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Wheel torque distribution optimization of four-wheel independent-drive electric vehicle for energy efficient driving

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Abstract—With the help of several in-wheel motors, four-wheel independent-drive electric vehicle (4WIDEV) has tremendous potential to improve vehicle performance. Except for the theme of stability enhancement during the past two decades, energy saving topic becomes more attractive recently. However, it is difficult to achieve stability and economy performance simultaneously. Given the commonly used control method of rigid yaw rate tracking may limit the energy saving potential of 4WIDEV in cornering. Therefore, be different from previous studies which often focus on a sole target, in this paper, a driving energy management strategy for 4WIDEV based on multi-objective online optimization of four-wheel torque distribution is proposed in this paper, which includes weighted yaw rate tracking error into its parameterized control objectives besides electric drive system efficiency, tire slip losses and wheel torque ripple. Meanwhile, for better coordination of stability and economy in different situation, fuzzy logic control is adopted to dynamically adjust the weight of the control objective indexes of the proposed multi-objective online optimization function. Then, a particle swarm optimization algorithm is adopted to solve the optimization function. Finally, the proposed strategy for energy-efficient driving is verified based on the built Simulink and CarSim co-simulation model. The results show that the proposed four-wheel torque distribution strategy based on the multi-objective online optimization is more effective than average distribution strategy and offline optimization strategy.

Keywords—Electric Vehicle, Independent drive, Torque distribution control, Multi-objective, Online optimization, Energy efficiency

I. INTRODUCTION

Along with the increasingly serious situation of the environmental problems and the energy crisis, there is an unprecedented development opportunity for new energy vehicle. Be different from the conventional centralized drive system, four-wheel independent-drive electric vehicle (4WIDEV) has been recognized as a break-through concept that will have a major impact on future electric and hybrid vehicle design, as its powertrain have the advantages of packing flexibility, all wheel drive, space-saving, fast torque response, etc. [1][2]. As advanced distributed drive system, the drive/regenerative brake torque of each wheel of 4WIDEV can be controlled separately through torque distribution among in-wheel electric motors[3][4], which serves more control flexibility with several different purposes of vehicle performance enhancement.

Torque distribution can apply an additional yaw moment to the vehicle, which affects its stability and maneuverability. Generally, optimal wheel torque distribution to generate yaw moment for vehicle stability was always discussed in bunch of literatures in the past two decades[5]-[10]. The additional yaw moment was generally calculated through yaw rate controller based on the 2 degree of freedom (DoF) linear vehicle model, using feedforward and feedback control[6][7], fuzzy control[3], linear quadratic optimal control[8], sliding model control[9] and finite-time control[10], etc. Part of literature found a interesting topic which discussed a method to assist steering by using torque distribution between two-side steerable driving wheels[1][11][12]. But as for the torque distribution for energy efficiency, the topic just comes out in recently years[13]-[23].

Torque distribution also allows the motor to operate in an efficient zone, which improves the economy of the vehicle. Torque distribution control strategies with hierarchical structure were proposed in several literatures to improve the economy of the vehicle[13][14][15]. The total yaw moment control was firstly guaranteed in the upper layer, then the torque distribution between the motors or between the motor and the internal combustion engine was determined at the lower layer, and both in [14] and [15], the multi-objective optimization for torque distribution based on offline optimization and two-hierarchy distribution structure was proposed. In [16], the optimal torque distribution was formulated as the solution of a parametric optimization problem depending on the vehicle speed, and an analytical solution was proposed under the hypothesis that the drivetrain power losses were strictly monotonically increasing with the torque demand. In [17], a computationally efficient energy management system based on offline optimization was discussed for a 4WIDEV with two-speed transmissions, and a “torque-fill” controller was developed to compensate for the torque gap during gearshifts. In [18], the composition and structure of a prototype for 4WIDEV were introduced and a front- and rear-axle torque distribution method was proposed to reduce the energy consumption of vehicle. The control logic of the in-wheel motor

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was obtained based on the least squares method, and the algorithm of optimal torque allocation could maintain the working point of the driving motor in high efficiency area. In [19], with a newly developed offline optimization procedure, the performance of alternative objective functions for the optimal wheel torque distribution of a four-wheel-drive fully electric vehicle was assessed, including functions based on the minimum tire slip criterion. In [20], it pointed out that further significant energy consumption reductions could be achieved through the appropriate tuning of the reference understeer characteristics, and the effects of drivetrain power losses and tire slip power losses were discussed. Besides, an easily implementable torque-vectoring controller to minimize the electric drivetrain power losses and a sub-optimal torque-vectoring controller including consideration of tire slip power losses were proposed. In [21], three different real-time energy-efficient control allocation schemes were proposed and compared for longitudinal speed tracking control of 4WIDEV, and an objective function characterized with the energy consumption and maneuverability of electric drive system was formulated. Some literatures also studied how to reduce cornering resistance by torque vectoring between the left and right wheels, thereby reducing the turning energy consumption[22][23]. In [22], an efficient direct yaw moment control that was capable of minimizing tire slip power loss for 4WIDEV was proposed. It pointed out that the change in steer characteristics during acceleration or deceleration while turning could obtain a significant power loss reduction with a direct yaw moment, and the proposed control method could ensure compatibility between vehicle dynamics performance and energy efficiency. In [23], the turning energy saving problem of the rear-wheel independent driving electric vehicle was studied, and it pointed out that based on offline optimization, torque vectoring could reduce the turning resistance, so as to further rise the energy saving potential of a rear-wheel independent driving electric vehicle.

In previous studies on energy saving through torque distribution, the energy efficiency problem is generally transformed into an optimal torque distribution problem, such as offline optimization, online optimization or multi-objective optimization. However, in these optimizations, vehicle's stability is often strictly guaranteed first through yaw moment control, which limits the torque distribution ratio between left- and right-side, and the impact on vehicle economy of side torque distribution is not considered. Besides, as the stability is strictly constrained, it cannot dynamically coordinate stability and economy, so the driving conditions are greatly limited. Therefore, in this paper, to maximize the energy saving potential of 4WIDEV by allowing a certain error in the control process of yaw rate without affecting the stability is proposed. In another word, the stability limit is transformed from a conventional constraint condition into an optimization objective. At the same time, in order to accurately and quickly achieve the vehicle's optimal performance of 4WIDEV, a multi-objective online optimization function, including the target of yaw moment control, power loss of electric drive system and tire longitudinal slip, wheel torque vibration control, etc. is proposed, and a driving energy management strategy based on this function is designed in this paper. Besides, in order to better coordinate the stability and economy, a weight coefficient fuzzy controller is designed, which can dynamically adjust the weight coefficients of these optimization objectives to adapt to the various driving conditions of 4WIDEV. It is noted that the topic of this paper is the energy-saving of 4WIDEV under steady state and the tire force is in the linear region. In order to achieve the goal of vehicle energy-saving to the greatest extent without affecting the handing stability, this paper innovatively proposes a driving force distribution control strategy based on multi-objective online optimization. When the tire force is in the nonlinear region and the vehicle is in an unstable state, the primary problem is to maintain the stability of the vehicle instead of focusing on energy-saving control strategy proposed in this paper.

The rest of the paper is organized as follows. In Section II, the modeling of 4WIDEV is introduced, and the power losses of electric drive system are analyzed. In Section III, the system architecture of driving energy management strategy is introduced. In Section IV, the proposed driving energy management strategy is verified through a co-simulation model. Finally, conclusions are presented in Section V.

II. 4WIDEV MODELING AND ITS POWER LOSSES ANALYSIS

A. Vehicle model

In this paper, a 4WID electric vehicle model is built based on CarSim and MATLAB/Simulink. The co-simulation model is shown in Fig.1. Vehicle chassis model and battery model are built in CarSim, speed-following driver model, four-wheel electric drive system model and drive controller model are built in MATLAB/Simulink. The detailed modeling process is detailed in the literature [23][24]. In Fig.1, T_{mi} is the output torque of the i th in-wheel motor, β is the sideslip angle at the center of gravity (CoG) of the vehicle, u is the longitudinal speed of the CoG, and γ is the yaw rate of the vehicle.

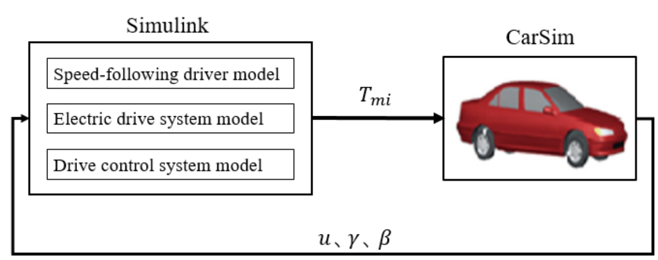


Fig.1. Co-simulation model

B. In-wheel motor model

The research object of this paper is a 4WIDEV, so the original transmission system in CarSim software is removed, and the input of torque command is given by in-wheel motor model built in MATLAB/Simulink. For reducing the computational complexity, the following two-order transfer function is used to simplify the torque response characteristics of the in-wheel motor[24]:

$$G(s)=\frac{T_m}{T_m^*}=\frac{1}{2\xi^2 s^2+2\xi s+1} \tag{1}$$

where, T_m is the actual output torque; T_m^* is the target output torque; ξ represents motor characteristic parameter.

