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Fastest with no-overshoot velocity control design of a two-differential robotic formation.

Julián-Aalejandro Hernández-Gallardo, E.-J. González-Galván, C.-F. Méndez-Barrios, and Adrian-Josue Guel-Cortez

Abstract This paper presents the control design for a multi-agent system (MAS) of land-based mobile robots on wheels formation, that also implements a PI control for the fastest with no-overshoot response at each agent's velocity. Regarding the robot formation tracking control design, a mathematical model constructed from a geometric approach is applied. In addition, the formation's asymptotic stability is guaranteed in the light of Lyapunov's theory. When discussing the agents' velocity control, the mentioned Proportional-Integral PI algorithm is designed by using different methodologies which include *σ-stability*, *λ tuning*, and *Haalman´s tuning* techniques. Finally, the theoretical results are confirmed by simulations.

1 Introduction

Within robotics research, mobile robotics [2, 3, 11] presents different issues of interest, such as kinematic modeling, dynamic modeling, motion control, planning, and environment perception. It also has a wide range of applications which include maritime exploration, terrain reconnaissance, search and rescue missions, medical and domestic assistance, to name a few [3]. Within the different scenarios where mobile robots can be applied, a Multi-Agent System (system that is composed of multiple intelligent agents/sub-systems interacting with each other) where several robots work together has significant advantages over the use of a single robot, such as robustness to failure of individual agents, reconfigurability, and flexibility to perform much more complex tasks [1]. Nonetheless, one of the problems in robotic multi-agent systems is the formation control's design. Specifically, it refers to the problem

J. A. Hernández-Gallardo · E. J. González-Galván · C. F. Méndez-Barrios
Facultad de Ingeniería, Universidad Autónma de San Luis Potosí. San Luis Potosí, S.L.P., México, e-mail: jhernandezg_@outlook.com

A.-J. Guel-Cortez
Centre for fluid and complex systems, Coventry University, UK

where the relative position and orientation of each of the mobile robots belonging to the multi-agent system must be controlled according to a formation that follows a desired trajectory [1]. To solve such a problem, various approaches have been proposed so far. For instance, the *leader-follower* [9, 10], *virtual leader* [8], *virtual structures* [6] and *cluster space* [15] techniques.

Regarding such methods, in [10] a *leader-follower* approach (a reference trajectory defined by the leader) is used. Besides, in such a work, the internal stability of the formation is implied by the stability of the control laws of the individual agents. The problem with this approach is that it does not have good robustness to perturbations. In [8], the *virtual leader* approach is used. Here, the agents in the training jointly synthesize a single, possibly fictitious, leader agent whose trajectory acts as a leader for the group. This approach improves the robustness to perturbations found in the *leader-follower* approach. Besides, in [23], a virtual leader approach that uses reduction of communication threads between the agents is introduced. By such communication protocol, the agents are allowed, under certain conditions, to stop communicating with its neighbors while still avoid collisions between them. As another example, [15] proposes a geometric interaction between the individual agents where the formation model has state variables that are a function of the state variables of the individual agents. Finally, in [17], a graph interaction approach is used for the solution of formation control in multirobot systems using the *consensus* algorithm.

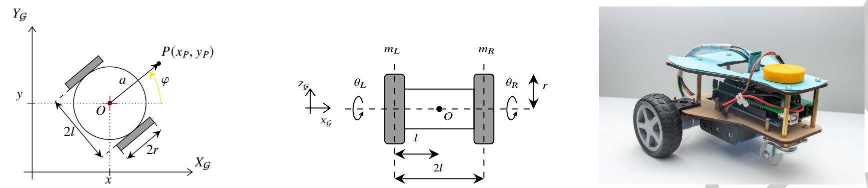
By taking into account what has already been reported in the literature, this paper presents a centralized control for the trajectory-tracking of a MAS composed of two-wheeled land mobile robots. The proposed control algorithm uses a geometric model for the estimation of the relative positions between the agents without taking into account their morphology. Furthermore, as a novelty of this work, a low-order PI type controller that ensures the fastest with no-overshoot response at each agent's velocity is implemented. To find such a control, a comparison between different design methodologies is performed. The analyzed methods are *σ-stability*, *λ tuning*, and *Haalman´s tuning*. In addition, we also apply an empirical tuning method, where only the stability of the response but not the performance is ensured. Finally, we briefly discuss the results in the form of numerical simulations.

