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## Development and application of New Rule Based Fuzzy Cognitive Maps

Zdanowicz, Pawel

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# DEVELOPMENT AND APPLICATION OF NEW RULE BASED FUZZY COGNITIVE MAPS

**PAWEL ZDANOWICZ**

Faculty of Engineering, Environment and Computing

Coventry University

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# Declaration

I declare that the work described in this PhD thesis, unless otherwise stated in the text, is my own work and has not been previously submitted for any academic degree.

Pawel Zdanowicz

January 2017

# Acknowledgments

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# ABSTRACT

Rule Based Fuzzy Cognitive Maps (RBFCMs) have been developed for modelling non-monotonic, uncertain, cause-effect systems. However, the standard reasoning and impact accumulation mechanisms developed for RBFCMs assume that the level of variation that a fuzzy set represents is directly linked with the shape of the fuzzy set. It poses a big restriction on how the corresponding fuzzy sets have to be constructed and limits flexibility of knowledge representation. This thesis presents a critical analysis of standard RBFCMs and defines its limitations. To address these limitations a new reasoning and impact accumulation mechanisms are proposed. They increase flexibility of the method, by reducing overall number of constraints of RBFCMs. The new mechanisms take into consideration standard semantics of fuzzy sets, where their uncertainty is measured by fuzziness and specificity. Introduction of new methods allows development of new type of complex fuzzy relationships and reasoning on them. Thanks to new type of relationships, RBFCMs can model complex systems, where a joint impact of several causal nodes on one effect node needs to be considered. Increased flexibility and modelling capability is achieved using mechanisms which are significantly less computationally demanding. New algorithms reduce by over 80% the number of operations that need to be performed to calculate one impact between two nodes. Advantages of using new RBFCMs are demonstrated using two new complex case studies: the modelling of the resource allocation on military capability of military units and the impact of investments into cyber security on the risk to the enterprise's business. Both case studies could not be modelled using standard RBFCMs as they require development of complex relationships. To demonstrate advantages of new RBFCMs a dedicated software packaged was developed for standard and new reasoning and accumulation mechanisms. Additionally, a new direction for the development of RBFCMs is outlined, i.e. integration with Discrete Event Simulation (DES), that allows combining abstract RBFCMs models with the operational perspective represented by DES.

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# List of Symbols

| Symbol | Description |
|---|---|
| $C_i$ | Fuzzy set $i$ |
| $core_{C_i}$ | Core of the fuzzy set $C_i$ |
| $support_{C_i}$ | Support of the fuzzy set $C_i$ |
| $bi_{C_i}$ | Inner base of the fuzzy set $C_i$ |
| $bo_{C_i}$ | Outer base of the fuzzy set $C_i$ |
| $xc_{C_i}$ | Centroid of the fuzzy set $C_i$ |
| $\hat{f}(C_i)$ | Normalized measure of fuzziness $f$ of the fuzzy set $C_i$ |
| $\hat{Sp}(C_i)$ | Normalized measure of specificity $Sp$ of the fuzzy set $C_i$ |
| $ILT$ | Interpolated Linguistic Term |
| $x_{ILT}$ | Input of the interpolation |
| $dist_{C_i}$ | Euclidean distance between $xc_{C_i}$ and $x_{ILT}$ |
| $p$ | Accumulation strength – dependent on uncertainty levels of fuzzy sets involved in accumulation of impacts |
| $s$ | Parameter used to control the strength of accumulation |
| $VOS$ | Variation Output Set (output of accumulation of impacts) |
| $VOS_S$ | Saturated Variation Output Set |
| $X_{\max}$ | Maximum of the universe of discourse $X$ |
| $X_{\min}$ | Minimum of the universe of discourse $X$ |
| $f_{\max}$ | Saturation parameter, when VOS extends beyond the maximum of the universe of discourse |
| $f_{\min}$ | Saturation parameter, when VOS extends beyond the minimum of the universe of discourse |
| $t$ | Iteration of the simulation |
| $impact^t_{C_i}$ | Impact received by a node in the iteration t |

# Chapter 1. INTRODUCTION

## 1.1 Introduction

Mathematical modelling of real-world systems is used every day to improve understanding of the environment we live in. This thesis contributes to the development of modelling using fuzzy sets. Fuzzy sets are particularly useful in systems where the available knowledge cannot be represented using crisp values or crisp sets, due to lack of information about the system or complexity of the information.

## 1.2 Modelling of uncertainty

Quantitative modelling methods have gained a lot of interest and have been improved in the last few decades. Great increase of the computational power of computers and expansion of distributed computations allowed the development of methods, which can replicate behaviour of very complex, physical systems in a very detailed way. The finite elements is one example of such method. It can model all the forces within the physical environment that are acting on the modelled object. This method is commonly used in engineering, to analyse the strength of materials and their deformation under different external conditions (Pavlou, 2015).

On the other hand, there are many systems whose behaviour can be analysed by looking at months or years of changes. Political, sociological and economical systems are difficult to predict as they do not abide the physical laws and are very often guided by unpredictable and uncertain man-made decisions. The increase of computational power and development of complex modelling methods does not solve problems inherent in these systems, such as lack of good understanding of processes driving these systems, difficulties in representing knowledge with required precision and limited ability to represent uncertainty.

Human decision making is a subject to different types of uncertainty but always results from deficiency of information (Klir and Wierman, 1998). Uncertainty related to randomness is well represented by probability theory - a branch of mathematics which developed a set of tools and methods to predict the likelihood of the occurrence of specific events. Uncertainty resulting from incompleteness, unreliability of data, vagueness or fuzziness of meaning is handled by a branch of mathematics called fuzzy set theory. Fuzzy sets are used to define objects that do not have sharp boundaries and therefore capture the lack of precision in the definition of a given object (Zimermann, 2001). Fuzzy sets allow assigning different degrees of membership, in the range from 0 to 1, to elements of the set, on the contrary to classical logic where membership degrees can be either 0 or 1. Most often fuzzy sets are represented using membership functions which define a mapping between all elements of a fuzzy set and their membership degrees.

The introduction of fuzzy sets developed a new way of handling uncertainty. It allows to represent knowledge using uncertain linguistic terms: *high, short, increase, hot, old,* etc. Such linguistic terms are every day used by humans to effectively exchange information about the world. An intrinsic property of linguistic terms is their subjectivity and imprecision, for example, term *cold* has drastically different meaning to somebody raised in the United Kingdom, than to someone who lives in tropical countries. In the recent years, new and more sophisticated methods of handling uncertainty were developed, such as Type 2 Fuzzy Sets or Interval Valued Type 2 Fuzzy Sets (Mendel, 2006). They allow representations of multiple subjective definitions of a linguistic term using one, complex fuzzy set.

Fuzzy sets contributed to the development of new modelling techniques – fuzzy systems, which try to handle uncertainty inherent in most systems. One of the first applications of fuzzy sets was the development of fuzzy rule based systems which allow representing relationships between concepts using fuzzy IF-THEN rules. (Zadeh, 1973)

The systems built using fuzzy IF-THEN rules are called Fuzzy Expert Systems (FES) (Zadeh, 1983). The reasoning in FES is as follows: first, a set of inputs into antecedent concepts triggers fuzzy sets representing the input values. In the next step, rules having respective fuzzy sets in the IF part of the rule are fired. Consequently, fuzzy sets representing linguistic terms in the THEN parts are determined. Based on these fuzzy sets the impact on the consequent concept is calculated using one of the inference methods: Mamdani (1977) or Takagi-Sugeno (Takagi and Sugeno, 1985).

The inference mechanism used to calculate the impact usually has an averaging nature. Therefore, the result of inference lies in between two fuzzy sets of the THEN part of the rule. Let us consider an example in which causal nodes A and B have positive impacts on the effect node C: *Increased Little* and *Increased*. In such case, the result of receiving these two impacts, calculated using the standard reasoning used in FES, would be somewhere in between fuzzy set *Increased Little* and *Increased*. In systems where reasoning has accumulative nature, the output should be *More than Increased.*

Fuzzy Cognitive Maps (FCMs) are a simplified approach to modelling using fuzzy sets (Kosko, 1986). Relationships between concepts in FCMs are defined using crisp values between [-1,1], and values of concepts using the intervals [0,1] or [-1,1]. Reasoning in FCMs has an accumulative nature. Impacts on the effect node are calculated by multiplying values of causal nodes and weights between causal nodes and the effect node. The new value of the effect node is determined as a sum of received impacts and the value of the effect node from the previous iteration. As a result, the cognitive map changes in every iteration, allowing analysis of dynamic systems.

In FCMs, linguistic terms are used to simplify knowledge extraction from experts developing the model. To represent *strong* impact of a causal node on an effect node, one could use weight equal to 0.8, while *weak* impact can be represented using weight equal to 0.2. Therefore, the use of linguistic terms in FCMs can be interpreted as an interface between the model and the expert and do not carry as much information about uncertainty as standard fuzzy sets.

FCMs gained popularity due to the simplicity of the method and the low computational cost - matrix multiplication – required to run simulations. It also allows the development of efficient optimisation and learning mechanisms, which are trending topics in this domain (Papageorgiou, 2012).

FCMs have several limitations specific to the nature of their mechanisms and multiple methods were developed to address these shortcomings: symmetricity of relationships, simplistic representation of uncertainty (single crisp value) and representation of dynamics which does not relate to actual time (Haigwara, 1992; Zhong et al., 2008; Stylios, 2008; Iakovidis and Papageorgiou, 2009; Yaman and Polat, 2009;).

In 1998, Rule Based Fuzzy Cognitive Maps (RBFCMs) were introduced (Carvalho and Tome, 1998). This modelling method takes the advantage of the strengths of FCMs and fuzzy rule based methods: the representation of knowledge in rule-based fuzzy systems and the accumulation mechanism, which has similar properties to accumulation of impacts in FCMs.

Knowledge in RBFCMs is represented by fuzzy relationships and fuzzy sets. Fuzzy relationships between concepts, defined using fuzzy IF-THEN rules, allow to specify the impact of a change in a causal node on an effect node. A causal node is one which causes a change (IF part of rule) on an effect node (THEN part of a rule). Values of a node are represented using fuzzy sets defined on the universe of discourse, which defines a range of possible crisp values a node can take. Fuzzy sets are defined using membership functions which specify to what degree a given element of the universe of discourse is represented by fuzzy sets.

An important element of cognitive modelling is ability to represent dynamic changes of the system over time. In RBFCMs every relationship has a base time unit assigned, which defines how many base time units need to pass to observe the impact of a causal node on an effect node (Carvalho, 2013). Such representation of dynamics is more advanced than the one used in FCMs, where every relationship has the same base time unit, equal to 1, and where all impacts are observable after every iteration of simulation.

To allow reasoning on rules, that has accumulative rather than averaging nature, the authors developed two additional mechanisms (Carvalho and Tome, 2009). The first new mechanism, Interpolation of Linguistic Terms was introduced to allow accumulation of fuzzy impacts. It calculates the fuzzified representation of the crisp impact received by an effect node. The second mechanism is the accumulation of impacts, which allows the accumulation of fuzzified impacts, when more than one impact is received by an effect node. The new approach to calculation of the output of multiple impacts allows accumulating impacts similarly to FCMs, but with much higher computational costs. It is because RBFCMs' accumulation of impacts mechanism recursively accumulates and shifts points of two fuzzy sets considering the surplus of memberships, until the surplus is equal to 0. As a result, many more operations have to be executed to accumulate two impacts for one node when compared to calculations of impacts in FCMs.

To enable the accumulation of impacts, new semantics of fuzzy sets was proposed in RBFCMs. It states that the area and support of a fuzzy set are related to the value that this fuzzy set represents. For instance, let us assume that fuzzy sets, defining a concept, represent different degrees of variation, i.e. *small increase* and *big increase*. According to the semantics used in RBFCMs, the fuzzy set representing a small increase of a concept needs to have smaller area and support than the fuzzy set representing a big increase. Such definition of fuzzy sets ensures that the output of accumulation is calculated correctly, using the proposed mechanism, as fuzzy sets with greater area allow accumulating and shifting more membership degrees.

Semantics proposed in RBFCMs is developed under a set of restrictions which, as demonstrated in this thesis, do not always ensure calculation of results that are expected. Many fuzzy rule based systems and methods require use of particular shape of membership functions sets:, to ensure fast calculations (e.g. to improve implementation of gradient decent optimisation (Subramanian, 2014)) or to improve knowledge representation (Barua at al, 2014) . Carvalho and Tome (2009) advise to use one of the two predefined sets of membership functions to model the variation. However, restrictions on the shape of fuzzy sets in RBFCMs are more substantial and have a highly negative impact on the flexibility of the method.

## 1.3 Aim and objectives

### 1.3.1 Motivations

Development of RBFCMs introduced new modelling capability and allowed addressing a range of limitations of FCM and FES. Increase of capability of representation of uncertainty, modelling of dynamics and accumulation of impacts used in RBFCMs, comes with a high computational cost of algorithms and restrictive knowledge representation.

This research work started in consequence of application of RBFCMs to model complex real-world problems. The RBFCMs were chosen due to their ability to model dynamic and nonlinear systems. When applied, they turned out to be difficult to use for modelling experts' knowledge and challenging to implement, due to the recursive nature of the accumulation of impacts mechanism and ambiguous semantics. Moreover, only simple relationships can be modelled, due to limitations of the reasoning mechanism.

Until now, RBFCMs have been used in a small number of applications such as forest fire modelling, pelagic fishery, student centred education, etc. This could be due to a low flexibility in representing knowledge and complex, not well defined, reasoning. So far there has not been critical analysis of RBFCMs, and their limitations, in the literature.

The aim of this thesis is to develop new RBFCMs with different semantics and reasoning mechanisms that have the potential to appeal to modellers and are based on concepts well defined in fuzzy sets theory. This aim can be achieved by: 1. analysing of the standard RBFCMs and their limitations, 2. development of new reasoning methods to increase flexibility and applicability of RBFCMs, 3. improvement of modelling of dynamics and 4. application of new RBFCMs in new case studies.

### 1.3.2 Objectives

**1. Analysis of the standard RBFCMs and their limitations**

RBFCMs have been used in a limited number of applications and there is no critical analysis of the method. The first objective of this thesis is analysis of constraints under which the reasoning and accumulation mechanisms in standard RBFCMs were developed. The next step is the analysis of the impact of restrictions used in RBFCMs on knowledge representation and results calculated by both mechanisms.

**2. Development of new reasoning methods to increase flexibility and applicability of RBFCMs**

Knowledge representation is the key aspect of modelling using fuzzy sets. Therefore, the number of restrictions enforced on experts, who define fuzzy sets, needs to be limited. This goal can be reached by removing semantics introduced in original RBFCMs, which links the area of a fuzzy set with the variation it represents. The original inference and accumulation of impacts mechanisms were developed in line with this semantics, therefore, they would also need to be redesigned. Introduction of new semantics of fuzzy sets and new reasoning mechanisms would allow to handle the reasoning on fuzzy sets, which do not comply with the semantics introduced in the RBFCMs.

Another aspect of knowledge representation that should be improved is the modelling of complex relationships. So far RBFCMs can model simple relationships, with one antecedent and one consequent concepts, e.g. IF Node A is *Increased Little* THEN Node B is *Increased.* In FES, relationships with multiple antecedent concept having an impact on one consequent concept are very common. They allow capturing the common characteristics of causal relationships, where more than one causal concept need to be considered in order to define the impact on one effect node. For example, to define the perception of the weather, one should jointly consider the current temperature and humidity. A change of temperature has a linear impact on the perceived temperature: the higher the temperature the higher the perceived temperature. On the other hand, an increase of humidity increases the perception of hot temperature and increases the perception of cold, making the relationship nonlinear. Such relationship cannot be modelled using two simple causal relationships.

**3. Improvement of modelling of dynamics**

The. RBFMCs introduced a new approach to the representation of dynamics in fuzzy rule based systems. The element of dynamics which is not addressed in RBFCMs, is handling of propagation of impacts in loops. In RBFCMs, similarly to FCMs and FES, every element of the model, in most applications, has a lower and upper limit of possible values. The existence of loops in RBFCMs can result in the reinforcement of impacts and its escalation over the minimum or maximum of allowed range of values. A mechanism is required to control the build-up of impacts in models that contain loops.

**4. Application of new RBFCMs in new case studies**

Until now, RBFCMs have been used in low number of case studies: forest fire modelling, modelling of fishermen behaviour, socio-economical modelling and the modelling of student-centred education. All but one application of RBFCMs were developed by authors of RBFCMS, Carvalho and Tome. Even though practitioners of cognitive maps recognize the modelling capabilities of RBFCMs (Papageorgiou and Salmeron, 2013), so far, RBFCMs struggled to gain popularity and to attract modellers (Zdanowicz and Petrovic, 2017) – so far it was applied to a small number of problems. It is important to apply the new RBFCMs into new domains and to solve new types of problems, to demonstrate the generalization ability and modelling capabilities of RBFCMs. The two new case studies that are presented in this thesis are: the modelling of the impact of resource allocation on military ability of military units and the modelling of impact of different investment strategies on the risk to the cyber security of an enterprise.

To enable the development of RBFCMs and their application the software to handle RBFCMs reasoning and accumulation of impacts needs to be developed. Currently there are no tools that allow using RBFCMs.

## 1.4 Publications during the research

As a result of this research work, the following publications have been produced:

**Presenting contributions of Chapters 5 and 6:**

Zdanowicz, P. and Petrovic, D. (2017) 'New Mechanisms for Reasoning and Impacts Accumulation for Rule Based Fuzzy Cognitive Maps'. *IEEE Transactions on Fuzzy Systems*

**Presenting contributions of Chapters 3 and 7:**

Zdanowicz, P., Petrovic, D., Irwin, C., Lucek, S., (2015) 'A new approach to modelling Cyber Defence based on a Rule Based Fuzzy Cognitive Map'. *27th European Conference on Operational Research*, 12 – 15 July, Glasgow, UK

Zdanowicz, P., Petrovic, D., Irwin, C., (2013) 'Modelling and Analysis of Defence Lines of Developments Using Fuzzy Cognitive Maps'. In *Proceedings of the 30th International Symposium on Military OR,* 29 July - 2 August, Royal Holloway, UK

Zdanowicz, P., Petrovic, D., Irwin, C., (2013) 'Modelling and Analysis of Different Localization of Joint Strike Fighter Squadron Using Fuzzy Causal Maps'. *26th European Conference on Operational Research*, 1-4 July, Rome, Italy

Zdanowicz, P., Petrovic, D., Irwin, C., (2012) 'Modelling and Analysis of Defence Lines of Development Using Causal Maps'. *29th International Symposium on Military Operational Research*, 28 – 31 August, Bishop's Waltham, UK

## 1.5 Thesis outline

In Chapter 2, literature review, core concepts of fuzzy sets and the stages of developments of cognitive modelling are outlined. In this chapter, the need of development of RBFCMs is defined in more detail.

Chapter 3 introduces two novel case studies for which the new RBFCMs are developed. Examples from these case studies are used in subsequent chapters to help to explain and understand new elements of RBFMCs.

Chapter 4 discusses semantics of fuzzy sets that is going to be used in RBFCMs, to allow more flexibility in modelling the experts' knowledge. One of the standard interpretations of fuzzy sets is introduced to represent fuzzy sets using membership functions. Advantages of using it, compared to the semantics of RBFCMs, is discussed. Additionally, a more in-depth analysis of the consequences of using the standard RBFCMs' semantics, when compared to the semantics introduced in this thesis, is presented.

Chapter 5 defines a new approach to inferring the output resulting from firing fuzzy IF-THEN rules. The reasoning process is split into two distinct steps, which allow the introduction of complex relationships. Furthermore, the disadvantages of using standard RBFCMs mechanism are discussed and a new *Interpolation of Linguistic Terms* mechanism is proposed.

Chapter 6 treats the core element of RBFCMs: handling causality and dynamics. In this chapter, a new acumulation mechanism based on uncertainty of fuzzy sets is proposed. Limitations of the standard accumulation of impacts mechanism are discussed and solutions achieved by using the new mechanism presented. Both mechanisms are also compared from the computational demand perspective, which is very important in modelling large or time critical systems. In the last part of this chapter a new method of handling loops in RBFCMs is proposed.

Chapter 7 presents the application of RBCMs for the two case studies discussed in Chapter 3. Two models are presented and results of running simulations are discussed. The outcome of using both models and their impact on the industry is outlined.

Chapter 8 introduces a novel Discrete Event Simulation model and its integration with the RBFCM. The impact of combining both models is demonstrated using three experiments. Analysis of results and guidance for the future work is also presented.

Chapter 9 summarizes the contributions to the modelling of dynamic systems achieved by the development of RBFCMs, presented in this thesis. The guidance for the future work related to the increasing flexibility and applicability of RBFCMs is also presented.

# Chapter 2. LITERATURE REVIEW

## 2.1 Fuzzy Set theory review

Fuzzy sets theory was proposed by Zadeh in 1964 (Zadeh, 1964) and has been continuously evolving for over five decades. The work of many researchers led to a great increase of applications of fuzzy sets, starting with simple fuzzy IF-THEN models, to complex use of fuzzy sets and fuzzy relations for pattern recognition or machine learning (Dubois and Prade, 2015). Validity of fuzzy set theory has been subject of a debate among researchers since their introduction (Elkan, 1984; Trillas and Alsina, 2001; Zadeh, 2008; Belohlavek, 2009). As there is no discussion about mathematical fundaments of fuzzy sets anymore, the semantics of the fuzzy sets is an area open to interpretations (Dubois, 1997).

### 2.1.1 Fuzzy sets and membership functions

**Fuzzy sets** are an extension of classical sets, where elements can either belong or not belong to the set. In the fuzzy sets theory, elements of the set have membership degrees in the interval of [0, 1], that define different levels of membership, where 1 defines full membership and 0, defines lack of membership.

Membership function of the fuzzy set $A$, $A(x)$, defines the mapping between elements of the **universe of discourse**, X, and the interval [0, 1], defining **membership degree**, $\mu_A(x)$, of a point $x$ to the fuzzy set A (Zadeh, 1965).

$$A: X \to [0,1] \tag{2.1}$$

### 2.1.1.1 Trapezoidal membership function

One of the most commonly membership function to represent fuzzy sets is trapezoidal function. It is defined using four points, $A(x;a,b,c,d)$, as in Figure 2.1, where:

- a and d are the lower and upper bounds, where membership degree is greater than 0, $\mu_A(x) > 0$
- b and c are the lower and upper bounds, where membership degree is equal to 1, $\mu_A(x) = 1$

The membership degree of any point x to the fuzzy set A for a trapezoidal function can be represented using the following function:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & \text{if } x \in [a,b) \\ 1 & \text{if } x \in [b,c] \\ \dfrac{d-x}{d-c} & \text{if } x \in (c,d] \\ 0 & \text{if } x > d \end{cases} \tag{2.2}$$

or using a shorter notation (Pedrycz and Gomide, 2007), as:

$$A(x;a,b,c,d) = \max\left\{\min\left[\frac{x-a}{b-a}, \frac{d-x}{d-c}, 1\right], 0\right\} \tag{2.3}$$

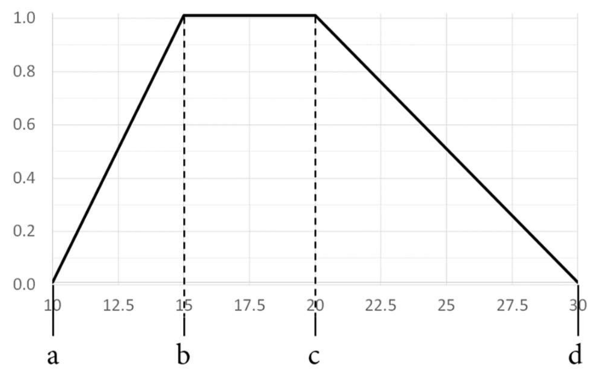



Figure 2.1 Trapezoidal membership function

Trapezoidal membership functions are used in this thesis to represent expert's knowledge. They can be conveniently used to extract expert's knowledge and have simple mathematical representation (Pedrycz, 1994; Barua at al, 2014), which simplifies development of reasoning mechanisms in RBFCMs and elicitation of knowledge from experts.

### 2.1.1.2 Triangular membership functions

Triangular membership functions are defined using three points a, b and c, $A(x;a,b,\mathrm{c})$ as in Figure 2.2.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & \text{if } x \in [a,b) \\ \dfrac{c-x}{c-b} & \text{if } x \in [\mathrm{b},\mathrm{c}] \\ 0 & \text{if } x > c \end{cases} \tag{2.4}$$

or using shorter notation (Pedrycz and Gomide, 2007), as:

$$A(\mathrm{x;a,b,c}) = \max\left\{ \min\left[ \frac{x-a}{b-a}, \frac{c-x}{c-b} \right], 0 \right\} \tag{2.5}$$

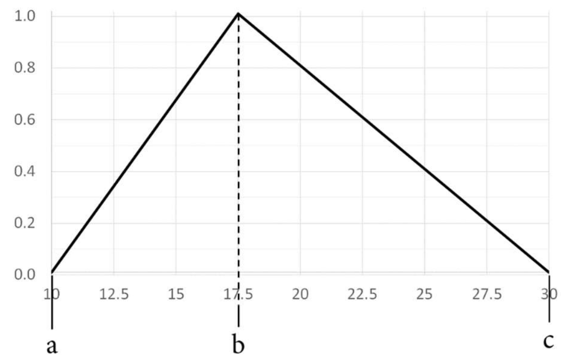



Figure 2.2. Triangular membership function

### 2.1.1.3 Gaussian membership function

Gaussian membership function is described using two parameters, $A(x;\sigma,c)$ and calculated using the following function (Figure 2.3):

$$A(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}} \tag{2.6}$$

where $c$ is the centre of a fuzzy set (mean) and

$\sigma$ is the width of a fuzzy set (variance)

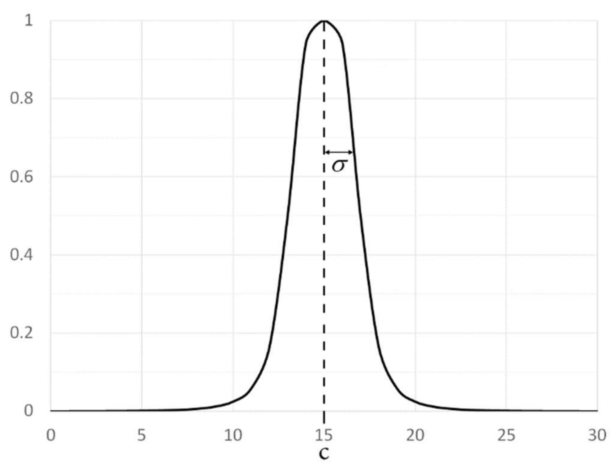



Figure 2.3 Gaussian membership function

## 2.1.2 Characteristics of fuzzy set

Every fuzzy set, regardless of the type of membership function, that defines its shape, has the same set of characteristics that allow its analysis and performing calculations using fuzzy sets.

**Complement** of the fuzzy set is defined as (Figure 2.4):

$$A^C(x) = 1 - \mu(x) \tag{2.7}$$

**α-cut** of a fuzzy set, represented as $A_\alpha$ defines the set of elements of the fuzzy set $A$, whose degree of membership is greater or equal to α.

$$A_\alpha = \{x \mid \mu_A(x) \geq \alpha\} \tag{2.8}$$

**Support** of the fuzzy set $A$, $support_A$, defines all elements of the universe of discourse $X$ that belong to the fuzzy set $A$ and have membership degree greater than 0. Support consists of all the elements of the α-cut of a fuzzy set $A$, where $\alpha$ equals to 0 (Figure 2.4). For the trapezoidal membership function the size of support of the fuzzy set $A$ is equal to:

$$support_A = A_0 = \{x \mid \mu_A(x) \geq 0\} = d - a \tag{2.9}$$

**Core** of the fuzzy set $A$, $core_A$, defines all elements of the fuzzy set $A$ that have maximum membership degrees of 1, where α-cut of a fuzzy set $A$ equals to 1 (Figure 2.4). For the trapezoidal membership function the size of core of the fuzzy set $A$ is equal to:

$$core_A = A_1 = \{x \mid \mu_A(x) \geq 1\} = c - b \tag{2.10}$$

**Inner** and **outer bases** of the fuzzy set are parts of the membership function where the membership degrees are between 0 and 1 (Figure 2.4). Inner base for the trapezoid membership function is calculated as follows:

$$bi_A(x) = \{x \mid \mu_A(x) \geq 0 \wedge x \leq b\} = b - a \tag{2.11}$$

and the outer base for the trapezoid membership function is calculated using the following formula:

$$bo_A(x) = \{x \mid \mu_A(x) \geq 0 \wedge x \geq c\} = d - c \tag{2.12}$$

Fuzzy set whose greatest membership degree is equal to 1, for at least one element on the universe of discourse, X, is a **normal** fuzzy set.

Fuzzy set is considered **convex** if and only if all its $\alpha$ $cuts$ are convex sets:

$$\mu_A(\lambda r + (1-\lambda)s) \geq \min[\mu_A(r), \mu_A(s)] \tag{2.13}$$

Fuzzy set which is normal and convex is called a **fuzzy number**. Examples of fuzzy numbers are presented in Figures 2.1 to 2.5. All fuzzy sets discussed in this thesis are normal and convex.

Set whose elements have the same membership degree equal to 1 is called a **crisp set**:

$$\mu_A(x) = \begin{cases} 1, & if\ x \in A \\ 0, & if\ x \notin A \end{cases} \tag{2.14}$$

Set with one element, $A = \{a\}$, is called a **singleton**:

**Centroid** of the fuzzy set represents its approximation using a crisp value on the universe of discourse. One way to calculate it is centre of gravity method (Braae and Rutherford, 1972):

$$xc_A = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx} \tag{2.15}$$

Centroid of the trapezoidal fuzzy set $A(x;a,b,c,d)$ can be calculated using the following formula (Figure 2.4):

$$xc_A = \frac{d^2 - a^2 + c^2 - b^2 + dc - ba}{3(d - a + c - b)} \tag{2.16}$$

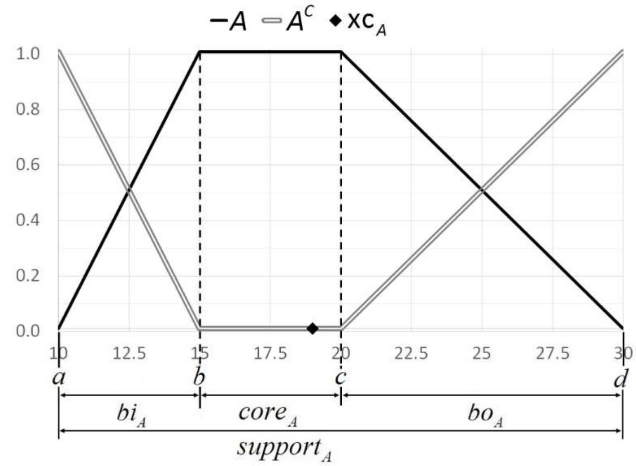



Figure 2.4 Definition of fuzzy set A, its complement $A^C$ and main characteristics: inner and outer base ($bi_A$ and $bo_A$), core and support ($core_A$ and $support_A$) and centroid ($xc_A$).

### 2.1.3 Fuzzy arithmetic

The fuzzy extension principle was introduced by Zadeh (1975) to allow definition of fuzzy operators, which are used to transform fuzzy sets into other fuzzy sets using functions. The fuzzy extension principle is a basis for arithmetic operations on fuzzy numbers. Examples of the main fuzzy arithmetic operations are presented in Table 2.1.

Table 2.1 Fuzzy arithmetic definitions

| **Operator** | **Syntax** | **Formula** |
|---|---|---|
| Addition | $A + B$ | $(a_A + a_B, b_A + b_B, c_A + c_B, d_A + d_B)$ |
| Subtraction | $A - B$ | $(a_A - a_B, b_A - b_B, c_A - c_B, d_A - d_B)$ |
| Multiplication | $A \times B$ | $(a_A \times a_B, b_A \times b_B, c_A \times c_B, d_A \times d_B)$ |
| Scalar multiplication | $m \times A$ | $(m \times a_A, m \times b_A, m \times c_A, m \times d_A)$ |
| Division | $A / B$ | $(a_A / a_B, b_A / b_B, c_A / c_B, d_A / d_B)$ |

where $A = (x; a_A, b_A, c_A, d_A)$ and $B = (x; a_B, b_B, c_B, d_B)$

### 2.1.4 Semantics

One of the characteristics of fuzzy sets theory is the fact that multiple meanings can be assigned to fuzzy sets. This created a debate among researchers regarding the validity of the fuzzy sets theory (Sauerland, 2011). At the same time, it has been considered as an advantage by researchers who have contributed to the development of fuzzy set theory and its applications (Dubois, 1997). They have emphasized the fact that ambiguity of the fuzzy set increases the range of potential use. As fuzzy sets can be interpreted in many ways, it is important to clarify the meaning of the fuzzy set for a given application (Dubois and Prade, 2012) and to make sure that the experts, from whom the knowledge is elicited, have a clear understanding of it.

There are three main interpretations of semantics of fuzzy sets as highlighted by Dubois and Prade (1997). The first semantics expresses the degree of similarity, $\mu(x)$, of a fuzzy set to the prototype object. It was first introduced by Bellman et al. (1966) for pattern classification. This semantics is appropriate when one would like to classify a set of objects into several categories, for instance classification of animals by their weights. In this case, three categorise could be distinguished: heavy, medium and light and for each category a prototype object needs to be defined that would be a reference point for assignment of degree of similarity.

The second semantics represents degree of preference or belief, $\mu(x)$, of object x belonging to a fuzzy set representing an object A, over other objects. This interpretation of fuzzy sets was proposed by Bellman and Zadeh (1970) to allow development of fuzzy sets based decision support systems. This semantics is used when the fuzzy set represent subjective opinion of an expert. For example, a definition of wealth could be different for every expert, if a reference point is not specified.

The third semantics defines degree of uncertainty, $\mu(x)$, that the object x represents object y. In this case $\mu(x)$ defines the degree of certainty when there is no definition of the object y, or it is very vague. This semantics is used when information about the object represented by the fuzzy set is very limited.

The two main types of fuzzy sets used in RBFCMs, proposed in this thesis, are representing variation of the attribute *(small increase, big increase*) or level of the attribute (*low level, high level*), defined by experts. The interpretation of fuzzy sets used is one where membership function defines degree of belief $\mu(x)$ that the point x belongs to the fuzzy set A; $\mu(x)=0$ means that the point does not belong to the fuzzy set A while $\mu(x)=1$ defines the full membership of point x to fuzzy set A.

### 2.1.5 Measures of uncertainty

Fuzzy measures were introduced by Sugeno (1974) as a generalization of the classical measures, allowing introduction of monotonic measures that do not require to be additive. Different classes of measures were developed over the course of the last five decades (Garmendia, 2005). The fuzzy measure allowing grading uncertainty of the experts' definition of a fuzzy set are measures of uncertainty (Klir, 1988).

According to Zhang (1998) there are three main categories of uncertainty that can be considered when analysing fuzzy sets: fuzziness, vagueness and ambiguity.

A term is considered **fuzzy**, when it is impossible to precisely quantify it. Let us consider following predicate:

*Last year the budget for military equipment increased a lot.*

In this statement, the linguistic variable *increased a lot* is a fuzzy statement as one cannot define it with a single number. Another example of a fuzzy term is around 20, as it could mean any number between 15 and 25, depending what level of precisions is applied

The statement is considered **vague**, when it has multiple meanings within the same semantic domain.

*This year military budget has changed.*

This statement has two possible meanings, i.e. the budget either increased or decreased and both interpretations relate to the same subject – change of the budget.

The third type of uncertainty – **ambiguity**, is considered when a statement has multiple meanings, in at least two distinct semantic domains.

*We saw a big part of the forest yesterday.*

This statement, depending on the interpretation of the verb saw, means visiting a forest or the activity of cutting the forest trees. Both interpretations give completely different and unrelated definition of the proposition.

Klir and Folger (1988) propose less detailed distinction of types of uncertainty and categorised it as: vagueness and ambiguity. The authors define vagueness as the lack of ability to make precise distinction of an object with sharp boundaries. Ambiguity, on the other hand, is related to the possibility of having multiple meanings. According to Klir and Folger, fuzziness is an example of vagueness, while ambiguity encapsulates a much broader definition of uncertainty than the one proposed by Zhang (Table 2.2).

Table 2.2 Comparison of definitions of types of uncertainty.

| | **Fuzziness** | **Vagueness** | **Ambiguity** |
|---|---|---|---|
| Lack of precise distinction | Zhang (1998) | Klir and Folger (1988) | |
| Multiple meanings – one semantic domain | | Zhang (1998) | Klir and Folger (1988) |
| Multiple meanings – different semantic domains | | | Zhang (1998) Klir and Folger (1988) |

In this thesis, membership functions represent expert's subjective definitions of linguistic terms. The linguistic terms used are known and well understood, therefore, vagueness and ambiguity of fuzzy sets, as defined by Zhang, are not further analysed. The only type of uncertainty further discussed in this thesis is fuzziness.

Fuzzy sets, developed based on the subjective interpretations of experts, carry a lot of information within its shape. The sizes of the core, support, and inner and outer bases allow analysis of uncertainty of fuzzy sets through different types of metrics. Metrics that are used to measure fuzziness of fuzzy sets are: level of fuzziness and specificity.

### 2.1.5.1 Fuzziness

Fuzziness level defines how different from crisp the fuzzy set is (Kaufmann, 1975). It can be used to assess the amount of uncertainty of experts regarding the lower and upper bounds of the membership function. There are three requirements that the function, $f$, measuring fuzziness level must satisfy:

Axiom 1. The fuzziness level is equal zero when set A is crisp: $f(A)=0$

Axiom 2. If fuzzy set A is considered less fuzzy than fuzzy set B, $A < B$ then $f(A) \leq f(B)$.

Axiom 3. $f(A)$ assumes the maximum value if and only if A is maximally fuzzy.

Different functions, which fulfil the three before mentioned axioms, can be used to measure fuzziness of fuzzy set (Klir and Folger, 1988; Garmendia, 2005).

Index of fuzziness $F(A)$ of the fuzzy set $A$ is defined by the difference of the fuzzy set from the crisp set defined on the support of fuzzy set A. The fuzzy set $A$ carries no fuzziness when $core_A = support_A$. Therefore, the smaller the core is with respect to the size of the support, the fuzzier the fuzzy set is.

Formula 2.17 defines Index of fuzziness for Minkowsky class of distances (Thompson, 1996), where $w \in [1, \infty)$.

$$f_{c,\mathrm{w}}(A) = (d-a)^{1/w} - \left( \int_a^d \delta_{c,A}^w(x)\, dx \right)^{1/w} \tag{2.17}$$

In this thesis, Hamming distance is used, where $w = 1$ (Klir and Folger, 1988):

$$f_{c,1}(A) = (d-a) - \int_a^d \delta_{c,A}^1(x)\,dx \tag{2.18}$$

where, $\delta_{c,A}^1(x) = \left|\mu_A(x) - \mu_{c(A)}(x)\right|$ is the Hamming distance between degree of belief of fuzzy set A and its complement, for a given point x,

a is the minimum of support of fuzzy set A,

d is the maximum of support of fuzzy set A and

$\mu_{c(A)}(x) = 1 - \mu_A(x)$.

In Figure 2.5, the grey area represents the integral $\int_a^d \delta_{c,A}^1(x)\,dx$. It can be observed that for any trapezoidal membership function, the Index of fuzziness can be calculated using the following formula:

$$\int_a^d \delta_{c,A}^1(x)\,dx = (d-a) - \left(\frac{bi_A}{2} + \frac{bo_A}{2}\right) \tag{2.19}$$

Therefore,

$$f_{c,1}(A) = (d-a) - \int_a^d \delta_{c,A}^1(x)\,dx = (d-a) - (d-a) + \left(\frac{bi_A}{2} + \frac{bo_A}{2}\right) = \frac{bi_A + bo_A}{2}$$

and normalized fuzziness is:

$$\hat{f}_{c,1}(A) = \frac{\frac{bi_A + bo_A}{2}}{(d-a)} = \frac{bi_A + bo_A}{2\ \text{support}_A} = \frac{bi_A + bo_A}{2\ (bi_A + bo_A + \text{core}_A)} \tag{2.20}$$

Factors that impact Index of fuzziness are the length of the core and inner and outer bases of the fuzzy set. If the length of the core is equal to 0 then the fuzziness is the highest (equal to 0.5). The longer the core is, the lower the fuzziness is. When inner and outer bases are equal to 0, i.e. fuzzy set is crisp – there is no fuzziness.

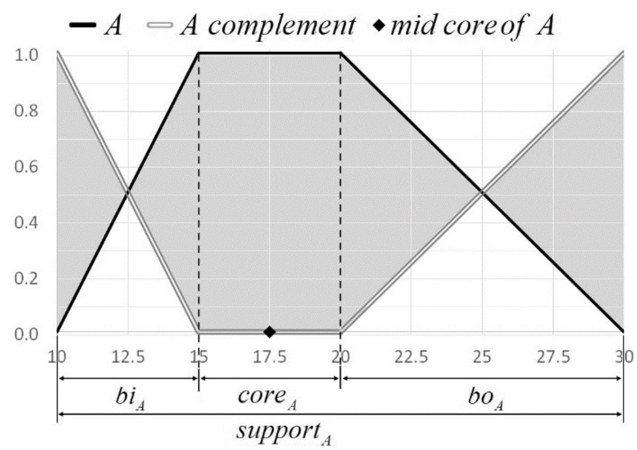



Figure 2.5. Fuzzy set A and its complement

### 2.1.5.2 Specificity

Another uncertainty measure that is useful to analyse in the RBFCMs context is specificity measure. It expresses how close to fuzzy singleton the fuzzy set is. It is useful to assess how specific the experts are; the wider the support and core of fuzzy set, the less specific it is (Yager, 1990). The measure of specificity needs to satisfy the following conditions:

Axiom 1. $Sp(A) = 1$ only if $A$ is a singleton

Axiom 2. $Sp(A) = 0$ when $A = \emptyset$

Axiom 3. $Sp(A) > Sp(B$ when $A \subset B$

Many different functions have been introduced to measure the specificity of fuzzy sets (Garmendia, 2003; Klir, 1988). In this thesis, specificity is calculated using a linear measure of specificity on a finite universe of discourse (Garmendia, 2003).

$$Sp(A) = a_1 - \sum_{j=2}^{n} w_j a_j \tag{2.21}$$

where $a_j$ is the j[th] greatest membership degree of fuzzy set A and $\{w_j\}$ is a set of weights verifying:

1. $w_j \in [0,1]$
2. $\sum_{j=2}^{n} w_j = 1$
3. $w_j \geq w_i$, for $1 < j < i$

#### 2.1.5.3 Relationship between the shape of the fuzzy set and uncertainty levels

Fuzziness and specificity measures are illustrated in Figure 2.6 using several examples of fuzzy sets. The greater the inner and outer bases are, for a given support (Figure 2.6 from (a) to (c)), the fuzzier and more specific the fuzzy set is. The same relation can be observed when the sizes of inner and outer bases remain unchanged but the size of the core and support become smaller (from (c) to (e)). The exception is a singleton fuzzy set, where specificity is the highest and fuzziness the lowest (f).

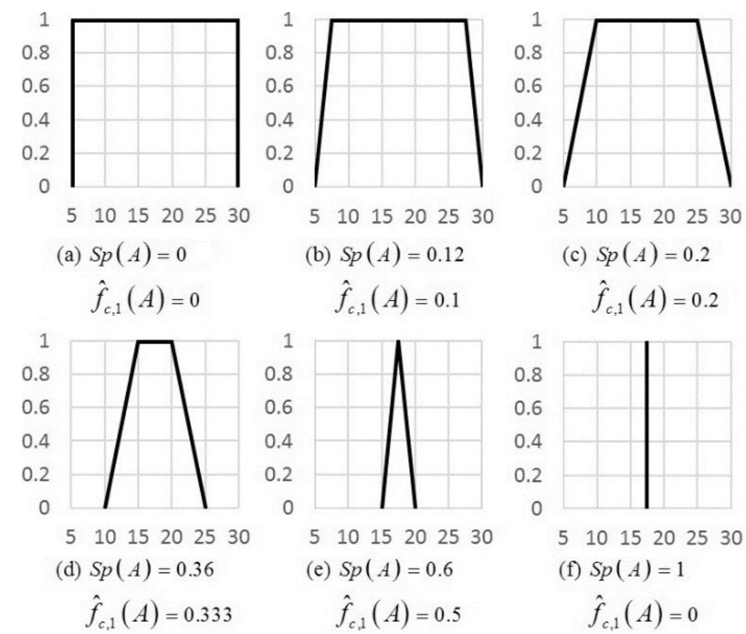



Figure 2.6. Comparison of specificity and fuzziness of fuzzy sets

### 2.1.6 Fuzzy rule base

Fuzzy sets based methods allow modelling systems where the knowledge cannot be represented using formulae, due to the lack of information or complexity of relationships. In this thesis, it is achieved by representing the knowledge using linguistic terms and relationships between concepts using fuzzy IF-THEN rules (Zadeh, 1973).

Let us consider a relationship between Node X and Node Z, as defined in Figure 2.7. Both nodes are defined in the same universe of discourse, defining variation, which is represented using seven linguistic terms: *Decreased Much, Decreased, Decreased Little, Maintained, Increased Little, Increased and Increased Much*. These linguistic terms are defined by the set of membership functions defined as in Figure 2.8.


Figure 2.7. Relationship between Node X and Node Z

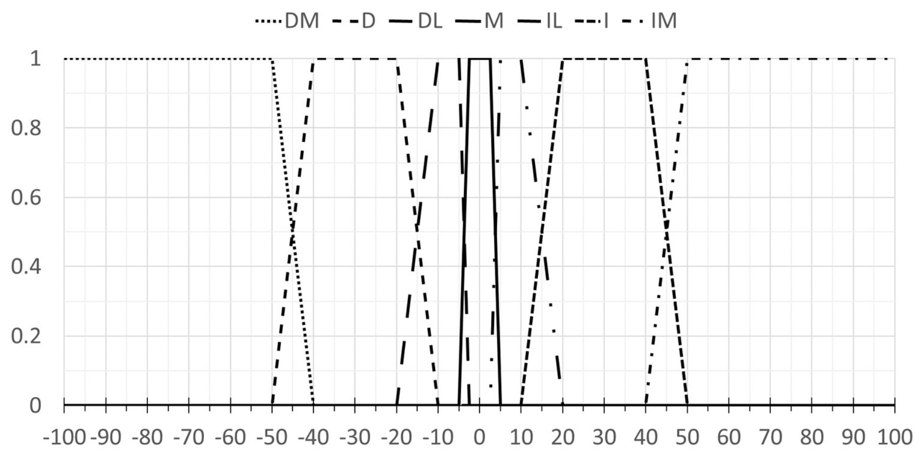



Figure 2.8. Definition of membership functions representing variations

The relationship between the Node X and Node Z is defined using prepositions that consist of IF part, called an **antecedent** and THEN part, called a **consequent** of a rule. These IF-THEN prepositions are called rules that build the relationship. An example of one IF-THEN rule defining relationship between Node X and Node Z can be expressed as follows:

IF Node X is *Increased Little* THEN Node Z is *Increased*

In this rule the statement: “IF Node X is *Increased Little*” is the antecedent and the statement: “THEN Node Z is *Increased*” is the consequent of the rule.

In case when more than one node has impact on the Node Z, e.g. Node X and Node Y, then the antecedent of the rule is composed by combining the linguistic terms of both nodes with AND or OR operator:

IF Node X is *Increased Little* **AND** Node Y is *Increased Little* THEN Node Z Is *Increased*

IF Node X is *Increased Little* **OR** Node Y is *Increased Little* THEN Node Z Is *Increased*

### 2.1.7 Approximate reasoning

The rule base defined by experts can be informative about the system or the perception of the experts but does not allow its in-depth analysis. To enable it a reasoning (Mamdani, 1977) on the gathered knowledge needs to be performed.

In the fuzzy rule based systems the reasoning consists of three phases (Figure 2.9):

1. fuzzification of the impact – where the crisp input into a node, $x$, is mapped on the corresponding fuzzy sets, having membership degrees for the input $x$ greater than 0,
2. inference – once the fuzzy sets having membership degree greater than 0 are identified, then the appropriate rules are fired, i.e. those that have the respective fuzzy sets in their antecedent. The consequent fuzzy sets are inferred using min (Mamdani, 1977) or algebraic product (Larsen, 1980) and then aggregated using max operator,
3. defuzzification of the impact – once the consequent fuzzy sets are aggregated the resulting fuzzy set is defuzzified to obtain a crisp output.

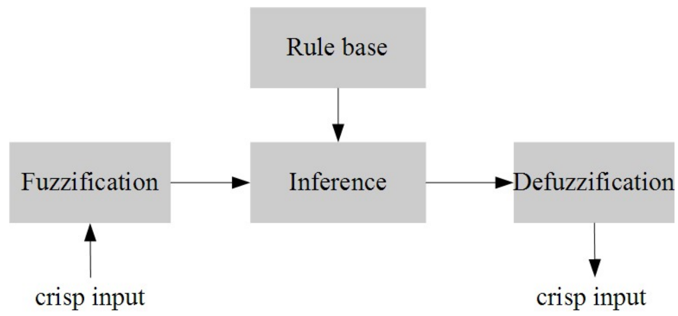



Figure 2.9 Approximate reasoning in rule based fuzzy systems

#### 2.1.7.1 Fuzzification of the crisp input

Let us assume a relationship between a causal node: Node X and an effect node: Node Z, as presented in Figure 2.7. The relationship between the two nodes is triggered by a crisp input into the causal node, Node X. The input, $x$, is mapped on the universe of discourse and all fuzzy sets to which the element $x$ belongs are determined.

#### 2.1.7.2 Inference

Let us assume, that the crisp input, $x$, into the Node X, falls within the support of two fuzzy sets, *Decreased Little* and *Maintained* (Figure 2.10).

The first step of inference is identifying rules that have fuzzy sets *Decreased Little* and *Maintained*, triggered by the input, in their antecedents. Let us assume that the following two rules are fired in consequence of the input $x$:

1. IF Node X is *Decreased Little* $(m_{A_i})$
   THEN Node Z is *Maintained* $(m_{C_i})$
2. IF Node X is *Maintained* $(m_{A_j})$
   THEN Node Z is *Increased Little* $(m_{C_j})$

The firing levels of rules correspond to the membership degrees of fuzzy sets triggered by the input. In the discussed example, rules 1 and 2 are fired with firing levels $m_{A_i}$ and $m_{A_j}$, respectively.

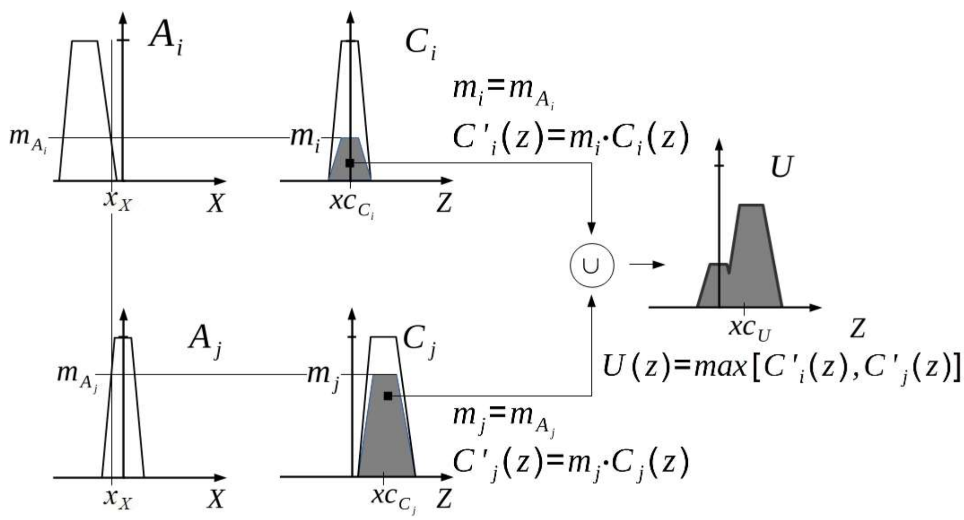



Figure 2.10. Mamdani inference when two rules are fired

The firing levels of consequents of the activated IF-THEN rules, are equal to the membership degrees of antecedents:

$$
\begin{aligned}
m_{C_i} &= m_{A_i} \\
m_{C_j} &= m_{A_j}
\end{aligned}
\tag{2.22}
$$

The fuzzy sets of the effect node are "cut", corresponding to their firing levels using product conjunction (Larsen, 1980):

$$
\begin{aligned}
C'_i &= C_i(z) \cdot m_i \\
C'_j &= C_j(z) \cdot m_j
\end{aligned}
\tag{2.23}
$$

Another conjunction method that is very often used in the rule based systems is the minimum conjunction (Mamdani, 1977) where fuzzy sets are "cut" as follows:

$$C'_i = C_i(z) \wedge m_i$$
$$C'_j = C_j(z) \wedge m_j \tag{2.24}$$

The union of two "cut" fuzzy sets are aggregated using maximum operator regardless of the conjunction method used:

$$U(z) = \max[C'_i(z), C'_j(\mathrm{z})] \tag{2.25}$$

#### 2.1.7.3 Defuzzification

Defuzzification is a method through which a crisp representation of a fuzzy set is calculated. There are many methods that can be used to defuzzify a fuzzy set, in this case the union of cut fuzzy sets. Methods the most often used are centroid, weighted average and mean of maxima (Roychowdhury and Pedrycz, 2001).

Centre of gravity (COG) defuzzification method (Braae and Rutherford, 1972) proved to be suitable in many applications due to high responsiveness to small changes in the input variables (Pham, 2002). In this method, the defuzzified value, $xc_U$, is highly affected by the areas of consequent fuzzy sets.

$$xc_U = \frac{\int z\mu_U(\mathrm{z})dz}{\int \mu_U(\mathrm{z})dz} \tag{2.26}$$

Weighted average (WA) defuzzification method weights centroids of the consequent fuzzy sets with their firing levels. The defuzzified value, $xc_U$, calculated using this method, changes linearly depending on the input, as this method does not consider the shape of the fuzzy sets.

$$xc_U = \frac{\sum_{i=1}^{n} m_i xc_{C_i}}{\sum_{i=1}^{n} m_i} \tag{2.27}$$

where $xc_{C_i}$ is the centroid of the fuzzy set $C_i$ and

$m_i$ is the firing level of i[th] fuzzy set.

Mean of maxima (MOM) defuzzification method calculates mean of all elements of fuzzy set U with the maximum membership degree.

$$xc_U = \frac{\int z\mu_{U,\max}(z)dz}{\int \mu_{U,\max}(z)dz} \tag{2.28}$$

where $z$ are elements of the fuzzy set U, for which $\mu_U = \mu_{U,\max}$ and

$\mu_{U,\max}$ is a max membership degree of a fuzzy set U

The defuzzification output for the three methods is presented in Figure 2.11.

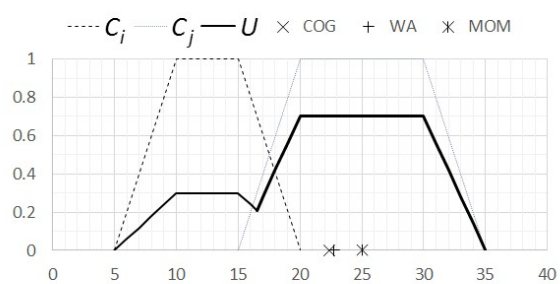



Figure 2.11 Outputs of defuzzification methods

In this thesis, centre of gravity (Braae and Rutherford, 1972) method is used to defuzzify fuzzy sets to obtain their crisp representation as it was the method authors of RBFCMs advised to use (constraint 3, section 2.4.1) and it is one of the most often used method (van Leekwijck, 1999) that proved to perform well in various applications (Runkler, 1997; Roychowdhury and Pedrycz, 2001). However, other defuzzification methods can be used to defuzzify fuzzy sets, in the RBFCMs proposed in this thesis, but impact of different defuzzification methods is not analysed in this research.

## 2.2 Cognitive Maps and Fuzzy Cognitive Maps

### 2.2.1 Cognitive Maps

Cognitive maps (CMs) have been introduced as a tool for modelling systems with a cause-effect type of relationships between entities (Axelrod, 1976). A CM is a graph where nodes represent concepts of a system, whereas the behaviour of the system is modelled by relationships among the nodes. There are two types of nodes; a node that has a causal influence on another node is the causal node, whereas a node that is impacted by this influence is an effect node. Relationships between the nodes in the CM are represented using adjacency matrix $[w_{ij}]$, where $w_{ij}=0$ means that there is no causal relationship, $w_{ij}=1$ means that there is a positive relationship between nodes i and j, i.e., if node i is *increased* then node j will *increase* and if $w_{ij}=-1$, there is a negative relationship, i.e. if node i is *increased* then node j will *decrease*. The CMs inference is done by analysing the pathways leading to the decision node (Wellman, 1994). Most systems modelled using CMs were socio-political and business systems (Axelrod, 1976; Axelrod, 1977; Levi and Tetlock, 1980; Bonham and Shapiro, 1986; Barr at al., 1992).

### 2.2.2 Fuzzy Cognitive Maps

To model more complex relationships between entities, Fuzzy Cognitive Maps (FCMs) were proposed (Kosko, 1986). In a FCM, uncertain relationships between entities are defined using fuzzy terms which represent degrees of causality: *a little, a lot, much, strong*, etc. These fuzzy relationships take values between [-1, 1], where -1 represents a maximally negative impact and 1 a maximally positive impact. Each node in the FCM has a value in the range [0, 1] or [-1, 1]. FCM can be represented using an adjacency matrix defining weights between concepts, where columns and rows represent nodes:

$$FCM = \begin{Bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & \dots & w_{1,n} \\ w_{2,1} & w_{2,2} & w_{2,3} & \dots & w_{2,n} \\ w_{3,1} & w_{3,2} & w_{3,3} & \dots & w_{3,n} \\ \dots & \dots & \dots & \dots & \dots \\ w_{n,1} & w_{n,2} & w_{n,3} & \dots & w_{n,n} \end{Bmatrix} \tag{2.29}$$

The value of the effect node, $A_j$, is calculated by summing values of the corresponding causal nodes, $A_i$ multiplied by degrees of causality – weights, $w_{i,j}$, of the corresponding relationships and the value of the node $A_j$ in the previous iteration.

$$A_j(t+1) = A_j(t) + \sum_{i=1}^{n} w_{i,j} A_i(t) \tag{2.30}$$

To ensure that the value of the node $A_j$ would not exceed the allowed range, [0, 1] or [-1, 1], a threshold function is used:

$$A_j = f\left( A_j(t) + \sum_{i=1}^{n} w_{i,j} A_i(t) \right) \tag{2.31}$$

The most common type of function to define a threshold is a sigmoid function (Stach et al., 2012):

$$f(x) = \frac{1}{1+e^{-5x}} \tag{2.32}$$

Sigmoid function allows nodes to have values in the interval of [0, 1] or [-1, 1], therefore leading to more insightful analysis of FCM simulations (Bueno and Salmeron, 2009). On the other hand, bivalent or trivalent functions allow nodes to have two or three of the following values: -1, 0 or 1.

