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Original citation & hyperlink:
Manarikkal, I, Elasha, F, Laila, D, delli Carri, A & Mba, D 2022, 'Simplified Single Stage Planetary Gearbox and Rolling Element Bearings Dynamic Analysis using Lagrange's Theorem and Comparison of Vulnerable Frequencies of Vibration', Journal of Vibration Engineering and Technologies , vol. 10, pp. 211-223.
https://doi.org/10.1007/s42417-021-00372-0

DOI 10.1007/s42417-021-00372-0
ISSN 2523-3920
ESSN 2523-3939

Publisher: Springer

The final publication is available at Springer via http://dx.doi.org/10.1007/s42417-021-00372-0

Simplified Single Stage Planetary Gearbox and Rolling Element Bearings Dynamic Analysis using Lagrange's Theorem and Comparison of Vulnerable Frequencies of Vibration

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Abstract— Planetary gearboxes are widely used in the aerospace, automotive and renewable energy sectors. These gearboxes are the critical component in a system's reliability; hence, monitoring this component is essential. In order to study the vibration characteristics, a novel dynamic modelling approach using Lagrange's equations is presented in this paper. Newtonian equations of motion have been developed and solved using Lagrange's theorem, and modal analysis has been performed to estimate the dynamic characteristics of a single-stage planetary gearbox. The torsional stiffness and time-varying mesh stiffness have been considered in this paper for dynamic modelling. This study applies the perturbation method to solve the conditions of the system. The significant resonance frequencies of the individual components in the gearbox have been identified using coordinates based on eigenvectors. The finite element analysis results have been considered for validation purposes and compared to the numerical model. The frequency range of the gearbox components' resonances and dynamic characteristics have been obtained from the FEA. The study shows that Lagrange's dynamic modes match with modes obtained from the finite element modal analysis. In addition, the resonance frequencies produced by the sun and planet gears and the bearings are detected. Therefore, the results show a positive potential in gearbox fault diagnostics and characteristics frequencies studies for individual components in the gearbox.

Keywords— Gearboxes, Vibrations, Resonance, FEM, Modal Analysis

I. Introduction

Gearboxes are a critical component in a variety of machines. From the reliability point of view, planetary gearbox failure can occur due to bearing defects and tooth crack initiation. Planetary gearboxes are also known as epicyclic gearboxes and are widely used in the aerospace, automotive and energy sectors due to their unique characteristics such as their high load bearing capacity, high transmission ratio, high torque to weigh ratio, compactness and resistance to shock. Gearboxes in wind turbines tend to fail at early stages due to varying load conditions. An epicyclic gearbox is often the critical component widely used in the aerospace and energy sectors [1] due to its high load-bearing capacity. Due to its complex mechanical structure and motion type, it is an essential mechanical component for transmission systems in most non-stationary operational environments. Gearbox associated failures and maintenance are among the main concerns in the power generation sector. An increase in global wind energy demand has made a significant change in the research and development of gearbox performance. According to specialist renewable energy statistics (2014), the gearbox constitutes 13% of the overall cost of the wind turbine's maintenance [2]. The maintenance and replacement of a gearbox could take time due to its complexity and installation procedures. By considering the failure rate of components in turbines, bearings failure is much higher at a rate of 70%, gears are at a rate of 26%, and other components are at 4% [3]. Planetary gearboxes have many vibration sources, and these vibrations mainly come from the bearings and gears during meshing. The failure of any parts in an epicyclic gearbox could lead to the catastrophic failure of the whole drive train [4]. A planetary gearbox comprises four main elements that produce a wide range of speed ratios, i.e. sun gear, ring gear, planet gear and carrier gear. The dynamic evaluation and condition monitoring are inevitably critical in high-speed train traction transmission systems [5]. Furthermore, the planetary gearbox used in cranes is prone to failure due to high loading factors and axial deformations. Therefore, a dynamic investigation plays a critical role in noise harshness and vibrations of planetary gearbox structure[6].

Vibration-based monitoring techniques have always been a research interest to estimate the life and diagnose the faults of bearings and gearboxes [7]. Past research in planetary gearbox diagnostics and prognostics has hugely contributed to the advancement of gearboxes and their reliability. He and Jia [4] developed a numerical model for studying the dynamic response of a single-stage planetary gearbox with a backlash effect using the Newtonian lumped mass parametric method. The study confirms a significant change in vibration present when there is an increment in backlash. Jungho [8] established a novel fault diagnosis approach using transmission error with a lumped mass parametric dynamic model. The transmission error of the planetary gearbox was diagnosed in both a healthy and a faulty state. The results show that the meshing of the gears is out of phase when there is a fault present on the gears. Inalpolat and Kahraman [9] proposed the power and gear mesh model. The modes are simulated by solving the eigenvalues and the eigenvectors. The research used the torsional case to detect the natural frequencies and vibrations. The forced vibration excitation stage was computed by the corresponding eigenvalues. Zhao and Mutellip [10] studied the dynamic numerical simulation for a wind turbine helical planetary gearbox along with FEM solutions. The gears' mode shapes and natural frequencies were studied in [11,11] for varying mesh stiffness values using time-synchronous averaging techniques. Time-frequency analysis and the dynamic response of the gears in the presence of a crack were investigated in [13]. The study concluded that the change in frequency response function FRF results from an increase in the crack's size. Dynamic modelling of the spur gear system using lumped mass Newtonian equations was presented in [14].

A gear model with torsional and translational vibration was modelled using the lumped parametric method [15]. The effect of a crack on the gear mesh stiffness observed in this study. Characteristics of a three-dimensional compound planetary gearbox vibration were studied in [16] from a lumped parameter non-linear model using TVMS (time-varying mesh stiffness) alterations and mesh phase relations on a faulty sun gear compared with a healthy sun gear. The result indicated a significant reduction in stiffness when the sun gears are in chipped conditions. Howard and Jia [17] modelled a 16 DOF system considering friction. The effect of torsional mesh stiffness during the gear meshing was analysed, and the effects with friction and without friction were compared for the fault diagnostics. The study showed more noise present in the frequency spectrum when there is friction due to defects. Lagrange's equations of planetary gear were modelled for studying the transient shift behaviour of gears. However, no frequency or resonance study was involved. A dynamic model of planetary gear with a varying pressure angle and contact ratio due to a faulty bearing module using the Lagrange method was modelled in [18]. The study considered non-linear gear mesh deformation, and the results showed that the time-varying pressure angle and contact ratio causes more frequency components in the spectrum than the constant pressure angle and contact ratio case. The dynamic modelling of a planetary gearbox using Newtonian equations of motion for studying the resonance and Modal properties was investigated in [19]. The model was developed using the lumped parametric method and the solved eigenvalues problem to study the resonant frequencies. The effects of stiffness and backlash were also reflected in studying the dynamic behaviour of the planetary gearbox. The results showed the relation between the backlash and the high-frequency responses in the resultant vibration.

