A VISCOELASTIC MODEL FOR DESCRIBING THE MECHANICAL RESPONSE OF THE SCAPHOLUNATE LIGAMENT UNDERGOING LARGE DEFORMATIONS

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CONTENT OF THE STUDY

ANATOMICAL DESCRIPTION OF THE SCAPHOLUNATE LIGAMENT

UTILITY OF THE STUDY

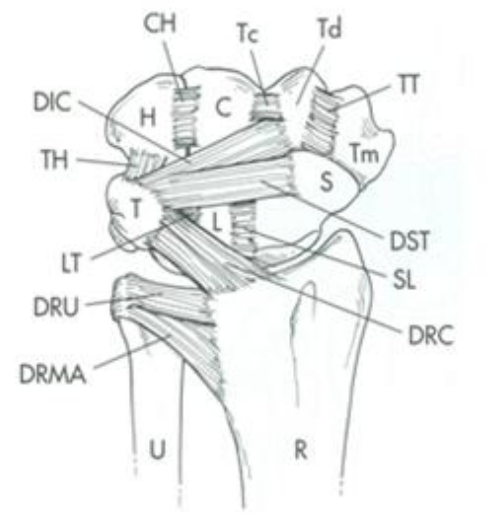
MATERIALS AND METHODS

RESULTS

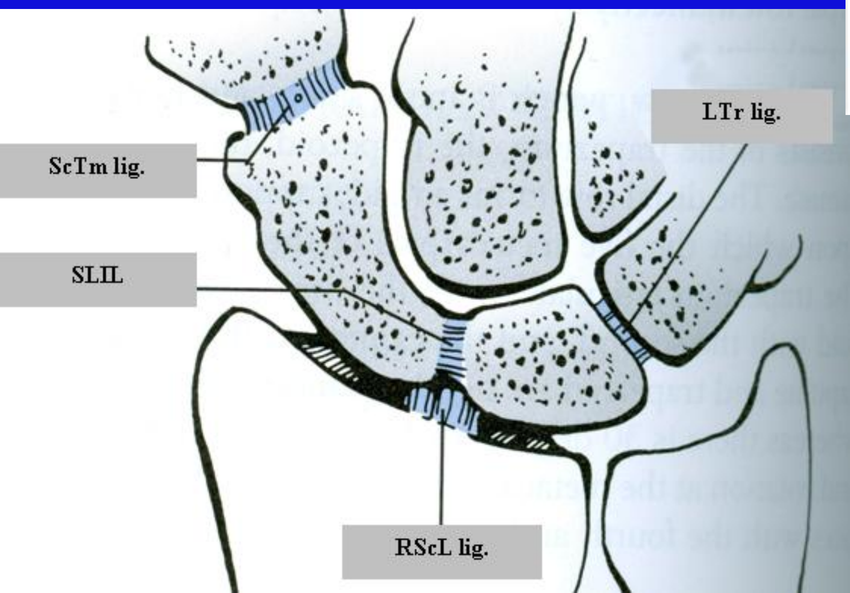
CONCLUSION

ANATOMICAL DESCRIPTION OF THE SCAPHOLUNATE LIGAMENT

Scapholunate ligament (S-L): main ligament of the wrist

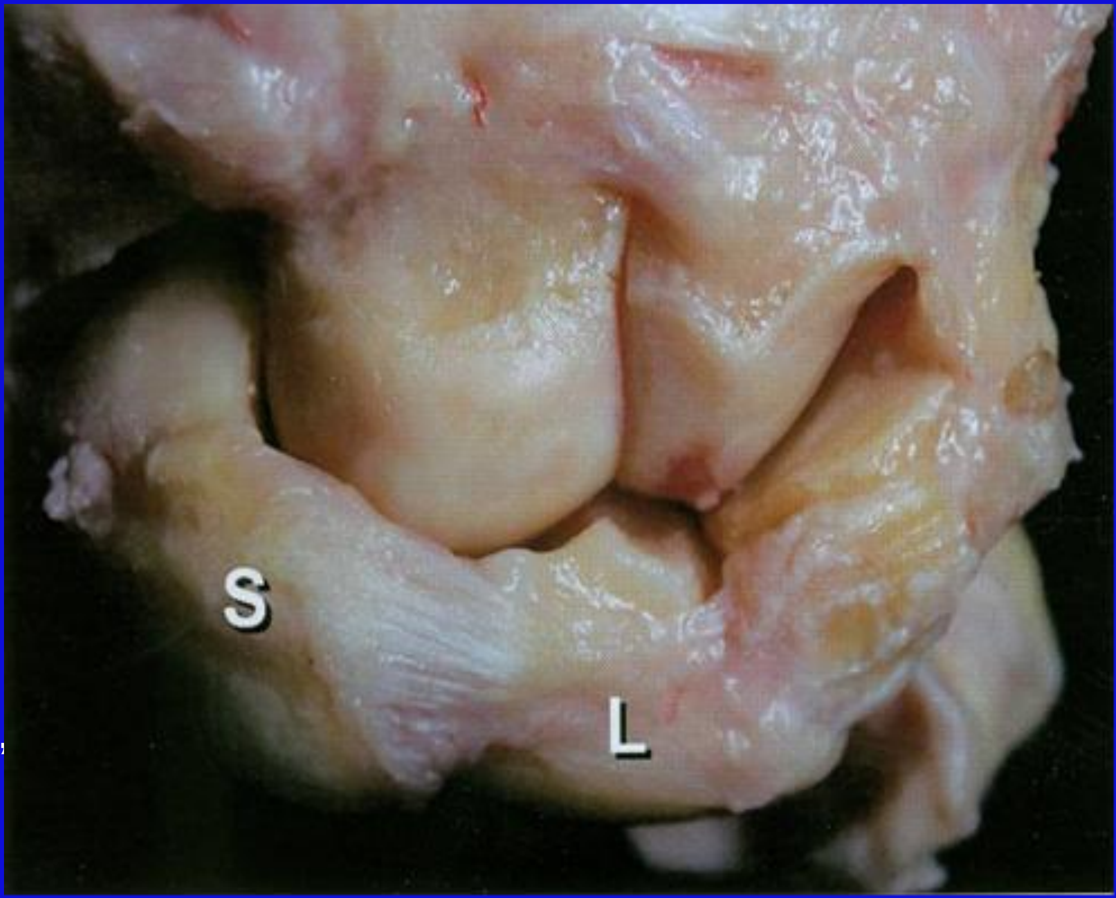


R	Radius	SL	Scapholunate
U	Ulna	LT	Lunotriquetrum
CH	Capitohamate	TH	Triquetrohamate
S	Scaphoid	TT	Trapezium-Trapezoid
L	Lunate	Tc	Triquetrocapitate
T	Triquetrum	C	Capitate



ORIENTATION OF S-L LIGAMENT FIBERS

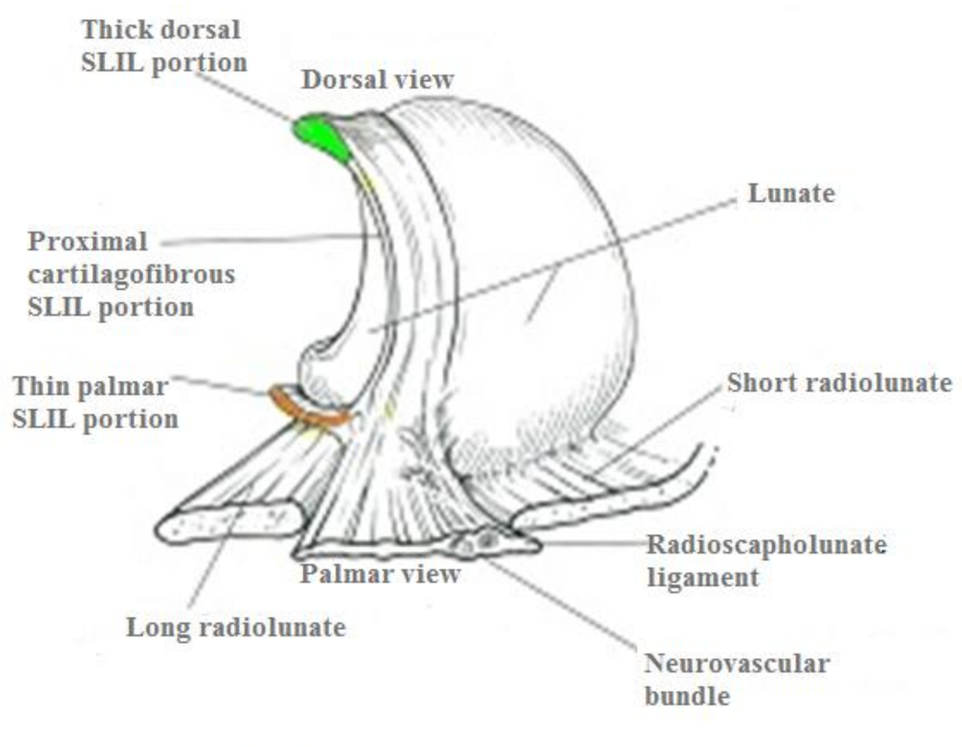
The S-L fibers are orientated along several directions; they cross dorsally the joint from the dorsal or vertical aspect while fibers placed in the circumferential part of the ligament are aligned along the arc of the joint; obliquely crossing from the scaphoid (Sc) to the lunate (L).



