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A Minimum Area Discrepancy Method (MADM)

for Force Displacement Response Correlation

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Abstract: With the increasing use of Computer Aided Engineering, it has become vital to be able to evaluate the accuracy of numerical models. Specific methods such as CORA were developed to objectively evaluate the correlation between a physical test and a numerical simulation results in terms of parameter vs time. However, no metric has so far been developed for Force Vs Deflection (FvD) signals often used in crashworthiness and biomechanics. A unique method called the Minimum Area Discrepancy Method, or MADM, is proposed to address this deficiency. This new method initially calculates a parameter 'R' which represents the area between numerical model and the average physical test response and then divides it by the average area generated by the upper and lower test corridors, based on the same standard deviation. The parameter 'R' is then normalized between 0 (no correlation) and 1 (perfect correlation) to become the MADM correlation rating. The MADM method was then validated by comparing a one dimensional Finite Element (FE) model of a chest model, under 2 impact velocities, against reference Post Mortem Human Subject (PMHS) data. The MADM method was further used to improve the correlation of this thorax model, by varying model parameters and generating 81 model variations. Based on the MADM ratings, a set parameter values leading to the best fit was identified. The best fit exhibits a response significantly better than the original chest model. MADM is novel, unique, easy to use and fulfills an important gap in objectively evaluating FVD correlation responses.

Keywords: MADM, Correlation, FvD, Minimum Area Discrepancy Method

Nomenclature:

MADM	Correlation rating value (Minimum Area Discrepancy Method)
$MADM_{n,m}$	MADM correlation rating using a specific scaling value of 'n' and power rating 'm'
FvD	Force versus Displacement
FvT	Force versus Time
DvT	Displacement versus Time
NM	Numerical model
PE	Physical Experiment
A_{model}	Area under the average signal and the Numerical Model
A_{upper}	Area under the average signal +1 standard deviation
A_{lower}	Area under the average signal -1 standard deviation
R	Ratio between A_{model} and the average of A_{upper} and A_{lower}

1. Introduction

The development of computing capability has led to an ever increasing use of numerical modelling in science and engineering. It is essential to validate any numerical model in order to ensure credibility of the results. Usually, the response of a model is compared to that of the represented system for a set of (physical) experimental test configurations. Often the responses from a physical test are a series of curves and the validity of the numerical model can be evaluated by comparing its response curves to those obtained via physical experimentation. Experimentation can take place in large scale like in the case of UNECE R94 [1], UNECE R95 [2], where the full vehicle is impacting or is being impacted by a deformable barrier or for a full assembly assessment, like protocol EN1317 [3] where a highway safety furniture fence is being impacted by vehicles. Such full-scale tests are complex, and most instrumentations are accelerometers and forces transducers, which respectively output accelerations, and forces, which can be post-processed and compared against a set of engineering criteria. These signals, readings from transducers as a function of time, are provided to the CAE analyst in order to perform a calibration of their computer models.

Before performing a full crash test, sub-component tests are usually conducted in the first place to provide the analysis team with the necessary structural local responses to design the structure. As an example, it is important to capture early the force-deflection characteristics of a steering column, an airbag or a crush can front crash structure. Calibrating a computer model on the acceleration on its own is not sufficient. Performing a correlation of a function as a function of time by itself is useful, but does not guarantee that the full event is correlated as deformation of the structure, a function of time, is also very important. As the displacement of the structure is linked with the second derivative of the acceleration, any small variation of this acceleration will have a major influence on the displacement. Consequently, correlating against a force-displacement response is a more stringent, physically representative and a more rigorous scientific approach. Acceleration versus time and displacement versus time both have to be correlated at the same time to perform a realistic and meaningful correlation; in this paper, it is proposed to perform a correlation when both output are combined as a force-displacement signal (FvD).

Several methods have been developed in order to achieve a quantitative assessment of the discrepancy between the physical test results and numeric simulations. The CORA [4] and EEARTH [5] methods were developed to evaluate the correlation of a Computer Aided Engineering (CAE) time history signal to a reference experimental signal. These methods are extensively used in industry; however, they are designed to evaluate time history signals and are not applicable to FvD signals, which are frequently used in experimental validation datasets within the field of crash safety and biomechanics. CORA only works for monotonic functions, and as such fails to be precise on the rebound phase of an impact, as the ordinate of the FvD function then has more than one value.

This paper will focus in validating a new method, which will correlate the response of a human thorax computer model against force-displacement signals obtained from literature test data.

This choice of model is based on the fact that human biomechanics is of particular importance to the improvement of automotive safety. The introduction of Anthropometric Test devices (ATDs) has greatly contributed to reducing casualties resulting from traffic collisions [4][7][8][9][10]. ATDs are biomechanical models intended to represent the mechanical behaviour of the human body in loading configurations representative of a car crash. More recently detailed numerical models of the human body (HBMs) have been developed in order to allow for a more accurate and detailed understanding of the mechanisms leading to injuries during car crashes [8][11]. Validation of ATDs and human models requires biomechanical data resulting from cadaver (PMHS) testing with impactor, drop or sled test configurations ([12], [13], [14]). The PMHS response is typically represented by a set of curves, which are either parameter vs time or Force vs. Deflection (FvD) signals. Often the variability in the response of the tested cadavers is represented using the average response curve with appropriate corridors [15] [16]. Recently, a proposal to define new biofidelity requirements for frontal crash dummies based on reference cadaver data was published [13] [15]. Out of the 21 proposed validation cases proposed, 15 consider the Force vs Deflection or Moment vs Rotation response of the tested body region.

In this study, a metric aimed at objectively and quantitatively evaluating the correlation between the FvD response from a numerical model and a reference physical test data curve is proposed. The study will develop a new correlation technique and test it against a numerical one dimensional model of a human thorax, frontally impacted by a pendulum at two different energy levels mimicking experimental tests performed on PMHS [5].

2. Method

2.1. MADM Rating

The proposed methodology is aimed at evaluating the level of correlation between a numerical FvD curve and a reference FvD curve. The methodology is particularly aimed at rating sub-component assemblies used in impact dynamics scenarios as well as biomechanical models by comparing their response to reference experimental data. The approach is called the “Minimum Area Discrepancy Method” or MADM. MADM will minimise the discrepancy between different sets of information, by matching the shape of the average test curve, whilst considering the testing dispersion about the standard deviations.

The first step of the correlation process is to process the Physical Experiment (PE) biomechanical data so that a FvD response and an upper and lower corridor can be defined. Several methods can be used [15][16] to generate the average curve and the corridors from the PE test data. The corridors are introduced to cater for variability in the PE data; typical corridors represent usually a spread of one standard deviations about the mean. Note that the MADM correlation process is valid for any standard deviation spread. In Figure 1 (left), a possible NM model response is plotted and overlaid so divergences and convergences can be observed.

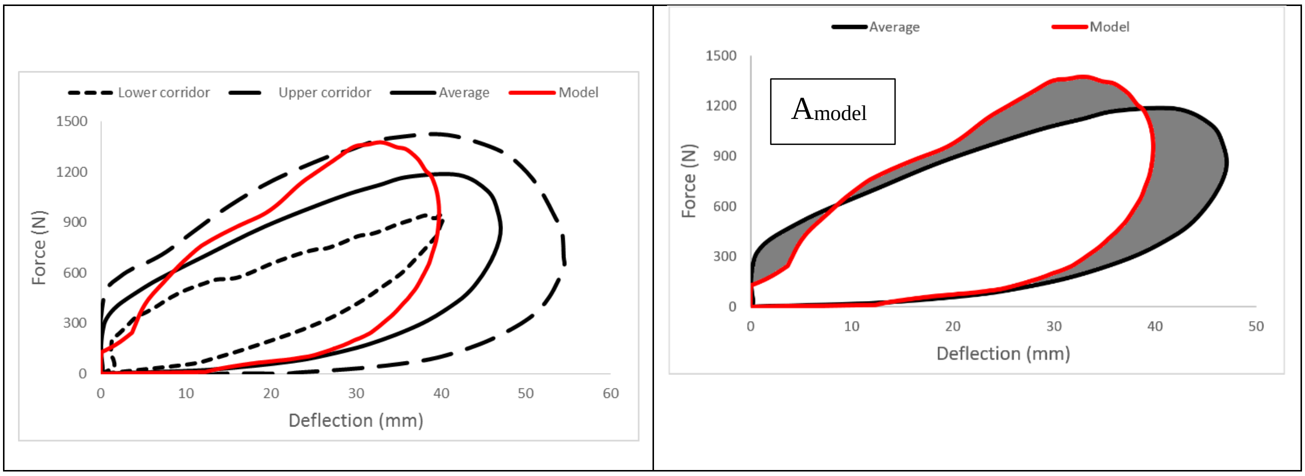


Figure 1 Left: Average PMHS tests curve and associated +/-1SD corridors compared to a model response. Right: Illustration of the difference between PMHS average and model (A_{model}).

In step 2, the difference (A_{model}) between the Numerical Model (NM) response and the average PE reference curve is evaluated by calculating the difference in graphical area between the reference PE FvD curve and the NM FvD curve (Figure 1). The experimental variability is evaluated by calculating the difference in graph area between the lower and the upper corridor to the average experimental curve, as per Figure 2, and subsequently dividing it by 2.