Motor inverter efficiency also has an impact on power loss characteristics of electric drive system. Therefore, both motor efficiency and inverter efficiency are considered when the total power loss of electric drive system is calculated as follows:

$$P_{lossi}=\frac{T_{mi}n_i}{9549\eta_i}(1-\eta_i) \tag{2}$$

where, 9549 is the unit conversion coefficient; the unit P_{lossi} is kW. n_i is the motor rotational speed, the unit is r/min; T_{mi} is the motor torque, the unit is Nm; η_i is the efficiency of the i_{th} in-wheel motor.

The 4WIDEV studied in this paper has a high requirement for dynamics, so four identical motors with large rated torque are selected, and different size motors for the front and rear wheels will be considered as a potential improvement in future work. The value of η_i can be interpolated from electric drive system efficiency map shown in Fig.2.

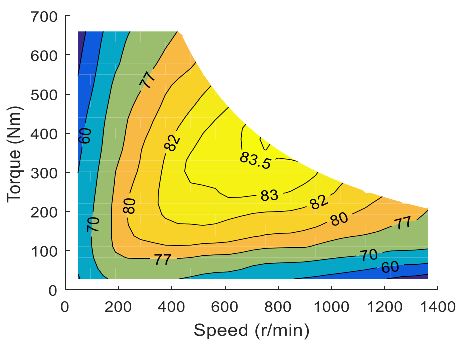


Fig.2. Electric drive system efficiency map

C. Power loss analysis of electric drive system

There are many factors that lead to power losses for 4WIDEV when driving, including electric drive system efficiency, tire rolling resistance, air resistance, tire longitudinal and lateral slip loss, etc. [25][26]. Among these factors, most of them are generally inevitable factors causing energy losses, such as air resistance and tire rolling resistance. But as for some other factors, such as electric drive system efficiency variation and tire slip, it can be found that appropriate torque vectoring can reduce their power-loss contribution. For example, the electric drive system efficiency variation, which has already been proved to be most important factor resulting in driving energy losses in many literatures[15][23][25], can be controlled by optimizing the operation points of these in-wheel motors. Specifically, the operation points optimization can only be realized by adjusting the torque distribution ratio either between the front- and rear-axle or between the right- and left-side, without changing the total driving torque requirement. Optimizing the torque distribution among four wheels in different driving situation can improve the overall efficiency of motor, thereby reducing power loss. When the vehicle is traveling straight, it can be approximated that the driving torques of left- and right-side are completely identical. Therefore, we only need to consider the axle torque distribution issue for energy saving by introducing an axle torque distribution coefficient:

$$k_a=\frac{T_2}{T_1+T_2} \tag{3}$$

where, T_1 is the wheel torque of the front-axle; T_2 is the wheel torque of the rear-axle. This paper prefers front-axle drive, so the range of k_a is [0,0.5].

With the goal of minimizing the total power loss of electric drive system, the offline optimization is performed for the situation of driving straight. The optimization function is written as follows:

$$\min P=\left(\frac{T_1}{\eta_1(n,T_1)}(1\text{-}\eta_1)+\frac{T_2}{\eta_2(n,T_2)}(1\text{-}\eta_2)\right)\times\frac{n}{9549}$$
$$\text{s.t.}\begin{cases}T_1+T_2=T_d\\ 0\le T_1\le T_{\max}(n)\\ 0\le T_2\le T_{\max}(n)\\ n\le n_{\max}\end{cases} \tag{4}$$

where, n is the even rotational speed of electric motor, these inequalities and equalities are the constraints that the optimization process has to be subjected to. T_d is the total driving torque demand, $T_{\max}(n)$ and $n_{\max}$ are the maximum torque at n speed and the maximum speed of in-wheel motor.

Fig.3 shows the optimal axle torque distribution coefficient result corresponding to the minimum total power loss obtained by offline optimization. As shown in Fig.3, single-axle driving mode is the most efficient way to drive for 4WIDEV only when the driving torque requirement is no more than 1000Nm. In most cases outside the region of low torque requirement, four-wheel driving mode almost with equal torque distribution is more preferred. In addition, the torque threshold of switching from single-axle driving mode to four-wheel driving mode comes earlier in both lower motor speed and high speed.

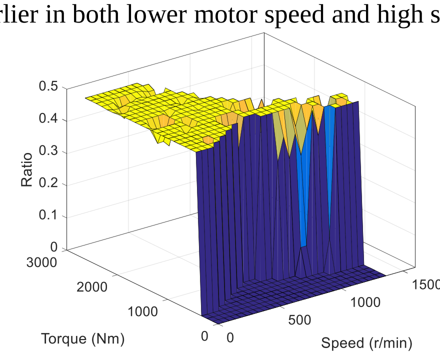


Fig.3. Axle torque distribution coefficient map

Similarly, if the vehicle stability is ignored, the operation points optimization for energy-saving also can be implemented through reasonable two-side driving torque redistribution of 4WIDEV. The side torque distribution coefficient is defined as:

$$k_s = \frac{T_l}{T_r + T_l} \tag{5}$$

where, T_r is the torque of the right-side of the vehicle; T_l is the torque of the left-side of the vehicle. The range of k_s is [0,1].

Assuming the torque distribution between the front- and rear-axle of the single side is equal, the side torque distribution coefficient result corresponding to the minimum power loss obtained by offline optimization is shown in Fig.4. As shown in Fig.4, the vehicle tends to be single-side drive when the axle torque is in low torque region and tends to equal torque distribution between the left- and right-side in high torque region, in this way, the minimum power losses can be achieved.

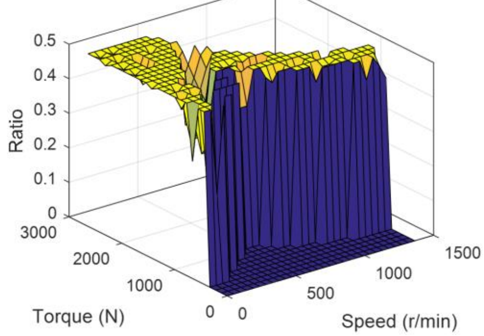


Fig.4. Side torque distribution coefficient map

However, in the actual application of torque distribution offline optimization process with consideration of stability control requirement, generally, the side torque distribution between the left- and right-side is firstly calculated based on the constraints of the total torque demand and the additional yaw moment required by the stability demand. The calculation process is shown in equation 6. After that, the two-side vehicle are decoupled. It means that the axle torque distribution between the front- and rear-axle in each side can be obtained from the offline optimization look-up table of axle torque distribution as shown in Fig.3 according to the distributed single-side total torque in the first step. Finally, the optimal torque for each wheel of 4WIDEV in cornering can be calculated based on equation 7. Under combined-slip condition, the yaw moment generated by the lateral force is relatively small, and the lateral force is usually uncontrollable [27], so this paper ignores the influence of the lateral force on the additional yaw moment.

$$\begin{cases} T_l + T_r = T_d \\ (T_l - T_r)\dfrac{B}{2R_w} = M_d \end{cases} \tag{6}$$

$$\begin{cases} T_{1r} = T_r(1-k_{ar}) \\ T_{2r} = T_r k_{ar} \\ T_{1l} = T_l(1-k_{al}) \\ T_{2l} = T_l k_{al} \end{cases} \tag{7}$$

where M_d is the additional yaw moment demand, B is the track with of the vehicle, R_w is the scroll radius for wheel, k_{ar} and k_{al} are the axle torque distribution ratios of right-side and left-side of vehicle body, respectively, T_{1r}, T_{2r}, T_{1l} and T_{2l} are the torque of right front wheel, right rear wheel, left front wheel and left rear wheel, respectively.

III. MULTI-OBJECTIVE ONLINE OPTIMIZATION DESIGNS

As mentioned above, both axle torque distribution and side torque distribution have an impact on the economy of the 4WIDEV. However, side torque distribution also has a great influence on the lateral stability due to generated additional yaw moment by the uneven torque distribution between the two sides. Besides that, the tire longitudinal slip and side slip have apparent influence on the driving energy dissipation especially in the situation of large lateral acceleration. It also should be taken into account in the optimization formulation. Moreover, frequent transfer in driving mode from single-axle drive to four-wheel drive will result in possible discontinuous torque output flow or even unstable torque ripple and vibration. Therefore, in this paper, all the possible energy losses affecting factors as well as the relatively elastic yaw tracking performance for the stability and smooth torque output as much as possible are all included in the proposed driving energy management strategy based on multi-objective online optimization, aiming to maximize the driving energy efficiency of 4WIDEV without affecting its stability at the same time.

A. Framework of multi-objective online optimization control

Fig.5 shows the overall control structure of multi-objective online optimization, which is composed of two layers, Layer 1 and Layer 2.