From the palmar aspect, the thickness of the S-L ligament is not larger than 1mm. The oblique aligned S-L fibers cross the palmar verge of the lunate. In addition to 3 portions of S-L (dorsal (d-SL),intermediate (i-SL), palmar portion of S-L (p-SL)) on the palmar verge of S-L; the proximal pole of Sc and L are stabilized due to the radioscapholunate ligament.
The S-L fibers are orientated in several directions in each of the S-L portions.

MORPHOLOGY OF S-L FIBERS

- The d-SL is the thickest portion of the S-L, constituted from vertically orientated collagen fibers. This is characterized of length (≈5 mm), thickness (3–4)mm and from the frontal aspect it seems as a triquetral bone shaped like the space presented between the dorsal linking of the Sc with L.



3 S-L portions

- The i-SL seems as a knee meniscus characterized of a prominent crest. There are not vessels, nerves in the i-SL and this is characterized totally from fibro-cartilageous tissue. A small number of collagen fibers, orientated on the palmar aspect cross the p-SL.

- The p-SL is thick and it is not often discernible from the radiocarpal and mediocarpal joint aspect.

INTENTION OF THE STUDY

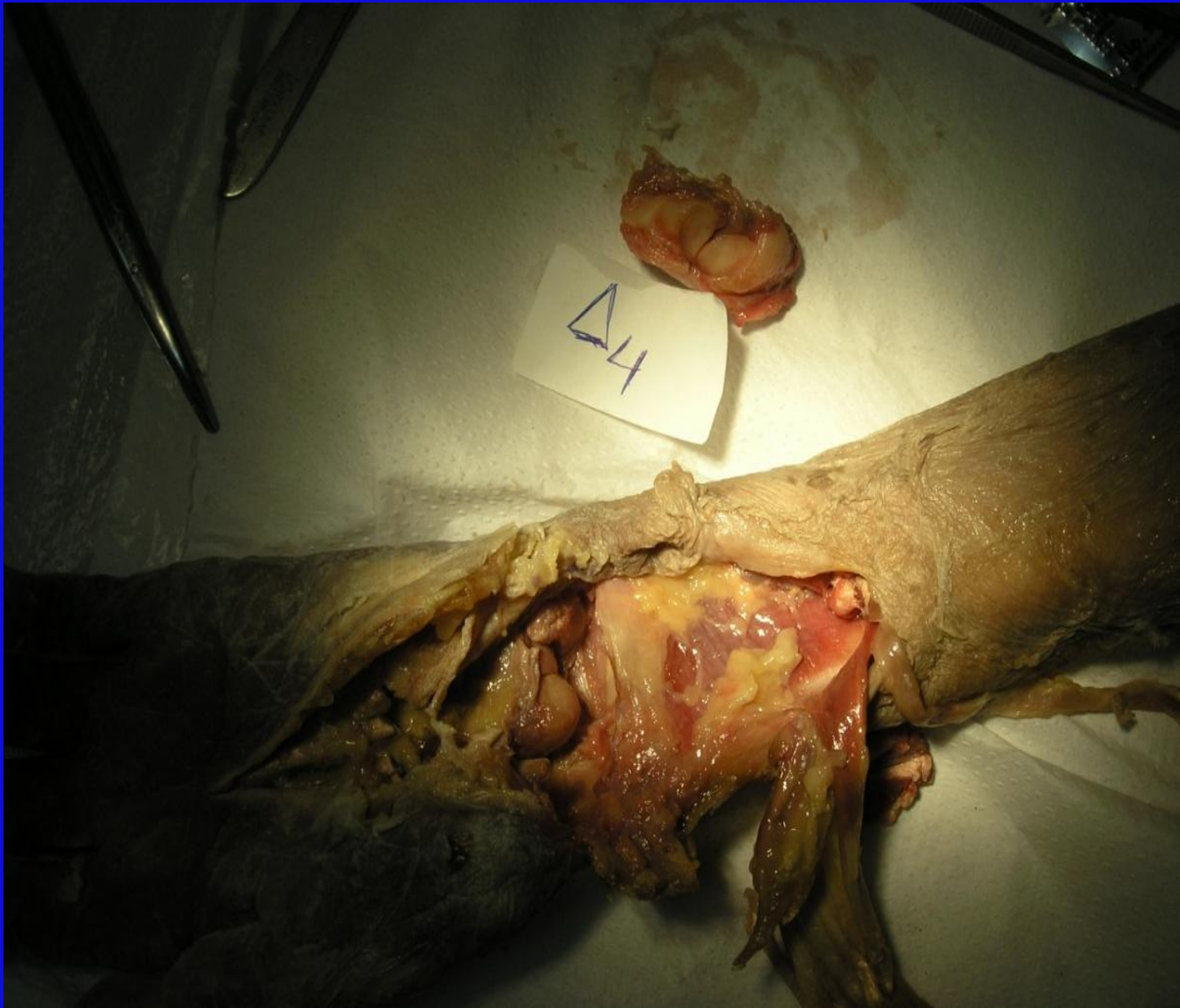
The evaluation of the mechanical behavior of scapholunate ligament up to fracture and the simulation of this behavior by using a simply formulated viscoelastic model and the equations describing the Normal (Gaussian) distribution.

The proposed model of this study could be used for used for the determination of the S-L biomechanical properties by aiming to facilitate the construction of the autologous grafts in the future.

MATERIALS AND METHOD

- The mechanical behavior of 8 S-L fresh cadaver specimens, dissected from the upper edge or circumferential from the elbow, was studied.

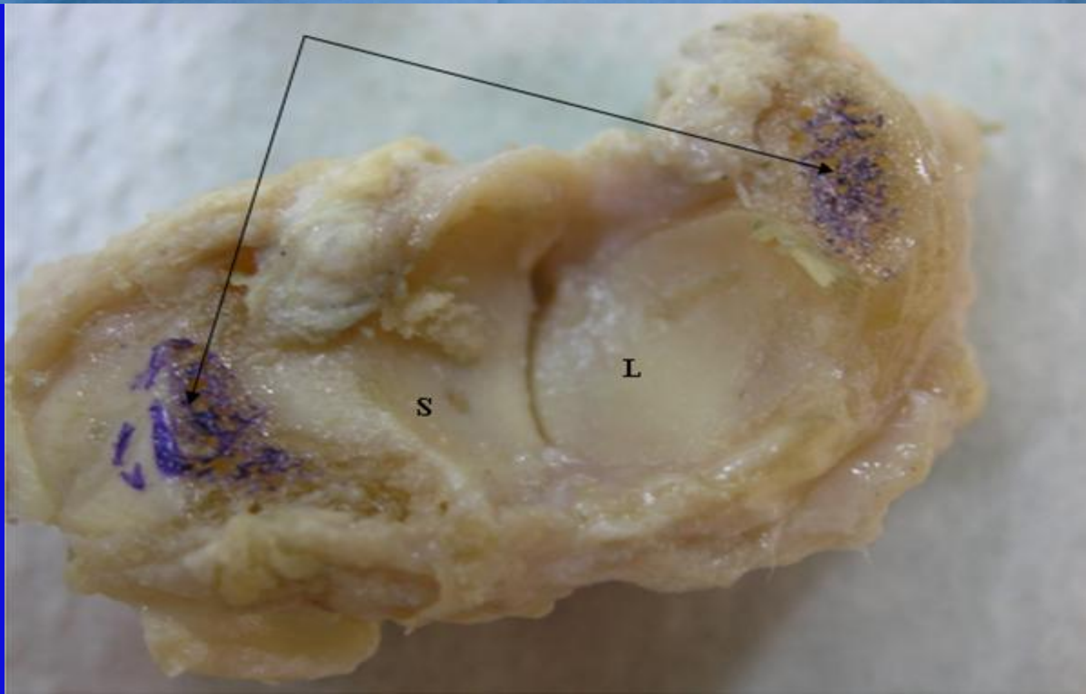
- Only 7 specimens were used during the accomplishment of measurements because one specimen was damaged.



