical lecture and homework elements in order to reflect the flipped classroom model where the teacher provides recorded online lectures explaining mathematical prompts for students to explore before entering the classroom (at home, in the library) while in-class time is devoted to exercises, projects and discussions. More insights and deeper discussion on the flipped classroom approach is given in the following subsection.

C. Enabling Pedagogy: Inquiry-based Learning and the Flipped Classroom model

Inquiry-Based Learning (IBL) describes a cluster of learning and teaching approaches in which students' inquiry or research activities drives the learning experience. Students conduct inquiries that enable them to engage actively with questions and problems with their subject and discipline. Coupling of inquiry-based learning and mathematics, is an empowering approach with benefits for the acquisition of problem solving skills as well as a wide range of high-order intellectual attributes.

The flipped classroom model is the ideal approach for bridging the online gamified content with the classroom environment; and thereby achieving a seamless transition from the PLE to the class where inquiry-based learning is enacted for creating mathematical quests. Flipped classroom is widely used to describe a class structure that provides pre-recorded online content followed by in-class exercises. The notion of a flipped classroom, introduced by Bergmann and Sams [2], draws on such concepts such as inquiry-based learning, student engagement and blended learning.

The value of the flipped classroom is in the repurposing of class time into an active learning activity where students can inquire about mathematical concepts, evaluate their skills in applying knowledge and interact with one another in hands-on activities. Therefore instead of students listening to a lecture on mathematics in class and then work on a set of assigned problems out of class (e.g. home), they access gamified mathematics videos and/or games before coming to class and then engage in class in active learning using inquiry-based learning activities integrated into mathematical statements exercises and mathematical prompts. A guiding principle of the flipped classroom is the work typically done in informal learning environments and how this type of learning can be empowered during in-class teaching through involving students in syntonic mathematics rather than just dystonic.

Freeman and Schiller [7] conducted a large scale study of 200 STEM university teachers who reported that they teach in a flipped classroom model and argued that there is more time to spend with students on authentic research. Students who miss class, may watch pre-recorded lectures while reflecting outside of the classroom. Challenges of the flipped approach might be that students new to the method may be initially resistant as they may need to do work out of class rather than be first exposed to the subject matter at the University. Another challenge is that the content for students to access before entering the classroom needs to be designed in a way that will motivate students and encourage their participation in the learning activities that will follow into the classroom. However, Freeman and Schiller argued, based on their survey that teachers perceived that the preparation of online content is a complex process which requires pedagogical and technical skills and hence, requires a significant amount of time for permeating the quality necessary to stimulate students' interest to further participate in the learning activities to be held in-class. Consequently there is a need to support teachers in their new role.

D. Learning and Teaching Mathematics through Games

Gamification is the use of game design elements and game mechanics in non-game contexts. Gamification has been incorporated with commercial success into platforms, especially social ones, as a way to create narrow relationships between the platform and the users, and to drive viral behaviors on them to increase platform popularity. This success has made some researchers theorize that it could also be used in education as a tool to increase student engagement and to drive desirable learning behaviors on them [8]. To create a **gamification system** that increases student motivation it is necessary to focus o**n the fundamental elements that make videogames appealing to their players.** We define a Gamified Learning Activity as a voluntary, intrinsically motivated structured activity that offers pleasure, enjoyment and new learning opportunities to the participants.

We perceive a Gamified Learning Activity in Mathematics as a combination of learning with play to learn universal truths regarding quantity, structure, space and change. The challenge is to infuse gamified elements to offer enjoyment and new learning opportunities. The foundational elements to enable such environments are gamified learning activities which we define as: "*inquiry-based voluntary, intrinsically motivated structured learning interventions that offer pleasure, enjoyment and new prompts for building sound knowledge in topics such as quantity (numbers), structure, space and change.*"

IV. GAMIFYMATHS SCENARIOS OF USE AND FRAMEWORK

To situate the GamifyMaths framework into a teaching and learning, we present two scenarios – the first refers to a school setting and the second to a higher education context.