The main advantages of FCMs that appeal to the modellers are: simplicity of the model, which simplifies knowledge extraction, FCMs are easy to build and interpret, mechanisms calculating states of nodes are easy to understand by nontechnical audience, low computational cost, ability to model maps with many nodes, possibility of adding optimisation and learning mechanisms (Papageorgiou, 2012).

The drawback of FCMs is linearity and monotony of relationships between causal node and effect nodes (Papageorgiou and Salmeron, 2013). Regardless of how the causal node changes, its impact is multiplied by a single weight defining its relationship with the effect node. Therefore, the impact received by the effect node is linearly dependent on the value of the causal node and weight of the relationship. A relationship in an FCM, is monotonic and symmetrical about midpoint of the range of values that a causal node can take when the threshold function is used to calculate the value of the effect node, i.e. for range equal to [-1,1], the relationship is symmetrical in point 0.

FCMs are a modelling tool that allows to model dynamic systems, however the representation of time is limited and rather simplistic. It is difficult to relate the actual time to iterations of the FCM simulation, therefore FCMs can only simulate long term trends. Moreover, FCMs cannot model time delays between nodes, therefore impact of the causal node on the effect node is always observable after one iteration.

The knowledge in FCMs is represented using intervals: [0,1] or [-1,1]. Such simplification of complex concepts reduces the significance of the outputs. The new value of the node represents the direction and strength of the change but not its actual value.

Despite the limitations of FCMs, they have become a widely used modelling tool. FCMs have been used in a diverse range of applications, such as:

- Engineering (Stylios and Groumpos, 1999),
- Strategic planning (Andreou et al., 2005),
- Information Technology (Rodriguez-Repiso et al., 2007 ; Salmeron and Lopez, 2012),
- Project management, and investment analysis (Lee and Choi, 2004),
- Medicine (Amirkhani, 2017)
    - Decision making: e.g. cancer risk assessment (Papageorgiou at al., 2015; Buyukavcu at.al, 2016)
    - Diagnosis: e.g. Parkinson's (Anninou and Groumpos, 2014), Meningitis (Mango at al., 2012)
    - Prediction: e.g. HIV (Nápoles at al., 2014)
    - Classification: e.g. brain and breast tumour (Papageorgiou at al., 2008; Subramanian et al., 2015)
- Environment/ecology/terrain domain (Liu, 2003),
- Military (Yaman and Polat, 2009),
- Education (Mendonca, 2015).

### 2.2.3 Extensions of Fuzzy Cognitive Maps

FCMs require precise knowledge as an input into the system. This becomes a problem when the data is uncertain and experts are not able to extract certain weights connecting the concepts. To overcome this limitation Salmeron (2010) proposed Fuzzy Grey Cognitive Maps (FGCMs). This solution is very useful in multiple meaning, uncertain environments where crisp weights between nodes cannot be specified. While fuzzy sets are expressing qualitative meaning using a set of data with degrees of belief, grey sets are expressing a range value of the object not the meaning itself. This allows to create nodes and relations between them that are indescribable using linguistic terms but by a range of possible values they can have. According to Grey Sets Theory there are three degrees of uncertainty: black (range with only one lower/upper limit or without limits), grey (when the range is finite and limited by two numbers), white (both limits have the same value – weight is expressed by a crisp value). In case when all relationships between nodes have white degree of uncertainty, the map looks the same as the traditional FCMs. The output of FGCMs are nodes with values expressed as a range. Such results are giving more freedom for interpretation for the person analysing the simulation. In case of very sensitive systems the results are more reliable, as in case of FCMs the crisp value can be misleading. Important thing to mention is that FCMs are less complex than FGCMs. FGCMs are becoming FCMs in very rare situation when all the relationships between nodes are white, thus FGCMs contain much more knowledge what is a benefit during analysis performed by the system.

FGCM approach introduced new way of managing uncertainty in FCMs but lacks ability to model dynamics and more complex (i.e. non-linear) relationships. Further development of those areas can lead to more comprehensive and very reliable model to be used by experts.

An alternative approach to handling uncertainty in FCMs was proposed by Iakovidis and Papageorgiou (2009; 2013), who developed Intuitionistic FCMs (iFCMs). iFCMs allow specifying not only the strength of the relationship but also a degree of hesitancy of the expert regarding the relationship between two concepts. The new approach was developed to enhance medical diagnosis using FCMs. It allows to reduce the impact of the symptom about which the experts are not certain. The preliminary analysis, performed by the authors, presented an improvement in diagnosis of a disease, over the standard FCMs.

Haigwara, (1992), started to explore the possibility of using more advanced dynamics in FCMs by applying delay to relationships between nodes. This concept was further investigated by Miao at. al. (2001), who further developed Dynamic Cognitive Maps (DCM), extension of FCM. DCM have two new types of dynamic relationship, allowing more complex representation of dynamics, but requiring specifying multiple parameters, such as inertia rate. The complexity of this approach makes DCMs very difficult to use, especially for systems based on experts knowledge, that require simplicity of knowledge representation.

Zhong et al., (2008), investigated more complex use of time in FCMs. They developed dynamic and balanced model of FCMs. In the dynamic model the change in the effect node is caused by the change in the causal node. On the other hand, in the balanced model the impact on the effect node is dependent on the state of the causal node, regardless of how it changed in the previous iteration. Moreover, authors introduced new type of function that defines the decaying of the state of the node and self-response relationship, where a node has an impact on itself. Novelties proposed by the authors significantly improved modelling of dynamics. It does not address other limitations of FCMs, such as linearity of relationships and limited ability to represent uncertainty.

Most of the systems nowadays are very complex and modelling them results in a big map with many relations. Architecture proposed by Stylios, (2008), allows developing medical decision support systems where many factors and inputs must be considered before making a final diagnosis. The main aim of the system is to analyse the data the user inputs and to provide most probable disease/disability the person could have. Authors of the research have outlined three main novelties in their system:

• the competitive FCM – which is a FCM map where one of many diagnoses has to be chosen basing on the factors provided,

• distributed m-FCM – m independent but interrelated (sharing with each other knowledge) FCMs forming together complex medical decision support system,

• two level architecture for decision support. The upper level of the architecture consists of m-FCM system and the lower one is responsible for handling input from the user or for initial analysis of some factors. The output of the lower level is being transferred as an input into the upper level, after being appropriately transformed in the interface layer.

This architecture has been successfully applied for medical diagnosis of multiple diseases and proved to be helpful in distinguishing problems that have very similar symptoms.

The disadvantages of this method is its simplicity of representation of dynamics, it can represent time in a similar way to standard FCMs, lack of ability to model non-linear and non-monotonic relationships between nodes.

Another interesting development of FCMs is military Effect Based Operations (EBO) planning, where the focus is concentrated on how to achieve desired effects by best use of existing military and non-military resources. Proposed by Yaman and Polat (2009) extended FCMs is designed to best fulfil requirements that are necessary to plan military actions using

EBO methodology. This required improving fuzzy relationships between causal and effect nodes with:

• influence possibility – not all changes in the causal node will cause change in effect node, this is managed by the built-in randomness,

• influence duration – the time it takes to observe full impact (which gradually raises with time), of the change in causal node, on the effect node,

• influence degree – the amount of change (weight) between the causal node and effect node that will be applied as a result of change in the causal node,

• influence permanence – decay of the influence (resulting from the change in the causal node) on the effect node.

Those features allowed to create dynamic system with time attribute, being able to provide a dynamic analysis what is not possible with traditional FCMs.

This approach was designed to solve specific type of problems but it could be adapted to other, non-military environments, where EBO planning (or similar methodology) is being used.

The drawback of this method is complexity of the knowledge representation required to model the system. To model the relationships between concepts information about strength, duration and decay is required. Also, the relationships between concepts are monotonic and linear what can be a limitation in some cases.

Rickard at. al. (2015) proposed a new method for aggregating impacts in FCM: Weighted Power Mean. Aggregation of impacts is more complex than in standard FCM and requires development of two matrices defining positive and negative causality between concepts. The new aggregation method does not solve the problem of linearity of relationships and limited representation of dynamics.

### 2.2.4 Fuzzy expert systems

Fuzzy Expert Systems (FES) (Zadeh, 1983) is a rule based fuzzy modelling technique that uses the approximate reasoning discussed in section 2.1.7.

Every node of the system has its own universe of discourse, which defines all possible values the node can take. On the same universe of discourse fuzzy sets are defined, using membership functions, which allow representing the value of the node, using linguistic variables. Relationships between nodes are defined using IF-THEN rules. In contrast to FCMs, fuzzy IF-THEN rules enable modelling of non-monotonic and non-symmetrical relationships among nodes. FES allow more complex knowledge representation than FCMs and allow eliciting knowledge in a way which is easier to comprehend for expert unfamiliar with any modelling tools.

The application specific FES are Fuzzy Control Systems (FCS) (Mamdani, 1977). FCS are used to control the state of the system based on its parameters and instructions defined using fuzzy IF-THEN rules – therefore they are primarily applied in engineering systems.

## 2.3 System Dynamics

System Dynamics (SD) is an approach to modelling dynamic systems with causal relationships (Forrester, 1971). Main elements of the model are stocks – representing accumulation of objects and flows – defining the rate of outflow from the stock or inflow into the stock (Figure 2.12 (a)).

Relationships between elements of the system dynamics model are defined using mathematical formulas representing rate of change over time, therefore they can capture nonlinear causality between concepts.

Building SD model starts with drafting influence diagram which defines relationships between concepts. If the relationship is positive, then the arrow, indicating direction of the causal relationship, has a plus symbol assigned. If the relationship is negative, there is the minus symbol next to the arrow (Figure 2.12 (b)). The next step of design is identification of the feedback loops within the model. There are two types of feedback loops: reinforcing and balancing (Figure 2.12 (b)). A reinforcing loop indicates that any impact introduced into the loop would result in magnification of this impact – positive or negative. A balancing loop, on the other hand, has a counter effect – introducing positive impact in the loop results in a decrease of a node over time. Identification of the feedback loops allow better understanding of the system and its flows.

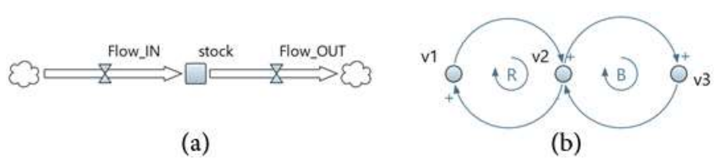



Figure 2.12. Core elements of SD: stocks and flows (a) and balancing and reinforcing loops (b).

The advantage of the SD is simplicity of the structure of models and calculations. They are based on differential equations which are easy to implement and are not computationally demanding. The relationships between objects can be nonlinear therefore they can represent complex relationships, on the contrary to FCMs.

The disadvantage of SD is requirement of precise mathematical functions that represent relationships between elements of the model. This becomes a significant problem when modelling systems, where precise information about relationships between concepts is not available. SD are difficult to use to model systems where the only source of the knowledge are experts.

## 2.4 Rule Based Fuzzy Cognitive Maps

An important step in the evolution of the cognitive modelling, was introduction of RBFCM. The new modelling approach allows modelling relationships between concepts using fuzzy IF-THEN rules and representing value of a concept using linguistic variables. RBFCMs allow more complex representation of knowledge, uncertainty and dynamics while achieving similar modelling capabilities to FCMs.

Nodes in the RBFCMs model are linked with a cause-effect type of relationships. Causal relationship between two nodes is represented using an arrow, where the head of the arrow points towards an effect node, which is impacted by a causal node, indicated by the beginning of the arrow.

### 2.4.1 Reasoning

The reasoning in RBFCMs consists of the following steps (Figure 2.13):

1. fuzzification and inference – executed in the same way as in the approximate reasoning (Chapter 2.1.7),

2. interpolation – interpolation of the output of the inference on the fuzzy set on the universe of discourse of the effect node,
3. accumulation of impacts – multiple impacts received by an effect node are accumulated to obtain two fuzzy sets – representing positive and negative impacts,
4. defuzzification of impacts – positive and negative fuzzy sets are defuzzified to obtain a crisp output.

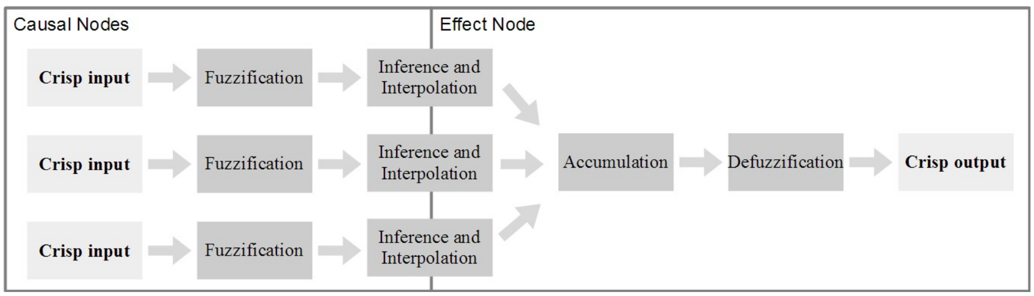



Figure 2.13. Reasoning steps in RBFCM

RBFCMs introduced two novel mechanisms: interpolation of the crisp value into a fuzzy set representing this value and accumulation of impacts mechanism, which allows adding multiple impacts.

The RBFCMs reasoning mechanism was developed under set of constraints:

1. The sum of membership degrees $\mu(x)$ of all membership functions $A_i$, defined on $\mathcal{F}(X)$ has to be equal to 1, for any point $x$: $\sum \mu(x) = 1$,
2. The membership functions of two consecutive fuzzy sets need to cross in point $\mu(x) = 0.5$,
3. The inference method must preserve both the shape and the centroid value of the consequent fuzzy set,

4. The output of the inference must be the union, U, of fuzzy sets of consequents of triggered rules, determined using Max-dot method,
5. The fuzzy set resulting from the interpolation must have the same area as the union U,
6. Fuzzy set representing greater variation needs to have a greater area and support than fuzzy set representing a smaller variation:

$$\forall A,B \in \mathcal{F}\left(X\right),\ A > B \Leftrightarrow support_A > support_B \wedge Area_A > Area_B \tag{2.33}$$

## 2.4.2 Semantics of fuzzy sets

To enable the development of new reasoning, Carvalho and Tome introduced a relationship between the shape and the variation (level) the fuzzy set represents. Fuzzy set A is representing greater variation than fuzzy set B when its support and area is greater than respective characteristics of the fuzzy set B (equation 2.33).

Upon this relationship, the interpolation mechanism and accumulation of impacts have been built. Such condition greatly affects the way the fuzzy sets must be constructed and limits expert's flexibility when defining fuzzy sets (as discussed in Chapters 4.3, 5.3 and 6.4).

## 2.4.3 Interpolation of Linguistic Terms

In the standard approximate reasoning methods, discussed in Chapter 2.1.7, the reasoning process finishes once the impact on the effect node is defuzzified. In the RBFCM, the received impact needs to have a fuzzified form to carry on the accumulation process. This can be achieved through interpolation of the crisp impact on the new fuzzy set. This process is called Interpolation of Linguistic Term (ILT).

Let us assume that the causal node: Node X is in causal relationship with the effect node: Node Z (Figure 2.14).

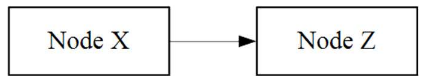



Figure 2.14. Relationship between Node X and Node Z

When the input in the Node X falls within the core of one of the fuzzy sets, $A_i = Maintained$, then only one rule, which has the linguistic variable *Maintained,* in its antecedent, is fired:

IF Node X is *Maintained* ($m_{A_i}$)
THEN Node Z is *Increased Little* ($m_{C_i}$)

Impact received by the Node Z, is determined by the consequent part of the fired rule. In this case, it is the fuzzy set $C_i = Increased\ Little$. In the discussed example, the impact received by the Node Z does not need to be interpolated, as it is represented by the fuzzy set *Increased Little* (Figure 2.15).

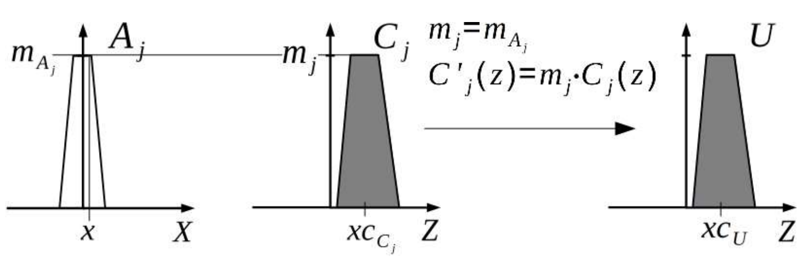



Figure 2.15. Calculation of the output set when one rule is fired.

Let us consider a different scenario, when the input into the node Node X falls in between supports of two fuzzy sets *Decreased Little and Maintained*. This triggers two rules *i* and *j*, with degrees of belief $m_{A_i}$ and $m_{A_j}$, respectively, where $m_{A_i} + m_{A_j} = 1$.

$$
\begin{aligned}
&\mathrm{R}_i\text{: IF Node X is } \textit{Decreased Little } (m_{A_i}) \\
&\quad \text{THEN Node Z is } \textit{Maintained } (m_{C_i}) \\
&\mathrm{R}_j\text{: IF Node X is } \textit{Maintained } (m_{A_j}) \\
&\quad \text{THEN Node Z is } \textit{Increased Little } (m_{C_j})
\end{aligned}
$$

where degrees of belief of the antecedent and the consequent are given in the brackets.

To handle the case when two rules are fired, the inference is carried on as described in Chapter 2.1.7. The output of the inference is the centroid $xc_U$ of the union of two fuzzy impacts, *U*, which represents the crisp impact received by the Node Z. The next step of the interpolation process is determining a new fuzzy set Causal Output Set ($COS_U$), which represents a combined variation and preserves the trapezoidal shape of the two consequent fuzzy sets. $COS_U$ is determined in such a way as to satisfy the following condition:

$$Area_{COS_U} = Area_U \tag{2.34}$$

The values of $core_{COS_U}$ and $bi_{COS_U}$ are calculated based on the distance between defuzzified consequent fuzzy sets $C_i$ and $C_j$, $xc_{C_i}$ and $xc_{C_j}$, respectively, and defuzzified union of the two "cut" fuzzy sets, $xc_U$, as follows:

$$core_{COS_U} = min\left\{core_{C_i}, core_{C_j}\right\} + \left| \frac{xc_U - xc_{C_i}}{xc_{C_j} - xc_{C_i}} \times \left(core_{C_j} - core_{C_i}\right) \right| \tag{2.35}$$

$$bi_{COS_U} = min\left\{bi_{C_i}, bi_{C_j}\right\} + \left| \frac{xc_U - xc_{C_i}}{xc_{C_j} - xc_{C_i}} \times \left(bi_{C_j} - bi_{C_i}\right) \right| \tag{2.36}$$

where fuzzy sets are defuzzified using centre of gravity method (Braae and Rutherford, 1972).

$bi_{COS_U}$ and $core_{COS_U}$ are used to calculate the remaining characteristics of $COS_U$: $support_{COS_U}$ and $bo_{COS_U}$ , as follows:

$$bo_{COS_U} = 2\,Area_{COS_U} - (2\,core_{COS_U} + bi_{COS_U}) \quad (2.37)$$

$$support_{COS_U} = bi_{COS_U} + core_{COS_U} + bo_{COS_U} \quad (2.38)$$

The centroid of new fuzzy set $COS_U$ is calculated using the centre of gravity defuzzification method and, then, the fuzzy set $COS_U$ is shifted so that $xc_{COS_U} = xc_U$ as in Figure 2.16. Fuzzy set $COS_U$ represents the impact of the causal node on the effect node.

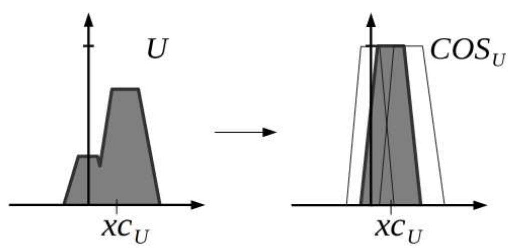



Figure 2.16 Transformation of union U into the fuzzy set $COS_U$

## 2.4.4 Dynamics

RBFCMs are dynamic systems with embedded time component. It is represented by a base time unit, which defines the minimum period of time at which the state of the system is analysed. Depending on the degree of sensitivity of the system, identified by the experts, the base time unit can take value of 1 hour, 1 day, 1 week, for accurate systems and 1 month, 6 months, 1 year, for less accurate systems. In the proposed RBFCM, every relationship has attached a number of base time units, that need to pass in order to observe the result of the impact of a causal node on an effect node. After every iteration, $t$ , equal to the duration of one base time unit, the values of all the nodes in the RBFCMs are analysed and all relationships are fired. All relationships need to be fired as some may have deteriorating effect: *maintaining* the value of the Node X can cause a *decrease* in the Node Z.

Let us assume that the base time unit is equal to 1 year and Node X is in a relationship with Node Z, with 3 base time units specified for this relationship. Such definition of the relationship means that it will take 3 years to observe the change on the Node Z if the Node X is varied.

### 2.4.5 Accumulation of impacts

Accumulation process is carried out when two or more causal nodes are affecting an effect node. Let us consider a case where two causal nodes: Node X and Node Y are in a causal relationship with one effect node, Node Z, as shown in Figure 2.17. If, for example, Node X is *Increased* and causes Node Z to *Increase* and Node Y is also *Increased* and causes Node Z to *Increase Little*, it seems appropriate that Node Z should change to *More than Increase*. However, the standard inference mechanisms, such as Mamdani, (1977), or Takagi and Sugeno, (1985), will result in Node Z changing somewhere between value *Increased Little* and *Increased* depending on the strength of each impact. Therefore, a different mechanism for accumulation of impacts in RBFCMs is required, as proposed in (Carvalho, 2004).

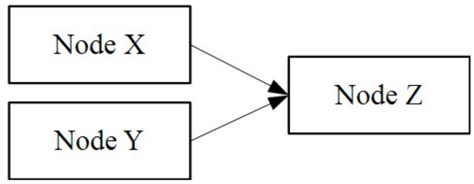



Figure 2.17. Impact of two causal nodes on the effect Node Z.

The accumulation of impacts mechanism requires shifting the fuzzy set representing a lower variation towards the fuzzy set representing a greater variation. Before shifting and accumulation are conducted, it is necessary to determine which fuzzy set represents lower variation. Fuzzy set A represents lower variation then fuzzy set B when:

$$min(core_A) < min(core_B) \tag{2.39}$$

Fuzzy set A is shifted towards fuzzy set B until the following condition is met (Figure 2.18 (a)):

$$min(support_{A_{shifted}}) = min(core_B) \tag{2.40}$$

Therefore, fuzzy set A is shifted towards fuzzy set B by distance:

$$shift_A = min(core_B) - min(support_A) \tag{2.41}$$

The standard RBFCMs accumulation process is a discrete, recursive algorithm. It accumulates degrees of belief of every point x as the sum of the corresponding degrees of belief of two fuzzy sets, shifted A and B; if it is greater than 1, then the surplus in degrees is carried forward towards points where the sum of degrees of belief is lower than 1:

$$\mu_{VOS_+}(x) = min\{1, \mu_B(x_i) + \mu_A(x_i - shift_A) + carry(x_{i-1})\} \tag{2.42}$$

where $VOS_+$ is a fuzzy set resulting from shifting of the fuzzy set A towards fuzzy set B and their accumulation.

$$\begin{aligned} carry(x_i) &= max\{0, \mu_B(x_i) + \mu_A(x_i - shift_A) + carry(x_{i-1}) - 1\} \\ carry(x_{-1}) &= 0 \end{aligned} \tag{2.43}$$

All positive impacts are accumulated resulting in positive Variation Output Set, $VOS_+$. The result of accumulating two positive impacts A and B, presented in Figure 2.18 (a), is shown in Figure 2.18 (b). The centroid of $xc_{VOS_+}$ is calculated using the centre of gravity defuzzification method (Braae and Rutherford, 1972).

Described algorithm performs accumulation of positive impacts. All accumulated negative impacts form negative Variation Output Set, $VOS_-$. In order to accumulate negative impacts, when fuzzy sets A and B represent negative variations, described algorithm needs to be altered accordingly, i.e. fuzzy set A is shifted to fuzzy set B until the following condition is fullfiled

$$\max\left(support_{A_{shifted}}\right) = \max\left(core_B\right) \tag{2.44}$$

The only characteristic of the involved fuzzy sets that is preserved during this shifting approach is the inner base of the fuzzy set representing the greater variation. In the given example, the inner base of $\mathrm{VOS}_+$ is the same as the inner base of fuzzy set B. The information about position of the shifted fuzzy set is lost.

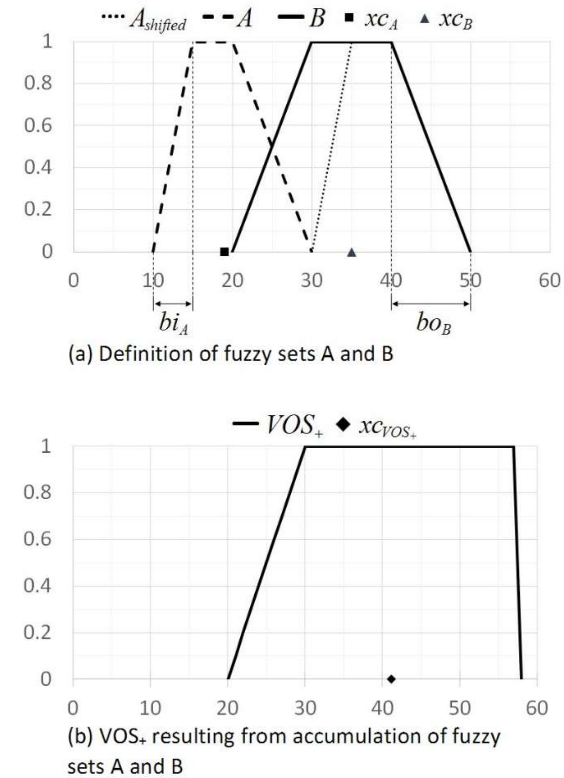



Figure 2.18 Accumulation of impacts using the standard accumulation mechanism

After accumulation of both positive and negative impacts, the total variation received by the effect node is determined as the sum of the centroids $xc_{VOS_+}$ and $xc_{VOS_-}$ of the two variation output sets, positive $VOS_+$ and negative $VOS_-$ respectively, weighted by their respective areas:

$$\text{variation} = \frac{xc_{VOS_+} Area_{VOS_+} + xc_{VOS_-} Area_{VOS_-}}{Area_{VOS_+} + Area_{VOS_-}} \tag{2.45}$$

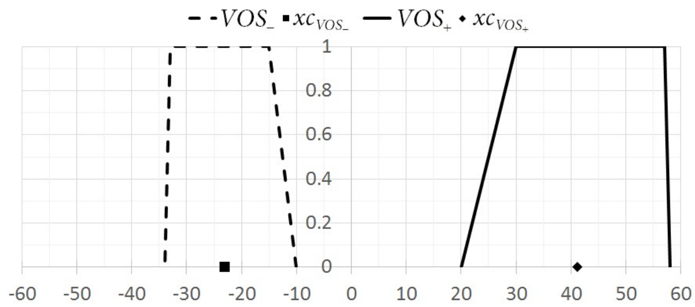



Figure 2.19. Example of VOS+ and VOS_

## 2.4.6 Saturation of impacts

The saturation is done when a value of a concept approaches its maximum acceptable value. In case of RBFCMs saturation is done when one of elements of accumulated fuzzy sets, $x$, is greater than the maximum or minimum of the universe of discourse $X$:

$$x > \max(X) \vee x < \min(X)$$

The saturation is done by shifting all the points of the fuzzy set $VOS$ by the same distance equal to the difference between the maximum of the universe of discourse and the furthest point defining the fuzzy set $VOS$:

$$VOS_S = \begin{cases} a_{VOS_S} = a_{VOS} + l \\ b_{VOS_S} = b_{VOS} + l \\ c_{VOS_S} = c_{VOS} - l \\ d_{VOS_S} = d_{VOS} - l \end{cases} \tag{2.46}$$

where $l = \max(support_{VOS}) - \max(X)$

Let us consider accumulation of two impacts $A$ and $B$, defined as in Figure 2.20 (a), whose accumulated value is represented by the fuzzy set $VOS$, as in Figure 2.20 (b). Point $d$ of the fuzzy set $VOS$ extends beyond the maximum of the universe of discourse.

Saturation of the resulting $VOS$ results in reduction of its size, therefore authors (Carvalho, 2004) suggest retaining the information about the size of the fuzzy set $VOS$ separately as it is required to calculate a defuzzified value of the concept (Formula 2.45).

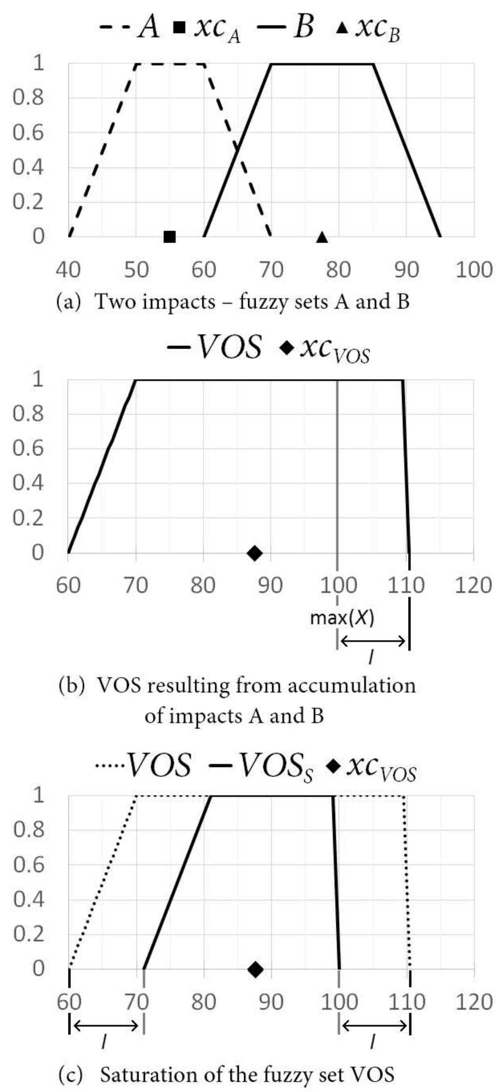



Figure 2.20. Saturation of VOS, resulting from accumulation of impacts A and B

The intrinsic characteristic of the saturation method discussed, is the fact that resulting fuzzy set $VOS_S$ does not preserve uncertainty of the fuzzy set $VOS$ as the points of fuzzy set $VOS_S$ are moved rather that scaled. It also has an impact on the centroid of the fuzzy set $VOS_S$, which is not equal to the centroid of the fuzzy set $VOS$.

### 2.4.7 Applications of RBFCMs

RBFCMs have been successfully used in a few applications.

The first application of RBFCMs was a generic socio-economical system where the input is the current economic situation and the RBFCM simulation calculates the future state of the various elements of the economy, e.g. growth, interest rates, inflation and oil prices.

The second application of RBFCMs was modelling of propagation of a forest fire (Tome et al., 2006). The forest is represented using multiple segments. Each segment can be in one of four states: intact, burning, extinct, burning. Transition between states, as well as propagation of the fire from one segment to another is controlled using RBFCMs.

The next area, where RBFCMs was used, is modelling of fisherman behaviour (Wise et al., 2012; Wise et al., 2015). The developed model simulates decisions of the fisherman: departure from the port, fishing and return to the port, ones the fishing activity is finished. Decisions are made based on multiple factors: time of the day, amount of fish caught, number of hours spent at the sea, etc. The modelled behaviour is compared with the actual behaviour of fishermen. RBFCM model accurately predicts the behaviour of fishermen.

The most recent application of RBFCMs is student-centred education (Peña-Ayala and Sossa-Azuela, 2014). Student-cantered education aims to tailor the content and presentation of learning materials, to the student's needs. In this application, RBFCM is used to help to choose the content that will be delivered to the student, therefore it can be considered as a decision support system.

## 2.5 Discrete Event Simulation

Discrete Event Simulation (DES) allows modelling systems on the operational level. DES models consist of events triggered in discrete points in time and are associated with activities of the entities that enter the system or interact with it. Every time an event happens the state of the system changes, i.e. number of entities in the system, availability of resources, etc. During time between events, the system is considered to be idle and its state remains unchanged.

DES models allow reproducing very detailed processes used by an organisation and usually focus on the isolated operation of a small subsystem. DES is very often used to represent operation of medical institutions (Caro and Moller, 2016), manufacturing processes (Jahangirian, 2010; Negahban and Smith, 2014) and other queue based systems (Kang and Choi, 2013).

DES models allow performing "what-if" analysis to investigate the impact of changes in the operation of a system. The output of the analysis is represented using Key Performance Indicators (KPIs) such as: average time required to process an entity, percentage of time a resource was busy, average length of the queue, etc.

### 2.5.1 Integration of DES and other methods

To increase modelling capabilities of DES, it is very often used in conjunction with SD (described in Section 2.3), which allows analysing long term, strategic changes in the system. Combining SD and DES (Morgan, 2017) allows analysing the impact of introducing strategic changes into the system on the operation of its subsystem.

As discussed in many papers (Morgan, 2017; Howick and Ackermann, 2011) both DES and SD, even though substantially different, can be used to model the same problems but most likely will lead to different conclusions. The area of focus of the methods and modellers is different, as well as modelling possibilities that both methods provide. Tako and Robinson (2009) summarised differences between both methods as presented in Table 2.3.

Table 2.3 Comparison of DES and SD

| Aspects compared | DES | SD | Author (s) |
|---|---|---|---|
| Nature of problems modelled | Tactical/operational. | Strategic. | Sweetser 1999, Lane 2000 |
| Feedback effects | Models open loop structures – less interested in feedback. | Models causal relationships and feedback effects. | Coyle 1985, Sweetser 1999, Brailsford and Hilton, 2001 |
| System representation | Analytic view. | Holistic view. | Lane 2000 |
| Complexity | Narrow focus with great complexity & detail. | Wider focus, general & abstract systems. | Lane 2000 |
| Data inputs | Quantitative based on concrete processes. | Quantitative & qualitative, use of anecdotal data. | Sweetser 1999, Brailsford and Hilton 2001 |
| Validation | Black-box approach. | White-box approach. | Lane 2000 |
| Model results | Provides statistically valid estimates of system performance. | Provides a full picture (qualitative & quantitative) of system performance. | Mak 1993 |

Tako and Robinson (2006) compared the process of development of DES and SD models to analyse the problem of sustainable fishery, based on the case study and historical data. Both models to some extent managed to identify problems and produce results that were in line with historical data using completely different mechanisms: DES, by emphasizing randomness and SD, by emphasizing deterministic complexity of the system. As discussed by Morecroft and Robinson, (2009), real world system contains both. Authors concluded that DES and SD can be considered complementary methods.

An example of a hybrid SD and DES system was developed for screening and treating sexually transmitted infections (STI) (Sadsad, 2014). The SD model is used to capture high level social relationships between different risk groups and factors affecting increase or decrease of population being infected. DES models the structure of the department of the hospital responsible for treatment of STI. Entities of the system are patients who come to the hospital for treatment or diagnosis. The department consists of reception, where several receptionists register patients and answer phones, waiting room and hospital rooms, where doctors treat patients. SD model provides an information about number of people in the area that are at risk of being infected what influences the rate of patients coming to the hospital. The DES model helps to monitor the performance of the hospital's departments. The aim of the study is to increase the number of patients that are screened and treated in the hospital while maintaining the low cost of operation of the hospital. The number of treated patients and number of patients that left hospital because of the too long queue is used as an input to the SD model. Some aspects of the problem are modelled by both models: number of patients that are successfully treated. It is used to compare the output of both models.

Another type of hybrid system is integration of DES and FES. There are few attempts of combining DES and FES described in the literature (Kunnathur at. al, 2004; Shaheen at. al., 2009; Corona-Suárez, 2014) and all of them have similar structure. The FES are used to alter parameters of the DES simulation based on experts' knowledge defined using fuzzy IF-THEN rules. The FES model is triggered either at the beginning or during the process of the DES simulation.

Integration of FES and DES models was used to recreate an uncertain environment of a ground penetration (construction) problem (Shaheen at. al., 2009). After each segment is penetrated the fuzzy expert system is executed to calculate new penetration rate. Some of the elements of the FES are fuzzy: weather, time of the day and some are crisp: use of the machine (meters) or type of ground. Relationship between factors and penetration rate is represented using fuzzy IF THEN rules. Inference method used by the authors is Mamdani.

There is one feedback from the DES simulation model to FES. After each segment is penetrated, an information about number of penetrated meters is updated (Figure 2.21).

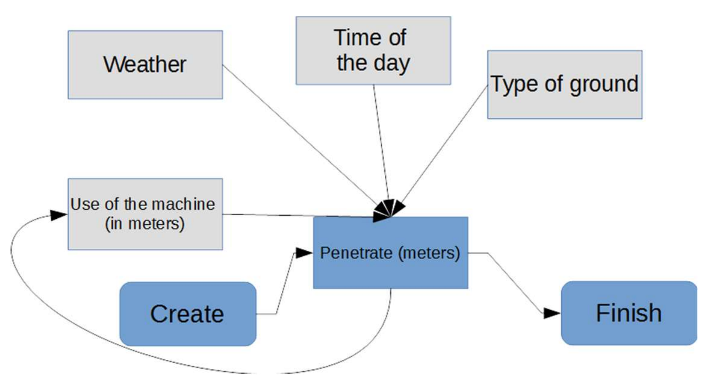



Figure 2.21. FES and DES models combined

## 2.6 Conclusions

This research work was prompted by the challenges encountered during the practical application of RBFCMs. Difficulty in representing experts' knowledge flexibly using fuzzy sets and lack of ability to model complex relationships between concepts significantly reduced modelling capabilities of RBFCMs. Use of standard mechanisms, proposed by Carvalho and Tome (2009) did not provide means to meet expectations of experts engaged in the project.

Improvement of RBFCMs is focused on the development of new mechanisms that are based on concepts that are well understood and widely accepted among practitioners of fuzzy sets: standard semantics of fuzzy sets and measures of uncertainty.

The advancements developed are demonstrated using real world case studies, where RBFCMs have not been implemented before. Moreover, a novel integration of RBFCMs and DES systems is presented as a possible direction of future work.

# Chapter 3. CASE STUDIES

## 3.1 Introduction

Until now, RBFCMs have been used in a limited number of case studies. The most common areas of applications have been: decision support systems, such as socio economical system, student-centred education and complex, dynamic, real world systems, such as modelling of fisherman behaviour and forest fire fighting. This chapter introduces two new applications in areas where RBFCMs have not been used before. Both case studies differ in the modelling subject as well as in the structure of the model.

In the first case study, RBFCMs are used to model the impact of investments on the military capability of military units. The second application investigates the impact of investments into cyber defence on the risk to the cyber security of an enterprise.

The significant difference between both applications lies in the structure of the models. The first system has a bottom up structure, where multiple, low level factors have indirect impact on the subject of the model, i.e. military ability of the military unit. Due to its bottom up structure, the model can be considered as an example of dynamic Fuzzy Expert System (FES).The second application is represented using five subsystems. The subject of the model is the risk to the business of an enterprise. In this case study, all the concepts have the same hierarchy and the subsystems are logical groupings of these concepts. The subsystems are interlinked; multiple concepts from one subsystem have impact on many concepts from another subsystem. The complicated interrelation of subsystems leads to the existence of multiple loops. As a result, the developed model resembles those created using FCMs or SDs.

This chapter consist of two sections 3.2 and 3.3, where two new applications are introduced.

## 3.2 Case study 1 – modelling of defence lines of development

The first case study, where the new RBFCMs have been developed, considers modelling of the Defence Lines of Development (DLoDs) for the Joint Strike Fighter (JSF) force. The purpose of the model is to analyse the impact of resource allocation to DLoDs on the military ability of the JSF, in respect to the location of the military base. In this case study, the resources allocated to the nodes are “budget” and “number of hours”, which through linear relationships contribute to an increase or decrease of the budget. Once the budget is allocated by military experts, it is possible to simulate its impact on the military ability of the military unit over time.

The literature on DLoDs is very limited and there is a great gap in the research on modelling of military ability of DLoDs (Rodrigues at al., 2015). The topics which are often covered is the importance and the structure of the DLoDs’ architecture (Newsome, 2003; Tatham at al., 2012) and the process of acquiring the military capability (Russel at al., 2008; Yue and Hanshaw, 2009). There were attempts made by Ministry of Defence (MoD) to model military ability using linear programming model. The complexity of the problem caused the models to be over complicated and difficult to analyse and maintain. This case study was triggered by the need of MoD to develop higher level, holistic, model to analyse military ability of force units.

### 3.2.1 Military structure

Capability Package (CP) is a military unit capable of delivering military tasks. It is considered as a composition of Force Elements (FEs), capable to perform specific military objectives through a very close cooperation.

To allow better understanding of this structure, let us consider an example of the Normandy Landing operation (commonly known as D-Day) (Badsey, 1990). The Normandy Landing operation required a cooperation of many different military forces from different countries to achieve the military goal of getting the foothold in France, then occupied by Germany. The main military units taking part in the operation were: infantry, airborne and naval divisions. A group of infantry soldiers, able to attack once on the shore of Normandy, represents one FE. Other examples of FE would be a group of ships transporting infantry soldiers, a nursing team and a communication team. While none of these FEs is able to achieve military goal of landing in Normandy on its own, combining their capabilities allows to reach specific military objectives. The combination of these multiple FEs is called a CP.

Every FE has a different level of readiness to perform its military tasks within the CP. UK MoD (UK Ministry of Defence, 2012) outlined eight core areas – DLoDs, that define the military capability of FEs. The visual depiction of the structure allowing analysis of the military capability is shown in Figure 3.1.

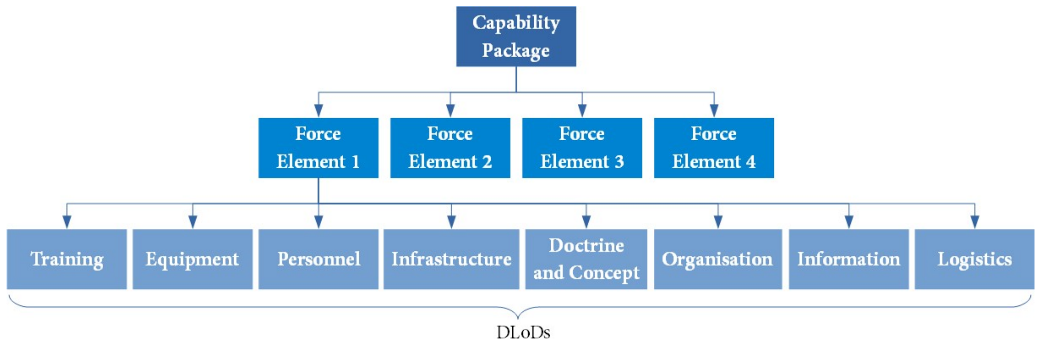



Figure 3.1. Structure of the CP

### 3.2.2 Capability Package

CP is a military unit, that consist of several FEs and is formed to perform specific military tasks. As an example, in order to perform a land attack on a target, the CP of a land army needs: soldiers, tanks, communication unit, special troops, medical unit, etc. The military capability of CP is expressed as its ability to perform specific tasks.

### 3.2.3 Force Elements

The smallest military unit within the MOD is the FE. It consists of groups of soldiers, or other personnel, and equipment capable of performing the same operation. Examples of FEs are as follows: medical unit, radio control unit, infantry, navy unit. Each of the FEs has a capability of performing a range of tasks, specific to this unit. Additionally, the FEs can perform other tasks when combined with other FEs.

The case study presented in this subchapter, is focused on modelling of DLoDs of a very specific FE –JSF. JSF is a FE within the Royal Air Force, the UK's aerial warfare force.

### 3.2.4 Defence Lines of Development

DLoDs are abstract concepts that allow categorising the factors contributing to the military capability of the FE. The eight DLoDs are: Training, Equipment, Personnel, Infrastructure, Doctrine and Concept, Organisation, Information, Logistics. Elements of these eight categories are interrelated.

## 3.3 Case study 2 – modelling of the Cyber Defence

The aim of the second case study is to analyse the impact of the high level, strategical investments into cyber security, on the risk to the business of an enterprise. This case study is in its nature very different from the JSF model, discussed in the subchapter 3.2.

The DLoDs model was developed for a very specific purpose of analysing the impact of resource allocations on one strategic, military decision. On the other hand, the cyber defence model is applied to analysis of the risk to the cyber security of any big enterprise whose security is closely related to the safety of their Communication and Information Systems (CIS) and their abilities to responding to cyber threats.

Modelling of the risk to enterprises has been a very important topic in the recent years (Yeo, 2012; Sawik, 2013; Yeo at al., 2014; Garvey and Patel, 2014; Zhuo and Solak, 2015). It is due to the increase of informatisation of the CISs as well as the increase of threats to these systems, resulting from vulnerability of IT technologies. Many elements of the cyber defence systems are supposed to mitigate the risk to the enterprise, resulting from vulnerabilities of CISs. Most authors focus on modelling of performance of individual aspects of the cyber security (Sawik, 2013; Yeo at al., 2014; Garvey and Patel, 2014; Zhuo and Solak, 2015), without looking at the cyber defence in a holistic way. Application of RBFCMs to model cyber defence problem emerged from the lack of methods able to capture the level of abstraction required by experts and ability of RBFCMs to easily extract knowledge from experts about intangible concepts such as: understanding or ability to assess.

This chapter discusses a novel approach to analysis of the impact of investments on the risk to the business of an enterprise. The model developed in this case study provides a holistic overview of many elements comprising the cyber security systems. It is not only focused on the quantitative side of the cyber defence but also on the qualitative one, usually related to the human factor.

### 3.3.1 Cyber defence structure

The cyber defence model consists of five functions, representing five core elements of the cyber defence: assess, protect, detect, respond and recover. Assess function is responsible for the assessment of cyber threats and vulnerabilities of the CIS. It facilitates a more effective management of the risk and increases network's defence abilities. Protect function defines the ability of the cyber defence teams and cyber defence tools to protect the CIS from compromises or protect information through mitigation of the compromises. Respond function represents the ability to respond to successful compromises, both internally – by taking actions to mitigate the impact of the compromise and externally – by responding with a counterattack to stop the threat. Detect function defines the ability of the systems and staff to detect new compromises. Recover function defines ability to recover the information, once it was compromised.

### 3.3.2 Investments into cyber defence

The cyber defence case study allows analysis of the impact of strategic investments on the risk to the business. During the research on this case study, the following six investment scenarios were defined:

- Improved Network Monitoring – helps to detect new compromises and to increase ability to assess the risk to an enterprise business,
- Increased Mandatory Cyber Hygiene Training – increases ability of the cyber security staff to apply correctly mitigation procedures and to avoid certain type of compromises, that can result from a human error (i.e. opening of infected email),
- Enhancements to Firewall – increases cyber security by improving access control to CIS,
- Through Life Software Management – keeping up-to-date with software patches and updates, security improvements, revisions, etc.,
- Detect and Deny Tools – acquisition, development and maintenance of technologically advanced, specialist tools to detect compromises,
- Specialists Skills Training – training of specialists responsible for security of the CIS.

An ultimate goal of the cyber defence model is to determine how specific investments will impact the Risk to the enterprise business (Figure 3.2).

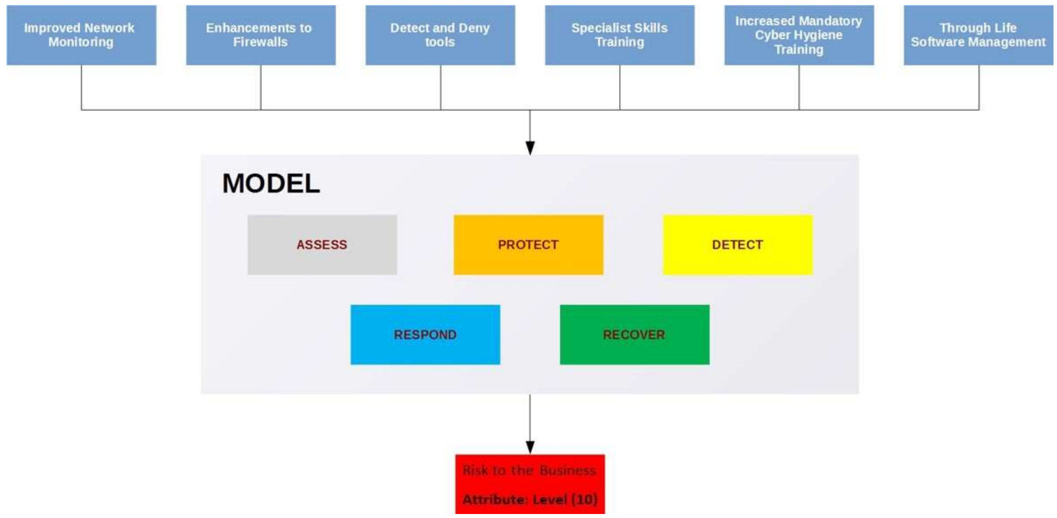



Figure 3.2 Impact of different investments on Risk to the business

# Chapter 4. SEMANTICS OF FUZZY SETS IN RBFCMS

## 4.1 Introduction

Semantics – a study of the meaning, is a very important element of the fuzzy set theory and any modelling technique using fuzzy sets. It defines the interpretation of mathematical representation of linguistic variable– membership function.

Development of RBFCMs (Carvalho, 1999) introduced a new way in which fuzzy sets can be interpreted. The new semantics assumes that the shape of a fuzzy set – the size of area and support, corresponds to the variation or level, it represents. This interpretation is not explained and justified formally. However, it can be induced that the reason for the use of such semantics lays in the reasoning mechanisms proposed by the authors. The accumulation of impacts mechanism is based on areas of fuzzy sets; the greater the area and support of a fuzzy set the greater the resulting accumulated value is. Therefore, to ensure correct performance of accumulation of impacts mechanism, size of the areas and supports of fuzzy sets need to reflect the variation they represent.

To increase flexibility of RBFCMs the standard semantics of fuzzy sets in RBFCMs, instead of semantics developed in (Carvalho, 2009), should be used. Standard semantics has well defined interpretations of fuzzy sets and a multitude of approaches to their analysis. Use of standard semantics allows to considerably increase flexibility of RBFCMs and to address issues identified in this thesis, which result from use of interpretation of fuzzy sets proposed in (Carvalho, 2009).

This chapter is organised as follows. In section 4.2, use of standard semantics in RBFCMs is discussed, including interpretation of fuzzy sets. In the subsequent section, the consequences of using semantics proposed in (Carvalho, 2009) are analysed. In the same section, additional restrictions resulting from adoption of nonstandard semantics in (Carvalho, 2009) are identified. The chapter finishes with highlighting advantages of using standard semantics in RBFCMs.

## 4.2 Semantics of fuzzy sets

Since multiple definitions of semantics of fuzzy sets exist, it is crucial to clearly outline the chosen meaning before modelling using fuzzy sets. As discussed in (Trillas, 2008), any application of fuzzy sets, should have well defined semantics due to availability of multitude of interpretations of fuzzy sets and reasonings on them. The semantics of fuzzy sets used in this thesis is one where degree of membership defines the degree of preference. Degree of preference, $\mu(x)$, describes the preference of the expert for the point $x$ to represent fuzzy set $A$, defined on the universe of discourse $X$. In other words, it expresses her/his subjective definition of the fuzzy set $A$.

## 4.3 Semantics of fuzzy sets used in RBFCMs

The standard RBFCMs are based on the novel semantics of the fuzzy set, which links the size of the area and support of the fuzzy set with its position on the universe of discourse, i.e. the variation/level it represents. However, this new approach of modelling of fuzzy sets, creates significant limitations to the use of RBFCMs.

One of the difficulties in using semantics proposed in the RBFCMs (Carvalho, 2004) results from the fact that the conditions, on how fuzzy sets should be build (discussed in section 2.5), do not ensure that the output of reasoning is in line with the common sense. Let us assume that two fuzzy sets, A and B, are defined on the universe of discourse, $X$, representing two variations. Fuzzy set A is considered to be representing greater variation than fuzzy set B when the following condition is fulfilled (Section 2.4.1, formula 2.33):

$$\forall(A,B)\in\mathcal{F}(X),\ A>B\Rightarrow support_A > support_B \wedge Area_A > Area_B \tag{4.1}$$

where $\mathcal{F}(X)$ is the set of all fuzzy subsets defined on the universe of discourse $X$

In consequence of the condition 4.1, in order to define fuzzy sets $F_1, F_2, F_3 \ldots F_n$, where $F_1 < F_2 < F_3 < \ldots < F_n$, the fuzzy sets have to fulfil the following requirements:

$$support_{F_1} < support_{F_2} < support_{F_3} < \ldots < support_{F_n} \tag{4.2}$$

$$Area_{F_1} < Area_{F_2} < Area_{F_3} < \ldots < Area_{F_n} \tag{4.3}$$

Though not expressed explicitly by Carvalho at al. (2004), it can be concluded that this semantics was introduced to enable creation of calculation of accumulation of impacts in RBFCMs. Accumulation of impacts is the element of the reasoning in RBFCMs that relies on the condition 2.33. Therefore, let us consider consequences that such definition of fuzzy sets has on the accumulation of the fuzzy sets.

Accumulation of impacts represented by fuzzy sets is the final part of the reasoning in RBFCMs (Carvalho and Tome, 2009). The proposed mechanism accumulates impacts using recursive algorithm, described in detail in section 2.4.5. As a result of accumulation, two elements of fuzzy sets involved in the process are carried forward to the new fuzzy set, the Variation Output Set (VOS): the sum of areas of accumulated fuzzy sets and the inner base of the greatest fuzzy set. The area of the fuzzy set VOS equals to the sum of areas of all accumulated fuzzy sets. Therefore, if all fuzzy sets defined in the universe of discourse, $X$, have insignificant differences of sizes of their areas and supports, it can be easily proved that the accumulation of such impacts will give incorrect results.

Let us assume, that:

$$F_1 < F_2 < F_3 < \dots < F_n \tag{4.4}$$

and

$$\begin{aligned} &Area_{F_1} < Area_{F_2} < Area_{F_3} < \dots < Area_{F_n} \\ &support_{F_1} < support_{F_2} < support_{F_3} < \dots < support_{F_n} \\ &Area_{F_1} \approx Area_{F_2} \approx Area_{F_3} \approx \dots \approx Area_{F_n} \\ &support_{F_1} \approx support_{F_2} \approx support_{F_3} \approx \dots \approx support_{F_n} \end{aligned} \tag{4.5}$$

Let us presume that a node receives two impacts: one representing maximal impact $F_n$, and another impact of any strength $F_i, i = 1,\dots, \mathrm{n}-1$. As a result, two accumulated impacts will give similar result for any strength of the second impact:

$$F_n \, accum \, F_1 \approx F_n \, accum \, F_2 \approx F_n \, accum \, F_3 \approx \dots \approx F_n \, accum \, F_n \tag{4.6}$$

where accum represents an operation of accumulation of impacts.

The equation 4.6 demonstrates that the standard accumulation approach, under certain assumptions, produces counterintuitive results even though fuzzy sets fulfil the condition 2.33.

To illustrate it using an example, let us consider three fuzzy sets, *Increased Little* (IL), *Increased* (I) and *Increased Much* (IM), defined as in Figure 4.1:

$$F_{IL} < F_I < F_{IM}$$

where $Area_{F_{IL}} < Area_{F_I} < Area_{F_{IM}}$ and

$$support_{F_{IL}} < support_{F_I} < support_{F_{IM}}$$

Fuzzy sets IL, I and IM are defined respecting condition 2.33. Nevertheless, the differences between supports and areas of fuzzy sets are so small, that fuzzy sets can be considered similar:

$Area_{F_{IL}} \approx Area_{F_I} \approx Area_{F_{IM}}$ and

$support_{F_{IL}} \approx support_{F_I} \approx support_{F_{IM}}$

As a result, if two impacts IM are received their accumulated value will be similar to receiving impacts IM and I, or impacts IM and IL.

$F_{IM}\,accum\,F_{IM} \approx F_{IM}\,accum\,F_I \approx F_{IM}\,accum\,F_{IL}$

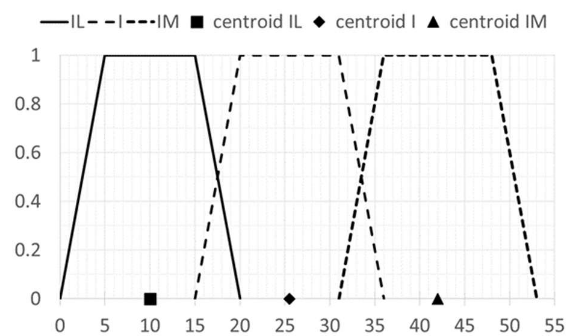



Figure 4.1. Definitions of fuzzy sets IL, I and IM

As it can be seen in Table 4.1, accumulation of two impacts *Increased Much* results in slightly higher accumulated value than accumulation of impacts *Increased Much* and *Increased* and *Increased Much* and *Increased Little.*

Table 4.1. Accumulation of impacts IM and IL, IM and I and IM and IM

| *Fuzzy set* | *centroid* |
|---|---|
| *IL* | 10 |
| *I* | 22.5 |
| *IM* | 40 |
| *IM accum IL* | 49.47 |
| *IM accum I* | 49.97 |
| *IM accum IM* | 50.47 |

This example demonstrates that conditions defined in equation 2.33 are not sufficient to ensure correct performance of RBFCMs accumulation mechanism. It can be concluded that there is a need for more conditions relating the size of the fuzzy set and the variation it represents.

