Scaling effects for piezoelectric energy harvesters

Zhu, D. and Beeby, S. P.

Author post-print (accepted) deposited in CURVE January 2016

Original citation & hyperlink:
Zhu, D. and Beeby, S. P. (2015) 'Scaling effects for piezoelectric energy harvesters' In: Proceedings of SPIE , ' Smart Sensors, Actuators, and MEMS VII; and Cyber Physical Systems'. Held 4 May 2015 at Barcelona, Spain.
http://dx.doi.org/10.1117/12.2178877

ISSN 0277-786X
DOI 10.1117/12.2178877

Copyright © and Moral Rights are retained by the author(s) and/ or other copyright owners. A copy can be downloaded for personal non-commercial research or study, without prior permission or charge. This item cannot be reproduced or quoted extensively from without first obtaining permission in writing from the copyright holder(s). The content must not be changed in any way or sold commercially in any format or medium without the formal permission of the copyright holders.

This document is the author's post-print version, incorporating any revisions agreed during the peer-review process. Some differences between the published version and this version may remain and you are advised to consult the published version if you wish to cite from it.

CURVE is the Institutional Repository for Coventry University
http://curve.coventry.ac.uk/open

Scaling effects for piezoelectric energy harvesters

D. Zhu*, S. P. Beeby
Electronics and Computer Science, University of Southampton, Southampton, SO171BJ, UK

ABSTRACT

This paper presents a fundamental investigation into scaling effects for the mechanical properties and electrical output power of piezoelectric vibration energy harvesters. The mechanical properties investigated in this paper include resonant frequency of the harvester and its frequency tunability, which is essential for the harvester to operate efficiently under broadband excitations. Electrical output power studied includes cases when the harvester is excited under both constant vibration acceleration and constant vibration amplitude. The energy harvester analysed in this paper is based on a cantilever structure, which is typical of most vibration energy harvesters. Both detailed mathematical derivation and simulation are presented. Furthermore, various piezoelectric materials used in MEMS and non-MEMS harvesters are also considered in the scaling analysis.

Keywords: scaling, piezoelectric, vibration energy harvesting

1. INTRODUCTION

A vibration energy harvester is a device that converts ambient mechanical vibration into electrical energy using certain transduction mechanisms. It can be used to power small electronic systems, such as wireless sensors, in environment where mechanical vibration is available. Systems powered by vibration energy harvesters tend to have longer life time and require less maintenance compared to battery powered systems. With latest development in microelectronics, sizes of wireless sensors are shrinking significantly. Therefore, there is an increasing demand to minimize vibration energy harvesters. In order to design effective energy harvesters to meet such requirements, it is crucial to understand scaling effects for vibration energy harvesters.

Common transducers used in energy harvesting include electromagnetic, electrostatic and piezoelectric[1]. Electromagnetic energy harvesters normally require bulky permanent magnets to provide a strong magnetic field to maximize their output power. Although magnets can be fabricated using standard MEMS process[2], their performance is very poor compared to conventional magnets. Furthermore, the performance of MEMS electromagnetic vibration energy harvesters is also limited by the use of micro-fabricated coils due to their large coil resistances. The scaling effect for electromagnetic energy harvesters were studied and reported by O'Donnell *et al*[3]. Electrostatic transducer, on the other hand, is easily achievable using MEMS fabrication process. However, electrostatic energy harvesters normally require an initial polarizing voltage or charge, which limits their applications. Piezoelectric transducer has received special attention[4] due to its high energy density and easy fabrication process[5]. More importantly, fabrication process of piezoelectric energy harvesters can also be easily integrated with traditional MEMS fabrication. This makes it a very promising candidate for providing localised power source for MEMS systems.

This paper studies scaling effects for piezoelectric vibration energy harvesters in terms of their mechanical properties and electrical output power. The mechanical properties investigated in this paper include resonant frequency of the harvester and its frequency tunability, which is important for maximizing output power under broadband excitations[6]. Electrical output power studied includes that under constant vibration acceleration and constant vibration amplitude. Various piezoelectric materials used in MEMS and non-MEMS harvesters are also considered in the scaling analysis. The energy harvester structure analysed is based on a cantilever structure, which is typical of most vibration energy harvesters.