Scenario 1 - Dimitris is a computer science teacher in a Lyceum in Greece. He is interested in providing opportunities for his students for deep learning in mathematics while, at the same time, work out a practical programming project that will allow them develop their programming and multimedia authoring skills. He sets up a space in the GamifyMaths PLE for his students with the aim to enable them develop educational games in mathematics in topics related to their mathematics class. The learning space integrates tools for the design of the software project and a software development tool for developing their games in tablets. The space accommodates a forum where student teams are expected to report on their weekly developments and upload presentations for the weekly meetings in classroom where their progress is presented; they exchange ideas with their peers and engage in creative discussions. A micro-blog is used by each team to report problems, solutions found and interesting links found in the web. At the end of their project, each team has to prepare a short video presenting the project. Videos are shared with the PLE and further reviewed via discussions in the classroom.

Scenario 2 - Sara is a mathematics university teacher. She is interested in adopting the flipped classroom model for engaging students in activities before and during lectures. She aims to flip her teaching for introducing short online learning activities such as gamified digital media (e.g. video presentation, virtual experiment, etc.), mathematical games and a game-based classroom response systems accessed from the GamifyMaths PLE, in order to repurpose classroom time into a workshop where students can inquire about different mathematical prompts. Sara selects, from the GamifyMaths PLE to reuse a particular gamified mathematics scenario provided from the GamifyMaths community or she designs her own gamified inquiry mathematics scenario/content using the authoring tools and game mechanics features provided via the PLE. Sara then uploads the gamified resources on the PLE for her students to watch before entering the classroom. Students have also the opportunity to respond to the questions via the game-based learning response system for evaluating their understanding of the mathematics topic to be discussed later with Sara.

It is evident from the above scenarios that the key elements of the GamifyMaths framework are the PLE which includes the learning content to be created and consumed by the students. Content such as mathematical notes, virtual mathematics experiments, videos, questions and answers and games may be created either by the student, by the teacher or the real-world scientist who scaffolds student's effort to emulate what a real scientist is doing [9] when mathematics are applied in a multidisciplinary context for creating a product or solving a real-world problem.

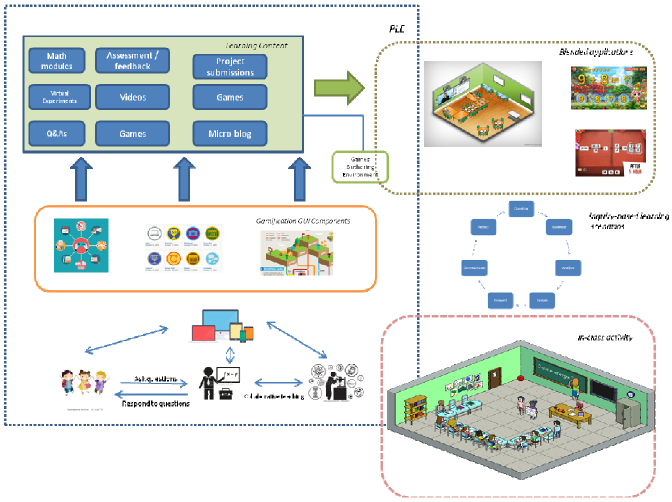

Fig. 1. The GamifyMaths Framework

Creative processes is key for learning how to solve problems [10] and therefore authoring environments for non-experts users are essential as means to help students to design their own content. As Young [11] argued, students tend to learn better, when they experience the process of creation. Creating therefore a mathematics games creates a process of programming the game to calculate mathematical problems whilst allowing the student to learn the inner-logic of mathematical thinking [3]. The user experience is complemented with a gamified Graphical User Interface (GUI) for adding motivational and engagement elements with the purpose of transforming the rather static content to stimulate empowerment, creativity, ownership, unpredictability and meaning. Game mechanics such as points, badges, leader-boards (PBL) progress bars, goals, achievements, digital storytelling [12] and levels are deployed to create a feeling of accomplishment and ownership of the learning that it is created from the student and negotiated with the teacher. Driver for the PLE activity are the in-class learning activities, where students are forming a large group for participatory learning, guiding their peers, giving feedback, articulating teacher's queries and finding solution to problems set by the teacher.