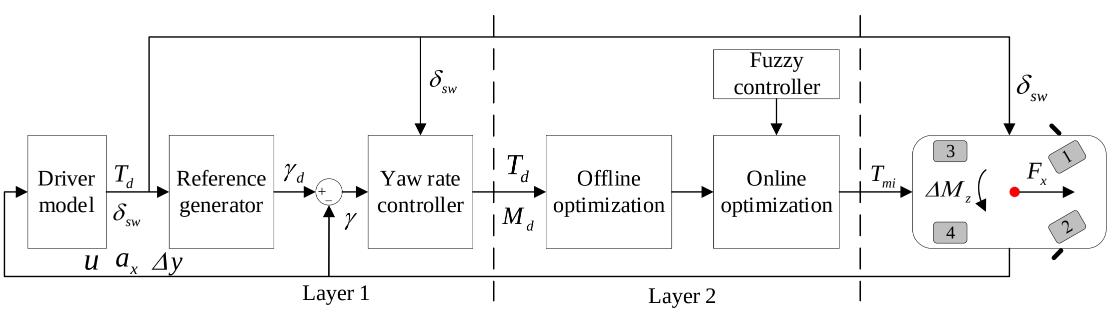


Fig.5. Overall framework of multi-objective online optimization driving energy management strategy

Layer 1 is yaw rate control layer. Yaw rate control is the core of research on handling stability. In [28], integration of vehicle yaw stabilisation and rollover prevention through nonlinear hierarchical control allocation, the relationship between longitudinal force and lateral force was described by friction ellipse theory. Reference [29] presented the design of a control strategy to coordinate active front steering (AFS) and differential braking to improve vehicle yaw stability and cornering control. The feasibility of the control strategy was verified by the test vehicle. In [30], a model predictive stability controller was designed using a combined-slip LuGre tyre model, and the proposed combined-slip controller takes advantage of the more accurate tyre model and can adjust tyre slip ratios based on lateral forces of the front axle.

In this paper, the yaw rate control process is described as follows. First, the driver model calculates the total torque demand Td and the steering wheel angle target δsw according to the real-time vehicle speed u, lateral acceleration ax and trajectory tracking error Δy. Then, the reference yaw rate generator calculates the ideal yaw rate required for the vehicle stability control according to the ideal linear 2DoF vehicle model. This method is commonly used in vehicle stability control, which can be found in many literatures[24][31]. The determined target yaw rate according to this method can be calculated as follows.

$$\gamma_d = \min\left\{\left|\frac{u/L}{1+K_s u^2}\frac{\delta_{sw}}{i_s}\right|,\left|\xi_\gamma \frac{\mu g}{u}\right|\right\}\cdot \operatorname{sgn}\left(\delta_{sw}\right) \tag{8}$$

where γ_d is the ideal yaw rate of the vehicle, u is the vehicle speed, L is the wheel base, K_s is the stability factor, δ_{sw} is the steering wheel angle, i_s is the steering system transmission ratio, ξ_r is a constant correction factor for the influence of longitudinal force on lateral force, it is set as 0.8, μ is the adhesion coefficient, g is gravitational acceleration. Next, the additional yaw moments demand M_d required by the yaw stability can be obtained by the yaw rate controller according to the error between ideal yaw rate and the actual yaw rate.

Layer 2 is multi-objective online optimization layer, which is composed of an offline optimization step and a further real-time online local optimization plus a fuzzy controller.

To reduce computing load of online optimization, the offline optimization discussed in section 2.C is implemented at first. Before that, the two-side torque demand of vehicle body based on the additional yaw moment demand M_d determined by yaw stability control is decoupled according to equation 6. Then the two-side axle torque distribution ratios are obtained from the look-up table as shown in Fig.3 calculated by the offline optimization. As a result, the four-wheel drive torques can be calculated according to equation 7.

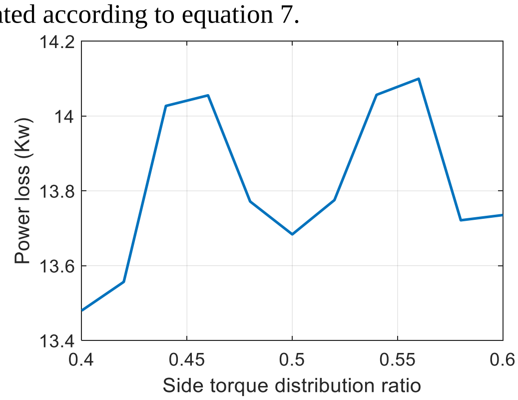


Fig.6. Electric drive system power dissipation with different side torque distribution ratio

Though each wheel torque has been obtained, this result may not be optimal due to the yaw rate tracking requirement determined according to the 2DoF linear vehicle model is over strict. In another word, the succeeding offline optimization makes a compromise to the preceding yaw stability control. However, in most cases, a slight deviation of the ideal yaw rate calculated from the 2DoF linear vehicle model will not affect the driving stability of the vehicle at all, especially on the good adhesion road or at low speed. Because at that time, the actual yaw rate or the target yaw rate of the uncontrolled vehicle is far less than the corresponding yaw rate of the neutral steering. The conclusion can be illustrated by Fig.6 and Fig. 7 as an example, which show the electric drive power dissipation and the yaw rate with different axle torque distribution ratio respectively at 70km/h vehicle speed.

It can be seen that different side torque distribution ratio between the two-side of vehicle all have obvious influence on power dissipation and yaw rate tracking error. To have good yaw stability performance, the side torque distribution ratio 0.55 is the best. But at this time, the power dissipation is the worst. The power dissipation will become almost the best when the side torque distribution ratio is 0.5. The offline optimization is hard to obtain the optimum value for the sake of compromising to rigid ideal yaw rate tracking requirement as shown in Fig. 7 which is determined by equation 8. In fact, if the ideal yaw rate tracking error is enlarged from zero to ±5%, then in this simulation case, the desirable range of side torque distribution ratio is expanded to 0.48-0.58 as shown in Fig. 7. As a result, the optimum side torque distribution ratio 0.5 which is respect to the local minimum power dissipation can be achieved, and so further 0.38kW (2.7%) power can be saved.

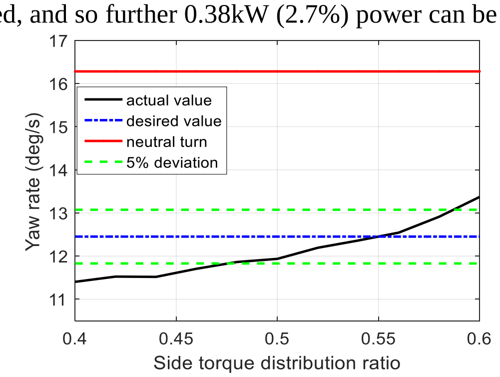


Fig.7. Yaw rate tracking result with different side torque distribution ratio

The torque of each wheel obtained by offline optimization is now used as the starting point of multi-objective online optimization function rather than as the final output directly. The reason to do so is that the speed of online optimization can be greatly improved. Then, in the following, the multi-objective weighted online optimization which integrates four parameterized control objectives is implemented. While doing the online optimization, a fuzzy controller firstly determines the weight coefficients satisfying the stability and economic requirements according to the vehicle driving state feedback, and then the final each wheel torque demand with respect to the optimum energy-saving performance of 4WIDEV without losing stability can be achieved by solving the multi-objective online optimization function. The core blocks in this overall framework of the proposed multi-objective online optimization driving energy management strategy as shown in Fig.5 will be introduced as following.

B. Desired yaw moment calculation

Yaw rate control in Fig. 5 can be achieved by applying an additional yaw moment. However, vehicle is a nonlinear system with large delay, so it is difficult to achieve precise and fast yaw rate tracking control by traditional PID feedback control method. Meanwhile, the subsequent online optimization process also needs the accurate desired yaw moment calculated by the yaw rate controller to minimize the tracking error of desired yaw moment which will be discussed later in Section 3.C Therefore, a feedforward plus feedback control is used in the design of the yaw rate controller [6][32]. Because the feedforward control does not need to rely on yaw rate error, a relatively accurate additional yaw moment can be directly predicted only according to the current driver operation and vehicle running state. So, the feedforward control has a fast response avoiding the delay and lag of feedback control. However, because the vehicle is a complex nonlinear system, and the vehicle model is simplified in the process of controlled system modelling, the additional yaw moment determined only by the feedforward control is just near the accurate value, which inevitably results in some errors. To correct the errors, the feedback control also has to be introduced at the same time to maintain the system stable and enhance the robustness. The feedforward controller is designed based on the linear 2DOF vehicle model as shown in Fig.8. Its dynamic equation at steady state can be expressed as follows.

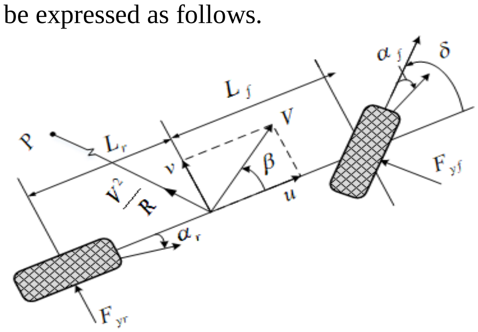


Fig.8. 2-DOF vehicle model

$$\begin{aligned}\dot{\beta} &= -\frac{C_f + C_r}{mu}\beta - \frac{L_f C_f - L_r C_r + mu^2}{mu^2}\gamma + \frac{C_f}{mu}\delta_f = 0 \\ \dot{\gamma} &= \frac{L_r C_r - L_f C_f}{I_z}\beta - \frac{L_f{}^2 C_f + L_r{}^2 C_r}{I_z u}\gamma + \frac{L_f C_f}{I_z}\delta_f = 0\end{aligned} \tag{9}$$

where, β the side slip angle of center of gravity (CoG) of the vehicle; C_f and C_r are the tire cornering stiffness of front- and rear-axle, respectively; m is the mass of the vehicle; L_f and L_r are the lengths from the CoG to the front- and rear- axle, respectively; δ_f is the steering angle of front wheels; I_z is the rotational inertia of the vehicle around the vertical axis.

