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THE BEHAVIOUR OF HOT PLUMES FROM TALL CHIMNEYS UNDER CONDITIONS OF STRONG THERMAL CONVECTION

By

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ABSTRACT

An examination is made of the consequences of a special type of convection on the behaviour of hot plumes emitted from tall chimneys. The type of convection envisaged is that in which thermals, originating at the ground, transport heat and other surface constituents to great heights. As they rise they amalgamate with each other and entrain air at all levels. The environment - that is the atmosphere excluding the thermals - is assumed to be slowly subsiding, compensating for the mass flux from the surface. The convection region is assumed similar at all heights and from the similarity laws governing the dynamics of isolated thermals we obtain $V_e(z) = -\beta z^{1/3}$, where $-V_e(z)$ is the subsiding velocity of the environment, z height above ground and β a constant.

The chimney plume is assumed either to be emitted into a thermal, in which case it does not reach the ground, or into the subsiding environment. In the latter case, it is depleted by thermals as it travels downwind and when passive, the plume is compressed vertically. If the plume remains buoyant long enough then maximum ground level concentrations can be parameterized by a single number $(F_o/U\beta^2)$, where F_o is the initial buoyancy flux parameter of the chimney plume, and U the wind speed. It is found that maximum ground level concentrations are increased over those calculated for a neutral environment. Alternatively, if the plume is assumed to become passive at a fixed distance downwind, maximum ground level concentrations are still increased.

It is realised that there may be occasions when there is thermal convection but no subsidence - e.g. at coastal sites where sea breezes may replenish the mass flux from the surface. Maximum ground level concentrations are decreased for these occasions.

Chimney Plumes in a Convective Atmosphere

1 The convection model

The problem under investigation is that of determining the behaviour of chimney plumes emitted into the air under strong convective conditions. Under such circumstances winds are generally light and organised mixing and heat transport from the surface is maintained through rising hot air masses called thermals, that originate near the ground. These thermals transport all surface air constituents (including pollutants) from the ground, they mix with the ambient air as they rise and consequently grow in size. The motion may extend to considerable heights - perhaps one or two kilometres above the surface and for this reason it is hypothesised that in order to maintain their buoyancy over such depths and also to ensure that the thermals do not occupy the whole sky at some height, they must amalgamate as they rise. To compensate for this upflow from the surface the environment - that is the air mass excluding the thermals - is slowly subsiding, and being stably stratified is warmed. There may also be conditions under which compensating inflow occurs through horizontal convergence into the convective field and then the environment would not necessarily need to sink to replenish this loss of air. These conditions may be typical of coastal areas where sea breezes readily form to replace the warmed air over the land. Again, large scale convergence into the low pressure centre formed within the convective region may also occur.

One way of describing the type of convection in which subsidence is the mechanism for mass conservation has been to invoke the principles of similarity in order to obtain a quantative understanding of the dynamics of the system. The field of convection is then determined by the buoyancy flux at the surface and the similarity relations governing the dynamics of isolated thermals.

The extent of the region must be infinite or the capping inversion at the top of the mixing layer must be at a height much greater than the region of interest where similarity is invoked. The thermals must occupy the same fraction of the sky at all heights and from the dynamics of isolated three dimensional thermals one result obtained is that the velocity of ascent of the thermals or descent of the environment is proportional to the cubed root of height above the surface. The environment decelerates as the ground is approached because its downward mass decreases through entrainment into the thermals, whilst it always occupies the same fraction of the sky. Fig. (1) shows schematically the type of convection envisaged here. This is taken from Scorer (1969) where the details of the calculations can be found.

It is recognised that the theory does not extend right down to the ground where the tiny thermals would theoretically have large temperature excesses. It is conjectured that the thermals burst from irregularities in the surface layer which may be 50-100 feet in depth and are of finite size at their inception. Their existence is beyond doubt but there remains confusion as to the detailed mechanisms of their generation.

Hence the theory is really only applicable above this surface layer and in conditions when the vertical extent of the mixing is much larger than that of the chimney plume itself. Most hot sources from relatively high stacks (greater than 250 feet say) would be expected to travel for considerable distances within this similarity region. Extrapolation down to smaller chimneys and heat sources would be speculative.

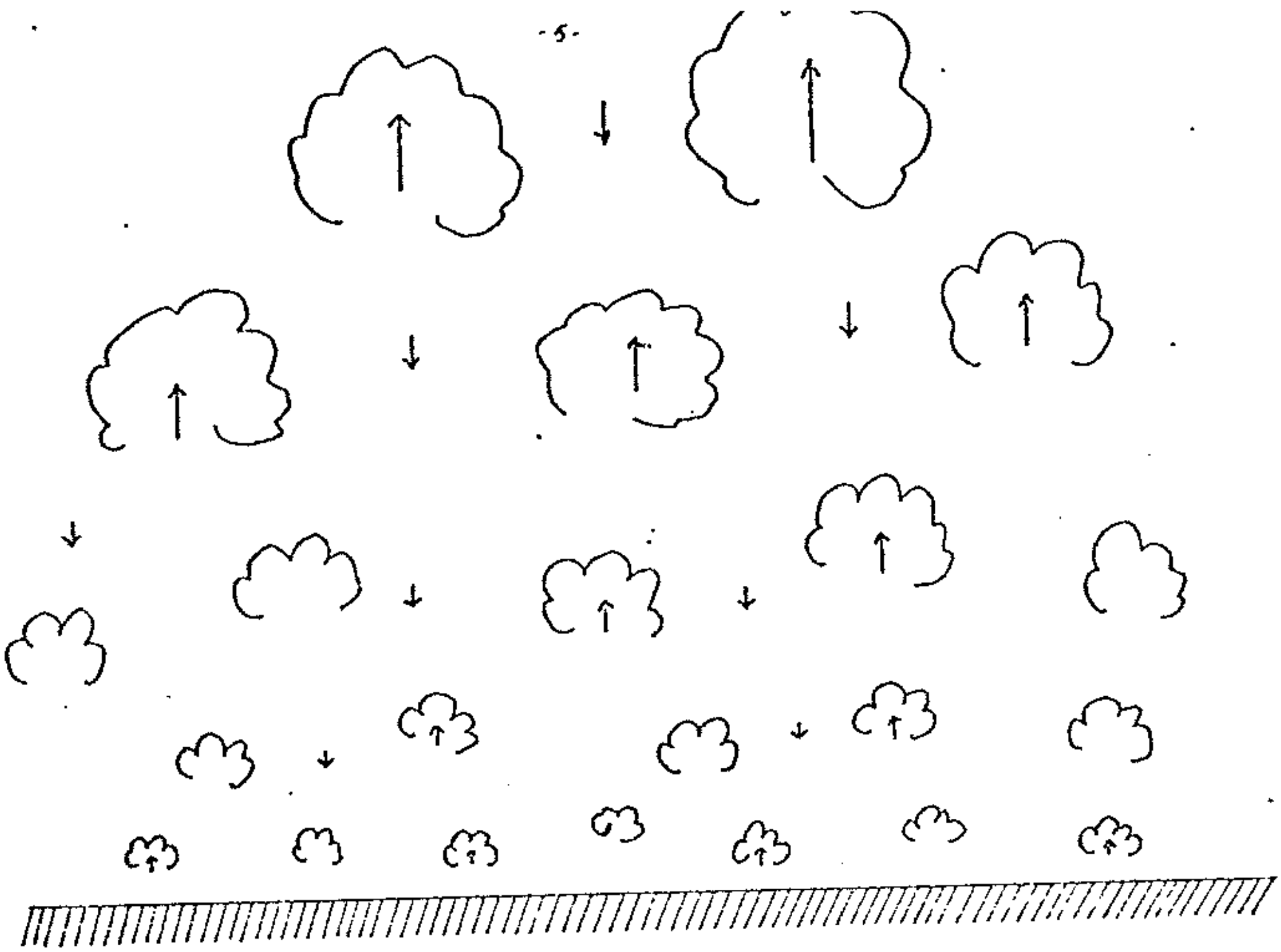

Fig (1) A similarity model of thermal convection. Thermals, originating at the ground, mix with their environment and with each other as they accelerate upwards. The environment slowly subsides, decelerating as it nears the ground.

For similarity -

(i) Thermals occupy the same horizontal fraction of the sky at all heights.

(ii) Thermal size is proportional to the height above ground.

2 Passive plume behaviour in the convection field

A model of plume behaviour under such conditions has been offered by F.B. Smith (Smith (1957a,1957b)). He considered passive ground level line sources emitted into a convective region which was capped by an inversion. He considered effectively two plumes. One was the original plume which was depleted as it travelled downwind and the other was the "convected plume".. This consisted of those entrained ground level plume elements which were now subsiding after having been carried to the top of the convection layer. Plume profiles were developed for the ground level plume by extending the solutions for non-convective conditions which were calculated from the 2-dimensional diffusion equation.

In our case we shall only be concerned with the original plume and assume that those elements which become entrained into the thermals after travelling to great heights will reappear at the surface at some considerable distance downwind, if at all, and will not appreciably affect maximum ground level concentrations (G.L.C). Here we are considering elevated 3-dimensional plumes and shall assume a concentration distribution for the plume in non convective conditions rather than solve the "diffusion equation". At great heights away from the surface it is not unreasonable to assume that conditions become more isotropic and in common with most other theoretical investigations we shall assume the plume distribution to be Gaussian. It will be shown that under certain conditions the concentration profile does not change from a non-convective to a convective atmosphere. However the organised mixing will modify certain plume features. The plume is considered either to be emitted into a thermal - whence it does not reach the ground - or it is emitted into the subsiding environment, through which thermals penetrate and part of which they entrain.