After the dissection, the specimens were placed on deep frozen conditions (-32ºC). Before the preparation of the carpal specimens they were thawed on room temperature for a 24-hour period.

The collection of the S-L specimens was accomplished through the dorsal approach.

After the preparation of the specimens and their removal from the carpus, wrapped in a wet gauze, impregnated in solution(0.9 % NaCl), they were preserved on room temperature before the accomplishment of the mechanical tests.



MECHANICAL MEASUREMENTS

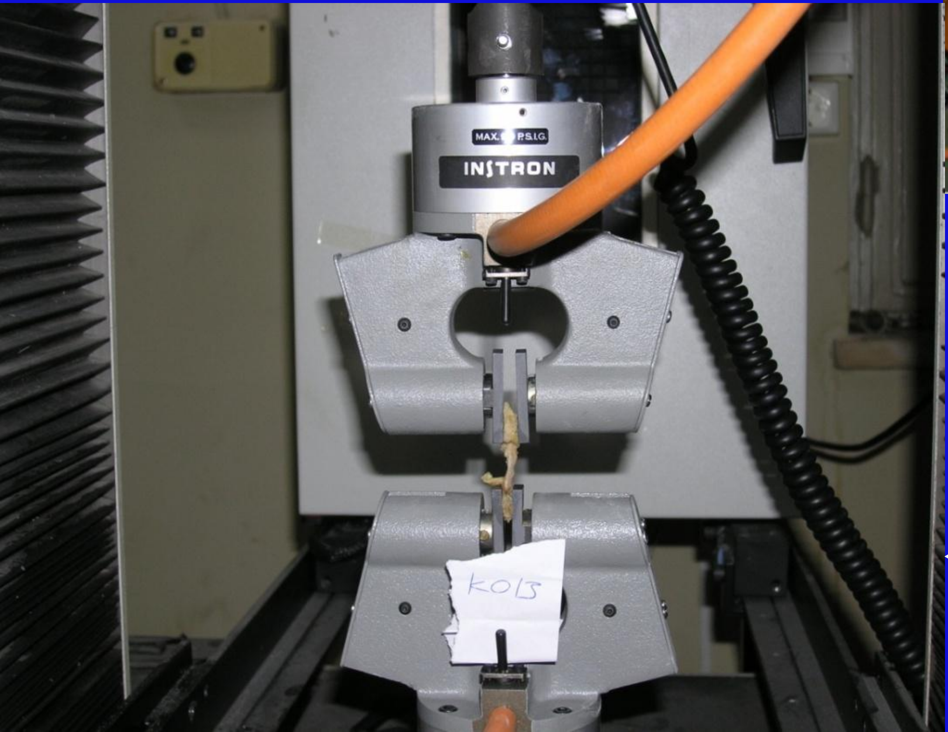
- The S-L ligament specimens were subjected to uni-axial tensile measurements (with constant displacement rate equal to 5mm/min)

- The determination of the loading conditions was considered as crucial due to the complicated architecture and geometry of the S-L ligament. It was accomplished on a such way that the S-L fibers are unfolded and not turned. Subsequently, the capture of the edge of the scaphoid (lunate) was accomplished by the upper grip. Then, the specimens were balanced and the verges of the scaphoid (lunate) were configured so that the bone surfaces are applied to the lower grip.

The accomplishment of this procedure was considered as essential to secure the loading of S-L fibers aligned in the same direction.

EXPERIMENTAL MACHINE

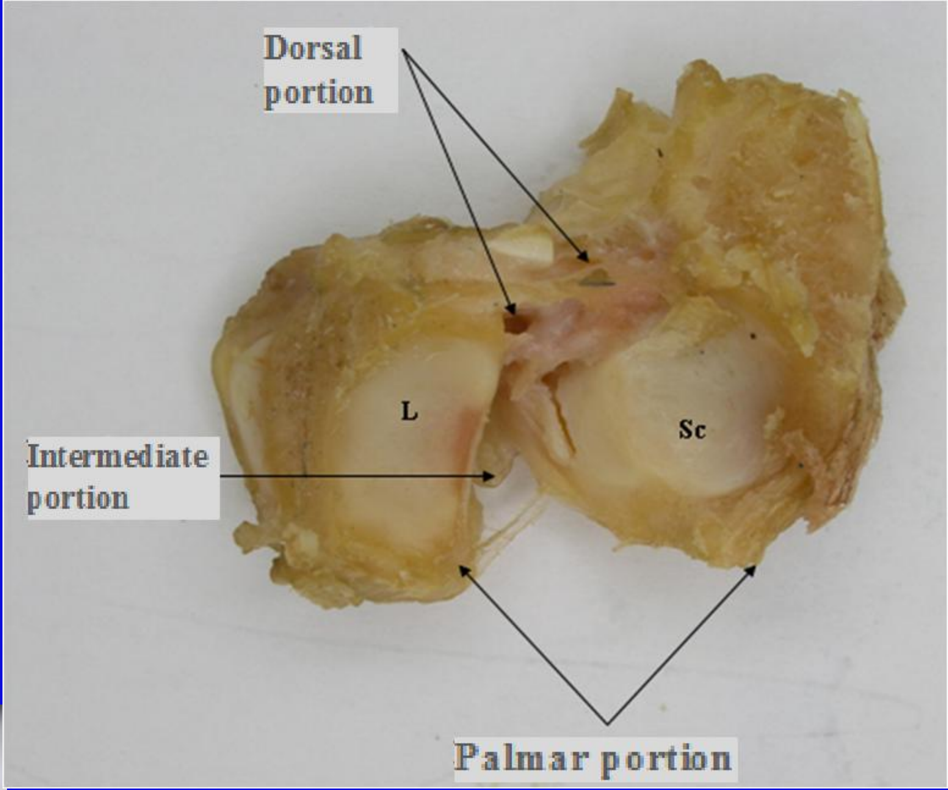
The tests were accomplished in a digitalized Instron 1121 US machine equipped with a load 1000N- sensor and pneumatic grips (pressure equal to 3 Bar).



Instron 1121 USA

S-L ligament applied on experimental machine

S-L ligament subjected in tension



S-L specimen displayed after being subjected in tension

RESULTS OBTAINED FROM THE MECHANICAL TESTS

Dimensions and mechanical properties of S-L				
Code No	**Average Dimensions (Length)x(Width)x(Thickness) (mm)**	**Maximum load (N)**	**Displacement at maximum load (mm)**	**S_{high} High stiffness (N/mm)**
ko5	5.2x15x1.5	105	8	37.8
ko6	5x16.5x1.5	104	5.5	48.6
ko40	5.5x15.5x1.5	210	11	25.6
ko41	5.5x21.5x1.5	161	7.3	43.6
ko51	4.3x14.2x1.5	68	6.3	25.2
ko52	5.6x16.1x1.5	193	8.9	41.6
ko53	4.8x14x1.5	187	9.1	27.2
	Average values	**147**	**8.0**	**35.7**

SIMULATIONS OF THE BEHAVIOR OF SOFT TISSUES

Models characterizing behavior of soft tissues (e.t.c. ligaments, tendons, muscles)

Mechanical behavior of tissues

Elastic

Viscoelastic

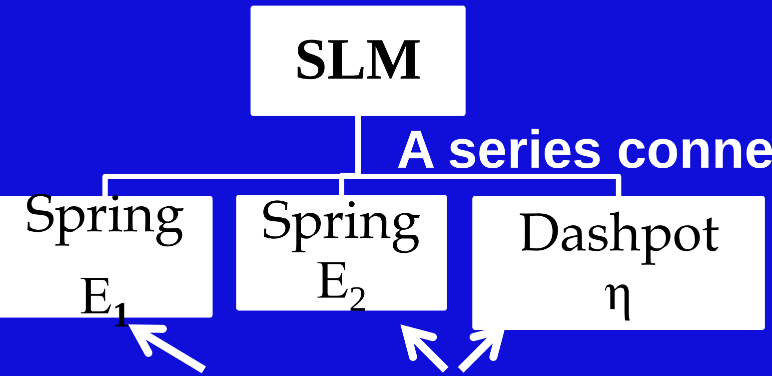
Model equations

Models (formulated by equations describing continuum material)