2. MECHANICAL PROPERTIES

Most reported vibration energy harvesters consist of a cantilever structure. It can be modelled using a spring-mass-damper system. Such a system has a resonant frequency. When the resonant frequency of the vibration energy harvester

*dz@ecs.soton.ac.uk; phone +44(0)23 8059 5161; ecs.soton.ac.uk

matches ambient vibration frequency, the output power of the harvester is maximal. In applications where vibration frequency varies occasionally, resonant frequency of the harvester needs to be tuned accordingly in order to maximize its output power. In this section, scaling effects for resonant frequency and frequency tunability of cantilever structures will be studied. Results in this section apply to any cantilever based kinetic energy harvesters, including piezoelectric, electromagnetic and electrostatic etc.

2.1 Resonant frequency

The spring constant of a resonator depends on its materials and dimensions. For a cantilever beam with an inertial mass at the free end as shown in Figure 1, its spring constant is given by[7]:

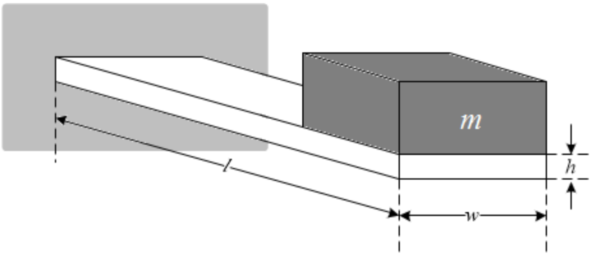


Figure 1. A cantilever beam structure.

$$k_s = \frac{Ywh^3}{4l^3} \tag{1}$$

where Y is Young's modulus of the beam material, w, h and l are the width, thickness and length of the cantilever beam, respectively. The resonant frequency of the resonator, f_r, is given by:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k_s}{m + 0.24m_c}} \tag{2}$$

where m is the inertial mass and m_c is the mass of the cantilever beam.

If the cantilever beam is scaled by a times (the structure is scaled up when $a > 1$ and the structure is scaled down when $0 < a < 1$), the scaled spring constant, k_{s_s}, is given by

$$k_{s\_s} = \frac{Y(a \cdot w)(a \cdot h)^3}{4(a \cdot l)^3} = a \cdot k_s \tag{3}$$

In the meantime, inertial mass, m_s, and mass of the cantilever beam, m_{c_s} are also scaled as

$$m_s = a^3 m \tag{4}$$

$$m_{c\_s} = a^3 m_c \tag{5}$$

Thus, the resonant frequency of the scaled cantilever beam, f_{r_s}, becomes

$$f_{r\_s} = \frac{1}{2\pi}\sqrt{\frac{k_s}{m' + 0.24m_c'}} = \frac{1}{a} \cdot f_r \tag{6}$$

It can be found in Eq. 6 that the resonant frequency increases as the structure is scaled down and decreases as the structure is scaled up.

2.2 Frequency tuning

Resonant frequency of a cantilever structure can be tuned by applying an axial load as shown in Figure 2.

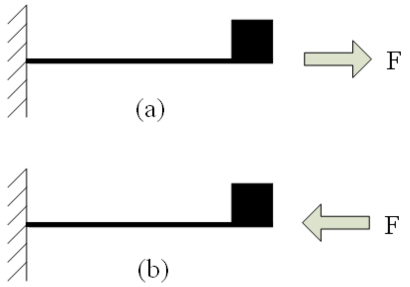


Figure 2. Resonant frequency of a cantilever structure can be tuned by applying an axial force (a) tensile (b) compressive.

The formula for the resonant frequency of a uniform cantilever in mode 1 with an axial load, f_{r1}', is given by[7]:

$$f_{r1}' = f_{r1} \cdot \sqrt{1 + \frac{F}{F_b}} \tag{7}$$

where f_{r1} is the resonant frequency in mode 1 without load and F is the axial load. F is positive if the load is tensile and F is negative if the load is compressive. F_b is the compressive axial load required to buckle the beam, i.e. when the resonant frequency drops to zero. The tuning ratio, which is defined as the ratio of the tuned frequency to the original frequency, of a cantilever beam is given by

$$\frac{f_{r1}'}{f_{r1}} = \sqrt{1 + \frac{F}{F_b}} \tag{8}$$

The buckling load F_b is given by:

$$F_b = \frac{\pi^2 \cdot Y \cdot w \cdot h^3}{48 \cdot l^2} \tag{9}$$

If the cantilever beam is scaled by a times (the structure is scaled up when $a > 1$ and the structure is scaled down when $0 < a < 1$), the buckling force for the scaled resonator, F_{b_s}, is given by

$$F_{b\_s} = \frac{\pi^2 Y (a \cdot w) \cdot (a \cdot h)^3}{48 (a \cdot l)^2} = a^2 \cdot F_b \tag{10}$$

It can be found in Eq. 10 that the buckling force of the cantilever beam increases as the structure is scaled up and decreases as the structure is scaled down.