F.B. Smith noticed that the convection will modify the "original" plume in two ways. First the thermals will deplete the plume and second the subsiding environment will compress it because the air higher up is descending more rapidly than that nearer the ground. He assumed that the rate of descent of the environment $(-V_e(z))$ was proportional to the height above the ground (z).

So

$$V_e(z) = -\lambda z \quad , \quad \lambda > 0 \text{ constant} \tag{1}$$

The rate of loss of volume per unit volume of the environment is given by

$$dV_e(z)/dz = -\lambda \tag{2}$$

So the rate of loss of material at any height is a fixed proportion of the concentration at that height and hence depletion does not alter the shape of the concentration profile.

With a Gaussian profile and compression given by equn. (2) the shape of the concentration profile again remains unchanged although the standard deviation will change. Equn. (2) says that the rate of separation between two particles is proportional to their separation. Consider a concentration profile C_1 at time $t = 0$ in fig. (2) given by $C_1(y) = (A/\Psi_0^2)\, e^{-y^2/2\Psi_0^2}$

A, constant and Ψ_0 the standard deviation at time $t = 0$.

Now consider the change in profile under compression alone given by equn. (2).

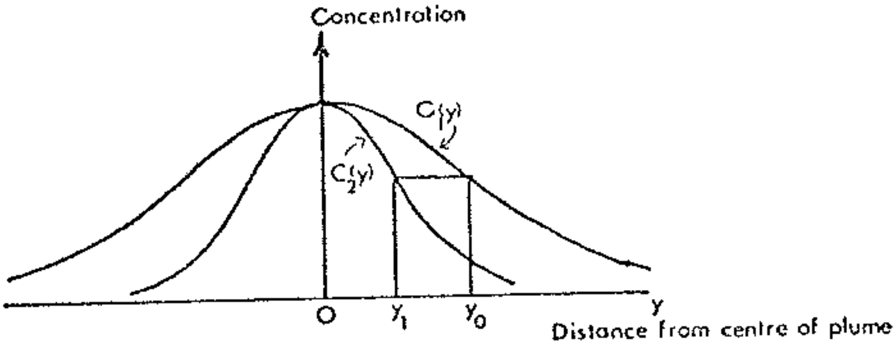

FIG. (2) Change in concentration profile by compression alone.

For any y, $dy/dt = -\lambda y$ so $y = y(t=0)e^{-\lambda t}$ with respect to the origin. For a particle at $y = y_o$ at $t = 0$ and $y = y_1$ at $t = t_1$ we have

$$y_1 = y_0 e^{-\lambda t_1}, \qquad C_2(y_1) = C_1(y_0) = (A/\Psi_0^2)\, e^{-y_0^2/2\Psi_0^2}$$

so

$$C_2(y_1) = \frac{A e^{-2\lambda t_1}}{(\Psi_0 e^{-\lambda t_1})^2} \cdot e^{-y_1^2/2(\Psi_0 e^{-\lambda t_1})^2}$$

Thus the Gaussian profile is preserved with the standard deviation $\Psi = \Psi_o e^{-\lambda t}$ i.e. $d\Psi/dt = -\lambda\Psi$.

The passive plume is also expanding under the influence of the eddies in the subsiding environment and hence the total rate of expansion is given by -

$$d\Psi/dt = (d\Psi/dt)_{\lambda=0} - \lambda\Psi \qquad (3)$$

The first term on the right hand side of equn. (3) gives the rate of expansion for the plume in the non-convective atmosphere appropriate to the standard deviation at any time and the second term is the compression term.

It is assumed that $(d\Psi/dx)_{\lambda=0} = a_o$ (constant) (4)

gives a reasonable estimate of eddy expansion. It assumes the plume spreads conically and is expected to apply reasonably well near the source at least up to the point of maximum GLC.

Hence with $\Psi = \sigma_z$, the vertical standard deviation of plume material, U wind speed, the solution of equn. (3) with $\sigma_z(t=0) = 0$ is given by

$$\sigma_z = (Ua_o/\lambda)(1-e^{-\lambda t}) \tag{5}$$

The horizontal spread of the plume remains unaffected by compression in a statistical sense because the loss of volume of the plume is taken up by the thermals. *Instantaneously*, however, the plume will take up an elliptical shape, preserving volume, because the motion is incompressible. So with x distance downwind, y acrosswind distance, z height above ground we have

$$\sigma_y = a_1 x \quad , \quad a_1 \text{ constant} \tag{6}$$

and with the Gaussian profile, concentrations are given by

$$C(x,y,z) = \frac{F(x)}{\sigma_y \sigma_z} e^{-y^2/2\sigma_y^2}\left(e^{-(z-h)^2/2\sigma_z^2} + e^{-(z+h)^2/2\sigma_z^2}\right) \tag{7}$$

The second term on the right hand side of equn. (7) is the image source and it is assumed that the image environment is a perfect reflection of the real environment with $V_e(z) = V_e(-z)$ for $z < 0$.

h is the centreline of the plume and is a function of x. Provided the rate of subsidence is much less than the wind speed the distribution in vertical planes about the plume centre line will be a good approximation.

To find $F(x)$ in eqυn. (7) we apply the conservation of mass to a vertical slice of the plume as below, integrated over all space.

From fig. (3)

Total flux of material through surface A - Total flux through B
= Rate of loss of plume material through entrainment into the thermals.

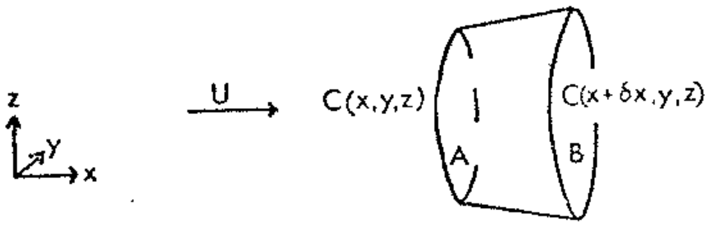

FIG. (3) Conservation of pollutant.

$$U\left[\iint_A C(x,y,z)\,dy\,dz - \iint_B C(x+\delta x,y,z)\,dy\,dz\right] = -\iiint_{Vol.} \frac{dV_e(z)}{dz}\cdot C(x,y,z)\,dx\,dy\,dz$$

and with $dV_e/dz = -\lambda$

$$d/dx\left[\iint C(x,y,z)\,dy\,dz\right] = -\lambda/U \iint C(x,y,z)\,dy\,dz$$

the solution of which is

$$\iint C(x,y,z)\,dy\,dz = \text{constant}.\ e^{-\lambda x/U} \qquad (8)$$

Now when $x = 0$, $U\iint C(x,y,z)\,dy\,dz = (1-\sigma)Q$ where Q is the amount of material emitted/second and σ is the proportion of the sky occupied by thermals. Hence

$$C(x,y,z) = \frac{(1-\sigma)Qe^{-\lambda x/U}}{2U\pi\sigma_y\sigma_z} \cdot e^{-y^2/2\sigma_y^2}\left(e^{-(z-h)^2/2\sigma_z^2} + e^{-(z+h)^2/2\sigma_z^2}\right) \qquad (9)$$

and for ground level axial concentrations

$$C(x,0,0) = \frac{Q(1-\sigma)}{U\pi\sigma_y\sigma_z} \cdot e^{-\lambda x/U} e^{-h^2/2\sigma_z^2} \qquad (10)$$

The results obtained in equations (5) and (10) apply only to an environment in which equn. (2) holds. As discussed earlier according to similarity theory $V_e(z) \propto z^{1/3}$ and if we are to apply the results of this theory we must justify the form of equn. (2). Now provided the gradient of $V_e(z)$ varies marginally over the plume depth it is fair to approximate the rate of subsidence over the plume through a linear dependence on height. As the plume expands this approximation becomes poorer. The rate of depletion as expressed in equn. (10) through $e^{-\lambda x/U}$ depends solely on the gradient of $V_e(z)$. The gradient changes slowly with height above the ground and the rate of change of gradient with height $(d^2V_e/dz^2 \propto z^{-5/3})$ at great heights is small. Fig. (4) shows the variation of $V_e(z)$ with height for perhaps strong convective conditions. The centreline of the plume $h(x)$ however depends on $\int V_e(z)$ directly and because $\int V_e(z)$ changes more rapidly with height than dV_e/dz it is a better approximation to assume h varies as –

$$dh/dt \propto h^{1/3}$$

So provided the major part of the plume trajectory is spent well above the surface layer there will not be any great error in assuming the exponential depletion rate. λ is taken as the appropriate average gradient over the plume trajectory.

A hot plume expands under its own buoyancy and equn. (10) could then only be applied after such a time as the plume becomes effectively passive. To calculate maximum GLCs we introduce variables.

x distance downwind measured from the point where the plume becomes passive.

$H(x) = h(x)$ centre line of the plume.

$H_o = H(x=0)$ chimney height + plume rise.

x_1 *actual* distance from chimney to $x = 0$.

σ_{yo} lateral standard deviation of plume at $x = 0$.

σ_{zo} vertical standard deviation of plume at $x = 0$.