Computational models

Stochasitc models(using probabilistic functions)

VISCOELASTIC STANDARD LINEAR MODEL (SLM)

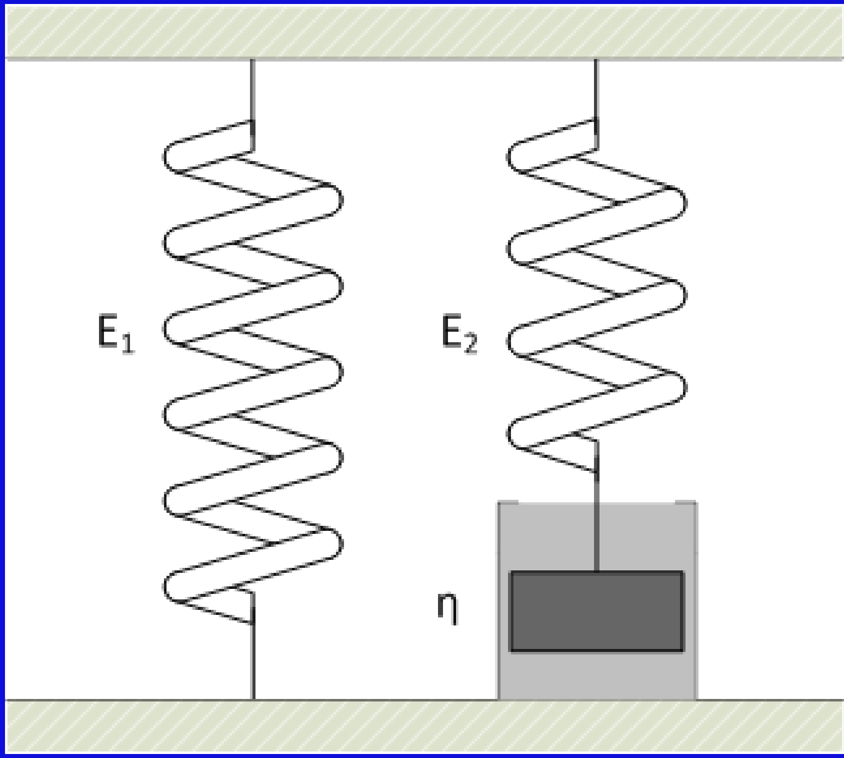


The spring (spring constant E_1) is in parallel connection with the system (spring (spring constant E_2) and dashpot (viscosity η).

✓Stress σ_1 causing strain ε_1 is applied to the edges of the spring (spring constant E_1).

✓ Stress σ_2 causing strain ε_2 is applied to the edges of the spring (spring constant E_2).

VISCOELASTIC STANDARD LINEAR MODEL



✓ Stress σ_A causing strain ε_A is applied to the edges of the dashpot (viscosity η).

SLM CONSTANTS

The spring constants E_1, E_2 characterize the elastic behavior of the ligament, but mostly the stiffness of the collagen fibers.

⊡ **Spring constant E_1**	Stiffness due to the recruitment of the fibers along the tensile direction of the collagen fiber bundles (CBF)
⊡ **Spring constant E_2**	Stiffness due to the configuration of the structure of the collagen fiber bundles based on the reinforcement of the crosslinks between the collagen fibers
⊡ **Dashpot (viscosity η)**	Viscosity based on the flow properties demonstrated by the ligament structure due to its displaying the viscoelastic response

STRESS-STRAIN RELATIONS AMONG THE SLM ELEMENTS

Constants and stress-strain relations among the SLM elements		
Element	**Constant of the element**	**Stress-strain function $\sigma=f(\varepsilon)$**
Spring	$\mathbf{E_1}$	$\sigma_1 = E_1\varepsilon_1$ ***(1)**
Spring	$\mathbf{E_2}$	$\sigma_2 = E_2\varepsilon_2$ ***(2)**
Dashpot	$\boldsymbol{\eta}$	$\sigma_A = \eta\frac{d\varepsilon_A}{dt}$ ***(3)**

* Code indicating the set of the equations

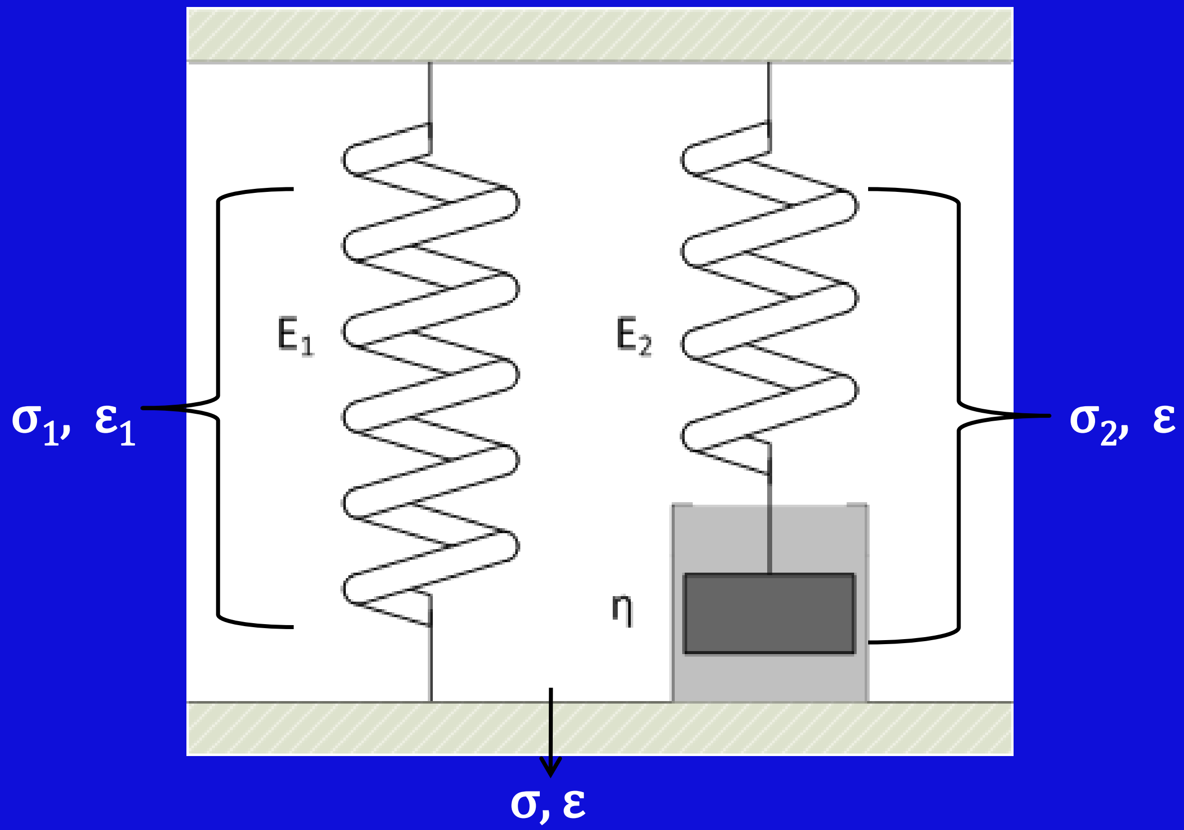
- Newton's law of viscosity describing the behavior of the linear viscoelastic solid materials

The balance of the compartment of SLM system (spring & dashpot connected in series) imposes the application of the equation: $\boldsymbol{\sigma_2 = \sigma_A}$ **(4)**

The strain $\boldsymbol{\varepsilon}$ characterizing the right part of the SLM system, being constituted by the spring and dashpot connected in series is expressed as: $\boldsymbol{\varepsilon = \varepsilon_2 + \varepsilon_A}$ **(5)**

Due to the geometry of the SLM system, the above defined strain $\boldsymbol{\varepsilon}$ is equal to the strain of the left string (string constant $\mathbf{E_1}$) of the system; it is written that: $\boldsymbol{\varepsilon = \varepsilon_1}$ **(6)**