The control framework of the yaw rate control system is shown in Fig.9, which has two control parts: feedforward and feedback. By solving the characteristic equation of the closed-loop transfer function of the system, it is easy to get the roots of the characteristic equations are on the left side of the complex plane. Therefore, the yaw rate control system is an internally stable system.

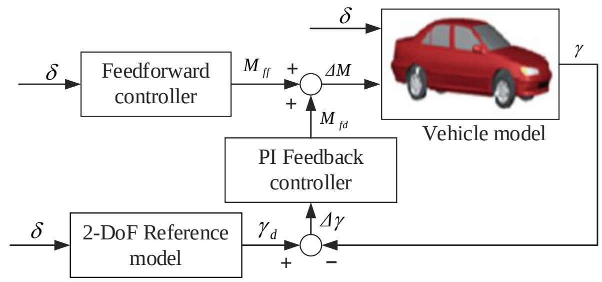


Fig.9. Framework of yaw acceleration control system

By applying the additional yaw moment M_{ff} to the vehicle body through the feedforward controller, the body state equation becomes as follows:

$$\begin{cases}\dot{\beta} = a_{11}\beta + a_{12}\gamma + b_1\delta \\ \dot{\gamma} = a_{21}\beta + a_{22}\gamma + b_2\delta + \dfrac{M_{ff}}{I_z}\end{cases} \tag{10}$$

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{C_f}{mu} \\ -\dfrac{L_f C_f}{I_z} \end{bmatrix} \tag{11}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{C_f + C_r}{mu} & \dfrac{L_f C_f - L_r C_r}{mu^2} - 1 \\ \dfrac{L_f C_f - L_r C_r}{I_z} & \dfrac{L_f{}^2 C_f + L_r{}^2 C_r}{I_z u} \end{bmatrix} \tag{12}$$

The above formula is transformed by Laplace, and its transfer function is as follows:

$$\begin{cases}\beta(s) = \dfrac{I_z(b_1 s - b_1 a_{22} + b_2 a_{12})\delta_f(s) + a_{12}M(s)}{I_z((s-a_{11})(s-a_{22}) - a_{21}a_{12})} \\ \omega_r(s) = \dfrac{I_z(b_2 s + b_1 a_{21} - b_2 a_{11})\delta_f(s) + (s-a_{11})M(s)}{I_z((s-a_{11})(s-a_{22}) - a_{12}a_{21})}\end{cases} \tag{13}$$

The purpose of the feedforward controller is to make the sideslip angle of center of gravity of the vehicle β equal to zero, so the yaw moment determined by the feedforward controller can be expressed as:

$$M_{ff} = \frac{\left(L_f + L_r\right)L_r C_f C_r - L_f C_f mu^2}{L_f C_f - L_r C_r + mu^2}\delta \tag{14}$$

The feedback control uses PI controller, which is stability. The additional yaw moment calculated by the feedback controller can be expressed as:

$$M_{fb} = g_{fb}(\gamma_d - \gamma) \tag{15}$$

where, g_{fb} is the feedback coefficient.

Therefore, through the analysis of the transfer function of the feedforward and feedback controllers, it can be seen that the designed yaw rate controller is stable. The total desired additional yaw moment determined by the feedforward plus feedback control can be expressed as:

$$M_d = M_{ff} + M_{fb} \tag{16}$$

C. Multi-objective online optimization formulation

The multi-objective weighted online optimization proposed in this paper synthesizes four parameterized control objectives, electric drive system efficiency variation, tire slip losses, wheel torque ripple while distribution and desired yaw moment control error. It can be treated as a secondary optimization of the driving wheel torque obtained by offline optimization according to the real-time state parameter feedback of the vehicle. Different from the solution of priority guarantee of stability in the torque distribution method based on the offline optimization, the yaw stability demand is only a part of the multi-objective weighting function when doing the multi-objective on-line optimization. By allowing a certain deviation in the yaw rate tracking control process, further energy-saving optimization space in the torque coordination distribution process can be enlarged. The multi-objective online optimization function is formulated as follows.

$$\begin{aligned}\min_{T_{mi}} J &= J_1 + J_2 + J_3 + J_4 \\ &= W_1 \sum_1^4 C_r(T_{mi}) + \sum_1^4 C_p(T_{mi}) \\ &\quad + W_2 \sum_1^4 C_t(T_{mi}) + \sum_1^4 C_v(T_{mi})\end{aligned} \tag{17}$$

where $J_1 = W_1 \sum_1^4 C_r(T_{mi})$ represents the weighted optimal objective of desired yaw moment control error, C_r is its cost function and W_1 is its weight coefficient; $J_2 = \sum_1^4 C_p(T_{mi})$ represents the optimal objective of electric drive system power losses, C_p is its cost function; $J_3 = W_2 \sum_1^4 C_t(T_{mi})$ represents the weighted optimal objective of tire longitudinal slip losses due to excessive driving torque, C_t is its cost function and W_2 is its weight coefficient; $J_4 = \sum_1^4 C_v(T_{mi})$ represents optimal objective of wheel torque ripple. T_{mi} represent each wheel driving torque. Special to note is that the weight of J_1 and J_3 take J_2 as reference in this paper, and J_4 is only used to limit the torque fluctuation of the motor in each control cycle, but not has too much influence on the result of online optimization. So, the weight coefficient of J_2 and J_4 are always set 1 in this paper.

As equation 13 shown, all the four optimal objectives can be expressed as the cost function of four wheel torques. The multi-objective optimization function is only related the online optimization of torque and motor drive efficiency, does not change the transfer function of the original system, and has no impact on the stability of the original system. The solution of this multi-objective online optimization function is just the optimum group of four wheel torques which integrates each performance requirements comprehensively. The specific cost functions are introduced one by one.

1) Yaw moment tracking performance cost function

If the desired yaw moment demand can be tracked well, the ideal yaw rate tracking performance of 4WIDEV can be indirectly controlled [9]. While turning, the actuators of yaw moment tracking control which generate an additional yaw moment are two-side driving torque of vehicle. The generated actual additional yaw moment can be expressed as:

$$M_r = \sum_{i=1}^{4} \frac{T_{mi}}{R_w} L_i \tag{18}$$

where, R_w is the tire rolling radius; L_i is the arm that rotates around the Z axis of the vehicle center of mass, which can be calculated as follows.

$$\begin{cases} L_1 = L_f \sin\delta_{fi} - B/2 \cos\delta_{fi} \\ L_2 = L_f \sin\delta_{fo} + B/2 \cos\delta_{fo} \\ L_3 = -B/2 \\ L_4 = B/2 \end{cases} \tag{19}$$

where L_i represents arm of inner front steerable wheel, outer front steerable wheel, inner rear wheel and outer rear wheel.

Therefore, the cost function of yaw moment tracking performance can be expressed as:

$$J_1 = W_1 (\sum_{i=1}^{4} \frac{T_{mi}}{R_w} L_i - M_d)^2 \tag{20}$$

where, W_1 is the weight coefficient of yaw moment tracking performance cost function. The influence of the wheel inertia force on the energy consumption of the whole vehicle is ignored in this paper. Because usually the acceleration of the vehicle during turning is very small or the acceleration time is very short. As a transient variable, the wheel inertial force is very small compared to the driving force. Even if the energy-efficient torque distribution does not have any corresponding response to the wheel inertia force due to short-term acceleration/deceleration, its resulting energy dissipation is still quite small when discussing energy-saving issue during a whole driving cycle.

2) Power loss of electric drive system

The second part of multi-objective online optimization function is the power loss cost function of electric drive system. It can be expressed as:

$$J_2 = \frac{n_i T_{mi} \left(1 - \eta(n_i, T_{mi})\right)}{9549 \eta(n_i, T_{mi})} \tag{21}$$

If the electric motor efficiency map is obtained like Fig.2, the power loss 3D map data at each operation point of in-wheel motor can be easily calculated according to equation 17.

3) Power loss of tire longitudinal slip

The third part of multi-objective online optimization function is the power loss cost function of tire longitudinal slip. Optimizing the slip power loss of each wheel can not only benefit the energy saving of 4WIDEV, but also benefit the stability improvement of 4WIDEV [33][34].

The energy consumption of tire slip can be expressed as based on the wheel rotational dynamics equation and slip ratio calculation equation.

$$J_3 = W_2 \sum_{i=1}^{4} F_{xi} v_{xi} = W_2 \sum_{i=1}^{4} \frac{n_i T_{mi}}{9549} \lambda_i (1-\lambda_i) \tag{22}$$

where, $W_2$is the weight coefficient of tire longitudinal slip power loss; F_{xi} , v_{xi} λ_i are the traction force , tire slip velocity, slip ratio of the i th wheel, respectively.

