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An investigation of the centreless grinding process with a view to improving geometric stability

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" AN INVESTIGATION OF THE CENTRELESS GRINDING PROCESS WITH A VIEW TO IMPROVING GEOMETRIC STABILITY "

BY

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" A THESIS SUBMITTED AS PARTIAL REQUIREMENTS FOR THE DEGREE OF MASTER OF PHILOSOPHY TO THE COUNCIL FOR NATIONAL ACADEMIC AWARDS FOLLOWING WORK CARRIED OUT IN THE DEPARTMENT OF PRODUCTION ENGINEERING, FACULTY OF ENGINEERING, LANCHESTER POLYTECHNIC, COVENTRY. "

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From 1953-58 he was a Senior Planning Engineer with Alvis Limited, responsible for the production of aircraft engines and fighting vehicles of a specialised nature. Whilst at Alvis he was elected a Member of the Institution of Production Engineers.

In 1958 he was appointed Assistant Lecturer at the Coventry Technical College and subsequently transferred as Lecturer in 1960 to the Lanchester Polytechnic where he is at present a Senior Lecturer in Production Engineering.

In December 1968 he commenced research on the present subject at the Lanchester Polytechnic, in the Department of Production Engineering. This project is supported by a Science Research Council Grant in collaboration with Wickman Limited.

Published Papers

(i) "Goemetric Stability Charts for the Centreless Grinding Process" by W.B. Rowe and D.L.Richards.
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(ii) "Geometrical Configurations for Stability in the Centreless Grinding Process". by D.L.Richards, W.B.Rowe, F.Koenigsberger.
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Advanced Studies

(i) Algol Programming Lanchester Polytechnic (12 x 2hr Lectures) January 1969.

(ii) Control System Analysis for Mechanical Engineers (six, 2day courses) January 1969.

CONTENTS

List of Illustrations and Plates

Figures

Plates

Nomenclature (See Figures 1,2 and 3)

α	Angle between the grinding wheel contact normal OA and the workblade contact normal OB.
α_1	Angle between the line joining the wheel centres and the inclined face of the workblade.
β	Included tangent angle between the grinding wheel contact normal and the control wheel contact normal.
ζ_i	Damping ratio in the i th dynamic mode of the process.
θ	Angle measured between the line of origin OX rotating with the workpiece and the grinding wheel contact normal OA.
$w, w_n, wt,$	Excited frequency, natural and resonant.
Ω	Workpiece rotational speed.
A	A stability parameter $(1 + K_2 \cos WT_2 - K_1 \cos WT_1)$
B	A stability parameter $(K_2 \sin WT_2 - K_1 \sin WT_1)$
DC, DG, Dw	Diameter of control wheel, grinding wheel and workpiece respectively.
h	Height of workpiece centre above axis of grinding and control wheel centres.
K_1	Depth of cut factor for workblade position $(\sin\beta/\sin(\alpha+\beta))$
K_2	Depth of cut factor for control wheel position $(\sin\alpha/\sin(\alpha+\beta))$.
n	Number of waves on workpiece (w/Ω).
$R(\theta)$	The apparent reduction in workpiece radius at the same position as $r(\theta)$.
$r(\theta), r(t)$	Reduction in radius from the workpiece reference circle in a direction determined by θ or t.

$R(s)$	The Laplace transformed value for reduction in radius at grinding point at time t.
S_i	The i th root $(a_i + jwi)$
t, T_1, T_2, T	Time for workpiece to rotate through an angle θ, α, $\pi - \beta$ and 2π respectively.
V	Ratio relating α to α_1 and β according to $\alpha = \pi/2 - \alpha_1 - V\beta$.
$X(\theta), x(t)$	The magnitude of the feed movement of the grinding wheel normal to the workpiece at the angle θ or at a time t respectively.
$X(s)$	The Laplace transformed value for $X(\theta)$.
δ_1	The magnitude of a negative error on the workpiece at the point where contact is made with the workblade $(-r(\theta - \alpha))$
δ_2	The magnitude of a negative error on the workpiece at the point where contact is made with the control wheel $(-r(\theta - \pi + \beta))$
f	Chatter frequency

SUMMARY

Previous research workers have shown the critical importance of the geometrical configuration of the basic elements of the centreless grinding process, on the ability to produce round workpieces. Resonances in the machine tool, may tend, in some cases, to worsen geometric instability, and it is therefore necessary to be able to predict these geometrical instabilities in order to avoid unfavourable operating conditions.

The results from this work indicate that it is now possible to present geometric stability charts for the configuration of the process. Justification for the stability charts is argued by reference to extensive experimentation. Optimum geometrical configurations and process conditions are suggested, by deduction from theory and experiment.

Conclusions are drawn regarding favourable grinding practice.

1. INTRODUCTION

1.1 History of Centreless Grinding

Centreless Grinding is a method of cylindrical grinding components eminently suited to mass production. As the name implies the work is not supported between centres. Instead it is supported in the Vee formed by the control wheel and workblade as shown in fig.1.

As early as the fifteenth century Leonardo Da Vinci had thoughts for an automatic needle grinding machine, yet it was not until 1820 that Wilkinson (1) made a small machine which consisted of a wooden roller rolling on a stone which turned the spindles, required for a 'Spinning Jenny', against the stone, hence grinding the spindles. This could have been the first centreless grinding machine. However the most critical inventions for the centreless grinding machine were the control wheel and workblade by Heim (1) in 1915. In 1921 the Cincinnati Grinding Machine Company developed the technique of the centreless grinding process so that it became the basic method of producing precision cylindrical workpieces, both hard and otherwise, with high rates of production at low unit cost.

1.2 The Centreless Grinding Process

The process is unique, as the workpiece is not positively supported or rotated as in most other grinding processes. Therefore its motions are not simply controlled. The workpiece is rotated by the combined action of the control wheel and grinding wheel, and may be held against or guided past these abrasive surfaces by any one of a number of types of devices. The contact of the workpiece with the grinding wheel rotates the work, in a direction as shown in figure 1., at a surface speed which would approach that of the grinding wheel were it not braked by its contact with the control wheel. Hence there is a difference in surface speed between the grinding wheel and the rotating workpiece, thus grinding the workpiece. It is, however, important that the workpiece is located and supported between the two wheels in the correct position for grinding and to give the correct pressure against the control wheel. The workblade performs this function as shown in figure 1.

There are several methods of feeding the workpiece. 'Through feed' was an early method and was used for grinding cold-drawn steel bars, to remove the flash and surface skin for inspection purposes to determine any surface defects. This method was later exploited for its resulting accuracy and low costs. In 'through feeding', the workpiece is drawn through the machine by the action of the wheels produced by a slight tilting or skewing of the control wheel axis. The greater the tilt, the faster the workpiece is drawn through the machine.

The 'in-feed' technique was developed in the early 1930's to permit profile grinding of the workpiece. Here the workpiece is supported between the control wheel and the workblade.

To achieve stock removal, the three elements are advanced together towards the grinding wheel. Alternatively the workblade may be clamped in position and left stationary whilst the control wheel advances, thus pushing the workpiece up the inclined face of the workblade towards the grinding wheel. For 'infeed grinding' the control wheel is tilted as in the 'through feed' method, but only slightly, less than 1°, to position the workpiece against an endstop. To ensure line contact of the control wheel to the workpiece, the control wheel must be dressed to an appropriate shape. One method is to dress the control wheel on its periphery at a height coincident with this contact line after the wheel has been tilted.

The directions of rotation of the grinding wheel, control wheel and workpiece, are as shown in figure 1. The main wheels may be rotated in the reverse directions but there is a tendency for the workpiece to lift, hence it is unusual to adopt this method. Another possibility is for the control wheel to rotate in the opposite direction to the grinding wheel. This would increase the downward component force on the workpiece and hence give greater stability to the system. However the friction between the control wheel and workpiece would have to be much higher to ensure steady rotation of the workpiece. Workblade wear would also be increased.

1.3 The advantages of the centreless grinding process

Centreless grinding has a tremendous appeal for high production rates of precision ground parts at low unit cost. The process lends itself to fully automatic operation and combination in machine groups. Although there is a need for skilled setters, the production of ground parts is maintained by easily trained operators.

1.4 The necessity for the project

As shown in figure 1. the work is located and supported with its axis slightly above the centreline of the axis of the grinding wheel and control wheel. Workpieces are ground above centre which is found to be a necessary condition to eliminate initial roundness errors on the workpiece. In the following discussion the centre height will be related to the angle β fig 1., in keeping with other research workers. The angle β is the included angle between the grinding wheel contact normal and the control wheel contact normal, which is also the angle between the grinding wheel contact tangent and the control wheel contact tangent. It is well known that if the included tangent angle β is zero, odd harmonics of the workpiece shape will not be eliminated by grinding. It may also be that for other geometrical configurations, there will be no improvement in roundness for certain workpiece shapes, more seriously, situations arise where roundness errors actually increase.

Since the workblade angle is also a parameter in the geometrical configuration, successful centreless grinding depends upon careful adjustment of these two angles in order to achieve a high order of geometrical accuracy.

1.5 The objectives and scope of the investigation

There are obvious causes of pronounced workpiece roundness errors which may arise in the centreless grinding process as a consequence of the geometrical configuration figure 1. This configuration is determined by the diameters of the grinding wheel, control wheel and workpiece, the angle of inclination of the workblade and the relative disposition of these caused by the variation in height of workpiece centre to a horizontal line joining grinding and control wheel centres. The present investigation has been carried out with the following objectives in mind:

(i) To find a convenient measure of geometric stability or instability to assist in achieving objectives (ii) and (iii)

(ii) to determine favourable grinding conditions for the process,

(iii) to determine the consequence of workpiece diameter, workpiece speed and machine resonance on geometric stability.

The scope of the investigation is primarily divided into two parts. Part I concerns the theoretical background leading up to the concept of "Geometrical Stability Charts". The charts are the result of numerous computed geometrical configurations investigated for stability based on the Nyquist Criterion.

Part II concerns the experimental studies and correlation with roundness and stability charts.

PART I

2. Theory

2. THEORY

2.1 Previous Work

As early as 1939, Sachsenberg and Kreher (2) recognised that although the centreless grinding process was basically accurate it did not necessarily produce round work and that the workpiece height (h), was an important parameter.

Dall (3), in his first analytical approach to the problem of roundness, used the two basic parameters, the included tangent angle (β) and the top workblade angle (α_1). He predicted the errors produced on the workpiece as a result of those already occurring at its point of contact either with the workblade or the control wheel. This consideration, however, only applied to a perfectly rigid machine.

Yonetsu (4,5) derived relationships between the pregrinding and postgrinding amplitudes of the harmonics of the workpiece profile. But the theoretical relations were obtained only for the case of a sudden infeed and these were compared with experimental results where the infeed was made over 0.5 sec. However, three main conclusions may be reached from his results:

(i) The lobed shapes related to odd harmonics of order below the 11th., 3,5, etc, are better removed with a large included tangent angle (β)

(ii) The lobed shapes related to even harmonics of order below the 10th., 2,4, etc, are better removed with a small angle (β)

(iii) Other errors are generated which include even and odd order harmonics which vary with (β) and these are observed to depend on the magnitude of the in-feed motions.