So equn. (10) becomes

$$C(x,0,0) = \frac{Q(1-\sigma)}{U\pi\sigma_y\sigma_z} \cdot e^{-\lambda(x+x_1)/U} \, e^{-H^2(x)/2\sigma_z^2} \qquad (11)$$

From equation (4)

$$\sigma_z = \frac{Ua_o}{\lambda}\left(1 - \delta e^{-\lambda x/U}\right) , \quad \delta = 1 - \frac{\lambda\sigma_{zo}}{Ua_o} \qquad (12)$$

$$\sigma_y = \sigma_{yo} + a_1 x , \quad \text{define} \quad \varepsilon = \frac{\lambda\sigma_{yo}}{Ua_1} \qquad (13)$$

To calculate $H(x)$, we have with $x = Ut$

$$dH/dt = -\beta H^{1/3}$$

$$H(x) = \left(H_0^{2/3} - (2\beta/3)\,t \right)^{3/2}$$

Introducing

$$\phi = a_0 x\sqrt{2}/H_0 , \quad \psi = \frac{\lambda H_0}{2\sqrt{2}\,U a_0} , \quad \alpha_0 = \frac{4\beta}{3\lambda H_0^{2/3}} \tag{14}$$

then the point of max GLC $x = x_m$ is given by $\frac{dC}{dx} = 0$ and the solution of

$$\phi_m^2 = \frac{2(\phi_m\psi)^2(1-\alpha_0\phi_m\psi)^2(\varepsilon+2\phi_m\psi)\left[3\alpha_0(1-\delta e^{-2\phi_m\psi})+4\delta e^{-2\phi_m\psi}(1-\alpha_0\phi_m\psi)\right]}{(1-\delta e^{-2\phi_m\psi})^2(1-\delta e^{-2\phi_m\psi}+2\phi_m\psi+\varepsilon)} \tag{15}$$

where $\phi_m = a_0 x_m\sqrt{2}/H_0$ N.B. if $\frac{\sigma_{z0}}{a_0} = \frac{\sigma_{y0}}{a_1}$ then $\delta = 1-\varepsilon$

Equation (15) can be solved graphically given ψ which in general will be a function of the convection (through λ) and the heat content of the gases (through H_0) .

N.B. as $\lambda \to 0$, $H_0 \to H_1$, $\phi_m \to 1 - \frac{\sigma_{z0}\sqrt{2}}{H_1}$ for $\frac{\sigma_{z0}}{\sigma_{y0}} = \frac{a_0}{a_1}$

With ϕ_m given from equn. (15) max. GLC Cmax is given by

$$C_{max} = \frac{8Q(1-\sigma)}{U\pi H_0^2} \cdot \frac{a_0}{a_1} \frac{e^{-\lambda x_1/U}\,\psi^2 e^{-2\phi_m\psi}\, e^{-4\psi^2(1-\alpha_0\phi_m\psi)^3/(1-\delta e^{-2\phi_m\psi})^2}}{(\varepsilon+2\phi_m\psi)(1-\delta e^{-2\phi_m\psi})} \tag{16}$$

Without convection max. GLCs will be given by

$$C'_{max} = \frac{2Q}{e\pi U H_1^2} \cdot \frac{a_0}{a_1} \qquad (17)$$

The effect of convection may be studied from examination of the ratio of equns. (16) and (17). H_1 is the plume rise plus chimney height in a non-convective atmosphere.

Having established equations (15) and (17) it is now necessary to obtain an expression for the rise of the buoyant plume in the convective atmosphere to determine ψ .

3 Buoyant plumes in the convection field and calculation of maximum ground level concentrations

Plume rise is determined by the instantaneous distribution of buoyant material within the plume elements rather than the statistical properties of the mean material distribution and so it is necessary to consider the detail of the thermal activity to describe the plume trajectory.

On emission buoyant plume elements either collide and become entrained into passing thermals when they will be carried to great heights or they will become part of the subsiding environment, expanding at a rate determined by their heat content and rising relative to the subsiding air. In the latter case it will be assumed that the plume may be considered as a bent over cylindrical thermal, its trajectory being calculated in a similar fashion to that in a non convective atmosphere. This will be all right provided the spacing between the environmental thermals is much larger than the plumes vertical size near the source. If σ is the fraction of sky occupied by the thermals, S the horizontal spacing between thermals and D the thermal diameter then

$$S^2/D^2 = (1-\sigma)/\sigma, \qquad S = D\sqrt{(1-\sigma)/\sigma}$$

Typically $\sigma \leq 1/5$ or $S \geq 2D$ (Scorer (1969)). With thermal dimensions D about 200 m at a height of 200 m above ground,(Warner and Tellford(1967)), initial plume lengths, or S , will be about 400 m. Hence the assumption made above will be satisfactory. The intermittancy of plume elements as they leave the chimney will be handled in the same way as for the passive plume described earlier.

As the plume travels downwind thermals will puncture it, entraining plume material, and it might be expected to become fragmented and 3-dimensional in nature. Then it will behave more like a series of discrete lumps than a continuous plume. However these elements must recombine again further downwind because they expand longitudinally, forming yet again a continuous plume albeit of reduced buoyancy per unit length because plume material has been lost to the thermals. Incidentally Richards (1963) has shown that the exterior flow field surrounding a thermal may be represented by a potential flow with no wake and so the plume would automatically reform behind the thermal without needed to diffuse longitudinally. Hence the plume may not be split up. Here it will be assumed that the plume may still be treated as a continuous entity even at greater distances downwind but its trajectory will be modified because of buoyancy depletion. This depletion rate may again be described in an identical fashion to the earlier passive plume treatment. The transition point between the initial and final trajectory will be difficult to determine. Their relative importance depends to a large extent on the size to which the plume is allowed to grow or on which plume rise formula is chosen.

The appropriate velocity of subsidence $V_e(z)$ for the buoyant plume will be the instantaneous velocity of subsidence. This will be equal to the mean velocity only if the thermals are homogenously spaced in horizontal planes. The *mean* plume rise is not expected to be too different from the rise calculated using the mean velocity of subsidence provided the above holds and average GLCs computed from this will not then be in serious error.

Further we have to discuss the compression of the buoyant plume due to the increasing velocity of subsidence with height. In a statistical sense compression did not affect the lateral spread of the passive plume because depletion through entrainment into the thermals compensated exactly for this volume loss. This is not true for the instantaneous plume. Because the compression will be volume conserving at any moment in time circular cross-sections of the plume will tend to be changing into elliptical forms. The consequences of this for the buoyant plume would be that the surface area would increase indefinitely and hence dilution too. It would then require knowledge of the rate of lateral and vertical diffusion before a complete solution could be found. However the problem does differ from that discussed for the passive plume because here the buoyant plume determines its own rate of mixing and its own particular shape. It is therefore assumed that at all stages in its development the hot plume tends to maintain its primary configuration having been subjected to small deformations, and therefore the effect of compression can be ignored.

Consequently the equations of motion will only be modified in that the rate of entrainment will be assumed proportional to the *relative* velocity of the plume with respect to the subsiding environment of which it is a part, for the initial plume trajectory, and with a buoyancy depletion term for larger distances from the source.

The conservation equations of mass, buoyancy and momentum are given below for a neutral environment with the following notation. Zero suffix denotes values at stack exit.

v gas exit velocity, ρ density of plume, ρ_e density of environment, z height above ground, r plume radius, $\Delta\theta$ potential temperature excess of plume. θ potential temperature of plume, F buoyancy flux parameter $= r^2 U \Delta\theta g/\theta$, F_o initial buoyancy flux parameter $= V_o \Delta\theta_o g/\pi\theta_o$, V_o volume flux from stack, g acceleration due to gravity, F_m initial momentum flux parameter $= \rho_o V_o v/\pi\rho_e$, w velocity of gases relative to subsiding environment, $-V_e(z)$ velocity of subsidence of environment, U wind speed, t travel time from chimney.

Density differences are important only for the generation of buoyancy forces.

Mass $\quad d/dt(\rho\pi r^2 U) = 2\pi r U \rho_e \alpha w$, $\quad \alpha$ constant $\quad$ (18)

Momentum $\quad d/dt(\rho\pi r^2 U w) = \pi r^2 U \rho_e \frac{\Delta\theta}{\theta} g$ $\quad$ (19)

Buoyancy (i) $\quad d/dt\left(\pi r^2 U \frac{\Delta\theta}{\theta}\right) = 0$ $\quad$ (20)

(ii) $\quad d/dt\left(\pi r^2 U \frac{\Delta\theta}{\theta}\right) = -\bar{\lambda}\pi r^2 U \frac{\Delta\theta}{\theta}$, $\quad \bar{\lambda}$ constant $\quad$ (21)

Plume velocity $\quad dz/dt = w + V_e(z)$ $\quad$ (22)

Solving first the set of equations with equn. (20) we have

$$r^2 U \frac{\Delta\theta}{\theta} g = F_o \quad , \quad r^2 U w = F_o t + F_m \tag{23}$$

from equn. (19). With $F_o t \gg F_m$ for most hot sources after a short time then

$r^2 U w = F_o t$ and from equn. (18) $\quad dr/dt = \alpha w$ $\quad$ (24)

Hence

$$W = (4F_0/9U\alpha^2)^{1/3}\, t^{-1/3} \tag{25}$$

and

$$dz/dt = (4F_0/9U\alpha^2)^{1/3}\, t^{-1/3} + V_e(z) \tag{26}$$

Equn. (26) is then solved for z once $V_e(z)$ is known.