Research in planetary gearbox vibration studies has increased dramatically over the past few years. Many studies have already been conducted in the area of planetary gearbox modelling with Newtonian equations [20] [27][13][14], but work performed in the area of planetary gearbox dynamic characteristics studies using Lagrange's theorem is rare. In this paper, numerical modelling through Lagrange's energy method for a single-stage planetary gearbox has been established and solved to perform modal analysis. In Newtonian mechanics, coordinate systems and all the constraint forces need to be considered while modelling. Lagrange's scheme avoids the considerations of the constraint forces deftly and can use any set of "generalised coordinates" consistent with the constraint relations. Moreover, the number of generalised coordinates are the same as the number of degrees of freedom of the system. This makes the modelling a lot easier to solve and study the behaviour of the system. However, a limitation in Lagrange modelling is that it is not ideal for non-conservative forces like friction. This study applies the perturbation method to solve the conditions of the system. The significant resonance frequencies of the individual components in the gearbox have been identified using the coordinates based on eigenvectors. This has made the system simple and easy to solve for studying the dynamic behaviours and characteristics. A gearbox with similar parameters was designed for performing the finite element analysis to validate the numerical modelling results. In the FE analysis, the modal analysis, resonance and characteristics frequencies were studied for validation. The possible faults and severity in the gears and bearings could be identified by using velocity vibration signal analysis in horizontal, vertical and axial directions. This simplified model's characteristics and modal frequencies can be used for the fault diagnostics of planetary gearboxes. The natural frequencies of modes will decrease if there is an increase in misalignment. Axial displacement increases with misalignment; the velocity is proportional to displacement. Therefore the velocity also increases with misalignment [21]. The finite element analysis of the model was also performed to analyse the dynamic behaviour of the system as well as the frequency response functions. This paper compares the modes and the characteristics resonance frequencies of a planetary gearbox which could lead to fatigue analysis, experimental research in diagnostics and the prognostics of the single-stage planetary gearbox. The mathematical model developed in this system incorporates the torsional and time-varying mesh stiffness, resonance characteristics of individual components through coordinate transformations. The novelty of this work is centred around resonance and vulnerable frequencies identification for fault detection and diagnostics of the gearbox. Although many degrees of planetary gearbox mathematical modelling has been studied in the past using Newtonian and Lagrange modelling, the validation using FEA and resonance frequency identification approach were not established in the past.

This paper is structured as follows. The first section discusses the developed dynamic modelling using Lagrange's method. The second part of this paper discusses the modal analysis for resonance studies using the finite element method.

II. LAGRANGE'S EQUATION FOR THE PLANETARY GEAR SYSTEM

A planet gears system with one planet and bearing module instead of 3 planets and 3 bearings, one sun gear, carrier gear and ring gear, the dynamic model representation is illustrated in Fig. 1. The arrangement of a single-stage gearbox can be seen in Fig. 2. Planet bearings on each of the planets are considered elastic media with stiffness equal to so, so the planet's centre can move near the equilibrium positions. The coordinates are

:

1. Sun angular coordinate is ϕ ,
2. Carrier angular coordinate is ψ ,
3. Planets' angular coordinates are θa ,
4. Number of planets is n,
5. C_{rp} and C_{sp} refer to the viscous damping of the ring-planet and sun-planet meshing.
6. Cartesian coordinates in planet bearings' displacements versus planet centre in carrier arex_a,y_a where x_a displacements in a tangential direction and y_a is displacement in a radial direction relative to the planet's centre motion.

K_t - Stiffness in teeth contact.

K_b - Stiffness in bearings.

The kinetic energy of the planet gear system E_{kin} is given by the formula;

$$E_{kin} = \frac{1}{2}\left\{I_c\dot{\psi}^2 + I_p\sum_{i=1}^{n}\dot{\theta_i}^2 + I_s\dot{\phi}^2 + m_p\sum_{i=1}^{n}((r_c\,\dot{\psi} + \dot{x_i})^2 + \dot{y_i^2}))\right\} \quad (1)$$

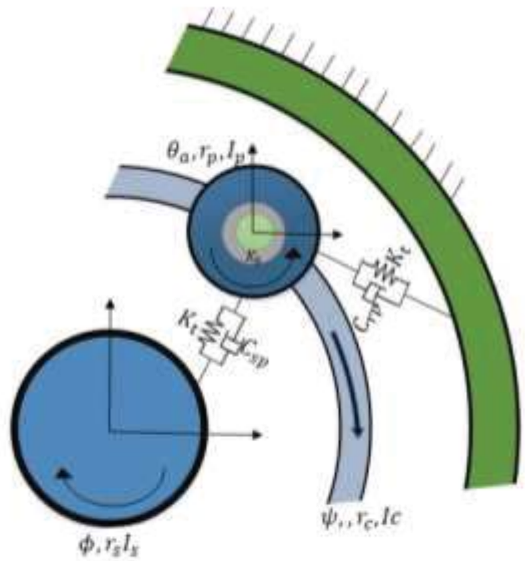


Fig.1 5 DOF Dynamic Model of a Planetary Gearbox

where I_s, I_c, I_p, are the moments of inertia of the sun, carrier and planet, $m_p$is mass of the planet. Displacements in the contacts of the sun planets δ_{spi} and ring planets δ_{rpi} are given by the expressions

$$\delta_{spi} = -(r_c\psi + x_a)\cos(\alpha) - r_p\theta_a + r_s\theta - y_a\sin(\alpha) \tag{2}$$

$$\delta_{rpi} = -(r_c\psi\_x_a)\cos(\alpha) + r_p\theta_a + y_a\sin(\alpha) \tag{3}$$

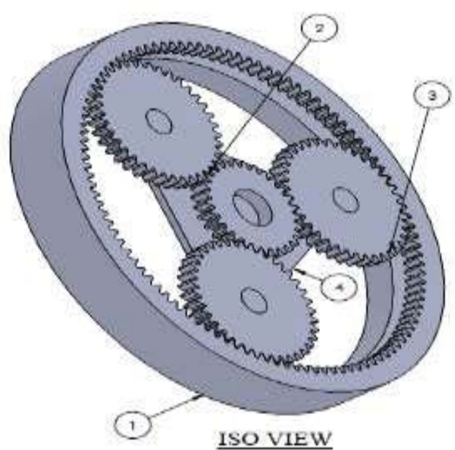


Fig.2 The gearbox model

where α is the pressure angle and deformations in the contacts results in the elastic force. The causes of the elastic force are the torsional stiffness (K_{body}) of the deformation of the pinion body, the torsional stiffness of the decline of the tooth (K_{tooth}) and the stiffness deformation of the tooth ($K_{contact}$) material in the place of contact [22]. All three stiffnesses are combined into the resulting stiffness K_t of gear contact, which will act like three springs in a row.