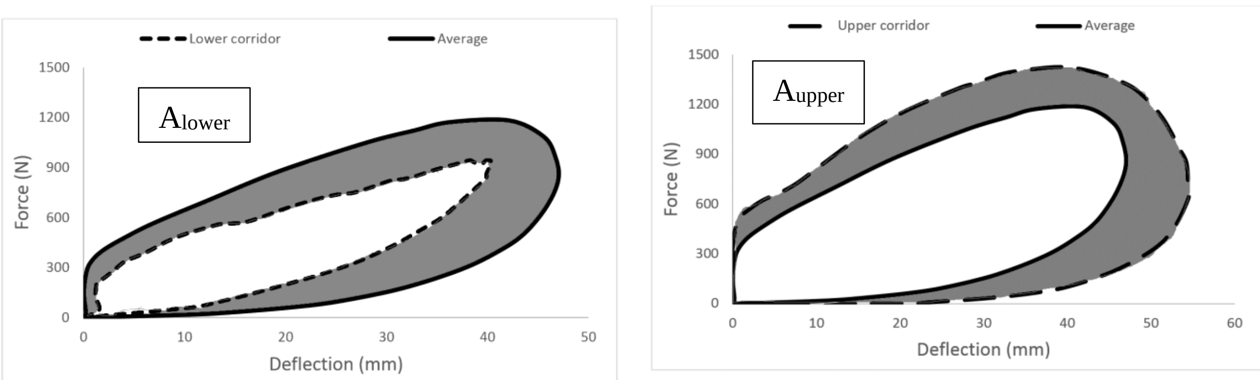


Figure 2: Calculating upper (A_{upper}) and lower (A_{lower}) areas from the standard deviation upper and lower corridors to the mean value.

The ratio R between A_{model} and the experimental variability is then calculated as per Equation 1.

Equation 1: Calculation of ratio between model area and corridors

$$R = \frac{A_{model}}{\frac{(A_{upper} + A_{lower})}{2}}$$

An R value of 0 simply means that there is perfect correlation between NM and FE; while a value of R greater than 0 relates to a weaker correlation. Finally, to ease understanding, the ratio R is translated to a value between 0 and 1 using Equation 2, which represents the general formulation of the MADM correlation methodology.

Equation 2: MADM general equation

$$MADM_{n,m} = \frac{1}{1 + nR^m}$$

The MADM equation is a function of two variables; hence, the necessity to cite the indices n and m as these values will affect the MADM rating outcome, as displayed in Figure 3.

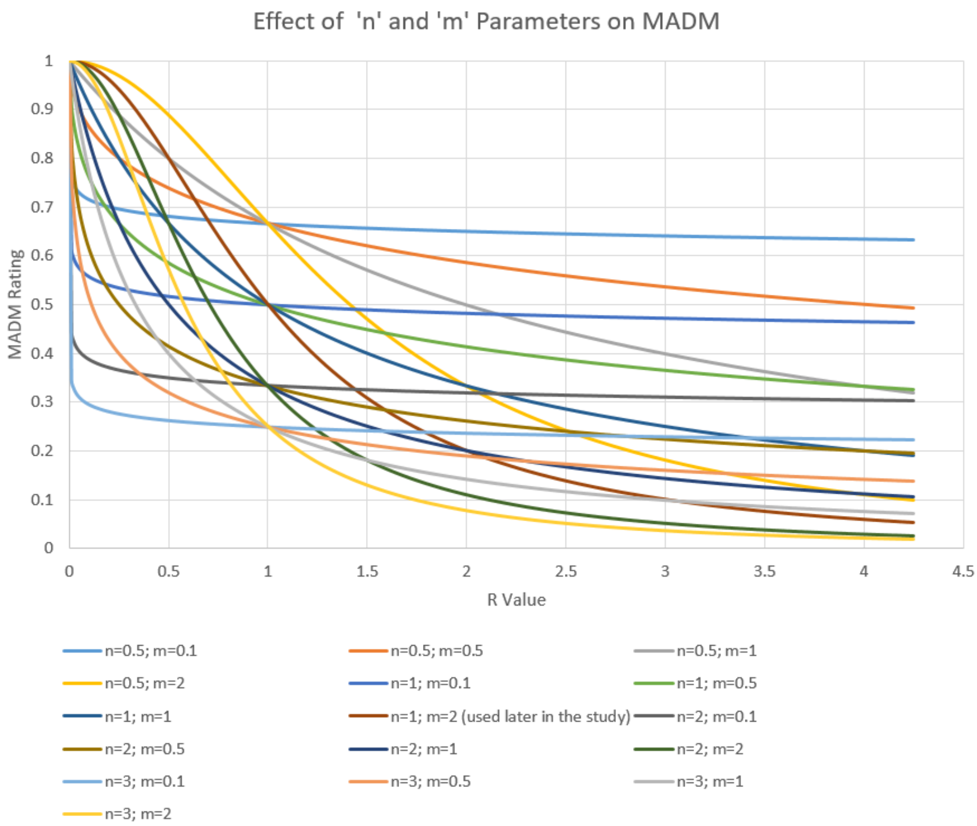


Figure 3: MADM correlation ratio parameters

The parameter values of ‘n’ and ‘m’ are user-defined parameters, which can be used to tune the correlation process. Depending on their values, the MADM ratio changes and can be more or less segregating. For the values n=1 and m=2, numerical models with an R value under 0.2 are not penalised significantly, as the slope of the MADM ratio is almost horizontal, whilst an R value of 0.2, models with a poorer correlation are more discretised because the slope of the MADM ratio response is more pronounced. For ‘m’ values lower than 0.5, the response suggested by MADM is not practical, as it penalises ‘good’ models more than less accurate ones. Excessive ‘n’ values also have this tendency, as illustrated in Figure 3 .

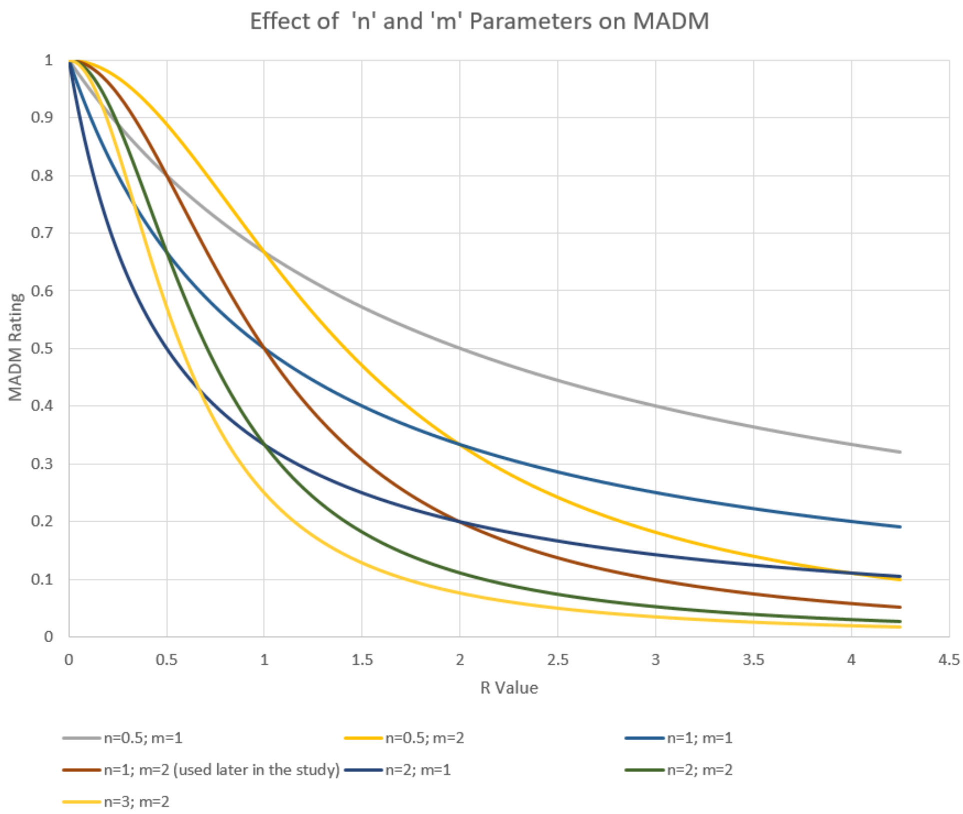


Figure 4: Recommended 'n' and 'm' values for MADM model differentiation

In order to allow a better differentiation between MADM results, Figure 4 was created by removing permutations from Figure 3, leaving the ranges of ‘n’ and ‘m’ allowing a clear differentiation between MADM ratings for a given value of R. By considering Figure 4, it can be proposed that the first ‘n’ and ‘m’ parameters to consider in a MADM correlation process would have the following range:

$$m \in \mathbb{R}^{+} \wedge 1 \leq m \leq 2$$

$$n \in \mathbb{R}^{+} \wedge 0.5 \leq n \leq 3$$

Obviously, the choice of ‘n’ and ‘m’ may be problem dependant and should be adapted accordingly. As the MADM function is decreasing monotonically, the rating order will not change, but its selectiveness will depend on the parameter chosen. Considering the pre-study performed in Figure 3 on the MADM ratio parameters, it was decided to use arbitrary $MADM_{1,2}$ as the most suited correlation method.