The paper is organized as follows, Section 2 presents the kinematic model. Then, Section 3 presents the PI speed control design while including the used mathematical dynamical model for the description of the agent's velocity. Finally, Section 4 presents the geometric model and the formation control for trajectory tracking.

2 Background

In this section, the kinematic model of the differential robot (including the constraints and conditions where it is valid) which is gonna be used for the control design is presented.

Differential Kinematic Model of a Decentralized Robot Point. Consider a mobile robot with unicycle or differential configuration (see Fig. 1), and a global coordinate axis $\mathcal{G} = \left[X_{\mathcal{G}}, Y_{\mathcal{G}}\right]^T$. Before constructing the kinematic model of our differential mobile robot, as in [18], we consider the following set of constrains: the robot moves on a flat surface, the steering axis of the wheels is always perpendicular to the ground, there are no flexible elements in the structure, there is no sliding between the wheel and the ground, in addition to disregarding any type of friction. Then, from Fig. 1, x represents the position on the $X_{\mathcal{G}}$ axis, y the position on the $Y_{\mathcal{G}}$



(a) $\{X, Y\}_{\mathcal{G}}$ Plane View Diagram.

(b) $\{X, Z\}_{\mathcal{G}}$ Plane View Diagram.

(c) Differential Mobile Robot Prototype.

Fig. 1: Schematic of the Differential Mobile Robot for the Holonomic Model.

axis, φ the orientation of the robot concerning the $X_{\mathcal{G}}$ axis, ω_L the angular velocity of the left wheel, ω_R the angular velocity of the right wheel, r the radius of the wheels, and $2l$ the perpendicular distance between the two wheels. Thus, the direct differential kinematics of a decentralized point P whose position and velocity is separated by a distance a from the robot's drive axis (see Fig. 1) are defined by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\varphi}(t) \end{bmatrix} = \begin{bmatrix} \cos(\varphi(t)) & -a\sin(\varphi(t)) \\ \sin(\varphi(t)) & a\cos(\varphi(t)) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix}. \tag{1}$$

Therefore, we will consider x, y, and φ to be our states and v, and ω to be our inputs. It is worth mentioning that this type of model is holonomic. This because the point $P(x_P, y_P)$ has no velocity constraints in the plane $\mathcal{G} = \left[X_{\mathcal{G}}, Y_{\mathcal{G}}\right]^T$.

3 Robot Speed Control Analysis and Design

The previous section presented the kinematic model that will be used for the formation's control but do not introduce the mathematical model utilized for the agents' velocity control. For such a purpose, this section describes the different performed tuning techniques for the PI agents' velocity controls and the implemented system identification method to obtain the dynamical model which describes each wheel's velocity in a mobile robot.

System parameter's identification. The experimental platform consists of a robot built with the necessary peripherals. In brief, the robot has a differential drive which consists of a chassis, a pair of independently moving motorized rigid wheels, and a freewheel (see Fig. 1c). To describe the mobile robot velocity dynamics, we consider a first-order model with time-delay [20] as follows

$$G(s) = \frac{\Omega(s)}{V(s)} = \frac{ke^{-hs}}{Ts + 1}, \tag{2}$$

where the output $\Omega(s)$ is the angular velocity, the input $V(s)$ is the reference voltage, and the constants $k > 0$ is the open-loop system gain, $h > 0$ is the system delay and $T \in \mathbb{R} \setminus \{0\}$ is the time constant. The parameters from the transfer function (2) are found by a system's parameter identification algorithm that uses the Particle Swarm Optimization (PSO) method (for further details, see [7, 19]). As a result, the following plants are obtained

$$G_R(s) = \frac{1.0646e^{-0.084s}}{0.056s + 1}, \qquad G_L(s) = \frac{1.1257e^{-0.0954s}}{0.0333s + 1}. \tag{3}$$

Where, G_R and G_L are the transfer functions for the right and left wheel of the mobile robot, respectively.

3.1 PI Controls design

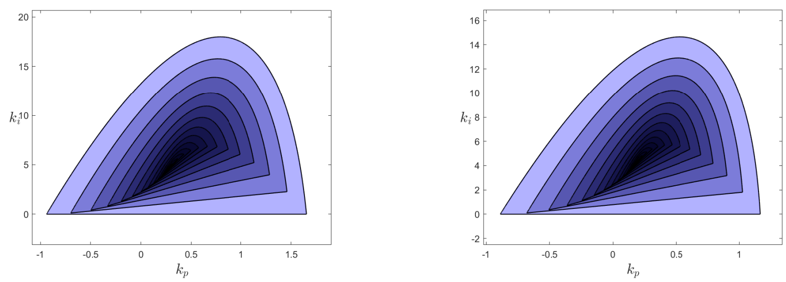
An important part of the formation control corresponds to the agent's velocity control design. Here, we explain some of the different methods that can be used for such a task. The comparison between these methods when applied to the system is given in Section 5.

