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$H_{\infty}$ loop shaping for the torque-vectoring control of electric vehicles: Theoretical design and experimental assessment

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**A B S T R A C T**

This paper presents an $H_{\infty}$ torque-vectoring control formulation for a fully electric vehicle with four individually controlled electric motor drives. The design of the controller based on loop shaping and a state observer configuration is discussed, considering the effect of actuation dynamics. A gain scheduling of the controller parameters as a function of vehicle speed is implemented. The increased robustness of the $H_{\infty}$ controller with respect to a Proportional Integral controller is analyzed, including simulations with different tire parameters and vehicle inertial properties. Experimental results on a four-wheel-driven electric vehicle demonstrator with on-board electric drivetrains show that this control formulation does not need a feedforward contribution for providing the required cornering response in steady-state and transient conditions.

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1. Introduction

A significant body of research is investigating electric vehicles with multiple motors, either with in-wheel or on-board installations. These vehicle configurations provide opportunities for torque-vectoring (TV) control, which consists of the variable front-to-rear and left-to-right wheel torque distributions in order to achieve enhanced vehicle response in steady-state and transient conditions [1].

Although an extensive literature on TV control and its potential impact on vehicle response exists, a conventionally accepted methodology for setting the objectives for such an application has not yet been established [2]. To address this knowledge gap, [3] proposes the definition of a set of achievable reference understeer characteristics (i.e., the graph of steering wheel angle as a function of lateral acceleration) at different longitudinal accelerations.

This systematic design approach of vehicle cornering response is adopted in [4] for defining different driving modes, each of them characterized by a set of understeer characteristics. Hence, the TV controller is used to continuously shape the understeer characteristic in common driving conditions. Moreover, the continuously actuated TV controller allows to significantly increase vehicle yaw damping during transients and, thus, enhances active safety.

Several control system formulations have been presented for the TV control of electric vehicles with multiple motors. For example, a non-linear feedforward yaw moment contribution is used for shaping the understeer characteristics in quasi-static conditions, and a feedback contribution, based on a PID, is used for providing the required tracking performance in transient conditions [3–5]. [6–9] discuss linear quadratic regulators, linear quadratic Gaussian controllers and optimal controllers. Their main limitation is the lack of robustness towards the unmodeled dynamics, which is a very significant issue for the specific application, characterized by the variation of the axle cornering stiffness as a function of slip angle, and the variation of vehicle yaw damping as a function of vehicle speed [10]. To enhance the performance of a linear quadratic regulator, [6] presents (without analyzing its stability) a gain scheduling formulation based on the variation of tire cornering stiffness as a function of the estimated slip angles. [11,12] discuss explicit model predictive control formulations [13], which have the advantage of good and robust tracking performance and low computational requirements, but the complexity of the procedure for the derivation of the controller could discourage their actual industrial implementation. At the moment the more conventional option of implicit model predictive control [14] is still characterized by an excessive computational demand for the current capability of automotive control units. [15–17] propose different sliding mode formulations, providing robustness with ease of tuning and simple control laws. Some of them are demonstrated through experiments (e.g., those in [16] and [17]) with very good results.

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## List of symbols

**Subscripts**
- `dem` demanded (reference) value
- `s` shaped plant
- `RF`, `LF`, `RR`, `LR` right front, left front, right rear and left rear drivetrains
- `AW` anti-windup configuration of the controller