4) Wheel torque vibration control

For 4WIDEV, the output torque of each wheel is independently controllable, so there may be multiple optimal solutions when solving the multi-objective online optimization function, and these optimal solutions may be very scattered. Fig.10 shows an example of the relationship between the total driving power loss and the axle torque distribution coefficient.

It can be seen that the relationship curve looks like a pair of wings, which the power loss reaches the lower value both at ends regions and in the middle region of axle torque distribution scope, but become higher value in the transitional regions among them. Therefore, during the online optimization, the axle torque distribution coefficient may fluctuate sharply from 0.5 at previous step k-1 to 0 or 1 at next step k, which affects both the stability of 4WIDEV and the motor's life.

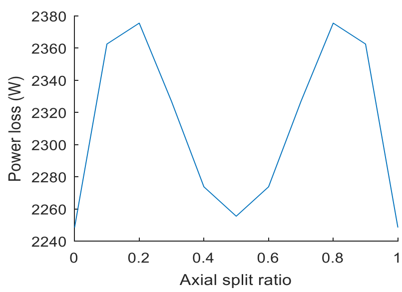


Fig.10. Influence of axle torque distribution coefficient on driving power loss

Therefore, it is necessary to add the fourth objective function in the multi-objective online optimization function, which will only limit the torque fluctuation of the motor in each control cycle, but not have too much influence on the result of online optimization. It can be expressed as:

$$J_4 = [T_{mi}(k) - T_{mi}(k-1)]^2 \tag{23}$$

5) Constrains

In the optimizing calculation process, the total driving torque demand should be guaranteed first. It can be expressed as:

$$\sum_{i=1}^{4} T_{mi} = T_d \tag{24}$$

Second, the driving torque of each motor should be within the output torque range of in-wheel motor as equation 21 shown. In this paper, only the driving condition is considered. In order to improve the efficiency of the electric drive system, the torque of each motor should be no less than 0. It can be expressed as:

$$0 \le T_{mi} \le T_{m\max}(n_i) \tag{25}$$

D. Weight coefficient fuzzy controller

In the online optimization process, whether the vehicle tends to be more stable or economical depends on the weight coefficient in the proposed multi-objective weighted optimization function. Obviously, the 4WIDEV has different performance needs in different driving states. For example, at medium and low speed, the risk of vehicle instability is relatively small, so more consideration should be given to improve vehicle economy during online optimization. While at high speed, the vehicle is easy to lose stability, so more consideration should be given to vehicle stability. Since the fixed weight coefficient obviously cannot meet the needs of different driving states of the vehicle. A weight coefficient fuzzy controller as Fig.5 shown is designed. This fuzzy controller can dynamically adjust the weight coefficient of yaw moment tracking cost function and the weight coefficient of tire slip power losses cost function according to the real-time state parameter feedback, thus to maximize the vehicle's energy saving potential without affecting stability [35].

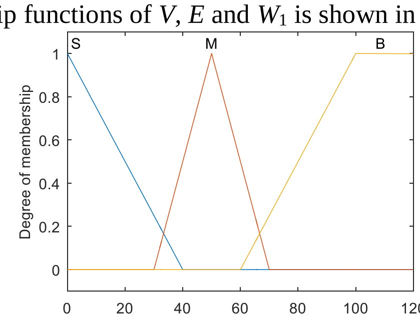
1) Weight coefficient of yaw moment tracking performance cost function

In the weight coefficient fuzzy controller, the weight coefficient of yaw moment tracking performance cost function is derived from two variables: vehicle speed (V) and yaw rate error rate (E), which is defined as below:

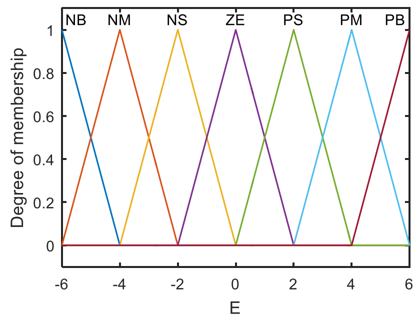
$$E = \frac{\gamma_d - \gamma}{\gamma_d} \tag{26}$$

Allowing a certain yaw rate following error can improve the economy of 4WIDEV, but in order to keep the vehicle's stability, the yaw rate following error should be controlled within a certain range. For better setting the weight coefficient of yaw moment tracking cost function, it is necessary to consider both the stability and economy of 4WIDEV. When the risk of instability is small, an appropriate yaw rate following error is allowed. Conversely, vehicle's stability is prioritized.

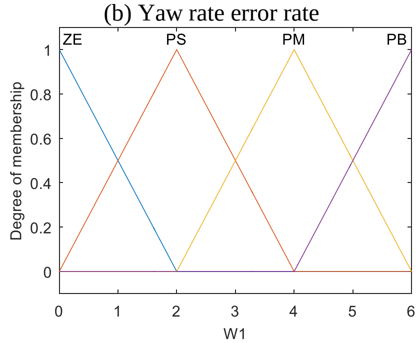
For a fuzzy research object, how to determine the appropriate membership function is the premise and key of fuzzy controller design [36]. The fuzzy controller in this paper is to determine the weight coefficient of yaw rate control and slip ratio control, thereby realizing the dynamic coordination of vehicle stability and economy. The requirements for control accuracy aren't very strict, and it is not sensitive to the mutation of membership function, but it must ensure a faster controller operating speed. Therefore, this paper selects the triangle membership function which is the most widely used, simple fuzzy inference, fast calculation speed and can meet the accuracy requirements. The membership functions of V, E and W_1 is shown in Fig.11.



(a) Vehicle speed



(b) Yaw rate error rate



(c) Weight coefficient of yaw moment tracking cost function

Fig.11. Membership functions of V, E and W_1

The final rule for the weight coefficient of yaw moment tracking cost function is as Table I.

TABLE. I

Rule for the weight coefficient of yaw rate control

W_1		E						
		NB	NM	NS	ZE	PS	PM	PB
V	S	PB	ZE	ZE	ZE	ZE	ZE	PB
	M	PM	PS	ZE	ZE	ZE	PS	PM
	B	PB	PM	PS	PS	PS	PM	PB

The fuzzy control surface of W_1 is shown in Fig.12.

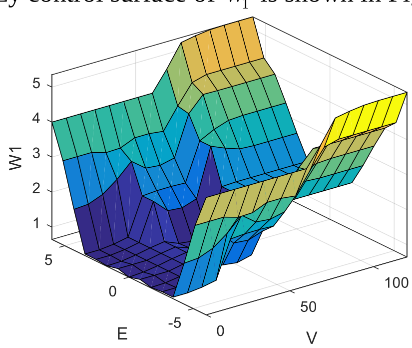
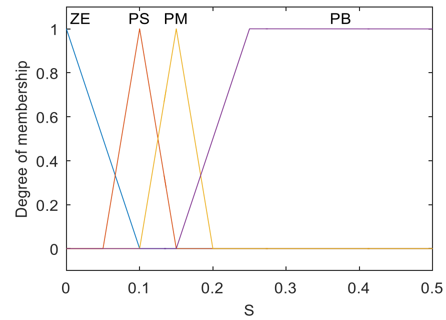


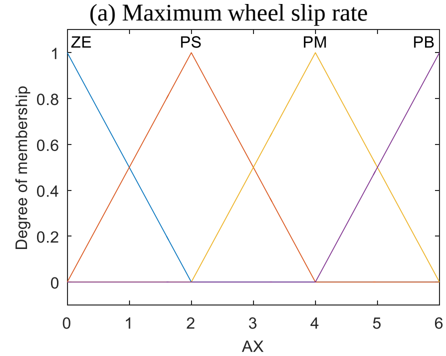
Fig.12. Fuzzy control surface of W_1

2) *Weight coefficient of tire longitudinal slip power loss cost function*

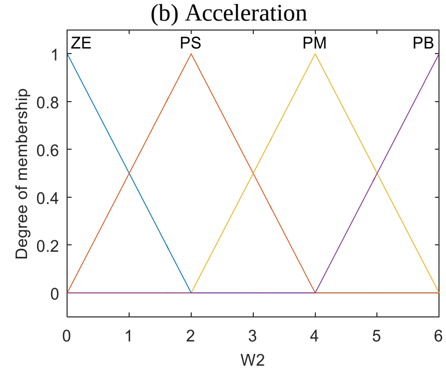
The weight coefficient of tire longitudinal slip power loss is related with two variables: maximum wheel slip rate (S) and vehicle acceleration (AX). If the wheel slip rate is small and the vehicle longitudinal acceleration is not so high, it generally means the vehicle does not have the tire-slip risk, now the weight coefficient of tire longitudinal slip power loss is minimum. If the wheel slip rate is large, the weight coefficient of tire longitudinal slip power loss should be appropriately increased to reduce the instability risk caused by wheel slip. The membership functions of S, AX and W_2 is shown in Fig.13.



(a) Maximum wheel slip rate



(b) Acceleration



(c) Weight coefficient of tire longitudinal slip power loss

Fig.13. Membership functions of S, AX and W_2

The rule for the weight coefficient of tire longitudinal slip power loss is as Table II.