The formula for the resonant frequency of a scaled cantilever beam in mode 1 with an axial load, f_{r1_s}', is given by:

$$f_{r1\_s}' = f_{r1\_s} \cdot \sqrt{1 + \frac{F}{F_{b\_s}}} \tag{11}$$

The tuning ratio of the scaled cantilever beam is given by

$$\frac{f_{r1\_s}'}{f_{r1\_s}} = \sqrt{1 + \frac{F}{a^2 \cdot F_b}} \tag{12}$$

Figure 3 shows the tuning ratio of various scaled cantilever beams. It is found that the buckling force is reduced and thus structures can bear less compressive forces as the structure is scaled down. Meanwhile, when the structure is scaled down, it can be tuned to a higher resonant frequency under the same tensile force.

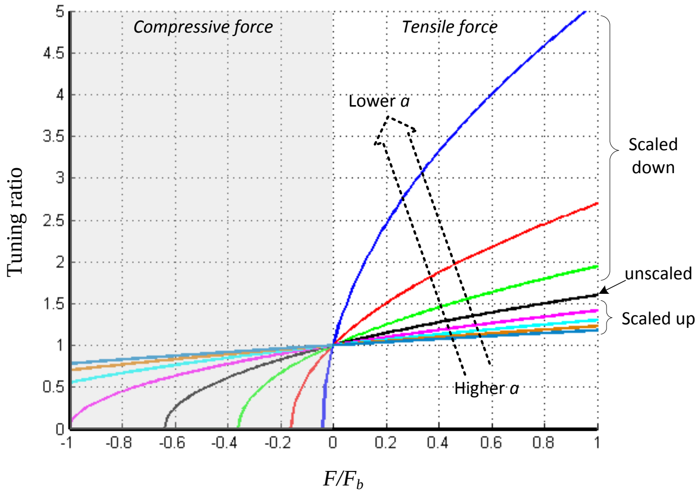


Figure 3. Tuning ratios of various scaled cantilever beams.

2.3 Simulation

As an example, a cantilever (device 0) with dimensions of 3 mm (length) × 2.7 mm (width) × 48 µm (thickness) was used in the simulation. Its mass dimensions are 1 mm (length) × 2.7 mm (width) × 0.2 mm (thickness). Device 1 is a cantilever that is scaled up based on device 0 by 10 times while device 2 is a cantilever that is scaled down based on device 0 by 10 times.

Simulation was carried out in ANSYS. Simulation results show that the resonant frequencies of device 0, 1, 2 are 696.781 Hz, 69.679 Hz and 6967.9 Hz, respectively. This agrees with Eq. 6 that resonant frequency is inversely proportional to the scale factor, a. The buckling forces of device 0, 1, 2 are 0.5072 N, 50.72 N and 0.005072 N, respectively. This agrees with Eq. 10 that buckling force is proportional to the scale factor squared, a^2.

Figure 4 compares tuning ratio of devices 0, 1, 2. F_{b0} is the buckling force of device 0. Simulation results agree with the trend shown in Figure 3.

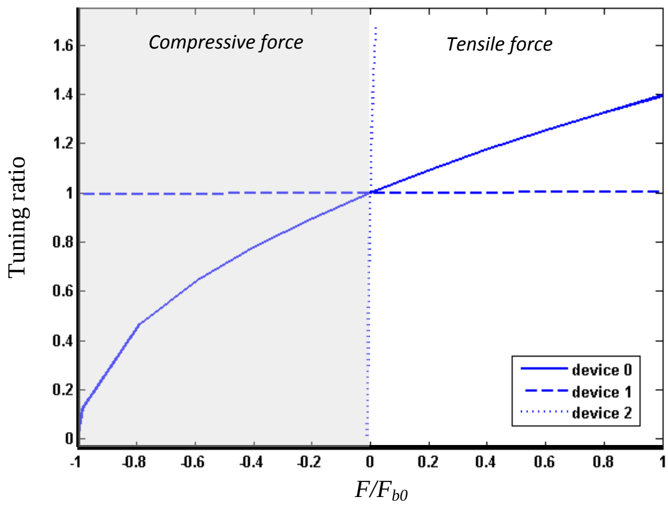


Figure 4. Comparisons of tuning ratios of scaled piezoelectric energy harvesters.