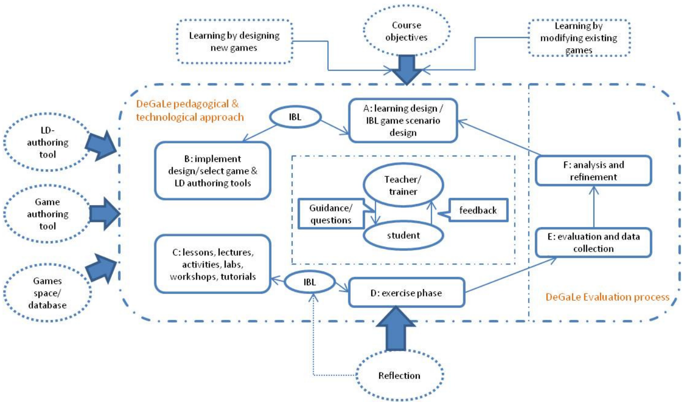

Fig. 2. Illustrating connections between IBL, content, games, course objectives, teacher and student interaction

Figure 2 is perpetuated as a 2nd layer of the GamifyMaths framework exemplifying the connections with the learning content, the gamified interventions, the roles of the teacher and the student and the importance of inquiry-based learning in designing new or existing content. It is evident that certain inquiry processes, such as data collection, refinement, analysis and reflection are central for applying inquiry-based learning in mathematics education.

V. Discussion and Concluding Remarks

GamifyMaths offers a conceptual framework for amplifying and enhancing mathematics learning that is taking place in the classroom by adopting inquiry-based learning, gamification strategies and the flipped classroom model for engaging students in immersive and interactive mathematics resources and processes realised both out-of-the classroom (designing and delivering a series of rich-mediated and gamified online videos with audio-visual explanations of mathematical topics and mathematical mini-inquiry-based games integrated in the PLE) and in-class for involving students into higher-order learning mathematics skills using inquiry-based learning activities. The vision of such trajectory is to help the educational community and the society in general to further pursue mathematics and mathematics education as a key skill which may be used in interdisciplinary contexts (e.g. science, computer science, engineering, humanities and art) as well as for purely mathematical settings.

We understand that solving mathematical problems necessitates knowledge, skills, creativity and resilience. To simplify the process of acquiring such skills, we propose an architecture that combines play, learning, motivation, participation and engagement. We aim to extend current work by empirically investigating the effects of such theoretical abstractions to actual teaching and learning manifestations. As long as the achievements of teachers are currently evaluated mainly by the scores of their students at traditional tests, and the regional experts check if they are on a specific topic at a specific time, the innovative strategies are hard to be revealed. To bypass this, GamifyMaths adopts a "competencies gained" measuring approach rather than "the scores in the traditional testing". To facilitate this, we are adopting competency maps (addressing both teachers and learners) integrating concepts from the underlying techno-pedagogical framework of the project (flipped classroom enriched with gamification elements to offer personalized learning spaces for mathematical inquiries). Impact measures are related to a reflective approach promotes teachers' and learners' self-reflection through exploiting learning scenarios as mediating artefacts to enable the expression of teachers' intentions (during design) and achievements (after the implementation of their learning interventions). This approach is facilitated by the use of the PLE approach that enables the seamless integration of tools (such microblogs, and forums) to enable structured discussions following a reflective approach to impact evaluation.

In terms of further future research, we are investigating the option of conducting empirical research for understanding how mathematics teachers and students may use the framework in practice and extract some substantial and significant findings on experiences and approaches to using inquiry-based learning, games and flipped teaching across the teaching and learning spectrum. In conjunction it would be essential to derive some quantifiable metrics as to conduct randomized trial experimentations.

However, experimentation cannot guarantee the uptake of the framework. Technologies and innovative pedagogical approaches in many cases fail as a consequence of the fact that their virtues remain unclear to the teacher, who predominantly plays the role of a gatekeeper and moderator of technology use. Consequently, there is a need to address the role of the teachers and the set of skills that they need to develop in order to have the full control of the pedagogical process and be able to support their students appropriately. This need calls for the establishment of a community of practice to support the teachers through learning materials and scenarios, a social network to promote their close collaboration and exchange of experiences and ideas. Several existing platforms and initiatives can serve as hosting infrastructures for this community building process such as the Open Discovery Space (http://www.opendiscoveryspace.eu/) and the Inspiring Science Education (http://www.inspiringscience.eu/) portals. Future work will focus on the selection of the most appropriate ones.

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