To ensure that the accumulation of impacts mechanism will perform as expected, an additional requirement needs to be defined. The difference of the sizes of areas and supports of fuzzy sets has to be proportional to the difference of variations/level they represent. This requirement has not been explicitly given by the authors of the standard RBFCMs.

If the universe of discourse, $X$, represents range of variations and fuzzy sets $F_1, F_2 \in \mathcal{F}(X)$, where $\mathcal{F}(X)$ represents set of all fuzzy subsets on $X$, are such that $F_1 < F_2$, then fuzzy set $F_2$ represents greater variation than fuzzy set $F_1$. Let us assume that semantically, fuzzy set $F_2$ represents a variation twice stronger than fuzzy set $F_1$. As a result, its area and support should be twice larger than those of fuzzy set $F_2$:

$$
\begin{aligned}
area_{\mathrm{F}_2} &= 2 \times area_{\mathrm{F}_1} \\
support_{\mathrm{F}_2} &= 2 \times support_{\mathrm{F}_1}
\end{aligned}
$$

Such a relationship between variations that fuzzy sets represent and their areas is required, as the method proposed for accumulation of impacts relies on the areas of fuzzy sets. Defining proportions between fuzzy sets $F_1$ and $F_2$ ensures that the result of accumulation of the fuzzy set $F_2$ is proportionally higher than the accumulation of the fuzzy set $F_1$, as twice as many degrees of belief would be shifted towards the fuzzy set representing greater variation.

It can be concluded that to define a valid set of membership functions representing variations, or levels, the following steps need to be taken:

1. the universe of discourse must be defined,
2. the number of fuzzy sets, necessary to represent the variations or levels, needs to be defined,
3. relationships between variations that fuzzy sets represent need to be specified.

It is apparent that the role of experts, who are modelling fuzzy sets, is very restrained in this process. Carvalho at al. (2009), propose to use a set of seven or eleven predefined membership functions, representing variation which satisfy condition 2.33, as presented in Figure 4.2.

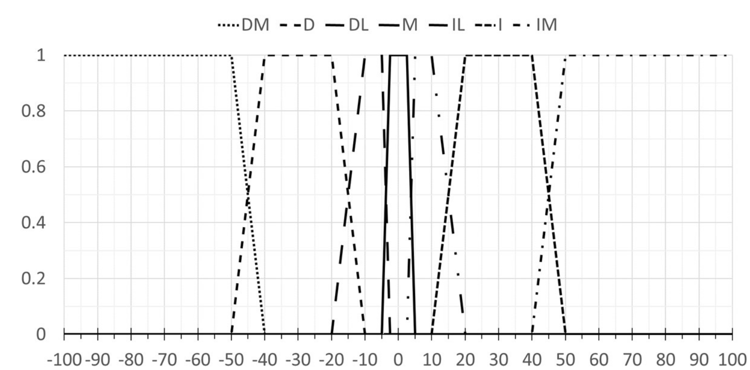



Figure 4.2. Definition of seven variations' fuzzy sets respecting condition 2.33

### 4.3.1 Benefits of using standard semantics

Using a predefined collection of fuzzy sets, defined above, ensures that RBFCMs generate expected results. Benefits of introduction of standard semantics of fuzzy sets which model variation or levels, and which do not have to satisfy the underlying prerequisites of RBFCMs, are discussed in detail in this section.

Let us consider two definitions of fuzzy sets: $F_{IL}$, representing variation *Increased Little*, and fuzzy set $F_I$, representing variation *Increased*. In Figure 4.3 (a), the definitions of these fuzzy sets respect the condition 2.33, as the area and support of the fuzzy set $F_{IL}$ are smaller than the area and support of the fuzzy set $F_I$ (Table 4.2 and Figure 4.3 (a)):

$F_{IL} < F_I$ where $Area_{F_{IL}} < Area_{F_I}$ and $support_{F_{IL}} < support_{F_I}$

Table 4.2. Definition of characteristics of fuzzy sets IL and I respecting condition 2.33

| | *Area* | *support* |
|---|---|---|
| $F_{IL}$ | 12.5 | 15 |
| $F_I$ | 15 | 20 |

On the other hand, in Figure 4.3 (b), different definitions violating the condition 2.33 of the same fuzzy sets, $F_{IL}$ and $F_I$, are given:

$F_{IL} < F_I$ where $Area_{F_{IL}} > Area_{F_I}$ and $support_{F_{IL}} > support_{F_I}$

Table 4.3. Definition of characteristics of fuzzy sets IL and I violating condition 2.33

| | *Area* | *support* |
|---|---|---|
| $F_{IL}$ | 15 | 20 |
| $F_I$ | 12.5 | 15 |

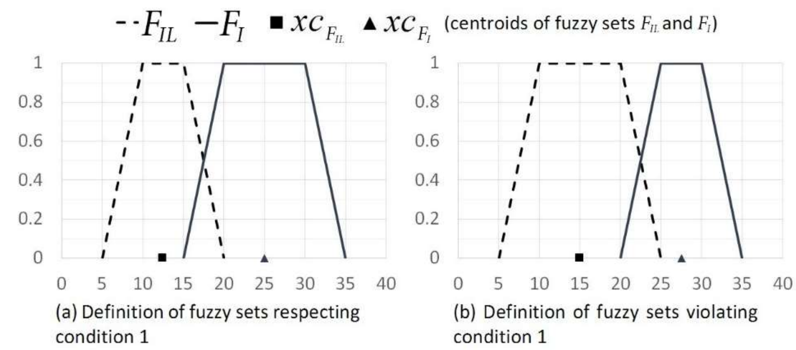



Figure 4.3. Definitions of fuzzy sets $F_{IL}$ and $F_I$

If two impacts $F_{IL}$ and $F_I$, defined in Figure 4.3 (b), are interpolated and accumulated, the resulting fuzzy set $VOS$, representing accumulated impacts, will be higher than if two impacts $F_I$ are received. Semantically receiving two impacts *Increased* should result in a higher increase than receiving impact *Increased* and *Increased Little*. Described problem is discussed in more detailed in section 6.3.1.

Moreover, if fuzzy sets are defined as in Figure 4.3 (b), the result of interpolation would give counterintuitive results by calculating the shape of interpolated fuzzy set in the incorrect way. The problem is discussed in more details in section 5.3.2.1.

As a result of these problems, when the semantics of fuzzy sets, proposed by Carvalho and Tome, is used, then the fuzzy sets defined in Figure 4.3 (b), cannot be used to model variation.

Therefore, in this thesis, it is proposed to consider a standard semantics of fuzzy sets, which represent variations or levels in RBFCMs, based on fuzziness and specificity. Consequently, new reasoning and accumulation mechanisms in RBFMCs need to be developed.

## 4.4 Conclusions

Semantics of fuzzy sets is one of the elements of fuzzy set based models that led to successful applications of fuzzy sets in various areas, regardless of scepticism of some researchers. Even though it is a subject that is not discussed often in the literature, semantics is important in every application of fuzzy sets, as it allows to understand the relationship between linguistic terms and their numerical representation – membership functions.

The semantics of fuzzy sets in RBFCMs proposed by Carvalho and Tome, can be used to model variation or level of a concept, once all the limitation it enforces are accepted. These restrictions are not well defined, and as demonstrated in this chapter, they do not ensure that the RBFCM's mechanisms would work correctly in all situations. Issues discussed in this chapter demonstrate that the use of semantics, proposed by authors of RBFCMs, reduces flexibility in modelling linguistic terms and forces modellers to use a set of predefined membership functions.

However, the undeniable strength of fuzzy sets lays in the multiplicity of their interpretations and subjectivity of definitions. It gives the experts a powerful mean to represent their knowledge, when other modelling methods fail to capture the required complexity. In this thesis, the use of standard semantics of fuzzy sets in RBFCMs is proposed. It allows better representation of expert's knowledge, and potentially increase the interest and areas of applications of RBFCM.

The new RBFCM, based on standard semantics, requires development of new reasoning mechanism; it allows correct interpolation of fuzzy sets and accumulation of impacts, regardless of the shape of fuzzy sets. The new mechanisms are described in the subsequent chapters 5 and 6.

# Chapter 5. NEW REASONING MECHANISM AND RELATIONSHIPS IN RBFCMS

## 5.1 Introduction

In this chapter, the new approach to reasoning in RBFCMs is proposed. As a result of the introduction of the standard semantics into RBFCMs, the mechanisms developed by Carvalho and Tome (2009) present drawbacks when handling the reasoning on fuzzy sets built using this semantics. The new approach presents a good improvement when compared to the standard method and opens a new, much greater range of possible applications by making the reasoning more flexible.

Reasoning is the key element of fuzzy systems, where the crisp input into a causal node is translated into fuzzy domain and through inference of rules, defining relationship between a causal and an effect node, a new crisp impact is calculated on an effect node. The most popular reasoning methods, used in fuzzy systems are Mamdani (1977) and Takagi-Sugeno (1985) allowing them to be used in wide range of applications.

Carvalho and Tome expanded the reasoning by introducing Interpolation of Linguistic Terms mechanism, that allows transforming the crisp impact received by an effect node into a fuzzy set representing this impact. The new reasoning is required to conduct accumulation of multiple impacts received by an effect node, discussed in the Chapter 6. The limitations of the mechanism proposed by Carvalho and Tome (1998), led to development of a new, more flexible and more intuitive mechanism discussed in this chapter.

The chapter is organised as follows: in Section 5.2 the new Interpolation of Linguistic Terms mechanism is introduced. In Section 5.3 comparison of the new and the standard mechanisms is presented and advantages of using the new mechanism are defined. In the following section, new types of relationships are introduced into RBFCMs and benefits of employing them are discussed. The chapter finishes with conclusions and directions for future work in the area of reasoning in the RBFCMs.

## 5.2 New reasoning mechanism in RBFCMs

Carvalho and Tome defined standard RBFCMs reasoning, discussed in Sections 2.4.1 and 2.4.3, as a single process starting from firing rules, triggered by an input into a causal node, to calculating the output fuzzy set representing the crisp impact received by an effect node. In the new reasoning, proposed in this thesis, it is split into two distinct processes (Figure 5.1):

1. Fuzzy inference, consisting of the following steps:
    a. fuzzification of the input of a causal node into the universe of discourse of a causal node,
    b. fuzzy inference from the rule base,
    c. calculation of the crisp impact received by an effect node.
2. Interpolation of the crisp impact received by an effect node, calculated in step 1.c, into the new fuzzy set on the universe of discourse of an effect node.

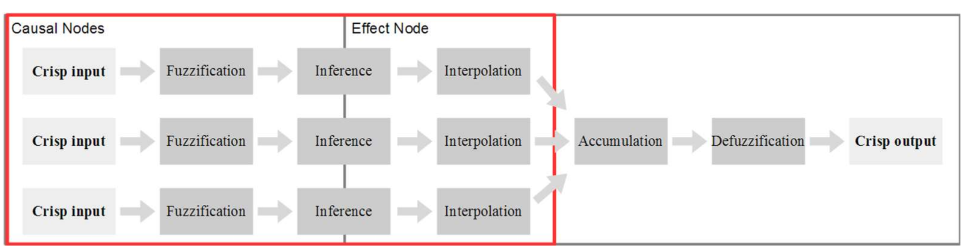



Figure 5.1. Reasoning process, inference and interpolation processes highlighted in red.

The separation of the reasoning into two sub mechanisms allows introduction of complex relationships, which enhance modelling capabilities of RBFCMs and allow capturing complex interdependencies between several causal nodes when determining their impact on one effect node. The separation of two processes also prevents calculations of multiple definitions of fuzzy sets on the universe of discourse of an effect node, when a fuzzy set defining the crisp value already exists.

The standard reasoning mechanism was developed under set of six constraints, discussed in section 2.4.1. The new mechanism, proposed in this thesis, reduces the number of constrains to the following:

1. The sum of membership degrees $\mu(x)$ of all membership functions $A_i$, defined on the universe of discourse $X$, has to be equal to 1, for any point $x$: $\sum \mu(x) = 1$,
2. The membership functions of two consecutive fuzzy sets need to cross in point $\mu(x)=0.5$.

### 5.2.1 Fuzzy Inference in the new mechanism

To explain in more details, the inference mechanism, let us consider an example of relationship between two nodes: a causal node “Node A” and an effect node “Node B” (Figure 5.2).

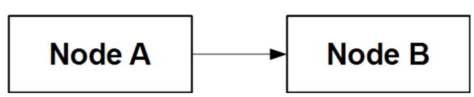



Figure 5.2. FCR relationship between a causal node and an effect node

The following rules are defining the rule base of this relationship:

1. IF Node A is *Decreased Much*
   THEN Node B is *Decreased*
2. IF Node A is *Decreased*
   THEN Node B is *Decreased Little*
3. IF Node A is *Decreased Little*
   THEN Node B is *Maintained*
4. IF Node A is *Maintained*
   THEN Node B is *Increased Little*
5. IF Node A is *Increased Little*
   THEN Node B is *Increased*
6. IF Node A is *Increased*
   THEN Node B is *Increased*
7. IF Node A is *Increased Much*
   THEN Node B is *Increased Much*

The first step of the reasoning is fuzzification of the input of the causal node. Due to constraint 1 and 2 (section 2.4.1), the fuzzified input can either fall within the core and support of one fuzzy set or falls within supports of two consecutive fuzzy sets as in Figure 5.3. Fuzzy sets within which supports the input falls are said to be triggered by the input.

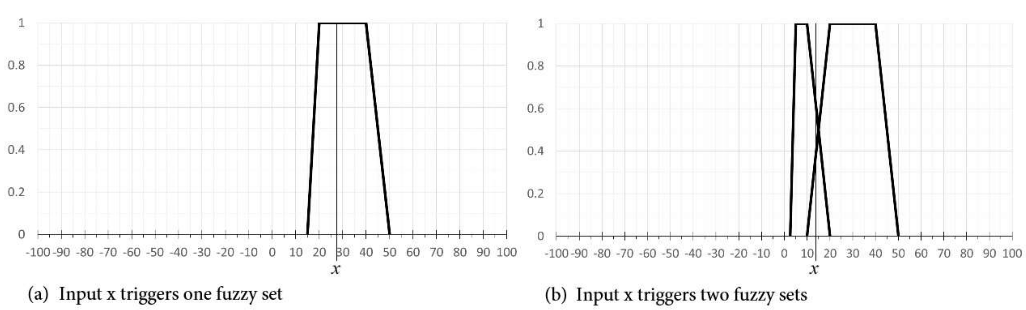



Figure 5.3 Examples of the input into a causal node triggering one or two fuzzy sets

The second step of the reasoning consists of deciding which rules, of the rule base, should be fired. This depends on which fuzzy sets are triggered by the input. Therefore, either one or two rules are fired, as discussed in section 2.1.7.

### 5.2.2 New Interpolation of Linguistic Terms

To further improve the rule based fuzzy reasoning, the interpolation of the crisp input is separated from the inference mechanism. Carrying out the reasoning in two stages, where the output of the inference is considered as an input of the interpolation, allows introducing complex FCRs. Interpolation of the input $x_{ILT}$ is carried out to obtain a new fuzzy set representing the input $x_{ILT}$, which is not defined by any fuzzy set on the universe of discourse $X$.

The input of interpolation, $x_{ILT}$, which is obtained through inference of rules, is mapped on the universe of discourse $X$, where a set of fuzzy sets $\varsigma_i$, where i=N, and N is a number of fuzzy sets in the set of all fuzzy sets $F(X)$, defined on $X$ (Figure 5.4).

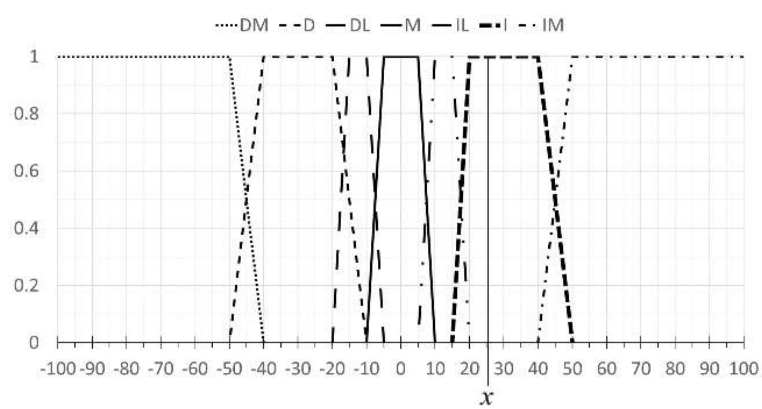



Figure 5.4. Mapping of the input $x_{ILT}$ on the universe of discourse $X$

#### 5.2.2.1 Interpolation when the input is equal to centroid of one of the fuzzy sets

Let us assume that the input $x_{ILT}$ is equal to the centroid of fuzzy set $C_i$, defined on the universe of discourse, $X$. In such case the input is considered to be represented by the convex and normal fuzzy set $C_i$ and the interpolation is not necessary.

Let us assume that as in section 5.2.1 the following rule is triggered by an input $x_{in}$ :

5. IF Number of CIS compromises detected is *Increased Little* $(m_{A_i})$
   THEN Ability to assess and learn from system compromises is *Increased* $(m_{C_i})$

The result of inference from rule 5 is the defuzzified output which is equal to the centroid of the consequent fuzzy set *Increased Little*. After mapping the output of the inference on the universe of discourse $X$ it can be concluded that the output of the inference is represented by fuzzy set *Increased* (Figure 5.5).

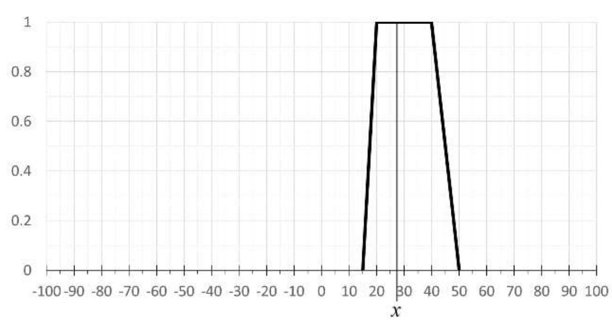



Figure 5.5. The result of the inference represented by the fuzzy set *Increased*

#### 5.2.2.2 Interpolation when the input is not equal to centroid of any fuzzy set

If the input is not equal to the centroid of any fuzzy set, defined on the universe of discourse, then two fuzzy sets, $C_i$ and $C_j$ within whose supports the output of inference, $x_{ILT}$, falls, are identified and interpolation is carried on based on these two fuzzy sets.

Let us consider an example from the Section 5.2.1, where two rules are triggered by an input that falls in between supports of two fuzzy sets (Figure 5.6). As a result, the defuzzified output, $x_{ILT}$ , is not equal to centroids of any fuzzy set defined on the universe of discourse $X$ and it has to be interpolated to define new fuzzy set on $X$ .

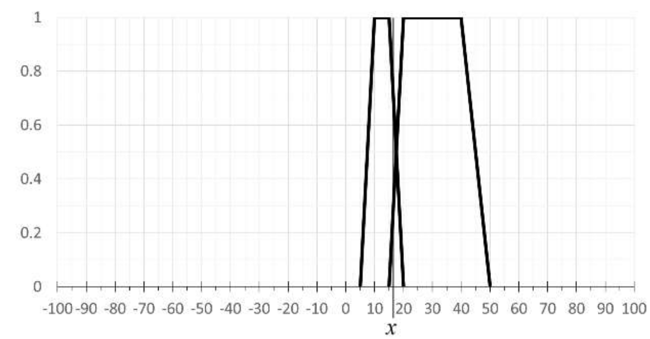



Figure 5.6. Input falling in between two fuzzy sets and requiring interpolation

For any point $x_{ILT}$ that belongs to the universe of discourse $X$ , where $C_i \subset F(X)$ for $i = 1,...,n$ and n is the number of fuzzy sets defined in the set of all fuzzy sets $F(X)$ defined on the universe of discourse $X$ , in such a way that two adjacent fuzzy sets cross in point where their degrees of belief are equal to 0.5 and in every point $x_{ILT}$ the sum of their degrees of belief is equal to 1, it is possible to define new fuzzy set $ILT$ such that:

1. Fuzzy set $ILT$ is convex and normal,
2. Fuzzy set $ILT$ has similar shape to neighbouring fuzzy sets $C_i$ and $C_j$ and

   $C_i \cong C_{ILT} \cong C_j$

3. uncertainty of $ILT$ is similar to fuzzy sets it is close to.

   $U(C_i) \cong U(C_{ILT}) \cong U(C_j)$ , where U is an uncertainty function.

The new ILT mechanism is defined based on the distance between the input $x_{ILT}$ and two fuzzy sets it falls in between. To calculate the impact that both fuzzy sets have on the shape of the new fuzzy set $_{ILT}$, the distance between the input $x_{ILT}$ and the two fuzzy sets $C_i$ and $C_j$ is used.

Let us assume that fuzzy sets $C_i$ and $C_j$ represent fuzzy sets *Increased Little* and *Increased* as in Figure 5.7 and that:

$dist_{C_i} = \left| x_{ILT} - xc_{C_i} \right|$ is a distance between centroid of $C_i$, $xc_{C_i}$, and the input $x_{ILT}$,

$dist_{C_j} = \left| x_{ILT} - xc_{C_j} \right|$ is a distance between centroid of $C_j$, $xc_{C_j}$, and the input $x_{ILT}$.

If the input $x_{ILT}$ is closer to the centroid of the fuzzy sets $C_i$, $xc_{C_i}$, then its characteristics should be similar to those of the fuzzy set $C_i$. If the input $x_{ILT}$ is closer to the centroid of the fuzzy set $C_j$, $xc_{C_j}$, then its characteristics should resemble those of the fuzzy set $C_j$. It can be concluded that the closer the input $x_{ILT}$ is to any of the two centroids, $xc_{C_i}$ or $xc_{C_j}$, the importance of the corresponding fuzzy set increases and the more the $_{ILT}$'s characteristic should resemble this fuzzy set. In the example presented in Figure 5.7 (a), the fuzzy set $_{ILT}$ should have similar core, inner and outer bases to the ones of the fuzzy set *Increased*, in example presented in Figure 5.7 (b), the fuzzy set is similar to fuzzy set *Increased Little* and 5.7 (c) it equally shares the features of both fuzzy sets *Increased Little* and *Increased.*

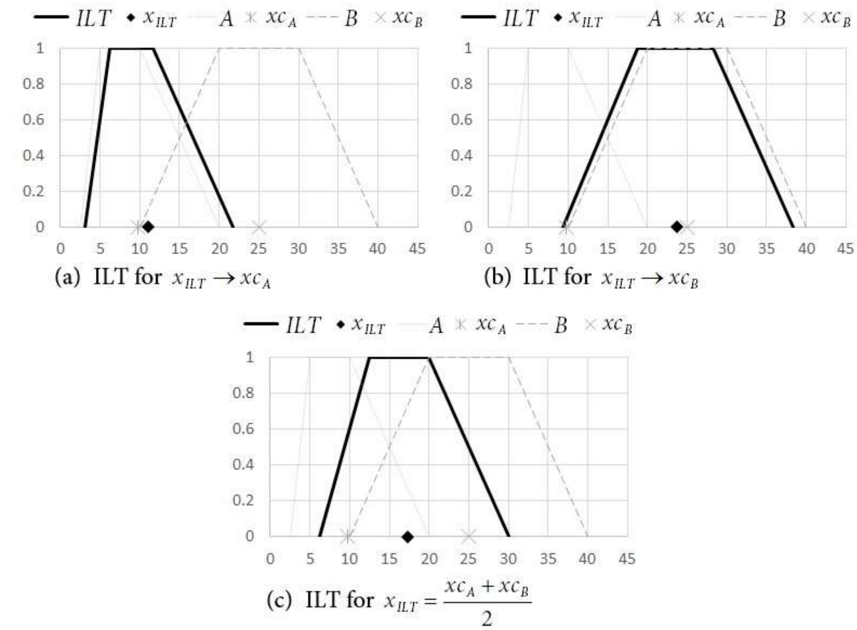



Figure 5.7 Definitions of ILT for different inputs x

The importance of fuzzy sets $C_i$ and $C_j$ is represented as an inverted distance between their centroids, $xc_{C_i}$ and $xc_{C_j}$, and the input $x_{ILT}$. The nearer the input to one of the fuzzy sets is, the higher its importance is. The importance is normalised using the normalising factor c as follows:

$$\frac{c}{dist_{C_i}} + \frac{c}{dist_{C_j}} = 1$$
$$c = \frac{1}{\frac{1}{dist_{C_i}} + \frac{1}{dist_{C_j}}} = \frac{dist_{C_i}\, dist_{C_j}}{dist_{C_i} + dist_{C_j}} \tag{5.1}$$

The characteristics of $ILT$ are determined using the following formulas:

$$core_{ILT} = c\left(\frac{core_{C_i}}{dist_{C_i}} + \frac{core_{C_j}}{dist_{C_j}}\right) = \frac{dist_{C_i}\, dist_{C_j}}{dist_{C_i} + dist_{C_j}}\left(\frac{core_{C_i} dist_{C_j} + core_{C_j} dist_{C_i}}{dist_{C_i} dist_{C_j}}\right) = \left(\frac{core_{C_i} dist_{C_j} + core_{C_j} dist_{C_i}}{dist_{C_i} + dist_{C_j}}\right) \tag{5.2}$$

$$bi_{ILT} = \left( \frac{bi_{C_i} dist_{C_j} + bi_{C_j} dist_{C_i}}{dist_{C_i} + dist_{C_j}} \right) \tag{5.3}$$

$$bo_{ILT} = \left( \frac{bo_{C_i} dist_{C_j} + bo_{C_j} dist_{C_i}}{dist_{C_i} + dist_{C_j}} \right) \tag{5.4}$$

Once the required characteristics of ILT are calculated, it is necessary to build the new fuzzy set ILT. It can be done by setting the point b of the trapezoidal fuzzy set at the beginning of the universe of discourse, point 0, and the point c, maximum of the core, to the calculated value of $core_{ILT}$. The point a of the fuzzy set, minimum of the support, is calculated by subtracting the value of the inner base from the calculated point b. The point c of the ILT is calculated by adding the size of the outer base to the calculated point c. Once four points of the trapezoidal membership function are calculated, it can be shifted so that its centroid matches the centroid of the $x_{ILT}$:

1. Set $\mathrm{b} = \min(core_{ILT}) = 0$ and $\mathrm{c} = \max(core_{ILT}) = core_{ILT}$
2. Calculate $\mathrm{a} = \min(\mathrm{support}_{ILT}) = -bi_{ILT}$ and $\mathrm{d} = \max(support_{ILT}) = \max(core_{ILT}) + bo_{ILT}$.

   At this step, all four points: a, b, c and d, of the trapezoid function are calculated.
3. Calculate the centroid of $ILT$ , $xc_{ILT}$
4. Shift the fuzzy set ILT by $shift = |xc_{ILT} - xc_U|$ so that $xc_{ILT} = x_{ILT}$.

Alternatively, the ILT can be calculated using following function:

$$ILT(C_i, C_j) = \begin{cases} a_{ILT} = x_{ILT} + shift - 0.5\, core_{ILT} - bi_{ILT} \\ b_{ILT} = x_{ILT} + shift - 0.5\, core_{ILT} \\ c_{ILT} = x_{ILT} + shift + 0.5\, core_{ILT} \\ d_{ILT} = x_{ILT} + shift + 0.5\, core_{ILT} + bo_{ILT} \end{cases} \tag{5.5}$$

where $shift = \dfrac{(2\, bo_{ILT} + 2\, bi_{ILT} + 3\, core_{ILT})(bo_{ILT} + bi_{ILT})}{6\,(bo_{ILT} + bi_{ILT} + 2\, core_{ILT})}$

## 5.3 Discussion on reasoning mechanisms

### 5.3.1 Preservation of uncertainty of fuzzy sets under standard semantics

Standard RBFCMs reasoning mechanism assumes that the linguistic variables are defined using a predetermined set of fuzzy sets that comply with constraint requiring fuzzy sets representing greater variation to have greater area and support than fuzzy sets representing a smaller variation. When fuzzy sets are defined violating the relationship between the shape of fuzzy sets and the degree of variation, then the standard reasoning mechanism is unable to generate outputs that can be considered as correct.

The RBFCMs standard reasoning mechanism relies on the distance between centroids of the $ILT$ and centroids of fuzzy sets $A$ and $B$, where $A$ and $B$ are fuzzy sets defined in the consequent part of the fired rules (Formulas 2.35 – 2.38).

The intuition behind the standard reasoning mechanism, defined by Carvalho and Tome (Carvalho, 2009), is the following: when the centroid of the union, $xc_U$, approaches the centroid of one of the consequent fuzzy sets, then the core and inner base of ILT should be similar to the core and inner base of the fuzzy set the centroid is closer to:

$$core_{ILT} \underset{xc_U \to xc_A}{=} core_{ILT} \approx core_A$$

$$core_{ILT} \underset{xc_U \to xc_B}{=} core_{ILT} \approx core_B$$

It can be observed that the core of $ILT$ takes the following values when centroid of union, $xc_U$, is approaching centroids of fuzzy sets A and B, assuming A<B, respectively:

$$if\ \ xc_U \to xc_A\ \ then\ \ core_{ILT} = \min\{core_A, core_B\} + \left|\frac{xc_A - xc_A}{xc_B - xc_A} * (core_A - core_B)\right| = \min\{core_A, core_B\}$$

$$if\ \ xc_U \to xc_B\ \ then\ \ core_{ILT} = \min\{core_A, core_B\} + \left|\frac{xc_B - xc_A}{xc_B - xc_A} * (core_A - core_B)\right| = \min\{core_A, core_B\} + |core_B - core_A| = \max\{core_A, core_B\}$$

### 5.3.1.1 Interpolation when fuzzy sets are defined using RBFCMs' semantics

Let us assume that the fuzzy sets $A = Increased\ Little$ and $B = Increased$, involved in the interpolation are defined as in Figure 5.8. It can be observed that the area, core, support and inner and outer bases of the fuzzy set B, representing a stronger variation, are greater than those of the fuzzy set A, which represents a weaker variation (Table 5.1). As the area and support of fuzzy set B are greater than those of the fuzzy set A, it can be concluded that fuzzy sets are defined respecting the condition 2.33. (section 2.4.1).

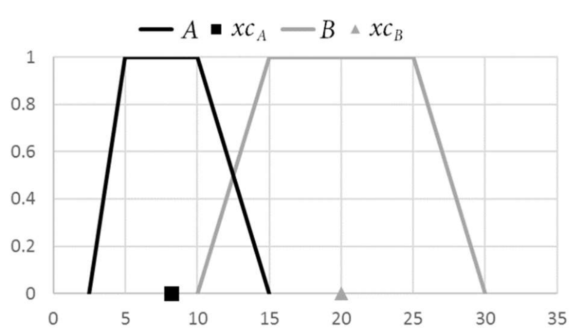



Figure 5.8. Definition of fuzzy sets A and B respecting condition 2.33 (section 2.4.1)

Table 5.1 Definition of main characteristics of fuzzy sets A and B

| | A | B |
|---|---|---|
| centroid | 8.2 | 20 |
| support | 12.5 | 20 |
| core | 5 | 10 |
| bi | 2.5 | 5 |
| bo | 5 | 5 |
| area | 8.75 | 15 |

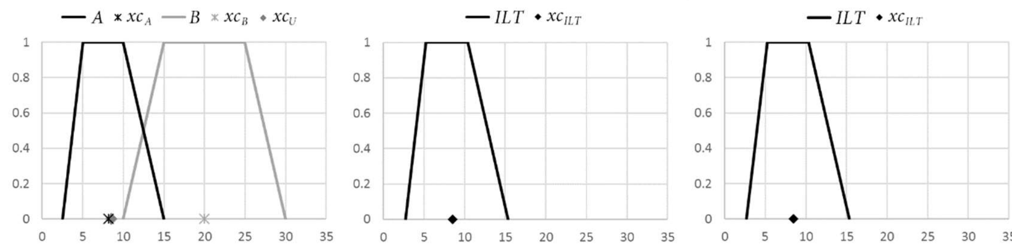
As it can be seen in the Table 5.1, the core of the fuzzy set $A$ is smaller than the core of the fuzzy set B, $core_A < core_B$, therefore:

$$xc_U \to xc_A \Rightarrow core_{ILT} \to \min\{core_A, core_B\} \Rightarrow core_{ILT} \to core_A$$

$$xc_U \to xc_B \Rightarrow core_{ILT} \to \max\{core_A, core_B\} \Rightarrow core_{ILT} \to core_B$$

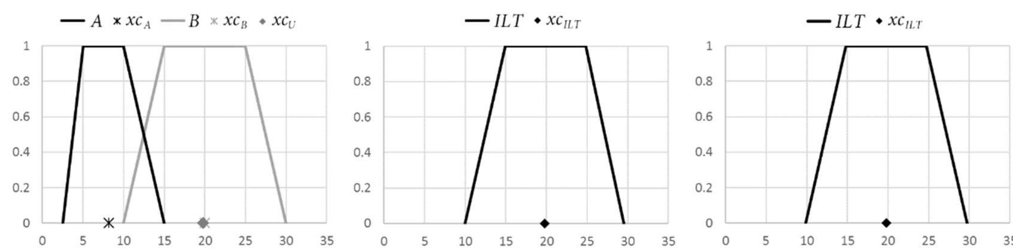
It can be observed that the $core_{ILT}$ takes correct values: when $xc_U$ approaches centroid of fuzzy set A, then the $core_{ILT}$ takes value of $core_A$ and it takes value of $core_B$ when the $xc_U$ approaches the centroid of fuzzy set B.

As a result, the $ILT$ has a similar shape to fuzzy set A when $xc_U$ approaches centroid of fuzzy set A (Figure 5.9) and similar shape to fuzzy set B when $xc_U$ approaches centroid of fuzzy set B (Figure 5.10).



(a) Definition of fuzzy sets $A$ and $B$ (b) $ILT$ calculated using standard interpolation mechanism (c) $ILT$ calculated using new interpolation mechanism

Figure 5.9 ILT calculated using standard and new mechanism when $xc_U \to xc_A$



(a) Definition of fuzzy sets $A$ and $B$ (b) $ILT$ calculated using standard interpolation mechanism (c) $ILT$ calculated using new interpolation mechanism

Figure 5.10 ILT calculated using standard and new mechanism when $xc_U \to xc_B$

### 5.3.1.2 Interpolation when fuzzy sets are defined using standard semantics

Let us assume that the fuzzy sets $A = Increased\ Little$ and $B = Increased$, involved in the interpolation are defined as in Figure 5.11. The area, support and inner and outer bases of the fuzzy set B, are greater than those of the fuzzy set A (Table 5.2). Even though the core of the fuzzy set B is smaller than the core of the fuzzy set A, the fuzzy sets A and B are considered to be defined respecting condition 2.33 (section 2.4.1), as the area and support of the fuzzy set B are greater than those of the fuzzy set A.

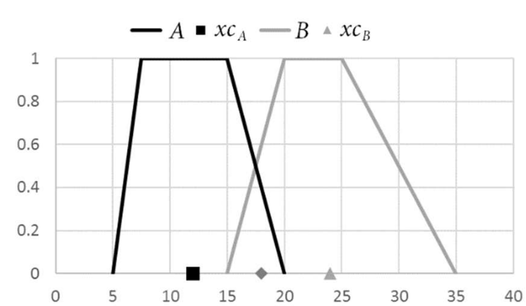


Figure 5.11. Definition of fuzzy sets A and B

Table 5.2 Definition of main characteristics of fuzzy sets A and B

| | A | B |
|---|---|---|
| centroid | 11.9 | 24 |
| support | 15 | 20 |
| core | 7.5 | 5 |
| bi | 2.5 | 5 |
| bo | 5 | 10 |
| area | 11.25 | 12.5 |

As it can be seen in the Table 5.2, the core of the fuzzy set $A$ is greater than the core of the fuzzy set B. As a result, the core of ILT takes the following values, when the standard mechanism is used:

$$xc_U \to xc_A \Rightarrow core_{ILT} \to \min\{core_A, core_B\} \Rightarrow core_{ILT} \to core_B$$

$$xc_U \to xc_B \Rightarrow core_{ILT} \to \max\{core_A, core_B\} \Rightarrow core_{ILT} \to core_A$$

The ILT takes the size of the core of the fuzzy set B, when it approaches the centroid of the fuzzy set A, therefore the ILT has the size of the core different than expected.

On the other hand, the inner base of the ILT is calculated as expected, as the inner base of the fuzzy set A is smaller than the inner base of the fuzzy set B:

$$xc_U \to xc_A \Rightarrow bi_{ILT} \to \min\{bi_A, bi_B\} \Rightarrow bi_{ILT} \to bi_A$$

$$xc_U \to xc_B \Rightarrow bi_{ILT} \to \max\{bi_A, bi_B\} \Rightarrow bi_{ILT} \to bi_B$$

As a result, the ILT has a shape similar to fuzzy set B when $xc_U$ approaches the fuzzy set A (Figure 5.12 (b)) and the shape similar to fuzzy set A when $xc_U$ approaches the fuzzy set B (Figure 5.13 (b)), which is contrary to common expectation.

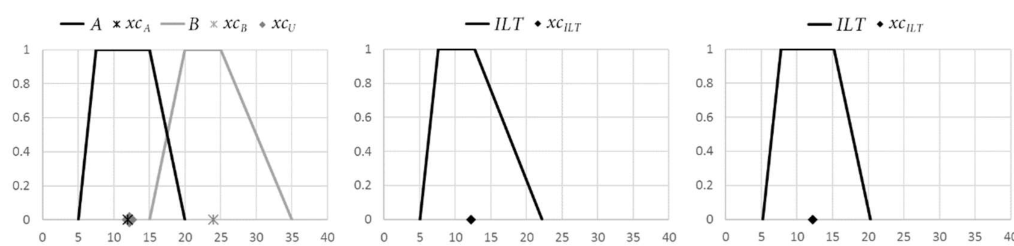
The new reasoning mechanism calculates the core of the fuzzy set in the same way, regardless of the shape of the fuzzy sets A and B:

$$xc_U \rightarrow xc_A \Rightarrow core_{ILT} \rightarrow core_A$$

$$xc_U \rightarrow xc_B \Rightarrow core_{ILT} \rightarrow core_B$$

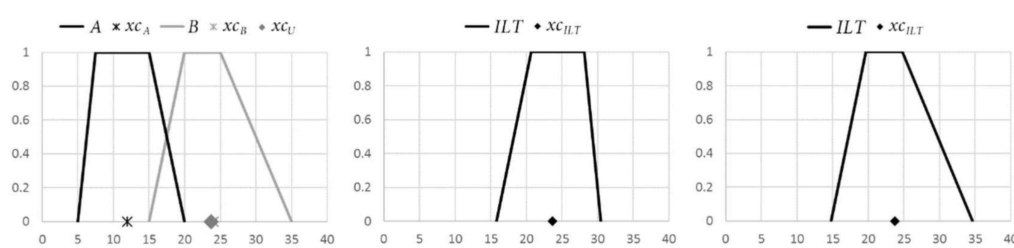
where $xc_U = x_{ILT}$

The new reasoning mechanism calculates the shape of the ILT as one would expect in both situations. When the $xc_U$ approaches the fuzzy set A then ILT has a shape similar to fuzzy set A (Figure 5.12 (c)) and when $xc_U$ approaches fuzzy set B then ILT has a shape similar to fuzzy set B (Figure 5.13 (c)).




(a) Definition of fuzzy sets $A$ and $B$ (b) $ILT$ calculated using standard interpolation mechanism (c) $ILT$ calculated using new interpolation mechanism

Figure 5.12. ILT calculated using the standard and the new mechanism when $xc_U \rightarrow xc_A$




(a) Definition of fuzzy sets $A$ and $B$ (b) $ILT$ calculated using standard interpolation mechanism (c) $ILT$ calculated using new interpolation mechanism

Figure 5.13. ILT calculated using standard and new mechanism when $xc_U \rightarrow xc_B$

Figure 5.14 presents how the size of the core and the inner base of ILT changes for different values of $xc_U = [xc_A, xc_B]$.

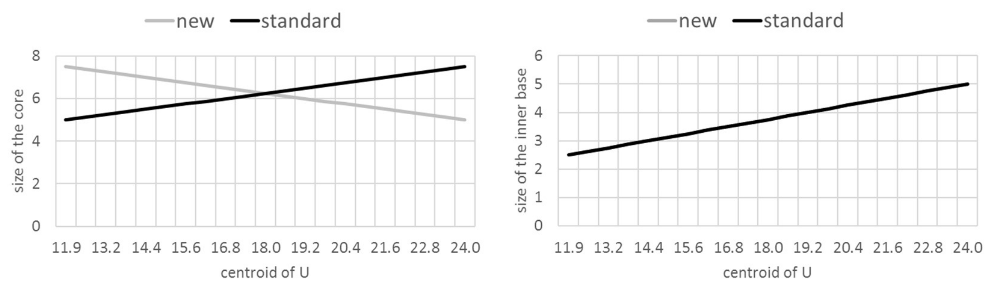



Figure 5.14 Relationship between $xc_U$ and the size of the core and inner base of ILT calculated using the standard and the new reasoning mechanisms

It can be observed (Figure 5.14) that the condition 2.33 (section 2.4.1), defined by Carvalho and Tome, on how the fuzzy sets should be constructed, do not ensure that the standard mechanism will handle calculations of ILT as expected. This limitation results from the calculation of the core and inner base of the fuzzy set ILT (Formulas 2.35 and 2.36). The size of the core of the ILT is equal to the sum of the smaller of the two cores and the difference between sizes of cores, of fuzzy sets involved in the interpolation, weighted with the normalised distance between the centroid of the union U, $xc_U$, and the centroid of the fuzzy set representing smaller variation.

To guarantee that the standard mechanism will calculate ILT correctly, every element of the fuzzy set defining a smaller variation, should be smaller than the corresponding feature of the fuzzy set representing a greater variation:

$$\forall A,B \in \mathcal{F}(X),\ A < B \Leftrightarrow core_A < core_B \wedge bi_A < bi_B \wedge bo_A < bo_B \wedge support_A < support_B \wedge Area_A < Area_B$$

As demonstrated in this section the new reasoning mechanism solves the problem of the incorrect calculations of the features of the fuzzy set ILT, when fuzzy sets are defined with violation of the condition 2.33.

The new algorithm for interpolation of linguistic terms is focused on maintaining the uncertainty levels between two fired fuzzy sets rather than the area of the union of consequent fuzzy sets.

### 5.3.2 Convexity and normality of the ILT

The standard RBFCMs are based on the idea that the area and variation represented by the fuzzy set are related, i.e. larger variations are represented by fuzzy sets with larger supports and areas. However, in some cases, the standard reasoning mechanism can result in $ILT$ being a trapezoid with a core longer than its support. In consequence, the obtained fuzzy set is no longer a fuzzy number.

To illustrate the problem let us consider a relationship between Node X and Node Z. Let us assume that for any input into a causal node, between 2.5 and 10, the following two rules will be fired:

1. IF Node X is *Maintained* $(m_{A_i})$
   THEN Node Z is *Increased Little* $(m_{A_j})$
2. IF Node X is *Increased Little* $(m_{C_i})$
   THEN Node Z is *Increased Much* $(m_{C_j})$

Let us assume that the input into the Node X, triggers the first rule with the firing level $m_{A_i}$ equal to 0.3 and the second rule with firing level $m_{C_i}$ equal to 0.7. As a result of the inference on rules 4 and 5, the $xc_U$ equals to 31. In the next step, interpolation is carried on two fuzzy sets *Increased Little, A,* and *Increased Much, B,* (Figure 5.15, Table 5.3) and ILT is calculated. The following values of ILT's features are calculated using the standard mechanism:

$core_{ILT} = 46.37$
$support_{ILT} = 37.37$
$Area_{ILT} = Area_U = 41.87$

The size of the support is smaller than the size of core of the fuzzy set $ILT$ (see Figure 5.16, when firing level = 0.3), therefore, the resulting set $ILT$ has incorrect shape and cannot be used as a fuzzy set, as it is not normal and not convex.

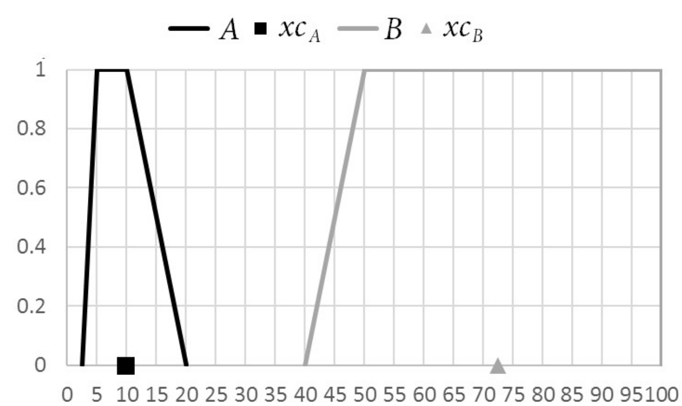



Figure 5.15. Definition of consequent fuzzy sets

Table 5.3 Definition of the main characteristics of fuzzy sets Increased Little and Increased Much

| | A | B |
|---|---|---|
| centroid | 10 | 72.4 |
| support | 5 | 50 |
| core | 17.5 | 60 |
| bi | 2.5 | 10 |
| bo | 10 | 0 |
| area | 11.25 | 55 |

Figure 5.16 presents the relationship between core and support of $ILT$ calculated using new and standard reasoning mechanisms, with respect to different firing levels $m_{A_i}$ of the two rules and consequent fuzzy sets defined as in Table 5.3. When the standard reasoning mechanism is used, for firing levels $m_{A_i}$ in the interval [0.15, 0.95], the size of the core is greater than the size of the support (Figure 5.16 (a)). As a result, for this definition of fuzzy sets the

standard reasoning mechanism generates the output function, which cannot be used as membership functions to represent a fuzzy set, as the core of the membership function cannot be greater than its support.

On the other hand, if the new interpolation mechanism is used, for fuzzy sets defined as in Table 5.3, the core of the $ILT$ is smaller than the support for all firing levels $m_{A_i}$. The resulting fuzzy set $ILT$ is a convex and normal fuzzy set (Figure 5.16 (b)), therefore can be used in RBFCMs.

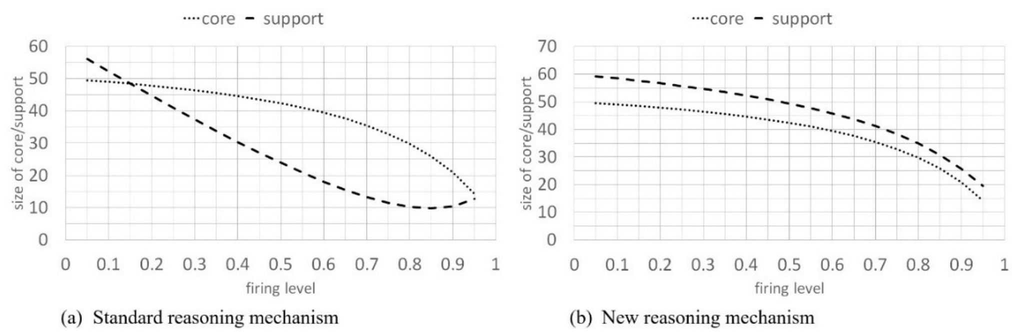



Figure 5.16. Relationship between the area, core and support for different firing levels using the standard and new reasoning mechanism

### 5.3.3 Separation of interpolation and inference

Let us consider the relationship between nodes Node X and "Node Z defined using two rules as follows:

1. IF Node X is *Maintained* $(m_{A_i})$
   THEN Node Z is *Increased Little* $(m_{A_j})$
2. IF Node X is *Increased Little* $(m_{C_i})$
   THEN Node Z is *Increased Much* $(m_{C_j})$

As a result of firing rules 1 and 2 with firing levels $m_{A_i}$ and $m_{C_i}$, respectively, the crisp output of the inference of rules takes values in the range $[9.7,\ 72.4]$ which is the range from the centroid of fuzzy set A, *Increased Little*, to the centroid of the fuzzy set B, *Increased Much* (Figure 5.17).

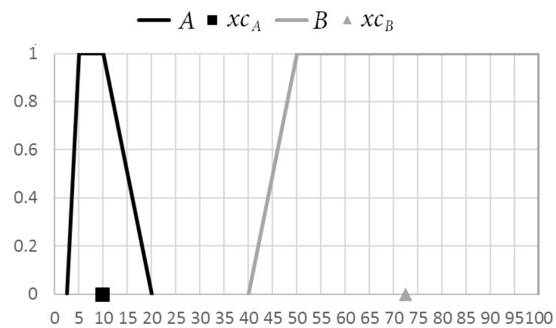



Figure 5.17. Definitions of fuzzy sets A and B

The main reason for interpolating the output of the inference is a need to represent the crisp value using a fuzzy set, in order to be able to accumulate several fuzzy impacts.

Let us assume that the output of the inference, carried on rules 1 and 2, is equal to 30. It can be observed that the crisp value 30 is already represented by the fuzzy set *Increased* (Figure 5.4), therefore there is no need to interpolate the output of inference. The standard mechanism would not consider the already existing definition of the fuzzy set on the universe of discourse and would create a new definition of fuzzy set representing the crisp value 30.

## 5.4 Inference in complex FCR

Introduction of the standard semantics of the fuzzy sets, development of the new Interpolation of Linguistic Terms mechanism and its separation from the inference mechanism, allowed development of a new type of fuzzy relationship, named complex Fuzzy Causal Relationships (cFCR). cFCRs allow modelling a joint impact of more than one causal node on a single effect node. An example of cFCR is presented in Figure 5.18.

The requirement for cFCRs was identified during modelling phase of the research on the case studies discussed in Chapter 3. The experts, who were defining relationships between concepts, were unable to capture some dependencies between nodes using the standard FCR where the information about the state of several causal nodes had to be taken into account collectively rather than individually, in order to calculate the impact on one effect node. Let us discuss two examples of relationships that, according to experts, required a use of cFCR.

Example 1

As discussed in Case study 1 (see Chapter 3.2.2) a military ability of a military unit greatly depends on its training, whose quality is determined by the staff that leads the training, the facilities, training equipment, time spent on training, etc. One of the key aspects of the training of Air Force units is number of hours that are spent practicing.

There are two main types of trainings available for an individual pilot: Live Flying, which is the training carried on in the real jet and the Synthetic Training, which is the training carried on in the simulator. With a greater accessibility of the synthetic environment and highly reduced costs, when compared to Live Flying, there is a tendency among decision makers to decrease number of Live Flying hours and increase number of hours spent on the Synthetic Training. Intuitively it can be assumed that increasing number of hours spent on Synthetic Training should have positive impact on the level of Training of the military unit. However, there is a point in the number of hours spent on the Live Flying when Synthetic Training cannot address the loss of skills caused by the lack of time spent in the air; that is to say, when the number of Live Flying hours is *Very Low*, then any increase of number of Synthetic Training hours cannot have positive impact on the level of Training of the military unit.

If one wanted to represent such relationship using two FCR relationships, it would require defining a positive relationship for any increase of the number of hours spent on the Synthetic Training and a highly negative impact when the Live Flying hours reach *Very Low* level. It is possible that the negative impact of *Very Low* level of Live Flying and *High* or *Very High* level of Synthetic training would balance out or even would manage to show a slightly negative impact but would not express it to a degree explained above.

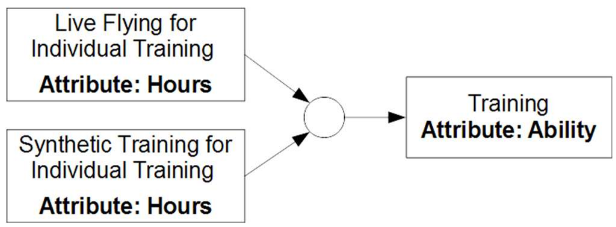



Figure 5.18. An example of cFCR

Following subset of rules defines the cFCR between two causal nodes: "Number of Hours of Live Flying for Individual Training" and "Number of Hours of Synthetic Training for Individual Training" and the effect node: "Ability of Training":

IF Number of Hours for Live Flying is Very Low AND
Number of Hours for Synthetic Training is Very Low
THEN Ability of Training DLoD is Decrease Much

IF Number of Hours for Live Flying is Very Low AND
Number of Hours for Synthetic Training is Low
THEN Ability of Training DLoD is Decrease Much

IF Number of Hours for Live Flying is Very Low AND
Number of Hours for Synthetic Training is Medium
THEN Ability of Training DLoD is Decrease

IF Number of Hours for Live Flying is Very Low AND
Number of Hours for Synthetic Training is High
THEN Ability of Training DLoD is Decrease

IF Number of Hours for Live Flying is Very Low AND
Number of Hours for Synthetic Training is Very High
THEN Ability of Training DLoD is Decrease

IF Number of Hours for Live Flying is Low AND
Number of Hours for Synthetic Training is Very Low
THEN Ability of Training DLoD is Decrease Much

IF Number of Hours for Live Flying is Low AND
Number of Hours for Synthetic Training is Low
THEN Ability of Training DLoD is Decrease

IF Number of Hours for Live Flying is Low AND
Number of Hours for Synthetic Training is Medium
THEN Ability of Training DLoD is Decrease

IF Number of Hours for Live Flying is Low AND
Number of Hours for Synthetic Training is High
THEN Ability of Training DLoD is Decrease Little

IF Number of Hours for Live Flying is Low AND
Number of Hours for Synthetic Training is Very High
THEN Ability of Training DLoD is Maintain

### 5.4.1 Definition of cFCR

Let us assume cFCR relationship between following two causal nodes: Node X and Node Y and an effect node: Node Z, as in Figure 5.19.

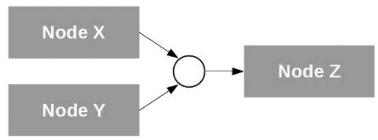



Figure 5.19. An example of cFCR

The cFCR consists of complex fuzzy IF-THEN rules whose antecedent part consists of definitions of all the causal nodes, involved in the relationship, joined with the AND operator.

$$cFCR(A_i \text{ and } B_i \rightarrow C_i)$$

Rules comprising cFCR are defined as follows:

$$R_i\text{: IF X is } A_j \text{ AND Y is } B_j \text{ THEN Z is } C_j$$

where $R_i$ is the $i^{th}$ rule, $i=1\ldots N \times N$,

X, Y, Z are nodes involved in the relationship and

$A_j$, $B_j$, $C_j$, $j=1\ldots N$, are fuzzy sets defining these nodes.

Let us assume that the Node X received an input x and the Node Y received an input y. In such case inputs of both nodes will fall in between, maximally, two fuzzy sets and therefore it will trigger up to four rules (Figure 5.20):

$$R_i\text{: IF X is } A_i\ (m_{A_i}) \text{ AND Y is } B_i\ (m_{B_i}) \text{ THEN Z is } C_i\ (m_{C_i})$$
$$R_j\text{: IF X is } A_j\ (m_{A_j}) \text{ AND Y is } B_j\ (m_{B_j}) \text{ THEN Z is } C_j\ (m_{C_j})$$
$$R_k\text{: IF X is } A_k\ (m_{A_k}) \text{ AND Y is } B_k\ (m_{B_k}) \text{ THEN Z is } C_k\ (m_{C_k})$$
$$R_l\text{: IF X is } A_l\ (m_{A_l}) \text{ AND Y is } B_l\ (m_{B_l}) \text{ THEN Z is } C_l\ (m_{C_l})$$

where $A_i = A_j, A_k = A_l, B_i = B_k, B_j = B_l$ and $C_i, C_j, C_k$ and $C_l$ are different fuzzy sets.

As the fuzzy sets are built in such a way that they cross in point where degree of belief is equal to 0.5 and for any point x on the universe of discourse the sum of degrees of membership functions is equal to 1, the rules defining the complex relationship between nodes X, Y and Z are fired with the following firing levels:

$$\begin{aligned} m_{A_j} &= m_{A_i} \\ m_{A_k} &= m_{A_l} = 1 - m_{A_i} \\ m_{B_k} &= m_{B_i} \\ m_{B_l} &= m_{B_j} = 1 - m_{B_i} \end{aligned} \tag{5.6}$$

As any input in a node falls in between two fuzzy sets, with degrees of belief m and 1-m. The rules can be rewritten as follows:

$$\begin{array}{llllll} R_i: \text{IF X is } A_i & (m_{A_i}) & \text{AND Y is } B_i & (m_{B_i}) & \text{THEN Z is } C_i & (m_{C_i}) \\ R_j: \text{IF X is } A_j & (m_{A_i}) & \text{AND Y is } B_j & (1 - m_{B_i}) & \text{THEN Z is } C_j & (m_{C_j}) \\ R_k: \text{IF X is } A_k & (1 - m_{A_i}) & \text{AND Y is } B_k & (m_{B_i}) & \text{THEN Z is } C_k & (m_{C_k}) \\ R_l: \text{IF X is } A_l & (1 - m_{A_i}) & \text{AND Y is } B_l & (1 - m_{B_i}) & \text{THEN Z is } C_l & (m_{C_l}) \end{array}$$

The firing level of the consequents of the rules are calculated as a product of firing levels of antecedent of the rules:

$$\begin{aligned} m_{C_i} &= m_{A_i} \cdot m_{B_i} \\ m_{C_j} &= m_{A_i} \cdot (1 - m_{B_i}) \\ m_{C_k} &= (1 - m_{A_i}) \cdot m_{C_k} \\ m_{C_l} &= (1 - m_{A_i}) \cdot (1 - m_{B_i}) \end{aligned} \tag{5.7}$$

The sum of firing levels of fuzzy sets of the consequent Node Z is equal to 1:

$$\begin{aligned} \sum_{i=1}^{N} m_{C_i} &= m_{C_i} + m_{C_j} + m_{C_k} + m_{C_l} = \\ & m_{A_i} m_{B_i} + m_{A_i} - m_{A_i} m_{B_i} + m_{B_i} - m_{A_i} m_{B_i} + 1 - m_{A_i} - m_{B_i} + m_{A_i} m_{B_i} = 1 \end{aligned} \tag{5.8}$$

In the next step, consequent fuzzy sets are "cut" corresponding to their firing levels:

$$\mathrm{C}'_i(z) = m_{C_i} . \mathrm{C}_i(z),\ for\, i = 1, \ldots, N \tag{5.9}$$

where $m_{C_i} = m_{A_i} . m_{B_i}$, $for\, i = 1, \ldots, N$

In order to calculate the value representing impact received by the effect node Z, z, when several rules are fired it is necessary to calculate the union $U$ of the consequent sets of the effect node using max function:

$$\mathrm{U}(z) = \max[\mathrm{C}'_i(z), \mathrm{C}'_j(z), \mathrm{C}'_k(z), \mathrm{C}'_l(z)] \tag{5.10}$$

Defuzzified value of the union represents received impact z:

$$z = xc_U \tag{5.11}$$

After the output of the cFCR is inferred it is possible to calculate the fuzzy representation of this crisp impact, ILT. Obtained crisp value z is transferred on the universe of discourse and two neighbouring fuzzy sets are identified and interpolation is carried out as described in Chapter 5.2.2.