Equation 3: Selection of MADM parameter for the following study (n=1 and m=2)

$$MADM_{1,2} = \frac{1}{1 + R^2}$$

The following sections will demonstrate an example application of MADM in the context of impact biomechanics. The NM presented relates to the modelling of a simplified thorax model [17], which has the capabilities to evaluate the chest force/deflection response upon pendulum impact. In automotive safety, chest deflections are critical in frontal and lateral accidents [18][19][20] [21], hence validating the accuracy of such model is of importance, should it be used to assess the occupants’ chest deflections under seatbelt or airbag loads.

2.2. Application of the method to the optimisation of a simplified thorax model

Lobdell [17] created a mathematical model representing the mechanical behaviour of the chest in frontal blunt impact. This model consists of an assembly of springs, dashpots and masses (Figure 5), and was optimised to correlate with cadaver responses obtained by Kroell [18], which were fitted with PMHS corridor responses from Lebarbé [21].

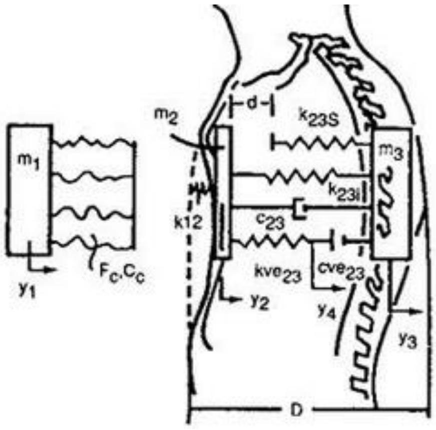


Figure 5: Original Simplified Thorax model [17]

The parameters values proposed by Lobdell are summarized in Table 1. In order to illustrate an application of MADM, it is proposed to initially rebuilt this NM and vary some of its parameters in order to generate an array of responses. The MADM algorithm will compute the R value for each permutation and the curve ratings obtained will be used to select an optimal model exhibiting the best correlation with the cadaver PE data.

m_2 (kg)	m_3 (kg)	k_{12} (kN/m)	k_{23i} (kN/m)	k_{23s} (kN/m)	k_{ve23} (kN/m)	c_{23} (kN.s/m)		c_{ve23} (kN.s/m)	d (mm)	D (mm)
						Compression	Tension			
0.45	27.2	281	26.3	52.6	13.2	0.525	1.23	0.18	38.1	222

Table 1: Table of values for Original Simplified Thorax model [17]

The NM was constructed using an explicit finite element model, following the exact build from Lobdell's mathematical model, and is illustrated in Figure 6. This NM was created, as per Figure 6, and included the stiffness and damping values listed in Table 1. The aim of the numerical model was to replicate the dynamic tests from Lebarbé [21], by extracting the chest compression and the impact force in two loadcases:

1. A Low Energy impact with a 23.4kg impactor travelling at 4.3m/s
2. A High Energy impact with a 23.4kg impactor travelling at 6.7m/s.

The chest compression was measured using the horizontal relative displacement between the solar plexus and the rear spine (between nodes A and B in Figure 6). In Figure 6, the front of the thorax is represented by the mass m_1, and the back represented by m_3. The chest impact force is simply extracted from spring K_{12}, which is the spring connecting the impactor mass to the thorax, i.e. at position m_1.

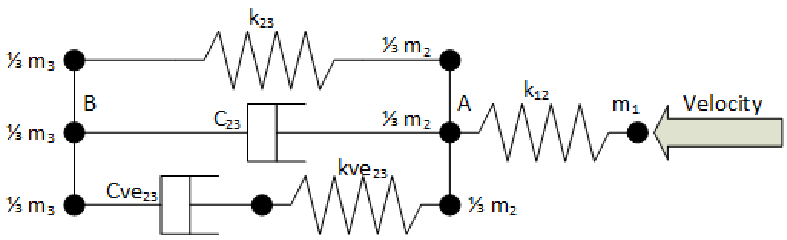


Figure 6: LS-Dyna model of original simplified thorax model

The NM thorax impact model is activated by subjecting m_1 to both low speed and high-speed impacts. Lebarbé [18] extracted thorax energy FvD corridors for comparable frontal pendulum impacts using available PMHS datasets and state of the art methods to develop +/- 1 standard deviation corridors. In order to capture a more biofidelic numerical model for both impact speeds, the NM is parametrised so that spring and damper stiffness as well as viscosity values are modified to capture a wider range of MADM response values which can be used to find a compromise fit.

The spring stiffness K_{12}, which represents the stiffness of the skin and flesh, and the distances d and D were kept constant to limit the number of simulations. As such, a Design of Experiment (DOE) is setup and arbitrary scale all springs and damper functions (Table 2), creating 81 permutations for which 162 force-displacement responses were extracted.

Table 2: DOE scaling values for each component from the Finite Element Model

Parameter	Lower value scaling factor	Middle value	Upper value scaling factor
K_{23}	0.5	1.0	1.5
C_{23}	0.5	1.0	1.5
K_{ve23}	0.5	1.0	1.5
C_{ve23}	0.5	1.0	1.5

3. Results

3.1. MADM Ratings for the Lobdel Model

The NM nominal response is illustrated in Figure 7, showing that its responses lay within or close to the PE test corridors. Using the MADM methodology previously described, the computed MADM rating value was lower for the Low Energy model (0.79) than for the High Energy model (0.89), suggesting that the correlation is better for the High Energy impact. This can be confirmed from Figure 7, as the force-displacement signal from the computer model is closer to the mean value of the High Energy impact than for Low Energy.

The higher the rating, the closer are the curves, as the value of R tends to zero, meaning that the area difference between the NM and the mean PE (A_{model}) is small. The higher the rating means that there is less difference in energy dissipation between the NM and the PE data. As the MADM correlation rating values are different, this suggest that the model parameters are not tuned to suit both loadcases. Consequently, it would be ideal to find a comparable level of correlation between the two NMs. As the fit is not perfect for both NM models, it is proposed to use $MADM_{1,2}$ to compute a compromise fit to improve the correlation to PE for both energy levels at the same time.

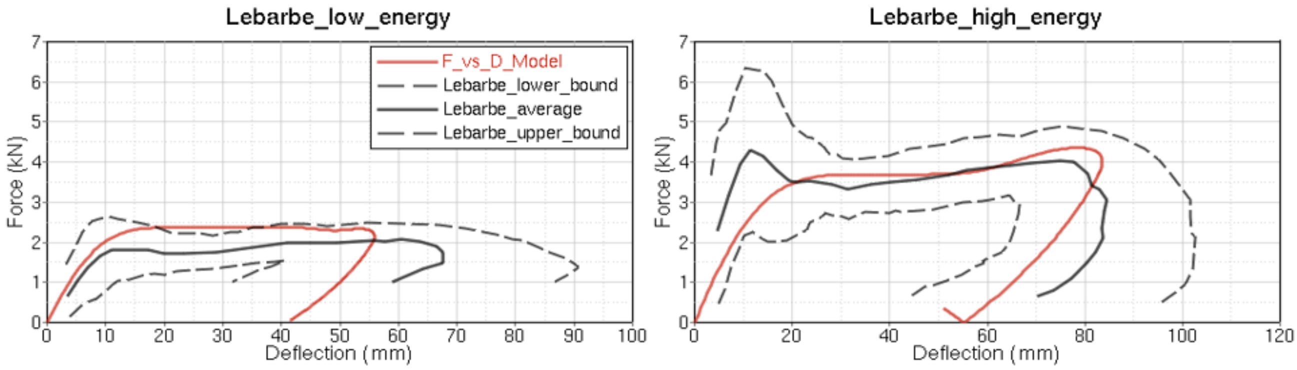


Figure 7: Low and High Energy Original Simplified Thorax model response compared to Lebarbé corridors [18]

3.2. Application of the method to the optimisation of a simplified thorax model

The full range of responses are illustrated in Figure 8 and Figure 9 , representing all the 81 permutations per computer model. All the results are available in Appendix A. From the computations, C_{23} and K_{23} appeared to have significantly more influence than C_{ve23} and K_{ve23}, consequently the results have been grouped based on the value of C_{23} and K_{23} with the following colour code:

-Nominal value, C23=0.5, K23=0.5 C23=0.5, K23=1.0 C23=0.5, K23=1.5 C23=1.0, K23=0.5 C23=1.0, K23=1.0 C23=1.0, K23=1.5- C23=1.5, K23=0.5 C23=1.5, K23=1.0 C23=1.5, K23=1.5

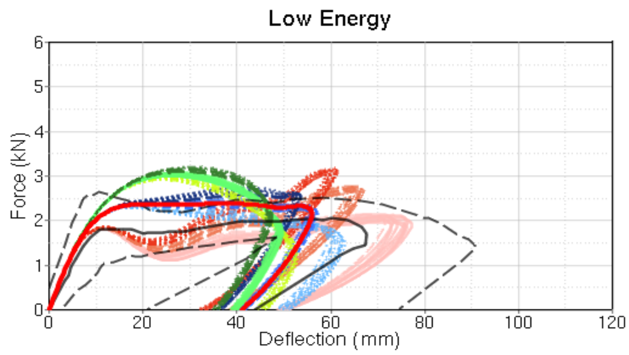


Figure 8: Low Energy force-displacement responses (DOE of 81 runs)

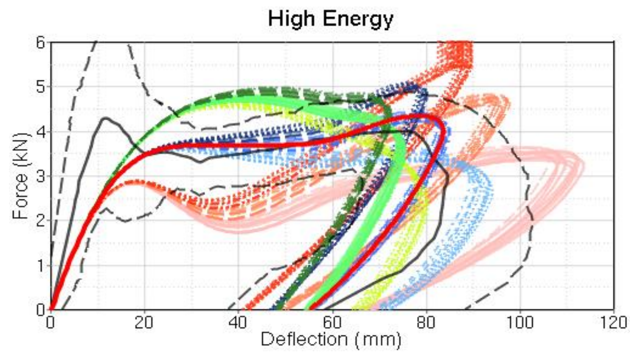


Figure 9: High Energy force-displacement responses (DOE of 81 runs)

From all the runs performed, the most relevant relate to the computer models with C_{23}=1.0 with K_{23}=0.5 (Figure 11).