For any but the very simplest functional forms for $V_e(z)$ equn. (26) is difficult to solve. Hence it was decided to take the simplest form which most nearly approximated to the form suggested from similarity theory i.e. $V_e(z) \propto z^{1/3}$. For this reason a linear approximation was assumed where -

$$V_e(z) = -(K + \lambda_0 z), \qquad \lambda_0, K > 0 \text{ constants}$$

Fig. (4) shows two linear approximations used. Each one is applied according to the anticipated trajectory of the hot plume. For example, approximation (i) gives a fit within 5% of curve A over a height range 200-1500 feet and similarly approximation (ii) gives a fit within 5% over the range 200–2000 feet. So provided the plume centreline is reasonably well anticipated the approximation can be made with small error. All the linear approximations of course diverge near the ground. However the theory itself does not apply at the surface where theoretically the infinitesimally small thermals have infinitely large temperature excesses. Indeed it is probably more realistic to assume a finite velocity of descent at the ground in accordance with the concept of large thermals "bursting" from

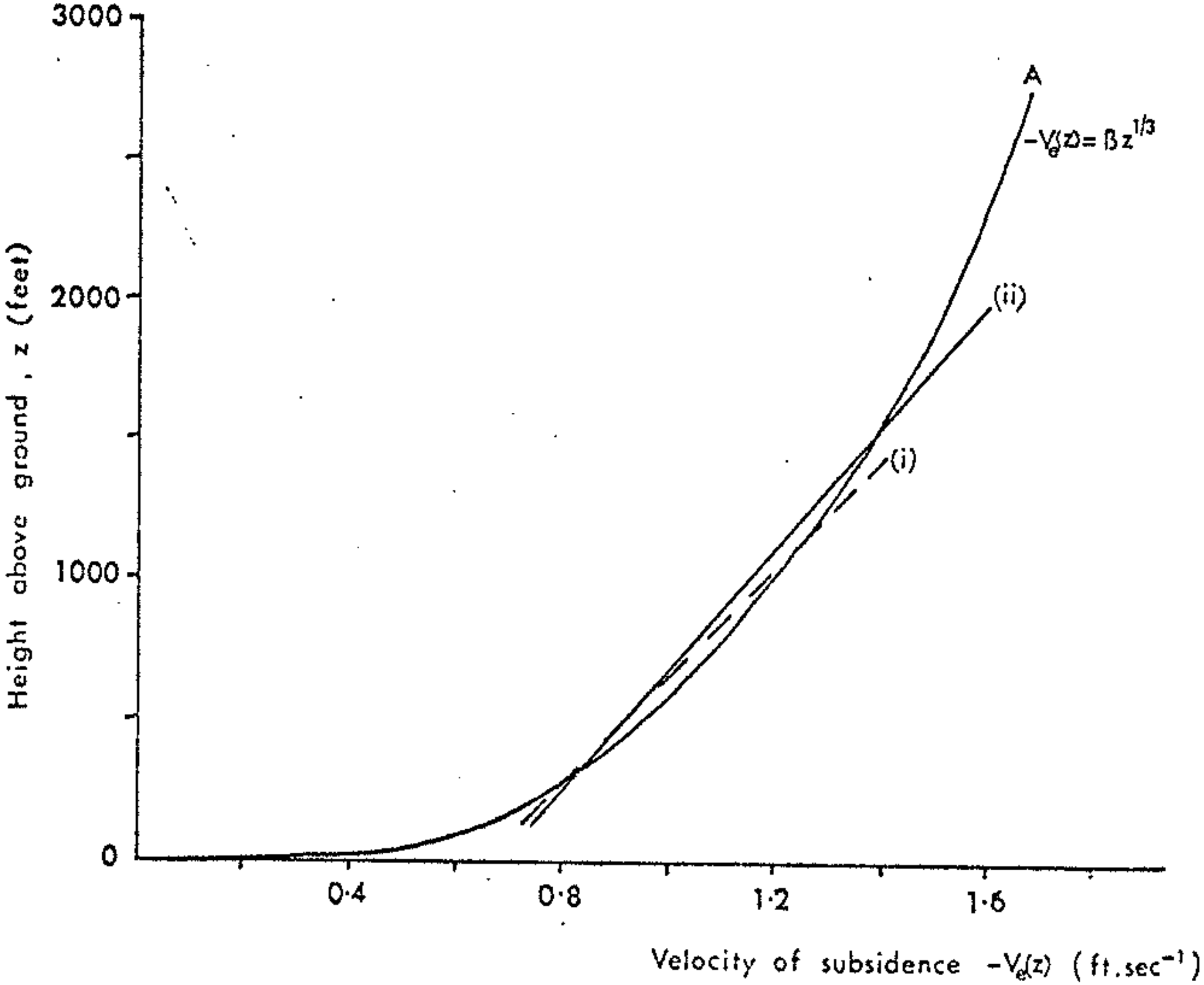

Fig (4) Velocity of subsidence of environment (curve A) for typically strong convective conditions. Curves (i) and (ii) are the linear approximations used to calculate the plume trajectory.

surface irregularities with non-zero velocities. Further, provided the plume expands and spends most of its trajectory above the surface layer, local anomolies at the surface are not expected to influence GLCs significantly. Hence we use a linear approximation in order to obtain a first rough approximation of the trajectory and so the height range of interest is established. Then a better linear fit is used to curve A to accord with the depth over which the trajectory is found by first approximation.

So , equn. (26) becomes

$$dz/dt + \lambda_0 z = -K + (4F_0/9U\alpha^2)^{1/3} t^{-1/3}$$

the solution of which is -

$$z = he^{-\lambda_0 t} - (K/\lambda_0)(1-e^{-\lambda_0 t}) + (4F_0/9U\alpha^2)^{1/3} e^{-\lambda_0 t} \int_0^t e^{+\lambda_0 s} s^{-1/3} ds$$

To evaluate the integral let $y = 1 - s/t$ then

$$z = he^{-\lambda_0 t} - (K/\lambda_0)(1-e^{-\lambda_0 t}) + (4F_0/9U\alpha^2)^{1/3} t^{2/3} \int_0^1 e^{-y(\lambda_0 t)} (1-y)^{-1/3} dy$$

and with $e^{-y\lambda_0 t} = \sum_{n=0}^{\infty} (-1)^n (y\lambda_0 t)^n / n!$ we have

$$z = he^{-\lambda_0 t} - (K/\lambda_0)(1-e^{-\lambda_0 t}) + (4F_0/9U\alpha^2)^{1/3} \sum_{n=0}^{\infty} (-1)^n (\lambda_0 t)^n \beta(n+1, 2/3)/n!$$

$$= he^{-\lambda_0 t} - (K/\lambda_0)(1-e^{-\lambda_0 t}) + (4F_0/9U\alpha^2)^{1/3} t^{2/3} \sum_{n=0}^{\infty} (-1)^n (\lambda_0 t)^n \frac{\Gamma(\frac{2}{3})}{\Gamma(n+\frac{5}{3})} \qquad (27)$$

Plume rise is assumed to terminate at some time $t = t_1$ and provide $\lambda_0 t_1$ is not too large the series converges quite rapidly. t_1 will be a function of the plume buoyancy and perhaps wind speed, stack height and turbulence depending on the criteria used for terminating the rise i.e. the plume rise formula used.

The alternative derivation - appropriate perhaps for larger sources - gives from equn. (21)

$$r^2 U \frac{\Delta\theta}{\theta} g = F_0 e^{-\bar{\lambda} t} \qquad (28)$$

$\bar{\lambda}$ is the average value of the gradient of $|V_e(z)|$ over the plume trajectory, and is taken as constant. As mentioned earlier, above the surface layers, this is not a bad approximation. As before a rough linear estimate of $V_e(z)$ and $\bar{\lambda}$ was taken. Having obtained the anticipated trajectory a better approximation is then made. $\bar{\lambda}$ is then taken as the average value of the gradient of $V_e(z)$ from the stack top to where the buoyant plume trajectory is terminated. i.e. with

$$V_e(z) = -\beta z^{1/3} \quad , \qquad \bar{\lambda} = \frac{\beta |z_1^{1/3} - h^{1/3}|}{|z_1 - h|}$$

where z_1 is the height at which the trajectory is terminated and h is the stack height. The plume trajectory is then recalculated and the second approximation proved satisfactory.

Equn. (19) gives $r^2 U w = F_m + \frac{F_0}{\bar{\lambda}}(1 - e^{-\bar{\lambda} t}) \xrightarrow{\text{large } t} \frac{F_0}{\bar{\lambda}}(1 - e^{-\bar{\lambda} t})$ (29)

Equns. (29) and (24) give

$$r = (3\alpha F_0 / 2U)^{1/3} t^{2/3} \left((2/\bar{\lambda} t) + (2/(\bar{\lambda} t)^2)(e^{-\bar{\lambda} t} - 1) \right)^{1/3} \qquad (30)$$

with initial conditions $r(t=0) = 0$.