**Main symbols**
- $\hat{\theta}$ estimated variable
- $a$ front semi-wheelbase
- $a_x$ longitudinal acceleration
- $a_{\ell}$ lateral acceleration
- $A$, $B$, $C$, $D$ matrices of a state-space formulation
- $APP$, $BPP$ accelerator pedal position and brake pedal position
- $b$ rear semi-wheelbase
- $C_A$ camber angle
- $C_F$ front axle cornering stiffness
- $C_R$ rear axle cornering stiffness
- $D_a$ approximated damping ratio of the drivetrain transfer function
- $F$ matrix of the observer form implementation of the feedback $H_\infty$ loop shaping controller
- $F_2(\cdot)$ lateral tire force
- $F_3$ vertical tire force
- $G_a(\cdot)$ transfer function of the simplified actuator model (with steady-state gain equal to 1)
- $G_m(\cdot)$ transfer function from yaw moment to yaw rate
- $G_p(\cdot)$ transfer function of the unshaped plant
- $G_s(\cdot)$ transfer function of the shaped plant
- $G_{sl}(\cdot)$ transfer function from steering angle to yaw rate
- $h$ indicator of the discrete intervals of vehicle speed adopted for the gain scheduling scheme
- $H$ matrix of the observer form implementation of the feedback $H_\infty$ loop shaping controller
- $i_{11}$, $i_{22}$ transmission gear ratios
- $I$ identity matrix
- $l$ integral of the absolute value of the control action
- $J_m$ mass moment of inertia of the rotating parts of the electric motor
- $J_{m1}$, $J_{m2}$, $J_{m3}$ mass moments of inertia of the transmission shafts
- $J_w$ mass moment of inertia of the wheel
- $k_{hs}$, $c_{hs}$, $\dot{J}_h$ half-shaft torsional stiffness, torsional damping coefficient and moment of inertia
- $k_1$ tuning parameter of the integrator reset condition
- $K$ generic feedback controller
- $K_{m,s}$/m, `dem` steady-state gain between electric motor torque demand and vehicle longitudinal acceleration
- $K_p$ proportional gain
- $K_i$ $H_\infty$ optimal controller
- $m$ vehicle mass
- $M_e$ yaw moment
- $M_y^{FB}$ feedback contribution of the yaw moment
- $M_y^{FF}$ feedforward contribution of the yaw moment
- $M_y^{TOT}$ total yaw moment
- $N_\delta$ stability derivatives of the yaw moment balance equation of the single-track vehicle model
- $P_o$ friction brake pressure
- $r$ yaw rate
- $r_{ref}$ reference yaw rate
- $R_{sh}$ radius of the skid pad test
- $RMSE$ root mean square value of the yaw rate error
- $s$ Laplace operator
- $SR$ slip ratio
- $t$ time
- $t_{man, \in}$ initial time of the relevant part of the maneuver
- $t_{man, fin}$ final time of the relevant part of the maneuver
- $Thr$ threshold for the integrator reset condition
- $T_i$ time constant of the integral term
- $T_m$ motor torque
- $T_{TOT}$ total wheel torque demand
- $u$ generic plant input
- $u_a$ actual plant input
- $v$ vehicle speed
- $W_f$ pre-filter
- $W_i$ constant gain
- $W_{PI}$ pre-compensator in the form of PI filter
- $x_0$ state of the shaped plant
- $X$ solution of the Riccati equation for $H_\infty$ controller design
- $y$ plant output
- $Y_\delta$ stability derivatives of the lateral force equation of the single-track vehicle model
- $Z$ solution of the Riccati equation for $H_\infty$ controller design
- $\alpha$ tire slip angle
- $\beta$ vehicle sideslip angle
- $\gamma$ parameter of the solution of the algebraic Riccati equation
- $\delta$ steering angle (of the wheel)
- $\delta_{SW}$ steering-wheel angle
- $\Delta \alpha$ increment of slip angle for the computation of the cornering stiffness during vehicle simulations
- $e_{max}$ maximum robust stability margin
- $\eta_{cv1}$, $\eta_{cv2}$ efficiencies of the inner and outer constant velocity joints
- $\eta_{11}$, $\eta_{12}$ efficiencies of the first and second transmission stages
- $\mu$ tire-road friction coefficient
- $\omega_{na}$ approximated natural frequency of the actuation system (drivetrain dynamics and tire relaxation)

However, according to the practical experience of some of the authors of this paper, sliding mode controllers can easily give origin to an excessively 'nervous' vehicle behavior when actually implemented on a vehicle. Various $H_\infty$ approaches, e.g., based on mixed sensitivity [18], are presented in [19–23], all of them evaluated through vehicle dynamics simulations. [20] points out the requirements of experimental vehicle tests. The main limitation of the proposed $H_\infty$ formulations is the complexity of the control synthesis procedure, often based on iterations, which restricts their tuning to control system specialists.

The novel contributions of this paper are

(a) The analysis of the required level of TV control system robustness based on the variation of the front and rear axle cornering stiffnesses in realistic operating conditions;

(b) A TV controller formulation based on $H_\infty$ loop shaping, a well-established robust control approach (its theory is discussed in [18] and [24]), to address the robustness issue related to the variation of axle cornering stiffness and vehicle parameters. To the authors’ knowledge this control approach...
has never been applied so far to the specific TV problem (but widely used in several fields with good results, e.g., in aerospace engineering, [25]). The presented controller also includes consideration of the experimentally measured actuation dynamics, which is an important insight into the features of the specific vehicle system;
(c) The simulation-based and experimental assessment of the controller on a four-wheel-drive electric vehicle with on-board drivetrains, including comparisons with other controller formulations.

The main advantages of the $H_\infty$ loop shaping approach are
(i) Absence of complex modeling requirements during the control system design;
(ii) Simple non-iterative design procedure based on Riccati equations and well-established software tools, while other $H_\infty$ control formulations, such as mixed sensitivity and $\mu$-synthesis, require complex iterative procedures;
(iii) $H_\infty$ robustness added to a conventional PI (Proportional Integral) compensator, which fits well with the current industrial practice of using PID controllers for vehicle yaw moment control;
(iv) Significantly closer to conventional control system design procedures than explicit model predictive control;
(v) Significantly lower computational load with respect to implicit model predictive control;
(vi) Robustness and quantifiable stability margins, while sliding mode formulations provide robustness and stability, which, however, are not quantifiable;
(vii) Design procedure in the frequency domain, including the visualization of the loop;
(viii) No risk of chattering, which is limited for advanced implementations of sliding mode, but which can occur in practice, especially in case of actuation delays or signal discretization;
(ix) Standard formulations are available for gain scheduling and anti-wind-up.

The paper is organized as follows. Section 2 describes the models for the simulation of the drivetrain and vehicle dynamics during the control system design and the assessment phase of the $H_\infty$ controller. Section 3 discusses the $H_\infty$ controller and the assessment of its robust stability. Section 4 presents a simulation-based analysis of the control system performance and its comparison with a more conventional TV controller based on the combination of feedforward and PID contributions. Section 5 presents the experimental results achieved on the four-wheel-drive electric vehicle demonstrator of the European Union FP7 project E-VECTOORC [26].