Unfortunately the mathematics are impracticable for application to a problem with a conventional method of stock removal.

Gurney (6) saw the problem of stability as one of two types, one of geometric instability and the other dynamic instability. He considered that while a given grinding arrangement may be good from one point of view, it may be poor from another. Hence there is a necessity to compromise. His analysis neglected the influence of the radii of the wheels. The fundamental parameters of Gurney's analysis were workblade angle ψ, and the "work angle" β_1 (fig1) (The "work-angle" β_1, was termed α in the original text). Gurney investigated the dynamic instability and simulated the equations of motion on an analogue computer. His method determined the dynamic stability threshold on the tacit assumption of geometric stability. His stability charts were quite complicated, and he introduced the term 'figure of merit', which was a measure of the ability of the grinding configuration to withstand dynamic instability.

His conclusions do not favour a particular value of 'β_1', but in his test on workblade angles was constant at 5^o. He considered the higher the frequency of motion, that is, the greater the number of workpiece lobes, the greater the accelerating and damping forces, hence restricting out of roundness. When the waves on the workpiece were equal to or less than ten, he considered the system unsatisfactory, therefore necessitating a change in the geometrical configuration. By increasing angle 'β_1' his 'figure of merit' for dynamic stability fell and only started to rise again for large values of 'β_1'. Hence indicating the advantages to be gained from using small values of 'β_1'. His work also investigated changes in workblade angle for 5,15,30 and 45^o with a work angle 'β_1' of 5^o, which is approximately equivalent to $\beta = 10^o$ in the present investigation. His 'figure of merit' favoured the 15^o workblade. Unfortunately, a centreless grinding machine was not available at the time for Gurney to correlate experimental values with his theoretical results.

Plainevaux (7) discussed the problem of dynamic stability for the case of $\beta = 0^o$, and the workblade having no effect, which however is not a usual operating condition. Small changes in the angle β can have considerable effect on the stability of the process, and to make the assumption that since it is small it can be considered zero as regards chatter behaviour is clearly an unjustified assumption.

Rowe & Barash (8) proposed a numerical simulation of the centreless grinding process on a digital computer. They discussed the effect of the type of infeed, sudden and continuously uniform motion. It was found that even with a perfectly round workpiece and a uniform infeed motion, some small errors must occur. The continuous infeed was made by a cord and pulley drive from an electric motor. The method of predicting the results of the centreless grinding process went much further than those of Dall (3) and Yonetsu (4,5). Dall did not take into account the machine elasticity, and the method of Yonetsu which was too complex for application to realistic problems. Some of the results obtained by Rowe & Barash (8) are verified in this investigation, experimentally, namely the number of workpiece waves for certain geometrical configurations.

Rowe & Koenigsberger (9) derived an analytical method for obtaining threshold stability charts for the relative stability of different grinding configurations, but these charts were more complex than those drawn by Gurney. However it was shown that instability must occur under particular conditions, independent of the machine's structural characteristics. This case was termed 'geometric' instability. The work by Rowe and his colleagues (8, 9 and 10) is used as a basis in this investigation.

The included tangent angle β of 7^{o} suggested by Furukawa Miyashita and Shiozaki (11) to minimise the out of roundness is verified in this investigation.

2.2 Mathematical Basis

The analysis on which this investigation is based, corresponds to the case of the method of 'in-feed' grinding and in particular where the workblade arrangement is attached to the control wheel carriage. Figure 1 shows the geometrical configuration of the basic elements in the process. A line OX on the workpiece is considered and this is followed during its rotation. The angle which this line makes with the line OA is always θ. Thus the angle θ together with a particular phase angle enables any position on the workpiece to be defined at any instant. For example, the point on the workpiece at the point B is given by the angular position $(\theta - \alpha)$ and similarly at position C by $(\theta - \pi + \beta)$, where β is given in the relationship:

$$\beta = \sin^{-1}\left(\frac{2h}{DG + DW}\right) + \sin^{-1}\left(\frac{2h}{DC + DW}\right) \quad ..(1)$$

and

$$\alpha = \frac{\pi}{2} - \alpha_1 - \nu\beta \quad ..(2)$$

where

$$\frac{1}{\nu} = 1 + \frac{DG + DW}{DC + DW} \quad ..(3)$$

See Appendix A

2. 2.1 Errors due to an irregularity on the workpiece

If an irregularity on the workpiece arrives at either the workblade (B) or at the control wheel (C) as shown in figure 2, the workpiece centre will be displaced. In order to analyse this movement the following assumptions are made.

(1) The workpiece always remains in line contact with both the workblade and the control wheel.

(2) An irregularity at (B) will cause the contact point at (C) to move in a direction tangential to the control wheel with negligible changes in the angles α and β, and by similar reasoning an irregularity at (C) causes the contact point at (B) to move along the inclined face of the workblade.

Such an irregularity at the contact point (B) will cause the point on the workpiece at (A) to move away from the grinding wheel. This motion is indicated by the vector diagram (a) in figure 2. The resultant of this motion is seen to be tangential to the control wheel surface, and the component of this movement normal to the grinding wheel surface at (A) is given by:

$$M = R.\sin\beta$$

where $$R = \frac{\delta_1}{\sin(\alpha + \beta)}$$

Hence $$M = + \frac{\sin\beta}{\sin(\alpha + \beta)} . \delta_1 \qquad ..(4)$$

It can be seen, as the workblade angle (α_1) increases, the angle (α) between the grinding wheel contact normal OA and the workblade contact normal OB, decreases. Hence the magnitude of component M increases, that is, the workpiece moves farther away from the grinding wheel. Therefore M has the same sign as δ_1 which is in fact a negative error according to the definitions of $r(\theta)$, $r(\theta - \alpha)$, $r(\theta - \pi + \beta)$

An irregularity at (C) causes a resultant movement in a direction parallel to the inclined face of the workblade, and has a component normal to the wheel surface given by:

$$M = R \, . \, \text{Sin}\, \alpha$$

where

$$R = \frac{\delta_2}{\text{Sin}\,(\alpha + \beta)}$$

Hence

$$M = - \frac{\text{Sin}\, \alpha}{\text{Sin}\,(\alpha + \beta)} \, . \, \delta_2 \qquad ..(5)$$

The direction of movement is towards the grinding wheel, resulting in a new error of opposite sign to δ_2 figure 2(b). An irregularity at the control wheel (C) has a much stronger influence than one at the workblade (B), as indicated by the following example.

Let $\beta = 7^{\circ}$ and $\alpha_1 = 30^{\circ}$, with grinding wheel, control wheel and workpiece diameters, 12in., 7in., and 1in respectively

$$\text{then} \quad \frac{1}{\nu} = 1 + \frac{12+1}{7+1}$$

$$\therefore \quad \nu = \frac{1}{2.625} = \underline{0.38}$$

$$\text{and} \quad \alpha = 90^{\circ} - 30^{\circ} - (0.38 \times 7)$$

$$\therefore \quad \alpha = 57.34^{\circ}$$

then δ_1 will produce an error $+ \dfrac{\text{Sin } 7^{\circ}}{\text{Sin } (57.34^{\circ} + 7^{\circ})} \cdot \delta_1$

$$= + \underline{0.135\ \delta_1}$$

and δ_2 will produce an error $- \dfrac{\text{Sin } 57.34^{\circ}}{\text{Sin } (57.34^{\circ} + 7^{\circ})} \cdot \delta_2$

$$= - \underline{0.934\ \delta_2}$$

2. 2.2 <u>The equation of Constraint</u>

If now, any reduction in radius from an initial reference circle is considered as an error, the apparent reduction in radius R (θ) at the grinding wheel contact point (A), may be calculated in terms of the infeed movement X (θ) considered in the direction OA and the δ_1 and δ_2 errors. Hence the basic equation to represent the system is as follows:

$$R(\theta) = X(\theta) - \frac{\text{Sin } \beta}{\text{Sin } (\alpha + \beta)} \cdot \delta_1 + \frac{\text{Sin } \alpha}{\text{Sin } (\alpha + \beta)} \cdot \delta_2 \qquad ..(6)$$

by substituting K_1 and K_2, the depth of cut factors, for workblade and control wheel positions respectively, into equations (4) and (5), and θ measured from the rotating line OX always defines the position on the workpiece occurring at the grinding wheel where the true radius is $r(\theta)$. Then δ_1 may be defined as $-r(\theta-\alpha)$ and δ_2 as $-r(\theta-\pi+\beta)$.

Hence equation (6) becomes;

$$R(\theta) = X(\theta) - K_1.r(\theta-\alpha) + K_2.r(\theta-\pi+\beta) \qquad ..(7)$$

which is an equation of Constraint.

As a function of time, this becomes;

$$-x(t) = r(t) - K_1 r(t - T_1) + K_2.r(t - T_2) \qquad ..(8)$$

The terms $r(t - T_1)$ and $r(t - T_2)$ may be explained by considering the time T_1 for the workpiece to revolve through an angle α, and the time T_2 for an angle $\pi - \beta$ figure 3. The error at the grinding point at time $(t - T_1)$ will have just reached the workblade at time t and similarly the error ground at the time $(t - T_2)$ will have reached the control wheel at time t.

A steady infeed motion is inconsequential to considerations of stability. Hence it is possible to neglect $X(\theta)$ or $x(t)$ when considering whether a system is inherently stable. Thus the dynamic reduction in radius from the workpiece reference circle with respect to time becomes;

$$r(t) = K_1.r(t-T_1) - K_2.r(t-T_2) \qquad ..(9)$$

In a discussion of stability it was found helpful to employ the mathematics of control systems. Equation (8) may be transformed into the Laplace domain so that,

$$-X(S) = R(S).(1 - K_1 e^{-ST_1} + K_2 e^{-ST_2}) \qquad ..(10)$$

The closed-loop response of the geometrical process is illustrated in figure 4a and the closed-loop transfer function by rearrangement of equation (10) is;

$$\frac{R(S)}{-X(S)} = \frac{1}{1 - K_1.e^{-ST_1} + K_2.e^{-ST_2}} \qquad ..(11)$$

The open-loop block diagram is illustrated in figure 4b from which it is apparent that the open-loop transfer function is;

$$\frac{\Theta_o}{\Theta_i} = K_2.e^{-ST_2} - K_1.e^{-ST_1} \qquad ..(12)$$

The method found most convenient for investigating the stability of the system represented by the closed-loop transfer function is based on the Nyquist Criterion.

2.2.3 The Nyquist Criterion for Stability

The Nyquist Criterion involves the examination or measurement of the open-loop frequency response to determine whether or not it is safe to close the feedback loop, that is, to determine the system's absolute stability.