$$\frac{1}{K_t} = \frac{1}{K_{body}} + \frac{1}{K_{tooth}} + \frac{1}{K_{contact}} \quad (4)$$

Here, linear dependence of force and deformation is accepted, so potential energy equation is expresses as,

$$U_{pot} = \frac{1}{2}(K_t \sum_{i=1}^{n} ((r_c\psi + x_i)\cos(\alpha) + \theta_i r_p - r_s\phi + y_i \sin(\alpha))^2 + K_t \sum_{i=1}^{n} (-(r_c\psi + x_i)\cos(\alpha) + \theta_i r_p + y_i \sin(\alpha))^2 + K_b \sum_{i=1}^{n} (x_i^2 + y_i^2)) \quad (5)$$

Lagrange's equation for the free motion of the planet gear system is;

$$L = E_{Kin} - U_{pot}$$

Lagrange's equations for free motion without damping are;

$$J_s\ddot{\phi} - \sum_{a=1}^{n} K_t r_s(-r_s\phi + \cos(\alpha)(r_c\psi(t) + x_a) + \sin(\alpha)\, y_a + r_p\theta_a) = 0$$

$$\left(2K_t\psi\cos^2(\alpha) + nm_p\ddot{\psi}(t)\right)r_c^2 + \sum_{a=1}^{n}(2K_t x_a \cos^2(\alpha) - K_t r_s\phi\cos(\alpha) + m_p\ddot{x_a})r_c + J_c\ddot{\psi} = 0$$

$$2K_t\theta_a r_p^2 - K_t(r_s\theta - 2\sin(\alpha)\, y_a)r_p + J_p\ddot{\theta_a} = 0$$

$$2K_t x_a \cos^2(\alpha) - K_t r_s\phi(t)\cos(\alpha) + K_b x_a + r_c(2K_t\psi(t)\cos^2(\alpha) + m_p\ddot{x_a})\_m_p x_a \ddot{=} 0$$

$$-K_t\sin(\alpha)\, r_s\phi + (2Kt_t\sin^2(\alpha) + K_b)y_a + 2K_t\sin(\alpha)\, r_p\theta_a + m_p\ddot{y_a}=0 \quad (6)$$

*A. **Modal analysis of gear motion in the case of varying constant stiffness***

The Lagrange approaches are highly convenient for modelling systems in arbitrary coordinate systems without the effect of damping, as shown in equation (7). In Lagrange's equations, the kinetic and potential energies act as a function of independent coordinates. Therefore, there is no need to trace the force to find their expressions (all calculations are derivatives), making the whole system's equations easy to solve.

Equations (6) can be written in matrix form;

$$M\ddot{X} + KX = 0 \tag{7}$$

where M is the mass matrix, K is the stiffness matrix, X is the physical coordinates of the system, $X = (\phi, \psi, \theta_a, x_a, y_a)^T$.

$$M = \begin{pmatrix} J_s & 0 & 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & nm_p r_c^2 + J_c & 0 & m_p r_c & 0 & & 0 & m_p r_c & 0 \\ 0 & 0 & J_p & 0 & 0 & \cdots & & & \\ 0 & m_p r_c & 0 & m_p & 0 & & & & \\ 0 & 0 & 0 & 0 & m_p & & & & \\ & & \vdots & & & \ddots & & \vdots & \\ 0 & 0 & & & & & J_p & 0 & 0 \\ 0 & m_p r_c & & & & \cdots & 0 & m_p & 0 \\ 0 & 0 & & & & & 0 & 0 & m_p \end{pmatrix} \tag{8}$$

$$K = \begin{pmatrix} K_t r_s^2 & -K_t \cos(\alpha) r_s r_c & -K_t \cos(\alpha) r_p r_s & -K_t \cos(\alpha) r_s & -K_t \sin(\alpha) r_s \\ -K_t \cos(\alpha) r_s r_c & 2K_t r_c^2 cos^2(\alpha) & 0 & 2K_t r_c cos^2(\alpha) & 0 \\ -K_t \cos(\alpha) r_p r_s & 0 & 2K_t r_p^2 & 0 & 2\,K_t \sin(\alpha) r_p \\ -K_t \cos(\alpha) r_s & 2K_t r_c cos^2(\alpha) & 0 & K_b + 2K_t cos^2(\alpha) & 0 \\ -K_t \sin(\alpha) r_s & 0 & 2\,K_t \sin(\alpha) r_p & 0 & K_b + 2K_t sin^2(\alpha) \end{pmatrix} \tag{9}$$

K is a 5 by 5 matrix. The rows and columns with θ_a, x_a, y_a must be repeated a number of times equal to the number of planets. The equation (7) can easily be solved if matrices M and K are constant. However, the tooth stiffness changes due to one or two teeth being in contact. We present stiffness as a constant part and a small fluctuation [23]:

$$K_t = k_0 + \epsilon f(t), \tag{10}$$

where ϵ is the small parameter [24] and stiffness matrix K is the sum of the two matrices K_0 and K_v

$$K = K_0 + \epsilon K_v \tag{11}$$

where K_0 is the 5 by 5 matrix (9) after replacing K_t by K_0 , Matrix K_v has this form;

$$K_v = \begin{pmatrix} f(t) r_s^2 & \cdots & -f(t)\sin(\alpha)\, r_s \\ \vdots & \ddots & \vdots \\ -f(t)\sin(\alpha)\, r_s & \cdots & 2f(t)\sin^2(\alpha) \end{pmatrix} \tag{12}$$

We will solve the equation (7) with the perturbation method [23] for finding the approximate solution to the problem. The vector X is written as zero and the first approximation is,

$$\mathrm{X} = \mathrm{X}_0 + \epsilon \mathrm{X}_1 \quad (13)$$

The equation (7) with approximations (12) and (13) give rise to equations of the first and the second orders;

$$M\ddot{X}_0 + K_0 X_0 = 0, \quad (14)$$

$$M\ddot{X}_1 + K_0 X_1 = -K_v X_0 \quad (15)$$

To solve equation (14), we must solve the eigenvalues and eigenvectors problem;

$$K_0 A_k = -\omega_k^2 M A_k \quad (16)$$

where ω_k are modal frequencies and A_k are eigenvectors. Eigenvectors form the modal matrix ϕ, where the eigenvectors are matrix columns.

$$\phi = (A_1, A_2, A_3 \dots \dots .. A_k) \quad (17)$$

The norm of eigenvectors is chosen to make from the matrix M unity matrix in modal space. Matrix K_0becomes a diagonal matrix:

$$M_m = \phi^T M \phi = diag(1,1, \dots .,1)$$

$$K_{0_m} = \phi^T K_0 \phi = diag\ (\omega_1^2, \omega_2^2, \dots \dots, \omega_k^2) \quad (18)$$