3. ELECTRICAL POWER

In this section, scaling effect of electrical output power of piezoelectric energy harvester is investigated. Output power of a cantilever based bimorph piezoelectric energy harvester as shown in Figure 5 when connected to a resistive load, R_L, at resonance is given by[8]:

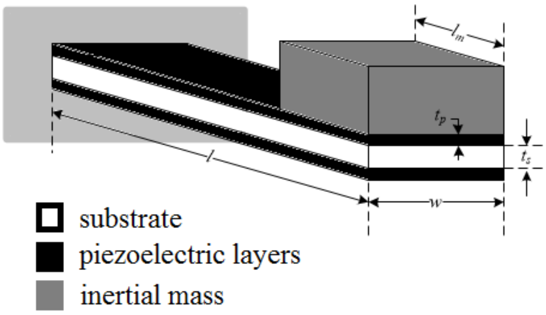


Figure 5. A cantilever based bimorph piezoelectric energy harvester.

$$P = \frac{R_L C_p^2 \left(\frac{2Y_p d_{31} t_p \gamma}{\varepsilon} \right)^2}{\omega_r^2 \left[\left(4\zeta^2 + k^4\right)\left(R_L C_p \omega_r\right)^2 + 4\zeta k^2 \left(R_L C_p \omega_r\right) + 4\zeta^2 \right]} \cdot A_{in}^2 \tag{13}$$

where Y_p is the Young's modulus of the piezoelectric material, t_p is the thickness of one piezoelectric layer. d_{31} is strain coefficient in 31 mode. ε is dielectric constant of the piezoelectric material. ω_r is the resonant frequency. ζ is the damping ratio. k is the coupling factor. C_p is the capacitance of one piezoelectric layer, respectively. A_{in} is the vibration acceleration. γ is the ratio of strain to vertical displacement which is given by:

$$\gamma = \frac{3\left(l + l_m\right)}{l^2 \left(2l + 1.5 l_m\right)} \cdot \frac{Y_s}{Y_p} \cdot \frac{t_s \left(t_p + t_s\right)}{2\left(t_p + \frac{Y_s}{Y_p} \cdot t_s \right)} \tag{14}$$

where l_m is the length of the inertial mass and Y_s is the Young's modulus of the substrate. t_s and t_p are thickness of the piezoelectric layer and substrate respectively.

The optimal load resistance can be found by differentiating Eq. 13 and is given by:

$$R_{opt} = \frac{1}{\omega_r C_p} \frac{2\zeta}{\sqrt{4\zeta^2 + k^4}} \tag{15}$$

Substitution of Eq. 16 into Eq. 13 results in the output power of the harvester when connected to its optimal load as:

$$P_{opt} = \frac{C_p \left(\frac{2Y_p d_{31} t_p \gamma}{\varepsilon} \right)^2}{\omega_r^3 \zeta \left(3\sqrt{4\zeta^2 + k^4} + 4k^2 \right)} \cdot A_{in}^2 \tag{16}$$

If the piezoelectric energy harvester is scaled by a times (the structure is scaled up when $a > 1$ and the structure is scaled down when $0 < a < 1$), the ratio γ becomes:

$$\gamma_s = \frac{3a(l+l_m)}{(a\cdot l)^2 a\cdot(2l+1.5l_m)}\cdot\frac{Y_s}{Y_p}\cdot\frac{a^2\cdot t_s(t_p+t_s)}{2a\left(t_p+\frac{Y_s}{Y_p}\cdot t_s\right)} = \frac{\gamma}{a} \tag{17}$$

and the capacitance of one piezoelectric layer, C_{p_s} becomes

$$C_{p\_s} = \frac{\varepsilon(al)(aw)}{at_p} = aC_p \tag{18}$$

For a given piezoelectric material, the coupling coefficient, k, is constant. It is assumed that the damping ratio, ζ, remains constant for simplicity in analysis.