The harmonic response to a sinusoidal input, θ_i = Sin. wt, may be obtained by substituting, S = jw, in equation (12). Separating the components of the response in phase with the input (real terms) and the components in quadrature (imaginary terms), defined as A - 1 and - B respectively, the following are obtained;

$$Re \ (\theta_o) = K_2 . Cos . W \ T_2 - K_1 . Cos . W \ T_1 = A - 1 \qquad ..(13)$$

$$Im \ (\theta_o) = K_1 . Sin . W \ T_1 - K_2 . Sin . W \ T_2 = - B \qquad ..(14)$$

A computer programme (See Appendix B) has been developed and the results obtained for these response co-ordinates covering a number of geometrical configurations for up to 50 waves on the workpiece were studied, by plotting the ordinates of the open-loop frequency response i.e. the Nyquist diagram.

2. 2.4

Nyquist Diagram of the open-loop system

A portion of a typical open-loop response is illustrated in figure 5 together with a representation of a hypothetical equivalent physical system. Physically the open-loop system might be visualised by considering an input surface wave continuously applied to the workpiece, thus successively meeting the workblade and the control wheel, finally being replaced at the grinding point by the output surface wave which is freshly ground. The output wave is erased before reaching the workblade in order to open the system loop.

Equations (13) and (14) may be written in terms of the number of waves on the workpiece, since $T = \frac{2\pi}{\Omega}$; $T_1 = \frac{\alpha}{2\pi} . T$ and $T_2 = \frac{\pi - \beta}{2\pi} . T$. The number of waves on the workpiece is $n = \frac{w}{\Omega}$ and hence;

$$A = 1 + K_2.\mathrm{Cos}\left[(\pi - \beta)\frac{w}{\Omega}\right] - K_1.\mathrm{Cos}\left(\alpha . \frac{w}{\Omega}\right) \qquad ..(15)$$

$$B = K_2.\mathrm{Sin}\left[(\pi - \beta)\frac{w}{\Omega}\right] - K_1\ \mathrm{Sin}.\left(\alpha \frac{w}{\Omega}\right) \qquad ..(16)$$

By plotting the open-loop response it is found that the locus approximately forms an infinite number of circular loops thus forming a helix with time as the third dimension. Each loop corresponds to one dynamic mode. The stability of the root corresponding to each mode may be examined by reference to the S- plane transformation (12)

<u>The Relationship between the Nyquist Diagram and the S- Plane</u>

Any linear system has solutions of the form:

$$x = A_1 e^{s_1 t} + A_2 . e^{s_2 t}$$

Roots may take the form $Si = a_i + jw_i$

The stability of a system depends on the transient modes. Either the transients may increase without limit or they may decrease to zero. If a system is unstable then an indefinitely small disturbance applied for an indefinitely small time causes some or all parameters to increase without limit, or else until saturation occurs. If the exponential terms in the complete solution have negative real parts, transient modes will decay wheras positive real parts correspond to instability.

In other words if all the closed-loop characteristic roots have negative real parts the system will be stable otherwise the system is unstable.

It is usually accepted that a plot of roots in the S- plane gives an indication of transient response since the positions may be interpreted as in figure 6, where at the root $a_i = -\zeta_i w \eta_i$, the term a_i is a measure of the damping or instability, depending on whether a_i is negative or positive respectively in order. The resonant transient operating frequency of the root is $w_i = w_t$

In the Nyquist Diagram the point $(-1 + jo)$ is the transformed position of all the roots and the frequency response locus is the transformed jw axis. Hence if the frequency response locus completely encircles the minus one point (-1,0) in a clockwise direction, while frequency is increasing, it follows that the corresponding root lies to the right of the frequency axis (jw) in the S- plane. Such roots are obviously unstable. In centreless grinding there is almost always an unstable situation. That this is not always apparent must be partly attributed to the small margin of instability. In some cases the regeneration of waviness takes so long that the grinding operation may be over before it reaches detectable proportions.

As a consequence there is the unusual situation that it is not sufficient just to know that the system is unstable, it is also desirable to know exactly the characteristics and severity of the instability by further examination of the Nyquist Diagram.

Figure 7 shows a construction for determining the co-ordinates of a root. The construction applies irrespective of whether the system is stable or not.

Although figure 6 may be taken as an interpretation of transient response from the position of the root, there is a limitation which is not immediately apparent. In most control systems the phase lag of any element does not exceed 360°. However in centreless grinding the phase lag may exceed 360° many times. The large phase lags do not affect the validity of the Nyquist Criterion, which is based on an assumption of a linear system, since a delay term represents the addition of further linear terms. However the phase lags do result in an increase in the period over which stable transients occur and also give rise to an appearance of irregularity in the process variations.

In an unstable system large phase lags increase the period of build up in amplitudes.

The effect of phase lags yield a partial explanation for the mildness so often apparent in centreless grinding instabilities. The phase lags are increased proportionally with increasing number of waves on the workpiece.

2. 2.5 Geometric Stability charts for Centreless Grinding

Ideally a measure of stability or instability should be simple and capable of fast computation because of the very large number of situations to be investigated. For a given geometrical configuration, and investigating from 2 to 50 waves on the workpiece, involves some 25 loops on the Nyquist Diagram. Obviously to carry out a graphical construction for each of these loops to evaluate 'a', would be very time consuming. However from an investigation of a number of examples, it appears that the construction is in any case unnecessary. Figure 8 shows one example. It is observed that the loop is approximately circular and centred at the point (0, 0). There is a very small error in calculating η at the root, or the value of 'a', if it is assumed to occur at the point where the locus crosses the negative real axis.

It is now possible to distinguish how the approach described in this investigation differs from the analysis offered in ref (9) In the earlier work it was concluded that if A were negative for any integer value of η, the number of waves, then the system was geometrically unstable. While that conclusion is not contradicted now, it is apparent that it is helpful to know more exactly what the unstable frequency is, and the indicated severity of the instability.

Figures 9 and 10 illustrate a method of representing the computed results of A and B in the forms of geometric stability charts.

Since the parameters A and B are approximately sinusoidal functions with unit amplitude, it is only necessary to plot the values of A corresponding with the number of waves on the workpiece. The instabilities are indicated whenever A becomes negative, and the maximum negative values of A may be taken as measures of theoretical geometric instability.

2.3 Interpretation of the Geometric Stability Charts - Rigid Machine

It is of interest to examine the significance of the parameter A more closely to determine the features of the geometric stability chart and the possible significance to grinding results. Some features may be listed as follows.

2.3.1

The roots of the system correspond closely with troughs on the stability chart.

2.3.2

At the extremity of a trough the parameter A may be either positive or negative. A positive value of A at a trough indicates geometric stability and a negative value indicates instability. The value of A at a trough will be termed A^*

2.3.3

A value of A^* if positive is a measure of the inherent rounding tendency in the process for the corresponding number of waves. If a value of A^* is negative, the magnitude of A^* is a measure of geometric instability.

2.3.4

There are circumstances where there is coincidence between a trough on the stability chart and an integer number of waves. This is a case which may be given a simple physical interpretation as in figure 11a. The dotted line represents the magnitude of a dynamically varying infeed, which produces a corresponding waviness on the workpiece. As the waviness passes the workblade and control wheel there will be a feedback producing further waviness. The full line indicates the amplitude of the additional waviness resulting from the feedback terms and which is in phase with the varying infeed. The value A* is the difference in amplitude between the waviness resulting from the infeed and the additional waviness due to feedback. In the practical closed loop system the feedback waviness adds to the infeed waviness so that in consequence the resulting waviness and the feedback contribution to waviness are modified.

It is possible to make an even simpler interpretation of the physical significance when the value A* = 0 and there is also coincidence between a trough and an integer number of waves, as in figure 11b. If the infeed variation motion is replaced by an initial waviness on the workpiece, it becomes apparent that the feedback waviness resulting from the workblade and control wheel effects, will be equal in magnitude to the initial waviness and also in phase. Thus the waviness remains unaltered as grinding proceeds. The best known example of this case is when $\beta = 0^\circ$ and odd orders of lobing may not be removed.

2.3.5

Figure 12a extends the previous physical interpretations to the case where there is a negative value of A* at a non-integer value of η . The dotted line represents an infeed variation at a frequency which gives rise to a non-integer number of waves. As this waviness passes the workblade and control wheel there is a consequent feedback contribution to waviness in phase with the infeed variation. The depth of cut variation at any instant is the difference between the waviness being ground $r(t)$ and the waviness being removed $r(t-T)$. Thus the effect of the non-integer root value of η is to cause a phase difference between depth of cut variation $\delta(t)$ and the waviness $r(t)$.

2.3.6

Any workpiece shape may be represented mathematically as the sum of a number of shapes involving integer number of waves by Fourier Analysis. It is therefore of interest to consider the relationship between the rounding action for an integer number of waves and the corresponding value of A. This case is illustrated in figure 12b. Because the value of η is not the trough value, the value of B will be significant. Hence the feedback contribution to waviness resulting from the control wheel and workblade effect, will be out of phase with the infeed variation. If there is initial integer order waviness on the workpiece it is suspected that the waviness will build up as a result of a negative value of A at the integer value of η . This prediction must be examined in the light of experiments and machine deflections.

2.4 Interpretations of the Geometric Stability Charts - with Machine deflections Resulting from the Grinding Force

In the foregoing analysis a rigid machine has been assumed. This assumption is clearly untrue since machine deflections may be of an order greater than the depth of cut (ref 10). It is therefore necessary to examine the interpretation of the geometric stability charts in the presence of deflections. The following points relate to a consideration of some particular combinations of circumstances.

2.4.1

η = integer at trough

x(t), s(t) and f(t) in phase

With an integer number of waves, the depth of cut vector s(t) will be in phase with the waviness vector r(t). The influence of machine deflections resulting from, and in phase with, depth of cut, will be to reduce its magnitude but not to completely eliminate the depth of cut or to cause a negative depth of cut variation which would imply out of phase deflections. It follows therefore, that in-phase deflection attenuates the instability if A* is negative but can not make the system stable. The converse is true if A* is positive.

If A* is positive the process is geometrically stable and therefore it appears that the roundness errors should be reduced during the course of grinding.

In this case the effect of in-phase deflection will be to reduce the rounding action.

At the marginal condition when $A^* = 0$, machine deflections have no effect on the roundness. Within the limitations of the basic assumptions there is complete independence between the dynamic system and the geometric system. However it cannot be over-emphasized that forced vibrations at a frequency corresponding to a condition of marginal stability, theoretically cause infinite response. Therefore such systems will be susceptible to machine disturbances. Such disturbances would provide an explanation for the occurrence of workpiece shapes which otherwise would not be expected. By indicating the coincidence of a trough and an integer value of η the stability charts help to interpret this case.

2.4.2

η = non-integer at trough

x(t), s(t) and f(t) in phase

For a non-integer value of η any waviness is constantly shifting around the periphery of the workpiece. Thus larger depths of cut are involved to maintain an amplitude of waviness than if η is an integer. It follows that the grinding force variations required are also much greater. The in-phase deflections will therefore tend to stabilise a system which is geometrically unstable if there is sufficient phase difference between $r(t)$ and $r(t-T)$. By an extension of this reasoning in-phase deflections may increase the margin of stability of a geometrically stable system. This is in agreement with a conclusion to which there was a reference in the recent work of Furakawa (11) from an earlier deduction by Miyashita (13).