In modal space, equations of motion are separated for each modal coordinate Y_{0k}. The solution is

$$Y_{0k} - C_k e^{i\omega_k t} \quad (19)$$

giving modal vector $Y_0$with projections Y_{0k} . Vectors $A_k$are columns of the modal matrix ϕ. In modal space, equation (15) becomes;

$$M_m \ddot{Y}_1 + 2C\dot{Y}_1 + K_{0m} Y_1 = \phi^T K_v \phi Y_0 \quad (20)$$

Small damping with coefficient C has been added to exclude significant amplitudes (a realistic approach) for resonance frequencies from the right-hand side of the equation (20) (which is the modal space general equation than 15). The equation (20) is nonhomogeneous linear equations. The left-hand part of the equation (20) contain modal frequencies ω_k. The right-hand part of the equation (20) contains a vector $Y_0$with modal frequencies ω_k and a variable part $f(t)$ of stiffness. The variable part is a periodic function due to the periodicity of the teeth contacts. The product of vector Y_0 and Fourier's series give rise to frequencies which are sums $\omega_k + \omega_f$and differences $\omega_k - \omega_f$[24]. Suppose one of those frequencies becomes equal to one of the modal frequencies on the left side. In that case, we will have a resonance that can damage the gear transmission since, in the modal space, all vibrations are one-dimensional vibrations with modal frequencies. Considering the right-hand side of equation (20) and its exponent with each resonance frequency, it is possible to calculate the amplitudes for the forced vibrations as in the one dimensional case and this can be transformed to the angles and x, y coordinates.

B. Case Study

For simulation purposes, possible deformation of the planet bearings was considered. The parameters for the elements of the planet gearbox are in Table 1. Assuming that the constant part of the teeth stiffness was equal $k_0 = 10^8\ N/m$, the bearing stiffness was taken as equal to $10^9\ N/m$, see reference [4] and [25] for details. The varying part of the teeth stiffness was taken as smaller $\epsilon =$

10^{-3} relative to the constant part of the form [26]. The parameters in the table.1 below are the values used to design the CAD model of the planetary gearbox which were obtained from the Rossi Gearbox EP Series Manual [28].

TABLE.1 Parameters for numerical calculations

Parameters	Sun	Planets	Carrier	Ring
No. of teeth	23	24	-	73
Pitch radius	22.85 mm	23 mm	-	149 mm
Mass	309g	1816g		1224g
Pressure Angle (P.A)	20^o			
Inertia	$0.39\ \mathrm{kg.m^2}$	$0.61\ \mathrm{kg.m^2}$	$3.00\ \mathrm{kg.m^2}$	$6.29\ \mathrm{kg.m^2}$

The system considered here has 5 degrees of freedom (one planet's translational and rotational motion, one bearing module's rotational motion, the sun gear's rotational motion and the carrier gear's rotation along with the planets). To study the vibrational characteristics, only one member of the multiple elements in the gearbox is considered. This has helped to minimise the complexity of the equation. Since the ring gear is at rest, it has no degree of freedom. There are 11 modal frequencies in zero approximation, and there are numerous modal frequencies that are equal. One of the modal frequencies is equal to zero, which means this is the rigid motion mode. After performing the modal analysis simulation, six modes have been obtained. The first three modes (Modes1, 2 and 3) are low band frequencies, modal frequencies of the sun, the carrier, and the planet's oscillations. The last three modes' (modes 4, 5 and 6) frequencies are from the bearings. The oscillation of bearings and the gears are obtained from simulation. The oscillations of the bearing module (figure.3) and sun, planet and carrier gears (figure.4) are represented below. The first six modal frequencies obtained from the simulation are listed in table.2.

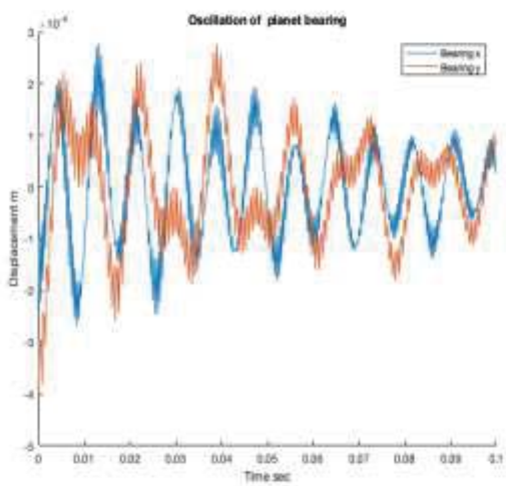


Fig.3 The planet bearing oscillations in x and y directions

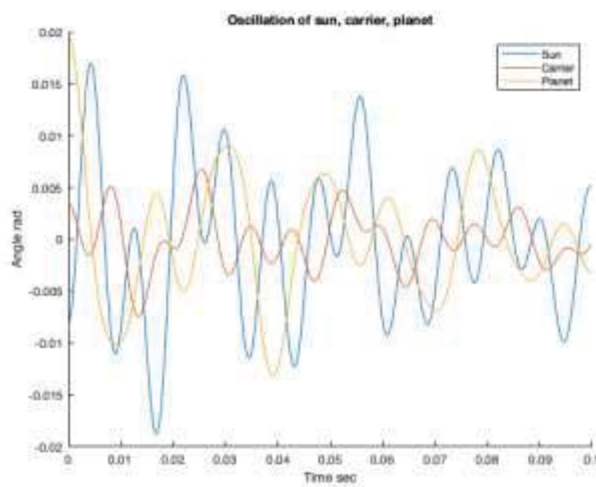


Fig.4 The sun, planet and carrier gear oscillations

The possible resonance frequencies (listed in table.3) have been calculated using the coordinates based on eigenvectors. The gear's motion can realise not all resonance frequencies. Also, not all resonance frequencies enter in the equation (20) due to the matrix in modal space. The resonance frequencies can be ranged as low band and high band frequencies. It is clear from Fig. 6 that all the resonance frequencies give resonance in the first approximation on the variable part of the stiffness. Here, the frequency is considered as the resonance frequency if its amplitude is in the range [0.8, 1] of the maximal amplitude for all frequencies. The simulation of the numerical model produced 37 resonance frequencies of the gearbox system, and the possible resonance frequencies of the individual components have been extracted by making 1200 coordinate points with respect to the planet's degree of freedom. The calculated possible resonance frequencies of the gears using the perturbation method are listed in table.3.