So

$$w = \frac{1}{\alpha} \frac{dr}{dt} = \left((F_0 \bar{\lambda} / 9 U \alpha^2)^{1/3} (1 - e^{-\bar{\lambda} t}) \right) / (\bar{\lambda} t - 1 + e^{-\bar{\lambda} t})^{2/3} \qquad (31)$$

Hence plume trajectory is given by solution of

$$dz/dt + \lambda_o z = -K + \left(\left(F_o\bar{\lambda}/U\alpha^2 q\right)^{1/3}\left(1-e^{-\bar{\lambda}t}\right)\right) / \left(\bar{\lambda}t - 1 + e^{-\bar{\lambda}t}\right)^{2/3}$$

giving

$$z = he^{-\lambda_o t} - (K/\lambda_o)(1-e^{-\lambda_o t}) + \left(\frac{F_o\bar{\lambda}}{U\alpha^2 q}\right)^{1/3} e^{-\lambda_o t}\int_0^t \frac{(1-e^{-\bar{\lambda}s})e^{\lambda_o s}\,ds}{(\bar{\lambda}s - 1 + e^{-\bar{\lambda}s})^{2/3}} \quad (32)$$

The integral is evaluated as an infinite series by expanding the integrand. Expressions (27) and (32) may be written as

$$z = he^{-\lambda_o t} - (K/\lambda_o)(1-e^{-\lambda_o t}) + (3F_o/2U\alpha^2)^{1/3} t^{2/3} f(\lambda_o t) \quad (27a)$$

$$z = he^{-\lambda_o t} - (K/\lambda_o)(1-e^{-\lambda_o t}) + (3F_o/2U\alpha^2)^{1/3} t^{2/3} g(\lambda_o t) \quad (32a)$$

The first two terms represent the centreline of a passive plume emitted at height h added to which we have the reduced plume rise. Both equns. (27a) and (32a) reduce to the familiar plume rise trajectory as $\lambda_o \to 0$ and $\bar{\lambda} \to 0$ and $f(\lambda_o t)$, $g(\lambda_o t) \to 1$ with K/λ_o independent of λ_o

$$z = h + (3F_o/2U\alpha^2)^{1/3} t^{2/3} \quad (34)$$

Functions f and g are shown in fig. (5) for typical values of β, F_o and U. The effect of buoyancy depletion on the plume rise is negligible over the distances of interest. Typically with strong convection $\lambda_o = 0.5 \times 10^{-3}\ \mathrm{sec}^{-1}$ and even when

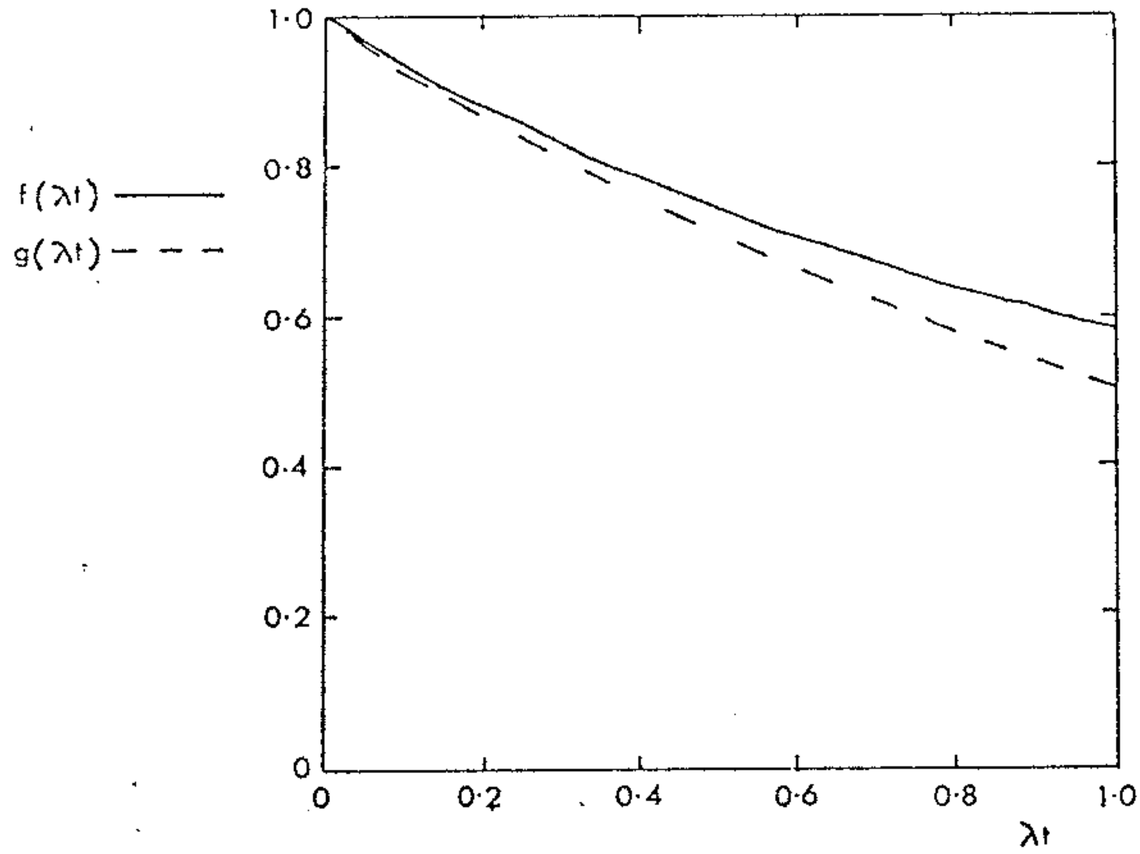

Fig (5) Functions g and f corresponding to plume rise with and without plume depletion respectively, for typical values of β, F_o and U. Typically, for strong convection, $\lambda = 0.5 \times 10^{-3}\ \sec^{-1}$.

t = 2000 seconds the influence of buoyancy depletion is small, reducing the plume rise as expected.

One result which becomes evident now is that if the plume continues to behave according to either equations (27a) or (32a) to great distances from the source then eventually the lower edge of the plume must reach the ground. This is because as the plume is diluted its relative rate of rise is reduced until it becomes about equal in magnitude to the velocity of subsidence of the environment whence through buoyant expansion it will soon reach the ground. At such a point the height of the plume will equal the plume "radius". For the purposes of this analysis we will assume that the plume has a top-hat profile and so maximum GLCs occur where it first reaches the ground. For a buoyant plume this is not such a drastic assumption to make and it has as much validity as the more usual Gaussian profile, in so far as neither profile really describes the shape configuration of a bent over buoyant plume. An advantage derived from describing the trajectory out to such large distances is that no artificial dodge need be made to terminate the rise in order to acquire the effective stack height from which GLCs can be calculated.

Suppose the plume reaches the ground at time $t = t^*$ then t^* is given from the solution of

$$r(t^*) = z(t^*) \tag{35}$$

$r(t)$ is given by either equations (24) and (25) or (30) and $z(t)$ given from either (27a) or (32a).

The solution of equn. (35) results in values of t^* which may be quite large especially for very hot sources and weak convection.

This suggests it is necessary to use equations ([illegible]) and ([illegible]) to describe the trajectory at these larger distances. It must further be added that because the equations describing the trajectory result in the plume remaining buoyant for considerable distances from the source there may be a conflict with some plume rise formulae. Some theories terminate the rise at distances much nearer the source, and if one decided to use these formulae then the appropriate procedure for calculating GLCs would be that discussed earlier in the passive plume theory. Under the type of conditions envisaged here i.e. light winds and low levels of turbulence within the *subsiding* environment then hot plumes might conceivably travel the distances required.

Hence from equns. (32a) and (30)

$$\left(\frac{3\alpha F_0}{2U\bar{\lambda}^2}\right)^{1/3}(\bar{\lambda}t^*)^{2/3} = \frac{h e^{-\lambda_0 t^*} - (K/\lambda_0)(1 - e^{-\lambda_0 t^*})}{\left[(2/\bar{\lambda}t^*) + 2(e^{-\bar{\lambda}t^*} - 1)/(\bar{\lambda}t^*)^2\right]^{1/3} - (c_j \lambda_0 t^*/\alpha)} \qquad (36)$$

Now λ_o, $\bar{\lambda}$ and K are all directly proportional to β. Hence given $(F_o/U\beta^2)$ there exists a unique value βt^* satisfying equn. (36).

With the top hat profile and assuming complete reflection at the ground then max GLCs are given from

$$C_{max} = \frac{2Q(1-\sigma)e^{-\bar{\lambda}t^*}}{U\pi r(t^*)^2} \cdot f \qquad (37)$$

The factor f takes into account wind directional changes causing lateral plume meandering over the sampling period. For sampling times of about 5 minutes $f \simeq 1$ and for periods of an hour - i.e. over the passage of many thermals $f \simeq \frac{1}{2}$. (N.B. f

is analogous to (σ_z/σ_y)). Plume depletion is accounted for by the exponential decay term and the probability that the plume will be emitted into the subsiding layer is quantified by the $(1-\sigma)$ factor. This follows in an identical way to that used in deriving equn. (10)

The point of max. GLC is given at

$$x^* = Ut^* \tag{38}$$

.4 An alternative model of buoyant plume behaviour

It is acknowledged above that the plume may be required to remain buoyant for periods longer than some popular plume rise theories predict. Hence an alternative treatment of calculating GLCs is now given where the plume rise is terminated nearer the source. Briggs (1969) developed a two-stage model of plume rise in which the initial plume trajectory is given by equn. (34). In the second stage mixing is dominated by the inertial subrange of atmospheric eddies. The final formula for the trajectory is complicated, but is simplified by Briggs when he notes that the final rise may be approximated by projecting the first phase trajectory up to a distance $x = 3x_R$ where x_R is the transition distance between both stages. Hence total rise plus stack height (h) is given by

$$H_1 = h + 1{\cdot}8 F_o^{1/3}(3x_R)^{2/3}/U, \quad \alpha = 0.5 \tag{39}$$

Now $x_R \propto F_o^{2/3} h^{3/5}$ and he notes that for many of the larger installations $3x_R \doteqdot 10h$. Then

$$H_1 = h + 1{\cdot}8 F_o^{1/3}(10h)^{2/3}/U \tag{40}$$

Implicit in equn. (40) is a proportional relationship between F_o and h, for larger plants. Here we shall extend equn. (40) to cover a wide range of heat emissions for a fixed chimney height (h = 400 feet). This will overestimate the plume rise from the smaller installations (assuming equn. (39) is true) but the differences within the model will be smaller than the differences which become apparent when using the procedure outlined earlier. Briggs's data upon which he bases equn. (39) barely covers distances greater than 4000 feet from the stack and it was not conclusively shown that any of the plumes from the smaller installations had terminated their rise before this distance. Hence there is as much justification to date in using equn. (40) with h = 400 feet as there is in using (39) exactly. The resulting formula is typical of other popular formula of the $1/u$ type e.g. Lucas (1967) takes plume rise $\propto F_o^{\frac{1}{4}} u^{-1}$.