Looking at all the MADM ratings in Appendix A, it can be observed that NM runs 32, 33, 35 and 36 have the highest MADM rating for Low and High Energy models, as shown in Figure 10 and Figure

11, consequently these permutations are the best candidates to suggest a compromise correlation between Low and High Energy models.

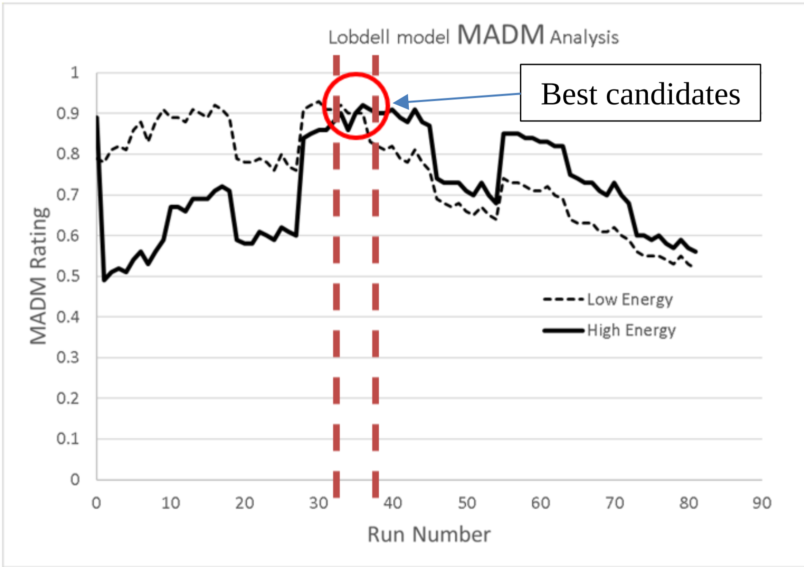
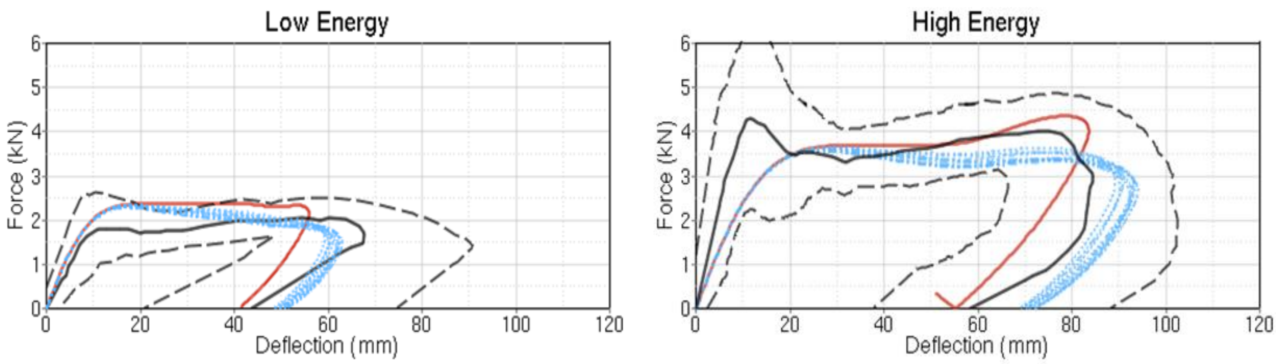


Figure 10: MADM response for Low and High Energy models

The results from Figure 10 suggest that a significant improvement has been made for the Low Energy model, because its rating increased from 0.79 to a rating between 0.90 and 0.92. The improvement can be observed in Figure 11. In the baseline run, the Low Energy response's maximum lies about the upper corridor response and the width usually narrower than the corridor, while in the improved model, the response is more centered on the average corridor whilst capturing a better width of the signal.



Blue: C_{23}=1, K_{23}=0.5. MADM Rating: 0.90-0.93
Red: Baseline model. MADM Rating=0.79

Blue: C_{23}=1, K_{23}=0.5. MADM Rating: 0.84-0.92
Red: Baseline model. MADM Rating=0.89

Figure 11: Improved computer results (with C23=1.0 and K23=0.5) vs Lebarbé corridors [18]

For the High Energy case, the gains are minimum, however the shape of the curve is more in-line with the average value of the corridor. Consequently, responses from Figure 11 are better than the baseline. This section illustrates how MADM can be used as a scientific tool to improve the correlation between a model and experimental reference data.

4. Discussion

4.1. Comparisons with CORA rating

The corresponding time histories for the computed models were extracted and compared using the CORA method. The obtained CORA scores for force vs time (FvT) and deflection vs time (DvT) for one model parameter set were averaged to form a global CORA score. The PE FvT and DvT average responses and corridors for the Low and High Energy impacts were obtained from [18].The global CORA score for a given model was then compared to the corresponding MADM score. CORA uses two independent sub-ratings: a corridor rating and a cross correlation rating to assess the correlation between 2 time-history signals.

- For the corridor rating, an inner and an outer corridor around the reference curve have to be defined. The model curve receives a score of one if the signal is within the inner corridor, the score decreases from 1 to 0 between the inner and the outer corridor.

  In addition, the CORA corridor score depends on the definition of the inner and outer corridor. Traditionally, PMHS results are presented with an average and +-1SD corridors encompassing about 68% of the variability in the data (based on the hypothesis of a normal distribution). No clear requirements exists to select inner and outer corridors, Vavalle [22] used 1SD as inner corridor and 2SD as outer corridor (representing 95% of the variability). Additionally, a more selective case was tested in which the inner corridor was +-0.5SD and the outer corridor +-1SD. Tested scenarios are summarized below:

  - Set 1: Inner corridor = +-0.5 SD, outer corridor = +-1 SD

 - Set 2: Inner corridor = +- 1 SD, outer corridor = +-2SD

- The cross correlation uses the three following sub-ratings: the phase, the size and the shape. The ISO TC22 SC10/12 Working Group 4 [6] proposed an improvement to the CORA cross correlation rating. Both the CORA scores obtained using the original CORA method and with the ISO improved version were calculated and compared to the MADM score.

For all simulated models, CORA values were calculated using the standard CORA cross correlation rating and the ISO improved version. Additionally, two sets of corridors were tested. Correlation plots for the ISO method and for the +-1SD inner +-2SD outer corridors set are presented in Figure 12. All the CORA scores are available in Appendix B and CORA vs MADM responses in Appendix C.

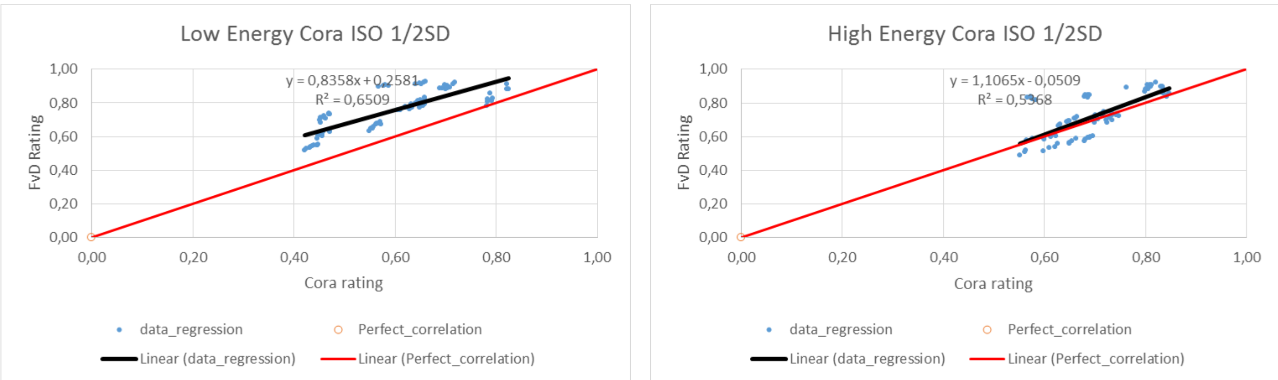


Figure 12: Correlation plots between MADM and CORA metrics. Left: Low Energy impact. Right: High Energy impact.

Generally, MADM and CORA ratings do correlate, however the r^2 coefficients remain fairly low (0.53-0.7). A summary of the three runs exhibiting the best rating values for each tested rating methods is presented in Table 3. As can be observed, the prediction is very consistent between all CORA rating variants, however MADM predicts a different set of best model fits. The MADM ratings for Run 6, 8 and 9 (0.88-0.91) at Low Energy despite not being the highest are among the best responses and close to run 29, 30 and 33 (0.92-0.93) which have the best MADM rating. The same can be observed at High Energy for run 29, 30 and 32 (0.85-0.88) when compared to run 36, 37 and 40 (0.91-0.92).