Due to the balance of the SLM the stress of the SLM system $\boldsymbol{\sigma}$ is distributed to the left string ($\mathbf{E_1}$) and compartment (spring and dashpot in series) is equal to: $\boldsymbol{\sigma = \sigma_1 + \sigma_2}$ **(7)**



Multiplying the equation (5) with the differential factor $\frac{d}{dt}$ and using the equations (2), (3) it is obtained that:

$$\frac{d\varepsilon}{dt} = \frac{d}{dt}\frac{\sigma_2}{E_2} + \frac{\sigma_A}{\eta} \qquad (8)$$

After the multiplying of the equation (7) with the factor $\left(\frac{1}{\eta} + \frac{1}{E_2}\frac{d}{dt}\right)$ it is obtained that:

$$\left(\frac{1}{\eta} + \frac{1}{E_2}\frac{d}{dt}\right)\sigma = \frac{\sigma}{\eta} + \frac{1}{E_2}\frac{d\sigma}{dt} = \frac{\sigma_1}{\eta} + \frac{1}{E_2}\frac{d\sigma_1}{dt} + \frac{\sigma_2}{\eta} + \frac{1}{E_2}\frac{d\sigma_2}{dt} \qquad (9)$$

SOLVING THE DIFFERENTIAL STRESS-STRAIN EQUATION OF THE SLM

The differential stress-strain equation of the SLM is obtained after processing the equation (8) based on the equations (1), (6):

$$\sigma + \frac{\eta}{E_2}\frac{d\sigma}{dt} = E_1\varepsilon + E_1\frac{\eta}{E_2}\frac{d\varepsilon}{dt} + \eta\frac{d\varepsilon}{dt} \qquad (10)$$

(relaxation time) $\tau = \frac{\eta}{E_2}$

where E_2 , η are the constants characterizing the SLM elements connected in series (spring, dashpot)

- **Under the influence of constant strain rate:** $\frac{d\varepsilon}{dt} = R$

<u>The solution of the differential equation of the SLM is expressed by</u>:

$$\sigma = (E_1\varepsilon + \tau(E_1 + E_2)R)\left(1 - \exp\left(-\frac{t}{\tau}\right)\right) \qquad (11)$$

STRUCTURE OF THE LIGAMENT

LIGAMENT

- **Water**
- **Elastine** (10-15% solid matter)
- **Collagen**
  Collagen I (60-86% solid matter)
- **Glycolipids proteoglycans fibroblasts**
- **Proteins glycoproteins** (actin, lamin, integrins)

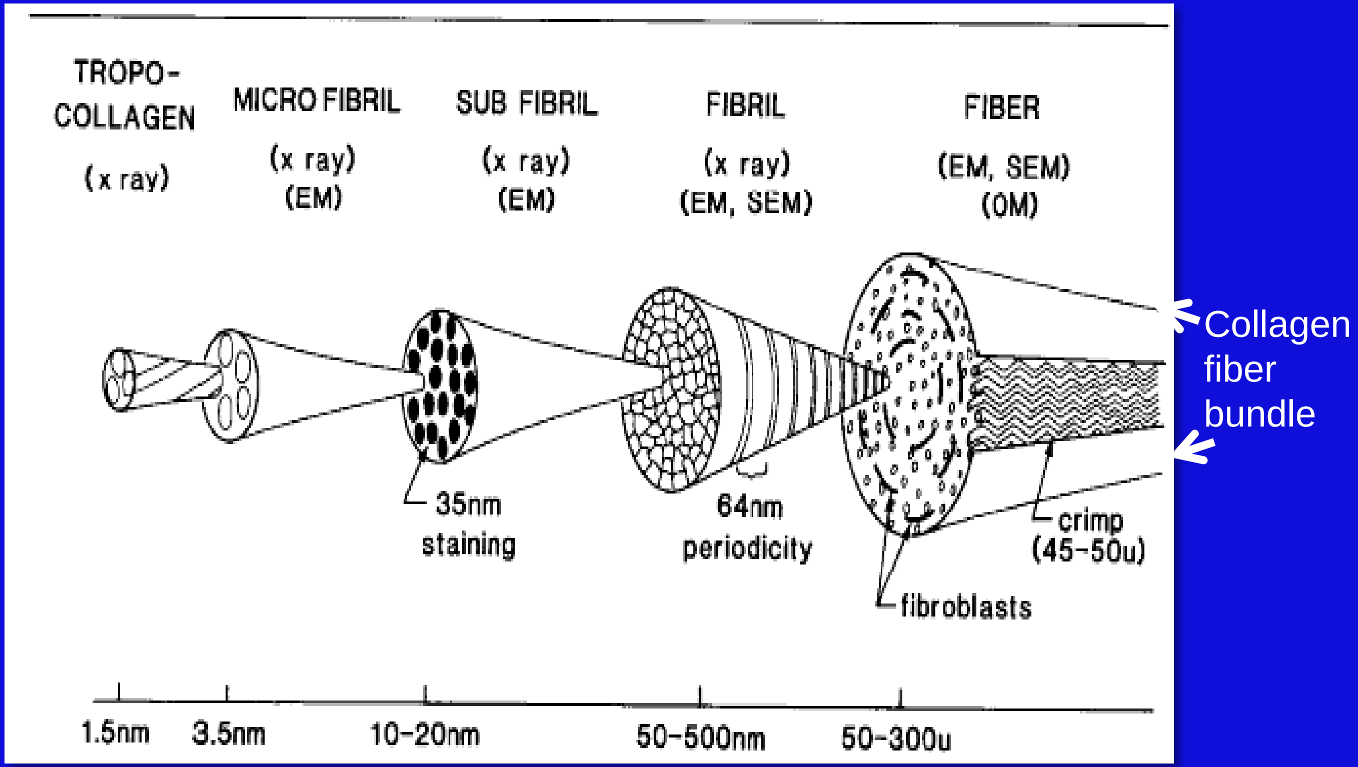
- **The collagen I (60–86% of solid matter) is organized into tightly packaged bundles, places one parallel to other, along the main axis of the ligament.**

- **The collagen (of which the main structural constituent is tropocollagen) is included in the set of the too rigid (tough) fibrous proteins.**
  The elastic modulus and the uphold structure of the ligament rendering a high resistance in tension to the ligament, are based on the mechanical and structural characteristics of the collagen material.

- **As the ligaments transpose the tensile load from one to the other, the main axis of the ligament is orientated along the length of the joint in order that the optimal placement of the system of the ligaments is achieved by serving the transmission of the load.**

STRUCTURE OF COLLAGEN

The hierarchically organized structure of ligaments is constituted from levels (collagen molecules, micro-fibrils, sub-fibrils (constructed due to tropocollagen molecules) packages of fibrils and fascicles orientated along the main axis of the ligament).



The collagen fibrils are characterized of periodic patterns in their structure (crimp).

Besides,

The enzyme **lysyl oxidase** promotes the construction of crosslinks (**crosslinks)** (presence of stable inter-collagenous and intra-collagenous crosslinks). **The creation of crosslinks is crucial for the high strengthening of collagen fibers.**