TABLE. II

Rule for the weight coefficient of tire longitudinal slip power loss

W_2		AX			
		ZE	PS	PM	PB
S	ZE	ZE	ZE	PS	PS
	PS	ZE	ZE	PS	PM
	PM	PS	PM	PM	PB
	PB	PB	PM	PB	PB

The fuzzy control surface of W_2 is shown in Fig.14.

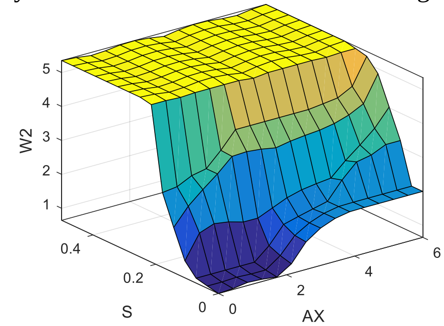


Fig.14. Fuzzy control surface of W_2

E. Multi-objective optimization function solution

After determining the weight coefficients of each weight index, the wheel torques satisfying the comprehensive optimal vehicle performance can be obtained by solving the multi-objective optimization function. The multi-objective optimization function is a nonlinear problem with constraints. The main methods for such problem include interior point method, sequential quadratic programming method, penalty function method, particle swarm optimization algorithm (PSO) and genetic algorithm etc. PSO has the ability of global optimization and the characteristic of easy implementation and quick convergence, since it has few setting parameters. Therefore, this paper chooses the PSO to solve the function. The flow chart of the PSO is shown in Fig.15. Where 'pBest' refers to the current individual optimal value and 'gBest' refers to the current global optimal value.

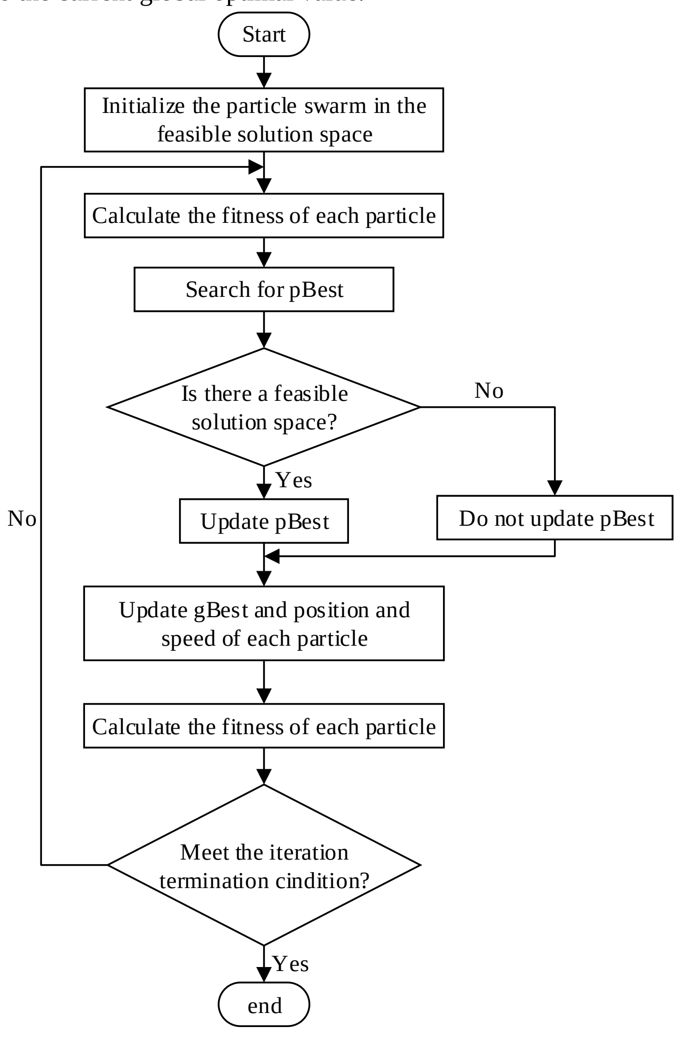


Fig.15. Flow chart of the PSO

In online optimization, the output torque of each motor should be within the external characteristic curve of the motor and not less than zero, which has two advantages. Firstly, the particle swarm is initialized only in the feasible solution space to ensure that all the particles obtained by the initialization are located in the feasible solution space. Secondly, only the particles in the feasible solution space are selected as pBest and gBest, so that even if some particles are out of the feasible solution space in the optimization process, the particle swarm can still return to the feasible solution space. The above two advantages can obviously improve the optimization speed of constrained optimization problems.

IV. Simulation Verification

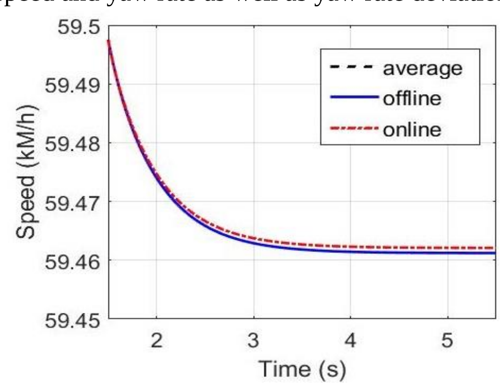
In order to verify the effect of the proposed driving energy management strategy, a co-simulation model based on CarSim and MATLAB/Simulink is built, then the turning condition with constant speed or acceleration is selected to compare the vehicle power losses in the case of proposed multi-objective optimization torque distribution strategy with other two conventional strategies, average torque distribution and offline optimization torque distribution strategies. In the average torque distribution, the drive torque is evenly distributed among four wheels. And as for the offline optimization torque distribution, the lateral force control of 4WIDEV should be firstly satisfied, which means the vehicle strictly follows the target yaw rate determined by linear 2DoF vehicle model. The basic parameters of the vehicle are shown in Table III.

TABLE. III
Parameters of the vehicle

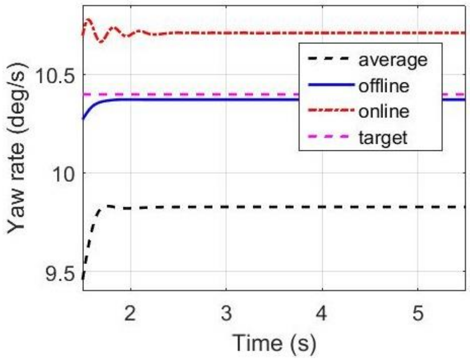
Parameter	Value
Vehicle mass(m/kg)	1274
Wheelbase(L/m)	2.578
Distance from center of mass to front-axle(L_f/m)	1.016
Distance from center of mass to rear-axle(L_r/m)	1.562
Scroll radius for wheel(r/m)	0.293
Yaw moment of inertia(r/kg.m^2)	1523
The track width(B/m)	1539
Center of mass from ground height(h/m)	0.540

A. Constant speed turning condition

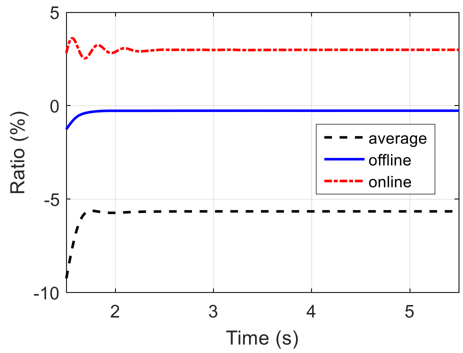
When turning at a constant speed, the road adhesion coefficient is set to 0.8, the initial vehicle speed is set to 60 km/h, and a 45 degrees step of steering wheel angel is input to the vehicle at the 0.5 second. The simulation results of these three cases are shown from Fig.16 to Fig.19. Fig.16 shows vehicle speed and yaw rate as well as yaw rate deviation ratio.



(a) Vehicle speed



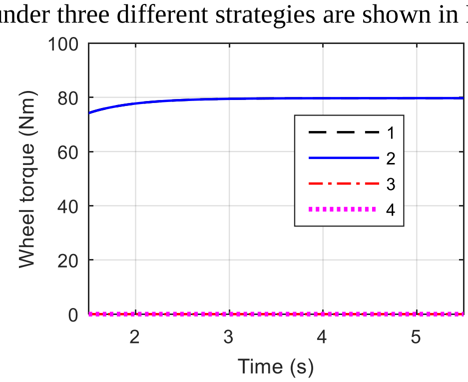
(b) Yaw rate



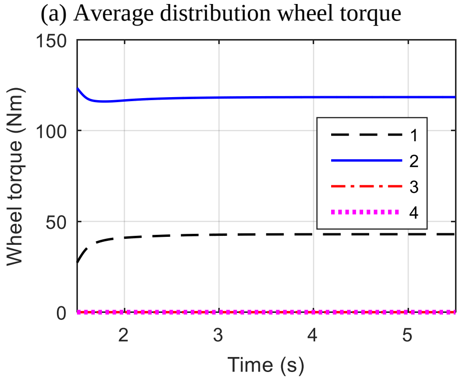
(c) Yaw rate deviation ratio

Fig.16. Vehicle speed and yaw rate

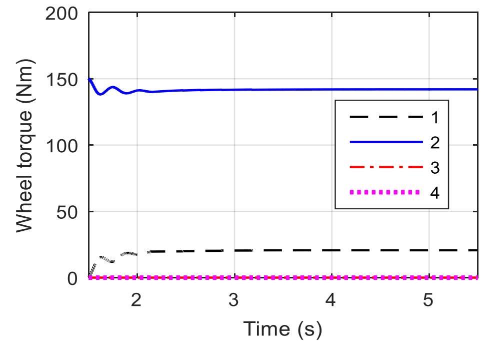
In Fig.16, it can be seen that all the three strategies have similar speed time history, and only the offline optimization torque distribution method can well follow the target yaw rate exactly. The online optimization torque distribution method has a certain yaw rate following error, but it can be effectively controlled within 5%, and all three strategies can ensure the vehicle stability. The wheel torque and required additional yaw moment under three different strategies are shown in Fig.17.