TABLE.2 Modal frequencies

Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
240 Hz	326 Hz	1083 Hz	1680 Hz	6688 Hz	6302 Hz

TABLE.3 Resonance frequencies of the gearbox members

Gear	Resonance Frequencies
Sun	14 Hz, 75 Hz, 88 Hz, 297 Hz
Carrier	310 Hz, 398 Hz
Planets	400 Hz, 473 Hz
Bearings	6065 Hz, 13404 Hz

The low band resonance frequencies 297Hz, 323 Hz and 473Hz appearing in the sun, the carrier and the planets are identified and shown in Figs. 7, 8 and 9. The high band frequencies 6065Hz, 6140Hz, 6153Hz, 6350Hz, 6463Hz, 6465Hz, 6538Hz, 6539Hz and 6938Hz produced by resonance in the bearings are also shown in Fig. 10 and Fig. 11. When the rolling element bearing defect comes in contact with another surface, an impulsive force is created. This impulsive force will possess larger amplitudes

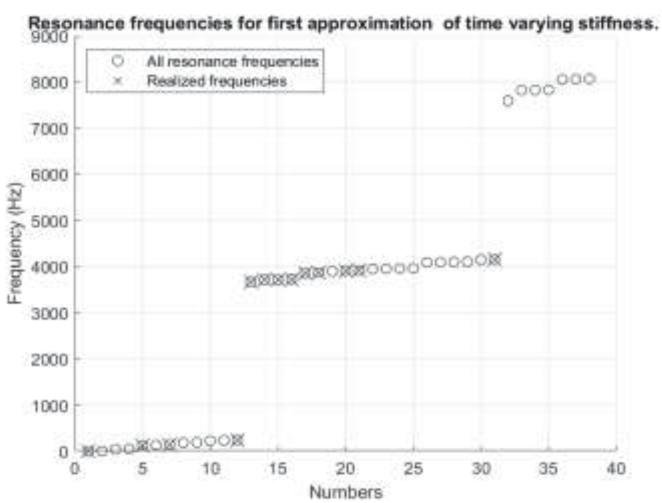


Fig.5 Resonance frequencies and realised resonance frequencies

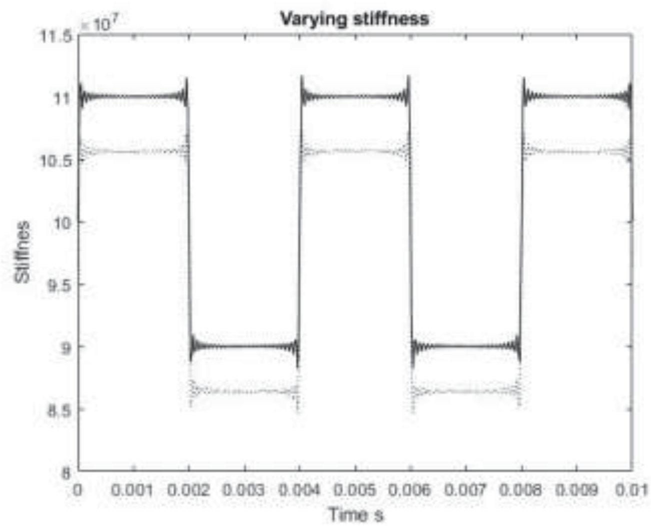
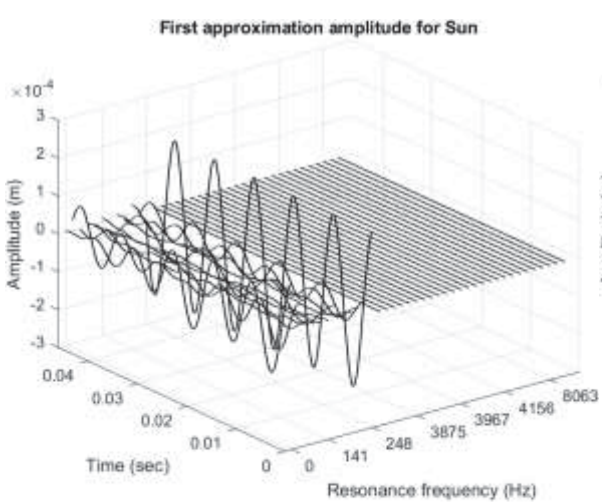


Fig. 6 Time Varying Mesh stiffness



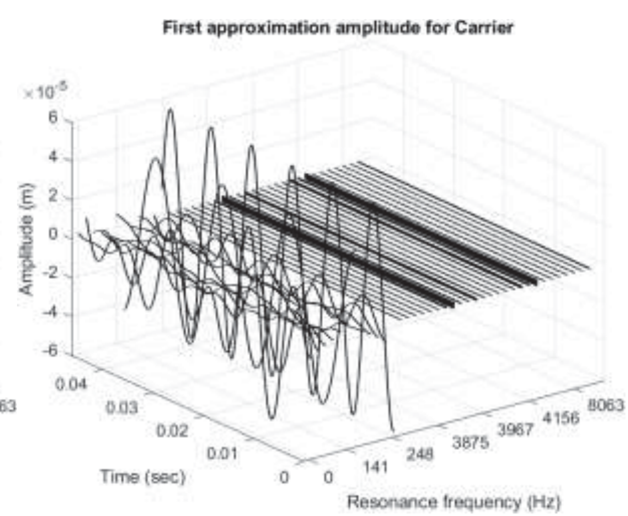


Fig.7 Resonance of the sun on 473Hz

Fig.8 Resonance of the carrier on 323Hz and 473Hz

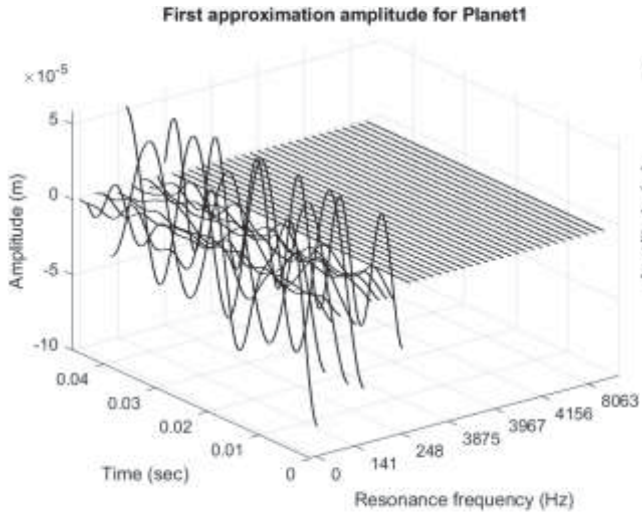


Fig.9 Resonance of the planet on 297 Hz and on 473 Hz

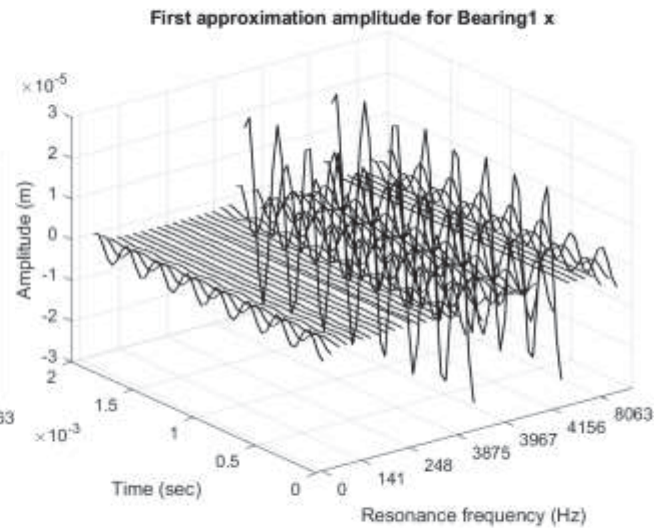


Fig.10 Resonance of x coordinates of the bearing on frequencies 6465Hz and 6938Hz

