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A two stage stochastic programming for asset protection routing and a solution algorithm based on the Progressive Hedging algorithm

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Abstract

In this paper, a two-stage stochastic programming model is developed for the asset protection routing problem (APRP) to be employed in anticipation of an escaped wildfire. In this model, strategic and tactical decisions are considered in a two-stage setting. The locations of protection depots are determined taking into account the routing decisions under different possible scenarios. To solve the proposed model, the Frank-Wolfe Progressive Hedging decomposition approach is employed. A realistic case study set in south Hobart, Tasmania, is considered. In this study, the scenarios for uncertain parameters are generated based on real data, considering different sources of uncertainties such as wind direction and speed, and total monthly rainfall. Computational experiments have been conducted to demonstrate the efficiency of the solution algorithm in solving the asset protection routing problem with a two-stage stochastic framework. The numerical results suggest that, by considering the proposed two-stage stochastic programming model, more assets with higher values can be protected. The value of the approach is particularly significant where resources are limited and uncertainty levels are high. Moreover, the model and solution procedure can be applied to other disaster situations in which protection activities take place.

Keywords: Asset protection, Location routing problem, Stochastic programming, Frank-Wolfe Progressive Hedging method, Wildfires.

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1. Introduction

Bushfires are natural disasters with devastating impacts on human lives and property. In Australia, the New South Wales Rural Fire Service (NSW RFS) reported that, in addition to 25 fatalities, 2162 homes were destroyed and 849 damaged by wildfires in 2019. Also, NSW RFS reported that during this time, 218 and 170 facilities were destroyed and damaged, respectively (Newsweek, 2020). However, the consequences of wildfires, in terms of property damage and loss of human life can be mitigated by proper planning. Incident Management Teams (IMT) respond to a range of different natural or anthropogenic disasters, including

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fires, floods, earthquakes, and tsunamis (Van der Merwe et al., 2015) and, thus, play an essential role in the management of a disaster. IMT focus their efforts on the four phases of a disaster response, namely mitigation, preparedness, response, and recovery (Altay and Green III, 2006). Analysing the potential danger and planning for activities that mitigate the effects of disaster is performed in the first two-phases, whereas in the last two phases, the IMT provides relief to the victims, and there is rebuilding of infrastructure (Altay and Green III (2006)). Although all phases are important, the first phase is that comprising strategic decisions, since proper preparation in the first phase can ease activities in the subsequent phases.

Wildfires have attracted intense research in recent years because of significant threats arising from the occurrence of wildfires in many countries. Based on a California Department of Forestry and Fire Protection (CAL FIRE) report in October 2017, 250 wildfires occurred in Northern California and lead to 43 fatalities (CAL FIRE, 2017). Also, the Thomas Fire in Southern California ruined more than 1300 assets in 2017 (CAL FIRE, 2018). In addition, there have been a number of fire incidents in the wildland urban interface (WUI) in Australia. The Ash Wednesday bushfires were a series of bushfires that occurred in south-eastern Australia on 16 February 1983 that caused widespread destruction across South Australia. The Black Saturday bushfires were a series of bushfires burning across the Australian state of Victoria on Saturday, 7 February 2009 where 173 people died and 414 were injured. Of further concern is the observed rise in both fire occurrence and area burnt in Canada (Podur et al., 2002) and in the US (Westerling et al., 2006). Other countries such as Russia have also experienced wildfire events in recent years (Kharuk et al., 2007).

These examples serve to highlight the need for fire incidents to be anticipated and appropriately planned for. In such cases, the IMT will manage suitable responses to a wildfire and the interventions in each of the aforementioned phases. In the first phase (the preparedness phase) strategies are implemented according to the available or predicted information in order to minimize the impact of fire. Fuel management and fire suppression preparedness planning are some examples of the tasks in this phase that has the goal of containing potential outbreaks. In the third phase (response), active or defensive actions are implemented in response to conditions and acquired information. Central to fire fighting is the management of resources, which is a tactical-level activity, while defensive activities comprise issuing warnings and evacuation of people. Asset protection is done in this phase as well. On days of extreme fire activity, fire suppression is ineffective and defensive tasks like protecting threatened assets, evacuating people and issuing warnings become the central task (CSIRO 2009). In the last phase (recovery), incident analysis, post-fire rehabilitation and fauna rescue are performed.

In several related studies, operations research tools have been used to plan for the different stages of wildfires. They are summarized and compared in Table 1. According to Table 1, it can be concluded that most of the studies have considered the problem in a deterministic setting. In these studies, the planning has been undertaken for all phases apart from the recovery phase. The nature of this problem is such that the preparedness and response phases are the most critical since they can influence the efficacy of the later phases. Moreover, from the studies reviewed in Table 1, it can be seen that in the preparedness stage, the two

Table 1: A summary of recent studies related to optimization for the wildfire planning

Reference	Stage					Parameters		Solution algorithm	Model Type
	Preparedness	Response			Recovery	Uncertain	Deterministic		
		Asset protection	Evacuation	Fire suppression					
Minas and Hearne (2016)	Burning units						*	E	IP
Shahparvari et al. (2016)			*				*	E	MIP
Rachmawati et al. (2015)	Fuel management						*	E and HA	MIP
Minas et al. (2015)	Fuel management and fire suppression preparedness						*	E	IP
Van der Merwe et al. (2015)		*					*	E	IP
Donovan and Rideout (2003)	Resource Allocation for Wildfire Containment						*	E	IP
Haight and Fried (2007)	Fire suppression resources allocation			*		Fd		HA	IP
Ntaimo et al. (2012)				*		Fl		-	MIP
Belval et al. (2015)	Suppression Resource planning			*		W		-	MIP
Kabli et al. (2015)	Fuel treatment planning					W and Fo		E	IP
Matsypura et al. (2018)	Fuel management						*	HA	MIP
Krasko and Rebennack (2017)			*			EM		E	MINLP
Shahparvari and Abbasi (2017)			*			Ep, Tw and Bp		HA	MIP

E:Exact, HA: Heuristic approach, Fd: Fire days, Fires locations, W:Weather condition, Fo: Fire occurrence, Em: effects of mitigation, Ep: Evacuee population, Tw: time windows, Bp: bushfire propagation main activities are fuel management and suppression planning. It can also be observed that there are studies with deterministic parameters for the response phase including planning for asset protection, evacuation and fire suppression. However, some factors that affect the parameters of a mathematical model, such as weather conditions, ignition point, and the intensity of fire, are unknown at the planning stage. Therefore, considering uncertainty in this problem is crucial for obtaining reliable recommendations in terms of decision support. There are a number of studies considering uncertain parameters and some of them are reviewed next.

Haight and Fried (2007) proposed a two-stage stochastic model by considering the number of resources required at different locations as an uncertain parameter. In their model, the number of resources deployed to each protection depot and the assignment of these resources to fire locations are the first- and second-stage decision variables, respectively. Belval et al. (2015) proposed a multi-stage stochastic model to determine suppression activities for minimizing the area destroyed by fire. Kabli et al. (2015) proposed a two-stage stochastic programming model for fuel treatments in rural forests by considering different scenarios for weather conditions and fire occurrence. Zhou and Erdogan (2019) proposed a two-stage stochastic model for the response stage by considering the wildfire spread as a stochastic parameter in allocating firefighting and resident evacuation. Two objectives including total cost and the number of people at risk were considered and a goal programming approach was applied to solve the proposed model. Shahparvari et al. (2017) proposed a capacitated vehicle routing problem for evacuation during a bushfire. They considered the population of evacuees at the town and its shelter capacity as uncertain parameters, which were represented by triangular fuzzy numbers. Most of these try to allocate optimal resources for efficient fire fighting with minimum cost, while one study Kabli et al. (2015) considers a two-stage stochastic programming approach to fuel treatment planning. To the best of our knowledge, no previous study has considered the asset protection routing problem with stochastic parameters based on a two-stage stochastic programming approach, which is the focus of this study.

We focus on the asset protection routing problem (APRP), in which the protection depots should be optimally located to achieve effective and efficient asset protection with the least cost. Since escaped wildfires destroy infrastructure and community assets, protecting such infrastructures in a timely manner should be the primary goal. Communication towers, power network dispatchers, hospitals, schools, hotels, historical buildings, bridges and factories are some examples of community assets. Therefore, a location routing model that considers the protection of key assets is proposed in this paper. Amideo et al. (2019) and Bruni et al. (2020) studied location routing and vehicle routing problems for an evacuation procedure under disaster situations, respectively. Moreover, according to the nature of an asset and its function and associated risks, a value may be assigned to each asset. To protect an asset, the required capabilities and resources should be available over a time window determined by the advancing fire. Protection time (called service time) differs from one asset to another. Service and travel times, routing cost, the value of assets, the time window upper bound are stochastic parameters which depend on the weather condition, wind speed, and so forth.

In most studies of related vehicle routing and location routing problems, two approaches, namely robust optimization and stochastic programming, were applied to address uncertainty. Schiffer and Walther (2018) proposed a robust location routing problem for electric vehicles by considering customers' patterns as an uncertain parameter. Also, they developed a parallelized adaptive large neighbourhood search algorithm. Shi et al. (2019) proposed a robust model for a vehicle routing problem in home health care planning by considering uncertainty in travel and service times. Lu and Gzara (2019) proposed a robust formulation for the vehicle routing problem model under demand uncertainty. Bertazzi and Secomandi (2018) proposed a model of a vehicle routing problem with stochastic demand and an extended roll-out algorithm and improved its efficiency. Zhang et al. (2019) proposed a two-stage stochastic model for a location and routing problem for electric vehicles with stochastic demand. Salavati-Khoshghalb et al. (2019) developed an exact method for solving a stochastic vehicle routing problem by considering demand.