Equns. (30) and (32a) given the final height and radius of the plume at time

$$t_T = x_T/U = 10h/U = 4000/U \quad \text{secs}$$

Hence

$$H_0 = 400\, e^{-\lambda_0 t_T} - \frac{K}{\lambda_0}\left(1 - e^{-\lambda_0 t_T}\right) + \left(\frac{3F_0}{2U\alpha^2}\right)^{1/3} t_T^{\,2/3}\, g(\lambda_0 t_T)$$

and

$$r(t_T) = \left(\frac{3\alpha F_0}{2U}\right)^{1/3} t_T^{\,2/3} \left[\left(2/\bar{\lambda} t_T\right) + \left(2/(\bar{\lambda} t_T)^2\right)\left(e^{-\bar{\lambda} t_T} - 1\right)\right]^{1/3}$$

To solve equns. (15), (16) and (17) we take

$$2\sigma_{z_0} = \sigma_{y_0} = 2r(t_T)\ , \quad a_1 = 0.1\ , \ a_0 = 0.05\ ; \ x_1 = 4000$$

$$\varepsilon = \bar{\lambda}\sigma_{z_0}/Ua_0\ , \ \delta = 1 - \varepsilon\ , \ \psi = \bar{\lambda}H_0/Ua_0 2\sqrt{2}\ ; \ \alpha = 0.5$$

5 Values of atmospheric parameters

Estimates of β where $V_e(z) = -\beta z^{1/3}$ and hence also λ_o, $\bar{\lambda}$ and K are now made.

From Scorer (1969) it can be shown that

$$V_e(z) = -\left(\frac{0.4\, g\, H_f}{T\rho\, C_p}\right)^{1/3} \cdot \left(\frac{\sigma}{1-\sigma}\right) z^{1/3}$$

where H_f is the heat flux from the surface, ρ is the density of air, c_p specific heat at constant pressure, T absolute temperature of the air, and the number 0.4 characterises the geometry of environmental thermals. For conditions of strong convection H_f might be 40 mw cm^{-2} giving

$$V_e(z) = -4.16\, z^{1/3}(\sigma/(1-\sigma)) \qquad \text{cm. sec}^{-1}$$

For a constant heat flux β increases as the proportion of sky occupied by thermals increases.

Also $w(z) = (1-\sigma)V_e(z)/\sigma$ where $w(z)$ is the velocity of ascent of the atmospheric thermals.

Values of $V_e(z)$ and w (z) for various σ are given in Table (1), for H_f = 40 mw cm^{-2}, and for heights of 100 and 500 m above ground.

Table (1)

σ		1/10	1/5	1/3
$V_e(z)$	100 m	10	22.9	45
cm s^{-1}	500 m	17.1	38.5	77
w(z)	100 m	89.7	89.7	89.7
cm s^{-1}	500 m	153	153	153

Scorer suggests that $\sigma < 1/3$ and so typical values of strong rates of subsidence would be 25 cm s^{-1} at 100 m above ground. The velocity of ascent of the thermals is close to values measured by [illegible] and Tellford (1957) of approximately 1 m. sec^{-1}.

6 Results of calculations of max. GLCs and distance of max. GLCs

Results derived from the theory leading to eqn. (35) are given in figs. (6) and (7). For this case given a chimney height h then $(C_{max} U\pi)/2Q(1-\sigma)$ becomes a function of a single parameter $\mu = F_0/U\beta^2$. We shall assume $r = 1/2$ and $\alpha = 0.5$

Curve I on fig. (6) shows $\Omega = (C_{max} U\pi)/2Q(1-\sigma)$ against μ. As μ decreases either through decreasing the heat content of the gases F_0, or increasing the intensity of subsidence β, or wind speed U, then Ω increases.

The curve on fig. (7) shows βt^* as a function of μ from which the point of max GLC can be found, for h = 400 feet. Comparison of the above results and those deduced from solutions of equations (15) and (16) are shown in fig. (6) by curve II. Here it is necessary to make estimates of the variables U, β etc. before a solution can be found for each case. Consequently typical values representing conditions of light winds and a strong velocity of subsidence were chosen. The values were –

$$U = 10 \text{ ft.s}^{-1}, \quad \beta = 0.12 \text{ ft}^{2/3} s^{-1}, \quad h = 400 \qquad (41)$$

The abscissa is given by the second scale which gives the heat content of the gases for these *fixed* meteorological conditions. The largest values are appropriate for large power stations. The results show that for the larger heat sources ground level concentrations are higher than those given in curve I. This is expected

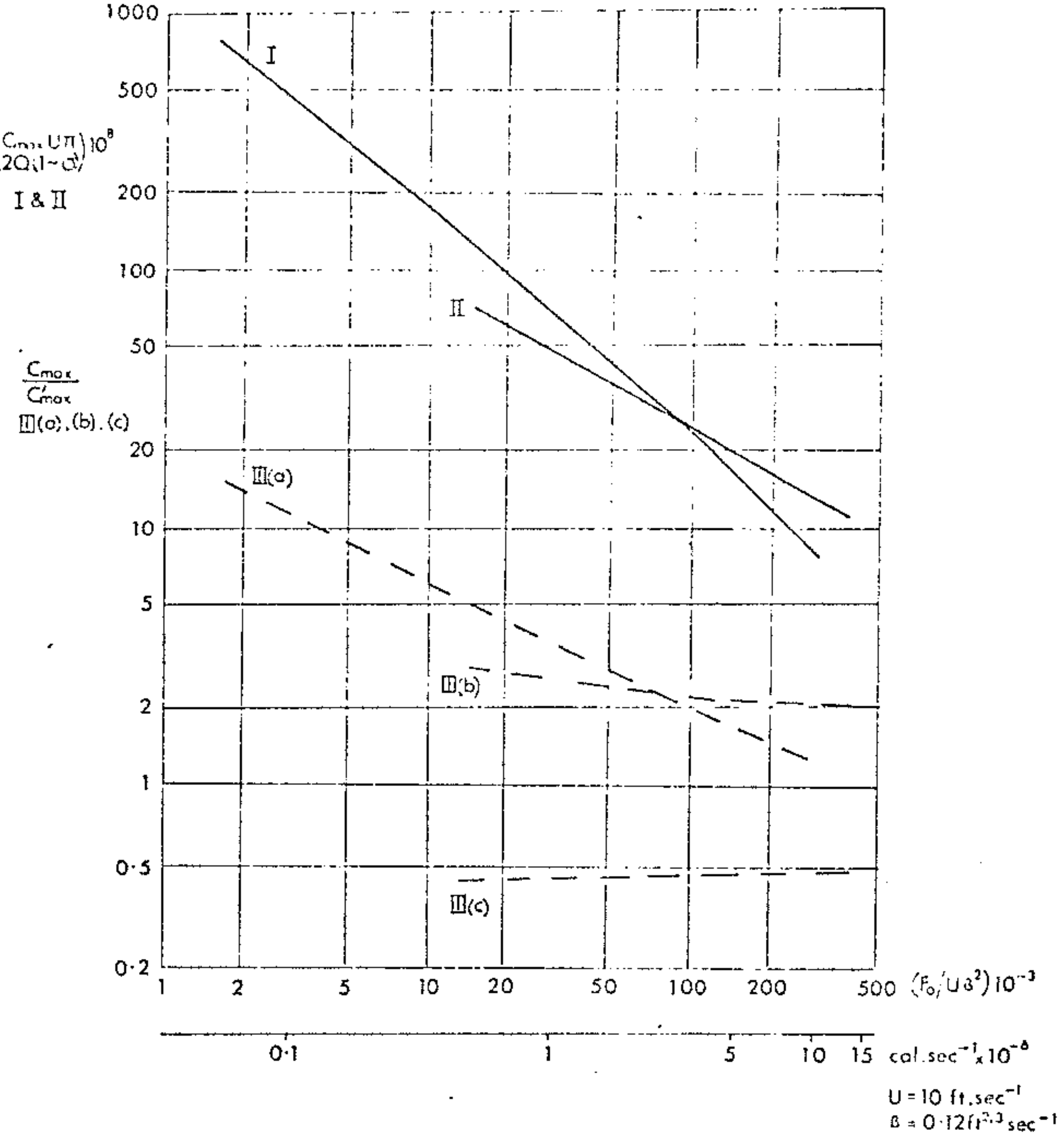

Fig (6) From curves I and II the maximum hourly ground level concentrations can be derived for given plume and atmospheric conditions. Curves III (a), (b), and (c) give the ratios of the maximum hourly ground level concentration with convection to that without convection. Two abscissa scales are given. The first is given in terms of $(F_o/U\beta^2)$ and refers only to curve I. The second scale is in terms of the initial heat content of the chimney gases for <u>fixed</u> values of U and β and refers to all the curves. Chimney height is 400 feet .