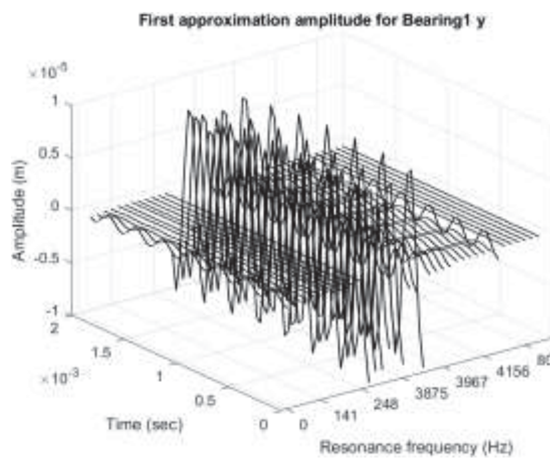


Fig.11 Resonance of y coordinates of the bearing on frequencies 6065Hz, 6140Hz, 6153Hz, 6350Hz, 6463Hz, 6538Hz and 6539Hz

For this research, modal space has been considered, and some modal frequencies have been obtained. If varying stiffness in the first approximation has resulted in such frequencies, there is a resonance present. Here, we proposed and proved that the resonance frequencies were all sums and subtracted pairs of modal frequencies. There were nearly 40 such frequencies. Figure. 5 represents them as circles on the graph in order from the lowest to the biggest X marks the frequencies that resonate. A resonance frequency is typically a significant increase in vibration, and it is essential to fetch this frequency range to study the vibration behavior of the gearbox. The frequency range of the individual components of the gearbox is shown in Figs. 7 - 11. The resonant frequency should be reduced to design an efficient gearbox through the elimination of noise and vibration. It is clear from Figs. 10 and 11 that the bearing resonant frequency bands are large compared to other components in the gearbox. Therefore, the fault will initiate from the planet bearings and transform the faults to the carrier and the planet gears. Bearing damage is a prevalent cause of machinery vibration due to local defects on the balls or the raceways. The super harmonic frequencies are the results of bearing inner and outer raceways defects. Low frequencies with a high amplitude are resonance frequencies originated from the sun, the planets and the carrier gears, whereas high frequencies with low amplitude resonances are from the bearings. Figure. 6 shows the comparison of time-varying mesh stiffness of gear tooth in healthy and unhealthy gear (reduced stiffness).

C. *Finite Element Analysis*

The single-stage planetary gearbox model was taken as the research object to perform the FE modelling and validate the results with numerical dynamic model simulation results. The gear was designed according to the parameters followed by Table 1 obtained from the gearbox data manual. Structural Steel material has been used for the analysis with the following properties as shown in Table 4 and is considered as a standard structural steel constant mainly used for gears. Structural analysis is critical because it can determine the cause and predict failure as to whether or not a specific structural design will be able to withstand the expected external and internal stresses and forces for the design. The model developed in Fig. 12 has been designed using the parameter values from Table.1 that match the description given in Section B as part of the case study.

As mentioned above, a finite element analysis is to be performed for a given design of planetary gearbox under design loading conditions. After performing a definite FEA of the system, results obtained for the stresses were justified with the available data for maximum allowable values in the properties' database mentioned below. Mechanical Properties of static structural are as standard (Fatigue Data at zero mean stress comes from 1998 ASME BPV Code, Section 8, Div. 2, Table 4-at Normal Temperature)[29].

TABLE.4 Properties of structural steel (ANSYS)

Structural Steel Constants	
Material Properties	Structural Steel
Density	7885 kg/m3
Modulus of Elasticity	200 GPa
Poisson's Ratio	0.30
Tensile Yield Strength	250 MPa
Ultimate Tensile Strength	460 MPa
Coefficient of Thermal Expansion	$1.2510^{-0.5}$C^{-1}

The general arrangement of the planetary gearbox and the loading conditions are shown in Fig. 12. Refinements have been applied to selected areas to improve the meshing quality. The dynamic characteristics of the planetary gear system have been studied extensively. However, most of them are of the spur planetary gear and the characteristics frequencies of the planetary gear are rarely considered.

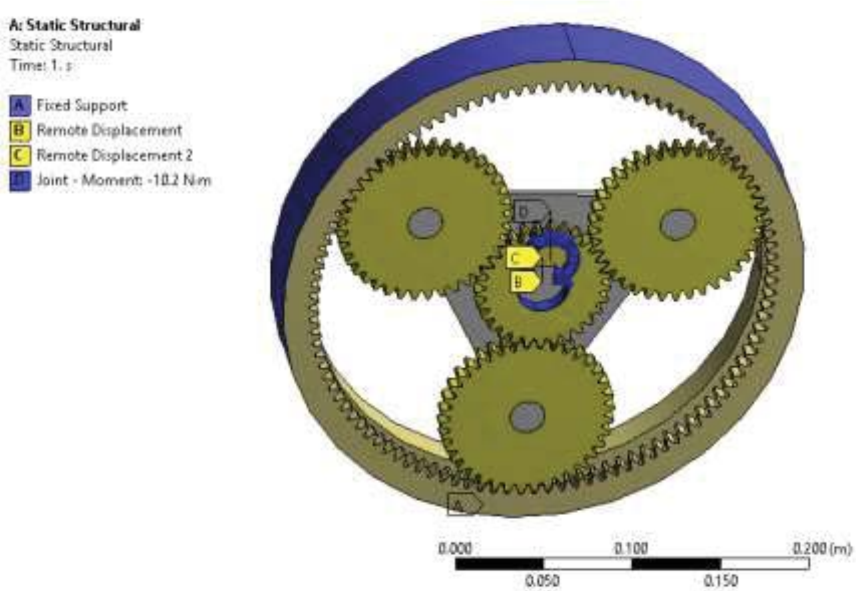


Fig. 12 Loading Conditions

The arms end supported as fixed, and the reference coordinate has been assigned to create the direction. Various contacts have been identified between the sun gear, ring gear, planet gear and arm. Teeth contacts were considered as a bonded contact from the beginning. Body to body contact bearing has been assigned for the planet gears. Three such bearings were applied to all the sun gears.

D. Boundary Conditions

The input is the sun gear and the output is the carrier gear. The ring gear was arrested, having no freedom of movement in this simulation. The assumptions made for setting up the boundary conditions of the planetary gear for performing the simulation are as follows.