In this study, a two-stage stochastic programming formulation is proposed to determine the locations of protection depots and the allocation of the assets to them based on the establishment and routing costs as well as the utility level associated with protecting the assets. The first-stage decision is the location of protection depots while the second-stage decisions are asset protection plans according to each scenario of fire spreading. The first- and second-stage decision variables represent the tasks in the preparedness and responses phases, respectively. Temperature, monthly rainfall, wind speed and the time of day that the fire started are the sources of uncertainty that are considered in generating scenarios in this study. A realistic set of scenarios for uncertain parameters are generated based on real data taken from south Hobart, Tasmania.

The complexity of two-stage and multi-stage stochastic programming models increases with the number of scenarios. Therefore, scenario-based decomposition algorithms, including Progressive Hedging (PH) and dual decomposition have been proposed to deal with the increase in computational time (Guo et al. (2015)). Progressive Hedging, originally proposed by Rockafellar and Wets (1991), is based on augmented Lagrangian dual problems and is guaranteed to converge to a global optimal solution for convex problems (i.e., only including continuous decision variables). However, there is no guarantee of convergence of PH for stochastic mixed-integer programming (SMIP) models. In recent years, the combinations of PH with other algorithms have been proposed for solving two-stage and multi-stage models with integer first-stage decision variables. Boland et al. (2018) proposed an algorithm that combines PH and the Frank-Wolfe method for solving SMIP models with integer decision variables in different stages. Boland et al. (2019) provided convergence guarantees for a close variant of FW-PH, to SMIPs under the perspective of a proximal bundle method. Barnett et al. (2017) proposed an algorithm based on a combination of PH and a branch-and-bound approach for solving multi-stage models. In this method, PH is used in each node of the branch-and-bound algorithm. Guo et al. (2015) proposed a method that used PH and dual decomposition algorithms in an integrated way. In this method, PH is used to speed up the convergence of the dual decomposition algorithm to the optimal solution.

In this study, a PH-based algorithm is used to achieve a tight dual bound of the problem while employing decomposition. The first-stage decision variables in the proposed model are binary variables representing the location decision of the protection depots. Because of the integrality of the first-stage decision variables, there is no guarantee that PH will converge to a globally optimal solution. Therefore, a combination of the PH algorithm and the Frank-Wolfe method, the Frank-Wolfe Progressive Hedging (FW-PH) algorithm as proposed by Boland et al. (2018), is used for obtaining dual (i.e., lower) bounds for the proposed SMIP model. Unlike PH, FW-PH is guaranteed to converge to the optimal Lagrangian dual bound of a SMIP and does not suffer from issues associated with premature convergence and cycling.

The paper has been organized as follows. The problem formulation is presented in the next section. A brief introduction to the FW-PH algorithm is given in Section 3. The scenario generation scheme developed for the purpose of our study is presented in Section 4. Numerical examples and analyses are presented in Section 5, and finally, concluding remarks are presented in the last section.

2. Problem definition

In the asset protection problem, it is assumed that there are assets that need to be protected by means of protection activities, such as clearing potential sources of ignition and hosing down structures by teams dispatched from depots. Each asset should be protected in a given time window related to the fire spread. The depots can be established in different locations and can be permanent structures, however they often are simple temporary bases comprising teams and vehicles located at strategically designated waiting places. The selection of good locations for establishing protection depots will determine the travel times to the assets and as a result, significantly affect the number of assets that can be protected.

Because of the limitation in the number of available resources and the time window for protecting each asset, often not all assets can be protected. Therefore, assets are selected for protection after considering their utility values and risk of damage. The utility value for each asset is determined in accordance with criteria such as monetary aspect, importance of the asset to the community, and the number of people affected.

Wildfire intensity is a significant hazard to assets, and depends on factors such as temperature, monthly rainfall, and wind speed. The risk of damage for each asset depends on how long the wildfire will take to reach it. This determines the latest time by which firefighters must depart from an asset and, hence, determines the upper bound of the time window for protecting a given asset. Assets with earlier (i.e., smaller) upper bounds in their protection time windows are at greater risk of damage. Figure 1 provides a schematic representation of the problem. Two types of assets, including schools and electricity substations are considered. As shown in Figure 1, three assets are selected to be protected according to their corresponding utility and risk values. The problem consists of locating protection depots over potential locations, allocating assets to located protection depots, selecting assets to be protected according to their protection time windows, utility values, or other resource limitations. Finally, the routing plan is used to protect the selected assets.

The utility value and risk of damage for each asset and the effect of these factors for protection decisions are illustrated in the example presented in Figure 1. Assets with greater utility values are selected for protection based on their corresponding time window. Moreover the routing order will be decided based on the travelling costs. In disaster conditions, mitigating damage and keeping people safe are more important than the traveling costs. Therefore, assets far from the protection depot might be selected for protection in case of high utility values.

Because of the inherent uncertainty, this problem is modeled as a two-stage stochastic programming model, in which the decisions of interest are the location of the protection depots, which has to be determined in advance. The assumptions of the proposed model are the following:

- The protection depots have equal protection capacity.
- It is assumed that the vehicles are heterogeneous in terms of travel time and cost but each vehicle can do all activities required for the asset protection.
- The upper bound of the time window for each asset is considered as an uncertain parameter and directly affected by temperature, monthly rainfall, and wind speed, while the lower bound of the time window for all assets is assumed to be the start time of the wildfire. An asset can be protected before the upper bound of its time window.
- The utility value of each asset depends on the starting time of the fire, therefore the utility value for each asset is considered as an uncertain parameter.
- The service time to protect an asset depends on the wildfire intensity and is affected by temperature and monthly rainfall. Therefore, the asset service times are also considered as uncertain parameters.
- Travel time and cost are considered as uncertain parameters.

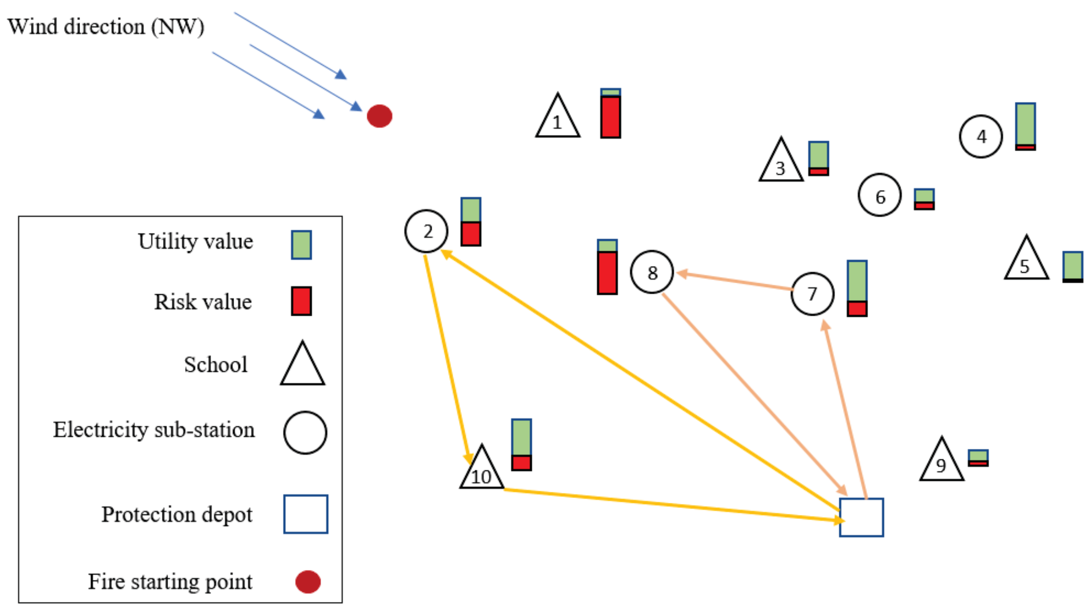


Figure 1: A schematic view of the problem.

2.1. Model notation

We present the notation used to formulate the proposed model.

Sets:

$\mathcal{I}$= { $1, 2, ..., I$}	Set of assets which may face bushfire hazard and need protection.
$\mathcal{J}$= { I+1, I+2, ..., I+J}	Set of potential nodes which may be selected to establish a protection depot.
$\mathcal{N}$= { $1, 2, ..., I+1, I+2, ..., I+J$}	Set of all nodes including assets and potential protection depots.
$\mathcal{K}$= { $1, 2, ..., K$}	Set of vehicles.
Ξ= { $1, 2, ...,$ S}	Set of scenarios.

Parameters:

E_j	Establishment cost of protection depot at node j ($j \in \mathcal{J}$).
A_i	Time window lower bound for protecting asset i ($i \in \mathcal{I}$).
$B_i(\xi)$	Time window upper bound for protecting asset i ($i \in \mathcal{I}$) under scenario ξ ($\xi \in \Xi$).
$V_i(\xi)$	Utility value of node i ($i \in \mathcal{I}$) under scenario ξ ($\xi \in \Xi$).
$C_{ii'k}(\xi)$	Routing cost from node i ($i \in \mathcal{N}$) to node i' ($i' \in \mathcal{N}$) by vehicle k ($k \in \mathcal{K}$) under scenario ξ ($\xi \in \Xi$).
$O_i(\xi)$	Operation time in protecting node i ($i \in \mathcal{N}$) under scenario ξ ($\xi \in \Xi$).
$T_{ii'k}(\xi)$	Travel time from node i ($i \in \mathcal{N}$) to node i' ($i' \in \mathcal{N}$) by vehicle k ($k \in \mathcal{K}$) under scenario ξ ($\xi \in \Xi$).
M	A large number $M = \max_{i,\xi}(B_i(\xi))$.

Decision variables:

x_j	1 if a protection depot is established at node j ($j \in \mathcal{J}$), 0 otherwise.
$z_{ij}(\xi)$	1 if node i ($i \in \mathcal{I}$) is serviced by protection depot j ($j \in \mathcal{J}$) under scenario ξ ($\xi \in \Xi$), 0 otherwise.
$y_i(\xi)$	1 if asset of node i ($i \in \mathcal{I}$) is protected under scenario ξ ($\xi \in \Xi$), 0 otherwise.
$r_{ii'k}(\xi)$	1 if vehicle k ($k \in \mathcal{K}$) travels along (i, i') ($i, i' \in \mathcal{N}$) under scenario ξ ($\xi \in \Xi$), 0 otherwise.
$t_{ik}(\xi)$	A positive variable indicates the arrival time to node i ($i \in \mathcal{I}$) by the vehicle k ($k \in \mathcal{K}$) under scenario ξ ($\xi \in \Xi$).