2. Electric vehicle model

2.1. Drivetrain dynamics and models

The case study vehicle is characterized by an on-board layout of the four electric drivetrains, each of them consisting of a switched reluctance electric motor drive, a single-speed transmission, constant-velocity joints and a half-shaft connecting the drivetrain to the respective wheel (Fig. 1). Owing to the substantial torsional compliance introduced by the half-shafts, located between the equivalent inertia of the drivetrain (consisting of the motor and the single-speed transmission) and the wheel inertia, the on-board drivetrain layout possesses non-negligible torional dynamics. As a consequence, this drivetrain configuration is the worst-case from a system control viewpoint, i.e., if a controller is functional for this drivetrain set-up, even better performance can be expected for in-wheel drivetrain layouts. Fig. 2 is an example of experimental frequency response characterization of the on-board electric drivetrains of the electric vehicle demonstrator, along three sweep tests of the electric motor drive demand. Sinusoidal torque demands with constant amplitude and increasing frequency (left graph of Fig. 2) were applied to the front electric motor drives and the resulting longitudinal acceleration ($a_x$) profiles of the vehicle were recorded with a very evident resonance peak for all tests. As discussed in [4] and [27,28], specific controllers can be implemented to shape the dynamic performance of the on-board electric drivetrains and reduce the resonance peak. In this paper, the $H_\infty$ controller is designed and assessed taking into account the actual passive behavior of the electric drivetrains (without any additional drivability controller) to demonstrate the effectiveness and robustness of the control system implementation.

The equations (i.e., the motor and transmission balance equation and the wheel balance equation) for modeling the dynamics of the electric drivetrains, which are implemented in the nonlinear model for the evaluation of the control system performance, are reported in [17]. During the $H_\infty$ controller design, the actuation dynamics are included in the form of a transfer function, $G_{\alpha_1}(s) = M_2/M_{\alpha_{dem}}$, summarizing the dynamics of the electric drivetrain and tire. In fact, the electric drivetrain is responsible for generating the wheel torque and, thus, the longitudinal tire force (with the additional dynamics relating to the relaxation length [29]) and the vehicle yaw moment. In practice, the dynamics associated with tire relaxation are much less influential than those related to the electric drivetrain. This simplifies the control system design procedure as tire relaxation dynamics are significantly dependent on the longitudinal slip condition of the tire [30], with tire relaxation length changing by an order of magnitude as a function of the slip ratio. The relaxation length variation makes very difficult to reliably simulate tire dynamics within a linear control system design. Based on the experimental frequency response characteristics $\frac{\alpha_1}{\alpha_{\text{dem}}}(s) = K_{\alpha_1}/\alpha_{\text{dem}}$, $G_{\alpha_1}(s)$, $\alpha_{\text{dem}}(s)$, has been defined as:

$$G_{\alpha_1}(s) = \frac{\omega_1^2}{s^2 + 2\nu_1\omega_1 \cdot s + \omega_1^2}$$

(1)

The natural frequency and damping ratio are selected according to the experimental results on the vehicle demonstrator, such as those of Fig. 2.

2.2. Vehicle dynamics model for controller assessment

During the virtual testing phase of the controller the vehicle chassis dynamics are modeled with the simulation package IPG CarMaker, which considers the six degrees of freedom of the unsprung mass, suspension elasto-kinematics, the degree of freedom associated with the suspension motion of each unsprung mass, and the rotational dynamics of the wheels.

The non-linear drivetrain model outlined in the previous subsection is implemented in Matlab-Simulink and linked to the CarMaker model. Tire behavior is modeled with the Magic Formula [29] and a variable relaxation model. The vehicle simulator includes consideration of signal discretization (as on the actual vehicle implementation) and the time-variant delays associated with
the vehicle communication buses (Controller Area Network, CAN, [17] and [22]).

To ensure accurate simulation results, the CarMaker-Simulink vehicle model was experimentally validated. Fig. 3 shows two examples of validation, in particular the simulated time histories of vehicle yaw rate, \( r \), against experimental results during ramp steer and step steer tests carried out with the electric vehicle (passive configuration, i.e., without any controller) at the Lommel proving ground (Belgium).

2.3. Single-track vehicle model for control system design

During the control system design, a single-track vehicle model [10] was adopted:

\[
\begin{align*}
    r(s) &= G_M(s)M(z) + G_d(s)\delta(s) \\
    G_M(s) &= \frac{mv \cdot s - Y_\beta}{f_1mv \cdot s^2 - (f_2Y_\beta + N_1mv) \cdot s - N_2Y_\beta + N_3mv + N_4Y_\beta} \quad (3) \\
    G_d(s) &= \frac{N_1mv \cdot s + N_2Y_\beta - N_3Y_\beta}{f_1mv \cdot s^2 - (f_2Y_\beta + N_1mv) \cdot s - N_2Y_\beta + N_3mv + N_4Y_\beta} \quad (4)
\end{align*}
\]