Section 2.3.6 was concerned with the rounding ability of an integer number of waves with a non-integer trough. In the particular case where A is negative for integer η it is suspected from consideration of section 2.4.1 that it will not be possible to eliminate an initial waviness even in the presence of in-phase deflections.

2.4.3

x(t), s(t) and f (t) not in-phase

It is no longer a simple matter to infer the effect of machine deflections due to the grinding forces since the complete system roots will not occur at the same frequencies as in the rigid machine system. A resonance tends to produce large amplitudes which may make a geometrically stable system become unstable or may sometimes stabilise an otherwise unstable condition (9 and 10)

Although the charts have an apparently clear interpretation at low frequencies, this must be justified by experiment. At higher speeds and at resonant conditions, the conditions governing stability must remain to be analysed by more sophisticated techniques or possibly inferred from particular grinding results.

2.5 Theoretical Results

Figure 10 illustrates a limited portion of several stability charts for various values of β and $\alpha_1 = 20^\circ$. The range of the stability chart covers values of n from 5 to 10. The chart for $\beta = 0^\circ$ reveals that all odd order lobings correspond exactly with roots as indicated by troughs in the value A. These roots correspond to a condition of marginal stability as indicated by zero values of A* and explained under 2.3.4. Deflections do not assist in the rounding process (section 2.4.1) and therefore such waviness is extremely difficult to eliminate as is well known (10).

For increasing values of β the roots are displaced away from the integer values of η a feature which should increase stability at moderate workpiece speeds (Section 2.4.2). There is an additional feature in the case of approximately fifth order lobing in so far as the value of A* becomes increasingly negative. It is therefore of interest to examine the variation of A at $\eta = 5$ exactly as shown in figure 9a. In the range $0^\circ < \beta < 3^\circ$ the parameter A is negative, which, it is suspected makes the system unstable (Section 2.4.2). This prediction is not contradicted by experiment. Rather it appears that the prediction may be confirmed.

Plotting the values of A with varying workblade angle for $n = 5, \beta = 8^\circ$ yielded figure 9b. This may suggest that the rounding ability for $\eta = 5$ may be improved with increasing workblade angle at moderate workpiece speeds. This is found to be in agreement with experiments.

A similar conclusion was reached by Rowe (Ref 9) who attempted to use the values of A at integer values of η as a measure of the rounding action.

However that attempt met with only partial success since it is necessary to know more about the variation of the parameter A as provided in the geometric stability charts. Figure 13, shows a selection of stability charts for various configurations and extended for up to 50 waves on the workpiece. The areas of potential instabilities to be avoided are indicated by the troughs on the stability charts. Because of the mildness of the instabilities it is found that in most cases these are serious only when there is coincidence between an integer number of waves and a maximum negative value of A. A coincidence of this nature must, therefore, be avoided by selecting a more favourable geometric configuration. An example of this in figure 13 is the case of 22 waves for $\alpha_1 = 20^\circ$, $\beta = 8^\circ$ indicating geometric instability. Hence theoretically, by changing the configuration to either $\alpha_1 = 15^\circ$ or 30° for $\beta = 8^\circ$ the system is now geometrically stable, thus indicating that roundness errors of a significant level should not arise. However, in practice if the frequency corresponding to the root is close to a resonant frequency of the machine, the workpiece lobes may be pronounced. This condition does not always arise, but in the event may be corrected by a change in the workpiece speed as shown later in the experimental results.

Table 1 gives a summary of the computed results for the most favourable configurations indicating the range and number of unstable areas. It compares the instabilities for the included tangent angles of $\beta = 7^{\circ}$ and 8° with workblade angles α_1 of 15°, 20° and 30° from the stability parameter A* and workpiece waves η . It can be seen that the most favourable configurations are when $\beta = 7^{\circ}$ and 8° and $\alpha_1 = 30^{\circ}$.

Workpiece Diameter

Although the workpiece diameter is a parameter which determines the geometrical configuration, it has been found from the theoretical results that variations have little significance in the system.

Where a case of instability did occur however, there was a slight improvement in the example given below by a reduction in workpiece diameter. This does not indicate however, that the larger the workpiece diameter the better the system's stability, as the workpiece waves and machine resonances also have their influences. Figure 14 shows the effect of variations in workpiece diameter on the stable and unstable areas for the favourable configuration of $\beta = 7^{\circ}$ and $\alpha_1 = 30^{\circ}$. Consider the trough at $\eta = 5.2$, the system is marginally stable, giving parameter A* a numerical value of +.00233 for 1 inch diameter workpiece. At .75 inch diameter, A* reduces to +.00207 and subsequently to +.00181 at .5 inch diameter. However at the trough $\eta = 19.76$, the system is marginally unstable having a numerical value of A* = -.01532. This case improves when the workpiece diameter is .75 inches, to -.01461 and finally to -.0139 at .5 inch diameter.

Table II summarises this marginal affect on the theoretical stability due to workpiece diameter.

2.6 Theoretical Conclusions

1. It is important to give adequate consideration to theoretical geometric stability to avoid unstable operating conditions. Geometric stability charts have been derived which are easy to compute and which may facilitate the interpretation of the grinding results.

2. Geometric configurations which give rise to large negative values of the parameter A particularly with integer number of waves n should be avoided.

3. The most favourable geometrical configuration was found to be $\beta = 7^{o}-8^{o}$ with $\alpha_1 = 30^{o}$

4. Workpiece diameter has a negligible effect on the systems stability.

$\alpha_1 = 15^\circ$

$\beta = 7^\circ$ n =	5.2	9.4	15.6	19.8	30.2	34.4	40.6	44.7	
A* =	-.09	-.058	-.05	-.09	-.08	-.06	-.041	-.09	
$\beta = 8^\circ$ n =	5.2	9.4	15.6	19.9	30.2	34.5	40.8	44.9	49.2
A* =	-.098	-.067	-.027	-.105	-.007	-.074	-.04	-.08	-.04

$\alpha_1 = 20^\circ$

$\beta = 7^\circ$ n =	5.2	11.4	15.6	21.8	26	32.2	36.4	42.6	46.8
A* =	-.084	-.027	-.075	-.05	-.057	-.063	-.032	-.07	-.002
$\beta = 8^\circ$ n =	5.2	11.4	15.6	21.96	26.16	32.4	36.4	42.9	49.16
A* =	-.089	-.004	-.080	-.074	-.05	-.086	-.004	-.096	-.035

$\alpha_1 = 30^\circ$

$\beta = 7^\circ$ n =				19.76	25.94	32	44.72		
A* =				-.015	-.004	-.015	-.029		
$\beta = 8^\circ$ n =			13.6		26.14	32.4	45		
A* =			-.010		-.024	-.026	-.037		

TABLE I INSTABILITIES OF β AND α_1, IN TERMS OF n AND A*

Stability	improves	at	n = 2	when	DW	is	small
"	"	"	n = 3,4,5,& 6	"	"	"	large
"	"	"	n = 7,8,&9	"	"	"	small
"	"	"	n = 10,11 & 12	"	"	"	large
"	"	"	n = 13,14 & 15	"	"	"	small
"	"	"	n = 16,17 & 18	"	"	"	large
"	"	"	n = 19,20 & 21	"	"	"	small
"	"	"	n = 22,23,24 &25	"	"	"	large
"	"	"	n = 26,27 & 28	"	"	"	small
"	"	"	n = 29,30 & 31	"	"	"	large
"	"	"	n = 32,33 & 34	"	"	"	small
"	"	"	n = 35,36 & 37	"	"	"	large
"	"	"	n = 38,39 & 40	"	"	"	small
"	"	"	n = 41,42 & 43	"	"	"	large
"	"	"	n = 44,45,46 &47	"	"	"	small
"	"	"	n = 48,49 & 50	"	"	"	large

TABLE II EFFECT OF Workpiece Diameter on Stability

PART II

3. EXPERIMENTAL STUDIES

3. Experimental Studies

3. 1 Objectives

3. 1.1.

To investigate some of the characteristics of the centreless grinding machine and their influence on the roundness accuracy.

3. 1.2

To correlate, if possible, theoretical with experimental results under practical grinding conditions.

3. 2. Equipment

3.2.1. The grinding machine

The grinding machine a Wickman Scrivener No. 0, is described in Appendix C and illustrated in plate 1. The machine used in the tests was in a near new condition and correctly mounted on a 'Tico' anti-vibration pad: Vibrations were kept to a minimum by the adjustment of belts, balancing of pulleys and wheels etc.

3. 2.2. Infeed Mechanism

This was an integral unit giving continuous controlled infeed by a hydraulic infeed mechanism incorporating a plate cam. Plate 2 and figure 24.

3. 2.3. Vibration Equipment

(i) Displacement Transducers

These were of the inductive type by Southern Instruments. The transducers were mounted on a bracket secured to the workblade attachment to pick up relative displacement of the grinding and control wheel spindles caused by wheelhead and grinding wheelhead inturn.

The arrangement and instrumentation is shown in plates 3 and 4 and the instrumentation link-up in figures 15 and 16. Transducers were also used to determine the rotational speed of the grinding wheel, control wheel and workpiece, the infeed motion of the cam and the relative displacement of the grinding wheel and control wheel during grinding as illustrated in plates 5 and 6.

(ii) Vibrator

The Philips Vibrator PR 9270 was mounted on the control wheelhead with its ram in contact with the grinding wheelhead, and again with their positions reversed.

(iii) Signal Generator

A Wayne-Kerr signal generator was used to provide a sinusoidal voltage of variable frequency.

(iv) Amplifier

A Philips Amplifier, 129-043, amplified the signal from the signal generator which was used to drive the vibrator.

(v) Accelerometer

Bruel and Kjoer 4330

(vi) Vibration Meter

Bruel and Kjoer 1606 (used as an integrator to give displacement)

(vii) Frequency Analyser

Bruel and Kjoer 2107

(viii) Frequency Counter

Venner type TSA 336 was used as a check on input frequency to amplifier.

(ix) Phase Meter

AD-YU Electronics type PM 406L

(x) Voltmeter

Bruel and Kjoer 2409 for measuring amplitude and vibrations.

(xi) Amplifier

Keno C B 2 was used to raise the output from the Southern Instruments F.M. unit to drive the phase meter (minimum input .05 volts).

(xii) Oscilloscope

Telequipment D54, was used as a sinewave overload monitor check on the waveform going into the phase meter.

(xiii) Velocity Transducer

Philips PR 9260 used for determing mode shapes at resonance.

3.2.4 Roundness Measurement

Before and after grinding, the shape of the specimens profile was measured on the 'Talyrond' Roundness testing machine, which has a guaranteed spindle rotation accuracy of 3 Mic.in. Plate 7.

3.2.5. Surface Texture

Before and after grinding, an assessment of the specimen's surface texture was made on a Model 3 'Talysurf', employing a skid type measuring pick-up. This instrument was also used to determine the depth and width of the wear crater on the workblades after the completion of their relavent tests. Plate 8

3.2.6 Size

The diameter of each specimen was measured before and after grinding on a 'Talymin' electrical comparator. Plate 9.

