# Saving Research Budgets by Improving Algebraic Tools for the Study of Population Dynamics

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## Introduction

**Our research:** Using algebraic tools to answer questions about the dynamic behavior of population models.

**Benefits:**

- Saving costs of research by reducing the number of experiments needed.
- Deterministic results guaranteeing that all dynamical behavior of the system is classified.

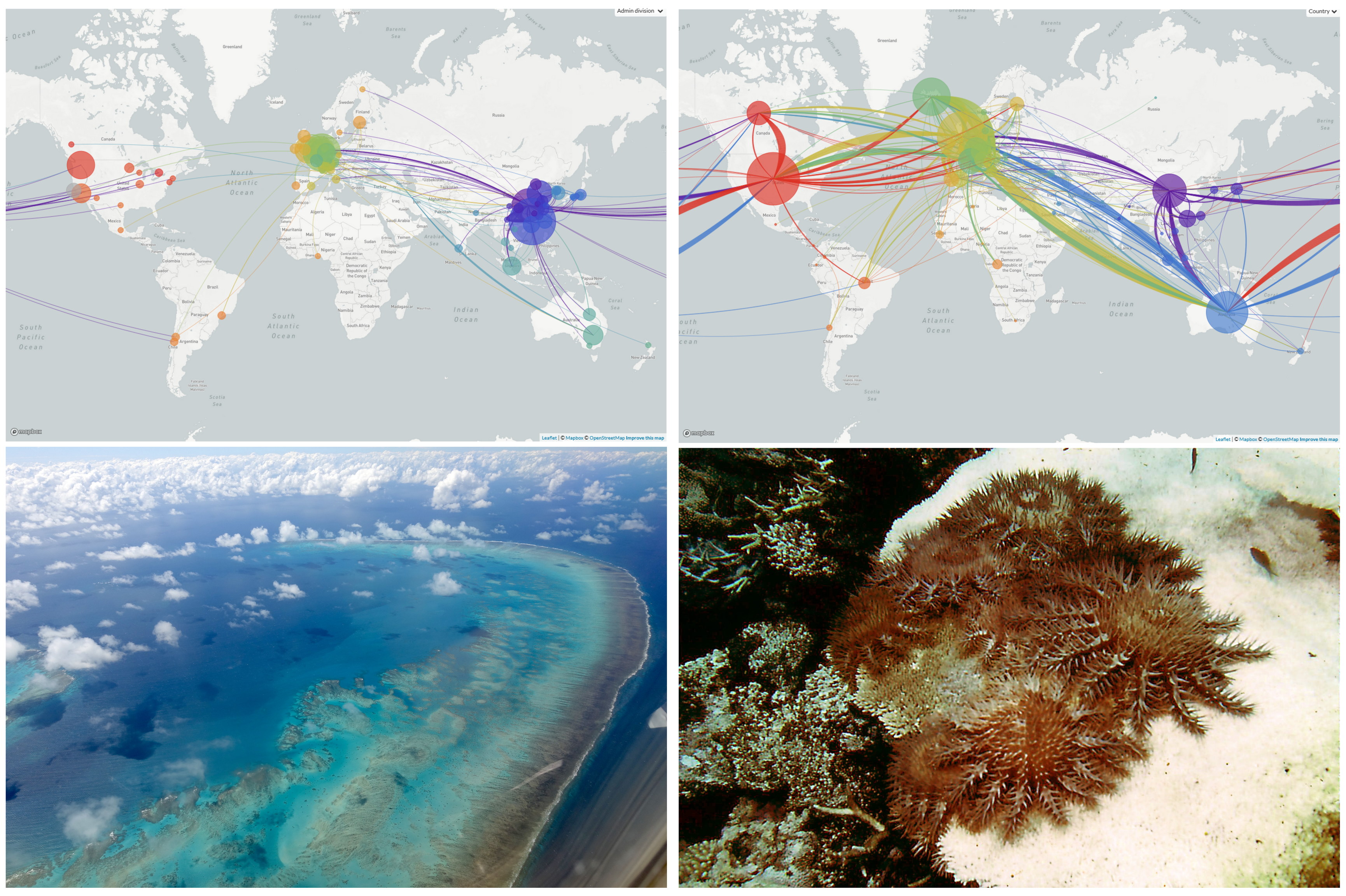
## Population Dynamics

### What are population dynamics?

They study the size and behaviour of a group of populations as variables of time.