The stability derivatives, \( N_1 \) and \( Y_\beta \), are functions of the front and rear axle cornering stiffnesses, \( C_F \) and \( C_R \):

\[
\begin{align*}
    Y_\beta &= C_F + C_R, \quad Y_\beta = \frac{aC_F - bC_R}{v}, \quad Y_\beta = -C_F \\
    N_1 &= a^2C_F - b^2C_R, \quad N_1 = \frac{a^2C_F - b^2C_R}{v}, \quad N_1 = aC_F \\
    N_2 &= aC_F - bC_R, \quad N_2 = \frac{a^2C_F - b^2C_R}{v} \\
    N_3 &= N_3, \quad N_3 = N_3 \\
    N_4 &= N_4, \quad N_4 = N_4
\end{align*}
\]

The actual values of \( C_F \) and \( C_R \) to be considered for the \( H_{in} \) control system design are obtained during maneuvers executed with the CarMaker model of the vehicle without TV controller, by using the definition of cornering stiffness as incremental ratio of lateral force with respect to slip angle:

\[
\begin{align*}
    C_F &= f_{2RF}(\alpha_{SR}, SR_{RF}, CA_{RF}, F_{RF}) + f_{3RF}(\alpha_{SR}, \Delta \alpha, SR_{RF}, CA_{RF}, F_{RF}) \\
        &= \frac{f_{2RF}(\alpha_{SR}, SR_{RF}, CA_{RF}, F_{RF}) + f_{3RF}(\alpha_{SR}, \Delta \alpha, SR_{RF}, CA_{RF}, F_{RF})}{\Delta \alpha} \quad (7) \\
    C_R &= f_{2RR}(\alpha_{SR}, SR_{RR}, CA_{RR}, F_{RR}) + f_{3RR}(\alpha_{SR}, \Delta \alpha, SR_{RR}, CA_{RR}, F_{RR}) \\
        &= \frac{f_{2RR}(\alpha_{SR}, SR_{RR}, CA_{RR}, F_{RR}) + f_{3RR}(\alpha_{SR}, \Delta \alpha, SR_{RR}, CA_{RR}, F_{RR})}{\Delta \alpha} \quad (8)
\end{align*}
\]

For each tire, the lateral forces are calculated by inputting the actual values of the incremented slip angle (the increment being \( \Delta \alpha \)), slip angle, slip ratio, camber angle and vertical load to the Magic Formula. Fig. 4 plots \( C_F(C_R) \) during a ramp steer test (quasi-static trajectory of the cornering stiffness) and a step steer test (transient trajectory of the cornering stiffness). Points 1–5 highlight the variety of possible operating conditions of the passive vehicle, thus demonstrating the requirement of a controller capable of providing robust stability. The lateral acceleration \( (\alpha_y) \) values corresponding to Points 1–5 are reported in Table 1, together with the damping ratio and natural frequency of the transfer function describing the input–output dynamics of the system.
Table 1
Properties of the transfer function $G_M(s)$ for the different $C_F$ and $C_K$ of Fig. 4 (at $v = 90 \text{ km/h}$).

<table>
<thead>
<tr>
<th>$C_F$, $C_K$ [N/deg]</th>
<th>Damping ratio</th>
<th>Natural frequency [Hz]</th>
<th>$\alpha_v$ [m/s²]</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>$C_F = -2.94 \times 10^3$, $C_K = -3.05 \times 10^3$</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>Point 2</td>
<td>$C_F = -2.00 \times 10^3$, $C_K = -2.37 \times 10^3$</td>
<td>0.92</td>
<td>0.93</td>
<td>6</td>
</tr>
<tr>
<td>Point 3</td>
<td>$C_F = -0.08 \times 10^3$, $C_K = -1.14 \times 10^3$</td>
<td>0.27</td>
<td>0.85</td>
<td>8.7</td>
</tr>
<tr>
<td>Point 4</td>
<td>$C_F = 0.09 \times 10^3$, $C_K = -1.01 \times 10^3$</td>
<td>0.2</td>
<td>0.86</td>
<td>7.8</td>
</tr>
<tr>
<td>Point 5</td>
<td>$C_F = 0.12 \times 10^3$, $C_K = -0.15 \times 10^3$</td>
<td>-</td>
<td>-</td>
<td>9.4</td>
</tr>
</tbody>
</table>

Fig. 4. $C_K$ as function of $C_F$ during a ramp steer (continuous line) and a step steer (dashed line).

Table 2
Damping ratio and natural frequency of transfer function $G_M(s)$ at different vehicle speeds ($C_F$ and $C_K$ are the values of Point 3).