3.2.7 Measurement of Included Tangent Angle β and Workblade angle α_1

The spindles of both the grinding and control wheels were set level in the horizontal plane by means of a Clinometer, and a plate was attached to the workblade and control wheel carriage, parallel to the wheel centres, to serve as a datum. This datum was used for setting the height (h) of the workpiece centre, whilst sitting on the workblade, to the corresponding included tangent angle β , by means of a dial indicator and slip gauges. All such settings were within .002 in. in height and parallelism.

The chilled cast iron workblades had their appropriate angles (α_1) ground by means of a precision surface grinder using a magnetic angle-setting block. The angles were checked after grinding by means of angle slip gauges and Angle Dekkor. All the blade angles were within 10 minutes of their nominal angle.

3.3. Preliminary Machine and Machining Tests

3.3.1 Vibration Tests.

In general a number of excitation and light running tests were made. Excitation was provided from one of two positions to cause vibration in the transverse horizontal plane of the machine either from the grinding wheelhead or from the control wheelhead. The two positions were employed to reduce the effect of vibrator mass on the resonant frequencies. At the lower frequencies best accuracy is obtained by mounting the vibrator on the grinding wheelhead. At higher frequencies it is better to mount the vibrator on the control wheelhead. Figures 15 and 16 indicate the instrumentation link-up for obtaining the relavent results of phase and amplitude respectively, of the relative vibrations at the wheels, as illustrated in plates 3 and 4.

3.3.2 Discussion of vibration results

It can be seen from figures 17 and 18 that the resonant frequencies altered with the position of excitation. These results indicated that there was a dominant mode in the region 71 to 76 Hz. . These frequencies were slightly depressed due to the vibrator mass. Vibrations in grinding suggested a resonance at approximately 82Hz, but the examples of associated waviness in grinding tests as discussed in later sections all indicated frequencies between 76-80 Hz . It was noticeable that the amplitudes were much larger at the control wheelhead, by a factor of four, with more moderate modes at the grinding wheelhead, base, tray and infeed attachment. Figure 19 illustrates the characteristics of the machine modes when the control wheelhead was excited from the vibrator mounted on the grinding wheelhead at 76 Hz. . It can be seen that the main machine base, tray and infeed attachment had a slight vertical oscillation with a twisting mode to right at front to left at rear.

However there was some considerable rocking of the control wheelhead toward and away from the grinding wheelhead with slight rocking from rear to front.

This is largely due to the lack of stiffness between the control wheelhead and its slideway. The grinding wheelhead had similar characteristics to the control wheelhead but of quarter the severity. Similar characteristics were also observed at 170 Hz . However more accurate determination of the critical frequencies were obtained from the light running and grinding tests as shown in figures 20, 21 and 22. These were obtained from an accelerometer positioned on the control wheelhead. Tests were carried out for each of the control wheel speeds of 23, 30 and 39 rev/min, indicating a control wheelhead mode between 80-83 Hz . Modes of 25 and 30 Hz were considered to be associated with the motor and wheel unbalanced respectively. From subsequent grinding tests it was found that work-regenerative chatter occurred at frequencies in the range 76 - 80 Hz . For predictions of waviness, it was therefore decided to base the calculations on machine instability at 78 Hz which is in the mid-range of the measured values.

3.3.3. Rotational speed of main wheels and workpiece, and main wheel deflections

Plates 5 and 6 show the arrangement and instrumentation for determining the rotational speeds of the main wheels and workpiece, and the main wheel deflections under actual grinding conditions.

The control wheel speed at 30 rev/min. was found to be 5% faster whilst the grinding wheel was approximately 2% slower. The workpiece slip under favourable geometrical configurations was less than $1\frac{1}{2}$% but under adverse conditions of a zero workblade angle and zero included tangent angle β , workpiece slip was as much as 15%.

Figure 23 shows the results obtained from a U.V. recorder, as illustrated in plates 5 and 6, indicating main wheel deflections, infeed motion, main wheel and workpiece speeds. The calibration graph for mainwheel deflections is shown in figure 42. The initial deflection of the grinding wheel as grinding commenced was in the order of .00008 in. whilst the control wheel deflection was .00018 in. There was also a slight oscillation of the main wheels toward and away from each other of a small magnitude of approximately .000015 in. The control wheel was the main offender. These deflections are in agreement with the mode indicated in section 3.3.2.

3.3.4 Infeed mechanism

Figure 24 shows the profile of the cam-plate used initially to cycle the grinding operation. The control wheel carriage was returned by reversing the cam-plate. However the specimen Talyrond traces, particularly at the lower workspeed, indicated a profile resembling a cam as shown in figure 25a.

The reason for this phenomenon was the action of the infeed mechanism. As the infeed cam-plate was reversed on the completion of sparkout, the workpiece was "pitched" towards the grinding wheel. A displacement transducer coupled to a U.V Recorder verified this "pitching" and showed it to be in the order of .005 in. figure 24b. The problem was overcome by modifying the cam-plate as shown in figure 24c, and to retract the control wheel carriage on the forward stroke. The specimen was removed before reversing the cam-plate in preparation for the next cycle. The resulting effect is shown in figure 25b.

3.4. Grinding Experiments

3.4.1 Experimental Conditions

Grinding Machine; Wickman Scrivener No. 0 model as described in Appendix c.

Grinding Wheel; 5A 46/54 -K5 - V50

Initially 11.6 ins. x 3ins. x 4ins. bore

Control Wheel; A 80 - R - R

Initially 6.7ins x 3ins x 3ins. bore.

Workblade Angles; 0° to 45° in steps of 5° chilled cast iron.

15°, 20° and 30° Carbide blades.

Specimen; 0.5, 0.75 and 1.0 ins diameter x 2.0 ins.

Hard and soft steel.

Control Wheel Speeds; 23, 30 and 39 revs/min.

Inclination $1/4^\circ$

Grinding Wheel Speed; 1,750 revs/min

Coolant; Fletcher Miller " Clearedge" water soluble oil 1:40

Wheel Dressing Procedure Traverse rate 1.5 ins/min

1 passwith .001 in depth of cut, return pass .001 in depth. 1pass without cut, return pass .0005 in cut. 1 passwithout cut, return pass without cut.

Stock Removal; .010 ins on diameter.

Spark out Time; 7 seconds.

Vibrations; were kept to a minimum by the adjustment of belts, balancing of pulleys and wheels etc.

Infeed; was continuous and controlled by a hydraulic infeed mechanism incorporating a plate cam (Section 3.3.4)

<u>Specimens</u>. A considerable number of both hardened and soft specimens were carefully prepared on a plain cylindrical grinding machine. Initially to within 70 Mic. ins M.Z.C. roundness error, but in later tests this was improved to 15 Mic. ins. All appropriate test specimens were kept within a diametrical tolerance of .0002 ins and surface texture to within 9 Mic. ins. which was again improved in later tests to within 7 Mic. ins. These parameters were checked respectively by a Talyrond, Talymin and Talysurf.

3.4.2 Grinding Preparations

Before each experimentation the grinding and control wheels were dressed by positioning the diamond horizontally opposite the point of contact with the workpiece after a "warming up" period of approximately one hour. The wheels were measured by means of a vernier, and the corresponding height of the workpiece centre to grinding and control wheel centres, for the appropriate included tangent angle β , was calculated. All height settings (h) were within .002 ins this gave an error in β of only 3 minutes. In order to standardise the pre-grinding conditions of the wheels, four specimens were ground before grinding the test specimens.

3.4.3. Experiment A

<u>The effect of varying the included tangent angle β with various control wheel speeds, on departures from roundness, Surface Texture, and size.</u>

(i) <u>Experimental Procedure</u>

It is well known that odd harmonics of workpiece shape will not be eliminated by grinding with an included tangent angle of zero degrees, and with increasing angle β a point will occur when chatter will result. Further increases in β may cause so much workspeed slip that it may approach the speed of the grinding wheel which would prove disastrous. Since the range of specimen varied from .5 to 1.0 ins diameter, it was decided to restrict the included tangent angle β to an equivalent height h equal to .5 in.

Hence the range was varied between 0° to 14° in steps of 2°.

Soft specimens of .5 in diameter were ground, supported on a 20° workblade. Ten specimens were ground at each test and the results averaged. At each subsequent test, i.e. a change in angle β for a given control wheel speed, the initial conditions were the same as in previous tests. The whole procedure was repeated for the other two control wheel speeds, grinding 240 specimens in all.

(ii) <u>Results</u>

Figure 26 shows graphically the resulting roundness errors for different values of the included tangents angle β and control wheel speeds.

The largest roundness error is shown to be at $\beta = 2^{\circ}$ for a control wheel speed of 23 revs/min, this agrees with the trend of the parameter A at the integer value $n = 5$ as indicated in figure 9b. The lowest average departure from roundness is shown at $\beta = 8^{\circ}$, and 75% of the tests indicated the most favourable control wheel speed to be 30 revs/min. This result is discussed in detail in example 2, section 3.4.6. The overall range of variation in roundness error was 72 Mic. ins. Figure 27 shows the corresponding surface texture values for the same test conditions. The results do not show any real significance for the control wheel speeds. The overall range of the C.L.A. value is only 5.2 Mic. ins. There is however a deteriation in the C.L.A. value for increasing β above 8°.

3.4.4. Experiment B

The effect of varying the workblade angle α, on workpiece roundness

(i) Experimental procedure

As the results in test A showed preference towards $\beta = 6^{\circ} - 8^{\circ}$ and a control wheel speed of 30 revs/min, these values, with the inclusion of $\beta = 7^{\circ}$, were used in turn with workblades of varying workblade angle α, from $0^{\circ} - 45^{\circ}$ in steps of 5°. Here four specimens were ground at each test and their averages plotted. All specimen were carefully pre-ground as before, and there roundness errors were initially within 25 Mic. ins. At each subsequent test the initial conditions were the same as in all previous tests. All the blades were accurately checked by angle slip gauges and Angle Dekkor and all were within 10 minutes of arc of their nominal angle.

(ii) Results

Figure 28 shows graphically the results of departure from roundness for various workblade angles when $\beta = 6^{\circ}$, 7° and 8°, with a control wheel speed of 30 revs/min. The complete range of roundness errors is within 55 Mic. ins for over 200 specimen. The best results were obtained with $\beta = 7^{\circ}$ and 8° and workblade angles 15° and 35°.

From the average trends of roundness errors for $\beta = 6^{\circ}$, 7°

and 8^{o} for varying workblade angles, it appears that the lowest roundness errors occur when $\beta = 7^{o}$, followed by $\beta = 8^{o}$, then $\beta = 6^{o}$. The errors also decrease as the workblade angle α_1 is increased up to the point were severe instability may occur, as illustrated in figure 29 and as predicted by the theoretical results in figure 9b. The effect on workpiece size and C.L.A. values was insignificant. For all the tests, covering over 200 specimen, the average size range was within .0001 in and their C.L.A. values within 5.8 Mic in.