The proposed APRP model is given by

$$\textbf{(APRP)} \ \min. \ \sum_{j\in\mathcal{J}} E_j x_j + \mathbb{E}_\xi \left(\sum_{i\in\mathcal{N}} \sum_{i'\in\mathcal{N}} \sum_{k\in\mathcal{K}} C_{ii'k}(\xi) r_{ii'k}(\xi) - \sum_{i\in\mathcal{I}} V_i(\xi) y_i(\xi) \right) \tag{1}$$

$$\text{s.t.:} \ \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{N}} r_{ii'k}(\xi) = y_{i'}(\xi), \qquad \forall i' \in \mathcal{I},\ \xi \in \Xi \tag{2}$$

$$z_{ij}(\xi) \le x_j, \qquad \forall j \in \mathcal{J}, i \in \mathcal{I}, \xi \in \Xi \tag{3}$$

$$\sum_{i'\in\mathcal{N}} r_{ii'k}(\xi) - \sum_{i'\in\mathcal{N}} r_{i'ik}(\xi) = 0, \qquad \forall i \in \mathcal{N}, k \in \mathcal{K}, \xi \in \Xi \tag{4}$$

$$\sum_{j\in\mathcal{J}} z_{ij}(\xi) = y_i(\xi), \qquad \forall i \in \mathcal{I}, \xi \in \Xi \tag{5}$$

$$t_{i'k}(\xi) + O_{i'}(\xi) + T_{i'ik}(\xi) \le t_{ik}(\xi) + M \times (1 - r_{i'ik}(\xi)), \quad \forall i' \in \mathcal{N}, i \in \mathcal{I}, k \in \mathcal{K}, \xi \in \Xi \tag{6}$$

$$A_i \le t_{ik}(\xi) \le B_i(\xi), \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}, \xi \in \Xi \tag{7}$$

$$\sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{J}} r_{ijk}(\xi) = 1, \qquad \forall k \in \mathcal{K}, \xi \in \Xi \tag{8}$$

$$\sum_{i'\in\mathcal{N}} r_{ji'k}(\xi) + r_{i'ik}(\xi) \le z_{ij}(\xi) + 1, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, \xi \in \Xi \tag{9}$$

$$x_j, z_{ij}(\xi), y_i(\xi) \in \{0, 1\}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, \xi \in \Xi \tag{10}$$

$$r_{ii'k}(\xi) \in \{0, 1\}, \qquad \forall i, i' \in \mathcal{N}, k \in \mathcal{K}, \xi \in \Xi \tag{11}$$

$$t_{ik}(\xi) \ge 0, \qquad i \in \mathcal{I}, k \in \mathcal{K}, \xi \in \Xi \tag{12}$$

The objective function (1) consists of three terms including the establishment cost and the expected value of both the routing cost and the associated value of protecting assets. Constraints (2) guarantee that if the i'^{th} asset is protected under scenario ξ it must be visited in the route of a vehicle. Constraints (3) state that the i^{th} asset can be protected by the protection depot established at the j^{th} node. Constraints (4) are vehicle balance equations and ensure that if the i^{th} asset is visited by the k^{th} vehicle, the vehicle must visit another asset after it or return to the protection depot. Constraints (5) ensure that if the i^{th} asset is protected, it must be allocated to a protection depot. Constraints (6) and (7) impose time window constraints for each asset. Also, constraints (6) are sub-tour elimination constraints. Constraints (8) ensure that each vehicle should return to its established protection depots at the end of the tour. It should be noted that if a vehicle visits a protection depot immediately after departure of the same protection depot, it means that the vehicle is not used for protecting any assets. Constraints (9) guarantee that if the i^{th} asset is allocated to the j^{th} established protection depot, it must be visited in the route that starts from that protection depot, otherwise the same vehicle can not visit protection dept j and asset i. Constraints (10)-(12) determine the types and domains of the decision variables.

3. The Frank-Wolfe Progressive Hedging algorithm

The scale, and consequently the computational requirements, of two-stage and multi-stage stochastic programming models increases with the number of scenarios. Therefore, if a solution strategy can solve a scenario-wise decomposed version of the problem, the computation time is likely to decrease significantly. Progressive Hedging (PH) and dual decomposition methods are two well known scenario-based decomposition methods for two-stage and multi-stage stochastic models (Guo et al., 2015). In these methods, the first-stage decision variables are artificially replicated for each scenario to then have non-anticipativity conditions imposed by means of constraints. These constraints are relaxed in these methods, thus yielding a model that can be solved independently for each scenario. As previously mentioned, PH can be used to obtain tight dual (i.e., lower) bounds for two-stage and multi-stage stochastic programming models. PH is suitable for solving the models with continuous decision variables but there is no guarantee for the convergence of this algorithm for SMIP. To deal with this issue, Boland et al. (2018) proposed a variant of PH that addresses convergence issues for the SMIP by incorporating a convexification strategy using polyhedral inner-approximations based on the Frank-Wolfe method. Specifically, the Simplicial Decomposition method (SDM), an extension of the Frank-Wolfe method, is used in FW-PH. This modified algorithm is guaranteed to construct a lower bound convergent to a tight Lagrangian relaxation (dual) bound on the (primal) objective value of the APRP. Moreover, feasible primal solutions can be constructed in each iteration of algorithm and so an estimate of a duality gap can be calculated. In FW-PH, the Lagrangian relaxation version of the APRP model is decomposed into S sub-problems and solved iteratively, which is possible once the non-anticipativity constraints have been relaxed. Sets of feasible solutions that typically do not satisfy non-anticipativity constraints are determined by solving these sub-problems. Then, an augmented Lagrangian reformulation of these sub-problems with inner approximation convex hull of the constraint set is used to find values approximately equal to the first-stage decision variables. The pseudo code of the Frank-Wolfe Progressive Hedging (FW-PH) algorithm used in this study is described in Algorithm 1. The additional sets and parameters in the FW-PH algorithm are as follows:

Parameters:

- k_{max} Maximum number of iterations in the FW-PH algorithm.
- ρ Penalty parameter in the FW-PH algorithm.
- $\omega_j^m(\xi)$ Dual variables associated with the non-anticipativity constraint of j^{th} first-stage variable in the FW-PH algorithm at the m^{th} iteration.
- ϵ A threshold value for tolerance of solution obtained by the FW-PH algorithm that determines the stopping condition.
- $\hat{x}_j^l(\xi)$ The solution obtained for the decision variable $x_j(\xi)$ in the l^{th} iteration of the FW-PH algorithm from Sub-1 or Sub-2 models.
- $\hat{z}_{ij}^l(\xi)$ The solution obtained for the decision variable $z_{ij}(\xi)$ in the l^{th} iteration of the FW-PH algorithm from Sub-1 or Sub-2 models.

$\hat{y}_i^l(\xi)$ The solution obtained for the decision variable $y_i(\xi)$ in the l^{th} iteration of the FW-PH algorithm from Sub-1 or Sub-2 models.

$\hat{r}_{ii'k}^l(\xi)$ The solution obtained for the decision variable $r_{ii'k}$ in the l^{th} iteration of the FW-PH algorithm from Sub-1 or Sub-2 models.

$\hat{t}_{ik}^l(\xi)$ The obtained solution for the decision variable t_{ik} in the l^{th} iteration of the FW-PH algorithm from Sub-1 or Sub-2 models.

Sets:

$\theta_x(\xi)$, $\theta_z(\xi)$, $\theta_y(\xi)$, $\theta_r(\xi)$, and $\theta_t(\xi)$ Sets of solutions obtained from Sub-1 and Sub-2 models ($\hat{x}_j^l(\xi)$, $\hat{z}_{ij}^l(\xi)$, $\hat{y}_i^l(\xi)$, $\hat{r}_{ii'k}^l(\xi)$, and $\hat{t}_{ik}^l(\xi)$, respectively) under scenario ξ ($\xi \in \Xi$) in different iterations of algorithm used as the extreme points in Sub-3 model.

Decision variables:

λ_l Multipliers used in taking the convex combinations of the solutions in the sets $\theta_x(\xi)$, $\theta_z(\xi)$, $\theta_y(\xi)$, $\theta_r(\xi)$, and $\theta_t(\xi)$ in the l^{th} iteration used in the Sub-3 model.