Table 3: Runs with the best ratings for CORA and MADM

	MADM	CORA 0.5/1SD	CORA 1/2SD	CORA ISO 0.5/1SD	CORA ISO 1/2SD
Low Energy	Run 29 Run 30 Run 33	Run 6 Run 8 Run 9	Run 6 Run 8 Run 9	Run 6 Run 8 Run 9	Run 6 Run 8 Run 9
High Energy	Run 36 Run 37 Run 40	Run 29 Run 30 Run 32	Run 28 Run 29 Run 30	Run 29 Run 30 Run 32	Run 29 Run 30 Run 32

Given that MADM and CORA are not predicting the same best fits, optimizing a model with either of these methods will lead to different results. MADM has the advantage of presenting a direct physical meaning relating to the stiffness of the response. However, the omission of the time dimension relative to the use of FvD signals could lead to undesired effects. In general, when both the FvD and the Force and deflection time histories are available, it is not clear if MADM or CORA should be used, or eventually a combination of both metrics, in order to evaluate a model response.

4.2. Evaluation of the MADM rating method

Barbat [6] listed a set of seven relevant criteria to evaluate the quality of a time history rating method, being **objective, generic, robust, symmetric, simple, provide physical meanings and provide ratings under uncertainty**. The same criteria can be used in order to asses MADM.

- Considering **Objectivity**, or the fact of producing the same results regardless who conducts the assessment, it can be proposed that MADM is an algorithm, which can be applied automatically, as long as the parameters m and n are set for the user.
- Regarding the **Generic** property, which reflects differences in the full distribution of the simulation and experimental outcomes and key features like phase, size and shape, MADM does not explicitly fulfils this criteria. Indeed, for FvD curves, the time dimension is omitted and therefore the phase is not relevant. In terms of the size and the shape, it is not directly evaluated by MADM. In the

case of FvD curves, the area under the curve (AUC) which could be interpreted as the size represents the energy transmitted to the structure during the impact and is not expected to vary as a function of the stiffness of the structure. In Figure 7 and 8, stiffer behaviour leads to more force and less deflection and the opposite is observed for softer structures, but the AUC remains constant. Shape is not directly evaluated by the algorithm even though if we consider the AUC constant, the Model Area used to calculate MADM is actually only related to the shape of the signal.

- Concerning **Robustness**, or the fact that consistent results are produced with different sampling rates, it can be said that this aspect has not been evaluated in the current work as the results produced from the simulations did not exhibit significant levels of noise.
- MADM meets the **Symmetric** requirement, which relates to the production of the same results when the simulation and experiment outcomes switch. Inverting the simulation and the experiment would produce the same A_{model}.
- Looking at **Simplicity**, i.e. comprehension and usage, the MADM method is based on simple calculation of areas between curves, it is easy to understand and apply.
- Considering the **clear physical meaning** and Subject Matters Experts (SMEs) knowledge, it can be suggested that A_{model} represents the difference between experimental average and simulation signals and $\frac{(A_{upper}+A_{lower})}{2}$ represents the variability in the experimental data. The MADM score is between 0 and 1, results close to 0 mean that the experimental and simulations are very different and 1 means that the two signals are identical.
- And finally, the **under-uncertainty** criterion, which accounts for uncertainties in the data both in the experiments and in the simulation. This uncertainty in the experimental data is accounted via the experimental corridors.

Considering these seven criteria, MADM meets six of them, only the Generic property criterion not being fully addressed.

5.0 Conclusions and Further Work

A new mathematical algorithm for multi-objective correlation of computer models to physical tests has been developed and trialled. This unique method entitled MADM addresses correlation deficiencies often found in PMHS biomechanical calibration tests. The method was successfully applied to the optimization of a one dimensional finite element model leading to significantly improved FvD responses.

Unlike any other correlation method commonly used for crashworthiness and human injury prediction MADM utilizes a FvD approach. This is very useful for future research as (publically available) test data is frequently provided in FvD format as opposed to acceleration over time. In this paper it has been demonstrated that MADM is capable of explicitly displaying the force response as a function of crush distance providing the user with a very useful visual check of the level of correlation. Consequently this offers the analyst a powerful tool linking crush distance with available packaging space enabling another avenues of investigation for example linking crash structure design with predicted human injury. These attributes make MADM a scientific, useful and powerful engineering tool, as it refers directly to the stiffness and strength of the system being analysed and correlated.

It was demonstrated that the the MADM method does not predict consistent results with ISO recommended methods for evaluating time history signals. Nevertheless, MADM meets six of seven criteria, which are proposed to assess the quality of a rating method. It is overall a credible correlation assessment mathematical model for FvD applications.

In this paper MADM has been successfully applied to correlate a human computer model to PMHS; but additional studies are required to investigate the robustness of the method and assess its suitability particularly for application in other fields of physics and engineering.

The MADM algorithm is available as a Python executable upon request.

References

[1] UNECE R94, "Frontal collision protection". Technical Information [online] available from <www.unece.org/?id=39145>

[2] UNECE R95, "Lateral collision protection". Technical Information [online] available from <www.unece.org/?id=39145>

[3] EN-1317, "Highways England List of EN 1317 Compliant Road Restraint Systems". Technical Information [online] available from < www.standardsforhighways.co.uk/ha/standards/tech_info/en_1317_compliance.htm>

[4] Gehre, C., Gades, H., Wernicke, P., 2009. Objective Rating of Signals Using Test and Simulation Responses; Paper Number 09-0407; 21st. ESV Conference; Stuttgart; Germany.

[5] Zhan, Z., Fu, Y., Yang, R.-J. 2011. Enhanced Error Assessment of Response Time Histories (EEARTH) Metric and Calibration Process; SAE 2011 World Congress; SAE 2011-01-0245; Detroit; MI; USA.

[6] Barbat, S., Fu, Y., Zhan, Z., Yang, R.J., Gehre, C., 2013. Objective Rating Metric for Dynamic Systems. Paper Number 13-0448, 21st. ESV Conference Seoul, Republic of Korea.

[7] Elvik; R., 2010. The stability of long-term trends in the number of traffic fatalities in a sample of highly motorised countries, Accident analysis and prevention, Volume 42, Issue 1, pp 245-260.

[8] Cuerden, R., Cookson, R., Massie, P., Edwards, M., 2007. A review of the European 40% offset frontal impact test configuration. Paper Number 07-0318, 20th. ESV Conference, June 18-21 Lyon, France.

[9] Viano, D.C., 1978. Evaluation of biomechanical response and potential injury from thoracic impact. Aviat Space Environ Med 49(1):125-135.

[10] Nusholtz G. S., Hsu T. P., Byers L. C., A proposed side impact ATD Bio-fidelity evaluation scheme using cross-correlation approach. Paper presented at 20th ESV Conference; June 18–21, 2007; Lyon, France.

[11] Iwamoto, M., Kisanuki, Y., Watanabe, I., Furusu, K.; Miki, K., Hasegawa, J., 2002. Development of a finite element model of the total human model for safety (THUMS) and application to injury reconstruction. IRCOBI, pp. 31-42.

[12] Prasad P., 1984. An overview of major occupant simulation models. SAE 84085. In: Mathematical simulation of occupant and vehicle kinematics. SAE Publications, Warrendale p146.

[13] Lebarbé, M., Donnelly, B., Petit, P., Moorhouse P., 2015. A Frontal Response Specification for Assessing the Biofidelity of an Anthropometric Test Dummy Part 2 -Lower Body. IRCOBI, Paper # IRC-15-59.

[14] Marcus, J.H., Morgan, R.M., Eppinger, R., 1983. Human response to and injury from lateral impact. Stapp Car Crash Journal, SAE 831634.

[15] Trosseille, X., Yoganandan, N., Pintar, F.A., 2015. Normalization and Scaling for Human Response Corridors and Development of Injury Risk Curves. In: Accidental injury: Biomechanics and prevention, pages 769-792.

[16] Lebarbé, M., Donnelly, B., Petit, P., Moorhouse P., 2015. A Frontal Response Specification for Assessing the Biofidelity of an Anthropometric Test Dummy Part 1 -Upper Body. IRCOBI, Paper # IRC-15-58.

[17] Lobdell, T.E., Kroell, C.K., Schneider, D.C., Hering, W.E., Nahum, A.M., 1973 Impact Response of the Human Thorax, In: King W.F., Mertz H.J. (eds) Human Impact Response. Springer, Boston, MA.

[18] Kroell C, Schneider D, Nahum A., 1971. Impact tolerance and response of the human thorax. Stapp Car Crash J., Paper #710851, pages 84-134.

[19] Brumbelow, M., Zuby, D., 2009. Impact and Injury Patterns in Frontal Crashes of Vehicles With Good Ratings for Frontal Crash Protection, in: 21st International Technical Conference on the Enhanced Safety of Vehicles (ESV), Paper No. 09-0257, Stuttgart.

[20] Sun, W., Jin, J.H., Reed, M.P., Gayzik, F.S., Danelson, K.A., Bass, C.R., Zhang, J.Y., Rupp, J.D., 2016. A method for developing biomechanical response corridors based on principal component analysis. Journal of biomechanics.

[21] Lebarbé, M., Petit, P., 2012. New Biofidelity Targets for the Thorax of a 50th Percentile Adult Male in Frontal Impact. Proceedings of the 2012 IRCOBI, Paper # IRC-12-89.

[22] Vavalle, N., Jelen, B., Moreno, D., Stitzel J. & Gayzik F. 2012. New Biofidelity Targets for the Thorax of a 50th Percentile Adult Male in Frontal Impact. IRCOBI, Paper # IRC-12-89.