<table>
<thead>
<tr>
<th>$v$ [km/h]</th>
<th>Damping ratio</th>
<th>Natural frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.76</td>
<td>0.91</td>
</tr>
<tr>
<td>60</td>
<td>0.40</td>
<td>0.86</td>
</tr>
<tr>
<td>90</td>
<td>0.27</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Fig. 5. Bode diagram of $G_M(s)$ at different $C_F$ and $C_K$.

i.e., $G_M(s) = \frac{1}{C_F(s)}$. The understeering behavior of the vehicle is evident, since $|\alpha_C| < |\beta_M|$ in quasi-static conditions. The adopted method for identifying vehicle understeer allows evaluating the variation of vehicle cornering behavior caused by the longitudinal tire forces and load transfers, as the slip ratios and vertical loads from the CarMaker model are input into the non-linear tire models of Eqs. (7) and (8). For example, positive longitudinal acceleration brings reduced $|C_F|$ and increased $|\beta_K|$, with the subsequent increase of vehicle understeer. In transient conditions, the actual trajectory $C_K(C_F)$ significantly deviates from that of steady-state conditions, which represents a limitation of the published gain scheduling schemes based on the steady-state $C_K(C_F)$ trajectory [6]. The Bode diagrams of $G_M(s)$ for the operating conditions of Points 1-5 are reported in Fig. 5, whilst Fig. 6 reports $G_M(s)$ for different vehicles speeds (with the typical decrease of yaw damping as a function of $v$), with the respective damping ratio and natural frequency values in Table 2.

Because of the fast variation of $C_F$ and $C_K$ during extreme maneuvers such as the step steer (which could compromise the effectiveness of a gain scheduling scheme applied to the $H_\infty$ controller), it was decided to limit the gain scheduling of the controller to $v$, which is a relatively slowly varying parameter. The controller design is focused on the parameters corresponding to Point 3 in Fig. 4, i.e., for an extreme steady-state cornering condition of the vehicle, with high $\alpha_v$ with respect to the friction conditions. The $H_\infty$ controller has to provide robust stability for the whole set of cornering stiffness corresponding to the real vehicle operation. It is expected that with the $H_\infty$ controller the transient $C_K(C_F)$ trajectory will be much less oscillatory than in Fig. 4 (and closer to the steady-state trajectory, e.g., without reaching the condition of Point 5), because of the beneficial action of the TV controller. The other vehicle parameters, such as $m$ and $J_z$, are selected at their nominal value, corresponding to the vehicle with a couple of passengers. Friction coefficient is not a parameter in the single-track vehicle model adopted for control system design. Moreover, the values of $C_F$ and $C_K$ in extreme cornering conditions are hardly influenced by the road friction value [10], which makes the controller robust against friction coefficient variations.
3. TV controller

3.1. Torque-vectoring control structure

Fig. 7 shows the schematic of the TV control structure, consisting of:

(i) A yaw rate reference generator, providing $r_{\text{ref}} = r_{\text{ref}}(\delta_{\text{SW}}, v, \text{APP}, \text{BPP}, \mu)$ from a multi-dimensional look-up table, created for a set of reference understeer characteristics using the procedure in [3–5];

(ii) The high-level controller, generating a reference yaw moment $M_{\text{TOT}}$, resulting from a feedback contribution (e.g., the $H_\infty$ controller discussed in this paper) $M_{\text{FB}}$ and (optionally) a non-linear feedforward contribution $M_{\text{FF}}$ designed according to [3–5];

(iii) The ‘Wheel torque distributor’, calculating the individual electric motor torque demands $T_{m, \text{dem}}$ and friction brake pressure demands $P_{b, \text{dem}}$. As the focus of the paper is on the performance of the high-level controller (advanced wheel torque allocation algorithms are discussed in [31]), the adopted wheel torque distributor equally splits the total wheel torque and yaw moment demands, respectively $T_{w, \text{dem}}$ and $M_{\text{dem}}$ between the front and rear axles.

The modular structure of the control system allows easy scalability for different drivetrain architectures, and is consistent with the state-of-the-art industrial design philosophy of vehicle control systems.

3.2. Feedback controller design using $H_\infty$ loop shaping approach

The formulation of the $H_\infty$ loop shaping robust stabilization problem is provided in [24]. The structure of the $H_\infty$ feedback controller is shown in Fig. 8. The plant $G_p(s)$ adopted for the control system design includes the yaw dynamics from the single-track vehicle model $G_M(s)$ linearized for Point 3 of Fig. 4 (design point of the specific controller) and the actuation dynamics $G_A(s)$ discussed in Section 2.1 (see Eq. (9)). Very interestingly, the simulations and experiments on the vehicle demonstrator showed that taking into account $G_M(s)$ in the controller design is essential for the correct operation of the TV controller with an on-board electric drivetrain layout. This gives origin to a relatively high order controller (order 5). However, without consideration of the actuation dynamics in the control system design, the control action would have to be designed in a very conservative way to avoid the excitation of drivetrain oscillations.

$$G_p(s) = G_M(s)G_A(s)$$

(9)

The $H_\infty$ controller consists of

(i) A pre-filter $W_f(s)$ for smoothing $r_{\text{ref}}$ and reducing the yaw rate overshoot in extreme maneuvers. $W_f(s)$ is implemented as a first order low-pass filter. The cut-off frequency of $W_f(s)$ is used for fine tuning the controller depending on the selected driving mode, i.e., a larger corner frequency of the pre-filter is adopted in the sport-oriented driving modes, in order to make the vehicle more responsive with respect to driver’s inputs;