The FW-PH method for the proposed model can be described as follows:

Step 1: The initial solutions of all decision variables ($\hat{x}_j^0(\xi)$, $\hat{z}_{ij}^0(\xi)$, $\hat{y}_i^0(\xi)$, $\hat{r}_{ii'k}^0(\xi)$, $\hat{t}_{ik}^0(\xi)$) are determined by considering copies of the first-stage decision variables to decompose the problem in to the sub-problems (Sub-1(ξ)) for each scenario. Notice that the difference between these sub-problems and APRP is the elimination of the non-anticipativity constraints. Therefore, there is no guarantee of obtaining the same values for the first-stage decision variables under different scenarios. Then, the expected value of the first-stage decision variables are calculated based on the value obtained for the first-stage decision variables. The dual variables in the first iteration for each protection depot under different scenarios $\omega_j^1(\xi)$ are calculated by $\omega_j^1(\xi) = \rho(\hat{x}_j^0(\xi) - \bar{x}_j^{m+1})$. Also, the solutions $\hat{x}_j^0(\xi)$, $\hat{z}_{ij}^0(\xi)$, $\hat{y}_i^0(\xi)$, $\hat{r}_{ii'k}^0(\xi)$, and $\hat{t}_{ik}^0(\xi)$ are included in the sets $\theta_x(\xi)$, $\theta_z(\xi)$, $\theta_y(\xi)$, $\theta_r(\xi)$ and $\theta_t(\xi)$, respectively. The Sub-1 model under each scenario is presented as follows:

$$(\textbf{Sub-1}(\xi))\ \min.\ \sum_{j\in\mathcal{J}} E_j x_j(\xi) + \left(\sum_{i\in\mathcal{N}}\sum_{i'\in\mathcal{N}}\sum_{k\in\mathcal{K}} C_{ii'k}(\xi) r_{ii'k}(\xi)\right) - \left(\sum_{i\in\mathcal{I}} V_i(\xi) y_i(\xi)\right) \tag{13}$$

$$\text{s.t.:}\ \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}} r_{ii'k}(\xi) = y_{i'}(\xi), \quad \forall i' \in \mathcal{I} \tag{14}$$

$$z_{ij}(\xi) \le x_j(\xi), \quad \forall j \in \mathcal{J},\ i \in \mathcal{I} \tag{15}$$

$$\sum_{i'\in\mathcal{N}} r_{ii'k}(\xi) - \sum_{i'\in\mathcal{N}} r_{i'ik}(\xi) = 0, \quad \forall i \in \mathcal{N},\ k \in \mathcal{K} \tag{16}$$

$$\sum_{j\in\mathcal{J}} z_{ij}(\xi) = y_i(\xi), \quad \forall i \in \mathcal{I} \tag{17}$$

$$t_{i'k}(\xi) + O_{i'}(\xi) + T_{i'ik}(\xi) \le t_{ik}(\xi) + M \times (1 - r_{i'ik}(\xi)), \quad \forall i' \in \mathcal{N},\ i \in \mathcal{I},\ k \in \mathcal{K} \tag{18}$$

$$A_i \le t_{ik}(\xi) \le B_i(\xi), \quad \forall i \in \mathcal{I},\ k \in \mathcal{K} \tag{19}$$

$$\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} r_{ijk}(\xi) = 1, \quad \forall k \in \mathcal{K} \tag{20}$$

$$\sum_{i'\in\mathcal{N}} r_{ji'k}(\xi) + r_{i'ik}(\xi) \le z_{ij}(\xi) + 1, \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ k \in \mathcal{K} \tag{21}$$

$$x_j(\xi),\ z_{ij}(\xi),\ y_i(\xi) \in \{0,1\}, \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ k \in \mathcal{K} \tag{22}$$

$$r_{ii'k}(\xi) \in \{0,1\}, \quad \forall i, i' \in \mathcal{N},\ k \in \mathcal{K} \tag{23}$$

$$t_{ik}(\xi) \ge 0, \quad \forall i \in \mathcal{I},\ k \in \mathcal{K} \tag{24}$$

Step 2: In this step, a series of sub-problems are constructed based on a scenario decomposition of the APRP model, by considering the Lagrangian relaxation of non-anticipativity constraints. The solutions obtained from Sub-2 model are added to the sets $\theta_x(\xi)$, $\theta_z(\xi)$, $\theta_y(\xi)$, $\theta_r(\xi)$ and $\theta_t(\xi)$. Determining sets of solutions for each set of decision variables by considering the non-anticipativity constraint as relaxed (or soft) constraints is the main aim of this step.

$$(\textbf{Sub-2}(\xi))\ \min.\ \sum_{j\in\mathcal{J}} E_j x_j(\xi) + \left(\sum_{i\in\mathcal{N}}\sum_{i'\in\mathcal{N}}\sum_{k\in\mathcal{K}} C_{ii'k}(\xi) r_{ii'k}(\xi)\right) - \left(\sum_{i\in\mathcal{I}} V_i(\xi) y_i(\xi)\right) + \sum_{j\in\mathcal{J}} \omega_j^m(\xi)(x_j(\xi) - \bar{x}_j^m) \tag{25}$$

$$\text{s.t.:}\ (14) - (24)$$

Step 3: Next, an augmented Lagrangian reformulation of Sub-2 with inner approximation convex hull of the constraint set (Sub-3) is employed. Considering the inner approximation convex hull of the constraint set and, as a result, relaxing the integrality requirements of decision variables, ensues the convergence behavior of the algorithm, as demonstrated in Boland et al. (2018). In this case, the convex hull is determined by the solutions from Sub-1 and Sub-2 models according to the sets $\theta_x(\xi)$, $\theta_z(\xi)$, $\theta_y(\xi)$, $\theta_r(\xi)$ and $\theta_t(\xi)$. The cardinality of these sets are same as each other in different iterations. The Sub-3 model under each scenario is as follows:

$$\textbf{(Sub-3}(\xi)\textbf{)} \ \text{min.} \ \sum_{j\in\mathcal{J}} E_j x_j(\xi) + \left(\sum_{i\in\mathcal{N}}\sum_{i'\in\mathcal{N}}\sum_{k\in\mathcal{K}} C_{ii'k}(\xi) r_{ii'k}(\xi)\right) - \left(\sum_{i\in\mathcal{I}} V_i(\xi) y_i(\xi)\right) + \sum_{j\in\mathcal{J}} \omega_j^m(\xi) x_j(\xi) + \frac{\rho}{2}\left(\sum_{j\in\mathcal{J}} (x_j(\xi) - \bar{x}_j^m)^2\right) \tag{26}$$

$$\text{s.t.:} \ x_j(\xi) = \sum_{l=0}^{|\theta_x|} \lambda_l \hat{x}_j^l(\xi), \qquad \forall j \in \mathcal{J} \tag{27}$$

$$z_{ij}(\xi) = \sum_{l=0}^{|\theta_z|} \lambda_l \hat{z}_{ij}^l(\xi), \qquad \forall i \in \mathcal{I},\ j \in \mathcal{J} \tag{28}$$

$$y_i(\xi) = \sum_{l=0}^{|\theta_y|} \lambda_l \hat{y}_i^l(\xi) \qquad \forall i \in \mathcal{I} \tag{29}$$

$$r_{ii'k}(\xi) = \sum_{l=0}^{|\theta_r|} \lambda_l \hat{r}_{ii'k}^l(\xi) \qquad \forall i' \in \mathcal{N}\ i \in \mathcal{N},\ k \in \mathcal{K} \tag{30}$$

$$t_{ik}(\xi) = \sum_{l=0}^{|\theta_t|} \lambda_l \hat{t}_{ik}^l(\xi) \qquad \forall i \in \mathcal{I},\ k \in \mathcal{K} \tag{31}$$

$$\sum_{l=0}^{|\theta_x|} \lambda_l = 1 \tag{32}$$

$$0 \le x_j(\xi) \le 1,\ 0 \le z_{ij}(\xi) \le 1,\ 0 \le y_i(\xi) \le 1,\ 0 \le r_{ii'k}(\xi) \le 1, \ t_{ik}(\xi) \ge 0,\ 0 \le \lambda_l \le 1 \tag{33}$$

4. Scenario generation scheme

In this study, we considered a bushfire emergency inspired by an actual event in the south region of Hobart, Tasmania (Australia), which is shown in Figure 2. To consider the problem under study, the selected area has been divided into a grid and a number has been assigned to each cell, as depicted in the Figure 2.

A square subregion has been selected and assets and potential locations for the protection depots have been determined. To generate more realistic scenarios, a list of variables at different levels have been considered, as depicted in Table 2. Some parameters of the proposed model such as the time window upper bound and utility value are affected by these sources of uncertainty. A combination of these sources of uncertainty is considered to determine these parameters. These sources of uncertainty have different scales, therefore, they are normalized to be scaleless and used to generate the values of parameters for each scenario. The wind speed, month's rainfall, and month's max temperature are normalized to lie between [1,2] and fire starting times are normal-

Algorithm 1: Frank-Wolfe Progressive Hedging algorithm for APRP

Input (ρ, ϵ, k_{max})

$m \leftarrow 0$

$\omega_j^m(\xi) \leftarrow 0$

$\theta_x(\xi) \leftarrow []$, $\theta_z(\xi) \leftarrow []$, $\theta_y(\xi) \leftarrow []$, $\theta_r(\xi) \leftarrow []$, $\theta_t(\xi) \leftarrow []$

Step 1:

for $\xi \in \Xi$ **do**

  Optimize Sub-1 model under each scenario and determine the initial values of the decision variables $(\hat{x}_j^0(\xi),\ \hat{z}_{ij}^0(\xi),\ \hat{y}_i^0(\xi),\ \hat{r}_{ii'k}^0(\xi),\ \hat{t}_{ik}^0(\xi))$;

Calculate the mean value of the first stage decision variables according the obtained solutions from Sub-1 model ($\bar{x}_j^{m+1} \leftarrow \sum_{\xi \in \Xi} p(\xi)\hat{x}_j^0(\xi)$);

Update the set of obtained solutions for each decision variable:

$\theta_x(\xi) = \theta_x(\xi) \cup \{\hat{x}_j^0(\xi)\}$;

$\theta_z(\xi) = \theta_z(\xi) \cup \{\hat{z}_{ij}^0(\xi)\}$;

$\theta_y(\xi) = \theta_y(\xi) \cup \{\hat{y}_i^0(\xi)\}$;

$\theta_r(\xi) = \theta_r(\xi) \cup \{\hat{r}_{ii'k}^0(\xi)\}$;

$\theta_t(\xi) = \theta_t(\xi) \cup \{\hat{t}_{ik}^0(\xi)\}$;

Update the weight based on the obtained solutions from Sub-1 model by

$\omega_j^{m+1}(\xi) \leftarrow \omega_j^m(\xi) + \rho(\hat{x}_j^0(\xi) - \bar{x}_j^{m+1})$;

$condition \leftarrow false$;

while *condition is false* **do**

  for $\xi \in \Xi$ **do**

    $m = m + 1$;

    Step 2:

    Optimize Sub-2 model under each scenario and determine updated trial solutions $(\hat{x}_j^m(\xi),\ \hat{z}_{ij}^m(\xi),\ \hat{y}_i^m(\xi),\ \hat{r}_{ii'k}^m(\xi),\ \hat{t}_{ik}^m(\xi))$;

    Update the set of obtained solutions for each decision variable:

    $\theta_x(\xi) = \theta_x(\xi) \cup \{\hat{x}_j^m(\xi)\}$;

    $\theta_z(\xi) = \theta_z(\xi) \cup \{\hat{z}_{ij}^m(\xi)\}$;

    $\theta_y(\xi) = \theta_y(\xi) \cup \{\hat{y}_i^m(\xi)\}$;

    $\theta_r(\xi) = \theta_r(\xi) \cup \{\hat{r}_{ii'k}^m(\xi)\}$;

    $\theta_t(\xi) = \theta_t(\xi) \cup \{\hat{t}_{ik}^m(\xi)\}$;

  Step 3:

  for $\xi \in \Xi$ **do**

    Optimize Sub-3 model under each scenario and determine updated proximal solutions $(\tilde{x}_j(\xi),\ \tilde{z}_{ij}(\xi),\ \tilde{y}_i(\xi),\ \tilde{r}_{ii'k}(\xi),\ \tilde{t}_{ik}(\xi))$;

  Calculate the mean value of the first stage decision variables according the obtained proximal solutions from Sub-3 model by $\bar{x}_j^{m+1} \leftarrow \sum_{\xi \in \Xi} p(\xi)\tilde{x}_j(\xi)$;

  Update the weights based on the obtained solutions from Sub-3 model by

  $\omega_j^{m+1}(\xi) \leftarrow \omega_j^m(\xi) + \rho(\tilde{x}_j(\xi) - \bar{x}_j^{m+1})$;

  if $\sqrt{\sum_{\xi \in \Xi} p(\xi) \left\| \tilde{x}_j(\xi) - \bar{x}_j^m \right\|_2^2} < \epsilon\ OR\ m = k_{max}$ **then**

    $condition \leftarrow true$;

Result: $((\tilde{x}_j(\xi),\ \tilde{z}_{ij}(\xi),\ \tilde{y}_i(\xi),\ \tilde{r}_{ii'k}(\xi),\ \tilde{t}_{ik}(\xi))_{\xi \in \Xi},\ \bar{x}_j^m\ )$

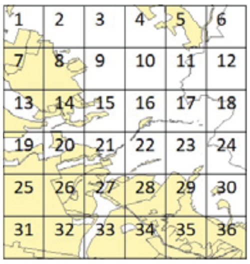


Figure 2: Selected area for the problem with assigned numbers

ized within values [1,3]. The normalized values of these coefficients are reported in Table 3. If the source of uncertainty affects the intensity of wildfire in the same direction, then the corresponding coefficients are calculated by $Coefficient = (Level\ of\ uncertainty\ \ source - minimum\ value)/(maximum\ value - minimum\ value)$. If there is a reverse effect on the intensity of wildfire, the corresponding coefficients are obtained according to following equation:

$Coefficient = (maximum\ value - Level\ of\ uncertainty\ \ source)/(maximum\ value - minimum\ value)$.

Table 2: Sources of uncertainties related to the bush fire event

Source of Uncertainty	Levels
Wind direction	West (W), Northwest (NW), Southwest (SW)
Wind speed (km/h)	55, 85, 115
Month's rainfall (mm)	9, 53, 115
Month's max temperature (^oC)	16, 26, 36
Fire starting point	1, 7, 13, 19, 25, 31
Fire starting time	Working hour, Non-school holiday (WN)- Non-working hour (N)- Working hour, School holiday (WH)

The related data and selected levels have been extracted according to the reported historical data in the period of Dec 2015 to Jan 2017 from the Australian government (Bureau of Meteorology) website[1]. Based on these levels, a collection of scenarios was generated. The scenario sets that form each instance generated were randomly sampled from this set (with 1458 scenarios).

Algorithm 2: Scenario generation procedure

Phase 1: Form a grid on the selected area and a number is assigned to each cell.
Phase 2: Fire spread patterns are determined based on fire starting cell and the wind direction.
Phase 3: The time window upper bound, service and travel times and asset values are obtained by using equations (34)-(37), respectively.

Considering the ignition cell and the wind direction, the fire spread patterns are defined. As an example, in the case of a NW wind, the fire spread patterns associate with the entering cell are shown in Figure

[1] http://www.bom.gov.au/climate/dwo/201606/html/IDCJDW7021.201606.shtml

Table 3: Specified coefficients for real simulation of time limit for asset protection during an escaped fire

Source of uncertainty	Levels	Coefficient
Temperature (^{o}C)	16	1
	26	1.5
	36	2
Total rain in a month (mm)	9	2
	53	1.6
	115	1
Wind speed (km/h)	55	1
	85	1.5
	115	2
Escaped fire time	Working hours, Non-school holiday (WN)	3
	Non-working hours (N)	1
	Working hours, School holiday (WH)	2

3. Furthermore, the upper limits of time windows,and the affected area are defined according to the wind direction. As an example, this is reported in Figure 4 in case of a NW wind. Finally to achieve the time window upper bound in the i^{th} asset under scenario ξ locating at a cell ($B_i(\xi)$), we consider.

$$B_i(\xi) = ITBC_i \times \alpha_1 \times TC(\xi) \times \alpha_2 \times TRC(\xi) \times \alpha_3 \times WSC(\xi), \qquad \forall i \in \mathcal{I}, \xi \in \Xi \tag{34}$$

where $ITBC_i$ is the initial time bound for the i^{th} asset that is determined based on wind direction, fire entering node and the cell that contains this asset. TC, TRC, WSC are temperature, total rainfall and wind speed coefficients, respectively. Terms α_1, α_2 and α_3 are weights to determine the impact of each source of uncertainty on the value of the time window upper bound at each asset.

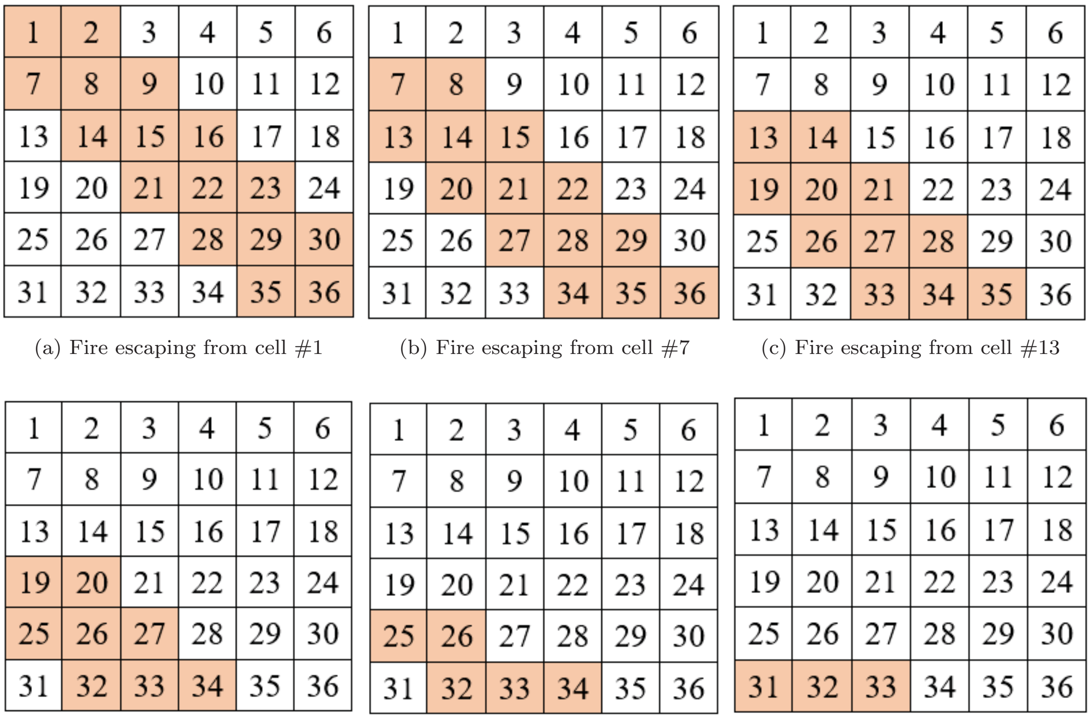
In each scenario, the travel time to assets in threatened cells are determined by predefined travel time coefficients which depends on the fire entering the cell and the wind direction. The travel time between assets are determined using the following equation (35):

$$T_{ijk}(\xi) = Itt_{ijk} \times TTC_{ij}(\xi), \qquad \forall i \in \mathcal{N},\ \xi \in \Xi \tag{35}$$

where, Itt_{ijk} is the initial travel time between two assets by vehicle k and $TTC_{ij}(\xi)$ is the travel time coefficient for each scenario. Moreover, by using equation (36), the protection time for an asset locating at a cell (O_i), in various scenarios is determined using

$$O_i(\xi) = IST_i \times \beta_1 \times TC(\xi) \times \beta_2 \times TRC(\xi), \qquad \forall i \in \mathcal{I},\ \xi \in \Xi \tag{36}$$

where IST_i is the initial expected servicing time to protect an asset i and reported in Table 4.The term β_1 and β_2 are the weight of the temperature and total rainfall factors in determining the operation time.



(a) Fire escaping from cell #1 (b) Fire escaping from cell #7 (c) Fire escaping from cell #13

(d) Fire escaping from cell #19 (e) Fire escaping from cell #25 (f) Fire escaping from cell #31

Figure 3: Escaped fire spreading pattern in case of wind direction of NW.

Finally, the asset utility value is calculated by equation (37).

$$V_i(\xi) = IV_i \times FTC_i(\xi), \qquad \forall i \in \mathcal{I}, \xi \in \Xi \tag{37}$$

where IV_i is the initial utility value of an asset i which is assumed to be as reported data in Table 3, and $FTC_i(\xi)$ is the fire time coefficient for each scenario. As previously mentioned, the initial utility value is determined based on monetary aspect, importance of asset, and residential population by equation (38)

$$\begin{aligned} IV_i = & \delta_1 \times monetary\ aspect\ of\ the\ i^{th}\ asset + \delta_2 \times importance\ of\ the\ i^{th}\ asset + \\ & \delta_3 \times residential\ population\ of\ the\ i^{th}\ asset \qquad \forall i \in \mathcal{I}, \end{aligned} \tag{38}$$

where δ_1, δ_2 and δ_3 are the weights of monetary, importance factors and residential population factors in calculating the value for each asset.