**Why is population dynamics important?**

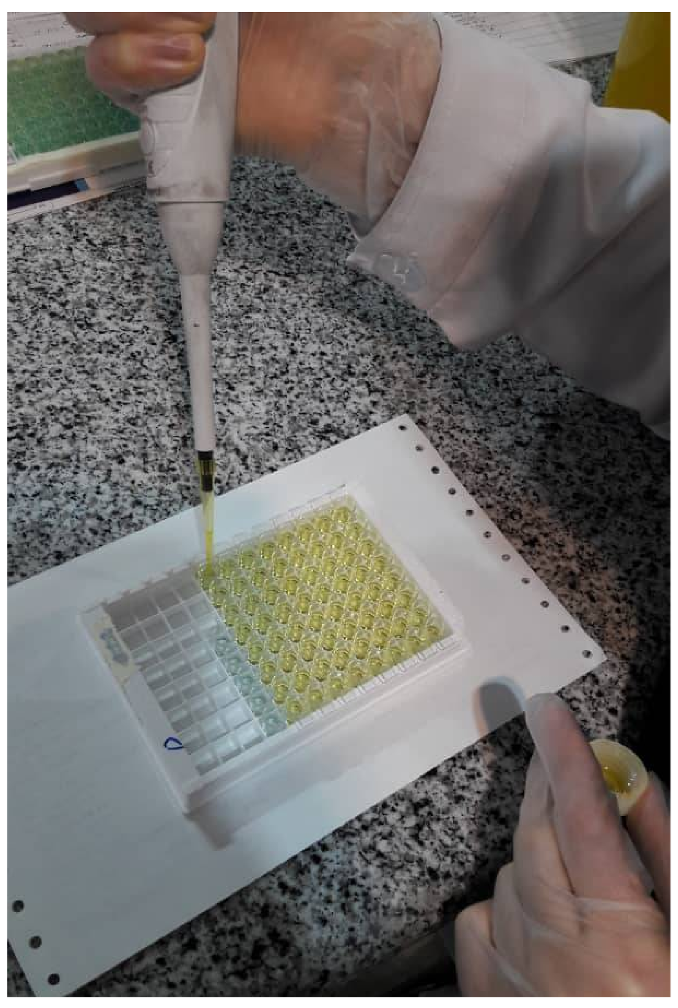
- May be used to analyse the relation between air traffic between countries and the spread of COVID19 varients (figures from Nextstrain).
- Allows us to understand the relation between ratios of different micro-organism communities in the human digestive system to better understand diseases.
- Can analyse the parameters that can control the outbreak in the population growth of crown of thorns starfish which is harming the Great Barrier Reef (photos from Wikipedia).



## Analysis by observations only

An experimentalist may repeat an experiment in a lab several times trying different parameters (photo by Rasoul Rezaei). What are the problems with this method alone?

- Repeating experiments increases costs.
- Only finitely many values of parameters can ever be tested.
- They can not do a continuous measurement and so they get a discrete approximation of the main model of interest.
- In the case of epidemics, waiting days in order to have more data could lead to inaction allowing the situation to get worse.