Appendix A: DOE graphical responses – MADM Rating

- C23=0.5 K23=0.5

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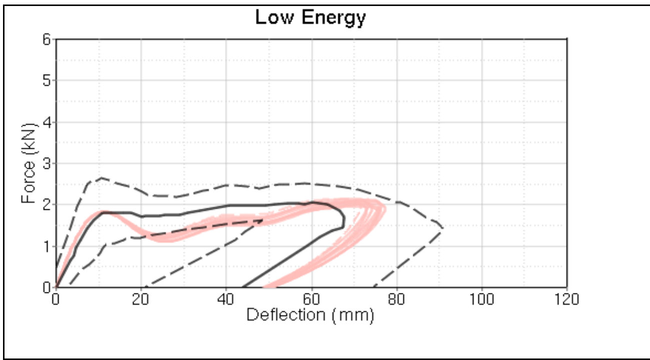

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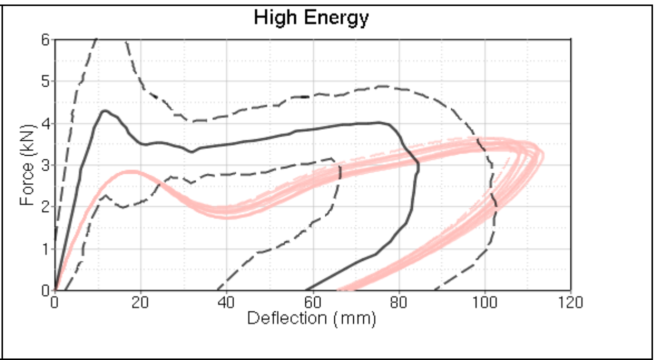

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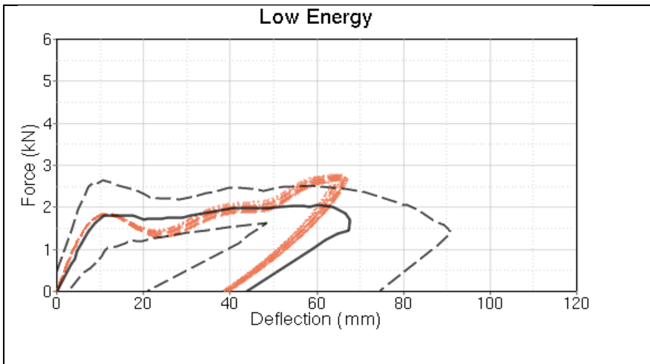

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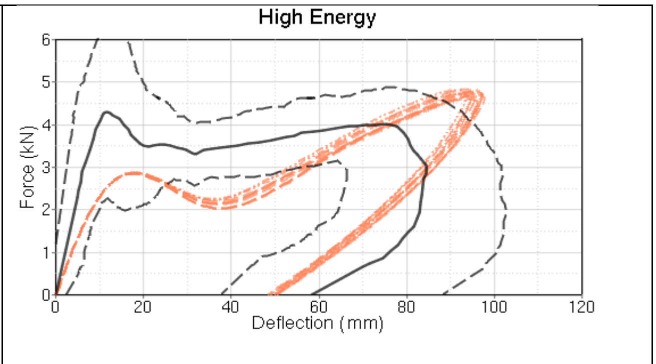

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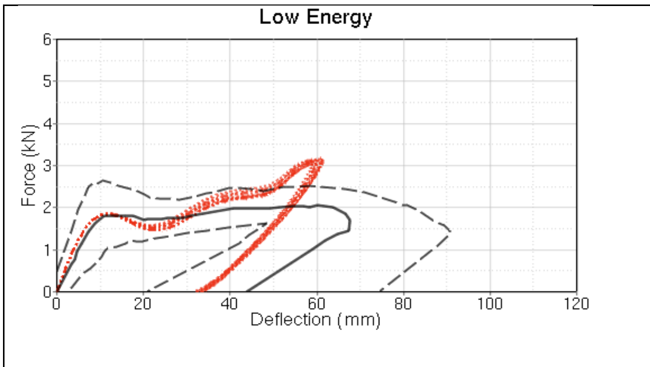

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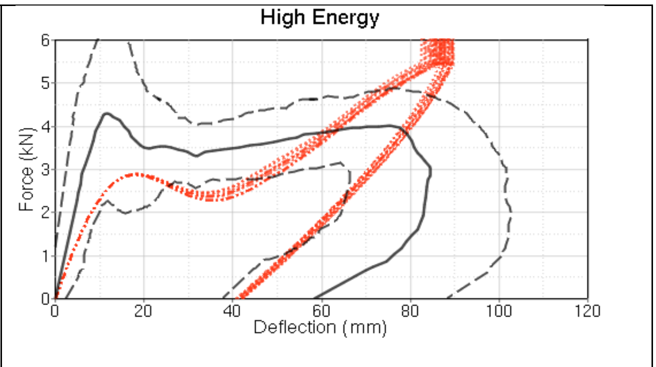

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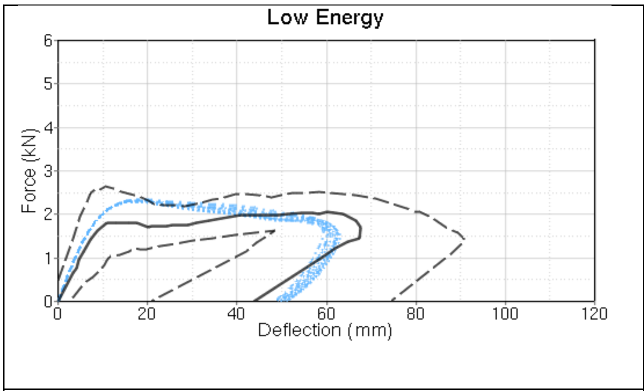

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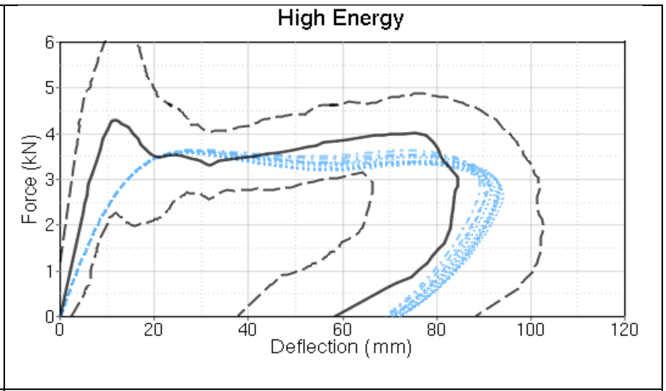

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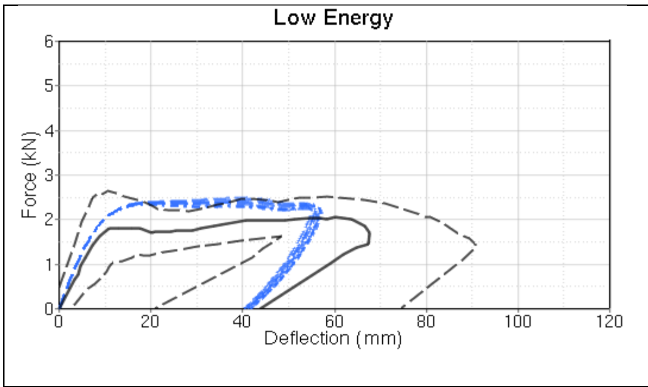

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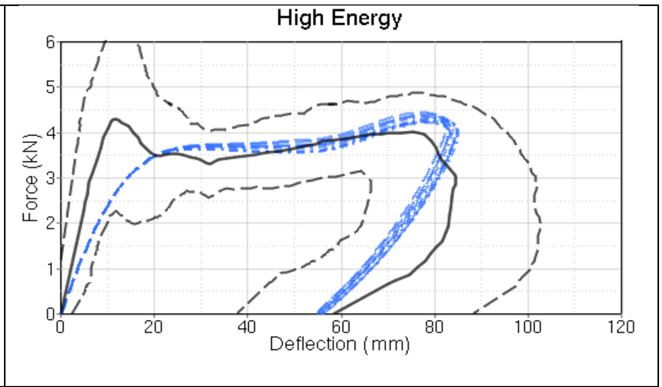

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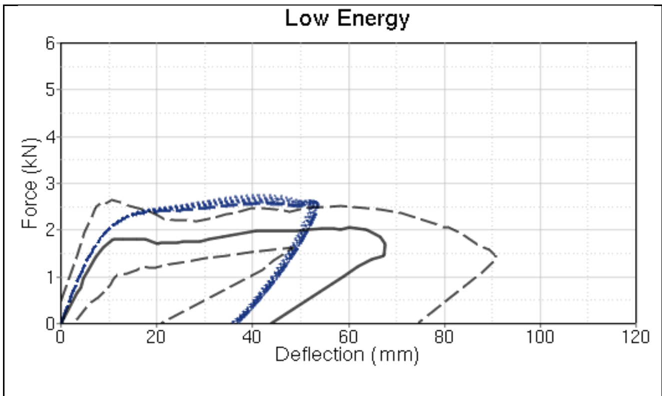