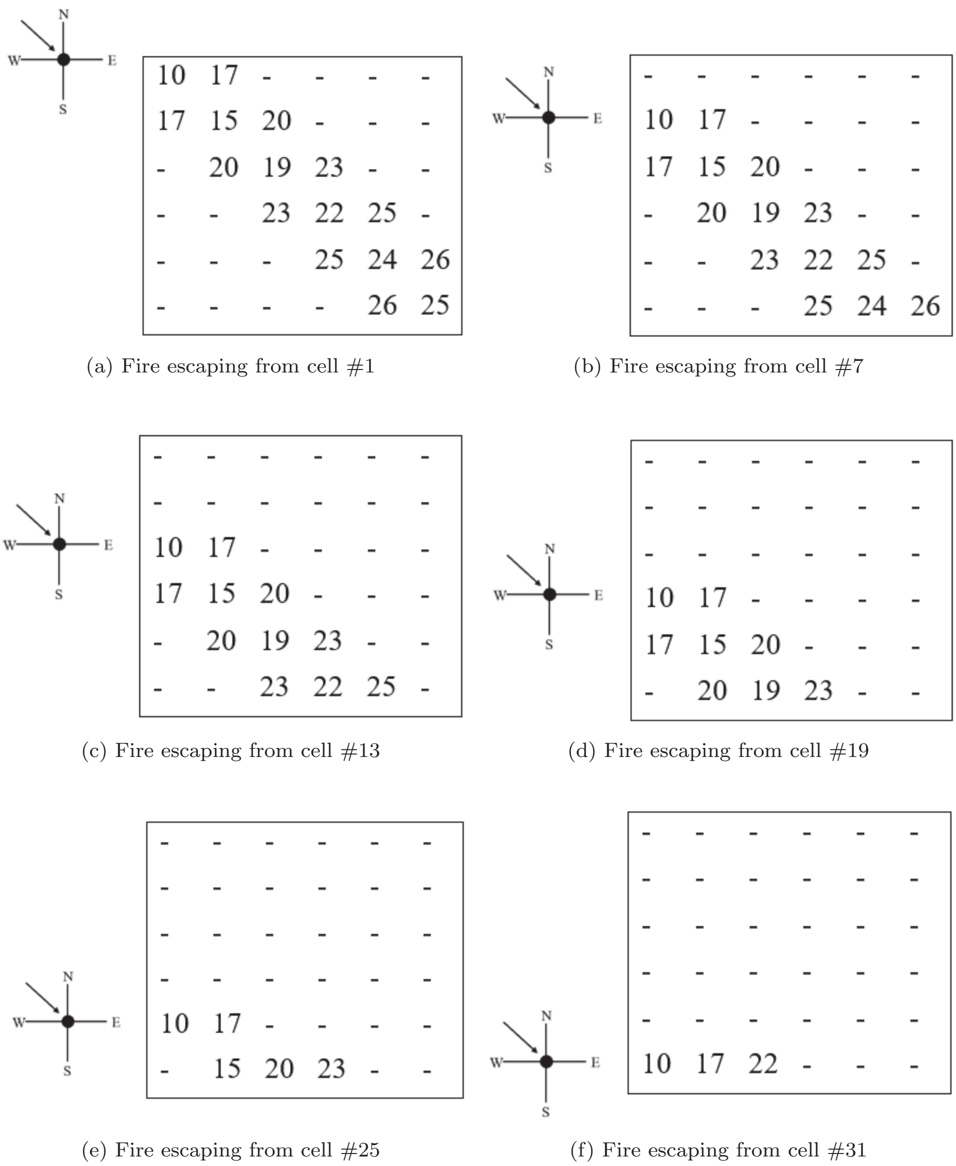


Figure 4: Time window upper bound for protection in the case of wind direction of NW. ”-” illustrates the fire does not spread to theses cells under each scenario.

Table 4: Specified initial values for assets servicing time bound, servicing time and their values

Asset	IST_i	IV_i	Asset	IST_i	IV_i
1	15	189	11	4	185
2	5	145	12	12	193
3	20	141	13	21	177
4	6	121	14	15	104
5	12	112	15	18	138
6	18	131	16	11	171
7	6	173	17	7	173
8	4	179	18	16	122
9	15	170	19	9	127
10	22	102	20	11	167

5. Computational study

In this section the validity of the proposed model and the necessity of considering uncertainty as well as the validity of the FW-PH algorithm are examined. Data for the parameters used for the computational study can be found in https://github.com/bashirimahdi/Asset_protection_routing. The computations were performed on a 3.5 GHz workstation with 32 GB RAM and 6 cores operating on Windows 10 (64-bit), using CPLEX.

In our model, the decisions related to location and routing are determined simultaneously. To show the importance of an integrated decision making, we suppose that the location decision has been done separately and we evaluate it in two different location decisions of [0 1 0 0] and [0 0 1 1] called (a) and (b), respectively. Then, the model is optimized with predefined locations. The results of routing costs and obtained values are compared in Figure 5 for both cases of (a) and (b). It shows that, although the routing part of the model is optimized in both cases it is definitely related to the first stage decision. This therefore confirms the necessity of integrated decision-making, as considered in the proposed model.

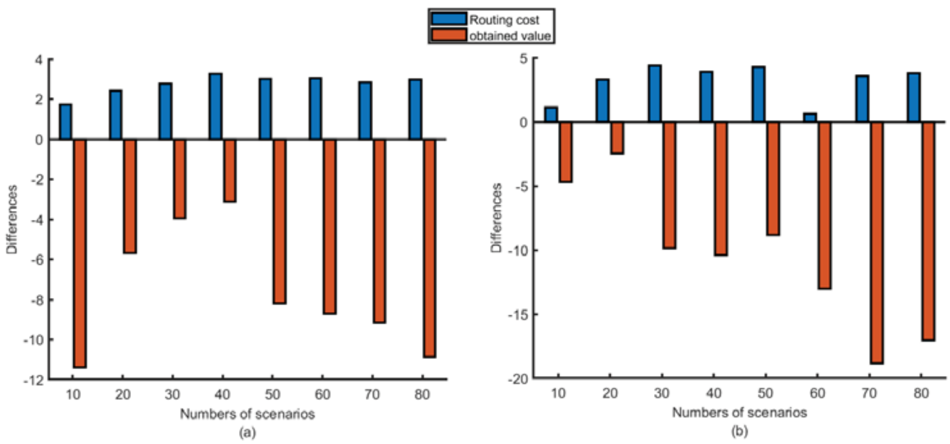


Figure 5: Routing cost and values obtained when the first-stage variables are equal to [0 1 0 0], [0 0 1 1], respectively

The results show how the routing cost increases and the obtained value decreases if the decisions in the first and second stage variables are determined separately.

5.1. Performance of FW-PH

The APRP model was solved using CPLEX and the first-stage decision variables obtained and respective computational times are compared with those of the FW-PH algorithm. Also, CPLEX is used in the FW-PH algorithm for solving the corresponding sub-problems (Sub-1, Sub-2, and Sub-3). This analysis is applied under different number of scenarios and the results are reported in Table 5. The computational time of CPLEX ($Time_C$) and the FW-PH algorithm ($Time_{FW-PH}$) are reported. In this table, Dis_{FS} shows the Hamming distance between the first stage decision variables that are obtained by CPLEX (which are integer) and the FW-PH algorithm. The results illustrate that the FW-PH algorithm find the optimal solution accurately for those instances that can be solved to optimality using CPLEX. Moreover, CPLEX can not find the optimal solution in 10000 seconds when the numbers of scenarios increase. Also the results show that the efficiency of the FW-PH algorithm increases when increasing the numbers of scenarios.

Table 5: Comparison between CPLEX and Progressive Hedging

J	K	S	$Time_C$	$Time_{FW-PH}$	$Dis_{FS}(\%)$	J	K	S	$Time_C$	$Time_{FW-PH}$	$Dis_{FS}(\%)$
		5	3.46	4.14	0.00			5	43.92	61.98	0.00
		10	5.13	55.98	0.00			10	51.47	50.61	0.00
		20	57.04	211.82	0.00			20	2436.56	102.78	0.00
		30	10000	231.80	-			30	10000	647.77	-
4	2	40	10000	273.86	-	10	3	40	10000	818.45	-
		50	10000	231.856	-			50	10000	1085.77	-
		60	10000	254.050	-			60	10000	3297.29	-
		70	10000	377.83	-			70	10000	3893.09	-
		80	10000	275.66	-			80	10000	7841.97	-
		5	9.12	13.97	0.00			5	29.06	54.46	0.00
		10	17.87	14.10	0.00			10	32.69	57.20	0.00
		20	711.59	24.43	0.00			20	3579.74	114.85	0.00
		30	10000	71.27	-			30	10000	1887.29	-
7	2	40	10000	117.07	-	10	4	40	10000	1640.36	-
		50	10000	1900.36	-			50	10000	2080.28	-
		60	10000	2360.80	-			60	10000	7394.83	-
		70	10000	2126.24	-			70	10000	8087.93	-
		80	10000	2979.51	-			80	10000	18084.75	-
		5	11.10	19.39	0.00			5	13.63	20.53	0.00
		10	59.96	42.77	0.00			10	97.78	85.30	0.00
		20	10000	105.44	-			20	10000	205.33	-
		30	10000	267.80	-			30	10000	1358.51	-
10	2	40	10000	354.42	-	15	2	40	10000	2033.34	-
		50	10000	622.75	-			50	10000	1754.63	-
		60	10000	3806.91	-			60	10000	1698.49	-
		70	10000	1286.17	-			70	10000	5682.24	-
		80	10000	1658.93	-			80	10000	6856.99	-