Therefore, the optimal load resistance of the scaled piezoelectric harvester, R_{opt_s}, is:

$$R_{opt\_s} = \frac{1}{\frac{\omega_r}{a}\cdot aC_p}\frac{2\zeta}{\sqrt{4\zeta^2+k^4}} = R_{opt} \tag{15}$$

3.1 Constant vibration acceleration

If the scaled harvester is excited under the same vibration acceleration as the unscaled harvester and assuming same substrate and piezoelectric materials are used, substitution of Eqs 3, 6, 17 and 18 into Eq. 16 leads to the output power of the scaled piezoelectric energy harvester at resonance, P_{opt_s}, as:

$$P_{opt\_s} = \frac{aC_p\left(\frac{2Y_p d_{31} a t_p \frac{\gamma}{a}}{\varepsilon}\right)^2}{\left(\frac{\omega_r}{a}\right)^3 \zeta\left(3\sqrt{4\zeta^2+k^4}+4k^2\right)}\cdot A_{in}^2 = a^4 P_{opt} \tag{19}$$

3.2 Constant vibration amplitude

Since vibration acceleration $A_{in} = Y\omega_r^2$, where Y is the vibration amplitude, Eq. 16 can be written as

$$P_{opt} = \frac{C_p\left(\frac{2Y_p d_{31} t_p \gamma}{\varepsilon}\right)^2}{\omega_r^3 \zeta\left(3\sqrt{4\zeta^2+k^4}+4k^2\right)}\cdot\left(Y\omega_r^2\right)^2 = \frac{\omega_r C_p\left(\frac{2Y_p d_{31} t_p \gamma}{\varepsilon}\right)^2}{\zeta\left(3\sqrt{4\zeta^2+k^4}+4k^2\right)}\cdot Y^2 \tag{20}$$

If the scaled harvester is excited under the same vibration amplitude as the unscaled harvester, the output power of the scaled piezoelectric energy harvester at resonance, P_{opt_s}', is:

$$P_{opt\_s}' = \frac{\left(\frac{\omega_r}{a}\right) aC_p\left(\frac{2Y_p d_{31} a t_p \frac{\gamma}{a}}{\varepsilon}\right)^2}{\zeta\left(3\sqrt{4\zeta^2+k^4}+4k^2\right)}\cdot Y^2 = P_{opt} \tag{21}$$

3.3 Simulation

Simulation was conducted in ANSYS with direct coupled field analysis between the mechanical and piezoelectric domain. Together with coupled physics circuit simulation in ANSYS, the performance of the piezoelectric energy harvester can be fully simulated[9].

Figures 6 and 7 compare output power of devices 0, 1, 2 when connected to various load resistances the constant vibration acceleration and amplitude, respectively. Output power and load resistances are normalized to those of device 0. It was found that, under constant vibration acceleration excitation, the optimum load resistances are the same for all three devices while the maximum output power is proportional to fourth powers of scaling factor, *a*. When excited under constant vibration amplitude, both optimum load resistances and maximum output power of the three devices are the same. Simulation results agree with Eqs. 19 and 21.

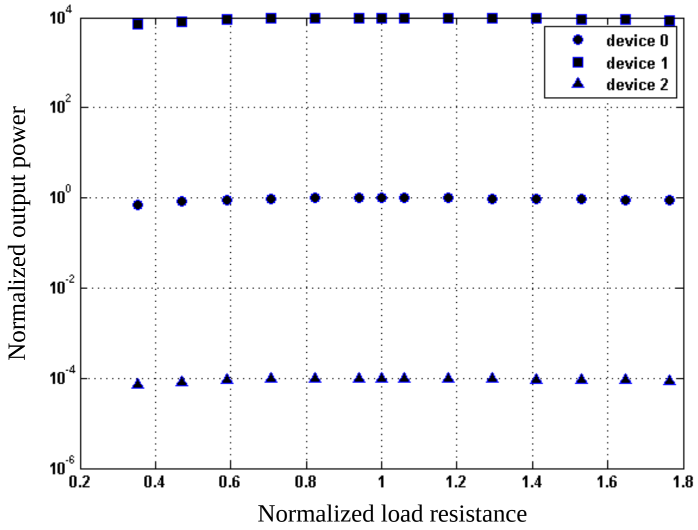


Figure 6. Comparison of output power of scaled piezoelectric energy harvesters under the constant vibration acceleration.