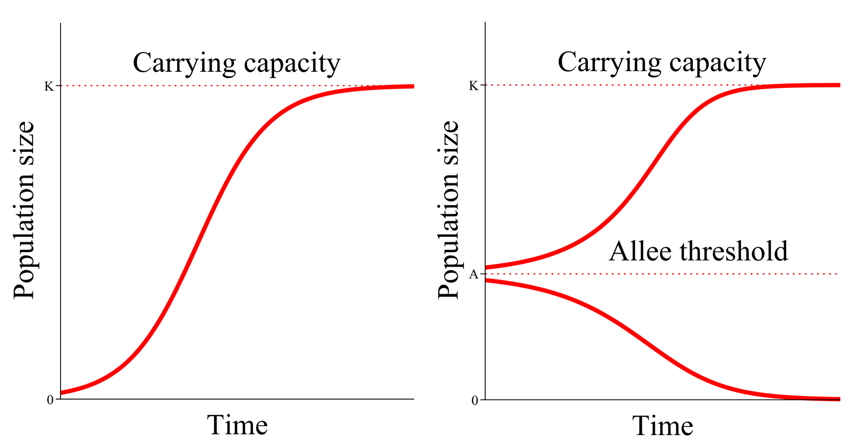


Thus we seek to help guide experimentalists with algebraic information on their systems of study. The algebra can provide the following.

- A finite number of experiments to run that would represent all possibilities, thus reducing costs.
- Parameter values that are safely away from areas where behaviour changes reducing the risk of errors from noisy data.
- A greater understanding of the properties of the system.

## Simple example

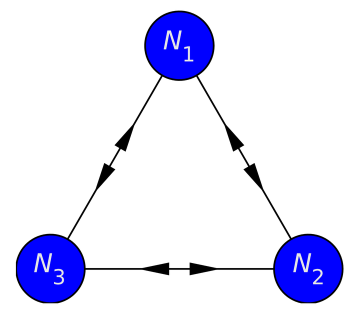
Studying a single population in isolation is the simplest case. In logistic growth, the population grows until reaching the maximum capacity of the environment. Sometimes populations exhibit the strong Allee effect, where the population needs to be above a threshold to survive, otherwise it will eventually become extinct.



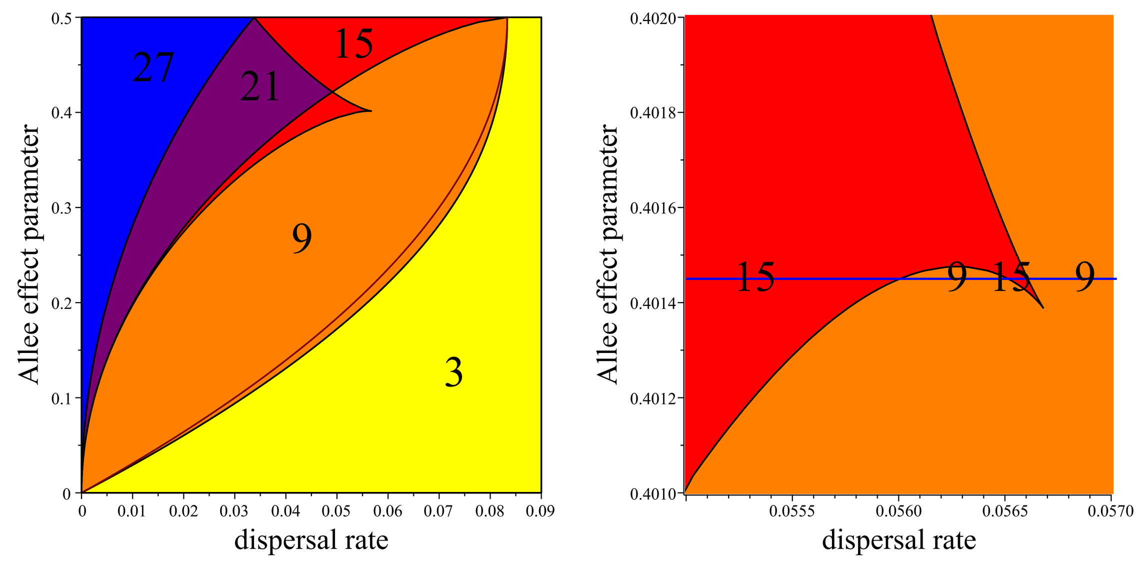

## Connected populations

Usually the situation of interest is more complicated, consisting of several populations with the possibility of migrations.