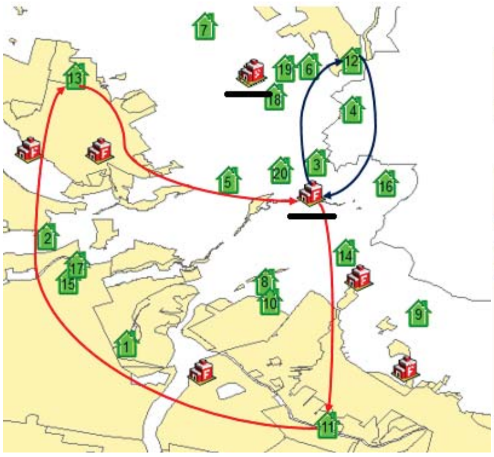
-: the optimal value of the first stage decision variable is not found by CPLEX in 10000 seconds

The objective value and solution time obtained via CPLEX and the FW-PH for the APRP model under different number of scenarios are reported in Table 6. In this table, Obj_C is the best objective value obtained by CPLEX within 10000 seconds and $Time_C$ reports the solution time of CPLEX. Also, the lower bound (LB_C) obtained by CPLEX is reported in this table. Obj_{FW-PH} is the objective value achieved by rounding and fixing the first stage-decision variables obtained using FW-PH in the Sub-3 model and optimizing it (which can be done independently for each scenario and recombined to obtain a upper bound for the APRP model). The lower bound obtained using FW-PH (LB_{FW-PH}) is reported as well. This lower bound is equal to expected value of obtained objective values from the Sub-3 model under each scenario without considering the terms related to relaxing the non-anticipativity constraints. The computational time of the algorithm ($Time_{FW-PH}$) is reported in this table. The gap between Obj_{FW-PH} and Obj_C is calculated by $Gap = (Obj_{FW-PH} - Obj_C)/Obj_C \times 100\%$ and reported in Table 6. Also, the gap between Obj_{FW-PH} and LB_{FW-PH} is calculated by $Gap_{FW-PH} = (Obj_{FW-PH} - LB_{FW-PH})/LB_{FW-PH} \times 100\%$ and is also reported. The results show that the FW-PH algorithm can find optimal or near-optimal solutions in a reasonable computational time (634.78 seconds in average) when the number of scenarios increases. In contrast, the commercial solver can find optimal solutions for 10 cases within 10000 seconds and the solution time of commercial solver increases exponentially with the number of scenarios. Therefore, using the FW-PH algorithm is particularly more efficient in cases with a large number of scenarios. Also, the lower bound obtained by FW-PH algorithm is almost equal to Obj_{FW-PH} in all cases. While it is not guaranteed in theory, the application of the FW-PH algorithm for our APRP model numerical experiments have always displayed convergence to a primal optimal solution.

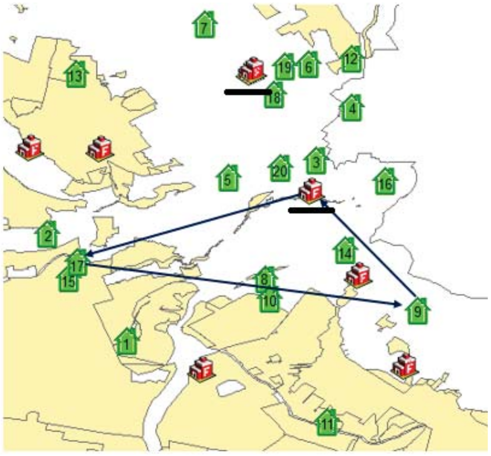
Moreover, the total difference between the value of first stage-decision variable under each scenario and their mean value ($\sum_{j \in \mathcal{J}} \sum_{\xi \in \Xi} |\tilde{x}_j(\xi) - \bar{x}_j|$ is reported in Table 7, which can be understood as a measure of primal infeasibility. The results show that the differences between the obtained first stage decisions under each scenario and their corresponding mean value is sufficiently small. Therefore, taking also into account the information in column Dis_{FS} (Table 5), it can be concluded that the algorithm can find solutions for the first stage decision variables that are feasible to the APRP model.

5.2. Necessity of considering uncertainty

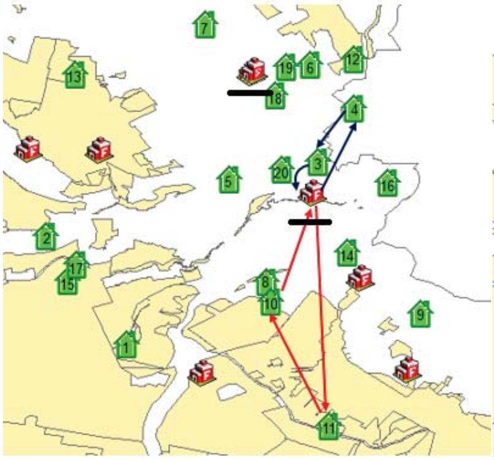
Figure 6 provides a schematic representation of an arbitrary solution in which two locations from 7 potential locations are selected for establishing protection depots. Also the routes corresponding to each vehicle under five different scenarios are indicated in this figure. The parameters including utility values, the time window upper bound, traveling and protecting times are different for these scenarios. As shown in this figure, one protection depot is established and two available vehicles are assigned to this protection depot. The assets selected for protecting are different for each scenario based on the trade off between the routing cost and the utility obtained by visiting an asset considering the risk of damage. As previously mentioned, in a disaster situation protecting assets and keeping people safe are more important than travelling cost. Therefore, it might be so that valuable assets are selected for protection although they are far from a depot.



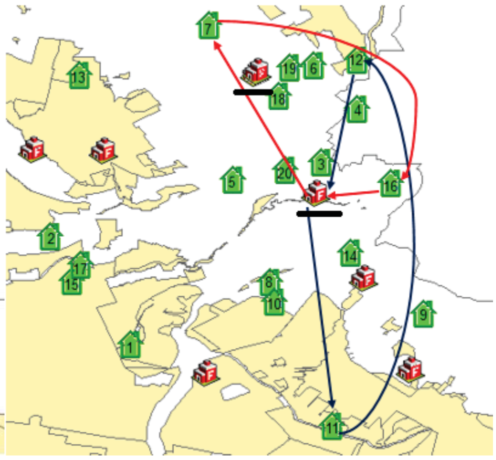
(a) First scenario



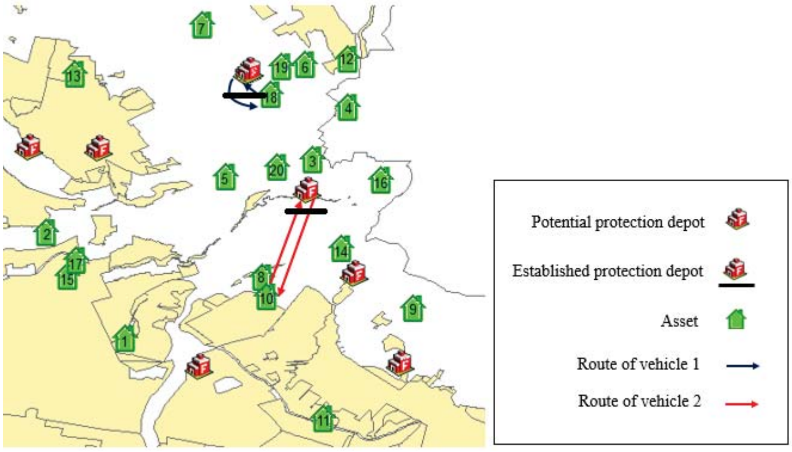
(b) Second scenario



(c) Third scenario



(d) Fourth scenario



(e) Fifth scenario

Figure 6: Obtained solutions from the proposed model under different scenarios.

Table 6: Computation results for different numbers of scenarios and potential number of locations for establishing protection depots

J	S	Obj_C	LB_C	$Time_C$	Obj_{FW-PH}	LB_{FW-PH}	$Time_{FW-PH}$	Gap(%)	Gap_{FW-PH}(%)
4	5	-564.98	-564.98	8.77	-564.98	-564.98	7.25	0.00	0.00
	10	-546.75	-546.75	27.82	-546.75	-546.75	39.12	0.00	0.00
	20	-539.91	-539.91	1727.79	-539.91	-539.91	177.24	0.00	0.00
	30	-540.51	-596.96	10000	-540.51	-540.51	178.44	0.00	0.00
	40	-528.41	-589.76	10000	-530.48	-530.49	195.28	0.38	0.00
	50	-542.69	-603.63	10000	-542.70	-542.70	158.86	0.00	0.00
7	5	-560.88	-560.88	10.48	-560.88	-560.88	14.28	0.00	0.00
	10	-556.74	-556.74	18.60	-556.74	-556.74	27.80	0.00	0.00
	20	-548.59	-562.15	10000	-548.59	-548.59	56.74	0.00	0.00
	30	-545.60	-591.51	10000	-545.87	-545.87	139.25	0.05	0.00
	40	-535.62	-592.56	10000	-535.95	-535.96	215.54	0.00	0.00
	50	-544.36	-623.44	10000	-544.36	-544.36	3728.75	0.00	0.00
10	5	-582.74	-582.74	14.40	-582.74	-582.74	26.99	0.00	0.00
	10	-568.78	-568.78	55.22	-568.78	-568.78	72.54	0.00	0.00
	20	-554.40	-571.86	10000	-554.40	-544.40	250.73	0.00	0.00
	30	-551.74	-638.26	10000	-551.74	-551.74	515.96	0.00	0.00
	40	-540.61	-628.28	10000	-540.63	-540.63	654.14	0.00	0.00
	50	-549.67	-639.60	10000	-549.69	-549.69	1163.99	0.00	0.00
15	5	-605.17	-605.17	19.01	-605.17	-605.17	36.43	0.00	0.00
	10	-576.80	-576.80	95.83	-576.80	-576.80	145.25	0.00	0.00
	20	-559.59	-606.04	10000	-559.59	-559.59	429.97	0.00	0.00
	30	-552.94	-639.58	10000	-553.33	-553.33	1848.51	0.07	0.00
	40	-542.95	-635.01	10000	-542.95	-542.95	2743.43	0.00	0.00
	50	-545.45	-641.83	10000	-552.12	-552.47	2408.25	1.22	0.06
Average							634.78	0.07	0.00