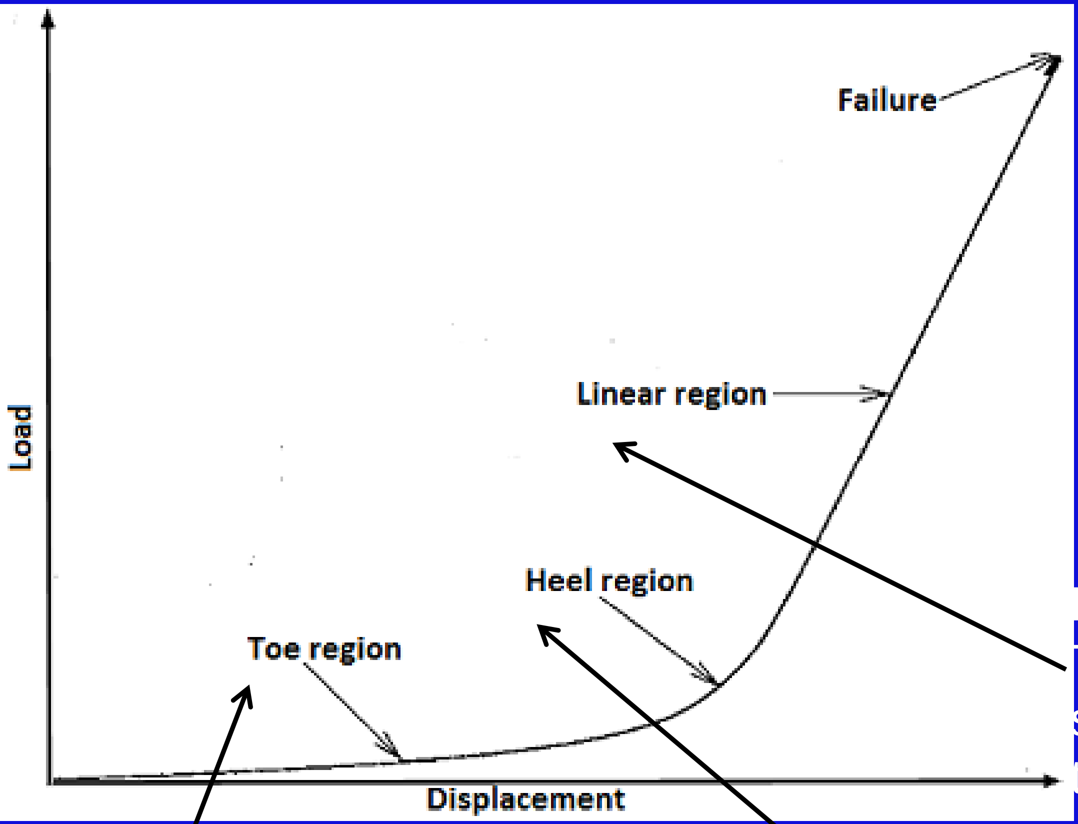
It is assumed through the formulation of the viscoelastic model that the Collagen Bundle Fiber (CBF) has the main impact on the elastic behavior of the ligament structure.

CBF is characterized as a cylindrical structure being constituted by fibers orientated along the main axis of the collagen structure.

The total (circumferential) length of the collagen fiber bundle (CBF) varies along the collagen fibers of the ligament.

LOAD-DISPLACEMENT CURVE

The load-displacement curve characterizing the tensile of ligament in tension is non linear.



Short, unequal reductions of the load are manifested on the end of the linear region of the curve indicating the gradual rupture (fracture) of the ligament.

Linear region of the curve is displayed. The fibers of the collagen I are strengthened along the direction where the tensile load is applied .

(≈ 2-3% of the displacement)
This region of the curve is characterized by the losing of the rigidity of the crimp-patterned structure of the Collagen I.

Strengthening of the cross-links being created inside the context of collagen structure.

MECHANICAL RESPONSE OF CBF UNDER LOADING

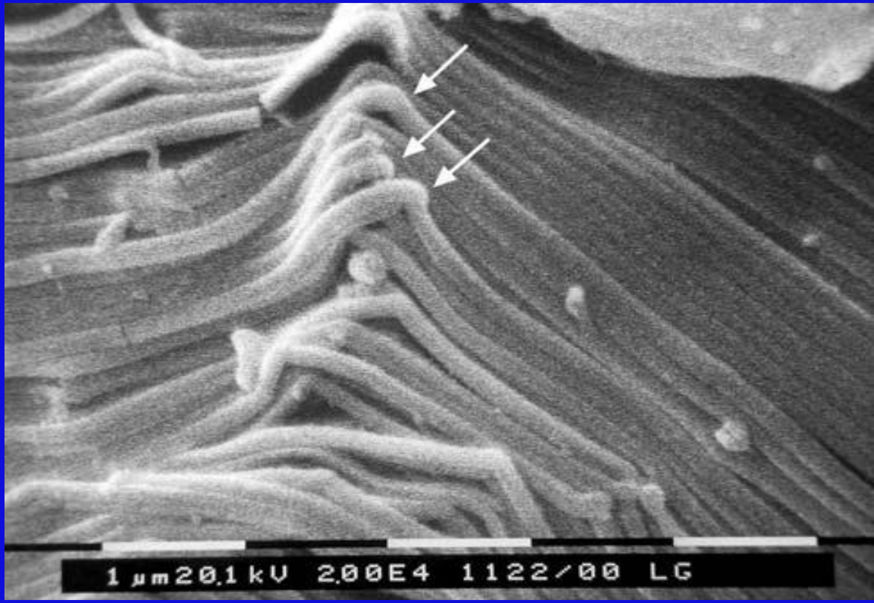
Under rest, CBF is characterized as folded and crimped.

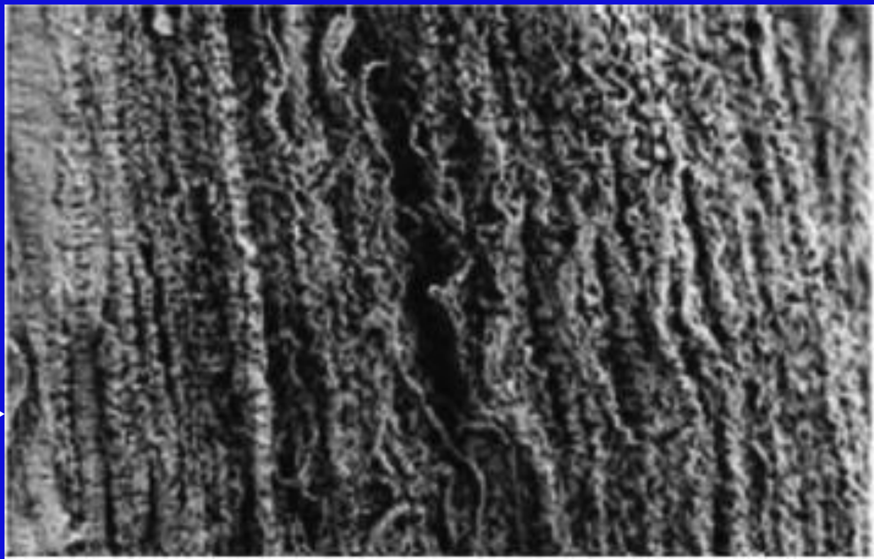
Assumption: CBF is considered being deformed on a relevantly easy way due to the imposed deformation leading to the unraveling of the CBF structure.

Under the implementation of the uniaxial loading the CBF structures are extended .

CBF of varied total(circumferential) length contribute in the mechanical response of the ligament under the imposing of various elongations. This situation occurs when each CBF is fully unraveled and elongated.

A rat MCL structure is displayed by SEM. (Collagen fibers constituted by parallel-structured collagen fibrils, form a planar crimped structure.)





Collagen fibril structure on the mature stage of their growing (a result displayed by using SEM). (The formation of the collagen fiber bundle indicates the crimped structure of a mature ligament)

RECRUITMENT OF CBF

Due to the externally caused deformation of the ligament, each CBF contributes due to its separate withstanding in elongation to the high stiffness value that characterized the ligament in under a certain extension.

The recruitment of CBF being characterized by different lengths in various extensions initiates to take place in the fibers of smaller length values proceeding to the longer ones.

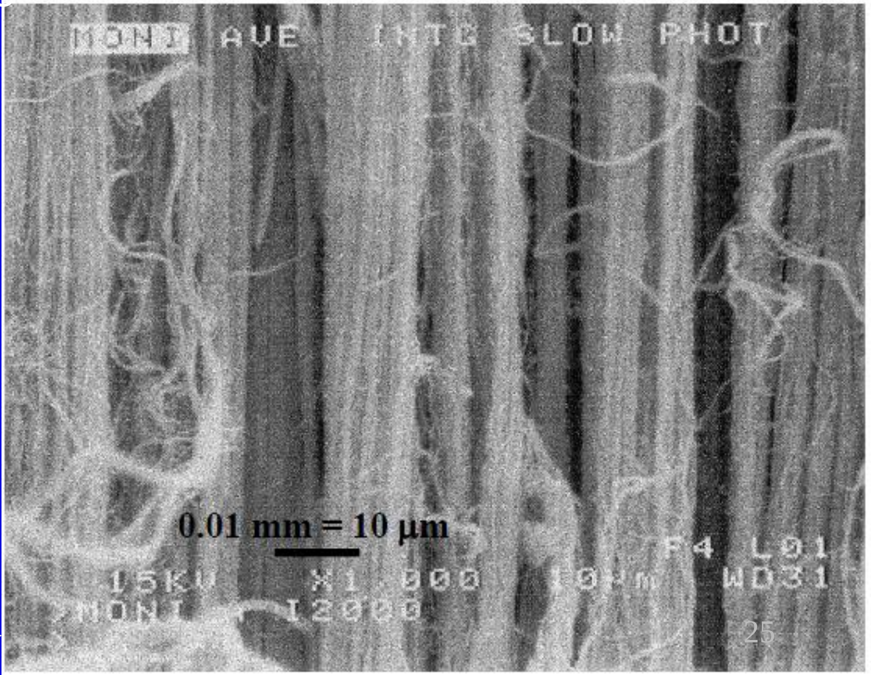
More and more CBF chains are recruited proportionally with the rising of the load value. Each CBF contributes due to its demonstrated mechanical response in the continual increasing of the modulus of Elasticity leading to the largest value of the modulus of Elasticity being caused (the mean deformation value approaches the value of the mean length of CBF).