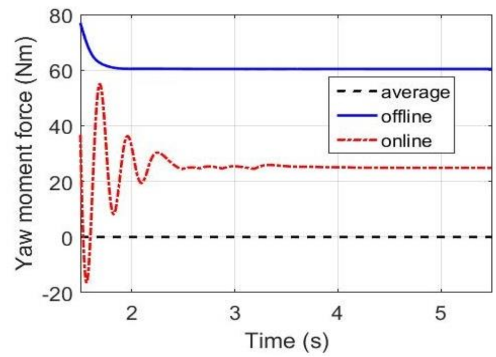
(a) Average distribution wheel torque



(b) Offline optimization distribution wheel torque



(c) Online optimization distribution wheel torque

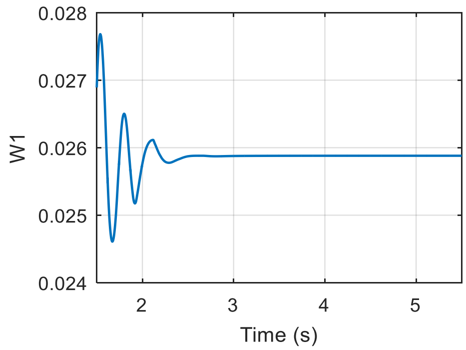


(d) Required additional yaw moment

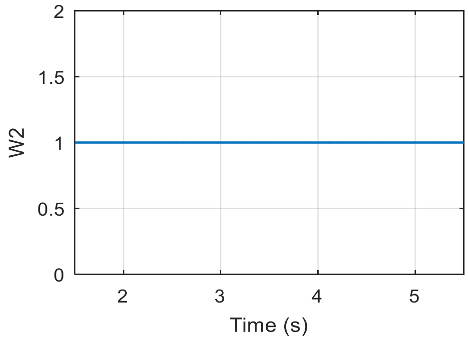
Fig.17. Wheel torque and required additional yaw moment

Considering the total demand torque of the vehicle is small under this constant-speed driving condition, the optimal distribution result of both the offline optimization strategy and the online optimization strategy is the front-axle drive. In order to better compare the energy-saving effect of side torque distribution, the vehicle is also set to front-axle drive in average distribution strategy. It can be seen from Fig.17 that the online optimization torque distribution method transfers more torque to the outer wheel than offline optimization. Because a certain yaw rate following error is allowed, the required additional yaw moment calculated by feedforward controller is corrected by feedback controller. Consequently, the required additional yaw moment of online optimization is lower than that of offline optimization. As the actual yaw rate of 4WIDEV is greater than the target yaw rate in the case of online optimization, the reduced required additional yaw moment is more representative of the optimization direction. At the same time, the final yaw rate following error is controlled by the weight coefficient of yaw moment tracking control.

The weight coefficient of yaw moment tracking cost function and weight coefficient of tire slip power loss cost function, which are determined by the weight coefficient fuzzy controller, are shown in Fig.18. The weight coefficient of yaw moment tracking control stabilize into 0.026 after a short-time oscillation, and the weight coefficient of tire slip power loss cost function remains as 1. It also reflects that the vehicle has less instability risk in this driving condition.



(a) Weight coefficient of yaw moment tracking cost function



(b) Weight coefficient of tire slip power loss cost function

Fig.18. Weight coefficient

The effect of energy saving control is shown in Fig.19. Compared to the average distribution, the proposed online optimization reduces the driving power loss by 13%; compared to the offline distribution, online optimization reduces the driving power loss by 8.8%. In a word, the online optimization better balances the stability and economy of 4WIDEV.

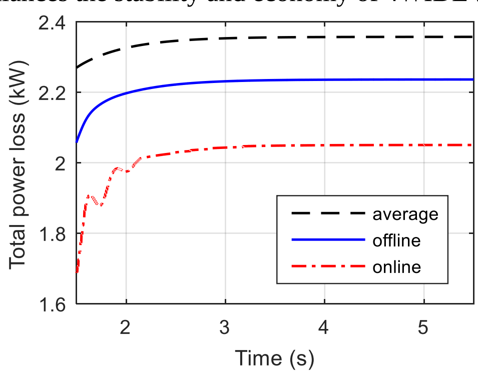
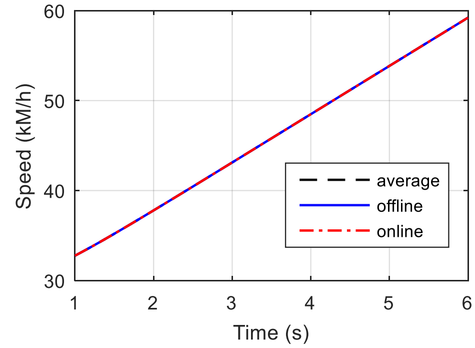


Fig.19. Total power loss

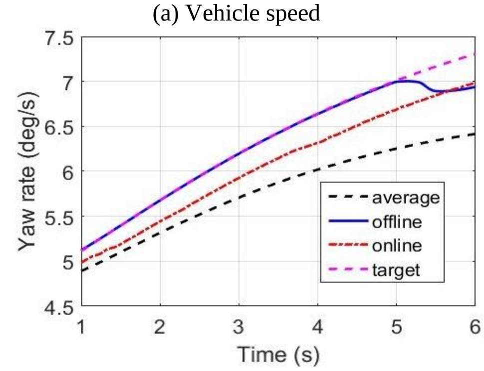
B. Accelerated turning condition

When accelerating the vehicle during turning, the road adhesion coefficient is set to 0.8, the initial vehicle speed is set to 30 km/h, the vehicle accelerates at 1.5 m/s^2, and 30 degrees step of steering wheel angel is input to the vehicle at the 0.5 second. The simulation results of these three cases are shown from Fig.20 to Fig.23. The vehicle speed and yaw rate are shown in Fig.20.

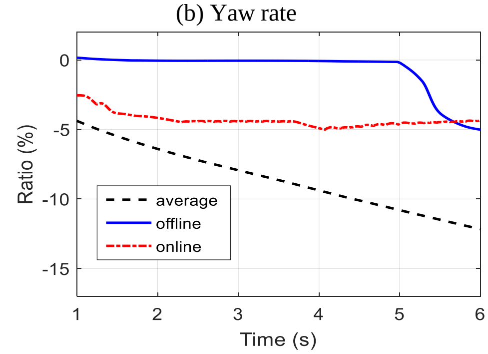
In Fig.20, it can be seen that all three strategies can well follow the target speed. Since the yaw rate control is not considered in the average distribution, its yaw rate following error is the largest, and the error increases continuously with the increase of vehicle speed, up to 12%, so the vehicle has a risk of instability. In the offline optimization, the target yaw rate is relatively well followed, except for the time history range after the 5th second, but still controlled within 5%. In the online optimization, although the target yaw rate is not accurately followed, the yaw rate following error can be always kept within the acceptable value, 5%.



(a) Vehicle speed



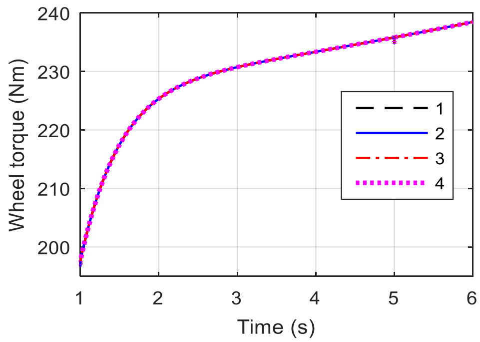
(b) Yaw rate



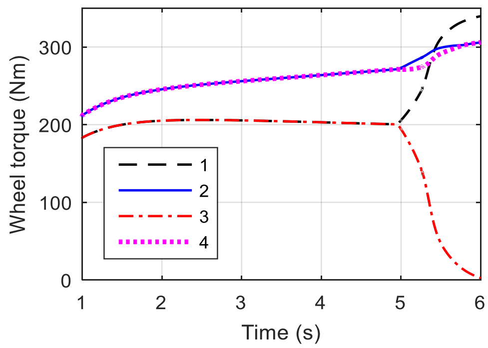
(c) Yaw rate departure ratio

Fig.20. Vehicle speed and yaw rate

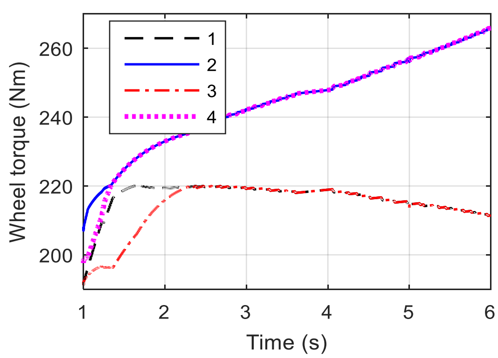
The wheel torque and required additional yaw moment under three different strategies are shown in Fig.21. It can be seen that all the wheel torques are the same in the case of average distribution, and both offline optimization and online optimization transfer torque to the outer wheels. The torque fluctuations are relatively small in the case of online optimization, while after the 5th second of the offline optimization, the wheel torque of the left-side suddenly changes from average distribution to front wheel drive, which also leads to a sudden increase of the yaw rate following error. And it is mainly because that the offline optimization does not consider the changes of the parameters, such as the wheel angle. The required additional yaw moment of the online optimization is greater than that of the offline optimization, and it is because that the additional yaw moment requirement is not fully satisfied in the case of the multi-objective online optimization, and the actual yaw rate is less than the target yaw rate, so the yaw rate controller increases the additional yaw moment to effectively follow the target yaw rate.