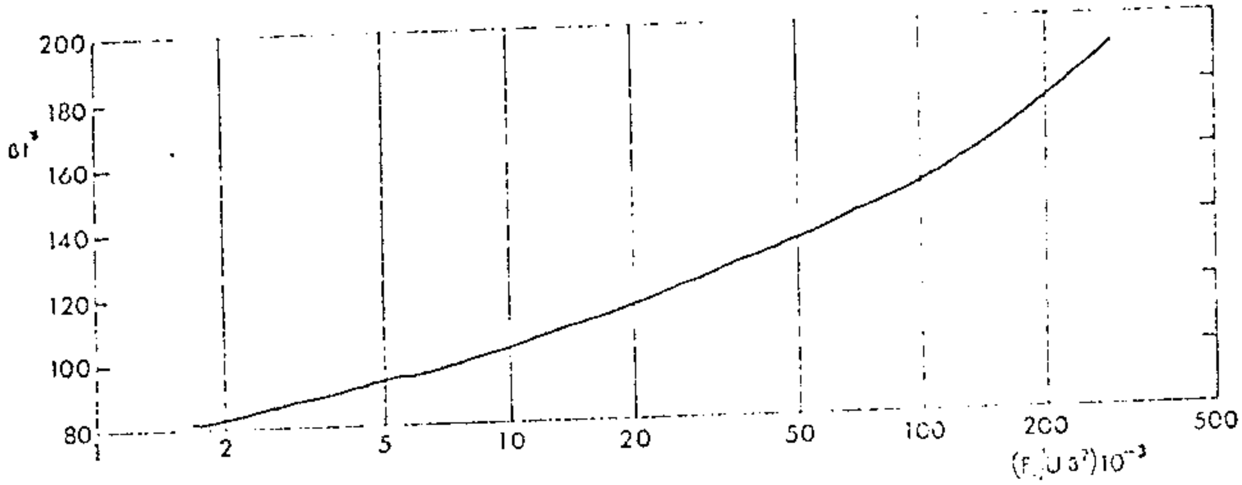

Fig (7) Time of travel to the point of maximum hourly ground level concentration for values of $(F_o/U\beta^2)$. Chimney height is 400 feet.

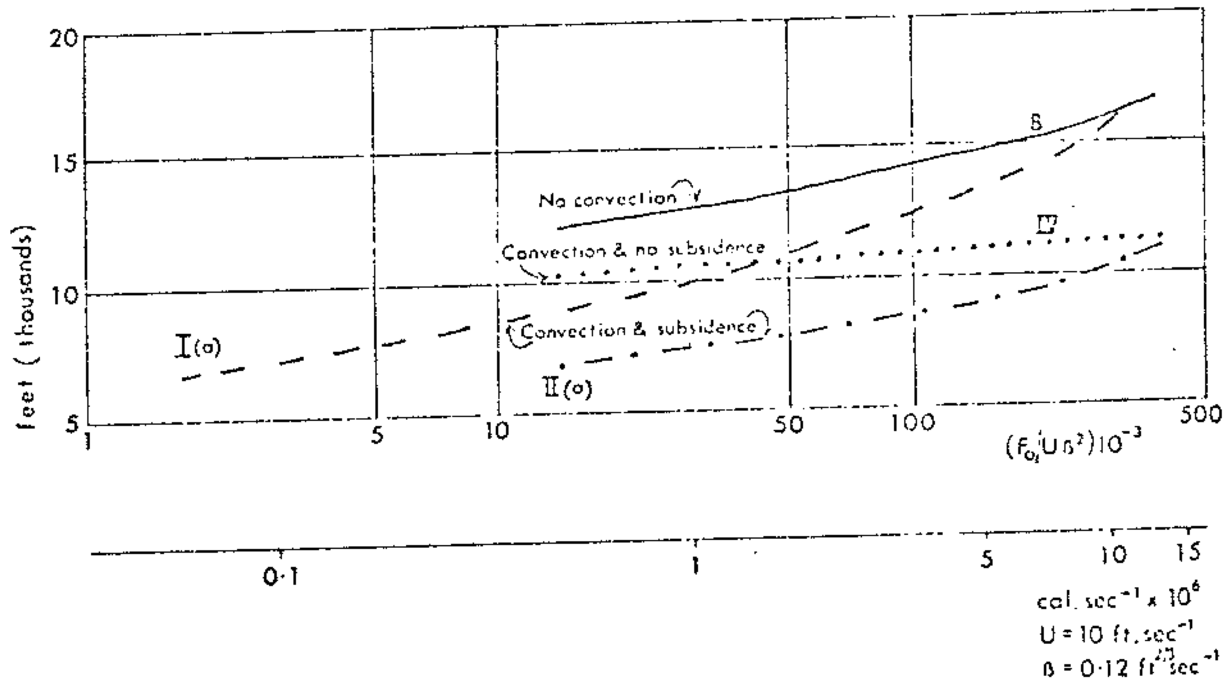

Fig. (8) Point of maximum hourly ground level concentration with and without convection. Two abscissa scales are shown for fixed values of U and β. The first gives values of $F_o/U\beta^2$ and the second is given in terms of the initial heat content of the chimney gases. Chimney height is 400 feet.

because the plumes are assumed to stop rising, when nearer the source.

The *effect* of convection is demonstrated by observing the ratio C_{max}/C'_{max} where C'_{max} is the max GLC obtained without convection and for comparative purposes again plume rise is assumed to terminate at a distance 10 stack heights (4000 feet) from the chimney.

Then with $\sigma_z = \frac{1}{2}\sigma_y = a_o x$ we have

$$C'_{max} = 2Q/(U\pi e H_1^2)\cdot(1/2)$$

H_1 given from equn. (40).

Fig. (6) shows the ratio with the ordinate scale remaining the same. Curves III(a) and III(b) show Cmax/C'max as a function of heat content of chimney gases, conditions (41) applying. These curves were compounded from curves I and II respectively. Both curves indicate that the larger heat sources are the least affected by convection. Even so for the larger sources in strong convective conditions it seems hourly max GLCs may be increased by a factor of 2 or 3.

Fig. (8) shows the distance of max GLC for convective and non-convective conditions. Without convection the point of max GLC is given by

$$X_{max} = \left(H_1/a_o\sqrt{2}\right) - X_o \tag{42}$$

where $\sigma_z/\sigma_y = \frac{1}{2}$, $\sigma_z = a_o x$ and x_o is the distance of the virtual origin to the real source. i.e. $x_o = (\sigma_{z_o}/a_o) - 4000$ (feet).

The abscissa is again given in terms of chimney heat flux for $U = 10$ ft/sec and $\beta = 0.12$ ft$^{2/3}$ sec^{-1}, $h = 400$ feet.

Curves I(a) and II(a) correspond to the max GLC as calculated from curves I and II in fig. (6). Curve B is the curve from equn. (42).

As expected, for curve II(a), the point of max GLC is much nearer the source in convective conditions due to subsidence. For curve I(a) the effect is not so marked because the plume is assumed to be continually rising.

7 Peak ground level concentrations

To estimate peak GLCs it is assumed that the gases in the subsiding environment are *not* depleted as they travel downwind i.e. they do not collide with any thermals. The probability that this occurs reduces as the chimney height or buoyancy flux of the gases is increased. Here because the plume always remains in the subsiding air we are essentially observing an instantaneous plume in so far as we are not integrating over the passage of many thermals. Hence, for the passive plume, the compression produced by subsidence not only reduces the vertical dimensions but increases the lateral spread. The motion is incompressible and so volumes are conserved. Hence the spread varies as -

$$d\sigma_z/dt = \left(d\sigma_z/dt\right)_{\bar{\lambda}=0} - \bar{\lambda}\sigma_z \qquad (43)$$

$$d\sigma_y/dt = \left(d\sigma_y/dt\right)_{\bar{\lambda}=0} + \bar{\lambda}\sigma_y \qquad (44)$$

With $(d\sigma_z/dt)_{\bar{\lambda}=0} = Ua_o$ and $(d\sigma_y/dt)_{\bar{\lambda}=0} = Ua_1$ then the solutions of equations (43) and (44) are

$$\sigma_z = \sigma_{z_0} e^{-\bar{\lambda}t} + (Ua_0/\bar{\lambda})(1 - e^{-\bar{\lambda}t}) \quad , \; \sigma_z(t=0) = \sigma_{z_0}$$

$$\sigma_y = \sigma_{y_0} e^{+\bar{\lambda}t} + (Ua_1/\bar{\lambda})(e^{+\bar{\lambda}t} - 1) \quad , \; \sigma_y(t=0) = \sigma_{y_0}$$

With σ_z and σ_y given above then axial GLCs are:-

$$C(x,0,0) = \frac{Q}{U\pi\sigma_y\sigma_z} e^{-H^2(x)/2\sigma_z^2} \qquad (45)$$

Where x is distance from where plume becomes passive = Ut .

Equn. (45) is identical to equn. (10) with the depletion terms missing. To find Cmax equn. (45) is differentiated w.r.t. x and equated to zero. For peak GLCs $a_0/a_1 \doteq 1$.

The alternative and less tedious approach is again to suppose the plume remains buoyant until its lower edge reaches the ground. Here the appropriate equations are (24), (25) and (27a). Because the buoyant plume is not depleted, it travels further before reaching the ground.