$\mathbf{E_1^*}$ Modulus of Elasticity determining CBF behavior in full extension

The total modulus of Elasticity characterizing The ligament structure under large deformations (**ε>>1**), where the strain unit $\boldsymbol{\lambda = 1 + \varepsilon}$ is defined, is given by:

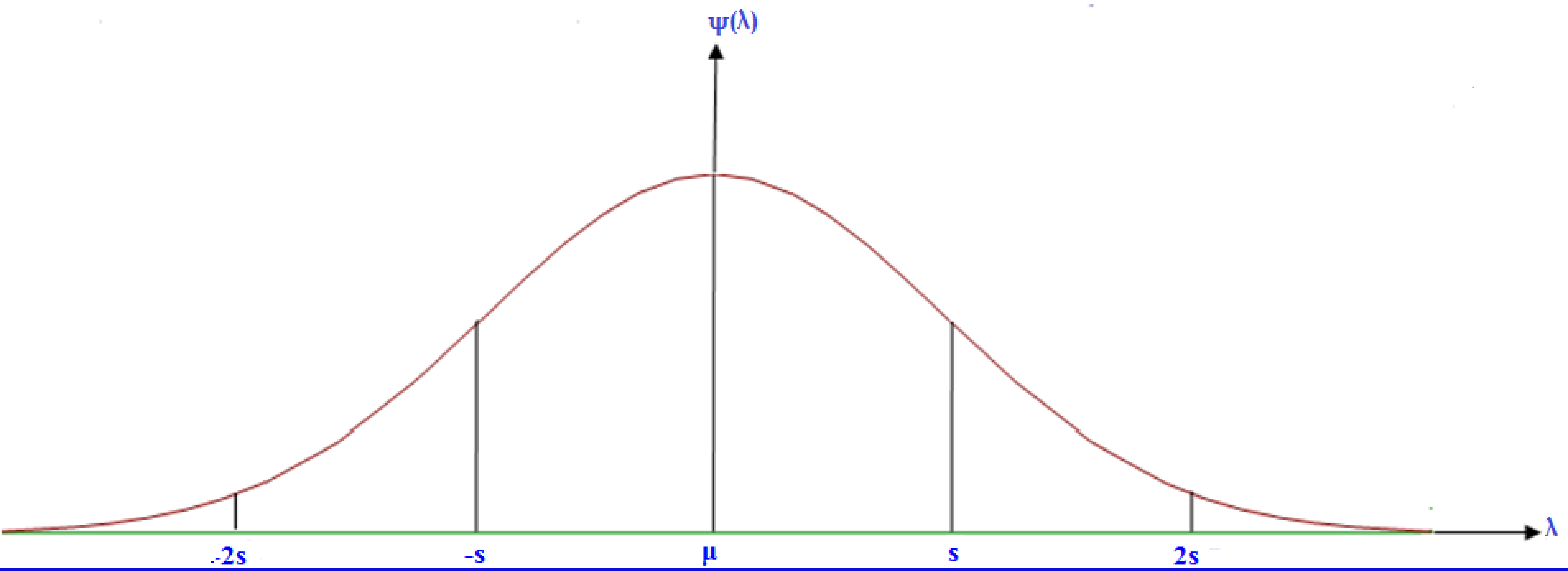
$$\mathbf{E_1 = E_1^* \Psi(\lambda)} \quad \textbf{(12)}$$

Ligament under tension →



The probability density function ψ(λ) corresponds to the Gaussian distribution of λ (characterized by mean μ and variance s=1) is formulated as:

$$\psi(\lambda) = \frac{1}{s\sqrt{2\pi}} \exp\left(-\frac{(\lambda-\mu)^2}{2s^2}\right) \quad (13)$$



The probability distribution function Ψ(λ) corresponding to the probability density function (Gaussian function) characterized by mean μ and variance s is expressed as:

$$\boldsymbol{\Psi(\lambda) = \frac{1}{s\sqrt{2\pi}}\int_{\lambda_i}^{\lambda_{i+1}} \exp\left(-\frac{(\lambda-\mu)^2}{2s^2}\right) d\lambda} \quad \textbf{(14)}$$

i: the number of the collagen fibers constituting CBF

It is considered that the total length of CBF follows the probability distribution function $\boldsymbol{\Psi(\lambda)}$

characterized by

mean $\boldsymbol{\mu}$

variance $\boldsymbol{s}$

Eventually, SLM equation

$$\sigma = (E_1\varepsilon + \tau(E_1 + E_2)R)\left(1 - \exp\left(-\frac{t}{\tau}\right)\right) \quad (15)$$

Because $E_1 = E_1^*\Psi(\lambda)$ (12)

and $$E_1^*\Psi(\lambda) = \frac{E_1^*}{s\sqrt{2\pi}}\int_{\lambda_i}^{\lambda_{i+1}} \exp\left(-\frac{(\lambda-\mu)^2}{2s^2}\right)d\lambda \quad (16)$$

The proposed viscoelastic model is eventually formulated as:

$$\sigma = \left(\varepsilon\frac{E_1^*}{s\sqrt{2\pi}}\int_{\lambda_i}^{\lambda_{i+1}} \exp -\frac{(\lambda-\mu)^2}{2s^2}d\lambda + \tau R\left(\frac{E_1^*}{s\sqrt{2\pi}}\int_{\lambda_i}^{\lambda_{i+1}} \exp -\frac{(\lambda-\mu)^2}{2s^2}d\lambda + E_2\right)\right)\left(1 - \exp\frac{-t}{\tau}\right) \quad (17)$$