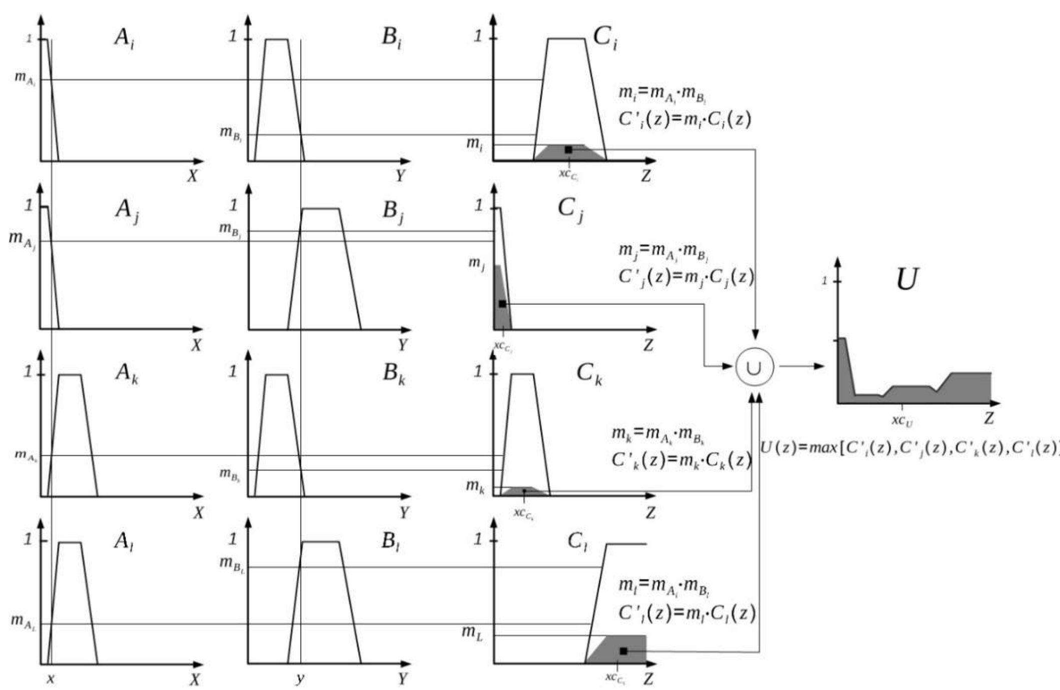



Figure 5.20. Inference on cFCR rules

## 5.4.2 Complex relationships and OR operator

In some definitions of FCR and cFCR between causal and effect nodes, two consecutive rules can have the same value in the THEN part of the rules. When consequent fuzzy sets of several rules that are fired by the input of a causal node are the same, then these rules can be combined and rewritten using OR operator to shorten and organise the definition.

Let us consider the following set of rules defining cFCR between two causal nodes: Node X and Node Y and one effect node: Node Z:

$$
\begin{aligned}
&R_i : \text{IF X is } A_i \quad \text{AND Y is } B_i \text{ THEN Z is } C_i\\
&R_j : \text{IF X is } A_i \quad \text{AND Y is } B_j \text{ THEN Z is } C_j\\
&R_k : \text{IF X is } A_k \quad \text{AND Y is } B_i \text{ THEN Z is } C_k\\
&R_l : \text{IF X is } A_k \quad \text{AND Y is } B_j \text{ THEN Z is } C_l
\end{aligned}
$$

If the fuzzy set, $C_l = C_k$ then the rules will have the following format:

$$
\begin{aligned}
&R_i : \text{IF X is } A_i \ (m_{A_i}) \quad \text{AND Y is } B_i \ (m_{B_i}) \quad \text{THEN Z is } C_i \ (m_i)\\
&R_j : \text{IF X is } A_i \ (m_{A_i}) \quad \text{AND Y is } B_j \ (1\text{-}m_{B_i}) \text{ THEN Z is } C_j \ (m_j)\\
&R_k : \text{IF X is } A_k \ (1\text{-}m_{A_i}) \text{ AND Y is } B_i \ (m_{B_i}) \quad \text{THEN Z is } C_k \ (m_k)\\
&R_l : \text{IF X is } A_k \ (1\text{-}m_{A_i}) \text{ AND Y is } B_j \ (1\text{-}m_{B_i}) \text{ THEN Z is } C_k \ (m_l)
\end{aligned}
$$

In the given example two rules k and l are fired with different firing levels and their consequent parts are the same. In such case, firing levels of rules k and l need to be summed, using algebraic sum and fuzzy set $C'_{k+l}$ is cut, instead of $C'_k$ and $C'_l$, using firing level $m_{i+j}$

$$m_{i+j} = m_i + m_j \tag{5.12}$$

It assures, that after aggregation of rules, the sum of all firing levels, will be equal to 1.

In before mentioned set of rules it can be observed that rules k and l have one of the antecedents and consequent equal:

$$
\begin{aligned}
&R_i : \text{IF X is } A_i \text{ AND Y is } B_i \text{ THEN Z is } C_i \\
&R_j : \text{IF X is } A_i \text{ AND Y is } B_j \text{ THEN Z is } C_j \\
&R_k : \text{IF X is } \underline{A_k} \text{ AND Y is } B_i \text{ THEN Z is } \underline{C_k} \\
&R_l : \text{IF X is } \underline{A_k} \text{ AND Y is } B_j \text{ THEN Z is } \underline{C_k}
\end{aligned}
$$

Rules whose fuzzy sets of the antecedent and the fuzzy set of consequent are the same, can be merged and in consequence the following rules can be defined:

$$
\begin{aligned}
&R_i : \text{IF X is } A_i \text{ AND Y is } B_i \quad \text{THEN Z is } C_i \\
&R_j : \text{IF X is } A_i \text{ AND Y is } B_j \quad \text{THEN Z is } C_j \\
&R_k : \text{IF X is } A_k \text{ AND Y is } (\underline{B_i} \text{ OR } \underline{B_j}) \text{ THEN Z is } C_k
\end{aligned}
$$

Reasoning on rules built with OR operator is presented in Figure 5.21.

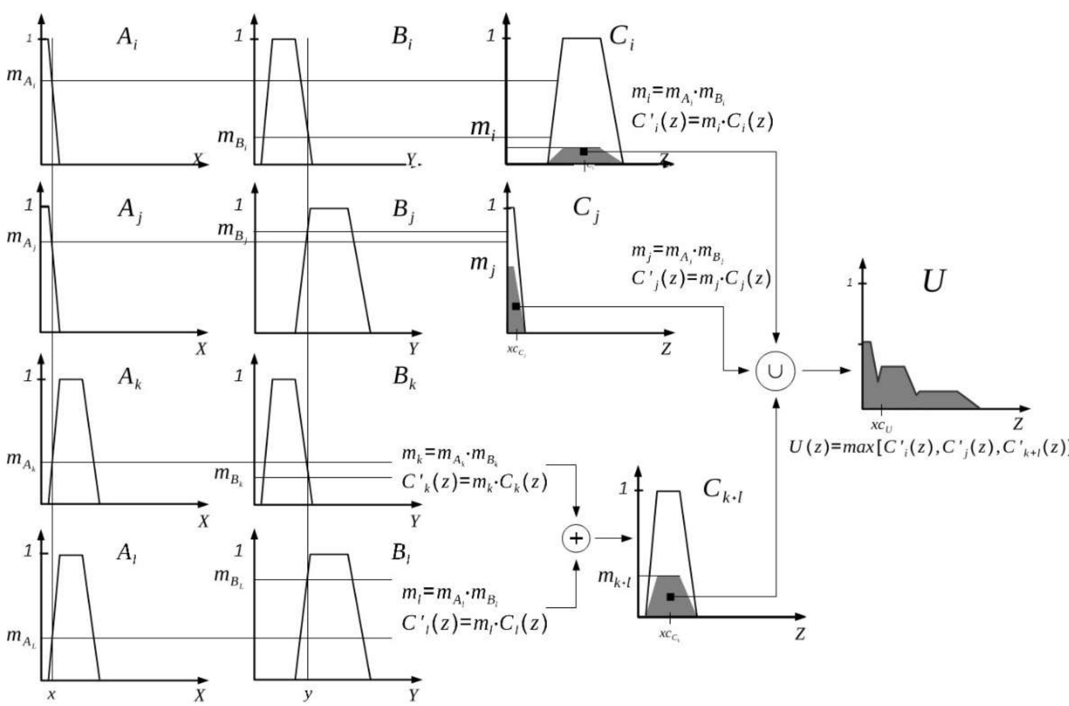



Figure 5.21. cFCR and use of OR operator

### 5.4.3 Conditional cFCR and NOT operator

Let us consider an example of the complex relationship between causal nodes "Budget for Advancements of equipment" and the effect node "Training ability", as in Figure 5.22. As it can be seen there is an additional input of the node "Budget for Training equipment", considered in the relationship.

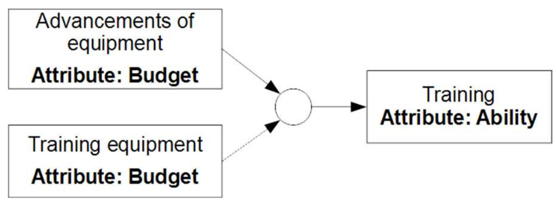



Figure 5.22. Example of conditional cFCR

Advancements of equipment represent improvements of the equipment used by soldiers in case of military or non-military intervention. Therefore, it should not have impact on the training ability of the military unit. However, if there is a great advancement of equipment used in combat, compared to the training equipment, it can have a negative effect on the training ability of the military unit as the equipment used in both are different. That is why the relationship between node "Budget for Advancements of equipment" and "Ability of training" is observable only when the budget for "Advancements of equipment" is *Increased* or *Increased Much* and the "Budget for Training Equipment" does not follow the change. The following set of rules is defining this relationship:

1. IF Budget for Advancements of Equipment is *Decreased Much*
   THEN Training DLoD is *Maintained*
2. IF Budget for Advancements of Equipment is *Decreased*
   THEN Training DLoD is *Maintained*
3. IF Budget for Advancements of Equipment is *Maintained*
   THEN Training DLoD is *Maintained*
4. IF Budget for Advancements of Equipment is Increased Little

THEN Training DLoD is *Maintained*

5. IF Budget for Advancements of Equipment is *Increased*
   AND (Training Equipment is *Increased* OR is *Increased Much*)
   THEN Training DLoD is *Maintained*
6. IF Budget for Advancements of Equipment is *Increased*
   AND NOT (Training Equipment is *Increased* OR is *Increased Much*)
   THEN Training DLoD is *Decreased Little*
7. IF Budget for Advancements of Equipment is *Increased Much*
   **AND (Training Equipment is *Increased* OR is *Increased Much*)**
   THEN Training DLoD is *Decreased Little*
8. IF Budget for Advancements of Equipment is *Increased Much*
   **AND NOT (Training Equipment is *Increased* OR is *Increased Much*)**
   THEN Training DLoD is *Decreased*

Conditional cFCR can be defined as follows:

$$\begin{array}{ll} \mathrm{R_i: IF\ X\ is\ A_i} & \mathrm{THEN\ Z\ is\ C_i} \\ \mathrm{R_j: IF\ X\ is\ A_k\ AND\ Y\ is\ B_i} & \mathrm{THEN\ Z\ is\ C_j} \\ \mathrm{R_k: IF\ X\ is\ A_k\ AND\ Y\ is\ B_j} & \mathrm{THEN\ Z\ is\ C_j} \end{array}$$

An example of reasoning on conditional cFCR is presented in Figure 5.23.

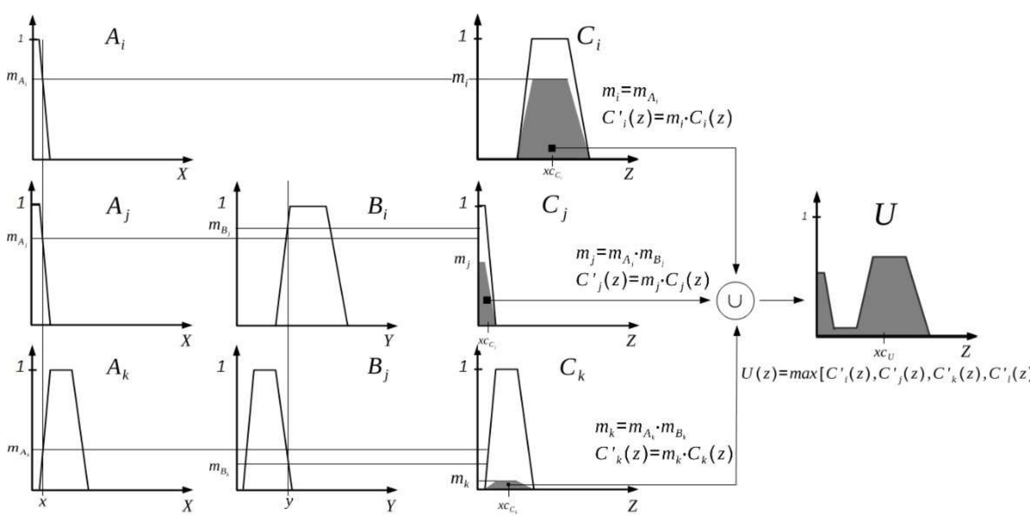



Figure 5.23. An example of conditional cFCR

Conditional relationship can be defined using OR and NOT operators, as follows:

$R_i$ :IF X is $A_i$ THEN Z is $C_i$
$R_k$:IF X is $A_k$ AND Y is ($B_i$ OR $B_j$) THEN Z is $C_j$
$R_k$:IF X is $A_k$ AND Y is NOT ($B_i$ OR $B_j$) THEN Z is $C_k$

The NOT operator acts as a logical negation:

$$B_i\ OR\ B_j \Rightarrow m_{i+j} = m_i + m_j$$

$$NOT(m_{i+j}) = 1 - m_{i+j}$$

Using NOT operator allows to shorten the definition of the rule base where only two distinctive values of consequent are used.

Let us consider the relationship as in Figure 5.22 and example of rules 7 and 8 defined for this relationship:

7. IF Budget for Advancements of Equipment is *Increased Much*
   **AND (Training Equipment is *Increased* OR is *Increased Much*)**
   THEN Training DLoD is *Decreased Little*
8. IF Budget for Advancements Advancement of Equipment is *Increased Much*
   **AND NOT (Training Equipment is *Increased* OR is *Increased Much*)**
   THEN Training DLoD is *Decreased*

If one would like to represent the rules 7 and 8 without NOT operator it would require a much longer definition:

1. IF Budget for Advancements Advancement of Equipment is *Increased Much*
   **AND Training Equipment is *Decreased Much***
   THEN Training DLoD is *Decreased*
2. IF Budget for Advancements Advancement of Equipment is *Increased Much*
   **AND Training Equipment is *Decreased***
   THEN Training DLoD is *Decreased*
3. IF Budget for Advancements Advancement of Equipment is *Increased Much*
   **AND Training Equipment is *Decreased Little***

THEN Training DLoD is *Decreased*

4. IF Budget for Advancements Advancement of Equipment is *Increased Much*
   **AND Training Equipment is *Maintained***
   THEN Training DLoD is *Decreased*
5. IF Budget for Advancements Advancement of Equipment is *Increased Much*
   **AND Training Equipment is *Increased Little***
   THEN Training DLoD is *Decreased*
6. IF Budget for Advancements Advancement of Equipment is *Increased Much*
   **AND (Training Equipment is *Increased* OR is *Increased Much*)**
   THEN Training DLoD is *Decreased Little*

## 5.5 Conclusions

The RBFCMs introduced the novel approach to interpolating the output of the inference on the universe of discourse of the effect node. It was developed under set of restrictions, ensuring, that the interpolated fuzzy set will have an area proportional to its position on the universe of discourse and will be proportional to other fuzzy sets it is close to.

The new interpolation mechanism, as well as the new approach to the reasoning, separating the inference and interpolation into two separate processes are introduced in this thesis. They can be considered to be a consequence of the decision of using the standard semantics of fuzzy sets. In the same time, development of the new reasoning, allowed great reduction of the number of constraints on how the reasoning should be conducted and what methods of inference or defuzzification can be used. The new approach gives the initiative to modellers, who can decide on methods that suit best the application.

The new approach addresses issues, prevalent in the standard RBFCM, but not identified in the literature before:

1. incorrect calculations of the shape of the interpolated fuzzy set when two consequent fuzzy sets differ greatly,
2. interpolation of the output of the inference even when the crisp output is already represented on the universe of discourse of the effect node, by the centroid of one of the fuzzy sets.

The first issue identified is especially critical, because the standard accumulation of impacts mechanism, discussed in Chapter 6, relies entirely on the shape of fuzzy sets calculated by the reasoning mechanism.

Additionally, in this chapter inconsistency was identified, in the restriction 2.33 of the standard mechanism (section 2.4.1), specifying that fuzzy sets representing a greater variation needs to have area and support greater than fuzzy sets representing smaller variation. Under some circumstances, the standard reasoning mechanism does not interpolate the output of the inference correctly. It is argued that, in order to ensure correct interpolation of fuzzy sets, all the features of the fuzzy set representing greater variation, need to be greater than those of the fuzzy set representing the smaller variation.

The next step of the research on the reasoning mechanism should be focused on further development of the method to eliminate the need of using any constrains on the shape of the fuzzy sets used in RBFCMs. The separation of the inference and interpolation of linguistic terms creates a potential for removal of the remaining constraints: 1 and 2, but it needs to be investigated further to confirm the initial finding.

The second area of further investigation is the analysis of the definition on which element of the fuzzy sets determines whether the input of interpolation is represented by the fuzzy set on the universe of discourse. One could argue that the core of the fuzzy sets defines the full membership of a given point to a fuzzy set, therefore, the core, instead of the centroid should be used as a reference point.

Given the current trends in fuzzy systems, the further research should be focused on increasing flexibility of the reasoning and accumulation of impacts, described in Chapter 6, to allow using Type 2 Fuzzy Sets. During the work on case studies multiple times the authors encountered situations where the use of Type 2 Fuzzy Sets would have been beneficial, i.e. when there was more than one opinion of experts on the shape of the fuzzy sets.

# Chapter 6. ACCUMULATION AND DYNAMICS IN RBFCMS

## 6.1 Introduction

The main purpose of the development of RBFCMs was the introduction of the accumulation mechanism to fuzzy rule based systems (Carvalho, 1999). Other fuzzy rule based systems were based mainly on two inference methods: Mamdani (1977) and Takagi-Sugeno (1985). Both methods do not allow accumulation of impacts but tend to average the value of multiple impacts received. To describe the need for accumulation of impacts, let us consider the following example based on the case study described in Section 3.3. Let us assume, that the node “Ability to respond appropriately to successful compromises” is affected by two nodes “Understanding of adversary intent” and “Understanding of adversary capability”. Let us assume that the value of the node “Understanding of adversary intent” is *Increased* and causes “Ability to respond appropriately to successful compromises” to *Increase*, and that “Understanding of adversary capability” is also *Increased* and causes “Ability to respond appropriately to successful compromises” to *Increase Little*. It seems appropriate that “Ability to respond appropriately to successful compromises” should change to *More than Increase*. However, the standard inference mechanisms, Mamdani or Takagi-Sugeno, will result in “Number of CIS compromises detected” changing somewhere between value *Increased Little* and *Increased* depending on the strength of each impact. Therefore, a different mechanism for accumulation of impacts in RBFCMs is required, as proposed by Carvalho and Tome (2009).

Accumulation of impacts in RBFCMs was built around the idea that the shape of the fuzzy set represents its position on the universe of discourse. When the fuzzy set is defined in such a way that it does not follow the condition relating the shape and the position of the fuzzy set, then the results of accumulation of impacts are counterintuitive to what one would expect, as discussed in Section 6.3.1. In consequence, a new accumulation of impacts mechanism is introduced in this thesis, to allow usage of fuzzy sets defined more flexibly.

The area of the fuzzy set, used in the standard RBFCMs accumulation mechanism is considered as a carrier of information on variation/level of the fuzzy set and its position on the universe of discourse (Section 2.4.2). In this way, the area doubles the role of the centroid of the fuzzy set. In the new accumulation approach the centroid of the fuzzy set is used as a standard expression of the position of the fuzzy set on the universe of discourse.

While the shape of the fuzzy set should not be used to determine its position on the universe of discourse, it can be used to analyse the uncertainty of the experts. It carries information about uncertainty, which can be analysed using different metrics, as discussed in Section 2.1.5. The new accumulation of impacts mechanism is developed with emphasis of the importance of the uncertainty, embedded in every fuzzy set.

Apart from new accumulation mechanism, this chapter introduces new approach to saturation of impacts when the accumulated impacts exceed the maximum/minimum of the universe of discourse, +100 and -100%, respectively. It also defines a new approach to handling propagation of impacts in loops in RBFCMs.

This chapter is organised as follows: first, the new accumulation of impacts mechanism is introduced in section 6.2, with a detailed explanation of all the steps that need to be taken. The next part discusses the new mechanism of saturation of the output of accumulation, when the accumulated fuzzy set expands beyond the limits of the universe of discourse. The following section, 6.2.4, presents several examples of accumulation of different numbers and compositions of impacts, as well as the saturation mechanism. The section 6.3 discusses the differences between the standard and new accumulation mechanisms and highlights advantages of using the latter in the context of flexibility of RBFCMs. The fourth part of the chapter is focused on improvement of dynamics in RBFCMs: handling time component and a mechanism to mitigate the impact of loops. The chapter finishes with conclusions and discussion on potential further developments of the accumulation and dynamics in the RBFCM.

## 6.2 Accumulation of impacts

The accumulation process is carried out when more than one causal node is affecting an effect node. Before the impacts received by the effect node are accumulated it is necessary to infer and interpolate impacts of causal nodes on an effect node (Section 5.2). Impacts received by an effect node are accumulated to represent the state of the node using a single fuzzy set (Figure 6.1).

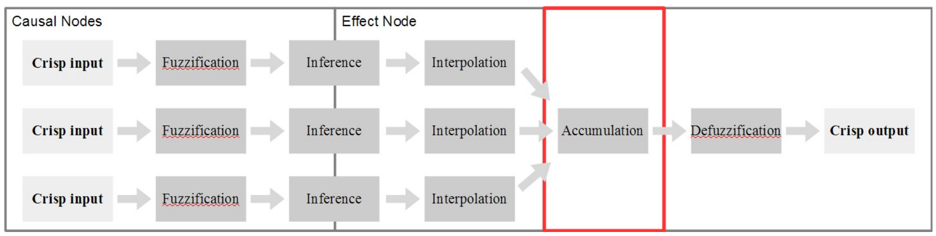



Figure 6.1. Reasoning in RBFCMs – accumulation of impacts

Let us assume that the effect Node X is in a causal relationship with N causal nodes (Figure 6.2). Every causal node, triggered by its own input, has an impact on the effect Node X, represented by a fuzzy set $C_i$, where i=1…N and N equals to a number of causal nodes. Fuzzy sets $C_i$ are calculated through interpolation (Figure 6.1).

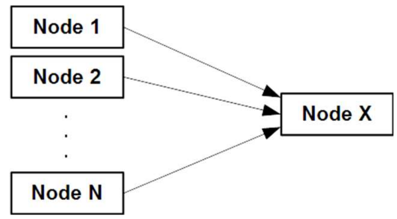



Figure 6.2. Example of N relationships between N causal nodes and one effect node

The fuzzy set resulting from accumulation of multiple fuzzy sets $C_i$, is called Variation Output Set (VOS). It represents the impact received by the effect node using a single fuzzy set. VOS is calculated in three steps:

1. Calculation of the shape of the fuzzy sets VOS,
2. Calculation of the centroid of VOS, $xc_{VOS}$,
3. Saturation of the fuzzy set if the centroid of VOS reached the minimum or maximum of the universe of discourse.

## 6.2.1 Calculation of the shape of the fuzzy set

The shape of the fuzzy set VOS is calculated using the standard fuzzy summation operation:

$$VOS(C_1,,...,C_N) = C_1\, accum,...,accum C_N = C_1 + ... + C_N \tag{6.1}$$

where N equals to number of fuzzy sets involved in the accumulation.

As a result, the four points defining a trapezoidal membership function of VOS are calculated as follows:

$$VOS(C_1+...+C_N)=\begin{cases} a_{VOS}=a_{C_1}+...+a_{C_N} \\ b_{VOS}=b_{C_1}+...+b_{C_N} \\ c_{VOS}=c_{C_1}+...+c_{C_N} \\ d_{VOS}=d_{C_1}+...+d_{C_N} \end{cases} \tag{6.2}$$

The summation of fuzzy sets ensures that the uncertainty of the VOS fuzzy set reflects uncertainty levels of all the fuzzy sets involved in the accumulation operation. As a result, the main characteristics of the trapezoidal fuzzy set have values as follows:

$$\begin{aligned} core_{VOS} &= core_{C_i}+...+core_{C_N} \\ bi_{VOS} &= bi_{C_i}+...+bi_{C_N} \\ bo_{VOS} &= bo_{C_i}+...+bo_{C_N} \end{aligned} \tag{6.3}$$

## 6.2.2 Calculation of the centroid of the VOS

The second step of accumulation is calculation of the centroid of the fuzzy set VOS. In this thesis, a mechanism for calculating the centroid of VOS, using fuzziness of the fuzzy sets involved in the accumulation operation, is introduced. It relates the accumulation strength to the uncertainty of the fuzzy set.

The centroid $xc_{VOS}$, of fuzzy set $\mathrm{VOS}$, is calculated considering centroids of fuzzy sets $C_i$, $xc_{C_i}$ as follows:

$$xc_{vos}=\left(\sum_{i=1}^{N} xc_{C_i}\right)\cdot p \tag{6.4}$$

where

$$p=1-\frac{\sum_{i=1}^{N}\left(\hat{f}(C_i)\right)}{N\times s} \tag{6.5}$$

$$\hat{f}(C_i)=\frac{bi_{C_i}+bo_{C_i}}{support_{C_i}} \tag{6.6}$$

$$s = (1, \infty) \tag{6.7}$$

The centroid of VOS, $xc_{VOS}$, is equal to the sum of centroids of fuzzy sets involved in the accumulation operation. It is weighted with parameter $p$, which depends on the fuzziness of fuzzy sets involved in the accumulation. The greater the fuzziness of fuzzy sets $\hat{f}(C_i)$ is, the smaller the parameter $p$ and $xc_{vos}$ are.

Parameter p takes the maximum value 1 when fuzzy sets $C_i$, $i = 1, ..., \mathrm{N}$ are crisp sets or singletons i.e., when $\hat{f}(C_1) = ... = \hat{f}(C_i) = ... = \hat{f}(C_N) = 0$. As a result, $xc_{vos}$ is equal to the arithmetic sum of centroids of the respective fuzzy sets and impacts are fully accumulated – 100% of the sum of centroids. Parameter p takes the minimum value 0.5 when fuzzy sets $C_i$ are triangular fuzzy sets, $\hat{f}(C_1) = ... = \hat{f}(C_i) = ... = \hat{f}(C_N) = 0.5$ and $s = 1$. In this case 50% of the sum of centroids of the respective fuzzy sets will be accumulated. The described relationship is displayed in Figure 6.3, which presents the relationship between different levels of fuzziness of fuzzy sets A and B, and the percentage of the maximal accumulation of two crisp fuzzy sets $xc_{VOS}$.

Parameter $s$ is introduced to increase the impact of accumulation. The higher the value of parameter $s$ is, the stronger the accumulation of impacts. Table 6.1 summarizes the impact of the parameter $s$ on the parameter $p$.

Table 6.1 Impact of parameter s on the strength of accumulation

| $s$ | $p_{\min}$ | $p_{\max}$ |
|---|---|---|
| 1 | 0.5 | 1 |
| 2 | 0.75 | 1 |
| 4 | 0.875 | 1 |
| 10 | 0.95 | 1 |

As it can be seen in the Table 6.1, regardless of the value of the parameter $s$, the maximum accumulation is equal to 1, i.e., 100% of the sum of centroids is accumulated in case of crisp fuzzy sets. The parameter $s$ has impact on the accumulation when fuzzy sets are not crisp. Increasing the value of the parameter $s$ from 1 to 2, increases the accumulation strength for two fuzzy sets with max fuzziness from 0.5 to 0.75, etc.

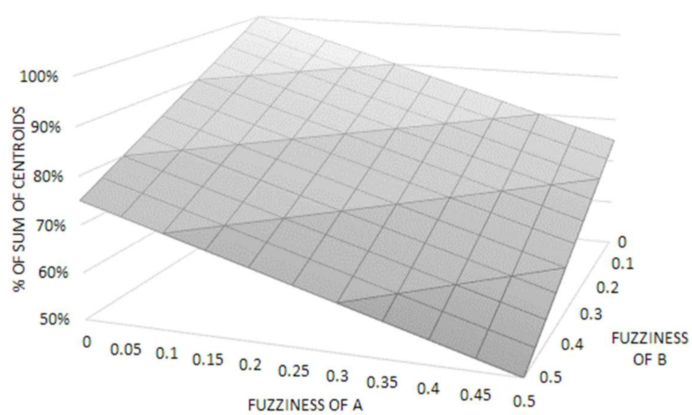


Figure 6.3. Relationship between fuzziness levels and accumulation strength

Figure 6.4 presents the impact of the parameter $s$ on the parameter p, for different levels of fuzziness of the fuzzy set B, assuming that the fuzziness of the fuzzy set A is equal to 0.5.

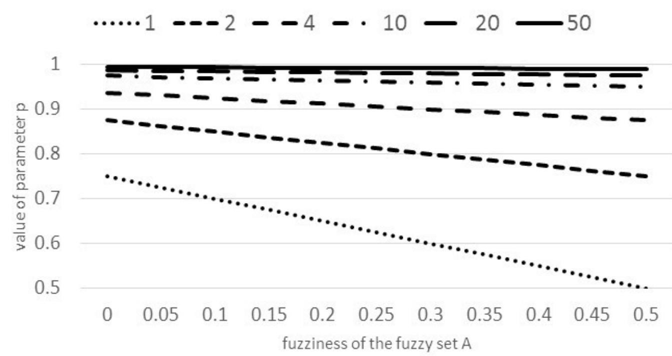



Figure 6.4. Impact of different values of parameter s (1 to 50) on the accumulation strength, where $\hat{f}(B) = 0.5$

## 6.2.3 Examples of accumulation

### 6.2.3.1 Accumulation of one impact

Let us consider an example of relationships from the Case Study 2, described in Section 3.3. Let us assume that the causal nodes: “Understanding of adversary intent” is in a causal relationship with the effect node, “Risk to the business of compromises and mitigation measures”, as shown in Figure 6.5.

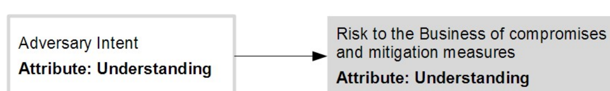


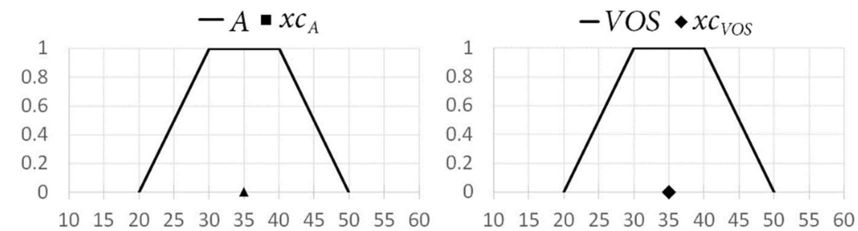

Figure 6.5. An example of relationship between one causal node and one effect node

Let us assume that the causal node “Risk to the business of compromises and mitigation measures” receives one fuzzy impact A, presented in Figure 6.6 (a).

When only one impact is received, it automatically becomes VOS and does not need to be accumulated.

$$VOS = A \tag{6.8}$$

$$xc_{VOS} = xc_A \tag{6.9}$$




(a) One impact – fuzzy set A

(b) VOS resulting from accumulation of impact A

Figure 6.6. Accumulation when one impact is received by an effect node

### 6.2.3.2 Accumulation of two impacts

Let us assume that the causal node "Risk to the business of compromises and mitigation measures" is in causal relationship with nodes "Understanding of adversary intent" and "Understanding of adversary capability" (Figure 6.7) receives two fuzzy impacts A and B, presented in Figure 6.8 (a).

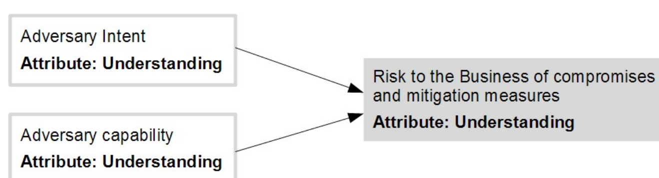



Figure 6.7. Accumulation of impacts using the new accumulation method

Two impacts received by a causal node need to be accumulated:

$$A\ accum\ B = VOS(A,B) = A + B \tag{6.10}$$

As a result of summation operation of fuzzy sets A and B, the resulting fuzzy set VOS is described by the following four points:

$$VOS(A+B) = \begin{cases} a_{VOS} = a_A + a_B \\ b_{VOS} = b_A + b_B \\ c_{VOS} = c_A + c_B \\ d_{VOS} = d_A + d_B \end{cases} \tag{6.11}$$

As a result of summation, the main elements of VOS are as follows:

$$core_{VOS} = core_B + core_A \tag{6.12}$$

$$bi_{VOS} = bi_A + bi_B \tag{6.13}$$

$$bo_{VOS} = bo_A + bo_B \tag{6.14}$$

$$support_{VOS} = bi_{VOS} + core_{VOS} + bo_{VOS} \tag{6.15}$$

After they are calculated, the fuzzy set is shifted so that the centroid of VOS is equal to centroid calculated as follows:

$$xc_{VOS} = (xc_A + xc_B) \times p \tag{6.16}$$

where the parameter $p$ is defined in Section 6.2.2.

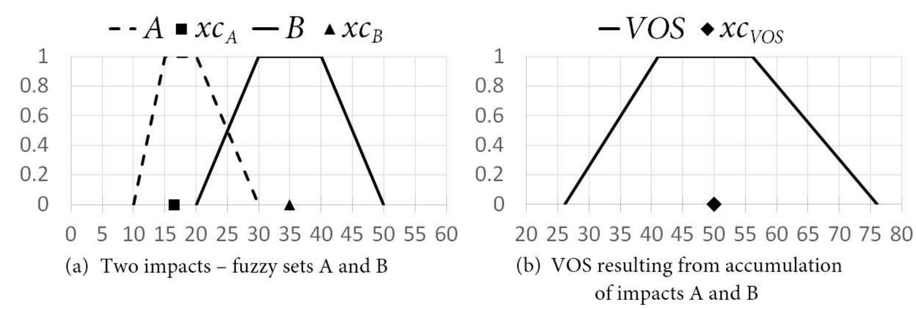



Figure 6.8. Accumulation when two impacts are received by an effect node

### 6.2.3.3 Accumulation of multiple impacts

In the next example, the node "Understanding of the risk to the business of compromises and mitigation measures" is impacted by the node "Understanding and awareness of CIS vulnerabilities" additionally to the two nodes given in the previous example (Figure 6.9).

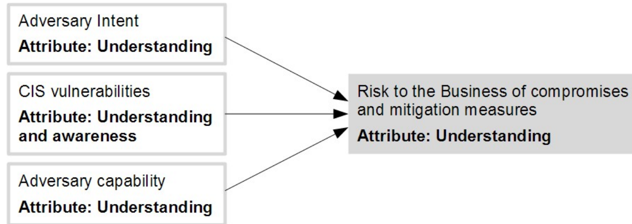



Figure 6.9. An example of relationship between three causal nodes and one effect node

In this example three impacts A, B and C are accumulated:

$$A \; accum \; B \; accum \; C = VOS(A,B,C) = A + B + C$$

The four points defining the trapezoid membership function, of the accumulated impacts, are calculated as follows:

$$VOS(A+B+C) = \begin{cases} a_{VOS} = a_A + a_B + a_C \\ b_{VOS} = b_A + b_B + b_C \\ c_{VOS} = c_A + c_B + c_C \\ d_{VOS} = d_A + d_B + d_C \end{cases}$$

The main characteristics of the fuzzy set VOS are as follows:

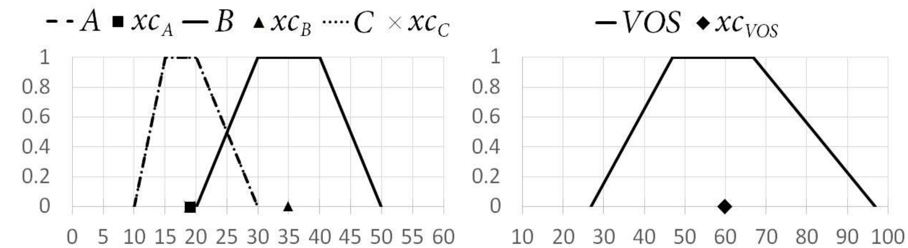
$$core_{VOS} = core_A + core_B + core_C$$

$$bi_{VOS} = bi_A + bi_B + bi_C$$

$$bo_{VOS} = bo_A + bo_B + bo_C$$

$$support_{VOS} = bi_{VOS} + core_{VOS} + bo_{VOS}$$

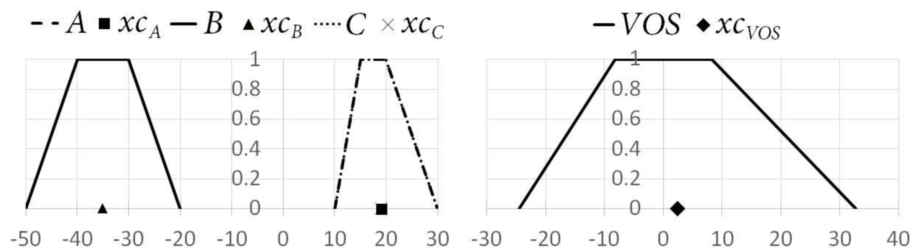
Figure 6.10 presents an example of accumulation of three positive impacts A, B and C. In Figure 6.11, different example of accumulation of multiple impacts is given, where two impacts A and C represent positive variation and the fuzzy set B, a negative one.




(a) Three impacts – fuzzy sets A, B and C

(b) VOS resulting from accumulation of impacts A, B and C

Figure 6.10. Accumulation of impacts when three positive impacts are received, where fuzzy set C = A, by an effect node




(a) Three impacts – fuzzy sets A, B and C

(b) VOS resulting from accumulation of impacts A, B and C

Figure 6.11. Accumulation of impacts when two positive and one negative impacts are received by an effect node, where fuzzy set C = A

## 6.2.4 Saturation

Saturation is a mechanism which allows handling the accumulation of impacts when its output, $VOS$, extends beyond the limits of the universe of discourse.

Let us consider the universe of discourse, $X$, representing variation from -100, being the maximally negative variation (decrease), to 100, being maximally positive variation (increase):

$$X_{\max} = 100$$

$$X_{\min} = -100$$

The saturation mechanism introduced in this chapter allows to reduce the shape of the fuzzy set to make sure that it stays within the boundaries of the universe of discourse and that its centroid is the same as the centroid of $VOS$:

$$xc_{VOS_S} = xc_{VOS} \tag{6.17}$$

where $VOS_S = sat(VOS)$ and $sat$ is saturation operation.

The important aspect of the saturation mechanism is that it maintains the same fuzziness of the saturated fuzzy set $VOS_S$ as the fuzziness of the fuzzy set VOS:

$$\hat{f}(VOS_S) = \hat{f}(VOS) \tag{6.18}$$

n the other hand, it reduces the specificity of the fuzzy set VOS:

$$Sp(VOS_S) > Sp(\mathrm{VOS}) \tag{6.19}$$

Every time a new impact is received and VOS needs to be saturated, its specificity increases, reaching maximum specificity, 1, and minimum fuzziness, 0, when the fuzzy set represents the maximum\minimum of the universe of discourse and takes value of the crisp singleton set.

In the new mechanism, the fuzzy set VOS is saturated by calculating the distance between the point situated the furthest away from the maximum, $X_{\max}$, or minimum, $X_{\min}$, of the universe of discourse, points d or a, respectively, and the centroid of the fuzzy set VOS. The calculated distance is weighted by the distance from the point d or a and the centroid of the VOS. The resulting parameter $f_{\max}$ or $f_{\min}$ is used to shift the respective points of the fuzzy set, resulting in shrinking of the fuzzy set, but maintaining the fuzziness and reducing specificity.

Parameter $f_{\max}$ and $f_{\min}$ are calculated as follows:

$$f_{\max} = \frac{d - X_{\max}}{d - xc_{VOS}} \tag{6.20}$$

$$f_{\min} = \frac{a - X_{\min}}{a - xc_{VOS}} \tag{6.21}$$

When fuzzy set VOS extends beyond the maximum of the universe of discourse, the following formula is used to calculate points of the saturated VOS:

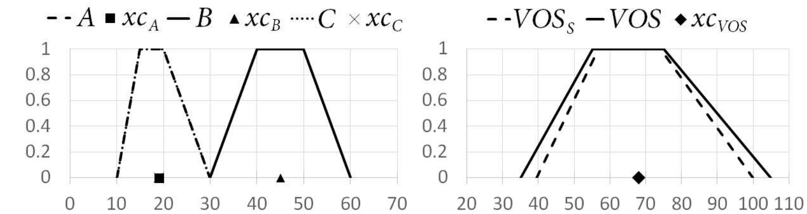
$$VOS_S = sat(VOS) = \begin{cases} a' = a + \left|a - xc_{VOS}\right| \times f_{\max} \\ b' = b + \left|b - xc_{VOS}\right| \times f_{\max} \\ c' = c - \left|c - xc_{VOS}\right| \times f_{\max} \\ d' = d - \left|d - xc_{VOS}\right| \times f_{\max} \end{cases} \tag{6.22}$$

When fuzzy set VOS extends beyond the minimum of the universe of discourse, then the following formula is used to calculate points of the saturated VOS:

$$VOS_S = sat(VOS) = \begin{cases} a' = a + \left|a - xc_{VOS}\right| \times f_{\min} \\ b' = b + \left|b - xc_{VOS}\right| \times f_{\min} \\ c' = c - \left|c - xc_{VOS}\right| \times f_{\min} \\ d' = d - \left|d - xc_{VOS}\right| \times f_{\min} \end{cases} \tag{6.23}$$

#### 6.2.4.1 Examples of saturation of impacts

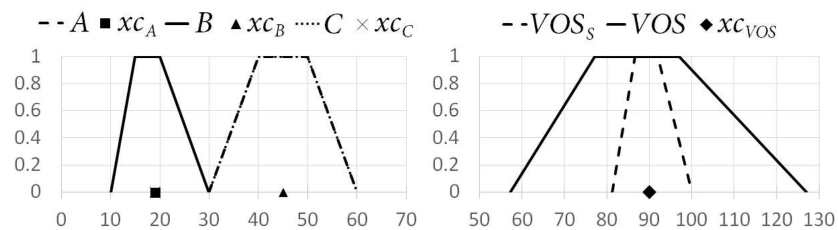
An example of saturation for fuzzy set VOS, extending beyond the maximum or minimum of the universe of discourse is presented in the Figure 6.12. In this example three fuzzy sets A, B and C, where C=A, representing positive variations, are accumulated, resulting in the fuzzy set VOS overflowing the maximum of the universe of discourse by 5. As a result of saturating the fuzzy set VOS, the specificity of the fuzzy set $VOS_S$ is slightly increased, while the centroid and fuzziness of the new fuzzy set are the same.




(a) Three impacts – fuzzy sets A, B and C (b) VOS and $VOS_S$ resulting from accumulation of impacts A, B and C

Figure 6.12. Saturation of VOS, where d=105

In Figure 6.13, a similar example is presented, where VOS is calculated based on three impacts A, B and C, where C=A. The strength of impacts is stronger than those presented in Figure 6.12, therefore the overflow of the VOS over the maximum of the universe of discourse is also greater. The saturated $VOS_S$ is much more specific than the VOS but the fuzziness and the centroid remain unchanged.




(a) Three impacts – fuzzy sets A, B and C (b) VOS and $VOS_S$ resulting from accumulation of impacts A, B and C

Figure 6.13. Saturation of VOS, where d = 128.

## 6.3 Propagation of impacts in loops

One of the challenges when modelling real world dynamic systems is existence of loops in causal maps. While modelling methods, such as SD, have multiple, well defined, techniques of taking advantage of loops, RBFCMs are not as well equipped to handle propagation of impacts within loops. In this thesis, the approach to mitigating the effect of loops in RBFCMs is proposed. It is achieved by deteriorating impacts received by an effect node, to ensure that the total accumulated impact in any node does not exceed the minimum or maximum of its universe of discourse.

An example of one loop in the cyber defence case study, introduced in Section 3.3, is presented in Figure 6.14. To understand how the impact propagates through this loop, let us consider an example of investments that has a positive impact on the “Understanding of Cyber tools” or the “Understanding of Cyber tradecraft” node. The result of increasing the understanding of either the tools or the tradecraft, is the increase of “Ability to detect compromises to CIS”, and in consequence the increase in “Number of CIS compromises detected”. Detecting more compromises means that there is more material for analysis of vulnerabilities of security and detection systems. More analysis of both allows increasing the “Ability to assess and learn from system compromises”. An increase in the assessment capabilities allows to use the cyber security tools more effectively and raise awareness of cyber security specialists.

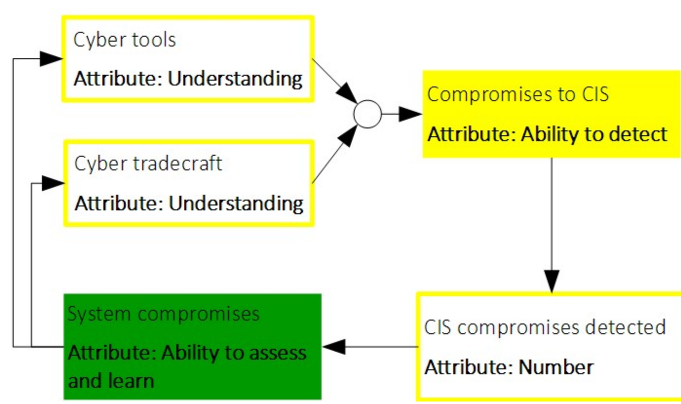


Figure 6.14. Example of a loop in the RBFCM cyber defence model

The discussed example presents a causal chain where a positive or a negative input in any node results in propagation of the impact through the loop and its infinite reinforcement, unless another impact is introduced in the system to counteract.

In order to control the build-up of impacts in loops, every new impact received by any effect node in the model, in time period t, is deteriorated using the following formulas:

- If the impact is positive, $impact_{C_i}^{t} > 0$ and the value of $VOS$ in iteration $t-1$ is positive, $xc_{VOS,t-1} > 0$, then:

$$xc_{VOS,\,t}^{d} = xc_{VOS,\,t-1} + impact_{C_i}^{t} \times (X_{\max} - xc_{VOS,\,t-1}) / X_{\max} \tag{6.24}$$

- For a negative impact, $impact_{C_i}^{t} < 0$ and the value of $VOS$ in iteration $t-1$ is negative $xc_{VOS,t-1} < 0$ then:

$$xc_{VOS,\,t}^{d} = x_{VOS,\,t-1} + impact_{C_i}^{t} \times (X_{\min} - x_{VOS,\,t-1}) / X_{\min} \tag{6.25}$$

where: $xc_{VOS,\,t}^{d}$ is a reduced value of $xc_{VOS,\,t}$ in iteration $t$

$impact_{C_i}^{t} = xc_{VOS,\,t} - xc_{VOS,\,t-1}$

$t$ is an iteration, $t = 1, \ldots, \mathrm{T}$ and $T$ is number of simulation iterations,

$X_{\max}$ is the maximal variation of the node and

$X_{\min}$ is the minimal variation of the node.

Both formulas ensure that the total value of the node in iteration $t$ does not exceed the maximum or minimum of the universe of discourse, $X_{\max}$ and $X_{\min}$, respectively. The closer the value of the node is to its maximal/minimal value, the less impact is accumulated.

The deterioration of impacts is done after the impacts are accumulated and saturated in iteration $t$. Once the deteriorated value of the centroid of $VOS$, $xc_{VOS}^{d}$, is calculated, then the fuzzy set $VOS$ is shifted so that its centroid is equal to its deteriorated value:

$$xc_{VOS} = xc_{VOS}^{d} \tag{6.26}$$

Table 6.2 and Figure 6.15 present a comparison of accumulation of impacts, with and without deterioration, assuming that in each iteration a node receives one impact of the same value equal to 10. The maximum of the universe of discourse of the effect node is $X_{\max} = 100$. When the impacts are not deteriorated, the value of the node reaches the maximum of its universe of discourse after 10 iterations. However, the deterioration function reduces the total accumulated impact in iteration 10 to 65.2. In iteration 40, the total accumulated impact, when impacts are not deteriorated, is equal to 400, extending beyond the maximum of the universe of discourse by 300. However, when impacts are deteriorated, the total accumulated impact is very close to the maximum of the universe of discourse, $X_{\max} = 100$, but is not extending it.

Table 6.2. Comparison of accumulation of impacts with and without deterioration of impacts

| *Iteration* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Received impact* | *0* | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| *Value (deteriorated)* | *0* | 10 | 19 | 27.1 | 34.3 | 41 | 46.9 | 52.2 | 57 | 61.3 | 65.2 |
| *Value (no deterioration)* | *0* | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

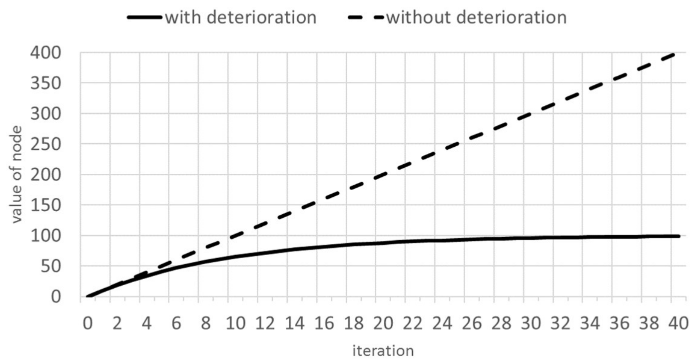



Figure 6.15. Comparison of accumulation of impacts with and without deterioration of impacts, assuming that the impact received in every iteration is equal to 10

## 6.4 Discussion on Accumulation of impacts

### 6.4.1 Commutativity and associativity

In an RBFCM, an effect node can be impacted by more than two causal nodes. Therefore, the possibility of receiving more than two fuzzy impacts simultaneously exists. To assure that the order of receiving impacts is not predominant, accumulation of impacts mechanism should have the following two properties:

- Commutativity – A accum B = B accum A
- Associativity – A accum (B accum C) = (A accum B) accum C

Both mechanisms, the standard and the new one, are commutative and associative. The standard mechanism, regardless of which of the impacts A, B or C is received first, shifts the fuzzy sets representing smaller variations towards the one representing greater variation. Therefore, the result of accumulation of two or more impacts is always the same. The new mechanism accumulates impacts based on the fuzziness and centroid of fuzzy sets involved, therefore it is not relevant which impact is received first. There is a very significant difference between the two approaches.

### 6.4.2 Accumulation sensitivity

In order to assure that the standard mechanism performs correctly, Carvalho and Tome use minimum of the core of fuzzy sets to define which fuzzy set represent greater variation, rather than centroid:

$$A < B \Leftrightarrow \min(core_A) < \min(core_B)$$

Let us consider three examples to demonstrate differences between the standard and the new mechanisms for accumulation of impacts.

In Example 1, definitions of fuzzy sets A – *Increased* and B – *Increased Little* violates condition 2.33 (Section 2.4.1), as presented in Figure 6.16 (a). Let us assume that an effect node receives both impacts A and B, and $xc_A$=15, $Area_A$=15, $xc_B$=27.5 and $Area_B$=10, respectively. In Example 2, presented in Figure 6.17 (a), the effect node receives two identical impacts *Increased* and $xc_A$=$xc_B$=27.5 and $Area_A$=$Area_B$=10.

Table 6.3 shows the results of the two accumulation mechanisms. As it can be seen, if the standard RBFCMs accumulation is used, the combined effect of two impacts *Increased* in Example 2 is equal to 32.45. However, it is smaller than the combined effect of two impacts *Increased* and *Increased Little*, in Example 1, which is equal to 36.07. This means that receiving two impacts representing a larger variation, such as *Increased*, results in a smaller accumulated value than if one smaller, *Increased Little*, and one larger impact, *Increased*, are received and accumulated using the standard accumulation algorithm. It is due to the larger area of the fuzzy set *Increased Little* than the area of the fuzzy set *Increased.* On the other hand, the new mechanism accumulates impacts based on their fuzziness levels, rather than areas. Therefore, in Example 1, the combined effect of *Increased* and *Increased Little* is equal to 36.3, and the combined effect of two impacts *Increased* is higher, equal to 45.83, when s = 4. In examples discussed the value of parameter s is set to 4 to increase the strength of the accumulation to obtain results comparable with the standard mechanism.

Table 6.3 Comparison of results obtained using the two accumulation mechanisms

| | Example 1 | Example 2 | Example 3 | Example 4 |
|---|---|---|---|---|
| $xc_A$ | 15 | 27.5 | 16.5 | 16.5 |
| $xc_B$ | 27.5 | 27.5 | 37.5 | 37.5 |
| $xc_{VOS}$ standard mechanism | 36.1 | 32.5 | 48.7 | 49.1 |
| $xc_{VOS}$ new mechanism | 36.3 | 45.8 | 51.5 | 48.2 |

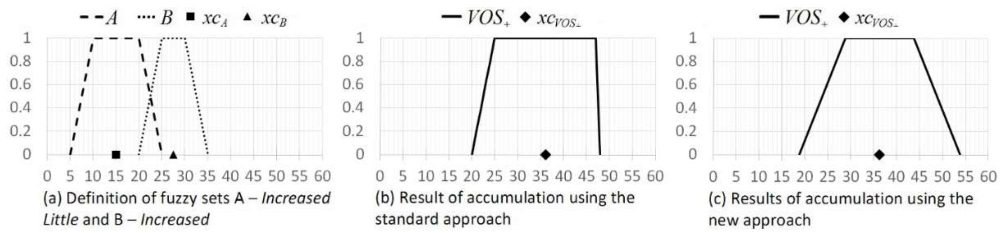



Figure 6.16. Comparison of results of the two mechanisms for accumulation of impacts in Example 1

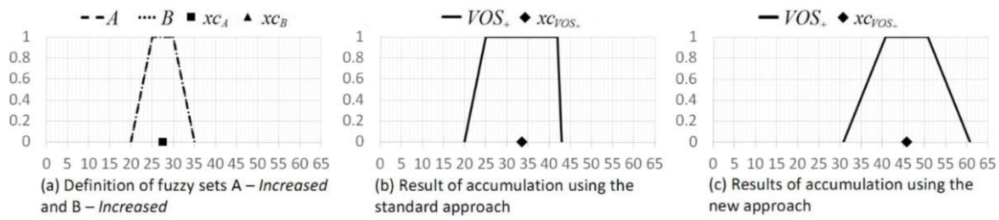
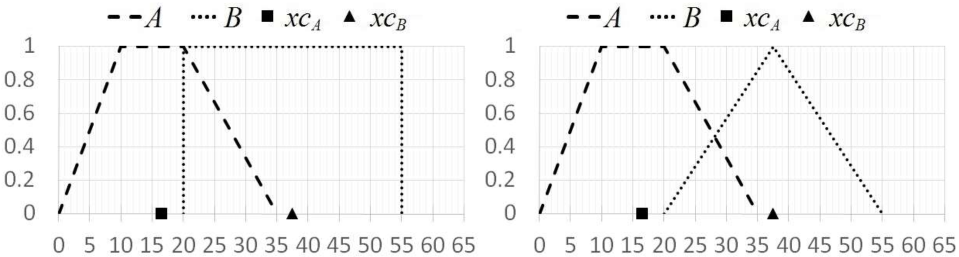



Figure 6.17. Comparison of results of the two mechanisms for accumulation of impacts in Example 2

Examples 3 and 4 demonstrate results of the two accumulation mechanisms when identical impacts, in terms of their centroid values, are received but the impacts have different indices of fuzziness. In Example 3, fuzzy sets A and B have centroids $xc_A$=16.5 and $xc_B$=37.5, and B is a crisp set with fuzziness 0 (Figure 6.18 (a)). In Example 4, fuzzy set B changes to a triangular fuzzy set with the maximum fuzziness of 0.5 and has the same centroid $xc_B$=37.5 (Figure 6.18 (b)). In both examples, fuzziness of fuzzy set A remains the same. Both mechanisms accumulate different impact when the degree of fuzziness of fuzzy set B is increased (Table 6.3). In Examples 3 and 4, the accumulated impacts are equal to 48.7 and 49.1 using the standard mechanism and 51.5 and 48.2 using the proposed mechanism, when $s = 4$. Even though the change in the accumulated value can be observed for both mechanisms, the one obtained by the standard mechanism is negligible.




(a) Definitions of fuzzy sets A ( $\hat{f}(A) = 0.286$ ) and B ( $\hat{f}(B) = 0$ )

(b) Definitions of fuzzy sets A ( $\hat{f}(A) = 0.286$ ) and B ( $\hat{f}(B) = 0.5$ )

Figure 6.18. Definitions of fuzzy sets A and B

The new mechanism is much more sensitive to changes in fuzziness of fuzzy sets involved in accumulation. Figure 6.18 presents the accumulation of impacts when fuzziness of the fuzzy set B changes from 0 to 0.5, for s=4, and the fuzziness of the fuzzy set A is $\hat{f}(\mathrm{A}) = 0.286$ . It can be observed that when the standard mechanism is used, the accumulated value abruptly increases when the degree of fuzziness of fuzzy set B reaches a value around 0.45. It is because the size of the area of the fuzzy set representing the greater variation, fuzzy set B, is smaller than the size of the area of fuzzy set A, representing a smaller variation, until the fuzziness of B is below 0.45. Therefore, the standard mechanism does not accumulate impacts correctly.