PIσ controller. Based on the concept of *sigma*-stability and the theory of $\mathcal{D}$-partitions [4, 5, 12, 13], we can design *PI* controllers that allows us to obtain the maximum achievable exponential decay in the closed-loop system response. A controller designed for such a goal is called PIσ. Here, to obtain the PIσ controller gains, we have used a design formula that can be found at [21]. Furthermore, in Fig 2, here we show the stability regions for the plants G_R and G_L. In the Figures, the level curves describe regions that contain different exponential decays, being the smaller one the region with the PIσ controller gains.

Remark 1 Instead of using the PIσ control, we can simply use a PI controller whose gains are arbitrarily chose from the stability areas shown in the Fig. 2. Such PI control would give us a stable closed-loop system response.

PIλ controller. A control that takes into account the uncertainty of the estimated system parameters would be of great benefit. A common alternative, to find such robust controller corresponds to the *Lambda tuning* technique (see, for further details [22]). In this regard, a PI control designed by such technique would be called PIλ and its gains will be given by the following rules

$$K_c = \frac{T}{k(h + \lambda)}, \qquad T_i = T, \tag{4}$$



(a) Right Motor. (b) Left Motor.

Fig. 2: Stability Region in the Parametric Plane K_p vs K_i of the Differential Mobile Robot.

where λ is a factor that affects the speed in the response. In brief, increasing its value decreases the speed, and vice versa. The author recommends $\lambda \approx 3T$ to obtain a robust controller [22].

PIH controller. Another popular method is the so-called Haalman method [14]. Such technique was designed for the iron rolling industry to provide a critically damped response with good reference tracking performance. This technique acceptably works in systems of type (2). The PIH controller constants are obtained from [16] the following equations

$$K_c = \frac{2T}{3kh}, \qquad T_i = T. \tag{5}$$

The main feature of the Haalman tuning method is that the poles and zeros of the process cancel with the poles and zeros of the controller. Nonetheless, as a drawback, often poor results may be observed for rejection of load disturbances when using the controller gains obtained by this method.

4 Two Robot Formation Control Analysis and Design

In this section, we first obtain the kinematic model of the formation when considering only two robots (see Fig. 3). In Fig. 3, P_C represents the center of the formation, P_i with $i \in \{1, 2\}$ is the decentralized point of each robot, Φ_C corresponds to the orientation of the formation concerning the $X_{\mathcal{G}}$ axis, ϕ_i with $i \in \{1, 2\}$ is the orientation of each robot concerning the $X_{\mathcal{G}}$ axis, and D is the Euclidean distance between the robots. Therefore, the geometric model in matrix form is as follows

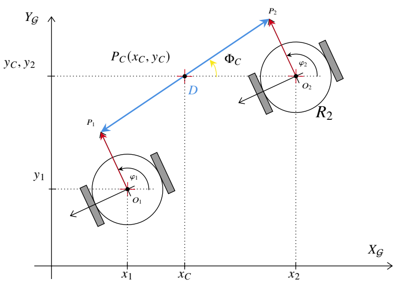


Fig. 3: Two Robot Formation Scheme.

$$\begin{bmatrix} x_C \\ y_C \\ D \\ \Phi_C \end{bmatrix} = \begin{bmatrix} \dfrac{x_1 + x_2}{2} & \dfrac{y_1 + y_2}{2} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} & \tan^{-1}\left(\dfrac{y_2 - y_1}{x_2 - x_1}\right) \end{bmatrix}^T . \quad (6)$$

Note that, by considering the geometric model, the differential kinematic model of the formation is obtained by computing the time derivatives of each of the previous equations [15]. Now, to meet the trajectory tracking goal, the block diagram shown in Fig. 4 is proposed. As it can be seen, the tracking control is applied for the total formation while each robot uses an individual tracking control algorithm. For the tracking control, the following result holds

Theorem 1 (Tracking Trajectory Control) *By considering the control law*

$$\dot{\mathbf{q}}_{ref} = \mathbf{J}^{\dagger}\left(\dot{\mathbf{p}}_d + \mathbf{K}\mathbf{e}\right). \quad (7)$$