(ii) A pre-compensator $W_p(s)$, in this case a PI controller, selected to produce a good tracking performance. For example, at 90 km/h, the PI is designed (according to the previous experimental experience of the authors) to achieve a good compromise between tracking performance and noise suppression (and thus smooth time history of the reference yaw moment):

$$W_p(s) = K_p\left(1 + \frac{1}{T_1s}\right), \quad K_p = 436 \text{ Nm/deg}, T_1 = 0.1 \text{ s}$$

(10)

(iii) A constant gain $W_i = K_i(0)W_p(0)$ to ensure a steady-state gain of 1 between $r_{\text{ref}}$ and $y_s$;

(iv) The actual $H_\infty$ compensator $K_c$ derived from the solution of the two algebraic Riccati equations reported in [24]. The order of the resulting compensator depends on the order of the system; in this case, by combining the second order dynamics of the single-track model and the second order dynamics of the actuator, i.e., the electric drivetrains, a fourth order system and a fifth order controller are obtained. For instance, at 90 km/h:

$$K_c(s) = \frac{-4.8 \times 10^5}{s^5 + (4.3 \times 10^5)s^4 + (2.1 \times 10^5)s^3 + (1.3 \times 10^5)s^2 + (5.1 \times 10^4)s + (5.0 \times 10^3)}$$

(11)

The resulting controller has higher order than the PI controller it derives from. The $H_\infty$ compensator provides robustness against
The gain scheduling controller is implemented in the observer/state feedback form [18]:

\[
\frac{d\hat{x}}{dt} = A_s \hat{x} + H_s (C_s \hat{x} - y_s) + B_s u_s
\]

where \( \hat{x} \) is the observer state. \( u_s \) and \( y_s \) are respectively the input and output of the shaped plant, and

\[
H_s = -Z_s^T C_s
\]

The shaped plant as a function of \( v \) is defined as:

\[
G_s(v) = W_{p0}(v)G_p(v) = \begin{bmatrix} A_v(v) & B_v(v) \\ C_v(v) & 0 \end{bmatrix}
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where the pre-compensator \( W_{p0}(v) \) is scheduled by using linear interpolation between the two pre-compensators at adjacent design points \( h \) and \( h+1 \). Table 4 reports the values of \( K_p \), the stability margin, and the cut-off frequency for the shaped plant, \( G_s(v) \), at the five selected design points.

In order to incorporate the gain scheduling scheme, the \( H_{\infty} \) loop shaping controller is implemented in the observer/state feedback form [18]:

\[
\frac{d\hat{x}_s}{dt} = A_s \hat{x}_s + H_s (C_s \hat{x}_s - y_s) + B_s u_s \]

where \( \hat{x}_s \) is the observer state. \( u_s \) and \( y_s \) are respectively the input and output of the shaped plant, and

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\]
The integral term of $W_p$ is characterized by a reset condition based on the linear combination of the reference yaw rate and its time derivative:

$$\left| \frac{dr_{ref}(t)}{dt} + k_1 |r_{ref}(t)| \right| > Thr$$

(17)

3.5. Anti-windup

In case of actuator saturation (e.g., high yaw moment demand in low friction conditions) the integrator in $W_p$ would continue to integrate the input and cause windup problems. Therefore, a self-conditioned anti-windup scheme [18] is employed to implement $W_p$, based on the state-space realization defined in (18) and (19). This prevents windup by keeping the states of $W_p$ consistent with the actual plant input at all times. When no saturation happens, $u = u_m$ and the dynamics of $W_p$ remain unaffected. When $u \neq u_m$, the dynamics of $W_p$ are inverted and driven by $u_m$ such that the states remain consistent with $u_m$. This scheme requires the pre-compensator to be invertible and minimum phase, which is satisfied by the chosen $W_{PI}$.

$$W_{PI} = \begin{bmatrix} A_{BW} & B_{BW} \\ C_{BW} & D_{BW} \end{bmatrix}$$

(18)

$$u = \begin{bmatrix} A_{BW} - B_{BW} D_{BW}^{-1} C_{BW} \\ \frac{B_{DW} D_{BW}^{-1}}{D_{BW}} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{C_{BW}}{D_{BW}} u_m \end{bmatrix}$$

(19)

4. Simulation analysis

The $H_\infty$ controller is evaluated along a set of simulations carried out with the validated CarMaker - Simulink vehicle model. The case study maneuver is a sequence of step steers at constant $v$, with positive and negative steering wheel angles exciting the vehicle well beyond its cornering limits for the given friction conditions. All the controlled vehicle simulations presented in this section are executed in the Normal mode of the TV controller (see also Section 5), which has a reference understeer characteristic for constant $v$ similar to that of the passive vehicle (i.e., the vehicle without any controller).

Firstly, a comparison between the passive vehicle, the vehicle with the $H_\infty$ controller without feedforward contribution, and the vehicle with more conventional controller formulations, i.e., a PI controller (with the same gains as for the PI compensator of the $H_\infty$ controller) and a PI + feedforward controller (with the non-linear feedforward contribution designed to achieve the reference understeer characteristic in quasi-static conditions, [3-5]) is performed. To assess the robustness of the $H_\infty$ formulation, the maneuvers are executed with: (a) two different values of the tire-road friction coefficient, 1.0 and 0.8. In order to make the test more demanding, the reference yaw rate is kept at the same level as for $\mu = 1$ for all simulations; (b) two different values of vehicle mass, 1725 kg and 2025 kg, with the subsequent variation of the other inertial parameters; and (c) two tire characteristics, called Tire A and Tire B, corresponding to different values of the Magic formula coefficients. Tire B is more sports-oriented, i.e. has a greater cornering stiffness.