After the completion of the tests on the workblade angles, their effect on the width and depths of crater wear on the workblade was checked with the aid of a Talysurf. The results of which are shown in figure 30. Figure 31, shows wear is greater with increasing workblade angle. This is indicative of increased forces on the blade. There seems to be little advantage in using a workblade with an angle α_1 greater than 30^{o}. In fact when using workblades with angles 35^{o} and 40^{o} there was a tendency to chatter and work bounce, to an extent that made it necessary to terminate the test as the conditions became dangerous.

Plates 10 and 11, and figure 30 show the position and extent of wear on the workblades in an ascending order 0^{o}, 5^{o}, 10^{o}, 15^{o} and 20^{o}, 25^{o}, 30^{o}, 40^{o} respectively. As can be seen, the wear crater moves towards the leading edge of the workblade as α_1 increases. This is explained in figure 32, since the position of the leading edge of the workblade was maintained constant for each test, it is obvious that the contact point B moves up the workblade towards the leading edge as α_1 increases.

An optical examination of the workblades lead to the following conclusions;

(i) the damange was caused primarily by grinding wheel debris.

(ii) the debris being hard relative to the cast iron workblade, caused an attrition effect as it was driven against the workblade. Embedded particles were identical to particles of the grinding wheel.

(iii) exposed surfaces of the debris were subjected to varying force causing breakdown of localised areas. Material had been torn away from the workblade, leaving relatively large cavities which were partially filled with debris from both grinding wheel and workpiece. The workblade material was particularly prone to failure due to cracking under the tensile stresses involved.

(iv) with the present arrangement of the grinding operation, this type of damage is always possible. Hence the practice of using carbide inserts in the workblades. Carbide workblades were used in the final tests.

3.4.5 Consideration of grinding results with reference to Geometric Stability Charts

From test A, the roundness errors for various included tangent angles β with a workblade $\alpha_1 = 20^{\circ}$ gave the following results as illustrated in figures 33,34,35, and 36. The ratio (r) of the diameter of the control wheel to the workpiece was 13.8, and with a control wheel speed of 30 rev/min, this gave a workspeed (Ω) of 6.89 rev/sec.

(i) When $\beta = 0^{\circ}$, the number of waves (n) around the specimen were predominantly odd and not exceeding 13. Figure 33(a) gives a typical trace of a specimen having 11 waves. From the geometric stability chart for this configuration as shown in figure 37a, it can be seen that the stability parameter A for these odd harmonics is identical in each case, i.e. marginally unstable, and are susceptable to forced vibrations of the machine. If forced at an odd frequency the amplitude would be expected to build up to infinity. The average C.L.A. value and roundness error for this configuration was 5.95 and 74.5. Mic.ins respectively.

(ii) When $\beta = 2^{\circ}$, figure 33(b), n was predominantly 5 and from figure 37b indicates that this configuration involves a geometric instability at $n = 5.06$. The value of A at $n = 5$ was negative. Examining the corresponding chart at figure 37b, it may be seen that there is an indicated geometric instability at $n = 11.12$. However in that case the value of A at $n = 11$ was positive and from section 2.4.2. it is suggested that this yields an explanation for the absence of 11 waves on any ground profiles. In this case the average C.L.A. and roundness values were 6.28 and 84.5 Mic. ins. respectively.

(iii) When $\beta = 4^{\circ}$, figure 34(a), the number of waves (n) was also predominantly 5.

The geometric stability chart figure 37c, shows a very slight margin of stability for $n = 5$, with strong instability at $n = 5.12$. The values of A were negative between $n = 5.02$ to 5.2. Hence any disturbance may cause errors which are not readily removed since the rounding tendency for this configuration is obviously very mild. The average C.L.A. and roundness values were 7.3 and 55 Mic. ins. respectively.

(vi) When $\beta = 6^{\circ}$, figure 34b, the number of waves was again 5 and the stability charts figure 37d, gives instability for this configuration at $n = 5.18$. Values of A were negative between 5.06 to 5.3. The stability chart does not definitely indicate whether five-waves should be a problem in this case. The value of A at $n = 5$ was again very slightly positive whilst the value of A^* at $n = 5.18$ was quite strongly negative. As discussed in section 2.4.2 it might be expected that this configuration would be geometrically stable for $n = 5$ although the configuration would not be expected to involve a strong rounding tendency and any disturbances again may cause errors which are not rapidly removed. In the case of grinding a workpiece with a flat (9), it was found that the predominant remaining harmonics, from computer simulations, was 32 waves, closely followed by 5 waves. Both 32 waves and 5 were experienced in practice, although some of the most accurate work was achieved with this geometrical configuration. The average C.L.A. and roundness values were 7 and 58.7 respectively.

When $\beta = 8^{o}$, figure 35(a) the roundness error was the lowest value in the tests. The number of waves on the specimen was most often 22, as would be expected from the stability chart in figure 13, since 22 waves is geometrically unstable. However the instability is obviously very mild: This configuration is further discussed in 4.3.4. The average C.L.A. and roundness values were 7.2 and 37.8 Mic. in., respectively.

(vi) When $\beta = 10^{o}$, figure 35(b) The number of waves was predominantly 16, this as shown in the stability charts of figure 38(a) is geometrically unstable, and the magnitude of roundness errors were larger than for $\beta = 8^{o}$. The average C.L.A. values and roundness errors were 7.9 and 71 Mic. in., respectively.

(vii) When $\beta = 12^{o}$, figure 36(a) The results were as for $\beta = 10^{o}$, i.e. 16 waves and geometrically unstable as shown in figure 38(b). The average C.L.A. values and roundness errors were 9.45 and 60 Mic.in., respectively.

(viii) When $\beta = 14^{o}$, figure 36(b) The number of waves apparent was 12, and from the stability charts figure 38 (c) this is also shown to be geometrically unstable. The average C.L.A. values and roundness errors were 10.18 and 54.8 Mic.in. respectively.

Therefore as β increases there is a noticeable variation in the roundness error of approximately 47 Mic.in., favouring $\beta = 8^{o}$, wheras the C.L.A. value seems to deteriorate as β increases, but only slightly, the variation being approximately 4 Mic. in.

3.4.6

Considerations of Geometric Stability Charts, Machine Resonance and workspeed for certain geometrical configurations

Example 1. $\beta = 8^{\circ}$, $\alpha_1 = 15^{\circ}$ hardened workblade

(r) ratio of control wheel and workpiece diameter = 7.2 DC = 6.7" dia, DW = .935" dia

Control wheel speed = 30 rev/min = .5 rev/sec

Workpiece speed (Ω) = 7.2 x .5 = 3.6 rev/sec

In this example 10 hardened specimen initially within 10 Mic.in., roundness error, were ground under the above conditions giving the following results;

Average roundness error	56 Mic. in.
Average C.L.A. value	8.7 Mic. in.
Average tolerance range	.0002 in.

All specimens had predominantly 22 waves, figure 39 (a). With such a geometrical configuration the stability charts indicate the system is geometrically stable for 22 waves, see figure 13. However with a workspeed of 3.6 rev/sec., the process became dynamically unstable a fact due to the machine resonance at approximately 80 Hz. This is an example of dynamic chatter.

i.e. $f = \Omega . n$

$= 3.6 \times 22$

$= \underline{79.2 \text{ Hz}}$

This frequency is remarkably close to the dominant machine frequency as discussed in 3.3.2 and as illustrated in figures 17-22.

Since the configuration is geometrically stable one looks to an improvement by changing the frequency, i.e. a change in the workspeed. Hence the control wheel speed in this example was increased to 39 rev/min giving a workpiece speed (Ω) of 4.68 rev/sec.

The corresponding area of interest on the stability charts is

$$n = \frac{80}{4.68} = 17$$

From the stability charts figure 13 the stability at n = 17 is much better than at n = 22 resulting in a 50% improvement in roundness as seen in figure 39 (b).

Example 2 $\beta = 8^{\circ}$, $\alpha_1 = 20^{\circ}$, hardened workblade

(r) ratio of control wheel and workpiece diameter = 7.45 (DC = 6.7" dia, DW = .9" dia.)

Control wheel speed = 39 rev/min = .65 rev/sec

Workpiece speed (Ω) = 7.45 x .65 = 4.85 rev/sec.

In this example the specimen had predominantly 16 waves as indicated in figure 40 (a). From the stability chart, figure 13 there is a geometric instability at n = 15.84, but the value of A is positive at n = 16. However here,

$$f = \Omega . n$$
$$= 4.85 \times 16$$
$$= 77.6 \text{ Hz}$$

which again is very close to the dominant machine frequency. Since from the forgoing sections 2.4.2 and 4.3.3., geometric instability is improbable in this example, the workpiece speed should be changed.

Therefore at a control wheel speed of 30 rev/min, (.5rev/sec)

$$\Omega = 7.45 \times .5 = 3.725 \text{ rev/sec}$$

$$\text{and } n = \frac{78}{3.725} = 21 \text{ waves}$$

From the stability chart figure 13, the stability parameter for 21 waves is at a peak. An example of this roundness is illustrated in figure 40 (b). From the stability charts figure 13, there is a geometric instability at n = 22 which corresponds to a frequency, at or slightly above, the resonant value. In almost all other cases instability occurs at a frequency slightly below resonance as predicted by Gurney (6). Possibly the moderate amplitudes at 22 waves may therefore be related toforced vibrations near the machine resonant frequency, rather than the work-regenerative chatter.

An extension to this example was to further reduce the control wheel speed to 23 rev/min (.38 rev/sec),

$$\text{hence} \quad \Omega = 7.45 \times .38 = 2.83 \text{ rev/sec}$$

$$\text{and} \quad n = \frac{78}{2.83} = 27.6 \text{ waves,}$$

From the stability charts, figure 13, at n = 27.6 it is found there is no trough in close proximity and hence no pronounced waviness was experienced at the resonant frequency. However 22 waves were again predominant due to the geometric instability at n = 22. The resulting roundness error is shown in figure 40 (c).

It is also interesting to note that a specimen having an initial shape of 16 waves, similar to that shown in figure 40 (a), was ground with the same configuration but with a control wheel speed of 30 rev/min., giving a workspeed (Ω) of 3.73 rev/sec, and the resulting roundness error was within 25 Mic. In., the 16 waves having been eliminated.

Example 3 A test comparison was made between $\beta = 7^{\circ}$ and 8° for $\alpha_1 = 30^{\circ}$ and a control wheel speed of 30 rev/min. The workblade had a carbide insert, and 10 hardened specimen were ground for each test. The test conditions were as previously. Diameter of specimen were .860 in and their average roundness error was 9 Mic. in., C.L.A. value 2 Mic. in., and size tolerance of .00046 in. The ratio (r) of the diameter of workpiece to control wheel was 7.5:1

Hence workpiece speed $\Omega = .5 \times 7.5 = 3.75$ rev/sec

and $$n = \frac{f}{\Omega} = \frac{78}{3.75} = \underline{20.8}$$

From the stability charts figure 14 for $\beta = 7^{\circ}$, the stability parameter for this configuration at 20.8 waves is at a peak and figure 13 for $\beta = 8^{\circ}$ is also very good. The results of this test are summarised as follows

For $\beta = 7^{\circ}$	Average C.L.A. value	= 7.4 Mic.in.
	Average roundness error	= 21 Mic.in
	Size tolerance	= .00012 in.
For $\beta = 8^{\circ}$	Average C.L.A. value	= 9 Mic. in.
	Average roundness error	= 28 Mic. in.
	Size tolerance	= .00026 in.