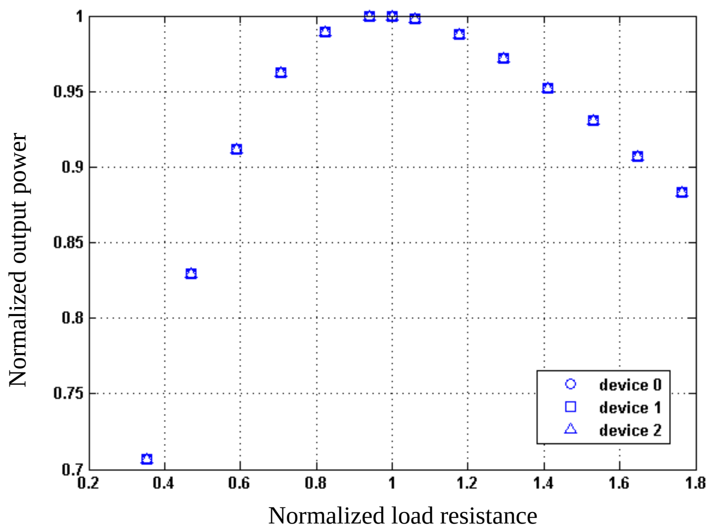


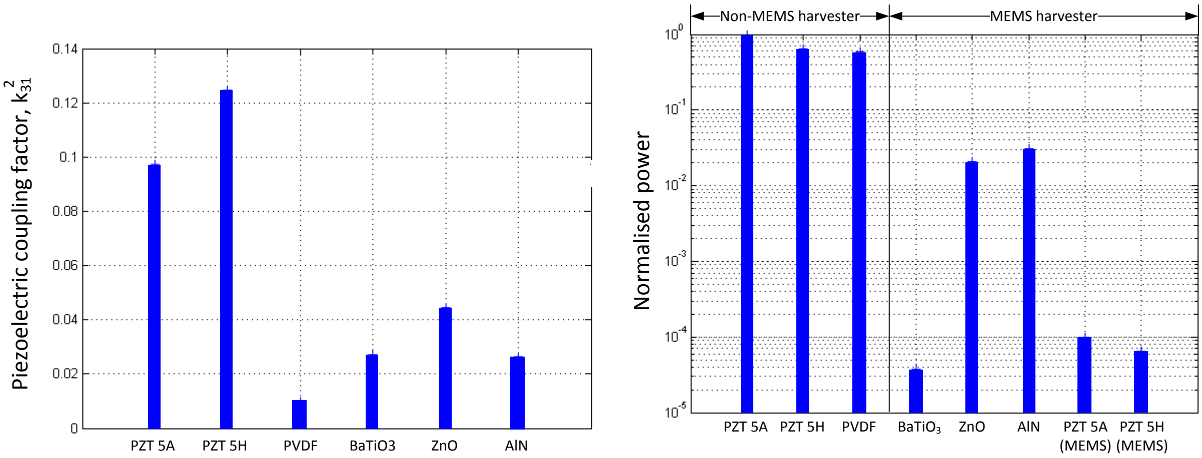
Figure 7. Comparison of output power of scaled piezoelectric energy harvesters under the constant vibration amplitude.

4. PIEZOELECTRIC MATERIALS

Common piezoelectric materials used in energy harvesters include (1) non-MEMS: PZT (types 5A and 5H) and PVDF (non-MEMS) (2) MEMS: $BaTiO_3$, ZnO, AlN and PZT. Table 1 summarizes properties of common piezoelectric materials used for energy harvesting applications. Figure 8a compares coupling factor of these piezoelectric materials, which is a function of Young's modulus (Y), piezoelectric coefficients (d_{31}) and dielectric constants (ε) ($k_{31}^2=d_{31}^2 \cdot Y/\varepsilon$). The simulation results show that PZT-5H has the highest coupling factor among piezoelectric materials. Figure 8b compares normalized maximum output power of harvesters with various piezoelectric materials and considering the scaling effects presented in Sections 2 and 3. In the simulation, MEMS harvesters were 1/10 in size compared to non-MEMS harvesters. It is found that the energy harvester with PZT type 5A has the highest output power among all harvesters while AlN has the highest output power among all MEMS scaled harvester.