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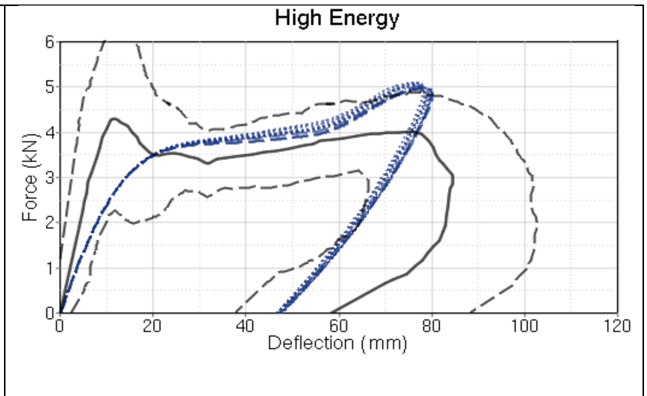

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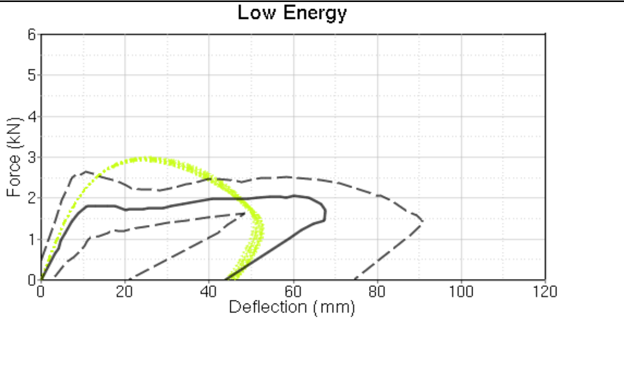

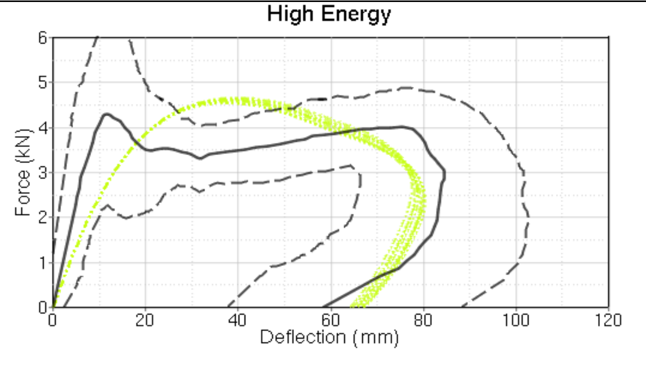

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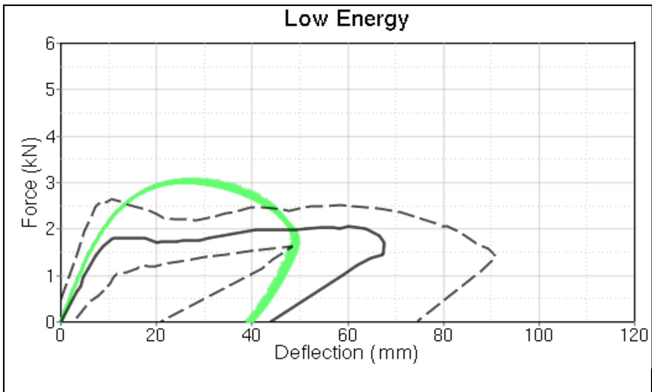

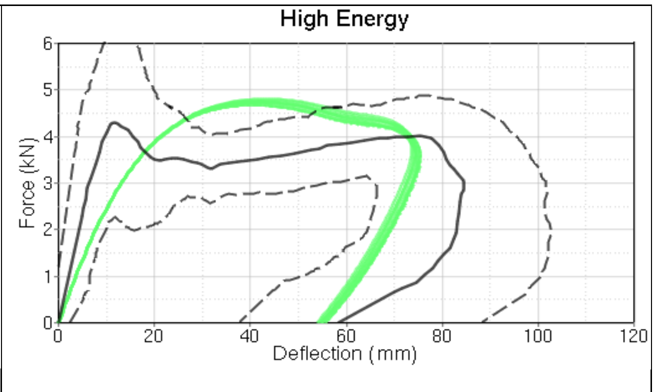

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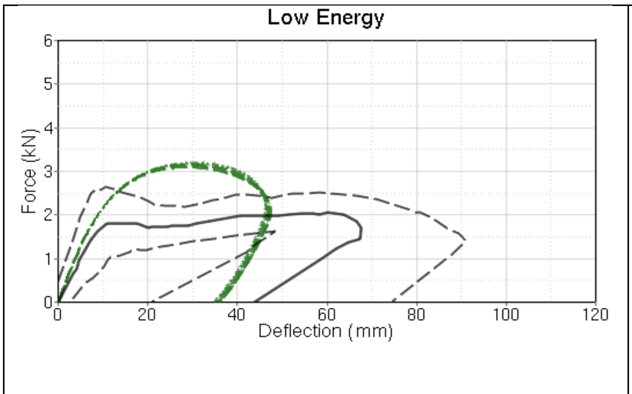
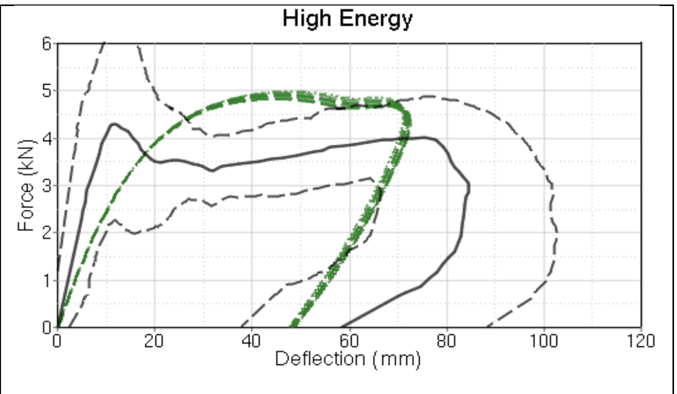
- C23=1.5 K23=1.5





Rating: 0.52-0.56	Rating: 0.56-0.60

Appendix B: Full DOE permutation at MADM values

DOE Run Matrix														
Parameters					MADM		CORA							
							Standard				ISO			
							Corridor 1		Corridor 2		Corridor 1		Corridor 2	
Run Id	**C32**	**K32**	**Kve**	**Cve**	**LE**	**HE**	**LE**	**HE**	**LE**	**HE**	**LE**	**HE**	**LE**	**HE**
Nom	1	1	0.0132	0.18	0.79	0.89	0.73	0.92	0.77	0.92	0.61	0.76	0.65	0.76
run_1	0.5	0.5	0.0066	0.09	0.78	0.49	0.86	0.58	0.91	0.68	0.74	0.47	0.78	0.55
run_2	0.5	0.5	0.0066	0.18	0.81	0.51	0.87	0.58	0.92	0.68	0.74	0.48	0.78	0.56
run_3	0.5	0.5	0.0066	0.27	0.82	0.52	0.87	0.58	0.92	0.68	0.74	0.48	0.78	0.56
run_4	0.5	0.5	0.0132	0.09	0.81	0.51	0.88	0.59	0.93	0.70	0.75	0.51	0.79	0.60
run_5	0.5	0.5	0.0132	0.18	0.86	0.54	0.90	0.60	0.94	0.71	0.76	0.53	0.79	0.62
run_6	0.5	0.5	0.0132	0.27	0.88	0.56	0.92	0.60	0.94	0.72	0.81	0.53	0.82	0.63
run_7	0.5	0.5	0.0198	0.09	0.83	0.53	0.90	0.59	0.94	0.70	0.76	0.52	0.79	0.61
run_8	0.5	0.5	0.0198	0.18	0.88	0.56	0.92	0.61	0.94	0.74	0.81	0.55	0.82	0.65
run_9	0.5	0.5	0.0198	0.27	0.91	0.59	0.92	0.62	0.94	0.75	0.81	0.56	0.82	0.66
run_10	0.5	1	0.0066	0.09	0.89	0.67	0.81	0.61	0.84	0.73	0.68	0.53	0.71	0.63
run_11	0.5	1	0.0066	0.18	0.89	0.67	0.80	0.60	0.83	0.73	0.68	0.53	0.70	0.63
run_12	0.5	1	0.0066	0.27	0.88	0.66	0.80	0.60	0.83	0.73	0.68	0.53	0.70	0.63
run_13	0.5	1	0.0132	0.09	0.91	0.69	0.82	0.62	0.85	0.75	0.70	0.54	0.72	0.65
run_14	0.5	1	0.0132	0.18	0.90	0.69	0.82	0.62	0.84	0.75	0.69	0.54	0.70	0.65
run_15	0.5	1	0.0132	0.27	0.89	0.69	0.81	0.61	0.83	0.75	0.68	0.54	0.69	0.64
run_16	0.5	1	0.0198	0.09	0.92	0.71	0.83	0.63	0.86	0.76	0.70	0.55	0.72	0.66
run_17	0.5	1	0.0198	0.18	0.91	0.72	0.82	0.63	0.85	0.77	0.68	0.55	0.70	0.66
run_18	0.5	1	0.0198	0.27	0.89	0.71	0.82	0.62	0.84	0.77	0.67	0.54	0.69	0.66
run_19	0.5	1.5	0.0066	0.09	0.79	0.59	0.79	0.57	0.81	0.71	0.64	0.48	0.66	0.60
run_20	0.5	1.5	0.0066	0.18	0.78	0.58	0.79	0.57	0.80	0.71	0.64	0.48	0.66	0.60
run_21	0.5	1.5	0.0066	0.27	0.78	0.58	0.79	0.57	0.80	0.71	0.64	0.45	0.65	0.56
run_22	0.5	1.5	0.0132	0.09	0.79	0.61	0.79	0.57	0.81	0.73	0.63	0.49	0.65	0.61
run_23	0.5	1.5	0.0132	0.18	0.78	0.60	0.78	0.57	0.81	0.73	0.62	0.49	0.63	0.61
run_24	0.5	1.5	0.0132	0.27	0.76	0.59	0.78	0.57	0.80	0.73	0.61	0.45	0.63	0.58
run_25	0.5	1.5	0.0198	0.09	0.80	0.62	0.79	0.58	0.81	0.74	0.63	0.49	0.64	0.62
run_26	0.5	1.5	0.0198	0.18	0.77	0.61	0.77	0.58	0.80	0.75	0.59	0.48	0.62	0.61
run_27	0.5	1.5	0.0198	0.27	0.76	0.60	0.76	0.58	0.79	0.75	0.58	0.49	0.61	0.62
run_28	1	0.5	0.0066	0.09	0.91	0.84	0.76	0.89	0.81	0.95	0.62	0.79	0.65	0.84
run_29	1	0.5	0.0066	0.18	0.92	0.85	0.77	0.93	0.81	0.95	0.62	0.83	0.66	0.85
run_30	1	0.5	0.0066	0.27	0.93	0.86	0.77	0.93	0.82	0.95	0.63	0.83	0.66	0.85
run_31	1	0.5	0.0132	0.09	0.91	0.86	0.74	0.91	0.79	0.94	0.54	0.81	0.58	0.84
run_32	1	0.5	0.0132	0.18	0.91	0.88	0.75	0.93	0.79	0.94	0.61	0.82	0.64	0.84
run_33	1	0.5	0.0132	0.27	0.92	0.90	0.76	0.93	0.80	0.94	0.62	0.82	0.65	0.83
run_34	1	0.5	0.0198	0.09	0.90	0.86	0.73	0.90	0.77	0.94	0.53	0.80	0.57	0.83
run_35	1	0.5	0.0198	0.18	0.90	0.90	0.74	0.91	0.77	0.93	0.55	0.81	0.58	0.83
run_36	1	0.5	0.0198	0.27	0.90	0.92	0.75	0.91	0.78	0.93	0.56	0.81	0.59	0.82
run_37	1	1	0.0066	0.09	0.83	0.91	0.84	0.93	0.87	0.94	0.64	0.80	0.66	0.81
run_38	1	1	0.0066	0.18	0.82	0.90	0.84	0.92	0.87	0.94	0.64	0.80	0.66	0.81
run_39	1	1	0.0066	0.27	0.81	0.90	0.84	0.92	0.86	0.94	0.64	0.80	0.66	0.81
run_40	1	1	0.0132	0.09	0.82	0.91	0.77	0.92	0.79	0.93	0.63	0.80	0.65	0.81