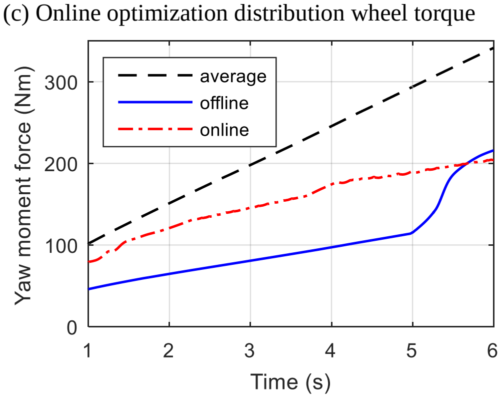
(a) Average distribution wheel torque



(b) Offline optimization distribution wheel torque



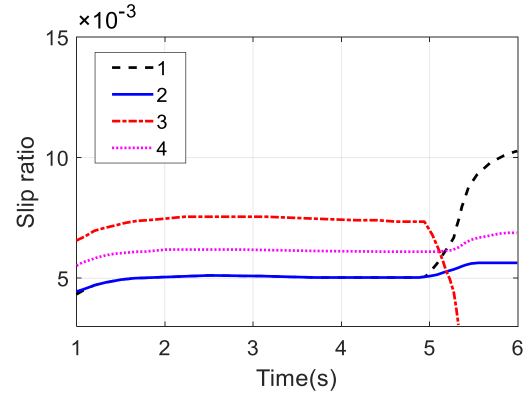
(c) Online optimization distribution wheel torque



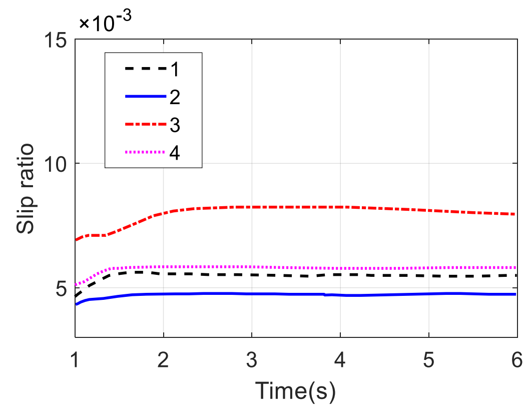
(d) Required additional yaw moment

Fig.21. Wheel torque and required additional yaw moment

The four-wheel slip ratio curves under offline optimization and online optimization control strategies are shown in Fig.22. While after the 5th second of the offline optimization, the left-side wheel torque suddenly changes from average distribution to front wheel drive, which lead to the slip ratio of the left-front wheel to increase rapidly and the left-rear wheel drops to 0, increased the risk of wheel slipping. The torque fluctuations are relatively small in the case of online optimization, the wheel slip ratio changes smoothly. Compared with offline optimization, the maximum wheel slip ratio of online optimization is reduced by 23.8%, which effectively reduces the risk of wheel slipping. Although the maximum wheel slip ratio under offline optimization and online optimization is very small and will not cause excessive wheel slipping, online optimization shows a great potential for maintaining the longitudinal stability of the vehicle.



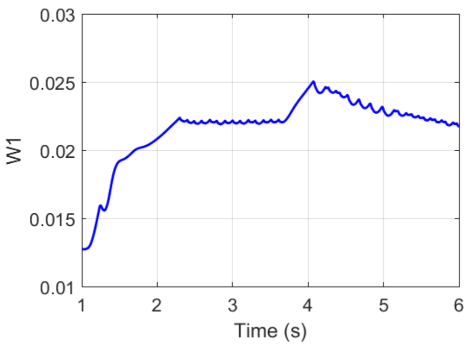
(a) Offline optimization



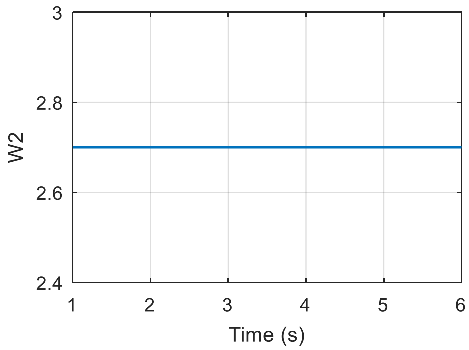
(b) Online optimization

Fig.22. Wheel slip ratio

Fig.23 shows the weight coefficient of yaw moment tracking cost function and weight coefficient of tire slip power loss cost function. The weight coefficient of yaw moment tracking cost function can be dynamically adjusted according to the vehicle speed, yaw rate following error and other information, so the yaw rate following error is controlled within a certain range. Although the vehicle acceleration is considered, the weight coefficient of tire slip control is greater than 1. However, since the maximum wheel slip ratio is still relatively small, the weight coefficient of tire slip power loss cost function does not change much and remains around 2.7.



(a) Weight coefficient of yaw moment tracking cost function



(b) Weight coefficient of tire slip power loss cost function

Fig.23. Weight coefficient

The effect of energy saving control is shown in Fig.24. Compared to the offline optimization, online optimization reduces the driving power loss by 0.23% in the precondition of maintaining the vehicle stability successfully. However, the online optimization increases the power loss of electric drive system by 0.13% compared to the average distribution strategy. That is because the online optimization makes a trade-off of pursuing energy-saving when facing the risk of losing stability. The online optimization better balances the stability and economy of 4WIDEV. This comparison results verify the competency and potential of online optimization in the aspect of improving the overall performance of the vehicle.

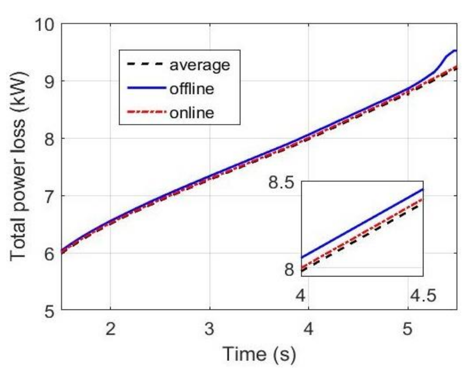


Fig.24. Total power loss

V. Conclusion

To maximum the driving energy efficiency without losing stability for a 4WIDEV, an optimal torque distribution method, characterized by using multi-objective online optimization and breaking away from the rigid yaw rate following required by 2DoF vehicle model, is proposed in this paper. This method takes four parametric control targets, yaw rate control error, power loss of electric drive system, power loss of tire slip and wheel torque vibration, as part of multi-objective online optimization function. In the optimal torque distribution, the left- and right-side of 4WIDEV are decoupled first, then the axle torque distribution coefficient corresponding to the minimum power loss of electric drive system can be obtained. Meanwhile, in order to better control the yaw rate following error in the multi-objective online optimization process, a weight coefficient fuzzy controller is also designed. The weight coefficient fuzzy controller can dynamically adjust the weight coefficient of the multi-objective online optimization function according to the online parameter feedback of 4WIDEV, which not only can effectively control the yaw rate following error within a reasonable range, but also improve the adaptability of the multi-objective online optimization function. Since the multi-objective optimization function is a nonlinear problem with constraints, a PSO is chosen to solve it.

Eventually, in order to verify the effectiveness of the proposed driving energy management strategy, typical co-simulations under constant speed condition and acceleration condition with steering angle excitation are carried out, and the simulation results of three different strategies are compared and analyzed. The results show that under the condition of constant speed, the multi-objective online optimization causes a certain yaw rate following error, but it can be effectively controlled within 5% and does not affect the stability of the vehicle. Besides, compared with average distribution strategy and offline optimization strategy, it can reduce the driving power loss by 13% and 8.8% respectively. Under acceleration conditions, compared to the average distribution, the multi-objective online optimization only causes a slight increase in energy consumption, but it can better ensure the stability of the vehicle, and always keeps the yaw rate error within 5%. At the same time, compared to offline optimization, the multi-objective online optimization can reduce energy consumption by 0.23%, and the maximum wheel slip ratio is reduced by 23.8%, avoid the risk of excessive wheel slipping, shows a great potential to maintain vehicle longitudinal stability. Therefore, the proposed driving energy management strategy based on multi-objective online optimization can maximize the vehicle's energy saving potential without affecting its stability.

Declaration of Competing Interest

None declared.

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