Even three connected populations with the strong Allee effect is too heavy for a normal computer using the prior state-of-the-art algorithms to analyse algebraically.




## Algebraic tools disprove conjecture

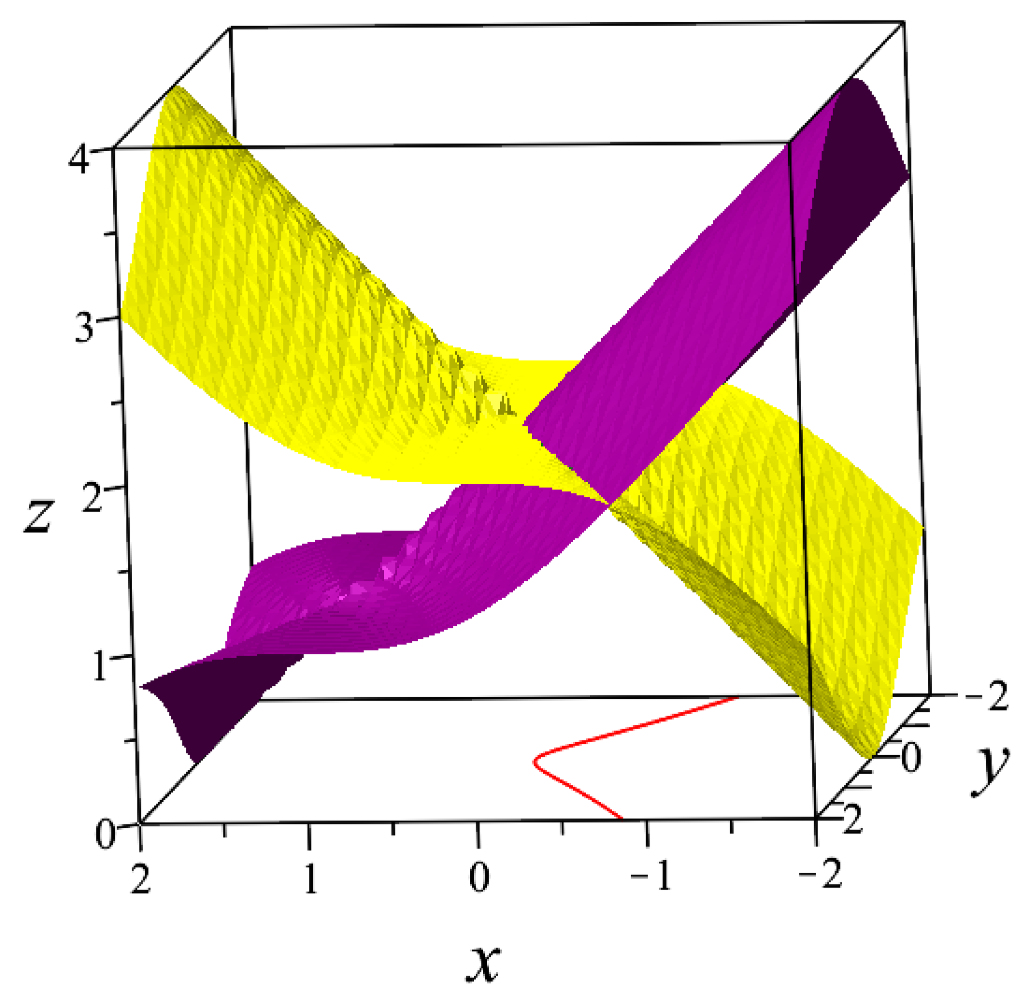



It was assumed that increasing the dispersal rate between patches must mean the behaviour of the system becomes simpler. However, using algebraic tools we disproved this: we showed that, for a suitable Allee effect parameter, it is possible to temporarily increase the number of steady states [1].