Then $r(t^*) = z(t^*)$ gives

$$\left(\frac{3\alpha F_0}{2U\lambda_0^2}\right)^{1/3} (\lambda_0 t^*)^{2/3} = \frac{h e^{-\lambda_0 t^*} - (K/\lambda_0)(1 - e^{-\lambda_0 t^*})}{(1 - (f(\lambda_0 t)/\alpha))}$$

and peak max GLCs given by

$$C_{max}^{peak} = \frac{2Q}{U\pi r(t^*)^2} \cdot f \quad \text{with} \quad f = 1$$

Peak to Mean max GLCs calculated over the whole range of heat emissions in fig. (4) are practically constant at a value of 4, although the point of peak max. GLC occurs slightly further from the chimney because the gases remain buoyant longer.

Peak max GLCs derived above may also be indicative of an environment which is gradually subsiding without any thermal activity taking place e.g. anticyclonic conditions. Assuming that the subsiding environment is still well described through $V_e(z) \propto z^{1/3}$ then max. 1 hourly GLCs would be about $\frac{1}{2}$ of the peak values given here, for these conditions.

8 Thermal convection without environmental subsidence

It is also possible to have a convective atmosphere in which there is *no* subsidence, as described in the introduction. Here the loss of air to the thermals is compensated for by a horizontal inflow into the convection field. If we assume that the same amount of material is entrained into the thermals as before then we can derive results for Cmax etc. Here we are obliged to use the two phase plume model because without subsidence a rising hot plume will not reach the ground in the manner described before. The plume trajectory is given by equn. (31) with $w = \frac{dz}{dt}$ the *actual* velocity of rise. So from equn. (31) we have

$$H_0 = h + \left(3F_0/2U\alpha^2\right)^{1/3} t_T^{2/3}\left(\left(2/\bar{\lambda}t_T\right) + \left(2/(\bar{\lambda}t_T)^2\right)\left(e^{-\bar{\lambda}t_T}-1\right)\right)^{1/3}$$

and plume rise is again terminated when $t_T = 10h/U$.

Strictly speaking the termination of plume rise criteria only applies to plumes in a non-convective field with no buoyancy depletion but the errors involved are insignificantly small.

Axial max GLCs are given by

$$C(x,0,0) = \frac{Q(1-\sigma)}{U\pi\sigma_y\sigma_z} \cdot e^{-\bar{\lambda}x/U} e^{-H_0^2/2\sigma_z^2} e^{\bar{\lambda}t_0}$$

x is measured from the *virtual* source. t_o is the time taken for the plume to travel from the virtual to the real source. $Qe^{\bar{\lambda}t_o}$ is the virtual source strength to take into account the fact that the plume does not undergo any depletion between the real and virtual chimney.

Here we have taken $\sigma_z = a_o x$, $\sigma_y = a_1 x$, $\sigma_{z_o}/\sigma_{y_o} = a_o/a_1$ and with

$$\phi = \sqrt{2}a_0 x/H_0 \quad , \quad \psi = \bar{\lambda}H_0/U a_0 2\sqrt{2}$$

for max GLCs $dC/dx = 0$ gives

$$\phi_m^2 + \psi\phi_m^3 = 1 \qquad (4.6)$$

where $\phi_m = a_0\sqrt{2}x_m/H_0$ and so

$$C_{max} = \frac{2Q(1-\sigma)}{U\pi H_0^2\phi_m^2}\frac{a_0}{a_1} \cdot e^{-(1+3\psi\phi_m)} e^{\bar{\lambda}t_0}$$

and

$$\frac{C_{max}}{C'_{max}} = \left(\frac{H_1}{H_0}\right)^2 \phi_m^{-2} e^{-3\psi\phi_m} e^{\bar{\lambda}t_0}\left(\frac{a_1}{a_0}\right)(1-\sigma) \qquad (4.7)$$

t_o is given from $t_o = u^{-1}(r(t_T)/a_o - 10h)$.

Hence solving equn. (4.6) for ϕ_m given ψ then equn. (4.7) is obtained.

Values of the ratio equn. (4.7) are shown in fig. (6) as a function of heat emission by curve III(c). Here we have taken

U = 10 ft/sec, β = 0.12 $ft^{2/3}$ sec^{-1} and h = 400 feet. The results show that max. GLCs will be reduced by up to 50%. The slight increase in the ratio with heat emission reflects the fact that the rate of depletion decreases with height above ground. Also the point of max. GLC approaches nearer the chimney. Curve IV in fig. (8) shows x max against heat emission with the above fixed meteorological parameters.

The greater vertical expansion ensures that x max may still approach as near to the chimney as observed for the case *with* subsidence (i.e. curve IIa).

9 Effect of increasing chimney height on max. GLCs

Increasing the chimney height will reduce max GLCs, the reduction being greatest for the smaller heat sources when plume rise is small. For large sources, because the plume rise is large, especially in light winds the advantages of increasing the stack height are small. These remarks apply to non-convective as well as convective conditions. With strong convection the improvements in increasing stack height will be greater because again plume rise is reduced. So assuming the plume remains buoyant until it reaches the ground the ratio of max GLCs for a 800 foot to a 400 foot chimney are shown in table (2). The ratios are given for various values of the parameter $(F_o/U\beta^2)$ and in terms of heat content of gases for U = 10 ft sec^{-1} and β = 0.12 $ft^{2/3}$ sec^{-1}. The ratios for a non-convective atmosphere are also shown for comparative purposes for the same fixed meteorological conditions.

As expected the improvement decreases the larger the source, or weaker the convection. The improvements are comparable with those expected in a non convective atmosphere, however in absolute terms the reduction in max GLCs is much greater for a convective atmosphere.

TABLE (2)

Ratios of max GLCs for a 800 foot to a 400 foot chimney

$10^{-3}F_o/\beta^2 u$	2.98	29.8	[illegible]98.1
Heat content cal s^{-1} (u,β fixed)	10^5	10^6	10^7
Ratio With convection	0.56	0.67	0.81
Ratio Without convection	0.42	0.55	0.69

10 Observations of plumes in convective conditions

Halliday (1968) presents measurements of the rise of plumes and GLCs around a power plant in Pretoria. The plumes were studied during summer afternoons under strong convective conditions. Plume trajectories were noted to be often convex rather than concave to the ground the reason being that thermals often take plume segments to greater heights. A series of photographs identifies this phenomenon quite clearly. It was noted that those segments not carried aloft reached the ground relatively near to the stack, in the manner suggested by the theory. Halliday compared his plume rise measurements with the Lucas Moore Spurr (1963) formula and demonstrated that the plume rise here was higher! Later he notes that this result came from including in the measurement of plume rise the upward convected plume elements and suggests that the "effective stack" height or that needed to calculate GLCs must be lower. This conclusion is borne out by measurements of GLCs made around the stack. His table 4 shows max. GLCs occurring between 800 and 1600 m from the chimney. (Plume rise measurements were made at 305 m and 1000m downwind - hence the apparent contradictions.) The chimney here has very much the same dimensions as the Tilbury plant discussed in Chapter 2. Both power plants have emissions of about 22 MW per stack. The Tilbury stack is 100 m high and the CASOL stack 76.3 m. From the data of Moore (1974) max. GLCs in the range of wind speeds 3-7 m s^{-1} occur at about 4,000 m from the Tilbury stack or at 4 times the distance from the CASOL chimneys.

It is also interesting to note that Halliday discarded nearly 1/3 of his plume rise measurements made at 305 m downwind in the wind speed class 2.68 m s^{-1}. He states that "In some cases the plume travelled with a fairly constant gradient, but in seven cases the plume rose from the stack vertically to almost its maximum height and then bent over. Thus the height at 305 m (downwind) was almost equal to the height at 1000 m (downwind)" This suggests that some of those 7 out of 23 cases were probably examples of the plume being emitted into a thermal and suggests values of $\sigma \leq 1/3$ here.

.11 Summary and discussion of results

We have examined the consequences of a special type of convection on the behaviour of hot plumes emitted from tall chimneys. The type of convection envisaged is that in which thermals, originating at the ground, transport heat and other surface constituents to great heights. As they rise they amalgamate with each other and entrain air at all levels. The environment - that is the atmosphere excluding the thermals - is assumed to be slowly subsiding, compensating for the mass flux from the surface. The convection region is assumed to be similar at all heights and from the similarity laws governing the dynamics of isolated thermals we obtain $V_e(z) = -\beta z^{1/3}$, where $V_e(z)$ is the subsiding velocity of the environment, z height above ground and β a constant.

The chimney plume is assumed either to be emitted into a thermal, in which case it does not reach the ground, or into the subsiding environment. In the latter case it is depleted by thermals as it travels downwind and when passive, the plume is compressed vertically. If the plume remains buoyant long enough then max. GLCs can be parameterised by a single number F_o/US^2 , where F_o is the initial buoyancy flux parameter of the chimney plume, and U the wind speed. It is found that max GLCs are increased over those calculated for a neutral environment. The effect of the convection increases as F_o decreases. Alternatively if the plume is assumed to become passive at a fixed distance downwind max GLCs are still increased.

It is realised that there may be occasions when there is thermal convection but no subsidence - e.g. at coastal sites where sea breezes may replenish the mass flux from the surface. Max. GLCs are decreased for these occasions.

The study represents a departure from conventional analyses where higher max. GLCs are derived by assuming larger value of σ_z . The treatment here also provides explanations for occasions when max GLCs may be reduced. Here we have assumed a non-preferential distribution of thermals. If the chimney is situated near a hot spot favourable for the formation of thermals then the plume may more often than not become entrained into them on emission and may rarely reach the ground. Also it is possible that for the larger sources the plume itself may entrain the environmental thermals becoming indistinguishable from them and rising to great heights. Conversely the plant may be situated in an area not conducive for the formation of thermals and the plume will be emitted into the subsiding air most of the time.

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