The system (7) *is Asymptotically Stable if and only if* $\mathbf{K}$ *is a positive definite diagonal matrix.*

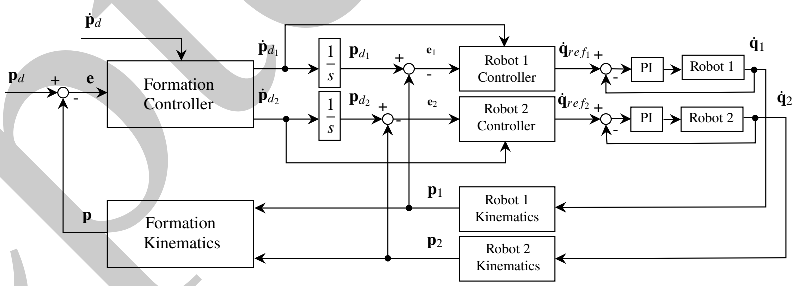
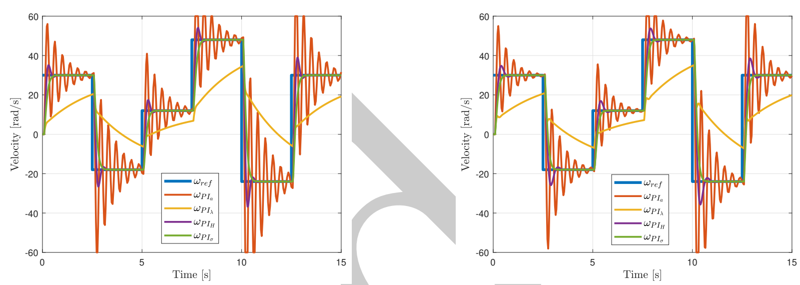


Fig. 4: Block Diagram for Trajectory Tracking Control of the Formation of Two Mobile Robots

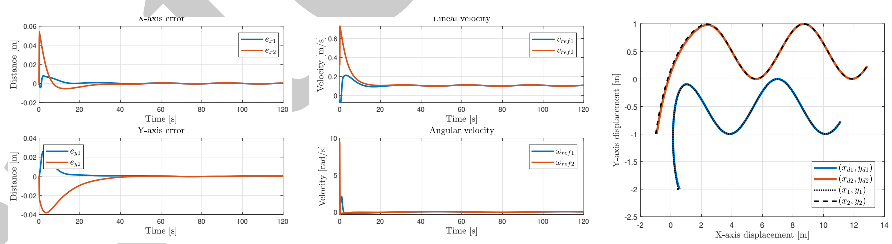
5 Numerical Results

First, simulations were performed to observe the performance of the speed response in the different controllers. The results are shown in Fig. 5. From the results, it is clear that the response with the highest speed and least over impulse corresponds to the system controlled by a PIσ algorithm. Thus, the speed control of each differential mobile robot in our formation uses this type of control. For the formation control, the group follows a trajectory parameterized by $x_d(t) = 0.1t$, and $y_d(t) = 0.5\sin(0.1 \cdot t)$. The simulation results of the robotic formation are shown in Fig, 6, where a correct motion response of the formation for trajectory tracking is clearly depicted. From Fig. 6, we can also see that the errors oscillate ±6 cm in the two axes of the plane, this allows us to conclude that the asymptotic stability is fulfilled. In addition, the error does converge to zero to a large extent. Finally, from the simulation, no collision between the agents is verified.



(a) Right Motor. (b) Left Motor.

Fig. 5: Closed Loop Response of PI Controllers.



(a) Position errors. (b) Velocities. (c) Trajectories.

Fig. 6: Response to Trajectory Tracking Control for a Sine function.

6 Concluding remarks

In this work, we have presented a methodology for the formation trajectory tracking control of a Multi-agent System (two mobile robots) by taking a geometrical approach. The methodology also applies the fastest with no-overshoot velocity control at each robot in the formation. This is achieved by a simple *PI* control tuned using different techniques. In this regard, the velocity control responses were compared, selecting the most optimal for our needs. In addition, according to our results, the used geometric approach permits us to not take into account the configuration of the mobile robot. This would enable us to consider different mobile robot configurations like differential, tricycles, Ackerman, or any combination between them within the multi-agent system. Finally, the presented numerical results permit us to conclude, that the proposed algorithm operates as expected. Leaving us with the opportunity to think of an experimental validation as part of the future work.

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