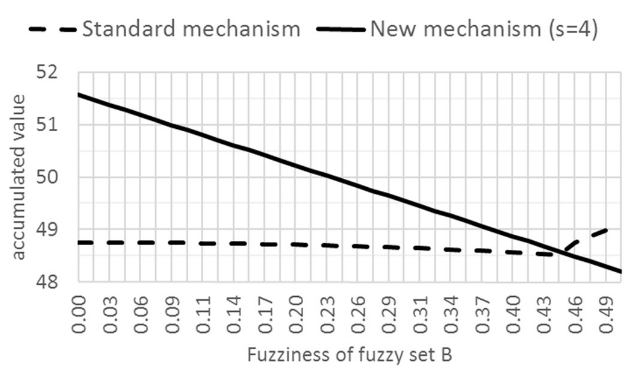



Figure 6.19. Comparison of results of the two mechanisms for accumulation of impacts in Example 3

### 6.4.3 Computational complexity

RBFCM's accumulation of impacts mechanism is a very complex recursive algorithm that requires a very high number of calculations. New accumulation of impacts mechanism is significantly less complex than the standard reasoning mechanism. The standard mechanism requires performing at least 29 operations for every fuzzy set which is accumulated. The new mechanism, requires only 5 operations to be performed, all of them being simple calculations, resulting in a reduction of computational complexity by at least 80%. This is due to the recursive nature of the standard mechanism, which needs to be performed for all the points on the universe of discourse, where two accumulated fuzzy sets overlap. The differences between both mechanisms are summarised in Table 6.4.

Table 6.4 Comparison of number of operations required to calculate the accumulated impact

| | Standard | New |
|---|---|---|
| Calculation of the shape of VOS | Depending on the size of area and support- min of 20 operations per accumulated fuzzy set | 1 |
| Calculation of the centroid | 2*2 (centroid of VOS+ and VOS-) | 2 |
| Saturation | 2*2 (for VOS+ and VOS-) | 2 |
| Defuzzification | 1 | 1 |

To demonstrate the difference between both mechanisms, let us consider the following example, where the effect node receives three impacts A, B and, C, defined as in Table 6.5, in the following order: A, B and C.

Table 6.5 Definition of fuzzy sets A, B and C

| | $xc$ (centroid) | $Area$ |
|---|---|---|
| $A(5,10,15,20)$ | 12.5 | 10 |
| $B(5,10,15,20)$ | 12.5 | 10 |
| $C(20,30,35,40)$ | 31 | 12.5 |

The first impact received, A, does not need to be accumulated. The second impact, B, requires 50 membership carryover operations: 25 calculations of the value of carryover and 25 calculations of the membership degrees. The accumulation of the third impact requires an extra 72 operations: 36 calculations of the value of carryover and 36 calculations of the membership degrees.

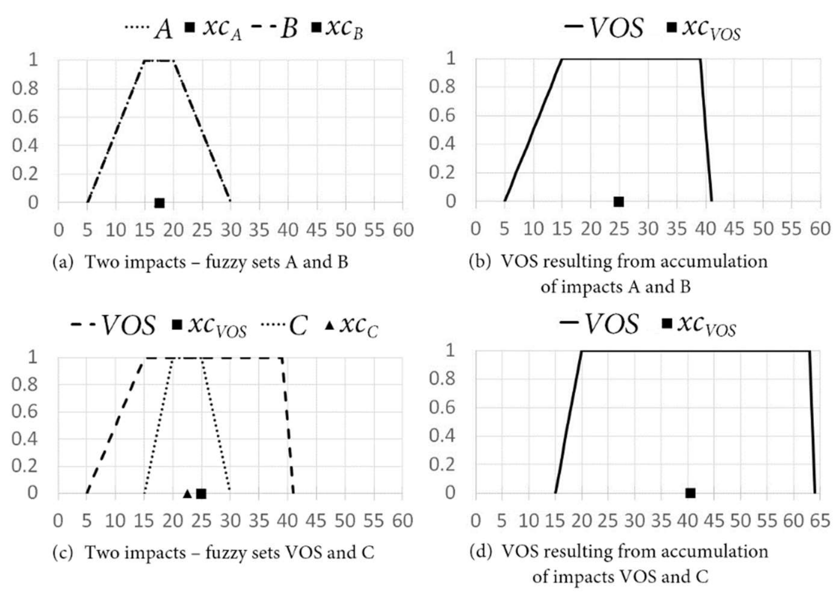



Figure 6.20. An example of accumulation of three impacts A, B and C

If the same set of impacts was accumulated using the new mechanism, only 2 operations would be required: calculation of the parameter p and calculation of the shape of the fuzzy set. Both operations are simple and easy to implement. The result of accumulation is a fuzzy set VOS defined as in Figure 6. 21.

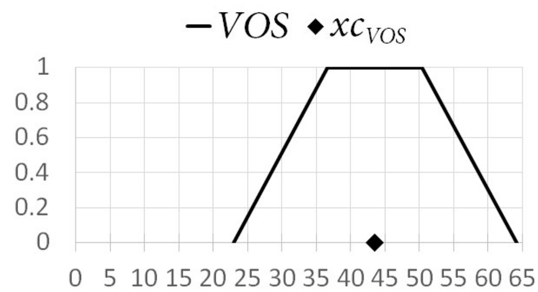



Figure 6.21. Fuzzy set VOS resulting from accumulating impacts A, B and C using new mechanism

## 6.5 Conclusions

RBFCMs introduced a different approach to reasoning in fuzzy rule based systems, where one of the main novelties is the accumulation mechanism. The new accumulation mechanism provides a new approach to defining the output of reasoning when multiple impacts are received by an effect node. The accumulation, like the rest of the reasoning process, is based on the relationship between the area of the fuzzy set and its position on the universe of discourse.

In this thesis, standard semantics is introduced to RBFCMs. In consequence, it is necessary to develop new accumulation mechanism. The use of standard semantics of fuzzy sets, in the new RBFCMs, allowed to use uncertainty to greater extent than before, and on uncertainty is based the new accumulation approach. This was possible to achieve due to the new interpolation mechanism, which is focused on maintaining fuzziness of the interpolated fuzzy set. As a result, the fuzzy sets that are accumulated in the final stage of the reasoning carry valid and meaningful information about uncertainty embedded into the shape of the fuzzy sets.

The new mechanism is based on the standard semantics and uncertainty of fuzzy sets, concepts well defined in the literature. This makes the new accumulation mechanism much easier to understand and to use for modellers familiar with the domain of rule based fuzzy systems. On the other hand, the standard RBFCMs rely on arbitrarily chosen shifting points and a vague semantic, relating an area of a fuzzy set with its position on the universe of discourse.

It is worth to emphasize that the output of accumulation in the new accumulation mechanism is one fuzzy set, which represents an accumulated impact and is reflection of all the impacts received. In the standard mechanism, the result of accumulations are two fuzzy sets representing a positive variation, VOS+, and a negative one, VOS-. In this case, both fuzzy sets reflect the total area of all the fuzzy sets involved in accumulation but not their uncertainty.

Another significant difference between both accumulation mechanisms is the computational complexity. The new mechanism requires much fewer number of calculations than the standard mechanism, which uses a complex recursive algorithm to calculate all the points belonging to VOS. On the other hand, the new accumulation mechanism requires few simple calculations. Decrease of complexity allows using RBFCMs in time-critical systems.

Further work in the accumulation of impacts area should be concentrated on development of accumulation mechanism for other types of fuzzy sets, e.g. Gaussian. They may require slightly modified saturation mechanism, as at the moment it relies on shifting of four points of the trapezoidal fuzzy set. This mechanism cannot be applied for other types of fuzzy sets, apart from the ones defined using trapezoidal and triangular membership functions. Another area of development of the accumulation mechanism is the analysis of the impact of different types of uncertainty measures, e.g. specificity or entropy, to allow a further increase of applicability and flexibility of the method.

# Chapter 7. DEVELOPMENT OF RBFCMs FOR THE TWO CASE STUDIES

## 7.1 Introduction

This chapter introduces RBFCMs models for the two new case studies presented in Chapter 3. To enable it, a dedicated software to handle the reasoning and accumulation of impacts proposed in this thesis, was developed, as there is no modelling tool allowing to implement RBFCMs. The short introduction to the interface of the software is presented in Appendix 1.

In the first case study related to Defence Lines of Development (DLoDs), an RBFCM is developed to model a Joint Strike Fighter (JSF) force, as Force Element (FE), which is a part of the United Kingdom's air force.  The developed model, is used to analyse the impact of investments on the military capability of the JSF. The second case study is a more generic one, where the subject is cyber security. The developed RBFCM is used to analyse the impact of investments on the risk to the business of an enterprise.

Both models were built with cooperation with external subject experts, who contributed their experience and knowledge to capture complexities of both systems. Also, the evaluation of both models was done by the same group of experts. The first case study was further verified by greater committee of military experts during a dedicated workshop. Both models were very positively reviewed and proved to be capable of modelling these two very different problems.

The use of new RBFCMs allowed development of complex fuzzy causal relationships, that greatly contributed to capturing complexities of both systems.

This chapter is organised as follows: Section 7.2 introduces a model for analysing the problem of resource allocation and choice of location of the JSF force. It discusses the results of multiple experiments carried on by military experts to verify the model. Section 7.3 presents a novel model for analysing resource allocation to cyber defence in order to minimise the risk to an enterprise. It also presents eight complex investment scenarios and their impact on the risk to the business. The chapter finishes with conclusions.

## 7.2 Modelling of Defence Lines of Development (DLoDs) using RBFCM

### 7.2.1 DLoDs model

During two workshops dedicated to building the causal map, military experts identified 54 nodes relevant to the analysis of the military capability of one FE: JSF. 49 of these nodes are contributing factors, and five are DLoDs: Training, Equipment, Personnel, Infrastructure and Logistics (Figures 7.1). Rules defining relationships between nodes of the DLoDs model are listed in Appendix 2.

Training DLoD is influenced by the number of hours spent on two types of training: synthetic and live flying. Synthetic training is a cheaper option than the live flying, as the latter is performed in flight simulators. Both types of training require development of the infrastructure, but it is more expensive for live flying. The live flying training also incurs the cost of logistics, as JSF requires fuel and maintenance. The relationship between different types of trainings and different DLoDs is shown in Figure 7.1. The second very important aspect of modelling of military ability of the JSF is Equipment. Ability of the Equipment DLoD is impacted by the technological advancements of equipment and by the degree of difference between technology used during the training and in actual military activity. Additionally, availability of equipment needs to be considered to determine readiness of the JSF. The third DLoD considered is Infrastructure, which represents development of the infrastructure of the military base, where JSFs are located. Aspects that need to be considered are as follows: quality of operating surfaces, number and development of buildings, such as buildings where trainings are performed, hangarages or accommodation of the personnel. Logistics DLoD is impacted by its available equipment, the airfield support and air to air refuelling capability. The last element modelled is military ability of the Personnel DLoD, which represents number of personnel as well as its mental state and life comfort. More detailed explanation of all the DLoDs and their contributing factors is provided in the following sections.

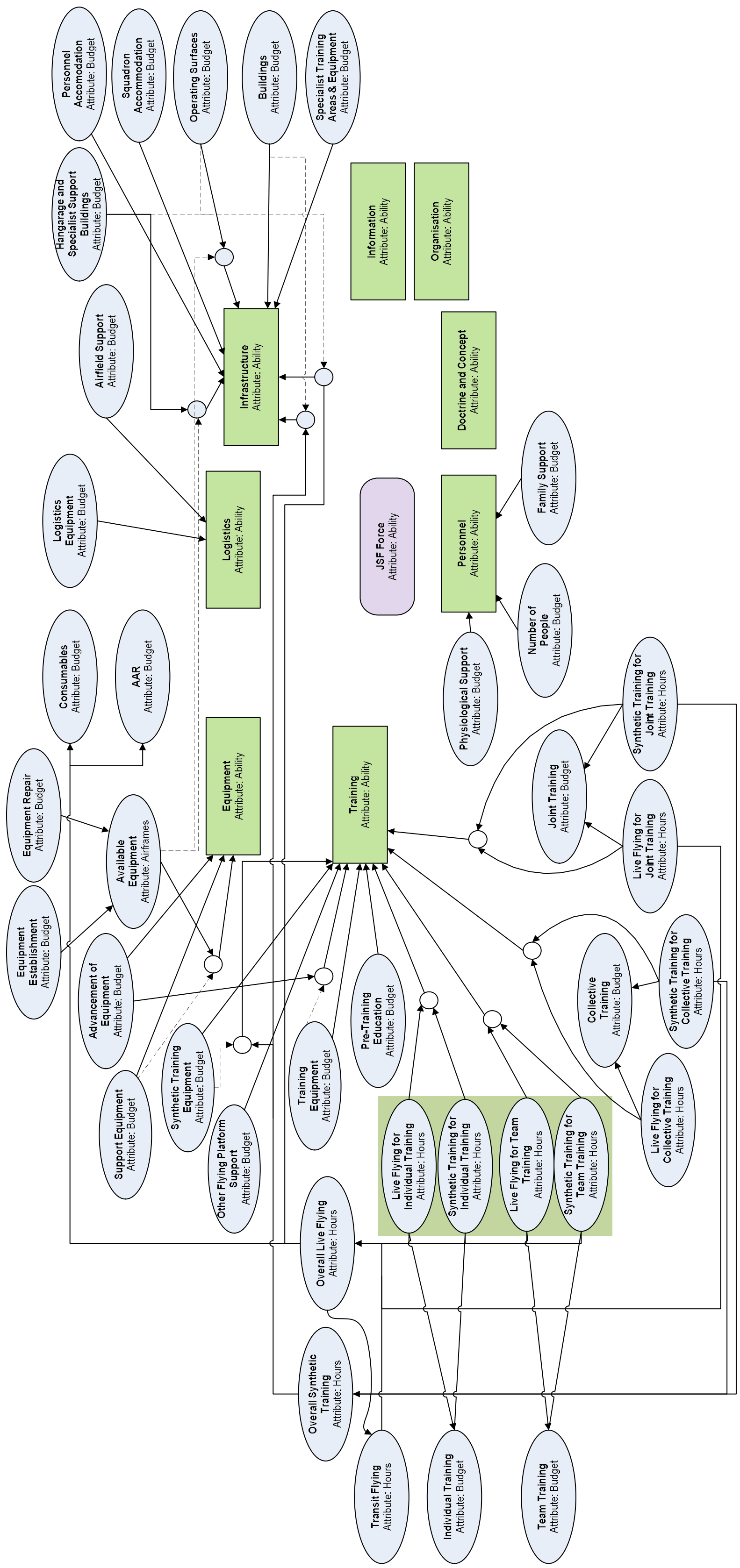



Figure 7.1. The structure of the JSF Force Element

#### 7.2.1.1 Training DLoD

In Figure 7.1 the Training DLoD is presented in its simplified form. In total, Training DLoD has 27 contributing factors, of which 22 represent different type of trainings the JSF can participate in. Detailed structure of the JSF Training DLoD is presented in Figure 7.2.

There are four types of trainings:

- Individual training – training performed by one member of the JSF unit. This training can be performed by:
  - Squadron – one JSF squadron,
  - OCU (operational conversion unit) student – training of a student of the unit supporting squadron,
  - OCU staff – training of the member of staff of the OCU,
- Team training – represents training of members of the JSF squadron
  - Squadron –JSF squadron,
  - OCU (operational conversion unit) student – training of the team of students of the unit supporting squadron,
  - OCU staff – training of the team of staff of the OCU,
- Collective training – represents collective training of the JSF squadron and other teams whose operation is related to the one of the JSF, i.e. AAR (air to air) fuelling unit – allowing fuelling of JSF in the air,
- Joint training – represents join training of the JSF squadron and other FEs, i.e. joint training of landing of naval, ground and air units on the enemy’s territory,

Each type of training can be performed in two ways:

- Live flying – training in live environment requiring use of real equipment and consumables,
- Synthetic training – training in the synthetic environment, i.e. simulator.

Synthetic training is a much cheaper training option than Live flying, as it does not involve usage of the expensive equipment and consumables.

Every type of training has number of hours assigned, which contributes to budgets of: Individual, Team, Collective and Joint trainings and to the total number of hours spent on Live and Synthetic Trainings. Overall Live Flying hours is used to calculate the number of Transit Flying, i.e. time spent on flying from the base to the training ground.

The second important aspect of training is equipment. There are 4 types of equipment that have impact on the ability of the Training DLoD:

- Synthetic Training Equipment – budget for equipment related to the synthetic environment; this node contributes to the impact of Overall Live Flying hours (dashed arrow),
- Training Equipment – budget for equipment related to the live flying environment,
- Other Live Flying platform support – budget for other elements of live flying training,
- Advancements of equipment – contributing factor of the Equipment DLoD, investment into this type of equipment needs to be followed by the investment into Training Equipment (dashed arrow) to have a positive impact on Training DLoD.

The last element of the Training DLoD is Pre-Training Education, which defines budget spent on the activities that help to encourage civilians or other members of military forces to join the JSF squadron.

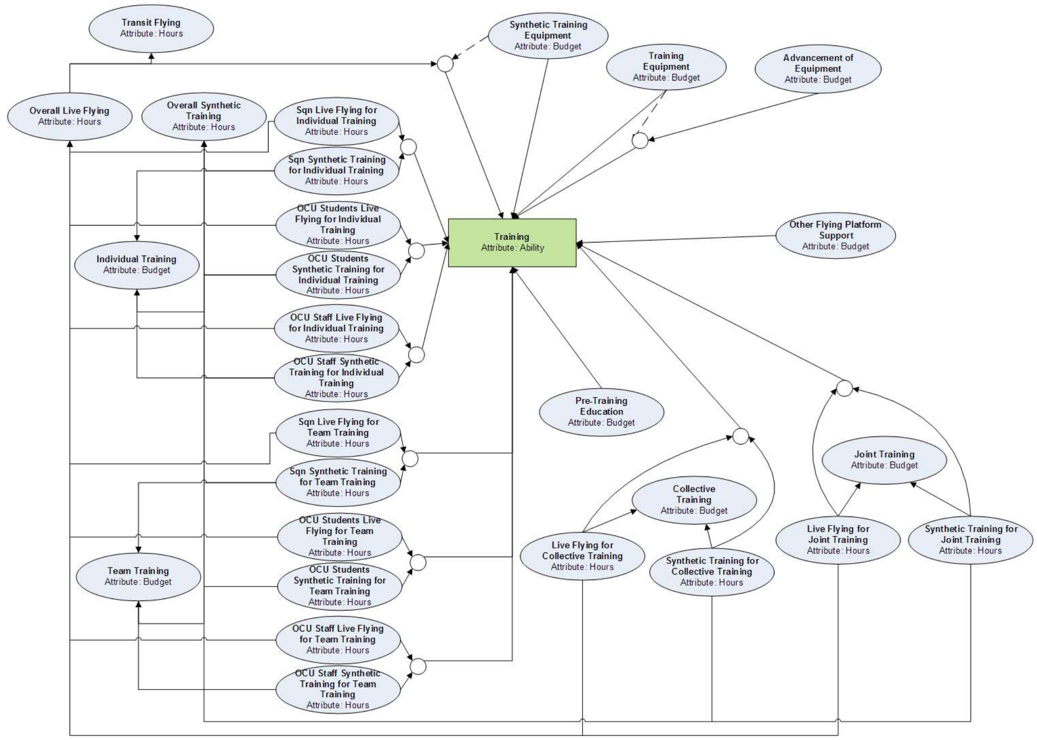



Figure 7.2 The structure of the Training DLoD

### 7.2.1.2 Equipment DLoD

Equipment DLoD is impacted by the following nodes (Figure 7.3):

- Advancements of Equipment – budget spent on improvement of used equipment,
- Equipment Establishment – budget spent on purchasing new equipment,
- Equipment Repair – budget of equipment repair,
- Available Equipment – number of airframes available to use, impacted by Equipment Establishment and Equipment Repair,
- Support Equipment – equipment used in addition to the Available Equipment, investment in this contributing factor is jointly considered (dashed arrow) with Available Equipment, e.g. high investment in the Support Equipment will not have positive impact on the Equipment DLoD if the number of Available Equipment is low.

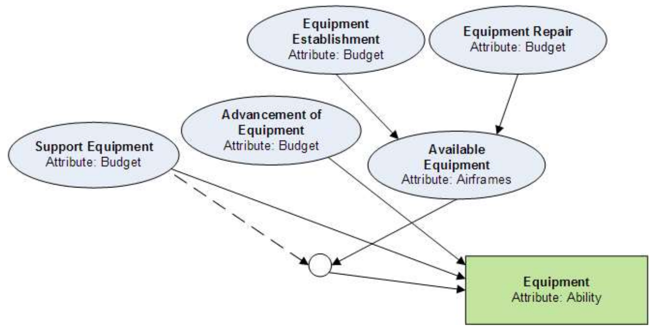



Figure 7.3 The structure of the Equipment DLoD

### 7.2.1.3 Infrastructure DLoD

Infrastructure DLoD represents the development of the infrastructure of the JSF base (Figure 7.4):

- Hangarage and Specialist Support Buildings and Operating surfaces – relate to the infrastructure allowing performing Live Flying Training and number of Available Equipment (airframes). The more time is spent on Live Flying training the more budget needs to be invested in these contributing factors,
- Buildings – represent budget spent on the infrastructure required for simulators; the more hours are spent on the Synthetic Training the more budget needs to be invested in this contributing factor,
- Specialist Training area and equipment – budget for infrastructure for training specialists and their equipment,
- Squadron Accommodation and Personnel Accommodation – accommodation for the squadron and related personnel.

Two nodes from the Traning DLoD: Overall Synthetic Training and Overall Live Flying and one node from the Equipment DLoD: Available Equipment contribute (dashed arrows) to the military ability of the Infrastructure DLoD.

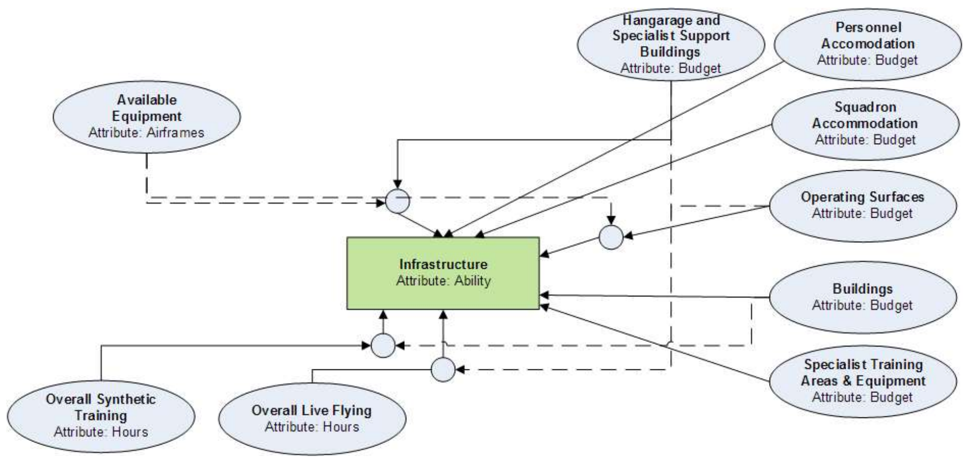



Figure 7.4 The structure of the Infrastructure DLoD

#### 7.2.1.4 Logistics DLoD

Logistics DLoD has two nodes: Consumables and AAR (air to air refuelling) which do not contribute to its military ability. These concepts are used to calculate the cost of hours spent on Live Flying training.

The two nodes which have impact on the military ability of Logistics DLoD are (Figure 7.5):

- Logistics Equipment – advancement of equipment facilitating logistics operations,
- Airfield Support – services related to the operation of military airfields, e.g. aircraft fuelling, fire and crash rescue, etc.

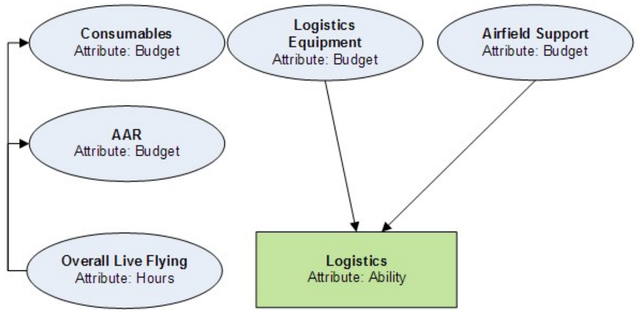



Figure 7.5 The structure of the Logistics DLoD

### 7.2.1.5 Personnel DLoD

Personnel DLoD represents aspects related to the number and quality of life of soldiers and staff employed to support the operation in the miliary base. The following nodes have impact ont the military ability of the Personnel DLoD (Figure 7.6):

- Number of People – the budget for employment of the JSF pilots and the operational support staff,
- Physiological Support – the budget on both physiotherapeutic and psychological support of the staff in the military base,
- Family Support – support for families who had to relocated to live close to the military base.

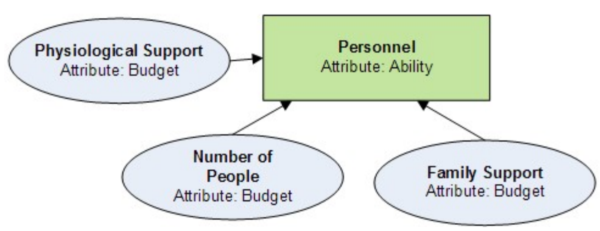



Figure 7.6 The structure of the Personnel DLoD

### 7.2.2 Military capability

The values of DLoDs and FE are measured using military capability – an abstract measure which is very difficult to define. It is scarcely described in the literature (Yue and Henshaw, 2009; Rodrigues at al., 2015), but is a core measure on the operational level of military units and was introduced as an alternative measure to counting numbers of units of specific FEs. Military capability defines what can be achieved from the military perspective, using available military units. In this case study, military experts defined it as the military ability on a scale from 0 to 100.

#### 7.2.2.1 Military capability of DLoDs

The value of the military capability of DLoDs is influenced by contributing factors. The strength of the impact of the contributing factor on the corresponding DLoD is modelled by three elements:

- strength of the relationship – defined by the rule base,
- definition of membership functions and
- the input into the contributing factor that triggers the appropriate rules.

Translation of the input of contributing factors into the impact on the DLoD is calculated using reasoning mechanisms, discussed in Chapters 5 and 6.

#### 7.2.2.2 Military capability of FE

The value of the military capability of the FE, $A_{FE}$, is calculated based on the values of DLoDs weighted with their importance. The importance of every DLoD was defined by the experts. The sum of the weights of all DLoDs needs to add up to 1.

$$A_{FE} = \sum_{i=1}^{N} A_{DLoD_i} \times w_{DLoD_i} \tag{7.1}$$

where $N$ is the number of DLoDs modelled, $A_{DLoD_i}$ is the military capability of the i$^{th}$ $DLoD$, $w_{DLoD_i} = (0,1)$ is a weight of the $DLoD_i$ and $\sum_{i=1}^{N} w_{DLoD_i} = 1$ .

### 7.2.3 Elements of the DLoD RBFCM model

#### 7.2.3.1 Membership functions

Every node in the DLoDs model has a name and an attribute, defining the domain it represents. In the JSF model, all the attributes, and therefore nodes, can be grouped into two types of attributes: variation and level.

A variation node represents the variation of the attribute of a given concept, defined on a scale from -100 to 100, where -100 represents maximal decrease of the attribute and 100 represents maximal increase of the attribute, e.g. increase or decrease of the budget. Variation nodes have one set of membership functions, defined using seven linguistic terms: *Decreased Much, Decreased, Decreased Little, Maintained, Increased Little, Increased and Increased Much*. All the nodes in the JSF model, whose attributes represent variation, have the same set of membership functions as defined in Figure 7.7.

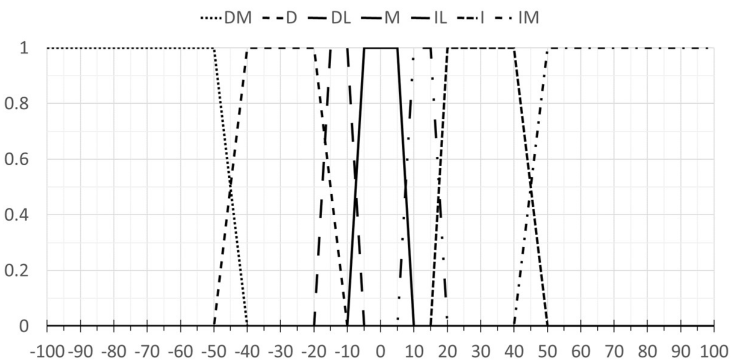



Figure 7.7. Membership functions defining variations in the JSF model

The second type of nodes represents the level of a given concept. Different scales can be used to define levels of attributes, e.g. the number of hours of live flying of the squadron can be measured on a scale from 0 to 40 hours per month, where 0 belongs to the lowest level of live flying and 40 belongs to the highest level of live flying (Figure 7.8).

Level nodes, due to the great variety of values that they can represent, have individual definitions of membership functions, defining the same set of five linguistic terms expressing the level of a node: *Very Low, Low, Medium, High* and *Very High*.

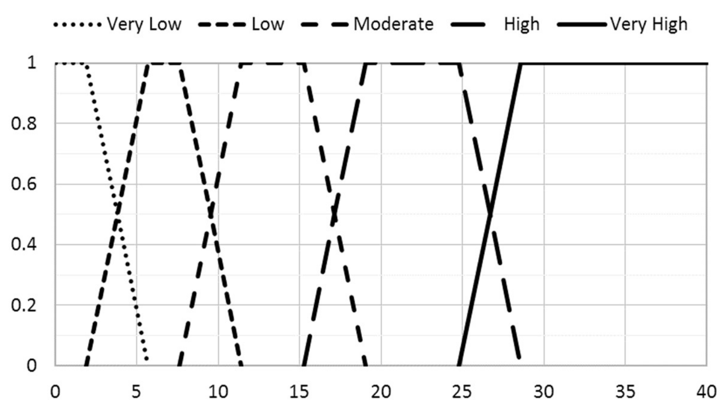



Figure 7.8. Membership functions representing the level of a node

### 7.2.3.2 Relationships

There are three types of relationships in the JSF model:

- linear relationship, where the value of an effect node depends linearly on the input into a causal node, e.g. the budget of the "Individual Training" node increases linearly depending on the value of the node, "Number of hours of Sqadron Live Flying for Individual Training".
- IF-THEN Fuzzy Causal Relationship between two nodes, with one antecedent and one consequent. An example of such relationship is presented in Figure 7.9, where, to determine the value of the effect node "Training", information about the budget of one effect node "Training Equipment" is required.

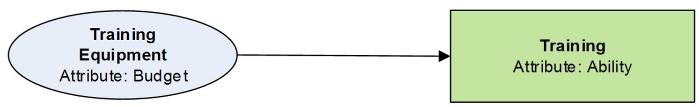



Figure 7.9. Fuzzy Causal Relationship

The following example of rules define relationship between "Budget for Training Equipment" and "Ability of Training":

IF Budget for Training Equipment is *Increased Little*

THEN Training Ability is *Increased,*

IF Budget for Training Equipment is *Increased*

THEN Training Ability is *Increased Much,*

- complex Fuzzy Causal Relationship, where the state of the effect node depends on inputs into several causal nodes. Examples of complex Fuzzy Causal Relationships are presented in Figures 7.10 and 7.11.

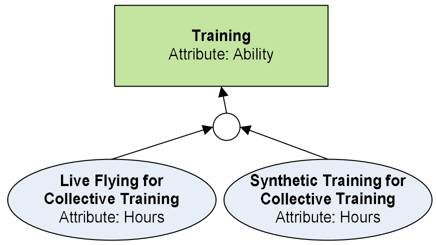



Figure 7.10. Complex Fuzzy Causal Relationship between training hours and Training DLoD

The following example of rules define relationship between two causal nodes: "Number of hours of Live Flying for Collective Training" and "Number of hours of Synthetic Training for Collective Training" and one effect node: "Ability of Training":

IF Number of Hours for Live Flying is *Very Low*
AND Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Decrease Much*
IF Number of Hours for Live Flying is *Very Low*
AND Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Decrease Much*

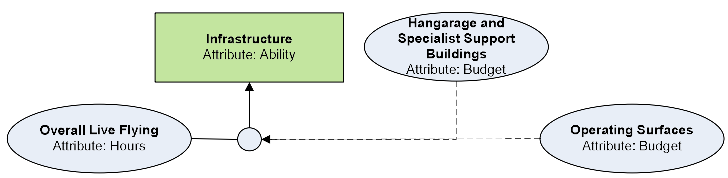



Figure 7.11. Complex Fuzzy Causal Relationship with conditional nodes

The following example of rules define relationship between three causal nodes: "Number of hours of Overall Live Flying", "Budget for Hangarage and Specialist Support Buildings" and "Budget of Operating Surfaces" and one effect node: "Ability of Infrastructure":

IF Number of Hours for Overall Live Flying is *Low*
THEN Ability of Infrastructure will *Maintain*
IF Number of Hours for Overall Live Flying is *Medium*
AND NOT (Budget for Operating Surfaces is *Increased* or *Increased Much*)
AND NOT (Budget for Hangarage and Specialist Support Buildings is *Increased Little* or *Increased* or *Increased Much*)

THEN Ability of Infrastructure will *Decrease Little*

#### 7.2.3.3 Time component and location impact

The time unit used in this case study is 1 year. It means, that the impact of contributing factors on DLoDs is observable after 12 months. The time unit is associated with Fuzzy Causal Relationships and Complex Fuzzy Causal Relationships. However, the linear relationships between number of hours and budget, as well as the linear relationships between DLoDs and JSF FE have no time associated with them and the impacts are considered to be happening as soon as one of the nodes changes. The simulation of the JSF model is run for 1 iteration only, therefore it simulates what the impact of resource allocation on the military ability of the JSF is after one year from implementing changes.

### 7.2.4 Analysis of the impact of investments

The JSF model was built to evaluate the impact of resource allocation with respect to the location of the JSF base. Therefore, every relationship reflects the state of the evaluated JSF base, e.g. number of training facilities available, development of the infrastructure, distance of the location of the base from the training location, etc.

The decision on the location of the JSF base and on the allocation of resources was taken in practice, before the model was built. The decision relied on the expertise of the military personnel and was not supported by any other model.

The military experts, supervising this case study, decided to evaluate the model by analysing several trials of the model. The JSF simulation required providing the following two inputs from the military experts:

1. Current ability levels of the JSF's DLoDs – it was assumed that the military ability in the iteration 0 is the same for all the DLoDs and it is equal to 80 (out of 100).

2. Changes in the allocation of the budget and the number of hours to contributing factors – the allocation of the budget/hours varies by each investment choice.

#### 7.2.4.1 No changes in resource allocation

The first analysed trial assumed that there are no changes in the resource allocation. The values of all nodes – changes in variations were maintained on the initial level 0. The expected output of this trial was high decrease of the military ability of the Infrastructure DLoD. It was a known fact that the infrastructure in the analysed location, required improvements and lacked facilities. This experiment demonstrated that the Training and Logistics DLoDs require some investments, as otherwise their military abilities would deteriorate slightly after a year. The results shown in Table 7.1 were in line with the expectations of military experts.

Table 7.1. The results of trial 1.

| **Node** | **AS IS ability** | **Trial 1 (%)** | **Trial 2 (%)** | **Trial 3 (%)** |
|---|---|---|---|---|
| Training DLoD | 80 | **-6.75** | **+1.5** | +1.5 |
| Equipment DLoD | 80 | 0 | 0 | 0 |
| Personnel DLoD | 80 | 0 | 0 | 0 |
| Logistics DLoD | 80 | -17.5 | -17.5 | -17.5 |
| Infrastructure DLoD | 80 | **-30.67** | **-36.16** | **-16** |
| Information DLoD | 80 | 0 | 0 | 0 |
| Organisation DLoD | 80 | 0 | 0 | 0 |
| Doctrine and Concept DLoD | 80 | 0 | 0 | 0 |
| JSF Squadron FE | 80 | -6.9 | -6.5 | -4.0 |

#### 7.2.4.2 Investment 1 – increase in synthetic training hours

The second trial was dedicated to analysis of improvements of the military ability of the Training DLoD. The solution identified by experts was the increase of the number of training hours by 10 for Individual Training of the squadron in the synthetic environment – flight simulator. It is a much cheaper option than increasing live flying hours, as no fuel or exploitation costs are incurred in the synthetic environment.

The consequence of the increase of number of training hours in the synthetics environment by 10, was the increase of military ability of the Training DLoD by 1.5% (Table 7.2), when compared to the starting military ability, and by 8.25% when compared to the lack of investment into training, discussed in the previous trial.

Table 7.2. The results of trial 2.

| Node | AS IS ability | Trial 1 (%) | Trial 2 (%) | Trial 3 (%) |
|---|---|---|---|---|
| Training DLoD | 80 | **-6.75** | **+1.5** | +1.5 |
| Equipment DLoD | 80 | 0 | 0 | 0 |
| Personnel DLoD | 80 | 0 | 0 | 0 |
| Logistics DLoD | 80 | -17.5 | -17.5 | -17.5 |
| Infrastructure DLoD | 80 | **-30.67** | **-36.16** | **-16** |
| Information DLoD | 80 | 0 | 0 | 0 |
| Organisation DLoD | 80 | 0 | 0 | 0 |
| Doctrine and Concept DLoD | 80 | 0 | 0 | 0 |
| JSF Squadron FE | 80 | -6.9 | -6.5 | -4.0 |

The drawback of increasing the number of hours for synthetic training is its very negative impact on military ability of the Infrastructure DLoD. The analysed base does not have enough facilities to allow an increase of number of hour in flight simulators, therefore military ability of Infrastructure decreased by further 5.5%, when compared with the first trial.

#### 7.2.4.3 Investment 2 – development of the infrastructure

The third investment is a continuation of the second one and is intended to increase the military ability of the Infrastructure DLoD, that was very negatively impacted by the increase of number of synthetic training hours.

In this experiment, the experts decided to increase the budget for Hangarage and Specialist Support Buildings and Buildings contributing factors, by 30% and to increase the budget for Specialists Training Areas and Equipment by 20%. The total cost of these investment was 0.1% of the total budget. As a result of increasing the budget of the infrastructure, the military ability of the Infrastructure DLoD increased by around 20%, when compared with the previous trial and decreased by only 16% when compared with the starting military ability (Table 7.3).

Table 7.3. The results of trial 3.

| Node | AS IS ability | Trial 1 (%) | Trial 2 (%) | Trial 3 (%) |
|---|---|---|---|---|
| Training DLoD | 80 | **-6.75** | **+1.5** | +1.5 |
| Equipment DLoD | 80 | 0 | 0 | 0 |
| Personnel DLoD | 80 | 0 | 0 | 0 |
| Logistics DLoD | 80 | -17.5 | -17.5 | -17.5 |
| Infrastructure DLoD | 80 | **-30.67** | **-36.16** | **-16** |
| Information DLoD | 80 | 0 | 0 | 0 |
| Organisation DLoD | 80 | 0 | 0 | 0 |
| Doctrine and Concept DLoD | 80 | 0 | 0 | 0 |
| JSF Squadron FE | 80 | -6.9 | -6.5 | -4.0 |

## 7.3 Modelling of risk to the cyber defence

A cyber defence RBFCM was built to model the risk to the enterprise business, considered as a potential disruption of services of an enterprise caused by compromises or by mitigations of compromises. For example, a potential risk to the operation of the business is a malware email sent to one of the employees of the company. If the attachment of the email is opened then the information targeted by the attack can be compromised, stolen or removed, and operation of the company can be at risk. On the other hand, strengthening mitigation options that prevent successfulness of the attack can also contribute to an increase of the risk to operation of the business. For example, an increase of the sensitivity of the firewall can result in slower delivery of some emails and therefore, can cause delays to the operation of the business. A potential solution to one issue can cause another problem.

### 7.3.1 Cyber defence model

Nodes used to build cyber defence functions of the cyber defence model can be grouped into two categories: function nodes and leaf nodes. Function nodes represent abstract core aspects of the cyber defence functions. Leaf nodes can be considered as contributing factors with less abstract attributes, having, in most cases, direct impact on the function nodes. Nodes from different functions are interrelated with each other and can have impact on more than one function. There are many different attributes of nodes in the cyber defence model, but all of them represent variations of the concept, on a scale ranging from -100 to 100.

Cyber defence model consists of five functions: Assess, Protect, Detect, Respond and Recover (Figure 7.12). Every function represents different aspect of the cyber security and in the same time they are strongly interrelated. Evaluation of the risk to the cyber security and the business starts with assessment of vulnerabilities of Communication and Information Systems (CIS), where information, crucial to the enterprise, is stored (Assess function). Alongside, it is important to understand and be aware of potential adversaries as different enemies could use different methods to compromise the information. Awareness of vulnerabilities helps to develop effective protection against compromises and efficient mitigation options to decrease impact of compromises on information and to decrease the impact of mitigations on the operation of the business (Protect function). If developed protection mechanisms fails, the cyber security needs to detect the compromise in time (Detect function). Ability to detect compromises depends on multiple factors: ability to assess vulnerabilities of the CIS, quality of detection systems and skills of cyber defence teams who use these systems. Additionally, detection abilities improve with gained experience from previously detected and analysed compromises. Once a compromise is detected cyber security responds on two levels: it tries to stop the compromise and mitigates its impact (Response function) and recovers the information if it got compromised (Recover function). Response to compromises depends on understanding of vulnerabilities of CIS, knowledge about adversaries' intent and availability of mitigation options. Recovery depends on the type of corruption of information and availability of appropriate tools to recover the information. Rules defining the relationships between concepts of the cyber defence model are listed in Appendix 3.

A goal of the cyber defence model is to determine how specific investments will impact the overall risk to the enterprise, modelled by the Risk to the business node (Figure 7.12). Eight investment scenarios are investigated, including Improved Network Monitoring, Increased Mandatory Cyber Hygiene Training, Enhancements to Firewall, Through Life Software Management, Detect and Deny Tools and Specialists Skills Training. Risk to the business is the only node that is part of the RISK function. It defines the risk to the operation of the whole enterprise, as an institution, rather than its cyber defence systems. This node is affected by only two other nodes: "Impact of Mitigation of CIS compromises or vulnerabilities on the business" (Node 21) and "Impact of CIS compromises on the business" (Node 23).

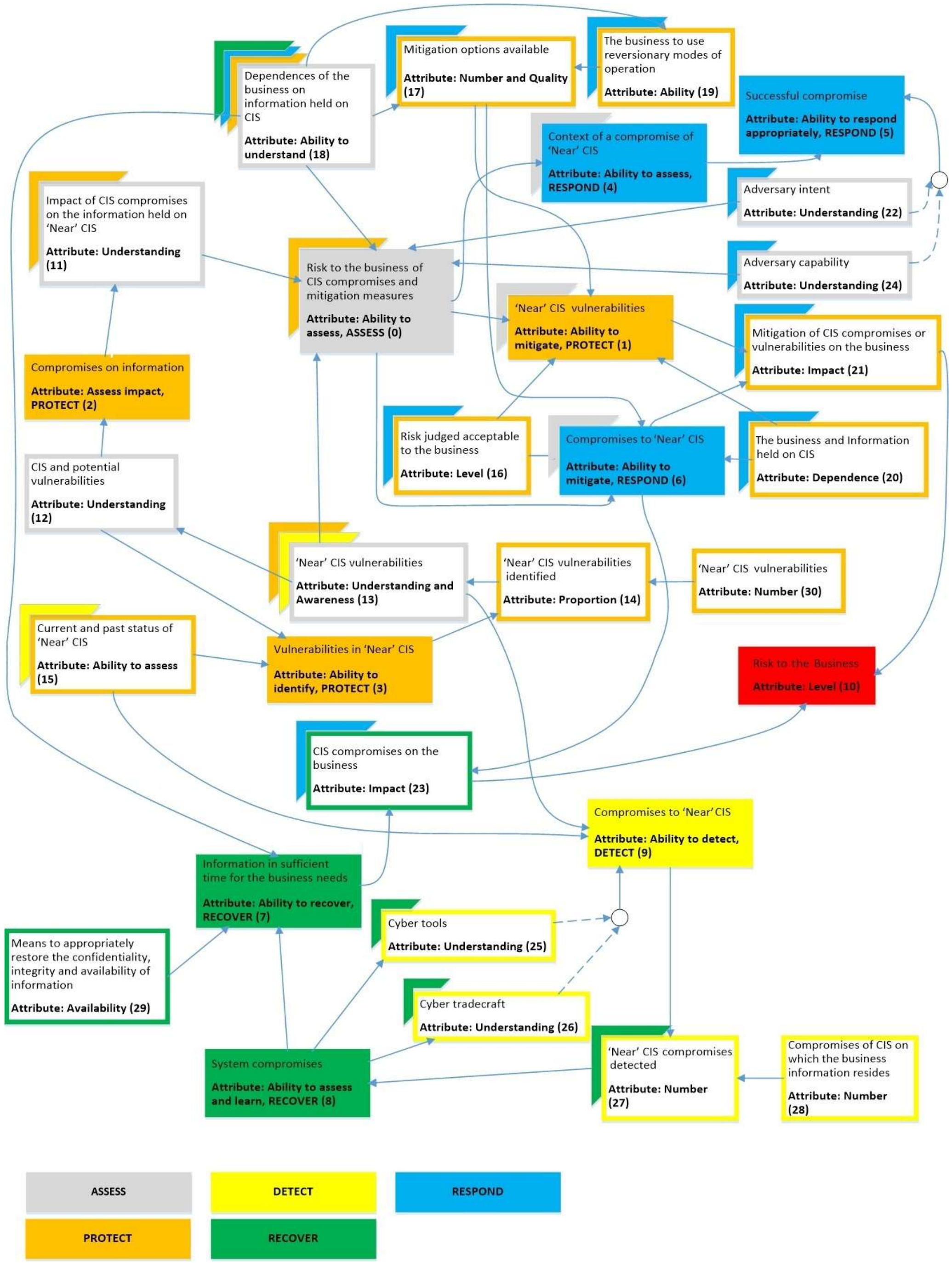



Figure 7.12 RBFCM model for cyber defence

#### 7.3.1.1 Assess function

The Assess function provides the "Ability to assess the risk to the business of CIS compromises and mitigation measures (Node 0)", which is the core role of this function (Figure 7.13). The assessment function allows to understand the current state of the network and potential threats resulting from the known vulnerabilities. In the same time, the business needs to be aware of the source of potential threats and tools that might be used by adversaries to compromise the network.

Ability to assess depends on:

- "Ability to understand dependences of business on information held on CIS (18)" – represents understanding of the relationship between the way information is stored on CIS and the potential vulnerabilities to the business it creates,
- "Understanding of the impact of CIS compromises on the information held on CIS (11) "– represents the understanding of the potential impact of compromises on the information stored on CIS,
- "Understanding of CIS and potential vulnerabilities (12)"
- "Understanding and Awareness of 'Near' CIS vulnerabilities (13)" – represents awareness of potential vulnerabilities of CIS,
- "Understanding of the adversary intent and capability (22 and 24)" – these two nodes represent understanding of the capabilities and intent of adversaries; they must be considered together to recognise correctly potential threats and understand the risk to the business.

In the same time, the ability to assess has impact on the following functions (described in more detailed in the following paragraphs, relevant to these functions):

- Ability to protect CIS:
    - Ability to mitigate 'Near' CIS vulnerabilities (1),
- Ability to respond to compromises:
    - Ability to assess the context of a compromise of 'Near' CIS (4),
    - Ability to mitigate compromises to 'Near' CIS (6).

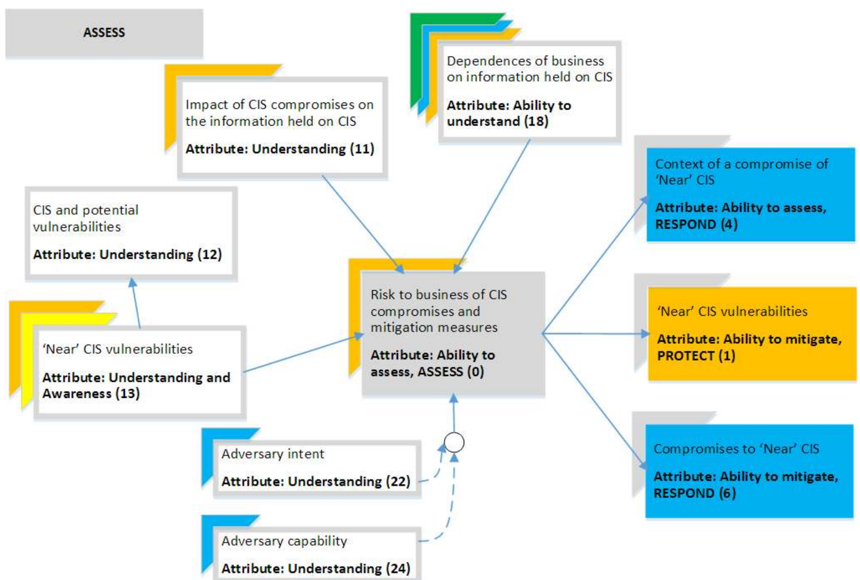



Figure 7.13 Nodes contributing to Assess function

### 7.3.1.2 Protect function

Protect function is the most complex cyber defence function and has impact on almost all other cyber defence functions as well as on the risk to the cyber defence (Figure 7.14). There are three protect functions:

- “Ability to mitigate ‘Near’ CIS vulnerabilities (1)” – represents ability of mitigating vulnerabilities of CIS before the compromise happens (e.g. ability to increase sensitivity of firewalls when it is known that one of employees received a phishing email),
- Ability to “Assess impact of Compromises on information (2)”,
- “Ability to identify Vulnerabilities to ‘Near’ CIS (3)” – represents ability of identifying new vulnerabilities in the CIS network.

The following nodes impact protect functions:

- “Number and Quality of Mitigation options available (17)” defines ability of the business to mitigate vulnerabilities and compromises. “Ability of the business to use reversionary modes of operation (19)” helps to improve mitigation options,
- “Ability to assess Current and past status of Business ‘Near’ CIS (15)” help to improve ability to identify vulnerabilities and assessment of the impact of compromises,
- “Level of Risk judged acceptable to the business (16)” and “Dependence of the business on Information held on CIS (20)” increase the ability to mitigate ‘Near’ CIS vulnerabilities,
- Indirectly “Number of Business ‘Near’ CIS vulnerabilities identified (14)” and “Number of Business ‘Near’ CIS vulnerabilities (30)” contribute to the ability of mitigation of compromises and assessment of their impacts as analysis of identified vulnerabilities helps allows eliminating them or developing new mitigation options,
- Assess function: “Ability to assess the risk to the business of CIS compromises and mitigation measures (0)” – increase of assessment ability improves the ability to mitigate vulnerabilities.

Protect functions have impact on the following concepts:

- “Impact of Mitigation of CIS compromises or vulnerabilities on the business (21)” – represents the impact of mitigation actions on the business, this node has direct impact on the risk to the business, e.g. increase of the sensitivity of firewalls increases safety of CIS and information but has impact on the delivery of emails, therefore increases the risk to the business.
- “Understanding of Impact of CIS compromises on the information held on Business ‘Near’ CIS (11)” – understating of the past impacts of compromises on the information allows to increase the understanding of the potential future compromises,

Protect function has one loop which reinforces impacts. Increase of “Ability to identify Vulnerabilities to ‘Near’ CIS (3)” has positive impact on “Number of Business ‘Near’ CIS vulnerabilities identified (14)”. Higher number of identified vulnerabilities increases “Understanding and awareness of ‘Near’ CIS vulnerabilities (13)” and “Understanding of CIS and potential vulnerabilities (12)”. Better understanding and greater awareness of vulnerabilities in CIS increases the “Ability to identify Vulnerabilities to ‘Near’ CIS (3)”, therefore closing the loop.

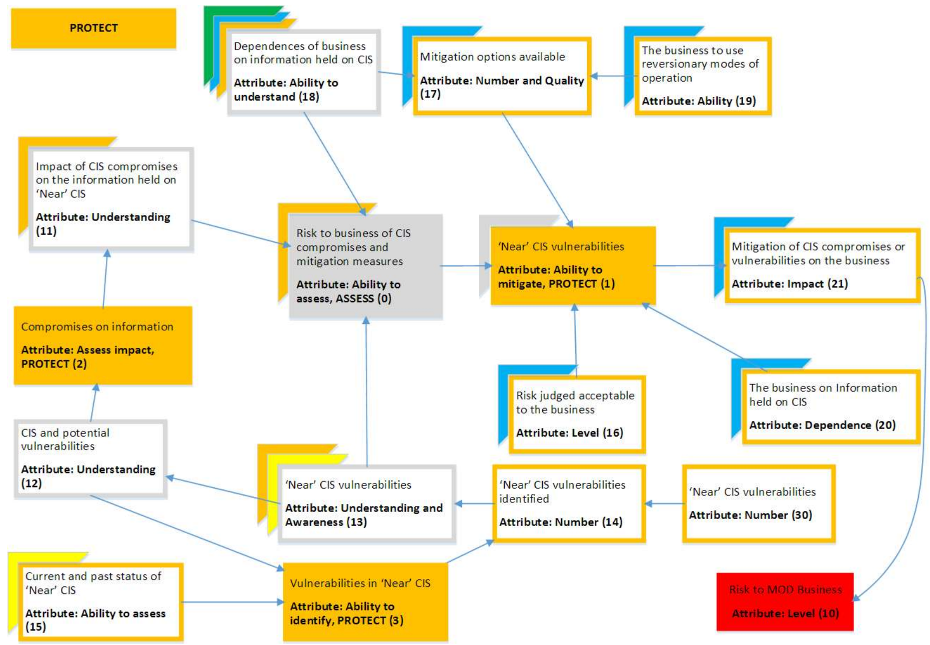



Figure 7.14 Nodes contributing to Protect function

### 7.3.1.3 Respond function

A very important aspect of the cyber defence system is its capability of responding to compromises. In the developed model, there are the following the following three functions defining response capability (7.15):

- "Ability to assess context of a compromise of 'Near' CIS (4)" which helps to understand the context of a compromise and increases the ability to respond to compromises,
- "Ability to respond appropriately to successful compromises (5)" – defines the response capability, including ability to counterattack,
- "Ability to mitigate compromises to 'Near' CIS (6)" – in parallel to responding to a compromise, the business needs to be able to mitigate it.

The response function does not have any contributing factors that belong to this function. However, it is affected by the following nodes from other functions:

- Assess function:
    - "Ability to assess the risk to the business of CIS compromises and mitigation measures (0)" – understanding of the risk helps to recognise the context of a compromise and to mitigate it,
    - "Understanding of the adversary intent and capability (22 and 24)" increases the likelihood of successful response to compromises.
- Protect function:
    - "Level of Risk judged acceptable to the business (16)" and "Dependence of the business on Information held on CIS (20)" increase the ability to mitigate CIS compromises,
    - "Number and Quality of Mitigation options available (17)" defines ability of the business to mitigate vulnerabilities and compromises.

Response function has impact on two key nodes, which both have impact on the risk to the business:

- "Impact of mitigation of CIS compromises or vulnerabilities on the business (21)" – successful mitigation of compromises is likely to increase the risk to the business as it usually involves actions that disturb business processes,
- "Impact of CIS compromises on the business (23)" – reduced response capability increases the probability of successful compromises which have negative impact on the business.

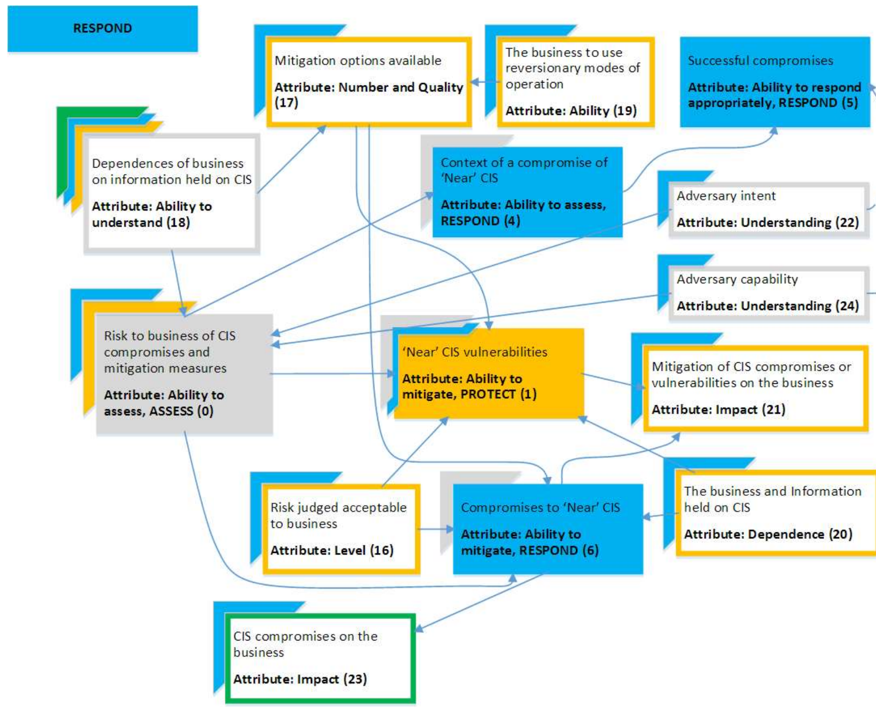



Figure 7.15 Nodes contributing to Respond function

#### 7.3.1.4 Detect function

The detect function represents the "Ability to detect compromises to 'Near' CIS (9)" of the cyber defence teams and systems (Figure 7.16). Detection abilities are affected by the following factors:

- "Understanding of cyber tools and tradecrafts (25 and 26)" – represent the understanding of how the available tools and skills of the cyber defence can increase detection of compromises.

Cyber detection abilities are also impacted by other functions:

- Protect function:

- “Ability to assess Current and past status of ‘Near’ CIS (15)” – ability to check the current and past status of CIS allows to observe anomalies in the network, that could result from successful compromise,

- Assess function:
  - “Understanding and Awareness of ‘Near’ CIS vulnerabilities (13)” – increased understanding of CIS vulnerabilities allows to concentrate the effort of detection systems on areas of the network which are the most prone to be successfully compromised.

Additionally, the ability to detect function has impact on:

- “Number of ‘Near’ CIS compromises detected (27)” – represents the number of compromises successfully detected by the detection systems and cyber defence team; this node is also impacted by “Number of compromises of CIS on which the business information resides (28)” – which represents the overall number of successful (also undetected) compromises of CIS; this node does not have direct impact on the ability to detect compromises.