## Algebraic Tool: Resultants

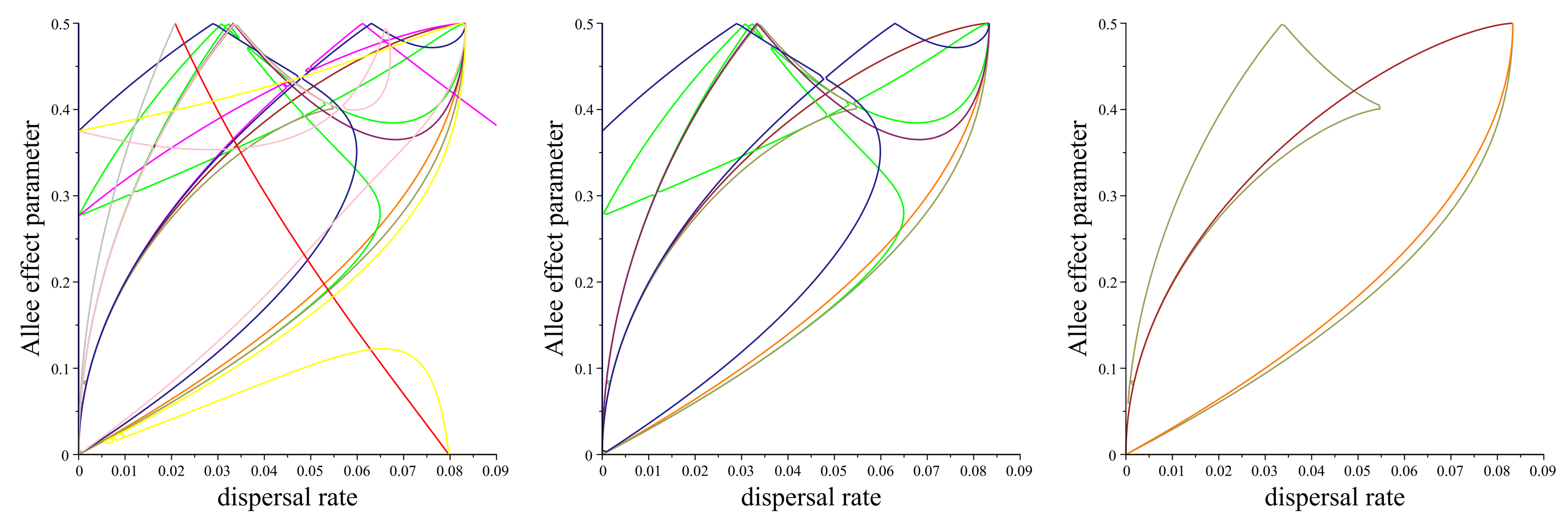
An important mathematical object we use is called the **resultant**. The resultant of the two 3-dimensional surfaces in the figure to the right is the projection of their intersection on the 2-dimensional plane at the bottom, i.e. the red curve.

Our new algorithmic methods are based on repeated use of resultants, but with a careful analysis to minimise the number required. They allowed for the symbolic solution of the three-connected population example which was infeasible with the prior state-of-the-art.




## Improved algebraic methods

The right figure contains the important curves which define the change in biological properties. Our new algorithm [2] detects these without detecting the biologically irrelevant surfaces in the other figures (additionally detected by the alternative methods).




## Conclusions

- One can reduce the cost of research by using algebraic tools.
- The algebraic algorithms have high computational complexity.
- Our research modifies and tunes these algorithms to the applications of interest so that their computations become feasible on a normal computer.

## References

[1] G. Röst and A. Sadeghimanesh, *Exotic bifurcations in three connected populations with Allee effect*, International Journal of Bifurcation and Chaos 31(13), 2150202, DOI: 10.1142/S0218127421502023 (2021).

[2] A. Sadeghimanesh and M. England, *Resultant Tools for Parametric Polynomial Systems with Application to Population Models*, Preprint: arXiv: 2201.13189, (2022).

## Acknowledgements

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