Table 1. Properties of common piezoelectric materials.

Material	**Young's modulus**	**d_{31}**	**Dielectric constant**
PZT-5A[10, 11]	50	171	1700
PZT-5H[10, 11]	50	274	3400
PVDF[10, 11]	2	23	12
$BaTiO_3$[10, 11]	67	78	1700
ZnO[12]	147.3	5.43	11
AlN[13]	300	2.65	9



(a) (b)

Figure 8. Comparisons of different piezoelectric materials used in energy harvesting (a) piezoelectric coupling factor (b) normalized power.

5. CONCLUSIONS

Performance of piezoelectric vibration energy harvesters depends on their dimensions and piezoelectric material used. Scaling of piezoelectric vibration energy harvesters will significantly affect these factors. Scaling effects for piezoelectric vibration energy harvesters found from mathematical analysis presented in this paper are summarized as follows:

- Resonant frequency increases linearly as scaling factor.
- When the structure is scaled down, it has a wider frequency tuning range under the same tuning force.
- With the same piezoelectric material, the optimal load resistance of a piezoelectric harvester remains unchanged no matter how it is scaled.
- When excited at constant vibration acceleration, maximum output power of a piezoelectric harvester is proportional to fourth powers of scaling factor.

- Maximum output power of a piezoelectric harvester remains unchanged when excited under constant vibration amplitude.
- PZT 5A and AlN work best for non-MEMS and MEMS piezoelectric harvesters, respectively.

It is worth mentioning that the first two rules listed above also apply to other vibration energy harvesters.

MEMS fabrication of piezoelectric energy harvesters offers a low cost and integrated solution to self-powered systems. However, scaling down piezoelectric harvesters results in a much lower output power due to size constraints and material properties. The main challenge in piezoelectric energy harvesting is how to balance device dimensions and its output power.

REFERENCES

[1] Beeby, S. P., Tudor, M. J. and White, N. M., "Energy harvesting vibration sources for microsystems applications," Measurement Science and Technology, 17 (12), 175-195 (2006).

[2] Han, M., Yuan, Q., Sun, X. and Zhang, H., "Design and Fabrication of Integrated Magnetic MEMS Energy Harvester for Low Frequency Applications" Journal of Microelectromechanical Systems 23(1), 204-212 (2014)

[3] O'Donnell, T., Saha, C., Beeby, S. and Tudor, J., "Scaling effects for electromagnetic vibrational power generators" Microsystem Technologies, 13(11-12), pp 1637-1645 (2007).

[4] Kim, H. S., Kim, J. H., Kim, J., "A review of piezoelectric energy harvesting based on vibration," International Journal of Precision Engineering and Manufacturing 12(6), 1129-1141 (2011).

[5] Roundy, S., Wright P. K. and Rabaey, J. M, [Energy Scavenging for Wireless Sensor Networks: With Special Focus on Vibrations], Kluwer Academic Publishers, Norwell (2004).

[6] Zhu, D., Tudor, J. and Beeby, S., "Strategies for increasing the operating frequency range of vibration energy harvesters: a review," Measurement Science and Technology 21(2), 022001 (2010).

[7] Blevins, R. D., [Formulas for natural frequency and mode shape], Krieger Publishing Company, Malabar, (2001).

[8] Roundy, S. and Wright, P. K., "A piezoelectric vibration based generator for wireless electronics," Smart Materials and Structures, 13, 1131 (2004).

[9] Zhu, D., Tudor, J. Beeby, S., White, N. and Harris, N., "Improving Output Power of Piezoelectric Energy Harvesters using Multilayer Structures" Proc. Eurosensors XXV (2011).

[10] "Piezoelectric Ceramics Data Book for Designers," Morgan Advanced Materials (1999).

[11] Gonzalez, J. L., Rubio, A. and Moll, F., "A prospect of the piezoelectric effect to supply power to wearable electronic devices," Proc. 4th Int. Conf. on Materials Engineering for Resources, 202–207 (2001).

[12] Ellmer, K., Klein, A. and Rech, B., [Transparent Conductive Zinc Oxide], Springer, Berlin Heidelberg New York (2008).

[13] Wasa, K., Kanno, I. and Kotera, H., "Fundamentals of thin film piezoelectric materials and processing design for a better energy harvesting MEMS," Proe. PowerMEMS 2009, 61-64 (2009).