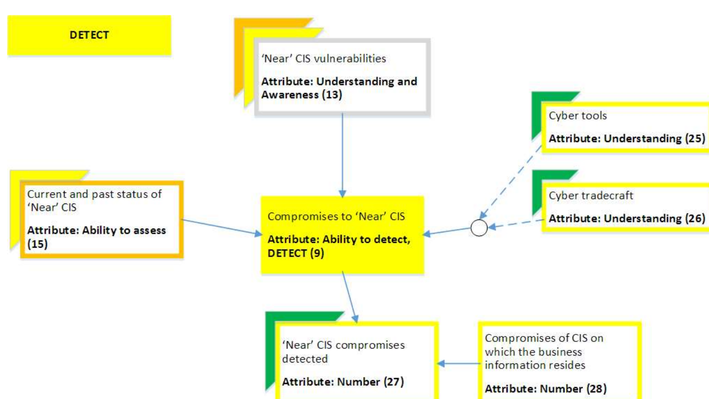



Figure 7.16 Nodes contributing to Detect function

#### 7.3.1.5 Recover function

Recover function has two main roles within the cyber defence system (Figure 7.17):

- It provides "Ability to recover information in sufficient time for business needs (7)" – which represents the ability of restoring the data in case of the successful compromise of information in a way that allows the minimal disruption to the business and
- Develops the "Ability to assess and learn from system compromises (8)" – educational role which allows to develop abilities to detect potential future threats.

Recover function is impacted by the following factor:

- "Availability of means to appropriately restore the confidentiality, integrity and availability of information (29)" – which defines the development of systems responsible for maintenance of the security of information in case of a successful compromise on the following three levels: availability (backup of data), integrity (ability to ensure that data has not been modified during the attack) and confidentiality (encryption of data to ensure that adversary will not be able to use sensitive information),
- Detect function: "Number of 'Near' CIS compromises detected (27)" – increase of number of compromises detected allows increase of ability to learn from compromises.

The recover function has impact on the following elements of the cyber defence system:

- Reduction of the "Impact of CIS compromises on the business (23)" – ability to recover information reduces the impact of compromises as they are unable to corrupt confidentiality, integrity and availability of information,

- "Level of Risk to the Business (10)"– the core aspect of the cyber defence model is impacted by the "Impact of CIS compromises on the business (23)". Ability to recover information is sufficient time reduces the risk to the business as the impact of successful compromises is mitigated.
- Detect function: "Ability to assess and learn from system compromises (8)" allows to increase the understanding of how to use "Cyber tools and tradecraft (25 and 26)" to improve detection capabilities.

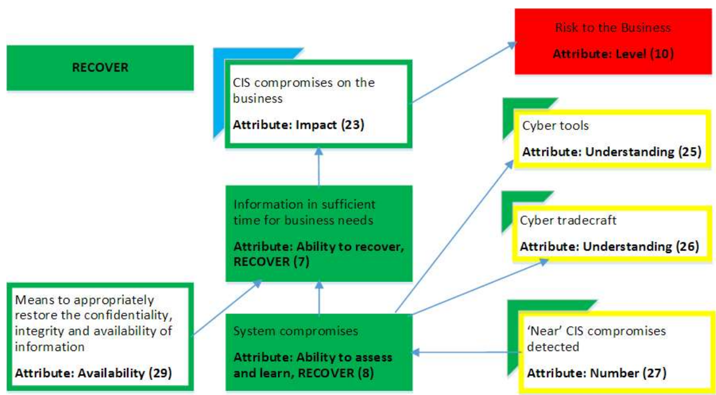



Figure 7.17 Nodes contributing to Recover function

## 7.3.2 Elements of the cyber defence RBFCM model

### 7.3.2.1 Membership functions

All nodes in the cyber defence model have attributes representing variations on the scale from -100 to 100. Variation of an attribute is defined using seven linguistic terms: *Decrease Much, Decreased, Decreased Little, Maintained, Increased Little, Increased, Increased Much*. All nodes have the same set of membership functions, defined as in Figure 7.18. These definitions satisfy condition 2.33 (Section 2.4.1)

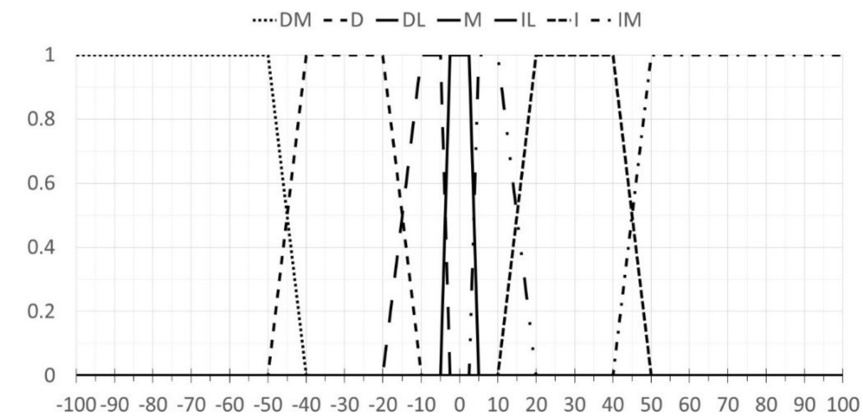



Figure 7.18. Membership functions defining variations in the cyber defence model

#### 7.3.2.2 Time component and relationships

In the developed model, the base time unit is 1 month. It is considered the shortest time period when the impact of the investment can be observed and is relevant for analysis. Simulations are run for 12 iterations, in order to allow an analysis of the impact of investments on the risk to the enterprise business over the course of 12 months.

All relationships in the cyber defence model are Fuzzy Causal Relationships or Complex Fuzzy Causal Relationships.

### 7.3.3 Analysis of the impact of investments

The cyber defence model was used to analyse the impact of six investments on the risk to the business. The following six investments and their combinations are analysed in this chapter: Improved Network Monitoring, Increased Mandatory Cyber Hygiene Training, Enhancements to Firewall, Through Life Software Management, Detect and Deny Tools, Specialists Skills Training and the following combinations: Enhancements to Firewalls and

Specialists Skills Training and Enhancements to Firewalls and Increased Cyber Hygiene Training.

The six investments have impact on different contributing factors. Investment in each of the options listed, is given in the form of a percentage increase or decrease of the allocated budget. The input into RBFCM model comes either through one investment or through the combination of six investment scenarios.

As all the investments represent positive impact on the cyber security, it was expected that all of them would decrease the overall risk to the MoD business in a long term perspective.

#### 7.3.3.1 Impact of the Enhancements to Firewall

An unexpected result was obtained when analysing the Enhancements to Firewall investment. Regardless of the level of investment, it had a negative impact on the risk to the business in long term, such as 1 year. The rationale for such a relationship between Enhancements to Firewall and the Risk, is that it protects the network from a number of potential compromises, but at the same time, it creates a risk to the way the enterprise works. There is a high probability that some of the communication, between intranet and internet, can be blocked by the firewall, resulting in the disruption of the service. Another problem is that firewalls represent a small portion of the overall security system of an enterprise. Such relationship between the investment and the risk to the business was defined by the technical expert.

To mitigate the negative impact of the Enhancements to Firewall investment on the risk to the business, multiple combinations of investments were investigated. The purpose of other investments is to complement the areas where Enhancements to Firewall fails to decrease the overall risk to the business.

In this analysis, the following three investments and two combinations of investments were analysed:

Investment 1. Enhancements to Firewalls,

Investment 2. Specialists Skills Training,

Investment 3. Enhancements to Firewalls and Specialists Skills Training,

Investment 4. Increased Cyber Hygiene Training,

Investment 5. Enhancements to Firewalls and Increased Cyber Hygiene Training.

For each investment option, the nodes in the RBFCM which are directly affected by the investment are listed in Tables 7.4, 7.5 and 7.6. For each investment option, three levels of investments are investigated: low, medium and high. These levels of investment have different direct impact on the respective nodes, which are evaluated subjectively by the expert. For example, low, medium and high level of investment into enhancements of firewall increase "Understanding and awareness of CIS vulnerabilities" (Node 13) by 5%, 20% and 30%, respectively, and decrease the "Number of CIS vulnerabilities" (Node 30) by -10%, -40% and -60%.

The impact of different levels of investments in Enhancements of Firewall, Specialist Skills Training, Cyber Hygiene Training and their combinations are simulated in 12 iterations. Their impact on the Level of Risk to the enterprise business (Node 10) is given in Figure 7.19. The x axis represents simulation iterations while y axis represents changes of the value of the Risk to the enterprise business (Node 10) over time.

Table 7.4. Definition of the impact of investment option 1 on the leaf nodes in the RBFCM

| | | **Investment 1** | | |
|---|---|---|---|---|
| | | ***low*** | ***medium*** | ***high*** |
| Node 13 | Understanding and awareness of CIS vulnerabilities | 5 | 20 | 30 |
| Node 21 | Impact of mitigation of CIS compromises or vulnerabilities on the business | 5 | 20 | 30 |
| Node 27 | Number of CIS compromises detected | 10 | 40 | 60 |
| Node 30 | Number of CIS vulnerabilities | -10 | -40 | -60 |

Table 7.5. Definition of the impact of investments 2 and 3 on the leaf nodes in the RBFCM

| | | **Investment 2** | | | **Investment 3** | | |
|---|---|---|---|---|---|---|---|
| | | ***low*** | ***medium*** | ***high*** | ***low*** | ***medium*** | ***high*** |
| Node 11 | Understanding of Impact of CIS compromises on the information held on near CIS | 5 | 20 | 40 | 5 | 20 | 40 |
| Node 12 | Understanding of CIS and potential vulnerabilities | 5 | 20 | 40 | 5 | 20 | 40 |
| Node 13 | Understanding and awareness of CIS vulnerabilities | | | | 5 | 20 | 30 |
| Node 15 | Ability to assess the current and past status of CIS | 5 | 20 | 40 | 5 | 20 | 40 |
| Node 21 | Impact of mitigation of CIS compromises or vulnerabilities on the business | | | | 5 | 20 | 30 |
| Node 22 | Understanding adversary intent | 5 | 20 | 40 | 5 | 20 | 40 |
| Node 24 | Understanding adversary capability | 5 | 20 | 40 | 5 | 20 | 40 |
| Node 25 | Understanding cyber tools | 5 | 20 | 40 | 5 | 20 | 40 |
| Node 26 | Understanding cyber tradecraft | 5 | 20 | 40 | 5 | 20 | 40 |
| Node 27 | Number of CIS compromises detected | 5 | 20 | 40 | 15 | 60 | 100 |
| Node 30 | Number of CIS vulnerabilities | | | | -10 | -40 | -60 |

Table 7.6. Definition of the impact of investment options 4 and 5 on the leaf nodes of the RBFCM

| | | **Investment 4** | | | **Investment 5** | | |
|---|---|---|---|---|---|---|---|
| | | ***low*** | ***medium*** | ***high*** | ***low*** | ***medium*** | ***high*** |
| Node 13 | Understanding and awareness of CIS vulnerabilities | 5 | 20 | 40 | 10 | 40 | 70 |
| Node 21 | Impact of mitigation of CIS compromises or vulnerabilities on the business | | | | 5 | 20 | 30 |
| Node 23 | Impact of CIS compromises on the business | -5 | -20 | -40 | -5 | -20 | -40 |
| Node 27 | Number of CIS compromises detected | | | | 10 | 40 | 60 |
| Node 30 | Number of CIS vulnerabilities | 5 | 20 | 40 | -5 | -20 | -40 |

The first investment analysed is Enhancements of firewall, which is expected to improve security of the business by controlling more strictly the communication between internal CIS and external systems. However, it may have a negative impact on the operation of the business as occasionally it can cause disruption to the service of the business.

Low and medium investments in Enhancements to Firewall (Investment 1) resulted in an increase of the Risk to the enterprise business. An increase of the Risk to the business can be explained by the fact that, due to increased firewall protection, the operation of the business will be impacted negatively. Benefits of increasing network protection do not balance out the negative impact caused by investments. High level of investment decreases the risk to the enterprise after four months but in the last two months the risk increases back to its initial state.

Two investments are analysed to counteract the negative impact of the investment into Enhancements to Firewalls: investment into Specialists Skills Training (Investment 2) and Increased Mandatory Cyber Hygiene Training (Investment 4).

An investment into Specialists Skills Training (Investment 2) is supposed to increase the skills of specialists responsible for identifying, responding and protecting CIS against compromises. The positive impact of the Specialists Skills Training is expected to improve the ability to assess the situation and to take mitigation and response actions against compromises in the CIS more effectively. For every investment level in Specialist Skills Training, the Risk to the business decreases significantly (Figure 7.19).

It can be intuitively assumed that the negative impact of the Enhancements of Firewalls will be partially mitigated when combined with investment in Specialist Skills Training (Investment 3). As it can be seen in Figure 7.19, regardless of the level of investment, the risk to the business initially increases due to the impact of Enhancements to Firewalls. However, for high and medium levels of investments, decrease of the risk to the business starts after four months and continues until the end of the year.

An investment into Increased Mandatory Cyber Hygiene Training, Investment 4, is expected to prevent employees from taking actions that could be impacted by the firewalls, resulting in the delays of their work (for example, send attachments with executable files could be stopped and undelivered to the recipient). Additionally, Increased Mandatory Cyber Hygiene Training would provide knowledge on the procedures of how to react if the firewall has interfered with their work. This investment option reduces the risk to the business after a month for high and medium investments (Figure 7.19). It has lower impact on the decrease of the risk than Investment 2. Low investment level into Mandatory Cyber Hygiene Training has negligible impact in the first ten months and increases in the last two.

As it can be seen in Figure 7.19, when Enhancements to Firewalls are implemented in parallel with Increased Mandatory Cyber Hygiene Training (Investment 5), the Risk to the enterprise business increases only when the level of investments is low. When the level of investments is medium or high, after a month, the Risk to the business decreases. Interestingly, for medium investment, after seven months, the level of the Risk increases again indicating that medium level of investment has temporarily positive impact on reduction of the Risk. The low investment causes a decrease in the risk to the business after six months. After additional four months, the risk increases again due to the increase caused by the Investment 4.

The significant difference between combining investment of Enhancements to Firewall with Specialists Skills Training (Investment 3) and Increased Mandatory Cyber Hygiene Training (Investment 5) is that when medium level of investments is applied, the level of Risk to the business does not increase after the eighth month but remains low for the Investment 3. It suggests that Specialists Skills Training has longer impact on the Risk, while Mandatory Cyber Hygiene Training needs to be repeated every few months. The low level of investment results in overall increase of the risk to the business in both scenarios. In the case of Investment 5 the Risk to the business decreases in months six to ten.

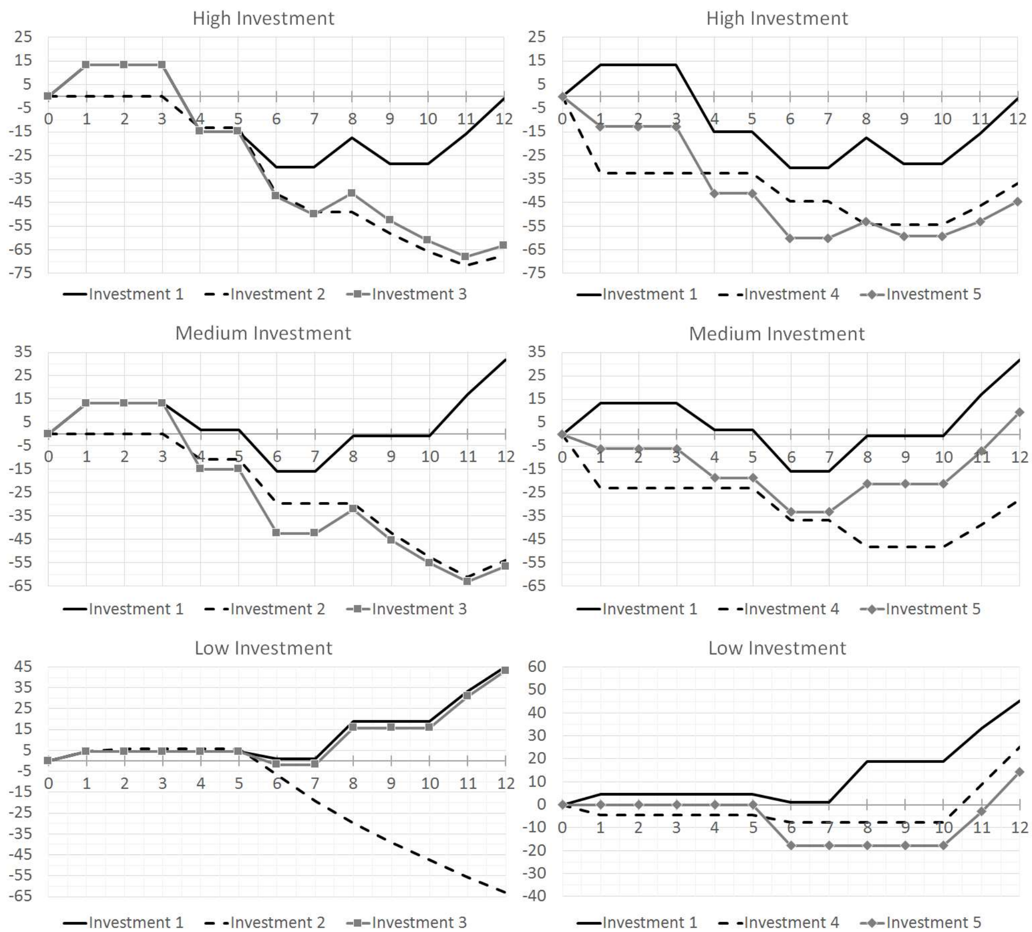



Figure 7.19. Results of simulations for investment options 1, 2, 3, 4 and 5

#### 7.3.3.2 Impact of investments in Improved Network Monitoring, Through Life Software Management and Detect and Deny Tools

In the second part of the analysis, remaining three investments were analysed:

Investment 6 – Improved Network Monitoring:

Investment 7 – Through Life Software Management,

Investment 8 – Detect and Deny Tools.

Nodes impacted directly by these investments are defined in Table 7.7.

Table 7.7 Definition of the impact of investment options 6, 7 and 8 on the leaf nodes

| | ***Investment 6*** | | | ***Investment 7*** | | | ***Investment 8*** | | |
|---|---|---|---|---|---|---|---|---|---|
| | ***low*** | ***medium*** | ***high*** | ***low*** | ***medium*** | ***high*** | ***low*** | ***medium*** | ***high*** |
| Node 12 | 5 | 20 | 30 | | | | | | |
| Node 13 | 5 | 20 | 30 | | | | 5 | 20 | 30 |
| Node 14 | | | | 10 | 40 | 60 | | | |
| Node 15 | 5 | 20 | 30 | | | | 5 | 20 | 30 |
| Node 17 | 5 | 20 | 30 | 10 | 40 | 60 | 5 | 20 | 30 |
| Node 22 | 5 | 20 | 30 | | | | | | |
| Node 23 | | | | -5 | -20 | -30 | | | |
| Node 24 | 5 | 20 | 30 | | | | | | |
| Node 25 | | | | | | | 5 | 20 | 30 |
| Node 26 | | | | | | | 5 | 20 | 30 |
| Node 27 | 5 | 20 | 30 | 10 | 40 | 60 | 5 | 20 | 30 |
| Node 30 | | | | -5 | -20 | -30 | | | |

The results of the three investments are presented in Figure 7.20. For all the three investments, the results are similar for the high level of investment. It causes risk to the business to decrease significantly in the third month after the investment is applied. Such state remains stable for the following months, until the month 12. Medium level of investment has a similar impact, though, the impact of the Investment 7 and 8 seems to be slowly deteriorating after the month 10. This indicates that all three investments have high contribution to the safety of the CIS but do not impact negatively the business. As medium and high levels of investments have comparable impact on the reduction of the Risk, it can be concluded that the objective of reduction of the Risk can be achieved without high spending for the three investments.

Interestingly, the low level of investment into Improved Network Monitoring and Detect and Deny Tools causes a significant increase of the risk to the business. For these two investments, the risk to enterprise business decreases marginally in the months 3 to 10, to substantially increase in the month 11. On the other hand, low level of investment into Through Life Software Management results in high decrease of the risk in the month 3 and increases in the month 11.

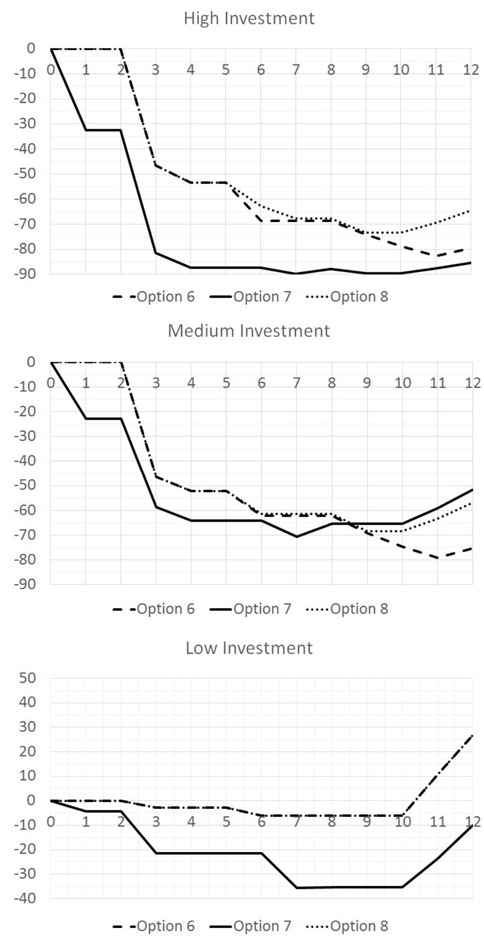



Figure 7.20. Results of simulations for investment options 6, 7 and 8

## 7.4 Conclusions

This chapter presents the results of development of RBFCMs for the two case studies discussed in Chapter 3. Both case studies are novel applications of RBFCMs and demonstrate how to use the RBFCM proposed for modelling complex real-world problems.

The first case study, modelling of the impact of resource allocation to DLoDs in respect to the location of the military base of the JSF force, was done to model a problem where the decision was already taken in practice. Therefore, the results generated by the model were verified against the decision based on the experts' knowledge. The second case study led to an in-depth analysis of the results. This model was not verified by any other model but by experts in cyber defence. The results were in line with the expectations of the cyber defence experts.

Both models were successfully developed, due to the new reasoning for RBFCM, i.e. interpolation of linguistics terms and accumulation of impacts, described in Chapters 5 and 6. The new RBFCMs provide much more modelling flexibility and more means of capturing complexities of the real-world systems. This proved to be useful in both case studies and made new RBFCMs more appealing to the experts, who helped to create the knowledge base for both case studies.

The results discussed in this chapter were presented at four conferences and received very positive reception on all of them. Moreover, the cyber defence model and the software developed, were further adapted by the Ministry of Defence to represent realities of their cyber security.

# Chapter 8. INTEGRATION OF RBFCMS AND DES MODELS

## 8.1 Introduction

RBFCM model for Cyber Defence, introduced in Section 7.3 is a strategic tool that gives a holistic overview of Cyber Security of an enterprise. The relationships between abstract functions: PROTECT, RESPOND, RECOVER, DETECT, ASSESS and RISK are expressed using qualitative relationships, defined using fuzzy IF-THEN rules, which describe how one node will be impacted by a change in another node, for example, IF Node A is *Increased Little* THEN Node B is *Increased*. The attributes of nodes are also of a qualitative nature: “Understanding and ability to assess”, “Ability to mitigate” and quantitative ones: “Number of vulnerabilities”, “Proportion of vulnerabilities identified”.

The proposed DES model for Cyber Defence is focused on the operation of the cyber defence and is modelling: detection, protection and response aspects of the system. It is not involved in the assessment of the vulnerabilities of the system or in the assessment of the risk to operation as these functions are too abstract and representing them in DES would be too complicated.

DES models the lifecycle of compromises and capability of the defence teams in detecting, neutralising, mitigating and responding to compromises. It allows usage of many KPIs: number of detected or undetected compromises, average time to respond to a compromise, number of times information was compromised, number of times the compromise was not neutralised by firewall but mitigation procedures or cyber team managed to respond to it, utilisation of resources, the length of queue of compromises that have not been neutralised, etc.

This chapter is structured as follow: first, in section 8.2, the method of integrating DES and RBFCMs models is explained, using simple example. In section 8.3 a novel DES model for cyber defence is introduced and definition of possible integration relationships between RBFCMs and DES models is outlined. The following section 8.4 presents results of simulations of integrated models using KPIs from the DES model. The chapter finishes with conclusions and directions for future work.

## 8.2 Method of integrating RBFCMs and DES

### 8.2.1 Discrete Event Simulation

A DES model consists of nodes representing activities. An example of the simplest DES model is presented in Figure 8.1.

A DES model consists of a start and an end node, “Create” and “Finish”, respectively and a “DES Process” module. They represent creation and destruction of an entity, or an entry and a departure of an entity from the system. The “Create” node creates entities at discrete times according to a statistical distribution that defines an inter arrival time of entities. The “DES Process” node, processes an entity for a time defined using a statistical distribution. To process the entity, the “DES Process” needs to allocate all its available resources; therefore, only one entity can be processed at any given time if the resource capacity is equal to 1. During processing of the entity, the state of the system is considered busy. If during processing of the entity, another entity is created, it joins the queue and waits for availability of the resources.

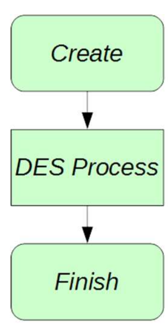



Figure 8.1. Example of very simple DES model.

Let us assume that the inter arrival time is defined using a uniform distribution $U\{a,b\}$, which means that the probabilities of having random interarrival time between a and b are equal. In a given DES model it is assumed that inter arrival time is defined using following distribution $U\{0,3\}$. Let us also assume that the time needed to process an entity in the "DES Process" module is described by a normal distribution $N_{DESProcess}\left(\mu_{DESProcess}, \delta^2_{DESProcess}\right)$, where $\mu_{DES\,Process}$ is the mean of the distribution and $\delta^2_{DESProcess}$ is the variance of the normal distribution and the processing time is measured in minutes. The processing time of the "DES Process" module is defined using the following distribution: $N_{DESProcess}(2, 0.01)$, which means that with probability of 95% the processing time is between 1.8 to 2.2 minutes, i.e. between two standard deviations from the mean– the three sigma rule (Pukelsheim, 1994).

### 8.2.2 Integration of RBFCMs and DES

In order to demonstrate how RBFCMs and DES models can be combined, let us consider an example of simple RBFCM integrated with the DES model presented in section 8.2.1 (Figure 8.2). The RBFCM model consists of 3 nodes, including 2 leaf nodes "RBFCM leaf node 1" and "RBFCM leaf node 2" and one function node "RBFCM function node".

The numbers on the arrows represent the time duration necessary for the impact to affect the node. If one iteration represents 1 month, then "RBFCM leaf node 2" will cause a change in "RBFCM function node" after one month, while "RBFCM function node" will modify "DES Process" module as soon as it is changed.

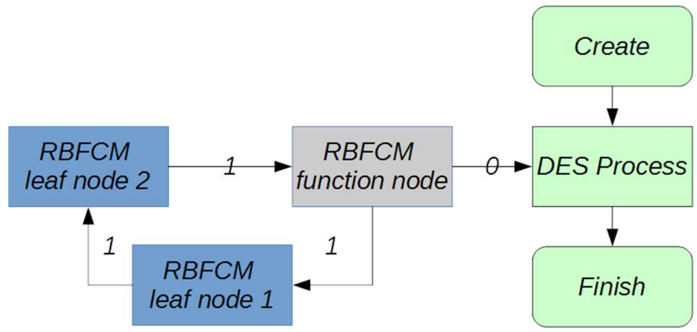



Figure 8.2. Relationship between RBFCMs and DES models

Let us assume that the relationship between RBFCM and the DES models is defined using a linear function. The change in the value of the “RBFCM Function node” causes a change in the behaviour of the DES model by impacting the “DES Process” module’s processing time. It is achieved by relating the value of the mean of the normal distribution, $\mu_{DES\ Process}$, which defines the processing time of the “DES Process” and the value of the “RBFCM Function node”, as follows:

$$\mu_{DES\ Process}(t) = 2 + \frac{RBFCM\ Function\ node(t)}{4}, \text{where } \ t > 0$$

Let us assume that the “DES Process” processing time is defined by the normal distribution with mean equal to 2, and variance equal to 0.01, $N(2{,}0.01)$. As a result of the impact of the “RBFCM Function node” the “DES Process” takes values in subsequent iterations as presented in Table 8.1.

Table 8.1 Change of the “RBFCM Function node” and “DES Process” module over time

| Iteration | Value of the “RBFCM Function node” | Processing time of the “DES Process” module |
|---|---|---|
| 0 | 0 | $N(2{,}0.01)$ |
| 1 | 2 | $N(2.5{,}0.01)$ |
| 2 | 4 | $N(3{,}0.01)$ |
| 3 | 6 | $N(3.5{,}0.01)$ |

The value of "RBFCM Function node" and the mean of "DES process" distribution, against time is presented in Figure 8.3.

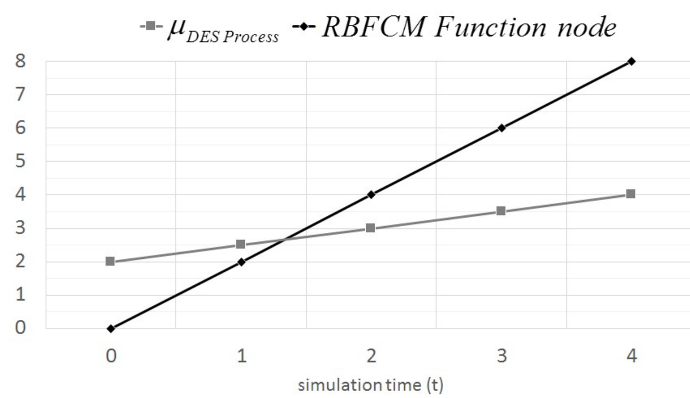



Figure 8.3. Relationship between RBFCM function node and DES process time.

Because of the impact of the "RBFCM Function node" the mean of the normal distribution defining the processing time of the "DES Process" changes over time. In the iteration 0, when the RBFCM model does not have impact on the DES model, the processing time of the "DES Process" is equal to $N(2,0.01)$. In the iteration 4, the value of the "RBFCM Function node" reaches value 8, therefore increasing the processing time of the "DES Process" to $N(4,0.01)$.

In Figures 8.4 to 8.7 four iterations of running DES model simulation are presented, for different processing times of the "DES Process" module and the following KPIs: information on the state of the system (busy or idle) and the length of the queue, at any point in time.

The following notation is used to define the DES simulation outputs (Figures 8.4 to 8.7):

$t_k$ – discrete time of arrival of the k-th compromise (event),

$A_k$ – inter arrival time between arrival of (k-1)-th and k-th compromise,

$S_k$ – overall time spent in the system of the k-th compromise, includes the process time and the queuing time,

$t_n$ – discrete time of arrival of the last, n-th compromise.

Queue – specifies how many compromises are waiting in a queue to be processed. In the simulation presented in Figure 8.4, the maximal queue length is equal to 2.

State – defines the state of the system: 0 – when no entity is processed, 1 – when an entity is processed by the "DES Process" module.

In Figure 8.4, at the beginning of the simulation the system is idle. The system's state changes as soon as the first entity 1 is created, at time $t_1$. At the same time it starts being processed by the "DES Process" module and it takes 1.9 minutes, $S_1$, to process it. After 0.4 minutes from starting processing entity 1, $t_2 - t_1$, an entity 2 is created and automatically joins the queue as the resource of the "DES Process" module is busy. After $t_1 + S_1$ time the processing of the entity 1 is finished and it leaves the system. In the same time the entity 2 is started to be processed and there are no entities in the queue of the "DES Process" module.

The queue of the "DES Process" module reaches the maximum of 2 entities at the time $t_4$, when the 4th entity arrives, while the entity 2 is processed and the entity 3 waits in the queue. The simulation continues until all entities are processed.

In the subsequent runs of the DES model, presented in Figures 8.5 to 8.8, the "DES Process" time increases due to impact of "RBFCM Function node", according to the changes presented in Figure 8.3 and Table 8.1. As a result, the processing time of entities in the "DES Process" module, changes according to the value of the "RBFCM function node". It can be observed that due to longer processing of entities, in the similar time, in the iteration 0 - 4 entities were processed, in iteration 1 – 3 entities were processed and in iterations 2 and 3 – only 2 entities were fully processed. Increasing of the processing time has influence on the length of the queue of entities waiting to be processed. In iterations 0, 1, and 2 the maximum queue length is equal to 2, while in iteration 3 the maximum queue length is equal to 3.

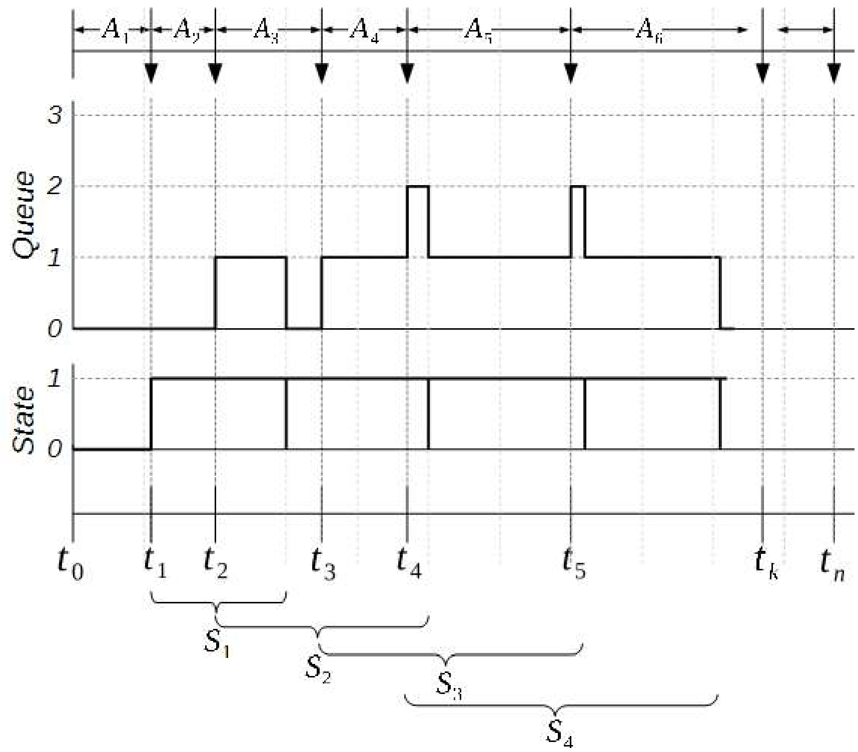



Figure 8.4 Results of the DES simulation for the iteration 0, where $N_{DES\,Process}(2{,}0.01)$

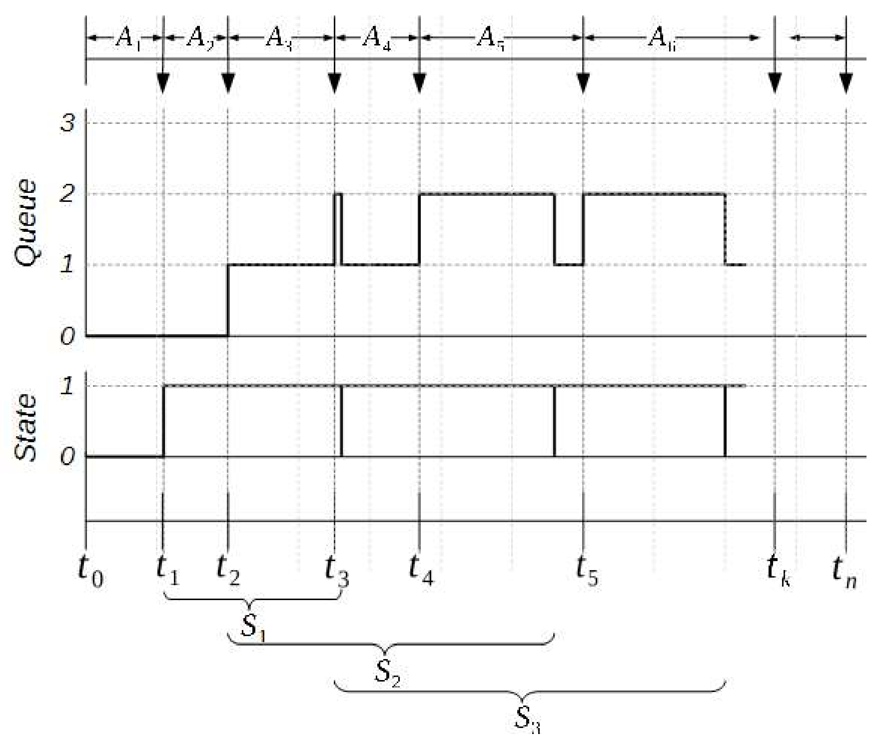



Figure 8.5 Results of the DES simulation for the iteration 1, where $N_{DES\,Process}(2.5{,}0.01)$

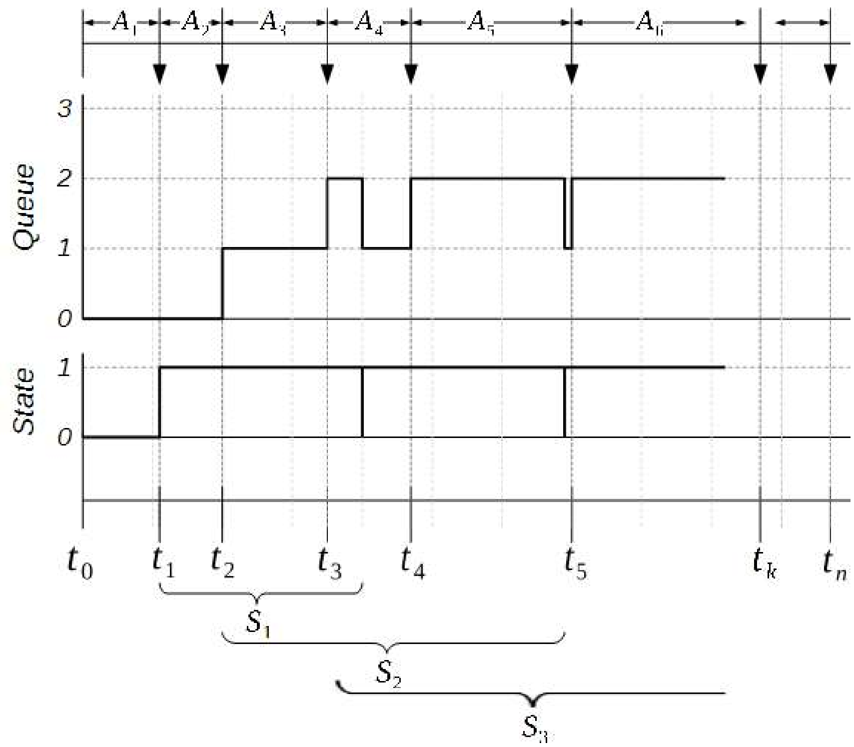



Figure 8.6 Results of the DES simulation for the iteration 2, where $N_{DES\,Process}(3,0.01)$

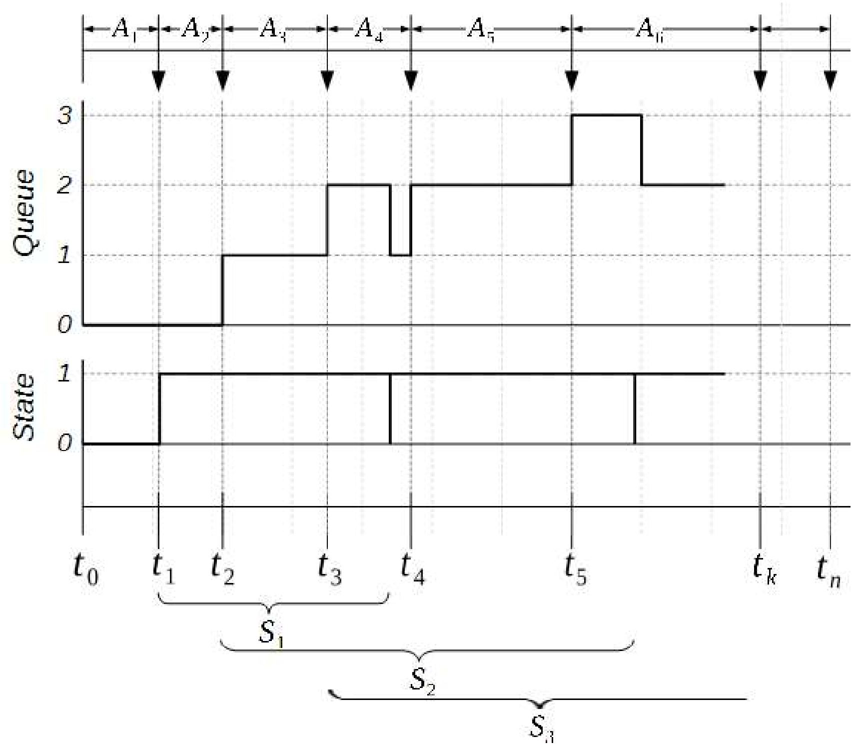



Figure 8.7 Results of the DES simulation for the iteration 3, where $N_{DES\,Process}(3.5,0.01)$

## 8.3 DES model for cyber defence

This section presents a novel DES model of the cyber defence. The model was built with the support of the cyber security expert and statistics gathered from the official cyber security reports (IBM 2015 Cyber security intelligence index, 2015; PricewaterhouseCoopers, 2015) were used to define behaviour of the system.

The developed model defines the lifetime of a compromise and the capability of the cyber defence teams and cyber security systems to detect, deny and respond to compromises. Entities that enter the system and interact with it, in discrete time, are compromises to cyber security with an aim to compromise the information. Every entity has an attribute: strength (Entity.Strength), which defines how difficult it is to respond and mitigate the compromise. The possible values of the strength of an entity are between 1 and 6, where 1 is the weakest compromise and 6 is the strongest compromise.

Every compromise is handled – responded to or mitigated, by a cyber defence team. In a developed model, there are 2 or 5 cyber teams available to handle compromises. The parameters of the activities of the model are defined in the Table 8.3.

A DES model, developed and proposed to be linked with the RBFCM, consists of the following types of modules, controlling the flow of the entities in the DES system, presented in Table 8.2.

Table 8.2 Definition of types of modules in DES model

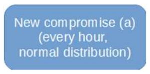
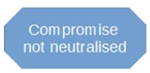
| Module | Definition |
|---|---|
|   | starting and finishing module |
|   | processes (activities): each process has its own duration and, if needed, resource assigned to it |
|   | indicates a decision module. As a result, the compromise can follow one of the two branches: TRUE or FALSE. Each node has probability that the compromise will follow the TRUE branch of the model. |
|   | indicates an assign module. In this module values of attributes or variables of a compromise or the system can be assigned or changed. |
|   | indicates a record module that collects information about entities (number of appearances of entity in the module). |
|   | module used to duplicate an entity in order to allow parallel processing. |
|   | queue |

Table 8.3 Definition of modules of the DES model for cyber defence

| ID | NAME | TYPE OF MODULE | PARAMETER | RESOURCE REQUIRED | VALUE | UNIT |
|---|---|---|---|---|---|---|
| **1** | New Compromise | Create | Inter-arrival time | | Exponential Distribution (0.138) | Hours |
| **2** | Assign Attributes | Assign | Entity Attribute | | Entity.Strength = Beta distribution(3, 6, 0, 6) | |
| **3** | Compromise detected | 2-way decision | Percent true | | 98% | |
| **4** | Compromise Undetected | Count | | | | |
| **5** | Identify Compromise | Process | | Cyber Team (1) | Normal Distribution (1, 0.25) | |
| **6,9** | Report detection | Process | Process time | Cyber Team (1) | Normal Distribution (0.5, 0.2) | Hours |
| **7** | Compromise Detected | Count | | | | |
| **8** | Compromise neutralised by Firewall or Deny tools? | 2-way decision | Percent true | | 85% | |
| **10** | Compromise Information | Process | Process time | Cyber Team (1) | (4 - Entity.Strength) * 0.3 | Hours |
| **11** | Information compromised and not recovered? | 2-way decision | Percent true | | 75% | |
| **12** | Mitigate Compromise | Process | Process time | Cyber Team (1) | Normal Distribution (Entity.Strength*0.167, 0.1) | Hours |
| **13** | Respond to compromise | Process | Process time | Cyber Team (1) | Normal Distribution (Entity.Strength*0.167, 0.1) | Hours |
| **14** | Information Compromised | Assign and Count | | | | |
| **15** | Information not compromised | Assign and Count | | | | |
| **16** | Compromise Mitigated or responded to | Assign and Count | | | | |
| **17** | Finish | Dispose | | | | |

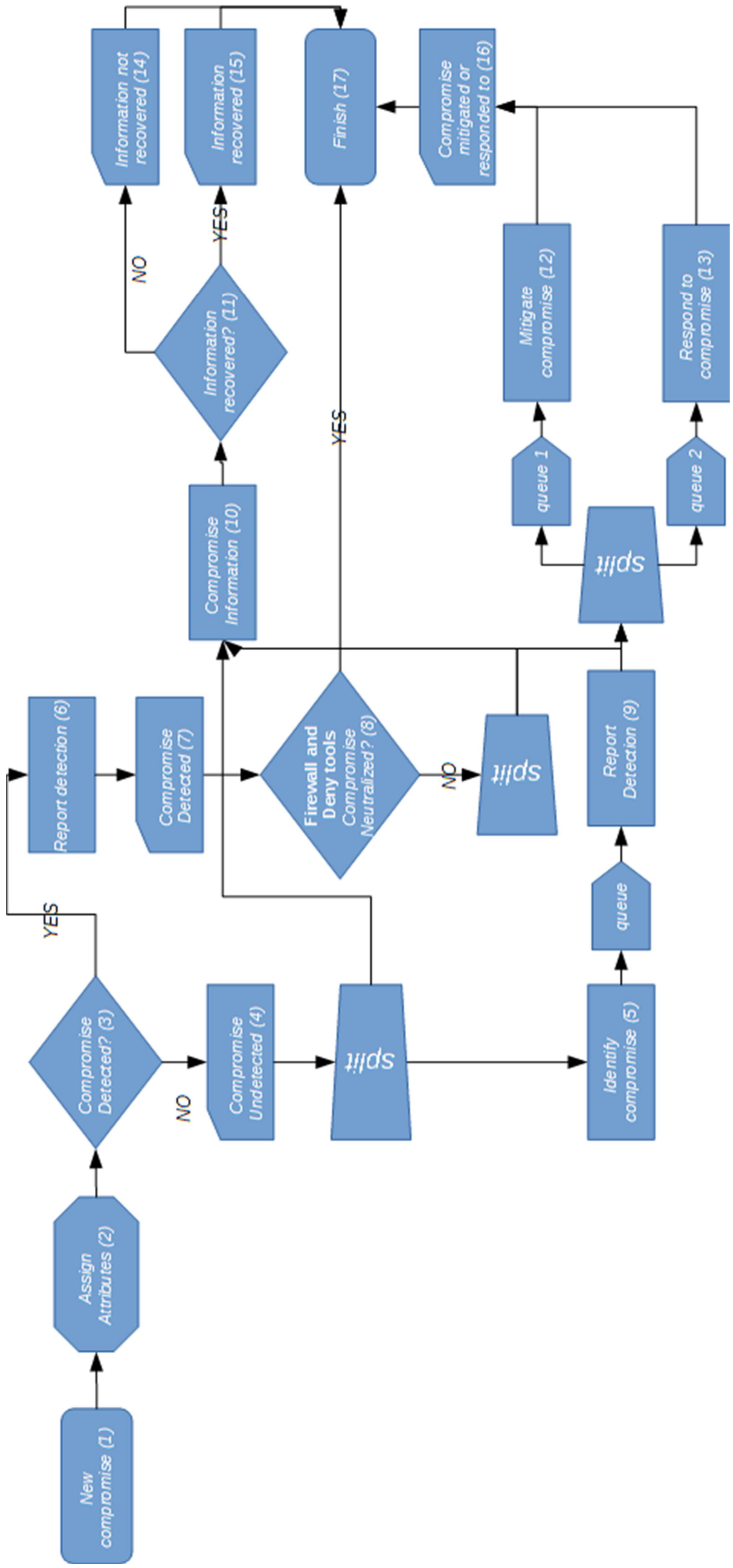



Figure 8.8. DES model for Cyber Defence

The lifecycle of a compromise in the DES model can be described using the following 8 steps:

1. It starts with creation of a new compromise (Module 1) in a discrete time ( $t_k$ ). This event represents arrival of a new compromise of CIS,
2. In the next step, values of attributes and variables are assigned (Module 2):
   2.1. Entity.Strength – the strength of the entity is assigned according to statistical distribution (Table 8.3),
3. In Module 3, it is decided whether the compromise was detected or not. In this module, there is 98% probability that the compromise will be detected. Information whether it was detected or not is recorded (modules 4 and 7),
4. If the compromise is not detected, it immediately starts to compromise information (Module 10). The amount of time it takes to compromise information depends on the strength of the compromise, defined in Module 2. In the meantime, while information is being compromised, the compromise is waiting to be identified (Module 5). This will take a different time for different types of compromises. Once the compromise is identified, the information about detection of the compromise is reported (Module 9) and the process continues to step 7.1,
5. If, in step 3, the compromise is detected, detection is reported (Modules 3, 6, 7) then the firewall is trying to neutralise the compromise (Module 8),
6. If the compromise is neutralised by firewall (Module 8), then the lifecycle of this compromise finishes (Module 17),
7. If the firewall did not neutralise the compromise (Module 8) then the compromise is split and takes two parallel routes:

7.1. The compromise is attempting to compromise information (Module 10). The time necessary to compromise information depends on the type, i.e. strength, of the compromise; the higher the strength the less time it needs to compromise information,

7.2. At the same time the compromise tries to impact CIS, Cyber Defence teams are trying to mitigate it and respond to the compromise (Modules 12 and 13). Both actions are performed concurrently and the time required to accomplish the task depends on the type of compromise, i.e. strength; the higher it is the longer it takes to mitigate and respond,

8. Depending on which process finishes first, the following three statistics are gathered:

8.1. Compromise is mitigated or responded to in time and information is not compromised (Module 16)

8.2. Information is compromised (Module 14)

8.3. Information is compromised but Cyber Defence team managed to recover it (Module 11) – probability 75%. Statistics gathered in Module 15.

### 8.3.1 Integration of RBFCM and DES models for cyber defence

The RBFCM model for Cyber defence is presented in Figure 7.8. RBFCM model consists of two types of nodes: leaf nodes – framed rectangles with white background and function nodes – rectangles with coloured background, where 5 different colours present 5 cyber defence functions. The function nodes represent higher level concepts, while the leaf nodes express more detailed aspects of the cyber system. The RBFCM model is described in detail in section 7.3.

Both models, RBFCM and DES, are dynamic models. One iteration in the RBFCM model represents one month of the real time system. The DES model changes its state whenever a new event is triggered. The DES model simulates one week of activities 50 times to calculate confidence intervals for KPIs, as the DES model relies on statistical data every run of simulation can yield different results. Confidence intervals of 95% define what was the maximum and minimum value of the KPI in the 95% of 50 runs.

After simulating one week in the DES model and one iteration of RBFCM model the information produced by both models is evaluated and exchanged (Figure 8.9).

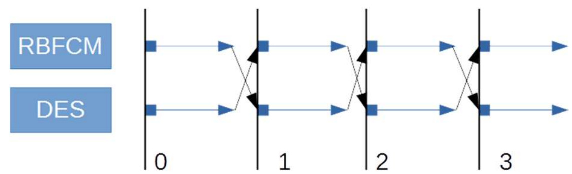



Figure 8.9. Exchange of the information between the DES and RBFCM models

The DES model was developed using AnyLogic software. The integration module was created to link the AnyLogic model with the software developed for RBFCMs.

### 8.3.2 Definition of relationships between the RBFCM and the DES models

Four relationships between RBFCM and DES models have been proposed, developed and analysed, as follows:

(a) Impact of "Ability to respond appropriately to successful compromise" (Node 5 in the RBFCM) has a direct impact on the time it takes to "Respond to Compromise" in the DES model. This RBFCM node has an impact on the mean and standard deviation of the DES activity through a proposed linear relationship as follows:

- Mean = Entity.Strength* Ability to RESPOND/500
- Standard Deviation = 0.1 * Ability to RESPOND/100

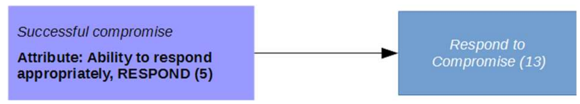



Figure 8.10. Linear relationship between node 5 (RBFCM) and activity 13 (DES model)

(b) "Ability to detect compromises to 'Near' CIS" (RBFCM node 9), has a direct impact on the probability that the compromise will be detected, in the DES decision module 3 – "Compromise detected?" (Figure 8.11)

- Percent True = 95 + Ability to DETECT/20

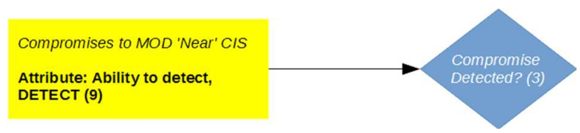



Figure 8.11. Linear relationship between node 9 (RBFCM) and decision node 3 (DES model)

(c) Two RBFCM nodes: "Risk to MOD Business" (RBFCM node 10) and "Ability to assess the risk to MOD business of CIS compromises and mitigation measures" (RBFCM node 0) have an impact on the Risk variable of the DES model through cFCR(Figure 8.12). It has however, insignificant impact on the model as the Risk variable is not currently used.

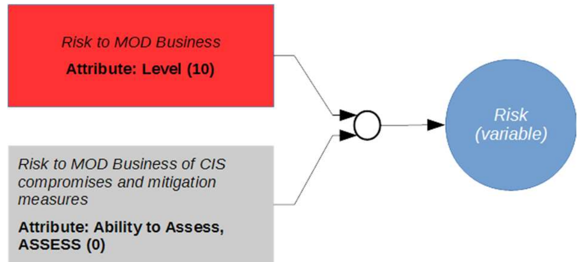



Figure 8.12. Complex Fuzzy Causal Relationship between two nodes of RBFCM and risk variable of the DES model.

(d) Node "Ability to recover information in sufficient time for MOD business needs" (RBFCM node 9), has a direct impact on the probability that the compromise will be detected, in the DES decision module 11 – "Information compromised and not recovered?" through a linear relationship defined as follows:

- Percent True = 75 - Ability to RECOVER/8


Figure 8.13. Linear relationship between Node 7 (RBFCM) and decision node 11 (DES)

## 8.4 Experimental design

Integration of the RBFCM and DES models is analysed using three scenarios. The first DES "AS IS" scenario runs without the impact of the RBFCM model. This scenario allows to understand the current state of the cyber defence operation and is treated as the reference point for other scenarios.

The impact of the RBFCM model on the DES model is evaluated using the following two investment scenarios:

1. Enhancements to Firewalls,

2. Specialists Skills Training.

These two investment options were introduced and their impact on the risk to the enterprise business discussed in section 7.3.3.1. For every scenario, including the “AS IS”, two simulations are executed, for different capacities of the cyber defence team resource: 2 and 5. Analysis of different capacities of the resource allows to observe the impact of its size on the operation of the cyber defence system.

For every scenario, eleven graphs are generated to measure the main KPIs generated by the DES model over 15 iterations:

1. Histogram representing distribution of levels of strengths of compromises. Most compromises have strength between 1 and 3. Distribution of levels of strengths is common for all scenarios.

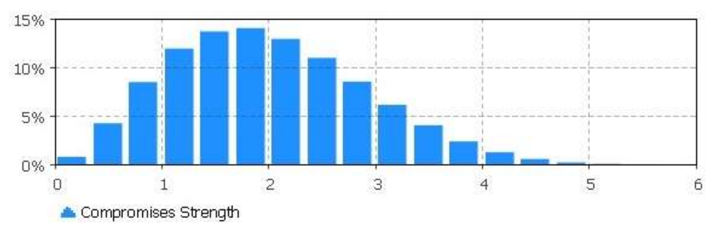



Figure 8.14 Compromises strength distribution

2. Number of all security incidents and number of security incidents detected. An incident becomes a compromise if it is not stopped by the detect and deny systems.

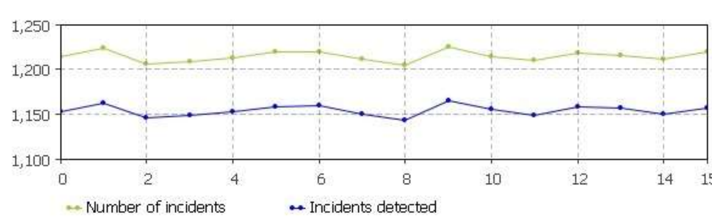



Figure 8.15 KPIs Total incidents and incidents detected

3. Number of undetected security incidents.

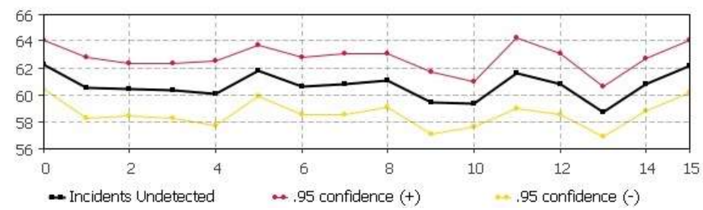



Figure 8.16 KPI Incidents undetected

4. Number of compromises – security incidents that were not detected and incidents that were not stopped by firewall and deny tools. Compromises attempt to compromise information.