Example 4 Finally an extensive experimentation was carried out under the conditions of $\beta = 7^\circ$, $\alpha_1 = 30^\circ$, found most favourable in experiments, with a control wheel speed of 30 rev/min for workpiece diameters of .5, .75 and 1.0 in. For the first two diameters, 100 specimen were ground in each case, and 57 specimen for the latter. All the specimen were hardened and the main wheels were only dressed at the beginning of each test.

Results. 1.0 in diameter workpiece

Average roundness error	30.8 Mic. in.
Average C.L.A. value	10.35 Mic. in.
Workpiece tolerance	.0005 in

There was a predominant number of workpieces having 24 waves albeit often ill-defined and of small amplitude.

.75 in diameter workpiece

Average roundness error	27.6 Mic. in
Average C.L.A. value	9.2 Mic. in
Workpiece tolerance	.00014 in.

18 waves could be distinguished on some workpieces, but not other orders of waviness.

.5 in. diameter workpiece

Average roundness error	19 Mic. in
Average C.L.A. value	8.5 Mic. in
Workpiece tolerance	.00014 in

Roundness accuracy was very good with occasional incidence of 12 waves.

The predominant modes of waviness described in this example appear to be associated with work regenerative chatter as influenced by the dominant machine resonance at approximately 78 Hz as follows:- figure 41,

<u>1in. diameter Workpiece</u>

Ratio, workpiece to control wheel diameter = 6.5:1

Control wheel speed = 30 rev/min = .5 rev/sec.

Hence workpiece speed Ω = 6.5 x .5 = 3.25 rev/sec.

$$\therefore \quad f = n \cdot \Omega$$
$$= 24 \times 3.25$$
$$= \underline{78 \text{ Hz}}.$$

<u>.75 in. diameter Workpiece</u>

$$r = 8.66 : 1$$
$$\Omega = 8.66 \times .5 = 4.33 \text{ rev/sec}$$
$$\therefore \quad f = 18 \times 4.33$$
$$= \underline{78 \text{ Hz}}$$

<u>.5 in. diameter workpiece</u>

$$r = 13 : 1$$
$$\Omega = 13 \times .5 = 6.5 \text{ rev/sec}$$
$$f = 12 \times 6.5$$
$$= \underline{78 \text{ Hz}}$$

From examination of figure 14 it may be observed that in each of the above three cases the troughs were slightly to the left of the integer number of waves (within ·25) corresponding to the frequencies at which work-regenerative chatter occurred. Further work would be required to determine whether this is a general rule.

3.4.7 Summary of Experimental Results

(i) The most favourable grinding configuration was, included tangent angle $\beta = 7^{\circ}$ and workblade angle $\alpha_1 = 30^{\circ}$, which agrees with the theoretical results.

(ii) Good results were also obtained with $\beta = 8^{\circ}$ and workblades with angles 15° and 20°.

(iii) Better roundness values were obtained with a control wheel speed of 30 rev/min than with other speeds.

(iv) If instability is apparent, by pronounced waviness of high amplitude, and the product of workpiece waves (n) and workpiece speed (Ω) coincides with the dominant machine resonance (f), then an improvement in the workpiece roundness error can be made by changing this frequency by a change in the workpiece speed.

If however the product of n and Ω does not coincide with f., with apparent geometric instability, then a change in the geometrical configuration is recommended.

The following points relate to the relationship between experimental grinding results and the nature of the geometric stability charts.

(i) Troughs on the geometric stability chart indicate modes of waviness which are least rapidly removed from the workpiece.

(ii) The coincidence of a trough with an integer value of n makes a waviness particularly difficult to eliminate and if the trough is negative the amplitude of waviness increases with time.

(iii) The geometric instabilities are very mild compared to other forms of instability in machine tools. Unless there is an appreciable initial roundness error the final error will not be large due to purely geometric regeneration. This is apparent since not all specimens of a group which have been ground to be a particular unstable configuration exhibit large roundness errors as might otherwise have been expected. However if there is a large initial roundness error or other disturbance corresponding to the unstable number of waves, there will be a large resulting error. A system which is either geometrically unstable or marginally unstable will be particularly susceptible to forced vibrations of the appropriate frequency. Amplitudes should build up rapidly although the maximum values will be limited by such factors as loss of contact between wheel and workpiece.

(iv) A system may also be susceptible to the effects of forced vibrations of the appropriate frequency if a stable trough is in coincidence with an integer value of n.

This may be explained since the rounding tendency is slight and the vibration amplitude is continually additive whilst contact is maintained.

(v) Roundness errors have been apparent when a negative trough on the geometric stability chart has been slightly removed from an integer value of n by not more than ¼ to the left. However the resulting errors have not been strongly apparent and it is assumed that the previous comments concerning the importance of initial errors and forced disturbances is again relevant. It is not known whether there is any significance in the fact that in all cases where roundness occurred the trough was to the left of an integer value of n, but not to the right. Neither is it known whether it would be correct to assume that this is a general rule.

(vi) Several examples of roundness errors have arisen as a result of the dominant machine resonance. These have occurred both where a trough has been in coincidence with an integer value of n, and also where a trough has been removed to the left of the integer number of the corresponding waviness by up to ½. The frequency corresponding to the waviness is always very close to the resonance. The effects of a resonance may be avoided by making the appropriate frequency coincide with a peak on the geometric stability chart.

4. Conclusions

(i) It is important to give adequate consideration to theoretical geometric stability to avoid highly unstable operating conditions. Stability charts are easy to compute and provide information which facilitates the interpretation of the grinding results. In particular it is possible to distinguish between geometric instability and dynamic chatter.

(ii) Geometric configurations which give rise to large negative values of the parameter A with an integer number of waves (n) should be avoided.

(iii) Resonant frequencies of the machine should be as high as possible so as to minimise the tendency towards large amplitude waviness.

(iv) The workpiece speed should be adjusted so that the resonant frequency corresponds to a peak on the geometric stability chart.

Suggestions for Future Work

(i) The effect of workblade and tray deflections on workpiece stability.

(ii) The effect of workpiece inertia in theoretical and experimental analysis.

(iii) To investigate a number of similar capacity machines regarding their resonant frequencies and effect on workpiece stability.

(iv) The consideration of variable speed control wheels to optimise grinding conditions.

(v) The development of adjustable angular graduated slides for accurate setting to the correct height of workpiece centre to main wheel axis for a given workpiece and main wheel diameters.

(vi) The study of an entirely different conception of workpiece location for the centreless grinding process. Possibly a roller or rollers, driven or free, in place of the conventional workblade.

APPENDIX A

From fig 1. $\beta_1 = \sin^{-1}\left(\frac{h}{RG + RW}\right)$ and $\beta_2 = \sin^{-1}\left(\frac{h}{RC + RW}\right)$

Hence $\beta = \beta_1 + \beta_2$ and with respect to diameters;

$$\therefore \quad \beta = \sin^{-1}\left(\frac{2h}{DG + DW}\right) + \sin^{-1}\left(\frac{2h}{DC + DW}\right) \qquad (1)$$

Again from fig.1.

$$\pi = \beta_1 + \alpha + \left(\frac{\pi}{2} + \alpha_1\right)$$

$$\therefore \quad \alpha = \frac{\pi}{2} - \alpha_1 - \beta_1$$

Let $V = \frac{\beta_1}{\beta}$ then $\beta_1 = V.\beta$

then
$$\underline{\alpha = \frac{\pi}{2} - \alpha_1 - V.\beta} \qquad (2)$$

And since $V = \frac{\beta_1}{\beta}$

then
$$V = \frac{\sin^{-1}\left(\frac{2h}{DG + DW}\right)}{\sin^{-1}\left(\frac{2h}{DG + DW}\right) + \sin^{-1}\left(\frac{2h}{DC + DW}\right)}$$

$$\therefore \quad V = 1 + \frac{2h\ (DC + DW)}{2h\ (DG + DW)} \qquad (3)$$

$$\therefore \quad \frac{1}{V} = 1 + \frac{DG + DW}{DC + DW}$$

APPENDIX B

(i) Computer programme for determining the values of the response co-ordinates A and B as explained in equations (13) and (14).

```
Values A,B;
begin real pi,alpha,beta,I,m,n,A,B,a,b,K1,K2;
      integer nf,nl;
      switch s:=again,stop;
      pi:=3.14159265;
      read nf,I,nl;
again: read alpha,beta;
      if alpha=0 then goto stop;
      print sameline,£alpha =?,alpha,£  beta =?,beta,££1??;
      print £values of n,A,B?,££1??;
      alpha:=alpha*pi/180;
      beta:=beta*pi/180;
      for n:=nf step I until nl+0.1 do
      begin a:=n*alpha;
            b:=n*(pi-beta);
            K1:=sin(beta)/sin(alpha+beta);
            K2:=sin(alpha)/sin(alpha+beta);
            A:=1+K2*cos(b)-K1*cos(a);
            B:=K2*sin(b)-K1*sin(a):
            print sameline,n,prefix(££s3??),A,B,££1??;
            if  A 0 then
            for m:=n-0.9*I step 0.1*I until n-0.11*I do
```

```
            begin a:=m*alpha;
                  b:=m*(pi-beta);
                  K1:=sin(beta)/sin(alpha+beta);
                  K2:=sin(alpha)/sin(alpha+beta);
                  A:=1+K2*cos(b)-K1*cos(a);
                  B:=K2*sin(b)-K1*sin(a);
                  print sameline,m,prefix(££s3??),A,B,££1??;
            end;
      end;
      goto again;
stop:
end;
```

(ii) Computer programme package for plotting the loops on the Nyquist diagram, as shown in figure 8, for those values of Aand B encircling the (-1,0) point in a clockwise direction.

```
 Values A,B.
 begin real pi,alpha,beta,I,m,n,A,B,a,b,K1,K2,x,y;
       integer nf,nl;
       switch s:=again,stop;
       pi:=3.14159265;
       read nf,I,nl;
 again:read alpha,beta;
       if alpha<0.0001 then
       goto stop;
       punch(1);
       print ££12??,sameline,£alpha=?.alpha,£beta=?beta,££1??;
       print £values of n,A,B,£1??;
```

```
alpha:=*pi/180;
beta:=beta*pi/180
for n:=nf step I until nl +0.1 do
begin a:=n*alpha;
      b:=b*(pi-beta);
      K1:=sin(beta)/sin(alpha+beta);
      K2:=sin(alpha)/sin(alpha+beta);
      A:=1+K2*cos(b)-K1*cos(a);
      B:=K2*sin(b)-K1*sin(a);
      punch(1);
      print sameline,n,prefix(££s3??),A,B,££1??;
      if A<0 then
      begin m:=n;
            for m:=m-(0.1*I) while A<0 do
            begin a:=m*alpha;
                  b:=m*(pi-beta);
                  K1:=sin(beta)/sin(alpha+beta);
                  K2:=sin(alpha)/sin(alpha+beta);
                  A:=1+K2*cos(b)-K1*cos(a);
            end;
            punch(1);
            print ££12??;
            plotter(15.1);
            elliott(7,2,7172,0,[illegible],[illegible],7172);
            setorigin(1250,2000,2000,1);
            axes(0.1,0.1,5,5,5,5);
            for x:=-0.5 step 0.1 until 0.51 do
            begin movepen(x-[illegible],-[illegible]);
```

```
for y:=-0.5 step 0.1 until C.51 do
begin movepen(0.0125,y);
      print aligned(1,1),y;
end;
movepen(0.375,0.5);
print £m:=?,sameline,freepoint(4),m;
movepen(0.375,0.45);
print £alpha:=?,sameline,freepoint(2),alpha;
movepen(0.375,0.4);
print £beta:=?,sameline,freepoint(2),beta;
B:=K2*sin(b)-K1*sin(a);
movepen(A,-B);
for m:=m+(0.1*I) while A<0 do
begin a:=m*alpha;
      b:=m*(pi-beta);
      K1:=sin(beta)/sin(alpha+beta);
      K2:=sin(alpha)/sin(alpha+beta);
      A:=1+K2*cos(b)-K1*cos(a);
```

```
    goto again;
stop;end;
end;
```