The controller performance is assessed with the root mean square value of the yaw rate error (RMSE), and the integral of the absolute value of the control action ($IAC_M_{\text{dem}}$) calculated during the relevant part of the test:

$$\text{RMSE} = \sqrt{\frac{1}{t_{\text{max,inf}} - t_{\text{max,inf}}} \int_{t_{\text{max,inf}}}^{t_{\text{ref}}(t) - r(t)} dt}$$

(20)

$$IAC_M_{\text{dem}} = \frac{1}{t_{\text{max,inf}} - t_{\text{max,inf}}} \int_{t_{\text{max,inf}}}^{t_{\text{max,inf}}} |M_{\text{dem}}(t)| dt$$

(21)

Table 5 reports the numerical values of the performance indicators for the different cases. The general conclusion is that the $H_\infty$ controller achieves better tracking performance than the PI and PI + feedforward controllers, with a significantly lower actuation effort. For example, for Tire A at $\mu = 1$ and $m = 1725$ kg, the RMSE for the $H_\infty$ controller is 11% and 20% lower than for the PI and PI + feedforward controllers, respectively, while the $IAC_M_{\text{dem}}$ is 9% and 18% lower than for the other two controller configurations. The benefits are significantly more evident for the test at reduced tire-friction conditions and Tire A, for which the $H_\infty$ controller brings a reduction of the RMSE of 21% and 45% with respect to the PI and PI + feedforward controllers. Fig. 10 reports the time histories of yaw rate, reference yaw moment and sideslip angle during this specific test.

Secondly, the benefits of the gain scheduling of the controller as a function of $v$ are investigated. To this purpose, Table 6 reports the RMSE and $IAC_M_{\text{dem}}$ for the sequence of step steers executed at 60 km/h and 150 km/h, for the $H_\infty$ controller with fixed gains designed for 90 km/h, and the $H_\infty$ controller with gain scheduling. The variation of the performance indicators is not negligible. For example, at 150 km/h the gain scheduling brings a reduction of 12% and 3% of the RMSE and $IAC_M_{\text{dem}}$, respectively.
The third analysis aspect is related to the anti-windup scheme. Fig. 11 shows the time histories of yaw rate and yaw moment and indicates the benefits of the selected anti-windup scheme, allowing an increase of the yaw damping during the transient, for a yaw moment saturation value of 5000 Nm.

5. Experimental results

The $H_\infty$ TV controller with gain scheduling and anti-windup was implemented on a dSPACE AutoBox system and experimentally assessed on the four-wheel-drive fully electric vehicle demonstrator (Fig. 12) of the European Union FP7 project E-VECTOORC, along two maneuvers – skid pad and step steer – as described further below. The controlled car was configured with two driving modes, Normal and Sport. As mentioned in Section 4, the Normal driving mode is set up with an understeer characteristic (with limited linear region and progressively increasing understeer gradient, which at 6 m/s$^2$ is approximately doubled with respect to its value at 3 m/s$^2$) similar to that of the passive vehicle, but a marginally higher level of maximum $a_y$ in high friction conditions (from 7.6 m/s$^2$ for the passive vehicle to 8.1 m/s$^2$ for the Normal mode). The Sport mode is characterized by a much more aggressive cornering response, with a substantially linear behavior until the maximum lateral acceleration of about 9.2 m/s$^2$. The yaw
moment characteristics reflect the different responses of the two modes, with consistently higher (destabilizing) yaw moments for the vehicle in Sport mode.

The skid pad tests [33] were carried out with \( R_{GP} = 60 \text{ m} \), with a test driver correcting the steering wheel input in order to follow the circular trajectory while progressively increasing \( v \). Figs. 13 and 14 show examples of understeer and yaw moment characteristics for the passive vehicle, and the TV-controlled vehicle in Normal and Sport modes. The subjective assessment of the test drivers was that the good tracking performance of the reference understeer characteristics, corresponding to values of RMSE between 0.4 deg/s and 0.5 deg/s for both driving modes, was achieved with smooth control action without any oscillation or drivability issue perceived within the car.

Fig. 12. The vehicle demonstrator during a step steer test at the Lommel proving ground (Belgium).

![Fig. 12](image1)

Fig. 13. Steering wheel angle, \( \delta_{SW} \), as a function of lateral acceleration, \( a_y \), during the skid pad tests (experimental understeer characteristics).

![Fig. 13](image2)

Fig. 14. Reference yaw moment, \( M_{yaw}^{ref} \), as a function of \( a_y \) during the skid pad test (experimental yaw moment characteristics).