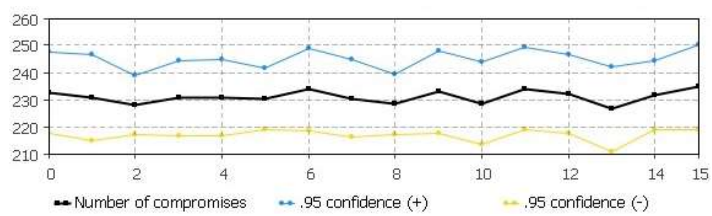



Figure 8.17 KPI Number of compromises KPI

5. Number of compromises that were responded or mitigated before information was compromised.

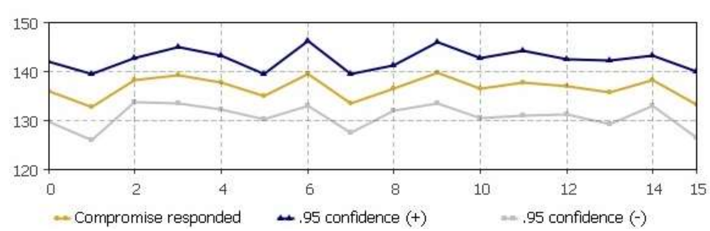



Figure 8.18 KPI Number of compromises responded to

6. Number of compromises not responded to and not mitigated, that managed to compromise information.

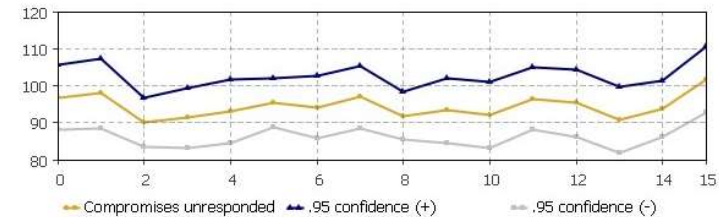



Figure 8.19 KPI Number of compromises unresponded to

7. Number of times information was recovered, after being successfully compromised.

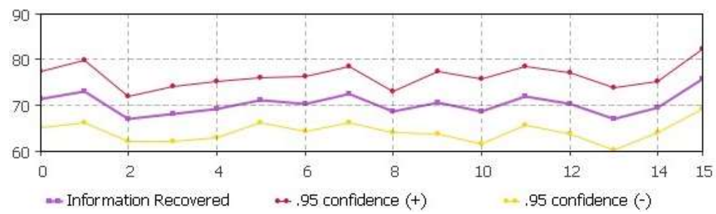



Figure 8.20 KPI Number of times Information was recovered

8. Number of times information was not recovered, after being successfully compromised.

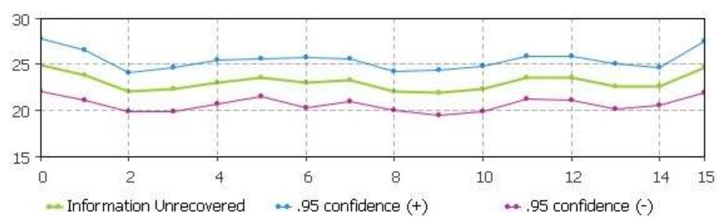



Figure 8.21 KPI Number of times Information was unrecovered

9. Detection rate – percentage of security incidents that are detected by detect tools.

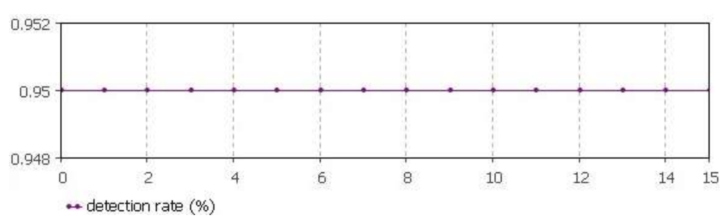



Figure 8.22 KPI Detection rate

10. Resource utility – average percentage of time, when resources are busy.

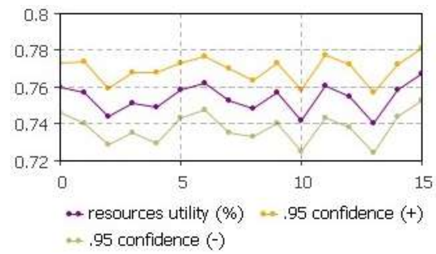



Figure 8.23 KPI Resources utility

11. Average queue size of three queues: queue – identification of compromises, queue1 – respond to compromises, queue2 – mitigate compromises.

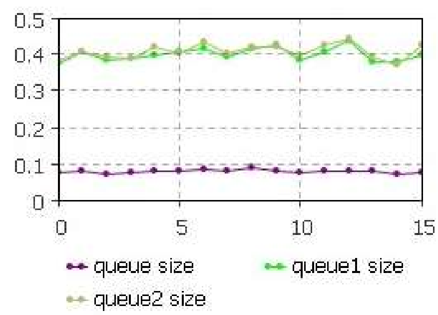



Figure 8.24 KPI Queues sizes

The results of the three scenarios are analysed with the focus on the three main KPIs produced by the DES simulation:

1. Number of incidents detected – depending on the value of "Ability of detecting compromises to 'Near' CIS" node, the number of compromises detected in the DES model is increased or decreased.
2. Number of compromises responded/unresponded – this KPI depends on the impact of relationships (a), "Ability to respond appropriately to successful compromise" on the time necessary to "Respond to Compromise" and (b), "Ability to detect compromises to 'Near' CIS" on the probability that the compromise will be detected. Increase of number of compromises detected allows quicker response to a compromise and increases

likelihood that a compromise would be stopped before corrupting information. The greater impact on reduction/increase of number of compromises responded to, has the node "Ability to respond successfully to successful compromise". It impacts that processing time of the "Respond to compromise" module.

3. Resources utilization and Queue length – both KPIs are, to some extent, related to each other and both depend on the number of compromises that need to be resolved and the time it takes to respond to them. It can be observed that the length of the queues corresponds to the "Ability to respond to successful compromises" and "Ability to detect compromises".

These three KPIs capture three important aspects of the cyber defence: ability of the cyber defence tools and teams to detect compromises, their ability to respond to compromises and efficiency of the modelled cyber defence for different capacities of available resources.

## 8.5 Results of DES simulations

### 8.5.1 AS IS scenario

The first scenario investigated assumes that the RBFCM cyber defence model does not impact the DES cyber defence model. The results of simulation of the DES model are presented in Figures 8.25 and 8.26. The obtained results for 15 iterations of DES simulation are very similar as the model does not change between iterations. Slight variations in the results between iterations can be observed as the model relies on statistical data.

The overall number of incidents, as well as numbers of incidents detected and undetected are the same for both simulations. The big difference can be noticed in number of compromises responded and not responded to. The difference results from different number of available resources. If only 2 resources are available, then the number of compromises responded to oscillates around 138 while number of compromises unresponded to oscillates around 92 (Figure 8.25); if 5 resources are available then number of compromises responded oscillates around 182, while number of compromises unresponded to oscillates around 48 (Figure 8.26).

Number of compromises responded to is higher in the simulation where the resource capacity is equal to 5 because cyber defence team has greater capability to respond to multiple compromises at the same time. In the scenario where the capacity of resources is equal to 2 only 2 compromises can be processed in the “Respond to compromise” module. In case when more than 2 compromises are attempting to compromise information in the same time the likelihood of failure to protect the network increases when only 2 resources are available compared to availability of 5 members of the cyber defence team.

The size of the cyber defence team and its capability to respond to compromises has an impact on the resources utility and the size of queues 1 and 2. It can be also observed that the average resource utility is equal to 71% for 2 resources available (Figure 8.25) and 41% for 5 resources available (Figure 8.26), similarly the average length of the queues for processes 9, 12 and 13 is lower when more resources are available. It is because having 5 team members of the cyber defence team allows to respond to multiple compromises at the same time. Therefore, there are fewer compromises that are not addressed as soon as detected. On the other hand, greater number of members of the cyber defence team reduces the overall utilisation of the team. In the scenario where the resource capacity is equal to 5, 60% of time the cyber defence team is not busy, while having smaller team means only 29% of the time the team is not occupied with their tasks.

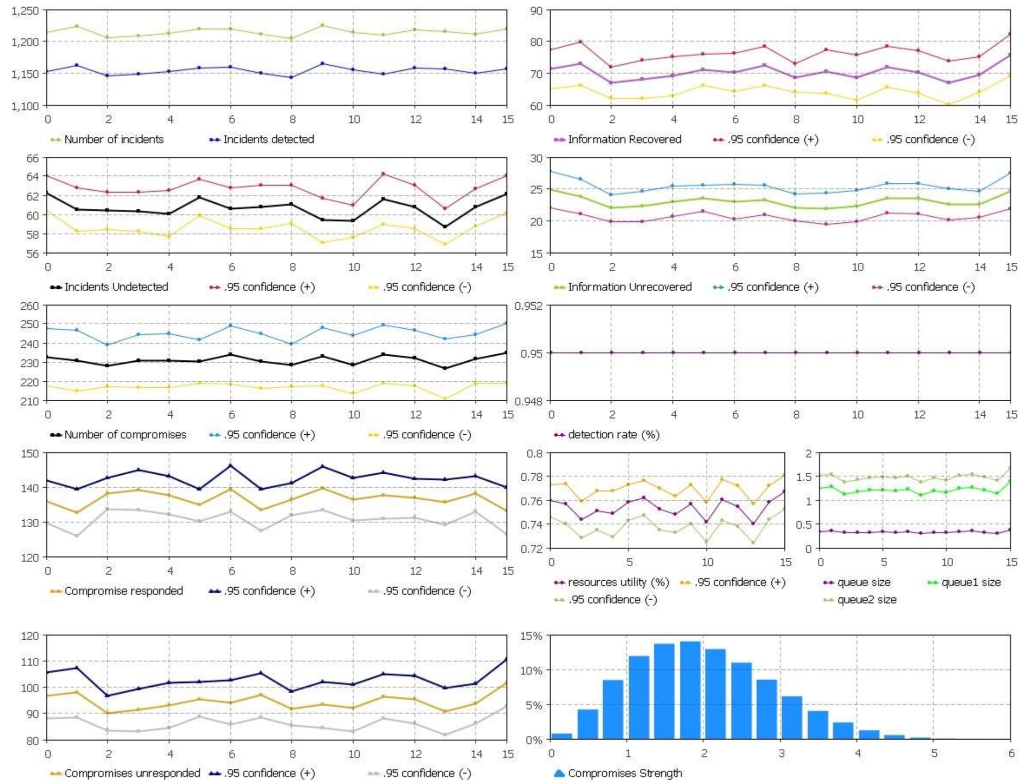



Figure 8.25 Results of the DES simulation for the scenario AS IS, the capacity of the resource equal to 2

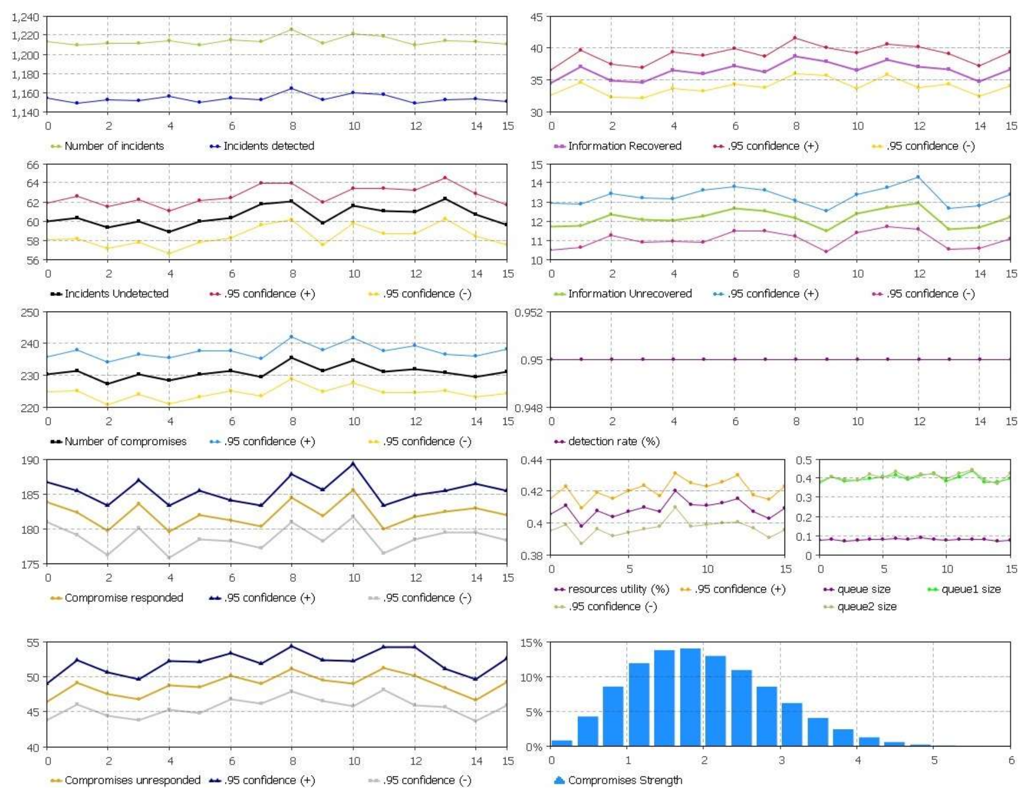



Figure 8.26 Results of the DES simulation for the scenario AS IS, the capacity of the resource equal to 5

### 8.5.2 Results of the Investment 1: Enhancements to firewalls

In this scenario, the DES simulation is impacted by the RBFCM after every iteration. The RBFCM simulation is stimulated by the Investment 1, as defined in section 8.4. The results of simulation of the DES model are presented in Figures 8.27 and 8.28.

Scenario 1, Investment in Firewalls, is supposed to improve the protection against cyber incidents but does not have impact on the protection against compromises that were not stopped by the firewall. Therefore, the impact of this investment is not expected to improve the response capability of the cyber defence team.

It can be observed that the overall number of incidents, as well as the numbers of incidents detected and undetected are similar in both simulations and similar to AS IS scenario. This is due to the fact that the detection capability is not changed significantly by the RBFCM model and the detection rate oscillates between 94.9% and 95.1%.

The RBFCM model has a significant impact on the number of compromises responded to. From the third iteration, the number of compromises declines and reaches a very low level of around 20 compromises responded to, in the 13th iteration. Very similar results are obtained regardless of the capacity of the cyber defence team resource.

Decrease of the capability of responding to compromises of the cyber defence team has a noticeable impact on the resource utilisation and the length of queues. The resource utilisation increases and reaches almost 100% in the iteration 14. It suggests that the cyber defence is unable to cope with incoming compromises in time. This observation is in line with the number of compromises waiting in the queue to be processed. The average size of the queues 1 and 2 in the final iteration reaches the size of around 25 and 13 for scenarios with 2 and 5 resources available, respectively. In the AS IS scenario these queues have the average size of 1.3 and 0.4 for scenarios with 2 and 5 resources available, respectively.

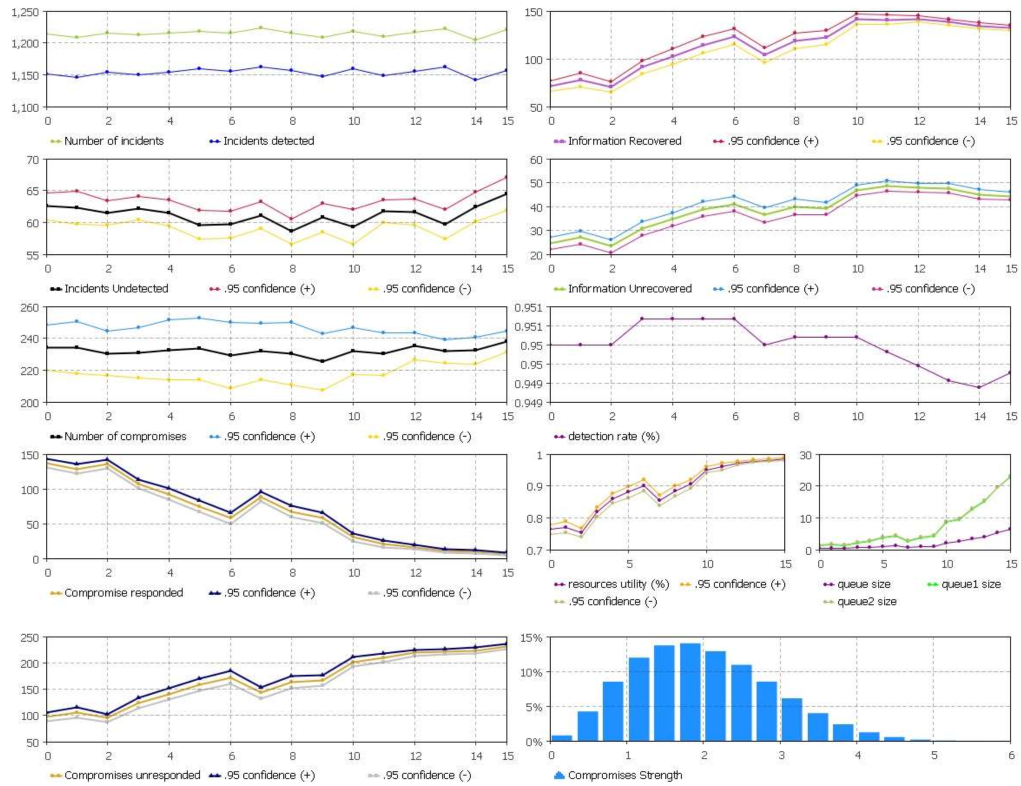



Figure 8.27 DES simulation results where capacity of the Cyber Team equals 2 and is under the impact from RBFCM model, triggered by the Investment 1

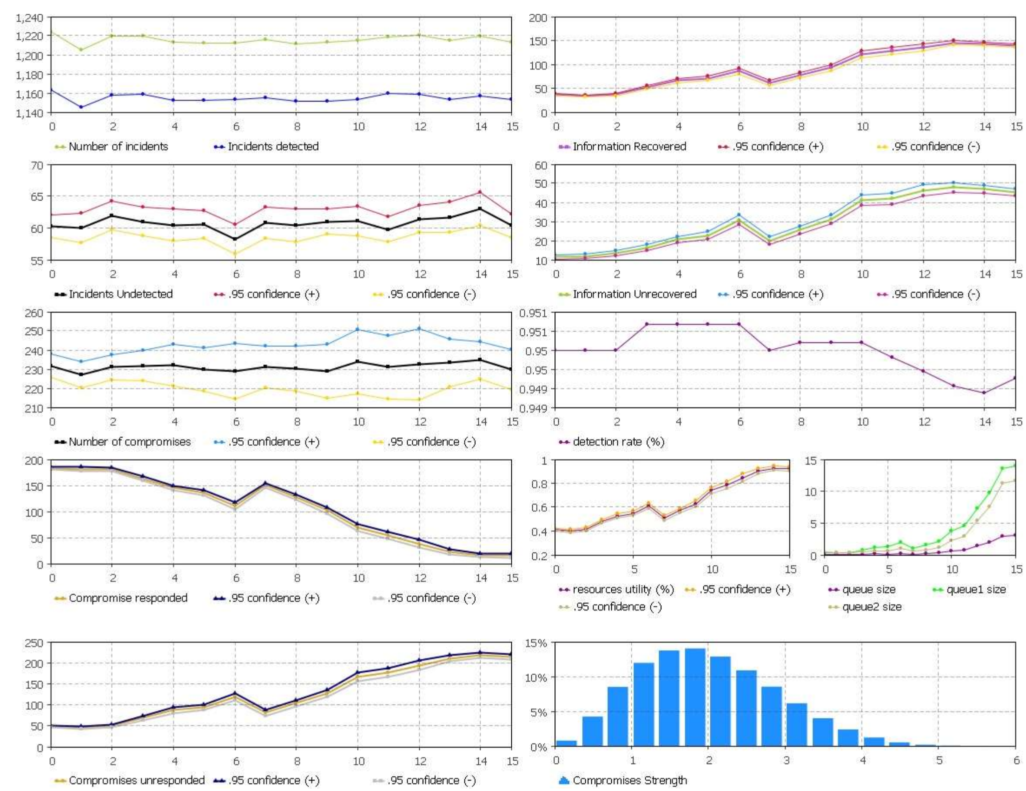



Figure 8.28 DES simulation results where capacity of the Cyber Team equals 5 and is under the impact from RBFCM model, triggered by the Investment 1

### 8.5.3 Results of the Investment 2: Specialist Skills Training

In this scenario, the DES simulation is impacted by the RBFCM after every iteration. The RBFCM simulation is stimulated by the Investment 2, as defined in section 8.4. The results of simulation of the DES model are presented in Figures 8.29 and 8.30.

The Investment option 2 – Specialists Skills Training assumes that the budget is invested in the training of the cyber defence team. This investment is expected to increase response capability of this team and in result an increase in the number of compromises responded to by the cyber defence team.

It can be observed that the overall number of incidents, as well as numbers of incidents detected and undetected is similar to both AS IS and Investment 1, even though the detection rate increases more considerably than in previous scenarios.

Number of compromises responded to is different than in the Investment 1. Initially, the number of compromises responded to increases and from the 5th iteration of the simulation, the number of compromises responded declines. Until iteration 8, the number of compromises responded to is greater or equal to number of compromises responded to in the AS IS scenario. It indicates that the positive impact of the investment in the training of the cyber team maintains for 8 iterations. It suggests that a greater investment in the response capability of the cyber team is required to ensure that the AS IS capability would not decrease over time. The number of compromises responded to is higher in case with 5 resources available than when only 2 members of the cyber team can respond to compromises. This pattern is similar to the one observed in the scenario AS IS and Investment 1.

Resources utilization and queue size is higher in the simulation when 2 resources are available (Figure 8.29) than when 5 resources are available (Figure 8.30). In both simulations, resource utilization increases over the utilisation from the AS IS scenario from the iteration 8. However, the maximum resource utilisation and the maximum of the size of queues 1 and 2, reached in iterations 12 and 14 is much smaller in Investment 2 than in Investment 1 (Tables 8.4 and 8.5).

Table 8.4 Comparison of Investments 1 and 2 when the capacity of the resource is 2

| | **Average size of queues 1 and 2** | **Resource utilisation** |
|---|---|---|
| Investment 1 | 22 | ~99% |
| Investment 2 | 6 | ~94% |

Table 8.5 Comparison of Investments 1 and 2 when the capacity of the resource is 5

| | **Average size of queues 1 and 2** | **Resource utilisation** |
|---|---|---|
| Investment 1 | 12 | ~95% |
| Investment 2 | 2.5 | ~70% |

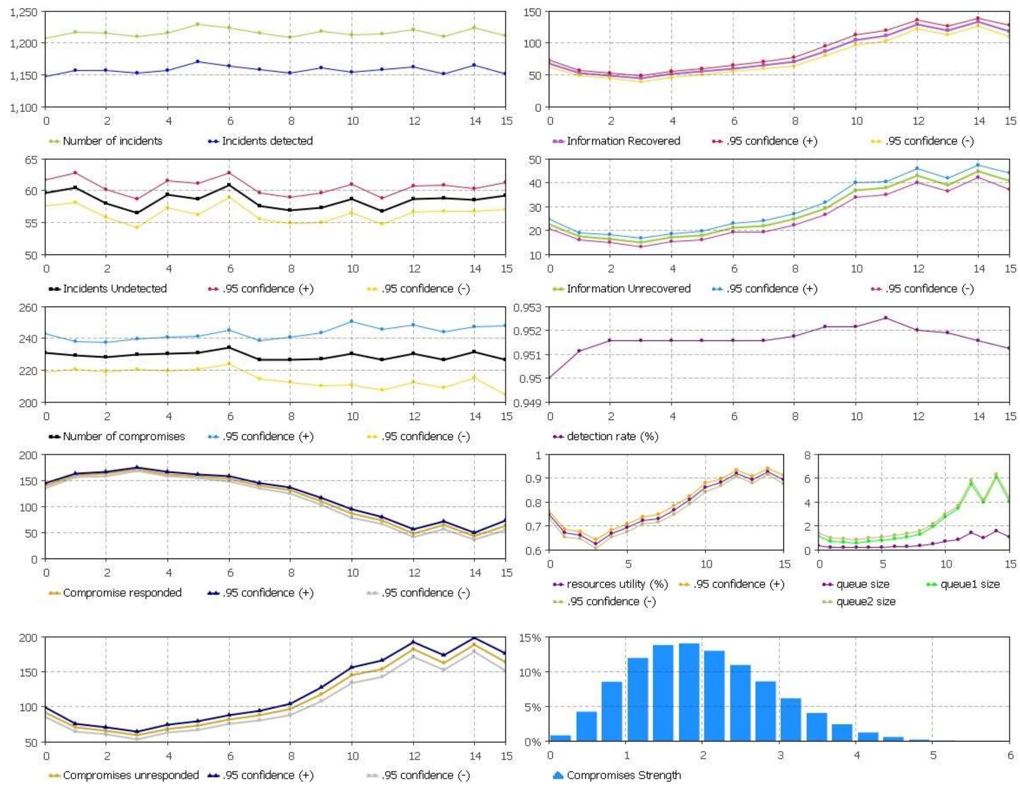



Figure 8.29 DES simulation results where capacity of the Cyber Team is 2 and is under the impact from RBFCM model, triggered by the Investment 2

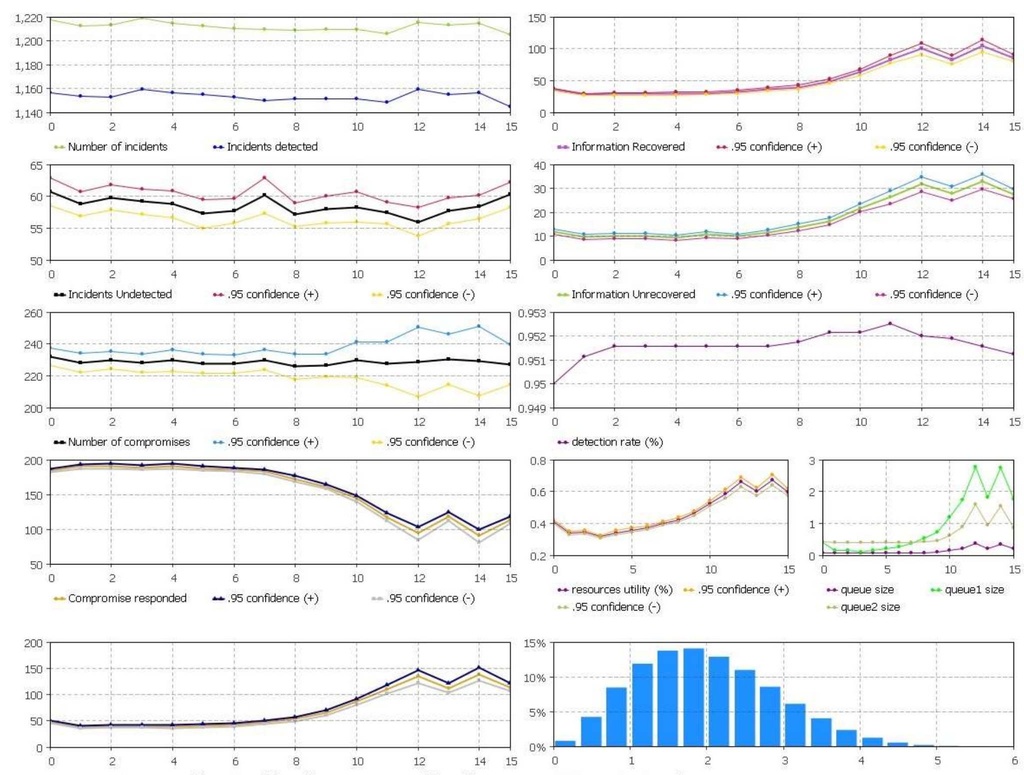



Figure 8.30 DES simulation results where capacity of the Cyber Team is 5 and is under the impact from RBFCM model, triggered by the Investment 2

## 8.6 Conclusions

This chapter introduced a novel DES model for cyber defence. It has been validated by cyber defence expert and can be considered as reasonable approximation of the operation of a small to medium size enterprise. The DES model for cyber defence is concentrated on modelling three aspects of the cyber defence: detection, denial and response to compromises. As a result, the DES model can be considered as a more detailed subsystem of a very abstract RBFCM Cyber Defence model, presented in the section 7.3.

Both models rely on completely different types of knowledge. The RBFCM model requires a knowledge elicited from experts about relationships between abstract, intangible concepts. On the other hand, DES models are based on the operation of real cyber defence teams. In DES the processing and inter arrival times are based on empirically collected, statistical data.

There are many benefits of integrating RBFCM and DES models, that cannot be achieved when running both models individually. One of the limitations of abstract models, such as RBFCM for cyber defence, is intangibility of outputs of simulations. Modelling techniques relying on inputs recognizable by experts tend to be more appealing and convincing to decision makers. Being able to present impact of different investments into RBFCM using KPIs produced by the DES model, significantly improves credibility of abstract RBFCM model.

On the other hand, integration of both models enhances modelling capabilities of DES, making them much more dynamic and able to model changing environment over time.

It allows decision makers to take more strategic decisions regarding their operation by visualizing potential future problems.

Another important benefit of integrating RBFCM and DES models is the possibility of optimisation of the RBFCM model using KPIs produced by DES model. As mentioned before, KPIs produced by DES are more tangible and trustworthy measure of the performance of the modelled system.

Additionally, both systems can exchange information to improve the accuracy of the model. For instance, in the RBFCM model, presented in section 7.3, the node “Number of NEAR CIS compromises detected” (Node 28) presents percentage increase of compromises detected. Similar KPIs are gathered by the DES model for cyber defence. Comparing outputs of both, in the given iteration, could guide modellers towards potential discrepancies between models. Moreover, the process of improving models, based on the difference in outputs, could be automatized by including a learning mechanism.

This chapter presents a direction for future research of integrating RBFCM and DES models. More complex and more detailed integration of both RBFCM and DES models should be explored and evaluated, to confirm initial findings presented in this chapter.

# Chapter 9. CONCLUSIONS

## 9.1 Conclusions

This thesis presents development and application of RBFCMs to model complex and dynamic real-world systems. As demonstrated in chapter 2, Literature review, modelling of dynamic systems faces many challenges: knowledge representation, definition of relationships between concepts, representation of dynamics and accumulation of impacts. RBFCMs provide solutions to these issues but it comes with a high computational cost and complicated semantics of fuzzy sets. So far, RBFCMs were used in a small number of applications and as demonstrated in this thesis, it could be due to its high number of limitations.

The aim of this thesis was the development of RBFCMs to increase their modelling capability and their application in new domains. This aim was achieved by meeting the following four objectives:

**1. Understanding of standard RBFCMs and their limitations**

So far RBFCMs have not been critically analysed in the literature. Hence, it was the first step of the research presented in this thesis. It allowed to understand RBFCMs limitations and areas of potential development. Four types of limitations are identified in this thesis:

- vague semantics of fuzzy sets used,
- incorrect interpolation of linguistic terms under some circumstances,
- very high computational cost of algorithms and
- inability to model complex relationships.

In Chapter 4 it is demonstrated that semantics of fuzzy sets, proposed by Carvalho and Tome is not in line with any of the standard interpretations of fuzzy sets. The use of fuzzy sets in RBFCMs is limited by a number of restrictions, which force modellers to use a set of predefined membership functions, hence reducing flexibility of the method. It is concluded that restrictions on the shape of fuzzy sets in RBFCMs is driven by the design of reasoning mechanisms. When fuzzy sets are defined using semantics proposed in RBFCMs, uncertainty measures do not provide information about uncertainty of experts, as their role in designing fuzzy sets is very limited. As discussed in Chapter 4, semantics introduced in standard RBFCMs requires modellers to use predefined set of membership functions, to ensure correct reasoning on fuzzy IF-THEN rules and accumulation of impacts. This significantly reduces flexibility of the method.

In Chapters 2, 5 and 6 consequences of using fuzzy sets defined under standard semantics are explored. It is demonstrated that standard RBFCMs algorithms produce incorrect outputs, as they are designed to be able to process fuzzy sets designed under before mentioned restrictions. This observation defined further direction of the research on new reasoning and accumulation of impacts mechanisms.

In Chapter 5, two important limitations of the standard reasoning mechanism are presented:

1. incorrect calculations of the shape of the interpolated fuzzy set when two consequent fuzzy sets differ greatly,
2. interpolation of the output of the inference even when the crisp output is already represented on the universe of discourse of the effect node, by the centroid of one of the fuzzy sets.

The first issue identified is especially critical, because the standard accumulation of impacts mechanism relies on the shape of fuzzy sets calculated by the reasoning mechanism. Therefore, incorrect interpolation of fuzzy sets results in incorrect accumulation of impacts.

In Chapter 6, computational complexity of accumulation mechanism is discussed. It is demonstrated that to accumulate two fuzzy sets, the standard mechanism requires performing at least 29 operations compared to only 5 required by the new RBFCMs. The standard RBFCMs mechanism uses a complex recursive algorithm to calculate all the points belonging to VOS, therefore depending on the size of fuzzy sets involved in accumulation the number of operations required can be even higher.

As demonstrated in Chapters 5 and 6, even when fuzzy sets are defined following restrictions defined by Carvalho and Tome, they do not ensure that the RBFCM's mechanisms would work correctly in all situations.

**2. Development of new reasoning methods**

One of the main limitations of RBFCMs is the high number of restrictions on the shape of fuzzy sets and the absence of well-defined semantics of fuzzy sets. The first step of development of new RBFMCs was the introduction of standard semantics to increase flexibility and applicability of the method. Standard semantics allows easy interpretation of fuzzy sets and membership functions defining them. The semantics proposed by Carvalho and Tome (1998) lacks formal justification and clear definition, making RBFCMs complicated to use.

In Chapter 4, benefits of using standard semantics are outlined:

- it gives much more flexibility when representing knowledge,

- allows using measures of uncertainty to analyse definitions provided by experts and
- has very well-known and formally defined interpretations.

The consequence of introducing standard semantics was a need for developing new inference and accumulation mechanisms to handle fuzzy sets defined more flexibly – violating restrictions enforced by Carvalho and Tome. Both mechanisms, proposed in this thesis, use measures of uncertainty and are designed to preserve them. Introduction of the standard semantics and new mechanisms, allows reducing number of constrains of RBFCMs, from 8 to 2, as discussed in Chapter 4.

The reasoning on rules, discussed in Chapter 5, is split into two stages – inference and interpolation. Separation of both mechanisms provides the following benefits:

- ensures that a new fuzzy set is calculated only when there is no fuzzy set representing the output of inference,
- allows introducing cFCR, a new type of relationships, that allow more than one causal node to be considered when calculating impact on an effect node.

In the same time, it addresses the issue of the standard mechanism, identified in Chapter 5:

- interpolates the output fuzzy set correctly, when both consequent fuzzy sets differ greatly in size.

Introduction of cFCRs significantly increases the modelling capabilities of RBFCMs, allowing better representation of complex relationships.

The new reasoning, gives the same results as the standard RBFCMs reasoning, when fuzzy sets are defined following the restrictions proposed by Carvalho and Tome.

The new accumulation of impacts mechanism, introduced in Chapter 6, significantly changes the way impacts are aggregated in RBFCMs. The shape of the accumulated fuzzy set is calculated using arithmetic summation rather than shifting fuzzy set representing smaller variation towards fuzzy set representing greater variation.

This provides the following benefits:

- Uncertainty of a new fuzzy set represents a joint uncertainty of all fuzzy sets involved in accumulation,
- Arithmetic summation is much easier to interpret than shifting of a fuzzy set towards arbitrarily chosen point –used in the standard accumulation of impacts,
- It allows using measures of uncertainty to calculate the output of accumulation and
- It significantly reduced computational complexity.

The position of the new fuzzy set on the universe of discourse, the centroid, is calculated using a measure of fuzziness. Its value is a sum of centroids of accumulated fuzzy sets, weighted with uncertainty levels of these fuzzy sets. It ensures that the impact of every fuzzy set is relative to its uncertainty. New calculation of the centroid and the shape of fuzzy set ensures that accumulation operation is commutative and associative, which is an important property when designing real time systems, where multiple impacts are received in the same time.

Another achievement is the significant reduction of complexity of accumulation of impacts, which contributes to increasing flexibility and applicability of the method. The new accumulation mechanism requires calculating only one output fuzzy set, $VOS$ , reducing by half number of calculations required to accumulate fuzzy sets. In addition, the new algorithm requires only 5 operations, in comparison with the standard accumulation mechanism which requires at least 20 operations – depending on the shape of fuzzy sets involved in the operation it can be much greater. Reduction of complexity of computations, required to calculate the output of simulations, makes new RBFCMs much more suited to model complex and time critical systems. Moreover, less computations would be beneficial when applying learning algorithms to RBFCMs. It is a well-known problem that rule based systems are more difficult to learn due to the great number of calculation steps required.

**3. Improvement of modelling of dynamics**

The third objective of this research was the improvement of representation of dynamics. It was achieved by introducing a mechanism of handling loops. Impacts received by effect nodes are reduced by the ratio of the difference between the current value of the node and its maximum/minimum value. The mechanism allows controlling the build-up of impacts within the loop and make sure that the total value of the node is not going to exceed the maximum value of the concept.

**4. Application of new RBFCMs in new case studies.**

The last objective of this research work was the application of RBFCMs in new areas. So far RBFCMs have been used in various, but limited in number, applications. In this thesis, two new case studies are presented in a completely new area: decision making in the military domain. Both, however, are used to tackle completely different problems and required a different approach to modelling. Therefore, the two developed models differ significantly in the modelling structure and in the subject of the model.

In both case studies, presented in Chapter 7, RBFCMs proved to be a valid modelling approach. Both models allow to understand systems in a holistic way and managed to capture complexities, which other modelling techniques struggled to capture.

The first case study, modelling of the impact of resource allocation to DLoDs in respect to the location of the military base of the JSF force, was done to model the problem where the decision was already taken. Therefore, the results generated by the model were verified against the actual consequences of the decision based on the experts' knowledge. The second case study, analysis of investments into cyber defence of an enterprise, finished with a more detailed analysis of the results. This model was not verified by any other model but by experts working on the project. The results were in line with the expectations of the cyber defence experts.

Both models were successfully developed, thanks to the new reasoning in RBFCM, i.e. inference, interpolation of linguistics terms and accumulation of impacts, described in Chapters 5 and 6. The new RBFCMs provide much more modelling flexibility and more means of capturing complexities of the real-world systems. This proved to be useful in both case studies and made new RBFCMs more appealing to the experts working on both case studies.

The results of the research on both case studies were presented at four conferences and received very positive comments on all of them. Moreover, the cyber defence model was developed further by the Ministry of Defence and tailored to analyse real life cyber security investment scenarios. It proves that RBFCMs can be used in domains as critical as the military defence of a country.

Additionally, an integration of RBFCMs and DES was proposed in this thesis. It is a preliminary study that needs further development and verification. The research conducted so far indicates that the RBFCM-DES hybrid model can be very helpful to analyse long term impact of complex abstract systems on operation of these systems or their subsystems. Results and benefits of combining both methods are demonstrated by combining RBFCM cyber defence system, proposed in Section 7.3 with the novel DES model, described in Section 8.3.

This research work and analysis could not be achieved without the development of a software for handling RBFCMs and the development of the integration module that linked RBFCMs with the DES model. The software for simulating RBFCMs and integration modules were developed using Microsoft .Net platform. The DES model was developed in the simulation software: AnyLogic.

## 9.2 Future work

The future work on RBFCMs can be divided into three categories: 1. Improvement of the knowledge representation and reasoning and accumulation mechanisms, 2. Development of learning and optimisation for RBFCMs and 3. Application of RBFCMs in new domains.

**1. Improvement of the knowledge representation and reasoning and accumulation mechanisms**

This thesis introduces significant changes to the mechanisms responsible for handling inputs into causal nodes and calculating the impact on effect nodes. It touches the core of modelling with fuzzy sets, i.e. semantics. The new mechanisms decreased the number of constraints on the shape of membership functions but it has not been verified whether all the conditions can be eliminated. The future work should be aimed to analyse the impact of removing all restrictions and whether results are in line with expectations.

Removal of restrictions and separation of reasoning into two stages: inference and interpolation, creates potential for using Interval Type 2 Fuzzy Sets (IT2FS), which allow more complex handling of uncertainty. IT2FS are a simplified version of Type 2 Fuzzy Sets and in recent years received a lot of attention thanks to relatively low computational requirements. IT2FS allow modelling knowledge of multiple experts, therefore incorporating them into RBFCMs would help to increase applicability of RBFCMs. As discussed before, new mechanisms proposed in this thesis reduced the number of constraints on definitions of fuzzy sets but it was not investigated whether IT2FS could be used.

The area of RBFCMs that could be improved, to increase modelling capabilities of RBFCMs, is the representation of knowledge. So far, the use of trapezoid or triangular fuzzy sets is advised. Adapting appropriate calculations to allow representing experts' knowledge using Gaussian or Bell shaped fuzzy sets would be an important advancement. It would require changing the way fuzzy sets are interpolated and accumulated – especially saturation.

In this thesis, index of fuzziness is used as a measure of uncertainty, to accumulate impacts represented by fuzzy sets. Depending on the application, other measures of uncertainty could be more appropriate to define degree of accumulation, such as: specificity or entropy. The future work should be focused on exploring the benefit of employing different measures of uncertainty and its impact on respective mechanisms.

**2. Development of learning and optimisation for RBFCMs**

The current trend in the literature on FCMs and rule based fuzzy systems is the development of learning mechanisms that allow creating causal maps based on the historical data. So far, no work has been done to incorporate one of the two main methods for learning of fuzzy systems: neural networks and evolutionary algorithms. Regardless of the method chosen to derive the shape and features of the RBFCMs model, it would need to be adapted to account for novel mechanisms of interpolation and accumulation of impacts. Learning could be focused on the improvement of the following: shape of membership functions, definition of rules between nodes and the value of the parameter $s$, which controls the strength of accumulation.

A significant improvement of analytical capabilities of RBFCMs would be the introduction of the optimisation module, to optimise choice of inputs to maximise or minimise objectives. So far, experts have to manually input different investments and using try and error method to find the most optimal solution. It seems appropriate that a meta heuristic approach would work best, due to the great search space that need to be explored to find the best set of inputs.

Decrease of the complexity of reasoning and accumulation, achieved in this thesis, makes new RBFMCs much more likely to perform fast enough, to make both learning and optimisation computationally feasible.

**3. Application of RBFCMs in new domains**

The third area of future research is the application of RBFCMs in new domains. In this thesis two new case studies of RBFCMs are proposed. The future work in this area should be focused on the application of RBFCMs in domains, where its usefulness has not been verified yet. Due to the improvements of RBFCMs, proposed in this thesis, they could be used in real time systems. It would be interesting to analyse, whether the decrease of complexity and reduction of computational power required to run the model, would allow usage of RBFCMs in real time systems that require fast response to the input.

Integration of RBFCMs and DES, proposed in Chapter 8, requires a more in-depth analysis. More relationships between nodes in RBFCMs and DES should be developed to model the integrated system more completely. Additionally, the feedback links from DES to RBFCMs should be developed to provide information about the state of the operational part of the system. Finally, a learning mechanism could be developed to improve either one or both models by calibrating relationships in RBFCMs or the parameters of modules in DES.

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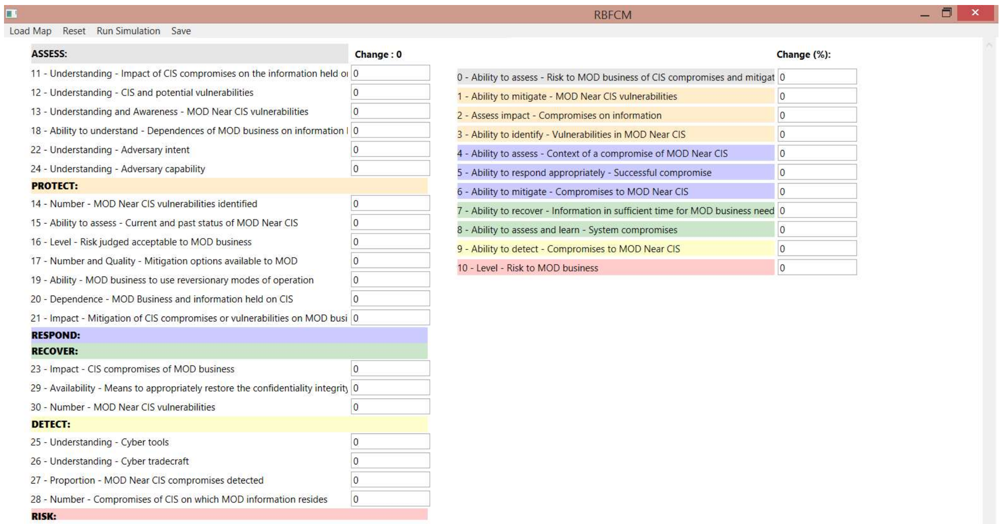
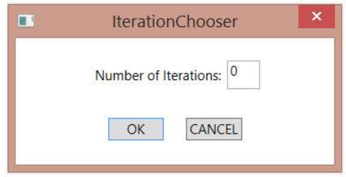
# APPENDIX 1. SOFTWARE INTERFACE



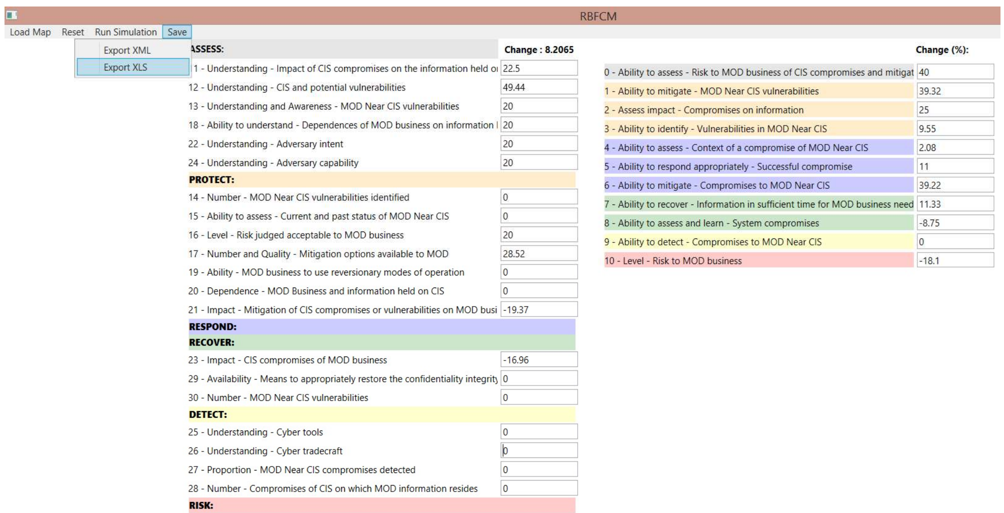

In order to load a map, the user needs to click on the **Load Map** button and choose an .xml file with the causal map. Nodes, the map consist of, are loaded in two columns: in the left column there are **nodes** which are sorted according to a function they belong to, in the right column there is a list of all **function nodes**.

The list of nodes consists of two fields: node id and name and percentage change (initially 0). User can modify percentage change text fields by inputting a value between -100 and 100.

In order to run simulation the user needs to click **Run Simulation** button. A new window will appear asking for number of iterations the simulation is supposed to run for.




After choosing the number of iterations, the simulation starts. The user can see the final result of the simulation (the last iteration). In order to run a new simulation it is necessary to reset the state of the window (**Reset** button)




In order to track how the values of nodes were changing, results can be saved to an Excel file (**Save** -> **Export XLS**) or to an XML file (**Save**->**Export XML**).

| ID | Node\Iteration | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| 0 | Risk to MOD business of CIS compromises and mitigation measures | 0 | 0 | 0 | 22.5 | 40 | 40 |
| 1 | MOD Near CIS vulnerabilities | 0 | 0 | 25 | 40 | 37.37 | 39.32 |
| 2 | Compromises on information | 0 | 0 | 0 | 0 | 25 | 25 |
| 3 | Vulnerabilities in MOD Near CIS | 0 | 0 | 0 | -2.5 | 9.55 | 9.55 |
| 4 | Context of a compromise of MOD Near CIS | 0 | 0 | -8.75 | -8.75 | -5.72 | 2.08 |
| 5 | Successful compromise | 0 | 0 | -7.5 | -2.23 | 5.63 | 11 |
| 6 | Compromises to MOD Near CIS | 0 | 0 | 25 | 40 | 32.5 | 39.22 |
| 7 | Information in sufficient time for MOD business needs | 0 | 0 | -7.5 | -0.66 | 11.33 | 11.33 |
| 8 | System compromises | 0 | 0 | -8.75 | -8.75 | -8.75 | -8.75 |
| 9 | Compromises to MOD Near CIS | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | Risk to MOD business | 0 | 0 | 0 | -18.1 | -18.1 | -18.1 |
| 11 | Impact of CIS compromises on the information held on MOD Near CIS | 20 | 20 | 20 | 20 | 22.5 | 22.5 |
| 12 | CIS and potential vulnerabilities | 20 | 17.5 | 17.5 | 19.44 | 34.44 | 49.44 |
| 13 | MOD Near CIS vulnerabilities | 20 | 20 | 20 | 20 | 20 | 20 |
| 14 | MOD Near CIS vulnerabilities identified | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | Current and past status of MOD Near CIS | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | Risk judged acceptable to MOD business | 20 | 20 | 20 | 20 | 20 | 20 |
| 17 | Mitigation options available to MOD | 0 | 17.5 | 25 | 22.5 | 16.62 | 28.52 |
| 18 | Dependences of MOD business on information held on CIS | 20 | 20 | 20 | 20 | 20 | 20 |
| 19 | MOD business to use reversionary modes of operation | 20 | 17.5 | 17.5 | 10.19 | 0 | 0 |
| 20 | MOD Business and information held on CIS | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | Mitigation of CIS compromises or vulnerabilities on MOD business | 0 | 0 | -12.38 | -12.38 | -12.38 | -19.37 |
| 22 | Adversary intent | 20 | 20 | 20 | 20 | 20 | 20 |
| 23 | CIS compromises of MOD business | 0 | 0 | 0 | 0 | -19.37 | -16.96 |
| 24 | Adversary capability | 20 | 20 | 20 | 20 | 20 | 20 |
| 25 | Cyber tools | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | Cyber tradecraft | 20 | 20 | 0 | 0 | 0 | 0 |

# APPENDIX 2. RELATIONSHIPS DEFINITIONS FOR CASE STUDY 1

**Infrastructure DLoD**

**Operating Surfaces – Infrastructure DLoD**

IF Current Budget allocated to Operating Surfaces is *Decreased Much*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Operating Surfaces is *Decreased*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Operating Surfaces is *Decreased Little*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Operating Surfaces is *Maintained*
AND (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Operating Surfaces is *Maintained*
AND NOT (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Decrease*

IF Current Budget allocated to Operating Surfaces is *Increased Little*
AND (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Increase Little*

IF Current Budget allocated to Operating Surfaces is *Increased Little*
AND NOT (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Operating Surfaces is *Increased*
AND (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will *Increase*

IF Current Budget allocated to Operating Surfaces is *Increased*
AND NOT (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Operating Surfaces is *Increased Much*
AND (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Increase Much*

IF Current Budget allocated to Operating Surfaces is *Increased Much*
AND NOT (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Increase Little*

**Squadron Accommodation – Infrastructure DLoD**

IF Current Budget allocated to Squadron Accommodation is *Decreased Much*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Squadron Accommodation is *Decreased*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Squadron Accommodation is *Decreased Little*
THEN Ability of Infrastructure DLoD will *Decrease*

IF Current Budget allocated to Squadron Accommodation is *Maintained*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Squadron Accommodation is *Increased Little*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Squadron Accommodation is *Increased*
THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Squadron Accommodation is *Increased Much*
THEN Ability of Infrastructure DLoD will *Maintain*

**Personnel Accommodation – Infrastructure DLoD**

IF Current Budget allocated to Personnel Accommodation is *Decreased Much*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Personnel Accommodation is *Decreased*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Personnel Accommodation is *Decreased Little*
THEN Ability of Infrastructure DLoD will *Decrease*

IF Current Budget allocated to Personnel Accommodation is *Maintained*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Personnel Accommodation is *Increased Little*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Personnel Accommodation is *Increased*
THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Personnel Accommodation is *Increased Much*
THEN Ability of Infrastructure DLoD will *Maintain*

**Hangarage and Specialist Support Buildings – Infrastructure DLoD**

IF Current Budget allocated to Hangarage and... is *Decreased Much*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Hangarage and... is *Decreased*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Hangarage and... is *Decreased Little*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Hangarage and... is *Maintained*
AND (Available Equipment is Decreased or Decreased Much)

THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Hangarage and... is *Maintained*
AND NOT (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will *Decrease*

IF Current Budget allocated to Hangarage and... is *Increased Little*
AND (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will *Increase Little*

IF Current Budget allocated to Hangarage and... is *Increased Little*
AND NOT (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will Decrease Little

IF Current Budget allocated to Hangarage and... is *Increased*
AND (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will *Increase*

IF Current Budget allocated to Hangarage and... is *Increased*
AND NOT (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will Maintain

IF Current Budget allocated to Hangarage and... is *Increased Much*
AND (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will *Increase Much*

IF Current Budget allocated to Hangarage and... is *Increased Much*
AND NOT (Available Equipment is Decreased or Decreased Much)
THEN Ability of Infrastructure DLoD will Increase Little

**Buildings – Infrastructure DLoD**

IF Current Budget allocated to Buildings is *Decreased Much*
THEN Ability of Infrastructure DLoD will *Decrease*

IF Current Budget allocated to Buildings is *Decreased*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Buildings is *Decreased Little*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Buildings is *Maintained*
THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Buildings is *Increased Little*
THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Buildings is *Increased*
THEN Ability of Infrastructure DLoD will *Increase Little*

IF Current Budget allocated to Buildings is *Increased Much*
THEN Ability of Infrastructure DLoD will *Increase*

**Specialist Training Areas & Equipment - Infrastructure DLoD**

IF Current Budget allocated to Specialist Tr. Areas & Eq. is *Decreased Much*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Specialist Tr. Areas & Eq.is *Decreased*
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Current Budget allocated to Specialist Tr. Areas & Eq.is *Decreased Little*
THEN Ability of Infrastructure DLoD will *Decrease*

IF Current Budget allocated to Specialist Tr. Areas & Eq.is *Maintained*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Specialist Tr. Areas & Eq.is *Increased Little*
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Current Budget allocated to Specialist Tr. Areas & Eq.is *Increased*
THEN Ability of Infrastructure DLoD will *Maintain*

IF Current Budget allocated to Specialist Tr. Areas & Eq.is *Increased Much*
THEN Ability of Infrastructure DLoD will *Increase Little*

**Overall Live Flying – Infrastructure DLoD**

IF Number of Hours allocated to Overall Live Flying is *Very Low*
THEN Ability of Infrastructure DLoD will *Increase Little*

IF Number of Hours allocated to Overall Live Flying is *Low*
THEN Ability of Infrastructure DLoD will *Maintain*

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Maintain*

IF Number of Hours allocated to Overall Live Flying is *High*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease*

IF Number of Hours allocated to Overall Live Flying is *High*
AND (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease*

IF Number of Hours allocated to Overall Live Flying is *High*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease*

IF Number of Hours allocated to Overall Live Flying is *High*

AND (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Maintain*

IF Number of Hours allocated to Overall Live Flying is *Very High*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Number of Hours allocated to Overall Live Flying is *Very High*
AND (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Number of Hours allocated to Overall Live Flying is *Very High*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Number of Hours allocated to Overall Live Flying is *Very High*
AND (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and... is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Maintain*

**Overall Synthetic Training – Infrastructure DLoD**

IF Number of Hours allocated to Overall Synthetic Training is *Very Low*
THEN Ability of Infrastructure DLoD will *Increase Little*

IF Number of Hours allocated to Overall Synthetic Training is *Low*
THEN Ability of Infrastructure DLoD will *Maintain*

IF Number of Hours allocated to Overall Synthetic Training is *Medium*
AND NOT (Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Number of Hours allocated to Overall Synthetic Training is *Medium*
AND (Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Maintain*

IF Number of Hours allocated to Overall Synthetic Training is *High*
AND NOT (Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease*

IF Number of Hours allocated to Overall Synthetic Training is *High*
AND (Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Maintain*

IF Number of Hours allocated to Overall Synthetic Training is *Very High*
AND NOT (Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Much*

IF Number of Hours allocated to Overall Synthetic Training is *Very High*
AND (Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Maintain*

**Logistics DLoD**

**Airfield Support – Logistics DLoD**

IF Current Budget allocated to Airfield Support is *Decreased Much*

THEN Ability of Logistics DLoD will *Decrease Much*

IF Current Budget allocated to Airfield Support is *Decreased*
THEN Ability of Logistics DLoD will *Decrease*

IF Current Budget allocated to Airfield Support is *Decreased Little*
THEN Ability of Logistics DLoD will *Decrease Little*

IF Current Budget allocated to Airfield Support is *Maintained*
THEN Ability of Logistics DLoD will *Maintain*

IF Current Budget allocated to Airfield Support is *Increased Little*
THEN Ability of Logistics DLoD will *Increase Little*

IF Current Budget allocated to Airfield Support is *Increased*
THEN Ability of Logistics DLoD will *Increase*

IF Current Budget allocated to Airfield Support is *Increased Much*
THEN Ability of Logistics DLoD will *Increase Much*

**Logistics Equipment – Logistics DLoD**

IF Current Budget allocated to Logistics Equipment is *Decreased Much*
THEN Ability of Logistics DLoD will *Decrease Much*

IF Current Budget allocated to Logistics Equipment is *Decreased*
THEN Ability of Logistics DLoD will *Decrease Much*

IF Current Budget allocated to Logistics Equipment is *Decreased Little*
THEN Ability of Logistics DLoD will *Decrease*

IF Current Budget allocated to Logistics Equipment is *Maintained*
THEN Ability of Logistics DLoD will *Decrease Little*

IF Current Budget allocated to Logistics Equipment is *Increased Little*
THEN Ability of Logistics DLoD will *Maintain*

IF Current Budget allocated to Logistics Equipment is *Increased*
THEN Ability of Logistics DLoD will *Maintain*

IF Current Budget allocated to Logistics Equipment is *Increased Much*
THEN Ability of Logistics DLoD will *Increase Little*

**Equipment DLoD**

**Support Equipment – Equipment DLoD**

IF Current Budget allocated to Support Equipment is *Decreased Much*
THEN Ability of Equipment DLoD will *Decrease*

IF Current Budget allocated to Support Equipment is *Decreased*
THEN Ability of Equipment DLoD will *Decrease Little*

IF Current Budget allocated to Support Equipment is *Decreased Little*
THEN Ability of Equipment DLoD will *Maintain*

IF Current Budget allocated to Support Equipment is *Maintained*
THEN Ability of Equipment DLoD will *Maintain*

IF Current Budget allocated to Support Equipment is *Increased Little*
THEN Ability of Equipment DLoD will *Maintain*

IF Current Budget allocated to Support Equipment is *Increased*
THEN Ability of Equipment DLoD will *Increase Little*