The effect of the number of available vehicles on the expected value of perfect information (EVPI) and on the value of the stochastic solution (VSS) indices are shown in Figure 7. The EVPI gap and VSS gap are calculated by using the following equations, respectively.

$$EVPIGap = \frac{z_{HN} - z_{WS}}{z_{WS}} \times 100 \tag{39}$$

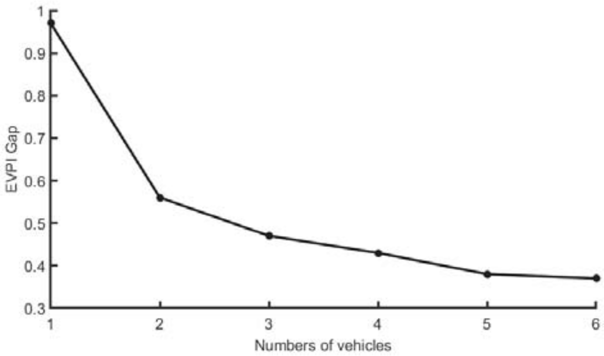
$$VSSGap = \frac{z_{EEV} - z_{HN}}{z_{HN}} \times 100 \tag{40}$$

where z_{HN} is the objective value of two-stage stochastic programming model. z_{EEV} and z_{WS} are calculated by using equations (41) and (42),respectively.

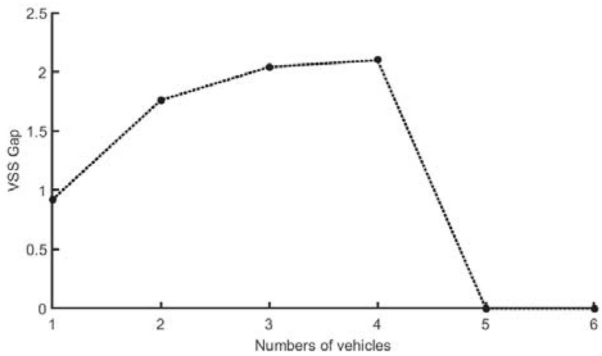
$$z_{WS} = \mathbb{E}_{\xi}\left(Min \sum_{j\in\mathcal{J}} EC_j \times x_j(\xi) + \sum_{i\in\mathcal{N}}\sum_{i'\in\mathcal{N}}\sum_{k\in\mathcal{K}} C_{ii'k}(\xi) \times r_{ii'k}(\xi) + \sum_{i\in\mathcal{I}} V_i(\xi) \times y_i(\xi)\right) \tag{41}$$

Table 7: Total error in the solutions obtained by the FW-PH algorithm

J	S	$\sum_{j\in\mathcal{J}}\sum_{\xi\in\Xi}\lvert\tilde{x}_j(\xi)-\bar{x}_j\rvert$	J	S	$\sum_{j\in\mathcal{J}}\sum_{\xi\in\Xi}\lvert\tilde{x}_j(\xi)-\bar{x}_j\rvert$
4	5	2.670×10^{-8}	10	5	2.988×10^{-6}
	10	2.567×10^{-5}		10	0.001
	20	0.0003		20	0.003
	30	0.0007		30	0.0003
	40	0.0007		40	0.001
	50	0.0002		50	0.001
7	5	2.965×10^{-6}	15	5	5.128×10^{-6}
	10	0.0007		10	6.733×10^{-6}
	20	9.896×10^{-6}		20	6.941×10^{-5}
	30	1.636×10^{-5}		30	0.0001
	40	2.863×10^{-5}		40	0.001
	50	0.06		50	0.003



(a) EVPI gap for different numbers of vehicles



(b) VSS gap for different numbers of vehicles

Figure 7: The effects of the available number of vehicles on EVPI and VSS indices

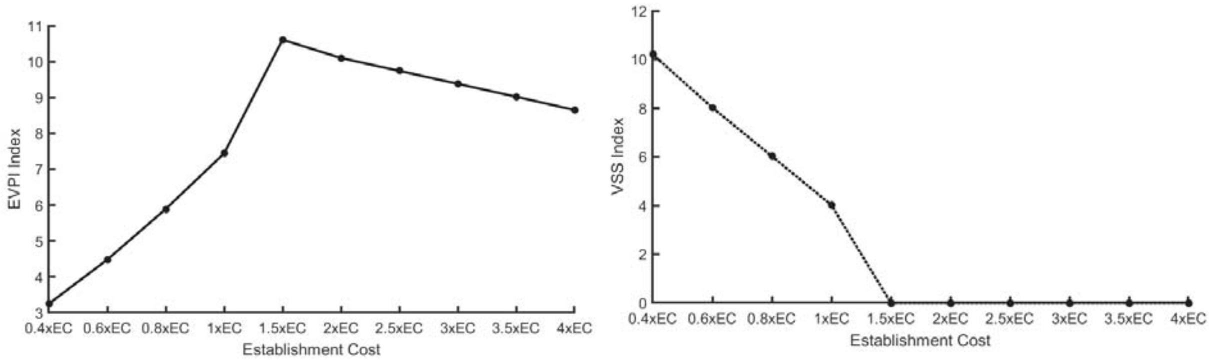
s.t: (2)-(12)

$$z_{EEV} = Min \sum_{j\in\mathcal{J}} EC_j \times \bar{x}_j + \mathbb{E}_\xi\left(\sum_{i\in\mathcal{N}}\sum_{i'\in\mathcal{N}}\sum_{k\in\mathcal{K}} C_{ii'k}(\xi)\times r_{ii'k}(\xi)\right) - \mathbb{E}_\xi\left(\sum_{i\in\mathcal{I}} V_i(\xi)\times y_i(\xi)\right) \quad (42)$$

s.t: (2)-(12)

In equation (42), $\bar{x}_j$ are the optimal values of the first stage decision variables extracted from solving a deterministic model with average values of uncertain parameters. Then $\bar{x}_j$ are considered as fixed parameters in equation (42). Figure 7a shows that the decision-maker is willing to pay less to obtain perfect information as the number of vehicles increases. Therefore, in the case of sufficient resources, perfect information is not as valuable.

In contrast, Figure 7b shows that the importance of considering uncertainty increases with an increasing numbers of vehicles until the number of vehicles exceeds some critical value. It can be concluded that, considering uncertainty does not have a significant effect when the number of resource are extremely small but its importance increases with increasing resources until some critical point is attained after which considering uncertainty does not have a significant effect. Therefore, the proposed model and consideration of uncertainty have greater importance in settings where the resources are limited. In practice, the lack of resources is both critical and commonplace, and thus by considering uncertainty deployment decisions can be improved significantly.



(a) EVPI index under different values of establishment cost (b) VSS index under different values of establishment cost

Figure 8: EVPI and VSS indices under different values of establishment cost

The EVPI and VSS indices under different values of establishment cost are shown in Figure 8. The VSS index decreases with increases in the value of establishment cost. On the other hand, in the face of a limited budget and high establishment costs, depot locations decisions are very similar with or without considering uncertainty.

The EVPI depends on the relationship between the establishment cost and other terms in the objective function, such as routing costs. On the other hand, when the establishment cost is considerably higher than the utility obtained from protecting the assets, the decision-maker is not willing to pay much to obtain perfect information. It can be observed that the model is most beneficial under conditions with small or medium establishment costs.

6. Managerial insights and conclusions

In this paper, a two-stage stochastic programming model is proposed for the asset protection routing problem. The first-stage decision determines the locations for establishing protection depots. The need for considering uncertainty in this problem is demonstrated by using EVPI and VSS indices. It is concluded that the proposed model is more beneficial when the number of vehicles is small and the establishment cost is not too large. Also, the importance of considering uncertainty of parameter values increases with increasing travel times between nodes.

As previously noted, the computation time of SMIP models increases as the number of scenarios increases. Scenario-based decomposition methods present a means for dealing with this issue and for reducing the computation time of two-stage and multi-stage stochastic programming models. PH is one of the well known scenario-based decomposition methods, but it cannot provide guarantees of convergence to the global optimal solution of a SMIP. In recent years, researchers focused on this issue and new algorithms based on a combination of PH and other algorithms were proposed. In this paper, an algorithm based on combining PH and the Frank-Wolfe method was used to solve the proposed model. The computation results show the effective performance of the FW-PH algorithm for the APRP problem. Also, the computational time of this algorithm is significantly smaller than the computational time of optimization solvers.

In situations where a wildfire becomes impossible to control, it is prudent to utilise resources to undertake protective tasks to mitigate the risk of losing community assets. By anticipating multiple scenarios, resources can be located to minimise expected travel distances to assets where protection activities are required. As a managerial insight, a similar model and solution method can be used in other disaster situations such as those arising from floods, hurricanes, and other natural disasters that allow for response preparation time to be issued from warnings such as meteorological reports or satellite survey. Using the proposed approach, relief organizations can locate protection centres and hopefully make relief operations more efficient while mitigating the damages incurred by the disaster.

The results of our numerical studies show that the proposed approach performs better than the equivalent deterministic model. Its value becomes more prominent as the level of uncertainty increases. This is important for relief organizations as they usually experience a high level of uncertainty. Furthermore, the value of the stochastic approach is more relevant the lower the establishment cost of the depots. Thus the approach is particularly pertinent in emergency or disaster situations where low cost temporary bases have to be set up speedily.

Moreover, there are significant benefits to the proposed approach for protecting assets in a two-stage stochastic programming framework in the face of limiting resources. In such conditions, it is suggested that the location of relief centers is planned ahead with the aim of maximising the total value of assets protected in a disaster. Extending this problem by simultaneously considering other criteria and issues relating to fire fighting stations in addition to asset protection is suggested as a direction for future study.

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