run_41	1	1	0.0132	0.18	0.79	0.89	0.74	0.92	0.79	0.93	0.60	0.80	0.63	0.81
run_42	1	1	0.0132	0.27	0.78	0.88	0.74	0.92	0.78	0.93	0.60	0.80	0.63	0.81
run_43	1	1	0.0198	0.09	0.81	0.91	0.76	0.92	0.79	0.93	0.63	0.80	0.65	0.80
run_44	1	1	0.0198	0.18	0.78	0.88	0.73	0.92	0.77	0.93	0.59	0.79	0.62	0.80
run_45	1	1	0.0198	0.27	0.76	0.87	0.73	0.92	0.76	0.93	0.58	0.79	0.61	0.80
run_46	1	1.5	0.0066	0.09	0.69	0.74	0.67	0.82	0.71	0.87	0.54	0.69	0.57	0.73
run_47	1	1.5	0.0066	0.18	0.68	0.73	0.67	0.81	0.71	0.86	0.53	0.70	0.56	0.73
run_48	1	1.5	0.0066	0.27	0.67	0.73	0.67	0.81	0.70	0.86	0.53	0.69	0.56	0.73
run_49	1	1.5	0.0132	0.09	0.68	0.73	0.67	0.82	0.71	0.87	0.54	0.69	0.57	0.74
run_50	1	1.5	0.0132	0.18	0.66	0.71	0.67	0.81	0.70	0.86	0.53	0.69	0.56	0.73
run_51	1	1.5	0.0132	0.27	0.65	0.70	0.66	0.81	0.69	0.85	0.53	0.69	0.55	0.72
run_52	1	1.5	0.0198	0.09	0.67	0.73	0.67	0.82	0.72	0.88	0.54	0.70	0.57	0.75
run_53	1	1.5	0.0198	0.18	0.65	0.70	0.67	0.81	0.70	0.87	0.53	0.69	0.56	0.73
run_54	1	1.5	0.0198	0.27	0.64	0.68	0.66	0.81	0.69	0.86	0.53	0.69	0.55	0.72
run_55	1.5	0.5	0.0066	0.09	0.74	0.85	0.61	0.73	0.67	0.77	0.42	0.65	0.47	0.68
run_56	1.5	0.5	0.0066	0.18	0.73	0.85	0.61	0.73	0.67	0.78	0.42	0.65	0.47	0.69
run_57	1.5	0.5	0.0066	0.27	0.73	0.85	0.61	0.73	0.67	0.78	0.42	0.65	0.47	0.69
run_58	1.5	0.5	0.0132	0.09	0.72	0.84	0.60	0.73	0.66	0.77	0.41	0.54	0.46	0.57
run_59	1.5	0.5	0.0132	0.18	0.71	0.84	0.60	0.73	0.66	0.78	0.41	0.65	0.46	0.68
run_60	1.5	0.5	0.0132	0.27	0.71	0.83	0.60	0.74	0.66	0.78	0.42	0.65	0.46	0.69
run_61	1.5	0.5	0.0198	0.09	0.72	0.83	0.60	0.73	0.65	0.76	0.41	0.54	0.45	0.57
run_62	1.5	0.5	0.0198	0.18	0.70	0.82	0.59	0.73	0.64	0.77	0.41	0.55	0.45	0.58
run_63	1.5	0.5	0.0198	0.27	0.69	0.82	0.59	0.74	0.65	0.78	0.41	0.55	0.45	0.58
run_64	1.5	1	0.0066	0.09	0.64	0.75	0.60	0.79	0.66	0.81	0.42	0.70	0.47	0.72
run_65	1.5	1	0.0066	0.18	0.63	0.74	0.61	0.79	0.66	0.81	0.43	0.70	0.47	0.72
run_66	1.5	1	0.0066	0.27	0.63	0.73	0.61	0.79	0.66	0.81	0.43	0.70	0.47	0.72
run_67	1.5	1	0.0132	0.09	0.63	0.73	0.59	0.79	0.65	0.81	0.41	0.69	0.46	0.71
run_68	1.5	1	0.0132	0.18	0.61	0.71	0.60	0.79	0.65	0.81	0.41	0.69	0.46	0.70
run_69	1.5	1	0.0132	0.27	0.61	0.70	0.60	0.79	0.65	0.81	0.41	0.69	0.46	0.71
run_70	1.5	1	0.0198	0.09	0.62	0.73	0.59	0.78	0.64	0.81	0.41	0.68	0.45	0.70
run_71	1.5	1	0.0198	0.18	0.60	0.70	0.58	0.78	0.64	0.80	0.41	0.68	0.45	0.70
run_72	1.5	1	0.0198	0.27	0.59	0.68	0.58	0.77	0.64	0.80	0.41	0.68	0.45	0.70
run_73	1.5	1.5	0.0066	0.09	0.56	0.60	0.59	0.75	0.64	0.80	0.41	0.66	0.45	0.70
run_74	1.5	1.5	0.0066	0.18	0.55	0.60	0.59	0.75	0.63	0.79	0.41	0.65	0.45	0.69
run_75	1.5	1.5	0.0066	0.27	0.55	0.59	0.59	0.75	0.63	0.79	0.41	0.65	0.45	0.69
run_76	1.5	1.5	0.0132	0.09	0.55	0.60	0.58	0.74	0.63	0.78	0.40	0.66	0.44	0.69
run_77	1.5	1.5	0.0132	0.18	0.54	0.58	0.58	0.74	0.62	0.78	0.40	0.65	0.43	0.68
run_78	1.5	1.5	0.0132	0.27	0.53	0.57	0.58	0.73	0.62	0.77	0.40	0.65	0.43	0.68
run_79	1.5	1.5	0.0198	0.09	0.55	0.59	0.58	0.74	0.62	0.78	0.40	0.66	0.44	0.69
run_80	1.5	1.5	0.0198	0.18	0.53	0.57	0.57	0.73	0.62	0.77	0.39	0.62	0.43	0.66
run_81	1.5	1.5	0.0198	0.27	0.52	0.56	0.57	0.73	0.61	0.76	0.39	0.62	0.42	0.65

<u>Legend:</u>
LE: Low Energy
HE: High Energy
Corridor 1: Inner corridors: +-0.5 SD – Outer corridors: +-1SD
Corridor 2: Inner corridors: +-1 SD – Outer corridors: +-2SD
Cells highlighted in red represent a 5% range from the maximum rating.
A highlighted cell represents a high correlation level for the range selected.

Appendix C: Correlation plot MADM vs CORA scores