IF Current Budget allocated to Support Equipment is *Increased Much*
THEN Ability of Equipment DLoD will *Increase*

**Available Equipment – Equipment DLoD**

IF Number of available Airframes is *Decreased Much*
THEN Ability of Equipment DLoD will *Decrease*

IF Number of available Airframes is *Decreased*
THEN Ability of Equipment DLoD will *Decrease*

IF Number of available Airframes is *Decreased Little*
THEN Ability of Equipment DLoD will *Decrease Little*

IF Number of available Airframes is *Maintained*
THEN Ability of Equipment DLoD will *Maintiain*

IF Number of available Airframes is *Increased Little*
THEN Ability of Equipment DLoD will *Increase Little*

IF Number of available Airframes is *Increased*
AND NOT (Support Equipment is Increased or Increased Much)
THEN Ability of Equipment DLoD will *Maintain*

IF Number of available Airframes is *Increased*
AND (Support Equipment is Increased or Increased Much)
THEN Ability of Equipment DLoD will *Increase*

IF Number of available Airframes is *Increased Much*
AND NOT (Support Equipment is Increased or Increased Much)
THEN Ability of Equipment DLoD will *Maintain*

IF Number of available Airframes is *Increased Much*
AND (Support Equipment is Increased or Increased Much)
THEN Ability of Equipment DLoD will *Increase Much*

**Advancement of Equipment – Equipment DLoD**

IF Current Budget allocated to Advancement of Equipment is *Decreased Much*
THEN Ability of Equipment DLoD will *Decrease Much*

IF Current Budget allocated to Advancement of Equipment is *Decreased*
THEN Ability of Equipment DLoD will *Decrease*

IF Current Budget allocated to Advancement of Equipment is *Decreased Little*
THEN Ability of Equipment DLoD will *Decrease Little*

IF Current Budget allocated to Advancement of Equipment is *Maintained*
THEN Ability of Equipment DLoD will *Maintain*

IF Current Budget allocated to Advancement of Equipment is *Increased Little*
THEN Ability of Equipment DLoD will *Increase Little*

IF Current Budget allocated to Advancement of Equipment is *Increased*
THEN Ability of Equipment DLoD will *Increase*

IF Current Budget allocated to Advancement of Equipment is *Increased Much*
THEN Ability of Equipment DLoD will *Increase Much*

**Equipment Establishment – Available Equipment**

IF Current Budget allocated to Equipment Establishment is *Decreased Much*
THEN Number of Available Airframes will *Decrease*

IF Current Budget allocated to Equipment Establishment is *Decreased*
THEN Number of Available Airframes will *Decrease*

IF Current Budget allocated to Equipment Establishment is *Decreased Little*
THEN Number of Available Airframes will *Decrease Little*

IF Current Budget allocated to Equipment Establishment is *Maintained*
THEN Number of Available Airframes will *Maintain*

IF Current Budget allocated to Equipment Establishment is *Increased Little*
THEN Number of Available Airframes will *Increase Little*

IF Current Budget allocated to Equipment Establishment is *Increased*
THEN Number of Available Airframes will *Increase Little*

IF Current Budget allocated to Equipment Establishment is *Increased Much*
THEN Number of Available Airframes will *Increase*

**Equipment Repair – Available Equipment**

IF Current Budget allocated to Equipment Repair is *Decreased Much*
THEN Number of Available Airframes will *Decrease*

IF Current Budget allocated to Equipment Repair is *Decreased*
THEN Number of Available Airframes will *Decrease*

IF Current Budget allocated to Equipment Repair is *Decreased Little*
THEN Number of Available Airframes will *Decrease Little*

IF Current Budget allocated to Equipment Repair is *Maintained*
THEN Number of Available Airframes will *Maintain*

IF Current Budget allocated to Equipment Repair is *Increased Little*
THEN Number of Available Airframes will *Increase Little*

IF Current Budget allocated to Equipment Repair is *Increased*
THEN Number of Available Airframes will *Increase Little*

IF Current Budget allocated to Equipment Repair is *Increased Much*
THEN Number of Available Airframes will *Increase*

**Personnel DLoD**

**Physiological Support – Personnel DLoD**

IF Current Budget allocated to Physiological Support is *Decreased Much*
THEN Ability of Personnel DLoD will *Decrease*

IF Current Budget allocated to Physiological Support is *Decreased*
THEN Ability of Personnel DLoD will *Decrease Little*

IF Current Budget allocated to Physiological Support is *Decreased Little*
THEN Ability of Personnel DLoD will *Maintain*

IF Current Budget allocated to Physiological Support is *Maintained*
THEN Ability of Personnel DLoD will *Maintain*

IF Current Budget allocated to Physiological Support is *Increased Little*
THEN Ability of Personnel DLoD will *Maintain*

IF Current Budget allocated to Physiological Support is *Increased*
THEN Ability of Personnel DLoD will *Increase Little*

IF Current Budget allocated to Physiological Support is *Increased Much*
THEN Ability of Personnel DLoD will *Increase*

**Skill profile– Personnel DLoD**

IF Current Budget allocated to Skill profile is *Decreased Much*
THEN Ability of Personnel DLoD will *Decrease*

IF Current Budget allocated to Skill profile is *Decreased*
THEN Ability of Personnel DLoD will *Decrease*

IF Current Budget allocated to Skill profile is *Decreased Little*
THEN Ability of Personnel DLoD will *Decrease Little*

IF Current Budget allocated to Skill profile is *Maintained*
THEN Ability of Personnel DLoD will *Decrease Little*

IF Current Budget allocated to Skill profile is *Increased Little*
THEN Ability of Personnel DLoD will *Maintain*

IF Current Budget allocated to Skill profile is *Increased*
THEN Ability of Personnel DLoD will *Increase Little*

IF Current Budget allocated to Skill profile is *Increased Much*
THEN Ability of Personnel DLoD will *Increase*

**Number of People – Personnel DLoD**

IF Current Budget allocated to Number of People is *Decreased Much*
THEN Ability of Personnel DLoD will *Decrease Much*

IF Current Budget allocated to Number of People is *Decreased*
THEN Ability of Personnel DLoD will *Decrease*

IF Current Budget allocated to Number of People is *Decreased Little*
THEN Ability of Personnel DLoD will *Decrease Little*

IF Current Budget allocated to Number of People is *Maintained*
THEN Ability of Personnel DLoD will *Maintain*

IF Current Budget allocated to Number of People is *Increased Little*
THEN Ability of Personnel DLoD will *Increase Little*

IF Current Budget allocated to Number of People is *Increased*
THEN Ability of Personnel DLoD will *Increase*

IF Current Budget allocated to Number of People is *Increased Much*
THEN Ability of Personnel DLoD will *Increase*

**Family Support – Personnel DLoD**

IF Current Budget allocated to Family Support is *Decreased Much*
THEN Ability of Personnel DLoD will *Decrease*

IF Current Budget allocated to Family Support is *Decreased*
THEN Ability of Personnel DLoD will *Decrease Little*

IF Current Budget allocated to Family Support is *Decreased Little*
THEN Ability of Personnel DLoD will *Decrease Little*

IF Current Budget allocated to Family Support is *Maintained*
THEN Ability of Personnel DLoD will *Maintain*

IF Current Budget allocated to Family Support is *Increased Little*
THEN Ability of Personnel DLoD will *Increase Little*

IF Current Budget allocated to Family Support is *Increased*
THEN Ability of Personnel DLoD will *Increase Little*

IF Current Budget allocated to Family Support is *Increased Much*
THEN Ability of Personnel DLoD will *Increase*

**Training DLoD**

**Training Equipment – Training DLoD**

IF Current Budget allocated to Training Equipment is *Decreased Much*
THEN Ability of Training DLoD will *Decrease Much*

IF Current Budget allocated to Training Equipment is *Decreased*
THEN Ability of Training DLoD will *Decrease*

IF Current Budget allocated to Training Equipment is *Decreased Little*
THEN Ability of Training DLoD will *Decrease*

IF Current Budget allocated to Training Equipment is *Maintained*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Training Equipment is *Increased Little*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Training Equipment is *Increased*
THEN Ability of Training DLoD will *Increase Little*

IF Current Budget allocated to Training Equipment is *Increased Much*
THEN Ability of Training DLoD will *Increase Little*

**Other Flying Platform Support – Training DLoD**

IF Current Budget allocated to Other Flying Platform Support is *Decreased Much*
THEN Ability of Training DLoD will *Decrease*

IF Current Budget allocated to Other Flying Platform Support is *Decreased*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Other Flying Platform Support is *Decreased Little*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Other Flying Platform Support is *Maintained*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Other Flying Platform Support is *Increased Little*
THEN Ability of Training DLoD will *Increase Little*

IF Current Budget allocated to Other Flying Platform Support is *Increased*
THEN Ability of Training DLoD will *Increase Little*

IF Current Budget allocated to Other Flying Platform Support is *Increased Much*
THEN Ability of Training DLoD will *Increase*

**Synthetic Training Equipment – Training DLoD**

IF Current Budget allocated to Synthetic Training Equipment is *Decreased Much*
THEN Ability of Training DLoD will *Decrease Much*

IF Current Budget allocated to Synthetic Training Equipment is *Decreased*
THEN Ability of Training DLoD will *Decrease*

IF Current Budget allocated to Synthetic Training Equipment is *Decreased Little*
THEN Ability of Training DLoD will *Decrease*

IF Current Budget allocated to Synthetic Training Equipment is *Maintained*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Synthetic Training Equipment is *Increased Little*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Synthetic Training Equipment is *Increased*
THEN Ability of Training DLoD will *Increase Little*

IF Current Budget allocated to Synthetic Training Equipment is *Increased Much*
THEN Ability of Training DLoD will *Increase Little*

**Training Personnel – Training DLoD**

IF Current Budget allocated to Training Personnel is *Decreased Much*
THEN Ability of Training DLoD will *Decrease Much*

IF Current Budget allocated to Training Personnel is *Decreased*
THEN Ability of Training DLoD will *Decrease*

IF Current Budget allocated to Training Personnel is *Decreased Little*
THEN Ability of Training DLoD will *DecreaseLittle*

IF Current Budget allocated to Training Personnel is *Maintained*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Training Personnel is *Increased Little*
THEN Ability of Training DLoD will *Increase Little*

IF Current Budget allocated to Training Personnel is *Increased*
THEN Ability of Training DLoD will *Increase*

IF Current Budget allocated to Training Personnel is *Increased Much*
THEN Ability of Training DLoD will *Increase Much*

**Advancement of Equipment – Training DLoD**

IF Current Budget allocated to Advancement of Equipment is *Decreased Much*
THEN Training DLoD will *Maintain*

IF Current Budget allocated to Advancement of Equipment is *Decreased*
THEN Training DLoD will *Maintain*

IF Current Budget allocated to Advancement of Equipment is *Maintained*
THEN Training DLoD will *Maintain*

IF Current Budget allocated to Advancement of Equipment is *Increased Little*
THEN Training DLoD will *Maintain*

IF Current Budget allocated to Advancement of Equipment is *Increased*
AND (Training Equipment is *Increased* OR is *Increased Much*)
THEN Training DLoD will *Maintain*

IF Current Budget allocated to Advancement of Equipment is *Increased*
AND NOT (Training Equipment is *Increased* OR is *Increased Much*)
THEN Training DLoD will *Decrease Little*

IF Current Budget allocated to Advancement of Equipment is *Increased Much*
AND (Training Equipment is *Increased* OR is *Increased Much*)
THEN Training DLoD will *Maintain*

IF Current Budget allocated to Advancement of Equipment is *Increased Much*
AND NOT (Training Equipment is *Increased* OR is *Increased Much*)
THEN Training DLoD will *Decrease*

**Overall Synthetic Training – Training DLoD**

IF Number of Hours allocated to Overall Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Maintain*

IF Number of Hours allocated to Overall Synthetic Training is *Low*
THEN Ability of Training DLoD will *Maintain*
IF Number of Hours allocated to Overall Synthetic Training is *Medium*
AND NOT (Synthetic Training Equipment is Increased or Increased Much)
THEN Ability of Training DLoD will *Decrease*

IF Number of Hours allocated to Overall Synthetic Training is *Medium*
AND (Synthetic Training Equipment is Increased or Increased Much)
THEN Ability of Training DLoD will *Maintain*

IF Number of Hours allocated to Overall Synthetic Training is *High*
AND NOT (Synthetic Training Equipment is Increased or Increased Much)
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours allocated to Overall Synthetic Training is *High*
AND (Synthetic Training Equipment is Increased or Increased Much)
THEN Ability of Training DLoD will *Maintain*

IF Number of Hours allocated to Overall Synthetic Training is *Very High*
AND NOT (Synthetic Training Equipment is Increased or Increased Much)
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours allocated to Overall Synthetic Training is *Very High*
AND (Synthetic Training Equipment is Increased or Increased Much)
THEN Ability of Training DLoD will *Maintain*

**Pre-Training Education for Individual Training – Training DLoD**

IF Current Budget allocated to Pre-Training Education is *Decreased Much*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Pre-Training Education is *Decreased*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Pre-Training Education is *Decreased Little*
THEN Ability of Training DLoD will *Maintain*
IF Current Budget allocated to Pre-Training Education is *Maintained*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Pre-Training Education is *Increased Little*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Pre-Training Education is *Increased*
THEN Ability of Training DLoD will *Increase Little*

IF Current Budget allocated to Pre-Training Education is *Increased Much*
THEN Ability of Training DLoD will *Increase Little*

**Pre-Training Education – Training DLoD**

IF Current Budget allocated to Pre-Training Education is *Decreased Much*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Pre-Training Education is *Decreased*
THEN Ability of Training DLoD will *Decrease Little*

IF Current Budget allocated to Pre-Training Education is *Decreased Little*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Pre-Training Education is *Maintained*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Pre-Training Education is *Increased Little*
THEN Ability of Training DLoD will *Maintain*

IF Current Budget allocated to Pre-Training Education is *Increased*
THEN Ability of Training DLoD will *Increase Little*

IF Current Budget allocated to Pre-Training Education is *Increased Much*
THEN Ability of Training DLoD will *Increase Little*

**Squadron (OCU Students/OCU Staff) Live Flying and Synthetic Training for Individual/Team/Joint/Collective Training – Training DLoD**

IF Number of Hours for Live Flying is ***Very Low*** and
Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *Medium*
THEN Ability of Training DLoD will *Decrease*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *High*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *Very High*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is ***Low*** and
Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Low* and
Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Low* and
Number of Hours for Synthetic Training is *Medium*
THEN Ability of Training DLoD will *Decrease*

IF Number of Hours for Live Flying is *Low* and
Number of Hours for Synthetic Training is *High*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is *Low* and
Number of Hours for Synthetic Training is *Very High*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is ***Medium*** and
Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Decrease*

IF Number of Hours for Live Flying is *Medium* and
Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is *Medium* and
Number of Hours for Synthetic Training is Medium
THEN Ability of Training DLoD will *Maintain*

IF Number of Hours for Live Flying is *Medium* and
Number of Hours for Synthetic Training is *High*
THEN Ability of Training DLoD will *Increase Little*

IF Number of Hours for Live Flying is *Medium* and
Number of Hours for Synthetic Training is *Very High*
THEN Ability of Training DLoD will *Increase*

IF Number of Hours for Live Flying is ***High*** and
Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is *High* and
Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Maintain*

IF Number of Hours for Live Flying is *High* and
Number of Hours for Synthetic Training is *Medium*
THEN Ability of Training DLoD will *Increase Little*

IF Number of Hours for Live Flying is *High* and
Number of Hours for Synthetic Training is *High*
THEN Ability of Training DLoD will *Increase*

IF Number of Hours for Live Flying is *High* and
Number of Hours for Synthetic Training is *Very High*
THEN Ability of Training DLoD will *Increase Much*

IF Number of Hours for Live Flying is ***Very High*** and
Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Maintain*

IF Number of Hours for Live Flying is *Very High* and
Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Maintain*

IF Number of Hours for Live Flying is *Very High* and

Number of Hours for Synthetic Training is *Medium*
THEN Ability of Training DLoD will *Increase Little*

IF Number of Hours for Live Flying is *Very High* and
Number of Hours for Synthetic Training is *High*
THEN Ability of Training DLoD will *Increase*

IF Number of Hours for Live Flying is *Very High* and
Number of Hours for Synthetic Training is *Very High*
THEN Ability of Training DLoD will *Increase Much*

# APPENDIX 3. RELATIONSHIPS DEFINITIONS FOR CASE STUDY 2

**ASSESS**
**(0, Risk to MOD business of CIS compromises and mitigation measures -> (1, MOD Near CIS vulnerabilities) Time periods: 1**

IF Risk to MOD business of CIS compromises and mitigation measures is Decreased Much
THEN MOD Near CIS vulnerabilities is Decreased

IF Risk to MOD business of CIS compromises and mitigation measures is Decreased
THEN MOD Near CIS vulnerabilities is Decreased

IF Risk to MOD business of CIS compromises and mitigation measures is Decreased Little
THEN MOD Near CIS vulnerabilities is Decreased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Maintained
THEN MOD Near CIS vulnerabilities is Maintained

IF Risk to MOD business of CIS compromises and mitigation measures is Increased Little
THEN MOD Near CIS vulnerabilities is Increased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Increased
THEN MOD Near CIS vulnerabilities is Increased

IF Risk to MOD business of CIS compromises and mitigation measures is Increased Much
THEN MOD Near CIS vulnerabilities is Increased

**(0, Risk to MOD business of CIS compromises and mitigation measures -> (4, Context of a compromise of MOD Near CIS) Time periods: 1**
IF Risk to MOD business of CIS compromises and mitigation measures is Decreased Much
THEN Context of a compromise of MOD Near CIS is Decreased

IF Risk to MOD business of CIS compromises and mitigation measures is Decreased
THEN Context of a compromise of MOD Near CIS is Decreased

IF Risk to MOD business of CIS compromises and mitigation measures is Decreased Little

THEN Context of a compromise of MOD Near CIS is Decreased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Maintained
THEN Context of a compromise of MOD Near CIS is Decreased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Increased Little
THEN Context of a compromise of MOD Near CIS is Maintained

IF Risk to MOD business of CIS compromises and mitigation measures is Increased
THEN Context of a compromise of MOD Near CIS is Increased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Increased Much
THEN Context of a compromise of MOD Near CIS is Increased
**(0, Risk to MOD business of CIS compromises and mitigation measures -> (6, Compromises to MOD Near CIS) Time periods: 1**
IF Risk to MOD business of CIS compromises and mitigation measures is Decreased Much
THEN Compromises to MOD Near CIS is Decreased

IF Risk to MOD business of CIS compromises and mitigation measures is Decreased
THEN Compromises to MOD Near CIS is Decreased

IF Risk to MOD business of CIS compromises and mitigation measures is Decreased Little
THEN Compromises to MOD Near CIS is Decreased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Maintained
THEN Compromises to MOD Near CIS is Maintained

IF Risk to MOD business of CIS compromises and mitigation measures is Increased Little
THEN Compromises to MOD Near CIS is Increased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Increased
THEN Compromises to MOD Near CIS is Increased Little

IF Risk to MOD business of CIS compromises and mitigation measures is Increased Much
THEN Compromises to MOD Near CIS is Increased
**(11, Impact of CIS compromises on the information held on MOD Near CIS -> (0, Risk to MOD business of CIS compromises and mitigation measures) Time periods: 3**
IF Impact of CIS compromises on the information held on MOD Near CIS is Decreased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased

IF Impact of CIS compromises on the information held on MOD Near CIS is Decreased
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased

IF Impact of CIS compromises on the information held on MOD Near CIS is Decreased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased Little

IF Impact of CIS compromises on the information held on MOD Near CIS is Maintained
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Impact of CIS compromises on the information held on MOD Near CIS is Increased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased Little

IF Impact of CIS compromises on the information held on MOD Near CIS is Increased
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased Little

IF Impact of CIS compromises on the information held on MOD Near CIS is Increased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased

**(12, CIS and potential vulnerabilities -> (2, Compromises on information)**
**Time periods: 3**

IF CIS and potential vulnerabilities is Decreased Much
THEN Compromises on information is Decreased

IF CIS and potential vulnerabilities is Decreased
THEN Compromises on information is Decreased

IF CIS and potential vulnerabilities is Decreased Little
THEN Compromises on information is Decreased Little

IF CIS and potential vulnerabilities is Maintained
THEN Compromises on information is Maintained

IF CIS and potential vulnerabilities is Increased Little
THEN Compromises on information is Increased Little

IF CIS and potential vulnerabilities is Increased
THEN Compromises on information is Increased Little

IF CIS and potential vulnerabilities is Increased Much
THEN Compromises on information is Increased
**(12, CIS and potential vulnerabilities -> (3, Vulnerabilities in MOD Near CIS)**
**Time periods: 3**
IF CIS and potential vulnerabilities is Decreased Much
THEN Vulnerabilities in MOD Near CIS is Decreased

IF CIS and potential vulnerabilities is Decreased
THEN Vulnerabilities in MOD Near CIS is Decreased

IF CIS and potential vulnerabilities is Decreased Little
THEN Vulnerabilities in MOD Near CIS is Decreased Little

IF CIS and potential vulnerabilities is Maintained

THEN Vulnerabilities in MOD Near CIS is Decreased Little

IF CIS and potential vulnerabilities is Increased Little
THEN Vulnerabilities in MOD Near CIS is Increased Little

IF CIS and potential vulnerabilities is Increased
THEN Vulnerabilities in MOD Near CIS is Increased

IF CIS and potential vulnerabilities is Increased Much
THEN Vulnerabilities in MOD Near CIS is Increased

**(13, MOD Near CIS vulnerabilities -> (12, CIS and potential vulnerabilities) Time periods: 3**

IF MOD Near CIS vulnerabilities is Decreased Much
THEN CIS and potential vulnerabilities is Decreased Much

IF MOD Near CIS vulnerabilities is Decreased
THEN CIS and potential vulnerabilities is Decreased
IF MOD Near CIS vulnerabilities is Decreased Little
THEN CIS and potential vulnerabilities is Decreased Little

IF MOD Near CIS vulnerabilities is Maintained
THEN CIS and potential vulnerabilities is Maintained

IF MOD Near CIS vulnerabilities is Increased Little
THEN CIS and potential vulnerabilities is Increased Little

IF MOD Near CIS vulnerabilities is Increased
THEN CIS and potential vulnerabilities is Increased

IF MOD Near CIS vulnerabilities is Increased Much
THEN CIS and potential vulnerabilities is Increased Much

**(13, MOD Near CIS vulnerabilities -> (0, Risk to MOD business of CIS compromises and mitigation measures) Time periods: 3**

IF MOD Near CIS vulnerabilities is Decreased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased

IF MOD Near CIS vulnerabilities is Decreased
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased Little

IF MOD Near CIS vulnerabilities is Decreased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased Little

IF MOD Near CIS vulnerabilities is Maintained
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF MOD Near CIS vulnerabilities is Increased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased Little

IF MOD Near CIS vulnerabilities is Increased
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased Little

IF MOD Near CIS vulnerabilities is Increased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased

**(13, MOD Near CIS vulnerabilities -> (9, Compromises to MOD Near CIS) Time periods: 1**

IF MOD Near CIS vulnerabilities is Decreased Much
THEN Compromises to MOD Near CIS is Decreased

IF MOD Near CIS vulnerabilities is Decreased
THEN Compromises to MOD Near CIS is Decreased Little

IF MOD Near CIS vulnerabilities is Decreased Little
THEN Compromises to MOD Near CIS is Decreased Little

IF MOD Near CIS vulnerabilities is Maintained
THEN Compromises to MOD Near CIS is Maintained

IF MOD Near CIS vulnerabilities is Increased Little
THEN Compromises to MOD Near CIS is Increased Little

IF MOD Near CIS vulnerabilities is Increased
THEN Compromises to MOD Near CIS is Increased Little

IF MOD Near CIS vulnerabilities is Increased Much
THEN Compromises to MOD Near CIS is Increased

**(18, Dependences of MOD business on information held on CIS -> (17, Mitigation options available to MOD) Time periods: 3**

IF Dependences of MOD business on information held on CIS is Decreased Much
THEN Mitigation options available to MOD is Decreased

IF Dependences of MOD business on information held on CIS is Decreased
THEN Mitigation options available to MOD is Decreased Little

IF Dependences of MOD business on information held on CIS is Decreased Little
THEN Mitigation options available to MOD is Decreased Little

IF Dependences of MOD business on information held on CIS is Maintained
THEN Mitigation options available to MOD is Maintained

IF Dependences of MOD business on information held on CIS is Increased Little
THEN Mitigation options available to MOD is Increased Little

IF Dependences of MOD business on information held on CIS is Increased
THEN Mitigation options available to MOD is Increased Little
IF Dependences of MOD business on information held on CIS is Increased Much
THEN Mitigation options available to MOD is Increased

**(18, Dependences of MOD business on information held on CIS -> (0, Risk to MOD business of CIS compromises and mitigation measures) Time periods: 3**

IF Dependences of MOD business on information held on CIS is Decreased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased

IF Dependences of MOD business on information held on CIS is Decreased
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased Little

IF Dependences of MOD business on information held on CIS is Decreased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Dependences of MOD business on information held on CIS is Maintained
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Dependences of MOD business on information held on CIS is Increased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Dependences of MOD business on information held on CIS is Increased
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased Little

IF Dependences of MOD business on information held on CIS is Increased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased

**(18, Dependences of MOD business on information held on CIS -> (7, Information in sufficient time for MOD business needs) Time periods: 3**

IF Dependences of MOD business on information held on CIS is Decreased Much
THEN Information in sufficient time for MOD business needs is Decreased

IF Dependences of MOD business on information held on CIS is Decreased
THEN Information in sufficient time for MOD business needs is Decreased

IF Dependences of MOD business on information held on CIS is Decreased Little
THEN Information in sufficient time for MOD business needs is Decreased Little

IF Dependences of MOD business on information held on CIS is Maintained
THEN Information in sufficient time for MOD business needs is Maintained

IF Dependences of MOD business on information held on CIS is Increased Little
THEN Information in sufficient time for MOD business needs is Increased Little
IF Dependences of MOD business on information held on CIS is Increased
THEN Information in sufficient time for MOD business needs is Increased

IF Dependences of MOD business on information held on CIS is Increased Much
THEN Information in sufficient time for MOD business needs is Increased

**(18, Dependences of MOD business on information held on CIS -> (19, MOD business to use reversionary modes of operation) Time periods: 3**

IF Dependences of MOD business on information held on CIS is Decreased Much
THEN MOD business to use reversionary modes of operation is Increased

IF Dependences of MOD business on information held on CIS is Decreased
THEN MOD business to use reversionary modes of operation is Increased Little

IF Dependences of MOD business on information held on CIS is Decreased Little
THEN MOD business to use reversionary modes of operation is Increased Little

IF Dependences of MOD business on information held on CIS is Maintained

THEN MOD business to use reversionary modes of operation is Maintained

IF Dependences of MOD business on information held on CIS is Increased Little
THEN MOD business to use reversionary modes of operation is Decreased Little

IF Dependences of MOD business on information held on CIS is Increased
THEN MOD business to use reversionary modes of operation is Decreased Little

IF Dependences of MOD business on information held on CIS is Increased Much
THEN MOD business to use reversionary modes of operation is Decreased

**(22, Adversary intent -> (0, Risk to MOD business of CIS compromises and mitigation measures) Time periods: 3**

IF Adversary intent is Decreased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased

IF Adversary intent is Decreased
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased Little

IF Adversary intent is Decreased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Adversary intent is Maintained
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Adversary intent is Increased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Adversary intent is Increased
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased Little

IF Adversary intent is Increased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased

**(22, Adversary intent -> (5, Successful compromise) Time periods: 3**

IF Adversary intent is Decreased Much AND Adversary Capability is Decreased Much
THEN Successful compromise is Decreased Much

IF Adversary intent is Decreased Much AND Adversary Capability is (Decreased OR Decreased Little OR Maintained OR Increased Little)
THEN Successful compromise is Decreased

IF Adversary intent is Decreased Much AND Adversary Capability is Increased
THEN Successful compromise is Decreased Little

IF Adversary intent is Decreased Much AND Adversary Capability is Increased Much
THEN Successful compromise is Maintained

IF Adversary intent is Decreased AND Adversary Capability is (Decreased Much OR Decreased OR Decreased Little OR Maintained)
THEN Successful compromise is Decreased

IF Adversary intent is Decreased AND Adversary Capability is (Increased Little OR Increased)
THEN Successful compromise is Decreased Little

IF Adversary intent is Decreased AND Adversary Capability is Increased Much
THEN Successful compromise is Maintained

IF Adversary intent is Decreased Little AND Adversary Capability is (Decreased Much OR Decreased)
THEN Successful compromise is Decreased

IF Adversary intent is Decreased Little AND Adversary Capability is (Decreased Little OR Maintained)
THEN Successful compromise is Decreased Little

IF Adversary intent is Decreased Little AND Adversary Capability is (Increased Little OR Increased)
THEN Successful compromise is Maintained

IF Adversary intent is Decreased Little AND Adversary Capability is Increased Much
THEN Successful compromise is Increased Little

IF Adversary intent is Maintained AND Adversary Capability is (Decreased Much OR Decreased)
THEN Successful compromise is Decreased

IF Adversary intent is Maintained AND Adversary Capability is (Decreased Little OR Maintained)
THEN Successful compromise is Decreased Little
IF Adversary intent is Maintained AND Adversary Capability is (Increased Little OR Increased)
THEN Successful compromise is Maintained

IF Adversary intent is Maintained AND Adversary Capability is Increased Much
THEN Successful compromise is Increased Little

IF Adversary intent is Increased Little AND Adversary Capability is Decreased Much
THEN Successful compromise is Decreased

IF Adversary intent is Increased Little AND Adversary Capability is Decreased
THEN Successful compromise is Decreased Little

IF Adversary intent is Increased Little AND Adversary Capability is (Decreased Little OR Maintained OR Increased Little)
THEN Successful compromise is Maintained

IF Adversary intent is Increased Little AND Adversary Capability is (Increased Little OR Increased)
THEN Successful compromise is Increased Little

IF Adversary intent is Increased Little AND Adversary Capability is Increased Much
THEN Successful compromise is Increased

IF Adversary intent is Increased AND Adversary Capability is (Decreased Much OR Decreased)
THEN Successful compromise is Decreased Little
IF Adversary intent is Increased AND Adversary Capability is Decreased Little
THEN Successful compromise is Maintained

IF Adversary intent is Increased AND Adversary Capability is (Maintained OR Increased Little)
THEN Successful compromise is Increased Little

IF Adversary intent is Increased AND Adversary Capability is (Increased OR Increased Much)
THEN Successful compromise is Increased

IF Adversary intent is Increased Much AND Adversary Capability is (Decreased Much OR Decreased)
THEN Successful compromise is Maintained

IF Adversary intent is Increased Much AND Adversary Capability is (Decreased Little OR Maintained)
THEN Successful compromise is Increased Little

IF Adversary intent is Increased Much AND Adversary Capability is (Increased Little OR Increased)
THEN Successful compromise is Increased

IF Adversary intent is Increased Much AND Adversary Capability is Increased Much
THEN Successful compromise is Increased Much

**(24, Adversary capability -> (0, Risk to MOD business of CIS compromises and mitigation measures) Time periods: 3**

IF Adversary capability is Decreased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased

IF Adversary capability is Decreased
THEN Risk to MOD business of CIS compromises and mitigation measures is Decreased Little

IF Adversary capability is Decreased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Adversary capability is Maintained
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Adversary capability is Increased Little
THEN Risk to MOD business of CIS compromises and mitigation measures is Maintained

IF Adversary capability is Increased
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased Little

IF Adversary capability is Increased Much
THEN Risk to MOD business of CIS compromises and mitigation measures is Increased

**PROTECT**

**(1, MOD Near CIS vulnerabilities -> (21, Mitigation of CIS compromises or vulnerabilities on MOD business) Time periods: 1**

IF MOD Near CIS vulnerabilities is Decreased Much
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Increased Much

IF MOD Near CIS vulnerabilities is Decreased
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Increased

IF MOD Near CIS vulnerabilities is Decreased Little
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Increased Little

IF MOD Near CIS vulnerabilities is Maintained
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Maintained

IF MOD Near CIS vulnerabilities is Increased Little
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased Little

IF MOD Near CIS vulnerabilities is Increased
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased

IF MOD Near CIS vulnerabilities is Increased Much
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased Much

**(2, Compromises on information -> (11, Impact of CIS compromises on the information held on MOD Near CIS) Time periods: 1**

IF Compromises on information is Decreased Much
THEN Impact of CIS compromises on the information held on MOD Near CIS is Decreased Much

IF Compromises on information is Decreased
THEN Impact of CIS compromises on the information held on MOD Near CIS is Decreased

IF Compromises on information is Decreased Little
THEN Impact of CIS compromises on the information held on MOD Near CIS is Decreased Little

IF Compromises on information is Maintained
THEN Impact of CIS compromises on the information held on MOD Near CIS is Maintained

IF Compromises on information is Increased Little
THEN Impact of CIS compromises on the information held on MOD Near CIS is Increased Little

IF Compromises on information is Increased
THEN Impact of CIS compromises on the information held on MOD Near CIS is Increased

IF Compromises on information is Increased Much
THEN Impact of CIS compromises on the information held on MOD Near CIS is Increased Much

**(3, Vulnerabilities in MOD Near CIS -> (14, MOD Near CIS vulnerabilities identified) Time periods: 1**

IF Vulnerabilities in MOD Near CIS is Decreased Much
THEN MOD Near CIS vulnerabilities identified is Decreased

IF Vulnerabilities in MOD Near CIS is Decreased
THEN MOD Near CIS vulnerabilities identified is Decreased

IF Vulnerabilities in MOD Near CIS is Decreased Little
THEN MOD Near CIS vulnerabilities identified is Decreased Little

IF Vulnerabilities in MOD Near CIS is Maintained
THEN MOD Near CIS vulnerabilities identified is Maintained

IF Vulnerabilities in MOD Near CIS is Increased Little
THEN MOD Near CIS vulnerabilities identified is Increased Little

IF Vulnerabilities in MOD Near CIS is Increased
THEN MOD Near CIS vulnerabilities identified is Increased

IF Vulnerabilities in MOD Near CIS is Increased Much
THEN MOD Near CIS vulnerabilities identified is Increased

**(14, MOD Near CIS vulnerabilities identified -> (13, MOD Near CIS vulnerabilities) Time periods: 1**

IF MOD Near CIS vulnerabilities identified is Decreased Much
THEN MOD Near CIS vulnerabilities is Decreased

IF MOD Near CIS vulnerabilities identified is Decreased
THEN MOD Near CIS vulnerabilities is Decreased Little

IF MOD Near CIS vulnerabilities identified is Decreased Little
THEN MOD Near CIS vulnerabilities is Decreased Little

IF MOD Near CIS vulnerabilities identified is Maintained
THEN MOD Near CIS vulnerabilities is Maintained

IF MOD Near CIS vulnerabilities identified is Increased Little
THEN MOD Near CIS vulnerabilities is Increased Little

IF MOD Near CIS vulnerabilities identified is Increased
THEN MOD Near CIS vulnerabilities is Increased Little

IF MOD Near CIS vulnerabilities identified is Increased Much
THEN MOD Near CIS vulnerabilities is Increased

**(15, Current and past status of MOD Near CIS -> (3, Vulnerabilities in MOD Near CIS) Time periods: 1**

IF Current and past status of MOD Near CIS is Decreased Much
THEN Vulnerabilities in MOD Near CIS is Decreased Much

IF Current and past status of MOD Near CIS is Decreased
THEN Vulnerabilities in MOD Near CIS is Decreased

IF Current and past status of MOD Near CIS is Decreased Little
THEN Vulnerabilities in MOD Near CIS is Decreased Little

IF Current and past status of MOD Near CIS is Maintained
THEN Vulnerabilities in MOD Near CIS is Maintained

IF Current and past status of MOD Near CIS is Increased Little
THEN Vulnerabilities in MOD Near CIS is Increased Little
IF Current and past status of MOD Near CIS is Increased
THEN Vulnerabilities in MOD Near CIS is Increased

IF Current and past status of MOD Near CIS is Increased Much
THEN Vulnerabilities in MOD Near CIS is Increased Much

**(15, Current and past status of MOD Near CIS -> (9, Compromises to MOD Near CIS) Time periods: 1**
IF Current and past status of MOD Near CIS is Decreased Much
THEN Compromises to MOD Near CIS is Decreased Much

IF Current and past status of MOD Near CIS is Decreased
THEN Compromises to MOD Near CIS is Decreased

IF Current and past status of MOD Near CIS is Decreased Little
THEN Compromises to MOD Near CIS is Decreased Little

IF Current and past status of MOD Near CIS is Maintained
THEN Compromises to MOD Near CIS is Maintained

IF Current and past status of MOD Near CIS is Increased Little
THEN Compromises to MOD Near CIS is Increased Little

IF Current and past status of MOD Near CIS is Increased
THEN Compromises to MOD Near CIS is Increased

IF Current and past status of MOD Near CIS is Increased Much
THEN Compromises to MOD Near CIS is Increased Much

**(16, Risk judged acceptable to MOD business -> (1, MOD Near CIS vulnerabilities) Time periods: 1**

IF Risk judged acceptable to MOD business is Decreased Much
THEN MOD Near CIS vulnerabilities is Decreased

IF Risk judged acceptable to MOD business is Decreased
THEN MOD Near CIS vulnerabilities is Decreased Little

IF Risk judged acceptable to MOD business is Decreased Little
THEN MOD Near CIS vulnerabilities is Decreased Little

IF Risk judged acceptable to MOD business is Maintained
THEN MOD Near CIS vulnerabilities is Maintained

IF Risk judged acceptable to MOD business is Increased Little
THEN MOD Near CIS vulnerabilities is Increased Little

IF Risk judged acceptable to MOD business is Increased
THEN MOD Near CIS vulnerabilities is Increased Little

IF Risk judged acceptable to MOD business is Increased Much

THEN MOD Near CIS vulnerabilities is Increased

**(16, Risk judged acceptable to MOD business -> (6, Compromises to MOD Near CIS) Time periods: 1**

IF Risk judged acceptable to MOD business is Decreased Much
THEN Compromises to MOD Near CIS is Decreased

IF Risk judged acceptable to MOD business is Decreased
THEN Compromises to MOD Near CIS is Decreased Little

IF Risk judged acceptable to MOD business is Decreased Little
THEN Compromises to MOD Near CIS is Decreased Little

IF Risk judged acceptable to MOD business is Maintained
THEN Compromises to MOD Near CIS is Maintained

IF Risk judged acceptable to MOD business is Increased Little
THEN Compromises to MOD Near CIS is Increased Little

IF Risk judged acceptable to MOD business is Increased
THEN Compromises to MOD Near CIS is Increased Little

IF Risk judged acceptable to MOD business is Increased Much
THEN Compromises to MOD Near CIS is Increased

**(17, Mitigation options available to MOD -> (1, MOD Near CIS vulnerabilities) Time periods: 1**

IF Mitigation options available to MOD is Decreased Much
THEN MOD Near CIS vulnerabilities is Decreased Much

IF Mitigation options available to MOD is Decreased
THEN MOD Near CIS vulnerabilities is Decreased

IF Mitigation options available to MOD is Decreased Little
THEN MOD Near CIS vulnerabilities is Decreased Little

IF Mitigation options available to MOD is Maintained
THEN MOD Near CIS vulnerabilities is Maintained
IF Mitigation options available to MOD is Increased Little
THEN MOD Near CIS vulnerabilities is Increased Little

IF Mitigation options available to MOD is Increased
THEN MOD Near CIS vulnerabilities is Increased

IF Mitigation options available to MOD is Increased Much
THEN MOD Near CIS vulnerabilities is Increased Much

**(17, Mitigation options available to MOD -> (6, Compromises to MOD Near CIS) Time periods: 1**

IF Mitigation options available to MOD is Decreased Much
THEN Compromises to MOD Near CIS is Decreased Much

IF Mitigation options available to MOD is Decreased
THEN Compromises to MOD Near CIS is Decreased

IF Mitigation options available to MOD is Decreased Little
THEN Compromises to MOD Near CIS is Decreased Little

IF Mitigation options available to MOD is Maintained
THEN Compromises to MOD Near CIS is Maintained

IF Mitigation options available to MOD is Increased Little
THEN Compromises to MOD Near CIS is Increased Little

IF Mitigation options available to MOD is Increased
THEN Compromises to MOD Near CIS is Increased

IF Mitigation options available to MOD is Increased Much
THEN Compromises to MOD Near CIS is Increased Much

**(19, MOD business to use reversionary modes of operation -> (17, Mitigation options available to MOD) Time periods: 1**

IF MOD business to use reversionary modes of operation is Decreased Much
THEN Mitigation options available to MOD is Decreased

IF MOD business to use reversionary modes of operation is Decreased
THEN Mitigation options available to MOD is Decreased

IF MOD business to use reversionary modes of operation is Decreased Little
THEN Mitigation options available to MOD is Decreased Little

IF MOD business to use reversionary modes of operation is Maintained
THEN Mitigation options available to MOD is Maintained

IF MOD business to use reversionary modes of operation is Increased Little
THEN Mitigation options available to MOD is Increased Little

IF MOD business to use reversionary modes of operation is Increased
THEN Mitigation options available to MOD is Increased

IF MOD business to use reversionary modes of operation is Increased Much
THEN Mitigation options available to MOD is Increased

**(20, MOD Business and information held on CIS -> (1, MOD Near CIS vulnerabilities) Time periods: 1**

IF MOD Business and information held on CIS is Decreased Much
THEN MOD Near CIS vulnerabilities is Increased
IF MOD Business and information held on CIS is Decreased
THEN MOD Near CIS vulnerabilities is Increased

IF MOD Business and information held on CIS is Decreased Little
THEN MOD Near CIS vulnerabilities is Increased Little

IF MOD Business and information held on CIS is Maintained
THEN MOD Near CIS vulnerabilities is Maintained

IF MOD Business and information held on CIS is Increased Little
THEN MOD Near CIS vulnerabilities is Decreased Little

IF MOD Business and information held on CIS is Increased
THEN MOD Near CIS vulnerabilities is Decreased

IF MOD Business and information held on CIS is Increased Much
THEN MOD Near CIS vulnerabilities is Decreased Much

**(20, MOD Business and information held on CIS -> (6, Compromises to MOD Near CIS) Time periods: 1**

IF MOD Business and information held on CIS is Decreased Much
THEN Compromises to MOD Near CIS is Increased

IF MOD Business and information held on CIS is Decreased
THEN Compromises to MOD Near CIS is Increased

IF MOD Business and information held on CIS is Decreased Little
THEN Compromises to MOD Near CIS is Increased Little

IF MOD Business and information held on CIS is Maintained
THEN Compromises to MOD Near CIS is Maintained

IF MOD Business and information held on CIS is Increased Little
THEN Compromises to MOD Near CIS is Decreased Little

IF MOD Business and information held on CIS is Increased
THEN Compromises to MOD Near CIS is Decreased

IF MOD Business and information held on CIS is Increased Much
THEN Compromises to MOD Near CIS is Decreased Much

**(21, Mitigation of CIS compromises or vulnerabilities on MOD business -> (10, Risk to MOD business) Time periods: 1**

IF Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased Much
THEN Risk to MOD business is Decreased

IF Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased
THEN Risk to MOD business is Decreased

IF Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased Little
THEN Risk to MOD business is Decreased Little

IF Mitigation of CIS compromises or vulnerabilities on MOD business is Maintained
THEN Risk to MOD business is Maintained

IF Mitigation of CIS compromises or vulnerabilities on MOD business is Increased Little
THEN Risk to MOD business is Increased Little

IF Mitigation of CIS compromises or vulnerabilities on MOD business is Increased
THEN Risk to MOD business is Increased

IF Mitigation of CIS compromises or vulnerabilities on MOD business is Increased Much
THEN Risk to MOD business is Increased Much

**RESPOND**
**(4, Context of a compromise of MOD Near CIS -> (5, Successful compromise) Time periods: 1**

IF Context of a compromise of MOD Near CIS is Decreased Much

THEN Successful compromise is Decreased

IF Context of a compromise of MOD Near CIS is Decreased
THEN Successful compromise is Decreased

IF Context of a compromise of MOD Near CIS is Decreased Little
THEN Successful compromise is Decreased Little

IF Context of a compromise of MOD Near CIS is Maintained
THEN Successful compromise is Maintained

IF Context of a compromise of MOD Near CIS is Increased Little
THEN Successful compromise is Increased Little

IF Context of a compromise of MOD Near CIS is Increased
THEN Successful compromise is Increased

IF Context of a compromise of MOD Near CIS is Increased Much
THEN Successful compromise is Increased

**(6, Compromises to MOD Near CIS -> (21, Mitigation of CIS compromises or vulnerabilities on MOD business) Time periods: 1**

IF Compromises to MOD Near CIS is Decreased Much
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Increased

IF Compromises to MOD Near CIS is Decreased
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Increased

IF Compromises to MOD Near CIS is Decreased Little
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Increased Little

IF Compromises to MOD Near CIS is Maintained
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Maintained

IF Compromises to MOD Near CIS is Increased Little
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased Little

IF Compromises to MOD Near CIS is Increased
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased

IF Compromises to MOD Near CIS is Increased Much
THEN Mitigation of CIS compromises or vulnerabilities on MOD business is Decreased Much

**(6, Compromises to MOD Near CIS -> (23, CIS compromises of MOD business) Time periods: 1**

IF Compromises to MOD Near CIS is Decreased Much
THEN CIS compromises of MOD business is Increased

IF Compromises to MOD Near CIS is Decreased
THEN CIS compromises of MOD business is Increased

IF Compromises to MOD Near CIS is Decreased Little
THEN CIS compromises of MOD business is Increased Little

IF Compromises to MOD Near CIS is Maintained
THEN CIS compromises of MOD business is Maintained

IF Compromises to MOD Near CIS is Increased Little
THEN CIS compromises of MOD business is Decreased Little

IF Compromises to MOD Near CIS is Increased
THEN CIS compromises of MOD business is Decreased

IF Compromises to MOD Near CIS is Increased Much
THEN CIS compromises of MOD business is Decreased Much

**RECOVER**
**(7, Information in sufficient time for MOD business needs -> (23, CIS compromises of MOD business) Time periods: 1**

IF Information in sufficient time for MOD business needs is Decreased Much
THEN CIS compromises of MOD business is Increased

IF Information in sufficient time for MOD business needs is Decreased
THEN CIS compromises of MOD business is Increased

IF Information in sufficient time for MOD business needs is Decreased Little
THEN CIS compromises of MOD business is Increased Little

IF Information in sufficient time for MOD business needs is Maintained
THEN CIS compromises of MOD business is Maintained

IF Information in sufficient time for MOD business needs is Increased Little
THEN CIS compromises of MOD business is Decreased Little

IF Information in sufficient time for MOD business needs is Increased
THEN CIS compromises of MOD business is Decreased

IF Information in sufficient time for MOD business needs is Increased Much
THEN CIS compromises of MOD business is Decreased Much

**(8, System compromises -> (25, Cyber tools) Time periods: 1**

IF System compromises is Decreased Much
THEN Cyber tools is Decreased Much

IF System compromises is Decreased
THEN Cyber tools is Decreased

IF System compromises is Decreased Little
THEN Cyber tools is Decreased Little

IF System compromises is Maintained
THEN Cyber tools is Maintained

IF System compromises is Increased Little
THEN Cyber tools is Increased Little

IF System compromises is Increased
THEN Cyber tools is Increased

IF System compromises is Increased Much
THEN Cyber tools is Increased Much

**(8, System compromises -> (26, Cyber tradecraft) Time periods: 1**

IF System compromises is Decreased Much
THEN Cyber tradecraft is Decreased Much

IF System compromises is Decreased
THEN Cyber tradecraft is Decreased

IF System compromises is Decreased Little
THEN Cyber tradecraft is Decreased Little

IF System compromises is Maintained
THEN Cyber tradecraft is Maintained

IF System compromises is Increased Little
THEN Cyber tradecraft is Increased Little

IF System compromises is Increased
THEN Cyber tradecraft is Increased

IF System compromises is Increased Much
THEN Cyber tradecraft is Increased Much

**(8, System compromises -> (7, Information in sufficient time for MOD business needs) Time periods: 1**

IF System compromises is Decreased Much
THEN Information in sufficient time for MOD business needs is Decreased Much

IF System compromises is Decreased
THEN Information in sufficient time for MOD business needs is Decreased

IF System compromises is Decreased Little
THEN Information in sufficient time for MOD business needs is Decreased Little

IF System compromises is Maintained
THEN Information in sufficient time for MOD business needs is Maintained

IF System compromises is Increased Little
THEN Information in sufficient time for MOD business needs is Increased Little

IF System compromises is Increased
THEN Information in sufficient time for MOD business needs is Increased

IF System compromises is Increased Much
THEN Information in sufficient time for MOD business needs is Increased Much

**(23, CIS compromises of MOD business -> (10, Risk to MOD business) Time periods: 1**

IF CIS compromises of MOD business is Decreased Much
THEN Risk to MOD business is Decreased Much

IF CIS compromises of MOD business is Decreased
THEN Risk to MOD business is Decreased

IF CIS compromises of MOD business is Decreased Little

THEN Risk to MOD business is Decreased Little

IF CIS compromises of MOD business is Maintained
THEN Risk to MOD business is Maintained

IF CIS compromises of MOD business is Increased Little
THEN Risk to MOD business is Increased Little

IF CIS compromises of MOD business is Increased
THEN Risk to MOD business is Increased

IF CIS compromises of MOD business is Increased Much
THEN Risk to MOD business is Increased Much

**(29, Means to appropriately restore the confidentiality integrity and availability of information -> (7, Information in sufficient time for MOD business needs) Time periods: 1**

IF Means to appropriately restore the confidentiality integrity and avail... is Decreased Much
THEN Information in sufficient time for MOD business needs is Decreased

IF Means to appropriately restore the confidentiality integrity and avail... is Decreased
THEN Information in sufficient time for MOD business needs is Decreased

IF Means to appropriately restore the confidentiality integrity and avail... is Decreased Little
THEN Information in sufficient time for MOD business needs is Decreased Little

IF Means to appropriately restore the confidentiality integrity and avail... is Maintained
THEN Information in sufficient time for MOD business needs is Maintained

IF Means to appropriately restore the confidentiality integrity and avail... is Increased Little
THEN Information in sufficient time for MOD business needs is Increased Little

IF Means to appropriately restore the confidentiality integrity and avail... is Increased
THEN Information in sufficient time for MOD business needs is Increased

IF Means to appropriately restore the confidentiality integrity and avail... is Increased Much
THEN Information in sufficient time for MOD business needs is Increased

**(30, MOD Near CIS vulnerabilities -> (14, MOD Near CIS vulnerabilities identified) Time periods: 1**

IF MOD Near CIS vulnerabilities is Decreased Much
THEN MOD Near CIS vulnerabilities identified is Decreased

IF MOD Near CIS vulnerabilities is Decreased
THEN MOD Near CIS vulnerabilities identified is Decreased

IF MOD Near CIS vulnerabilities is Decreased Little
THEN MOD Near CIS vulnerabilities identified is Decreased Little

IF MOD Near CIS vulnerabilities is Maintained
THEN MOD Near CIS vulnerabilities identified is Maintained

IF MOD Near CIS vulnerabilities is Increased Little
THEN MOD Near CIS vulnerabilities identified is Increased Little

IF MOD Near CIS vulnerabilities is Increased
THEN MOD Near CIS vulnerabilities identified is Increased

IF MOD Near CIS vulnerabilities is Increased Much
THEN MOD Near CIS vulnerabilities identified is Increased

**DETECT**
**(9, Compromises to MOD Near CIS -> (27, MOD Near CIS compromises detected) Time periods: 1**
IF Compromises to MOD Near CIS is Decreased Much
THEN MOD Near CIS compromises detected is Decreased

IF Compromises to MOD Near CIS is Decreased
THEN MOD Near CIS compromises detected is Decreased Little

IF Compromises to MOD Near CIS is Decreased Little
THEN MOD Near CIS compromises detected is Decreased Little

IF Compromises to MOD Near CIS is Maintained
THEN MOD Near CIS compromises detected is Maintained

IF Compromises to MOD Near CIS is Increased Little
THEN MOD Near CIS compromises detected is Increased Little

IF Compromises to MOD Near CIS is Increased
THEN MOD Near CIS compromises detected is Increased

IF Compromises to MOD Near CIS is Increased Much
THEN MOD Near CIS compromises detected is Increased Much

**(25, Cyber tools -> (9, Compromises to MOD Near CIS) Time periods: 2**

IF Cyber Tools is Decreased Much AND Cyber Tradecraft is Decreased Much
THEN Compromises to MOD Near CIS is Decreased Much

IF Cyber Tools is Decreased Much AND Cyber Tradecraft is (Decreased OR Decreased Little OR Maintained OR Increased Little)
THEN Compromises to MOD Near CIS is Decreased

IF Cyber Tools is Decreased Much AND Cyber Tradecraft is Increased
THEN Compromises to MOD Near CIS is Decreased Little

IF Cyber Tools is Decreased Much AND Cyber Tradecraft is Increased Much
THEN Compromises to MOD Near CIS is Maintained

IF Cyber Tools is Decreased AND Cyber Tradecraft is (Decreased Much OR Decreased OR Decreased Little OR Maintained)
THEN Compromises to MOD Near CIS is Decreased

IF Cyber Tools is Decreased AND Cyber Tradecraft is (Increased Little OR Increased)
THEN Compromises to MOD Near CIS is Decreased Little

IF Cyber Tools is Decreased AND Cyber Tradecraft is Increased Much
THEN Compromises to MOD Near CIS is Maintained

IF Cyber Tools is Decreased Little AND Cyber Tradecraft is (Decreased Much OR Decreased)
THEN Compromises to MOD Near CIS is Decreased

IF Cyber Tools is Decreased Little AND Cyber Tradecraft is (Decreased Little OR Maintained)
THEN Compromises to MOD Near CIS is Decreased Little

IF Cyber Tools is Decreased Little AND Cyber Tradecraft is (Increased Little OR Increased)
THEN Compromises to MOD Near CIS is Maintained

IF Cyber Tools is Decreased Little AND Cyber Tradecraft is Increased Much
THEN Compromises to MOD Near CIS is Increased Little
IF Cyber Tools is Maintained AND Cyber Tradecraft is (Decreased Much OR Decreased)
THEN Compromises to MOD Near CIS is Decreased

IF Cyber Tools is Maintained AND Cyber Tradecraft is (Decreased Little OR Maintained)
THEN Compromises to MOD Near CIS is Decreased Little

IF Cyber Tools is Maintained AND Cyber Tradecraft is (Increased Little OR Increased)
THEN Compromises to MOD Near CIS is Maintained

IF Cyber Tools is Maintained AND Cyber Tradecraft is Increased Much
THEN Compromises to MOD Near CIS is Increased Little

IF Cyber Tools is Increased Little AND Cyber Tradecraft is Decreased Much
THEN Compromises to MOD Near CIS is Decreased

IF Cyber Tools is Increased Little AND Cyber Tradecraft is Decreased
THEN Compromises to MOD Near CIS is Decreased Little

IF Cyber Tools is Increased Little AND Cyber Tradecraft is (Decreased Little OR Maintained OR Increased Little)
THEN Compromises to MOD Near CIS is Maintained

IF Cyber Tools is Increased Little AND Cyber Tradecraft is (Increased Little OR Increased)
THEN Compromises to MOD Near CIS is Increased Little

IF Cyber Tools is Increased Little AND Cyber Tradecraft is Increased Much
THEN Compromises to MOD Near CIS is Increased
IF Cyber Tools is Increased AND Cyber Tradecraft is (Decreased Much OR Decreased)
THEN Compromises to MOD Near CIS is Decreased Little

IF Cyber Tools is Increased AND Cyber Tradecraft is Decreased Little
THEN Compromises to MOD Near CIS is Maintained

IF Cyber Tools is Increased AND Cyber Tradecraft is (Maintained OR Increased Little)
THEN Compromises to MOD Near CIS is Increased Little

IF Cyber Tools is Increased AND Cyber Tradecraft is (Increased OR Increased Much)
THEN Compromises to MOD Near CIS is Increased

IF Cyber Tools is Increased Much AND Cyber Tradecraft is (Decreased Much OR Decreased)
THEN Compromises to MOD Near CIS is Maintained

IF Cyber Tools is Increased Much AND Cyber Tradecraft is (Decreased Little OR Maintained)
THEN Compromises to MOD Near CIS is Increased Little

IF Cyber Tools is Increased Much AND Cyber Tradecraft is (Increased Little OR Increased)
THEN Compromises to MOD Near CIS is Increased

IF Cyber Tools is Increased Much AND Cyber Tradecraft is Increased Much
THEN Compromises to MOD Near CIS is Increased Much

**(27, MOD Near CIS compromises detected -> (8, System compromises) Time periods: 1**

IF MOD Near CIS compromises detected is Decreased Much
THEN System compromises is Decreased Much

IF MOD Near CIS compromises detected is Decreased
THEN System compromises is Decreased

IF MOD Near CIS compromises detected is Decreased Little
THEN System compromises is Decreased

IF MOD Near CIS compromises detected is Maintained
THEN System compromises is Decreased Little

IF MOD Near CIS compromises detected is Increased Little
THEN System compromises is Maintained

IF MOD Near CIS compromises detected is Increased
THEN System compromises is Increased Little

IF MOD Near CIS compromises detected is Increased Much
THEN System compromises is Increased

**(28, Compromises of CIS on which MOD information resides -> (27, MOD Near CIS compromises detected) Time periods: 1**

IF Compromises of CIS on which MOD information resides is Decreased Much
THEN MOD Near CIS compromises detected is Decreased

IF Compromises of CIS on which MOD information resides is Decreased
THEN MOD Near CIS compromises detected is Decreased Little

IF Compromises of CIS on which MOD information resides is Decreased Little
THEN MOD Near CIS compromises detected is Decreased Little

IF Compromises of CIS on which MOD information resides is Maintained
THEN MOD Near CIS compromises detected is Maintained

IF Compromises of CIS on which MOD information resides is Increased Little
THEN MOD Near CIS compromises detected is Increased Little

IF Compromises of CIS on which MOD information resides is Increased
THEN MOD Near CIS compromises detected is Increased Little

IF Compromises of CIS on which MOD information resides is Increased Much
THEN MOD Near CIS compromises detected is Increased