MEANING OF PROPOSED (VISCOELASTIC) MODEL CONSTANTS

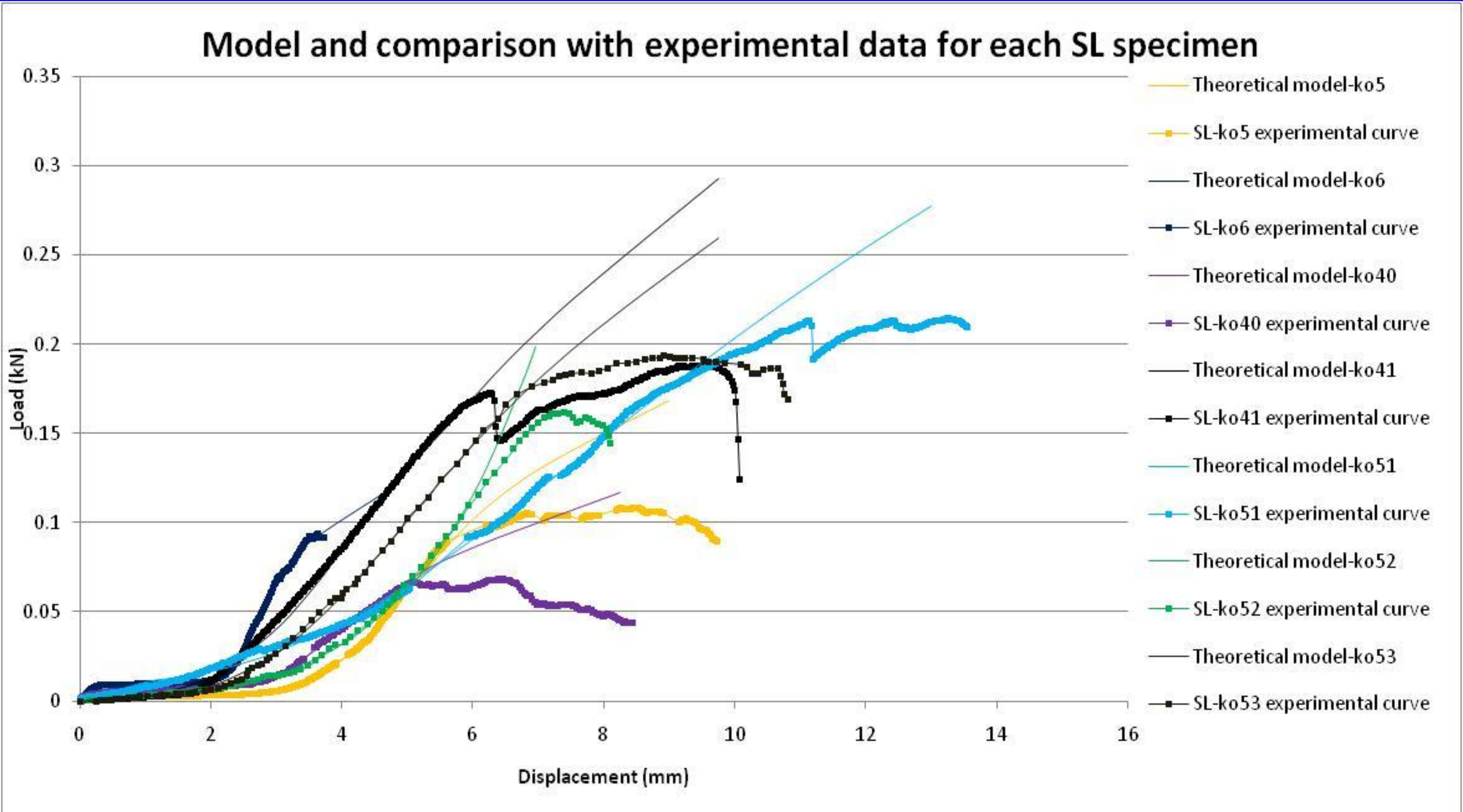
Assessment of the meaning of the viscoelastic model constants		
Constant	**Characterization of the constant**	**Meaning of the constant**
E_1^*	Mechanical	Stiffness due to the recruitment of fibers along the tensile direction of CBF
E_2	Mechanical	Stiffness due to the configuration of the CBF structure caused by the strengthening of the crosslinks between the fibers
η	Mechanical	Viscosity of the ligament characterizing viscoelastic conditions presented in the structure of ligament
μ	Structural	Mean value of the Normal distribution function $\Psi(\lambda)$ characterizing the total mean length* of CBF
s	Structural	Variance of the Normal distribution function $\Psi(\lambda)$ characterizing the total mean length* of CBF

Proposed (viscoelastic) model constants

- **Mechanical properties (characterizing the mechanical behavior of the ligament)**
- **Structural properties (characterizing the ligament structure)**

*Contour length of Collagen Fiber Bundle (CBF)

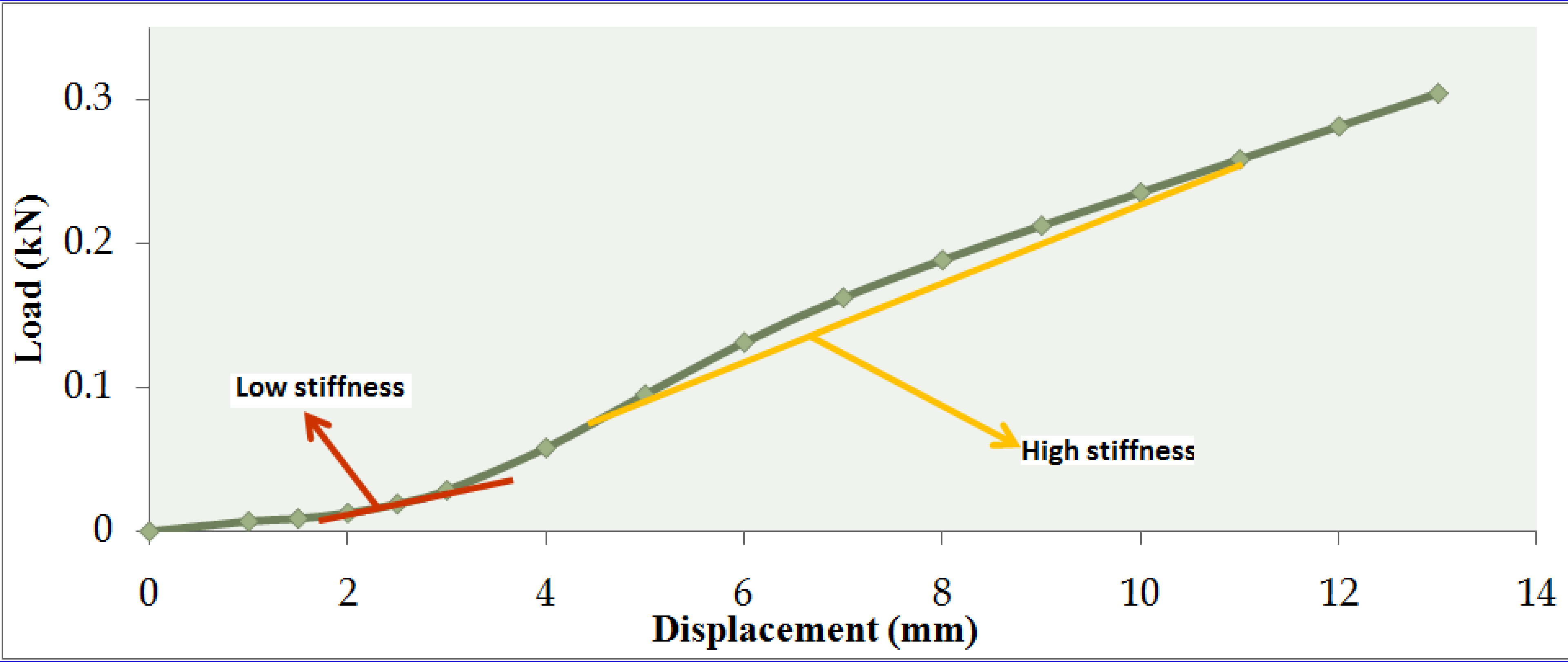
SIMULATION OF THE LOAD-DISPLACEMENT CURVES BY USING A THEORETICAL MODEL



STATISTICAL ANALYSIS OF THE MODEL CONSTANTS

Model constants for S-L					
Code No	Constants of the model				
	E_1^* (MPa)	E_2 (MPa)	η (sec^{-1})	Mean μ	Variance s
ko5	0.02	0.79	0.028	5.00	1.10
ko6	0.02	0.792	0.091	2.84	0.40
ko40	0.01	0.792	0.036	4.00	0.99
ko41	0.03	0.792	0.010	3.60	1.65
ko51	0.03	0.792	0.055	4.29	3.95
ko52	0.03	0.792	0.010	4.80	2.50
ko53	0.03	0.792	0.013	4.10	1.80
Mean value	**0.02**	**0.79**	**0.035**	**4.09**	**1.77**
Standard deviation (S.D.)	**0.01**	0.00	**0.030**	**0.73**	**1.17**

MEAN MECHANICAL BEHAVIOR OF S-L SPECIMENS



MECHANICAL PROPERTIES OF S-L LIGAMENT

- The **Maximum load** characterizes the ability of ligament to resist fracture. This property is characteristic of the strength of the ligament.

- **Stiffness**

  The slope of the load-displacement curves varies from point to point of the curve. The **stiffness of the high linear part of the load-displacement curve** (**High stiffness** that corresponds approximately to 85% maximum load) constitutes the main parameters used for characterizing the different sorts of ligaments.

- **Maximum displacement**: the maximum value of the displacement of S-L ligament before it is subjected to rupture.

CONCLUSION

- The simulation of the S-L mechanical tests by using the proposed model of this study could be considered as utile for the assessment of the viscoelastic and structural parameters of the ligament. It is obtained through the results of the study that the proposed model fits quite efficiently the S-L mechanical behavior in large deformations and it could be considered as a utile tool due to its dealing with predicting the mechanical response of S-L being subjected to various displacement rates.

- The results of the presented study could be evaluated as a versatile tool for the prediction of S-L biomechanical properties aiming to the production of autologous grafts in the future. On this way, clinicians could be helped to select proper biomaterials for the restoration of ruptured S-L ligaments.

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Thank you