1) Ring Gear is a fixed constraint with all six degrees of freedom.
2) Sun Gear is a fixed constraint with five degrees of freedom, except the rotational in the X direction DOF.
3) Planetary Gear is a fixed constraint with X linear direction, which is normal to the rotational Plane.
4) All the contact is assumed to be bonded for maximum stress concentration conditions.

E. Contact Conditions

In a dynamic process, the contact conditions are complex to analyse. If the contact's set conditions are not very accurate, the gears can easily penetrate into one another. This may cause catastrophic failure. The surface to surface and nodes to surface contact set has been used for this simulation. The static and dynamic frictional coefficient is 0.01 in this process (hard steel on hard steel value [32]). The contact set of the planet and carrier gear can be seen in Fig. 13, for example. The mesh consisted of both tetrahedral and hexahedral finite elements, 1291155 nodes and 472888 linear elements. The mesh refinement of the contact region between planet and sun is presented in figure.14.

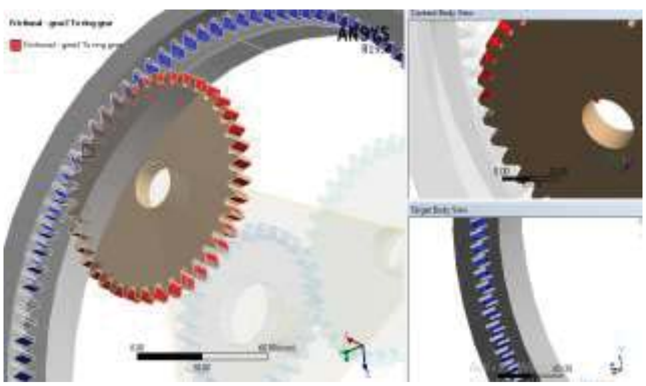


Fig. 13 Contact set of ring and planet

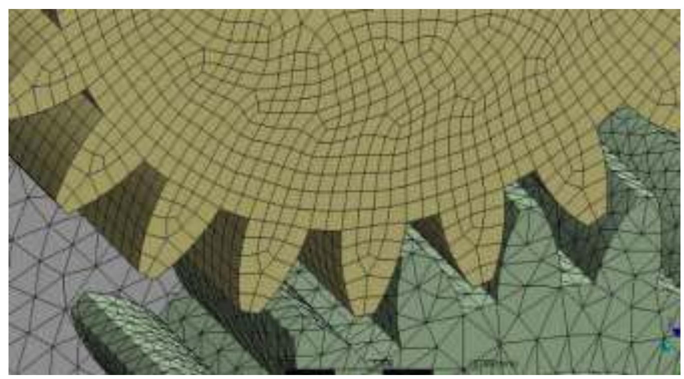


Fig. 14 Refinement on Gearbox

In this work, different types of analyses have been performed to check the behaviour of the planetary gear under other conditions. Modal analysis has been performed to identify the critical speed of the gear system at which larger deflection occurs. A mesh refinement on the structure is essential and will have a significant impact on results. As shown in the above figures, a structure mesh refinement was concluded for the sun, planet and ring gear with a 2 mm body sizing. The mesh shown in Fig. 14 is refined into fine elements due to the maximum contact stress.

Modal analysis has been performed using the modal analysis module with basic boundary conditions and structural steel material. A direct solver has been used instead of program control to improve the results based on software suggestions. The explicit dynamic simulation result at 334.02 Hz is shown in Fig. 15. The model's accuracy and stability have been checked using static structural analysis before simulating the dynamic analysis. Deformation in all the directions is moderate, which does not enforce added tensile stress on individual components. So, the overall design is safe for construction at the given boundary conditions and specified temperature as per the performed FEA. The meshing quality of the model is greater than 0.75, which shows that the model is accurate to produce better results. Six modal frequencies have been identified in the FE simulation. This fundamental natural frequency can be used to determine the critical speed of the system. Deformation values corresponding to the mode shape calculated shown in Fig. 15. The default settings for the mode shape are six but can be changed as per requirements. In this case, the maximum ratio of effective mass to the total mass was in the direction of x. The natural frequency considered here was 334.025 Hz, at which the frequency maximum mass deviated from the original position.

TABLE.5 Participation Factor (Max Ratio Effective Mass to Total Mass) ROT X Direction

MODE	Frequency	Period	Participation Factor	Ratio of Effective Mass to Total Mass
1	242.06	4.1303×10^{-3}	3.10×10^{-4}	9.22×10^{-7}
2	334.025	2.99×10^{-3}	-0.318	0.9795
3	554.147	1.80×10^{-3}	-1.26×10^{-3}	1.54×10^{-5}
4	558.907	1.78×10^{-3}	-6.12×10^{-4}	3.61×10^{-6}
5	1092.23	9.16×10^{-3}	-5.98×10^{-4}	3.45×10^{-6}
6	1518.31	6.58×10^{-3}	9.90×10^{-4}	9.52×10^{-8}
SUM				0.980

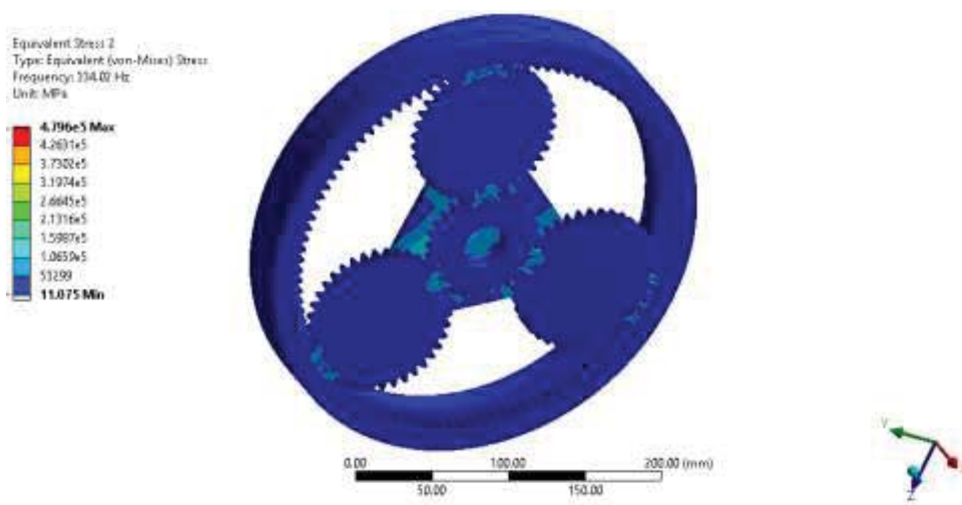


Fig. 15 Stress on gears at 334 Hz

III. Discussion

The dynamic characteristics of the planetary gear systems were analysed, taking into account the bearing deformation of the planet bearings and the time-varying stiffness for the teeth deformations—the equations of motion written as Lagrange's equations. The time-varying stiffness was considered small periodic perturbations due to periodic changes in the number of teeth in contact. The stiffness was deemed to be constant in zero approximation and identified the modal frequencies in zero-order approximation. The modal frequencies divided into low and high band frequencies; low band frequencies describe gears' angular oscillations, high band frequencies describe linear oscillations in the bearings. Resonance frequencies increase oscillation amplitude. The resonance frequencies were also both low band and high band. Meshing has improved by a convergence study identifying the complex sections in the geometry. Six mode shapes have been identified, and observed that mode 4 was the suitable mode of consideration for the working of the planetary gear system.