(iii) An example of manually calculating co-ordinate A, for:

$n = 5$, $\beta = 4^\circ$ $\alpha = 68.5$ $(\alpha_1 = 20^\circ)$

$$Re(\theta_o) = K_2 \cos w T_2 - K_1 \cos W T_1 = A - 1 \quad (13)$$

$$K1 = \frac{\sin\beta}{\sin(\alpha+\beta)} = \frac{\sin 4^\circ}{\sin 72.5^\circ}$$

$$= \frac{.06976}{.95372}$$

$$\therefore K1 = .07315$$

$$K2 = \frac{\sin\alpha}{\sin(\alpha+\beta)} = \frac{\sin 68.5^\circ}{\sin 72.5^\circ}$$

$$= \frac{.93047}{.95372}$$

$$\therefore K2 = .97562$$

$$W.T1 = W\alpha$$

$$= 5 \times 68.5^\circ$$

$$= 342.5^\circ$$

and $360^\circ - 342.5^\circ = 17.5^\circ$

$$\therefore \cos W\, T1 = +.9537$$

$$W.T2 = W\ (\pi - \beta)$$

$$= 5 \times 176^\circ$$

$$= 880^\circ$$

and $880^\circ - 720^\circ = 160^\circ$ ie -20°

$$\therefore \cos W.T2 = -.93969$$

$\therefore$ A - 1 = K2 Cos W T2 - K1 Cos W T1

= (.97562 x -.93969) - (.07315 x .9537)

= -.91678 - .0696

$\therefore$ A = -.9864 +1

A = +.0136

Shoreline	Gradient of Linear TSA (m/km)	Direction of Linear TSA (declines towards)		Sum of squares (m)	Degrees of freedom	Variance (m)	F	Confidence level (%)
ILG_{4A}	0.417	27.16° N of E	Due to Linear	216.576	2	108.288	315.03	> 99.9
			Deviation from Linear	5.156	15	0.344		
			Due to Quadratic	1.376	3	0.459	1.45	< 95
			Deviations from Quad.	3.778	12	0.315		
			Due to Cubic	1.806	4	0.452	1.18	< 95
			Deviations from cubic	3.051	8	0.381		
ILG_{5A}	0.412	26.82° N of E	Due to Linear	331.990	2	165.995	757.30	> 99.9
			Deviation from Linear	3.945	18	0.219		
			Due to Quadratic	2.783	3	0.928	11.97	> 99.9
			Deviation from Quad.	1.163	15	0.078		
			Due to Cubic	0.387	4	0.097	1.43	< 95
			Deviation from Cubic	0.745	11	0.068		
ILG_{8B}	0.212	24.56° N of E	Due to Linear	72.428	2	36.214	363.92	> 99.9
			Deviation from Linear	1.393	14	0.099		
			Due to Quadratic	0.637	3	0.212	3.09	< 95
			Deviation from Quad.	0.756	11	0.069		
			Due to Cubic	0.578	4	0.145	1.61	< 95
			Deviation from Cubic	0.629	7	0.089		

TABLE 21 The gradient and direction of decline of the linear trend surface for the 4 best developed Lateglacial shorelines in the Inner Moray Firth area and the calculation of F-ratios for the contribution of successively higher-order trend surfaces.

Grid Ref.	Alt.
T122 (cont.)	
NH 5017 4196	25.18
NH 5017 4190	25.44
NH 5017 4185	26.05
NH 5017 4180	26.53
NH 5017 4176	26.97
NH 5017 4173	27.50
NH 5016 4171	28.08
NH 5080 4176	28.53
NH 5079 4180	27.89
NH 5077 4183	26.97
NH 5075 4187	26.32
NH 5073 4189	25.54
NH 5071 4193	24.94
NH 5069 4196	24.44
NH 5067 4199	24.14
NH 5067 4204	23.81
NH 5067 4208	23.49
NH 5066 4212	23.15
NH 5065 4215	22.99
NH 5065 4220	22.60
NH 5065 4223	22.13
NH 5099 4236	22.00
NH 5100 4232	22.22
NH 5102 4229	22.81
NH 5105 4226	23.43
NH 5107 4222	24.02
NH 5113 4217	24.52
NH 5113 4214	24.77
T123	
NH 5149 4282	17.82
NH 5148 4285	17.72
NH 5148 4287	17.81
NH 5146 4291	17.59
NH 5146 4295	17.82
NH 5145 4299	17.72
NH 5144 4304	17.97

Grid Ref.	Alt.
T123 (cont.)	
NH 5142 4307	17.69
NH 5141 4311	17.79
NH 5140 4314	17.84
NH 5139 4318	18.04
NH 5139 4323	17.98
NH 5138 4328	18.19
NH 5136 4333	18.29
NH 5134 4337	28.48
NH 5149 4344	17.24
NH 5152 4340	17.52
NH 5156 4338	17.39
NH 5160 4335	17.17
NH 5162 4332	16.97
NH 5163 4329	16.78
NH 5159 4325	16.52
NH 5163 4324	16.37
NH 5167 4322	16.32
NH 5171 4319	16.31
NH 5174 4316	16.22
NH 5177 4316	16.22
NH 5180 4298	17.08
NH 5176 4295	17.11
NH 5173 4292	16.93
NH 5170 4290	17.06
NH 5168 4286	17.37
NH 5165 4284	17.47
NH 5163 4281	17.54
NH 5161 4278	17.93
NH 5160 4274	18.19
NH 5160 4270	18.28
NH 5161 4265	18.46
NH 5161 4261	18.80
NH 5160 4250	19.06
NH 5160 4247	20.00
NH 5163 4236	21.97
NH 5161 4233	22.61
NH 5157 4230	23.07

Grid Ref.	Alt.
T123 (cont.)	
NH 5157 4226	23.95
NH 5157 4223	24.75
NH 5157 4218	25.19
NH 5155 4215	25.96
T124	
NH 5131 4229	25.24
NH 5129 4225	25.01
NH 5128 4227	24.67
NH 5126 4230	24.15
NH 5125 4233	23.66
NH 5124 4237	23.08
NH 5124 4240	22.50
NH 5125 4243	21.73
NH 5128 4250	20.77
NH 5129 4254	20.41
NH 5128 4257	20.04
T125	
NH 5081 4276	21.89
NH 5082 4279	21.43
NH 5083 4283	21.80
T127	
NH 5059 4267	26.44
NH 5058 4270	26.75
NH 5052 4289	27.66
NH 5043 4294	27.75
NH 5040 4295	27.81
NH 5057 4291	27.71
T128	
NH 5122 4248	18.59
NH 5120 4245	19.13
NH 5116 4242	19.42
NH 5118 4238	19.68
NH 5119 4235	20.24

Grid Ref.	Alt.
T128 (cont.)	
NH 5120 4231	20.91
NH 5122 4228	21.29
NH 5124 4225	21.96
NH 5126 4223	22.96
NH 5128 4222	24.13
NH 5130 4218	24.61
NH 5131 4216	25.11
NH 5133 4213	25.33
NH 5135 4210	26.02
NH 5134 4207	26.68
NH 5134 4203	27.37
NH 5134 4199	27.97
NH 5134 4196	28.44
NH 5128 4185	30.05
NH 5130 4182	30.68
NH 5128 4178	31.61
NH 5125 4175	32.09
NH 5126 4171	33.12
NH 5128 4167	33.74
NH 5130 4164	34.67
T129	
NH 5039 4147	29.13
NH 5040 4151	28.38
HN 5042 4155	27.70
NH 5043 4157	26.87
T130	
NH 5114 4270	15.29
NH 5116 4268	15.85
NH 5119 4264	16.10
NH 5120 4261	16.69
NH 5121 4256	17.43
NH 5119 4252	17.77
NH 5116 4249	18.28
NH 5113 4246	18.70
T130 (cont.)	
NH 5110 4244	19.00
NH 5109 4239	19.48
NH 5111 4236	20.21
T131	
NH 5116 4277	14.98
NH 5118 4274	15.66
NH 5121 4272	16.12
NH 5125 4271	16.74
NH 5128 4268	17.52
T132	
NH 5101 4283	13.34
NH 5102 4281	13.67
NH 5101 4277	14.44
NH 5097 4275	14.77
NH 5096 4273	14.97
NH 5090 4271	15.21
NH 5087 4269	15.40
NH 5134 4252	17.80
NH 5135 4255	17.58
NH 5132 4249	18.24
NH 5131 4246	18.77
T133	
NH 5737 4451	28.70
NH 5732 4447	31.06
NH 5730 4443	31.37
NH 5728 4439	32.44
NH 5726 4436	33.33
NH 5725 4433	34.20
NH 5723 4431	34.56
T134	
NH 5809 4507	29.02
NH 5806 4506	29.20
T134 (cont.)	
NH 5802 4504	29.75
NH 5799 4503	30.30
NH 5795 4500	30.54
NH 5794 4525	23.29
NH 5790 4524	23.25
NH 5786 4523	23.27
NH 5782 4523	22.47
NH 5778 4523	22.03
NH 5793 4498	31.25
NH 5790 4495	32.03
NH 5789 4491	32.82
NH 5786 4488	33.44
NH 5785 4483	34.41
NH 5786 4479	35.41
NH 5788 4475	36.25
NH 5786 4469	26.65
NH 5774 4481	25.06
NH 5768 4481	32.24
NH 5764 4479	31.87
NH 5760 4477	31.09
NH 5755 4473	29.32
T135	
NH 5565 4382	27.32
NH 5571 4386	27.08
NH 5577 4390	25.79
NH 5580 4393	25.24
NH 5583 4394	24.55
NH 5586 4397	24.09
NH 5589 4399	24.03
NH 5593 4402	23.84
NH 5596 4405	23.34
NH 5600 4407	22.71
NH 5604 4409	22.50
NH 5608 4412	22.26
NH 5610 4415	21.86