![Fig. 14](image3)

The step steer tests were performed at 100 km/h and consisted of a fast (500 deg/s) steering wheel angle application with an amplitude of 100 deg, imposed through a steering robot [34] for achieving repeatability of the test results, while the torque demand was electronically set (i.e., driver input on the accelerator pedal was bypassed through the dSPACE AutoBox system) at the constant level required for keeping the vehicle at constant speed before the steering wheel angle application. As a consequence, \( v \) reduced after the steering wheel input. Figs. 15 and 16 show the yaw rate and sideslip angle (\( \beta \)) response (measured through a CORRSYS DATRON sensor) of the passive and TV-controlled vehicle. The relevant benefits in terms of yaw and sideslip damping enhancement with the TV controller can be summarized in: (a) a reduction the first (high) peak of \( r \) (critical for vehicle stability), from 36 deg/s for the passive vehicle to 25 deg/s for the controlled vehicle; (b) an increase of the second (low) peak of \( r \) from marginally negative values for the passive vehicle to 10 deg/s for the TV-controlled vehicle; and (c) consistent and smooth sideslip response for the TV-controlled vehicle with \( |\beta| \) consistently < 5 deg, against the \( \beta \) peak of -15 deg for the passive vehicle. The marginal increase of \( r \) after its stabilization following the steering wheel input is caused by the reduction of \( v \).

![Fig. 15](image4)

Fig. 15. Experimental time history of \( r \): passive vehicle and controlled vehicle (\( H_\infty \) controller without feedforward contribution) in Normal mode.

![Fig. 16](image5)

Fig. 16. Experimental time history of side slip angle: passive vehicle and controlled vehicle (\( H_\infty \) controller without feedforward contribution) in Normal mode.
skid pad tests in Sport mode was 0.45 deg/s with the feedforward contribution and 0.42 deg/s without the feedforward contribution, without any particular vibration or lack of smoothness of the control action in case of deactivated feedforward contribution.

### 6. Conclusions

A torque-vectoring controller based on an H∞ loop shaping formulation was designed, implemented and assessed through a comprehensive set of simulations and experimental results. The main conclusions are:

(i) H∞ loop shaping represents a control system configuration characterized by general simplicity and good compatibility with the conventional engineering practice of adopting gain-scheduled PID controllers for vehicle yaw moment control. In fact, PID-based control structures are easily tunable, especially by vehicle testing engineers on the proving grounds, which is an essential requirement for the industrial adoption of any automotive controller;

(ii) The inclusion of the simplified model of the actuator dynamics in the H∞ control system design proved to be effective on the four-wheel-drive electric vehicle demonstrator with on-board electric drivetrains, signal discretization and delays associated with the communication buses;

(iii) The significant robust stability benefit of the H∞ formulation with respect to a more conventional PI formulation was demonstrated through the evaluation of the maximum robust stability margin for a significant variety of operating conditions;

(iv) The H∞ controller showed enhanced yaw rate tracking performance with reduced control effort, compared to conventional PI and PI + feedforward yaw moment control formulations, along a sequence of step steers, for two tire parameterizations, and different values of vehicle inertial parameters and tire-road friction coefficients;

(v) The experimental results confirmed the excellent performance of the H∞ controller in shaping the understeer characteristic in quasi-static conditions, without the requirement of a non-linear feedforward contribution, even for tracking sets of reference understeer characteristics significantly different from those of the vehicle with even torque distribution among the four wheels;

(vi) In general, torque-vectoring control for electric vehicles can be effectively adopted for further enhancing the level of yaw damping allowed by conventional stability control systems based on the actuation of the friction brakes. The fine tuning of the reference yaw rate filter can be used in order to shape the transient response of the different driving modes.

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References

[1] De Castro R, Tanelli M, Araújo RE, Savarese SM. Minimum-time manoeu-
[2] Crolla DA, Gao D. The impact of hybrid and electric powertrains on ve-
[3] De Novellis L, Sorniotti A, Gruber P. Optimal wheel torque distribution for a four-
[5] De Novellis L, Sorniotti A, Gruber P. Wheel torque distribution criteria for elec-
wheel motor drive electric vehicle using gain scheduling based on tire corner-
[9] Geng C, Mostafal S, Denai M, Hori Y. Direct yaw-moment control of an in-
[16] Chen Y, Wang J. Adaptive energy-efficient control allocation for planar motion control of over-actuated electric ground vehicles. IEEE Trans Control Syst Tech-
4011;22(4):1362–73.
[18] Sokograd S, Postlethwaite I. Multivariable feedback control – analysis and de-
[22] Shuai Z, Zhang H, Wang J, Li J, Ouyang M. Combined AFS and DYC control of four-wheel-independent-drive electric vehicles over CAN network with time-
[23] Mammar S. Two-degree-of-freedom H∞ optimization and scheduling, for ro-
[25] Hyde RA, Glover K. The application of scheduled H∞ controllers to a vstol air-
[27] Bottiglione F, Sorniotti A, Sheal L. The effect of half-shaft torsion dynamics on the performance of a traction control system for electric vehicles. Proc Institu-
[30] Gianguio E, Arosio D. New validated tire model to be used for ABS and VDC simulations. In: Proceedings of the 3rd International Colloquium on Vehicle-
Tire-Road Interaction, Stuttgart, Germany; 2006.
[31] de Castro R, Tanelli M, Araújo RE, Savarese SM. Design of safety-oriented con-
trol allocation strategies for electric vehicles. Vehicle Syst Dyn 2014;52(8).