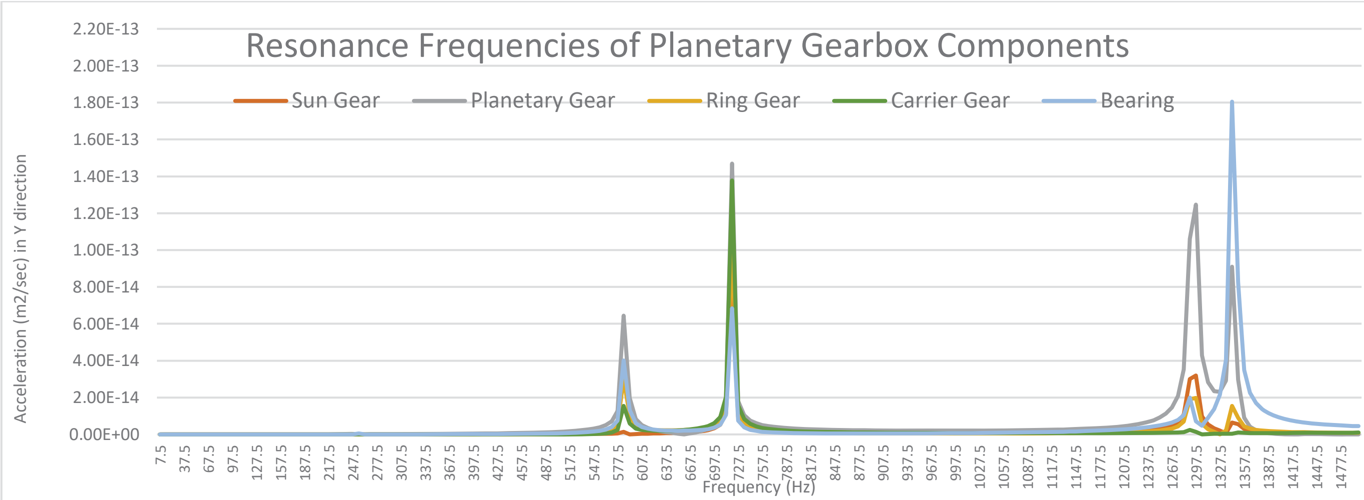


Fig. 16 Resonance Frequencies of Gearbox Components from the FE Analysis

Based on the modal analysis from the finite element model, six mode shapes and corresponding frequencies have been captured and listed below in the comparison table of Lagrange and FEA modal frequencies. In Lagrange modelling, the effect of damping and friction is neglected. The bearing module was not designed in the FE model, but the body to body contact bearing was applied to match the conditions. The FE used an implicit shape function that worked as an increase in stiffness; the Modal values of the FE tended to be relatively higher and deviated from the numerical or actual Modal frequencies. Nevertheless, the numerical simulation did not consider friction and had many boundary conditions compared to the FE analysis (such as material, supports, remote displacements etc.). The points above are the primary reasons for the deviations of modal frequencies obtained from FE and numerical analysis. The absence of friction and numerical modelling limitations could not match some torsional modes listed in table.6.

TABLE.6 Matching Modes comparison table

Modes Frequency (Hz)		Mode Shape	Δ(Hz)
Lagrange's Modes	FE Modes		
254.	242.06	Swing vibration and deformation on planets and ring gear	4.2 %
326	334.02	Rotational Shear stress on assembly	2.4 %
--------	554.15	Torsional stress on planets and sun	No matching modes
--------	558.91		
1083	1092.2	Torsional stress on sun	0.8 %
1680	1518.31	Torsional stress on planets	9.5 %
6688	---------	Torsional stress on planets	No matching modes

These frequencies are the fault frequencies that can damage the sun gear and then transform the defects to the mating gears in contact. The vibration generated from the ring gear is minimal compared to the components in the gearbox. The carrier's impact action and the tribology aspects were not taken into considerations in this dynamic model. The resonance or excitation frequencies obtained from the FE analysis (Fig. 16) matched with the results from Lagrange's numerical modelling resonances frequencies of the bearings, sun, planets and carrier gears. The high order frequencies from the bearings obtained from this simulation are vulnerable for the gearbox. This will affect the dynamics of the gearbox [30] since the planet bearings exhibit a high rate of failure [31].

IV. CONCLUSIONS

The condition monitoring of gearboxes is essential in carrying out the maintenance before failure. This paper proposed a dynamic model for a single-stage planetary gear system and studied the vibrational properties. The dynamic model for the planetary gear system was modelled using Lagrange's energy method. Instead of developing a complicated 11 DOF model, a 5 DOF model has been established, which performed the numerical simulation and the finite element analysis to study the resonance characteristics frequencies of all the elements in the gearbox that can cause catastrophic failure. The gear's vibrational response and resonance frequencies were studied for the sun, planets, bearings, carrier and ring gear. The resonance characteristics of the planetary gearbox were studied in Lagrange's dynamic model with modal analysis using eigenvalue equations. The modal analysis and resonance studies performed using the FE method were validated with the numerical modelling results obtained. The six different modal frequency values obtained from the simulation identified the origin of the resonance frequencies from the components. The result of mechanical structural defects will change the natural frequency and the modes in a gearbox. Therefore, Langrage's analytical model and FE method were adopted for the resonance and modal analysis of a planetary gearbox. It was concluded that the low band frequencies originated from the sun, carrier and planets, whereas the high band frequencies originated from the bearings. The significant high band resonance frequencies obtained from the numerical modelling were validated with the FE model using FRF. Based on the Lagrange energy theorem, a novel resonance frequency identification approach of components established. The results are then checked against the FE model analysis for identifying the deviation of modes. The vulnerable frequencies of vibration are well studied. The analysis results demonstrate the proposed dynamic model is a suitable approach for modal frequency identification of planetary gearbox. The resonance identification approach for fault diagnosis in a planetary gearbox established in this research will guide the future designs of planetary gearbox simultaneously. The results achieved from this study will be used for future experimental research for fault diagnosis and the prognostics of the planetary gearbox. In future, the Lagrange dynamic model in this study will be used for further non-linear and coupling effects mathematical model development of planetary gearbox.

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