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DOI 10.1016/j.jebo.2021.10.031
ISSN 0167-2681
ESSN 2328-7616

Publisher: Elsevier

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Waseem A. Toraubally*

Abstract

This paper analyses the failure of the traditional Ricardo–Haberlerian (1817; 1936) theory of comparative advantage (RTCA) in a strategic market game à la Shapley–Shubik (1977). In this model, trade is driven, not by comparative advantages, but by strategic considerations. We prove, in a Ricardian framework, the *simultaneous* existence of two types of equilibria, at both of which active international trade takes place. In the first type of equilibrium, both countries specialise based on comparative advantages. In the other, *each* country produces *only* its comparative-disadvantage good. The welfare properties, and policy implications of this result (using the examples of the China–US trade war and Venezuela), are discussed at length in two dedicated sections. We show that the predictions of the RTCA depend, not on the number of agents in the economy, but on the *nature* of agents: the RTCA fails to obtain even with an infinite number of large players in each country. We prove that the RTCA prevails when agents are price-takers, and establish the conditions under which equilibria of our market game coincide with Walrasian ones.

JEL Classification: C72, D43, D50, F10, F12.

Keywords: Oligopoly; Shapley–Shubik Market Games; Endogenous price formation; Cournotian Foundations to Comparative Advantage; China–US trade war

1 Introduction

The traditional *theory of comparative advantage* (RTCA) à la Ricardo–Haberler (1817; 1936), in its most basic two-country two-good form,[1] states that free trade is driven by differences in the ratios of labour productivity in the two goods across the two countries—due to differing constant-returns-to-scale technologies. Thus, countries produce and export the good in which they have a comparative advantage.[2]

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[1] For a comprehensive review of the RTCA à la Ricardo–Haberler (1817; 1936), see Haberler (1936), Dornbusch et al. (1977), and Feenstra (2015).

[2] The RTCA does not imply that specialisation must always be complete. In particular, it is possible for only one country to specialise completely in the production of one good, while the other produces both goods. This will occur if the former country cannot meet the total world demand for the good in which it specialises. For more on outcomes which the RTCA admits, please refer to the penultimate paragraph of this section.

Amongst others, recent support has been lent to the RTCA by Costinot and Donaldson (2012), who conclude that it is "... mathematically correct and non-trivial ..." (p. 6). In pioneering work, Eaton and Kortum (2002) construct a parsimonious trade model incorporating technological heterogeneity and geography, and they also demonstrate that the RTCA is buttressed by real-world evidence. Costinot et al. (2012) also derive theoretical foundations to guide empirical tests of the RTCA, and use the resulting framework to prove the paramount relevance of Ricardian trade. Despite the well-known fact that the essential ingredient of the RTCA is differing technologies/productivity, it is interesting to note that for tractability reasons, individually insignificant agents have been a mainstay of most Ricardian models of trade, as a result of which goods and factor prices are taken as given.[3] Yet, it is common knowledge that countries, and their constituents, behave strategically in the real world (see, e.g., Freund and Pierola, 2015; Siebert, 2007: p. 382, and; Jacquemin, 1982). We therefore deem it important—in fact, necessary, especially for policymakers—to theoretically reappraise the predictions of the RTCA, in a context where workers/countries are strategic in nature.

In this paper, we develop a Ricardian market game and show that the predictions of the RTCA fail. Indeed, in the two-country two-commodity Ricardian model that we consider, it is no longer comparative advantages, but *only* strategic interactions among agents, that drive trade. Our main contribution is twofold. First, we prove the existence of two types of equilibria; one in which countries specialise in accordance with the RTCA, and another in which each country produces and exports to the other *only* the good in which it has a comparative *dis*advantage. Second, we establish that the failure of the RTCA is intimately linked to the strategic nature of agents. Indeed, the RTCA fails even in economies consisting of an infinite number of agents,[4] each of whom wields a non-negligible amount of market power. We prove that specialisation as per the RTCA finally takes place when we consider economies with a continuum of small agents (see §5). The above claims are established in the context of *strategic market games* (SMG) of the Shapley–Shubik (1977) tradition. An SMG is a general equilibrium,[5] noncooperative game-theoretic model which features an explicit description of the commodity-price formation process.[6] Prices are de-

[3]This issue is discussed at length in Toraubally (2020), where we explain that none of perfect competition, perfect mobility of factors of production, or even perfect divisibility of the latter are crucial characteristics of the RTCA.

[4]Cordella and Gabszewicz (1997) provide an example in which the RTCA fails in oligopolistic markets, but holds in markets with large numbers of players, markets which need not be Walrasian. See §§2 and 4 of the current paper for more details.

[5]SMGs have been applied to a vast variety of frameworks, ranging from imperfect competition in unregulated electricity markets (Spear, 2003), providing solid micro-foundations for macroeconomic analysis (Goenka, Kelly and Spear, 1998), analysing the asymptotic convergence of Nash equilibria to Walrasian ones with uncertainty and asymmetric information (Meo and Koutsougeras, 2018), to experimental implementations (among others, Jiang et al., 2021; Ding and Puzzello, 2020; Barreda-Tarrazona et al., 2018; Duffy et al., 2011).

[6]The interested reader is referred to the seminal papers by Dubey and Shubik (1978), Dubey and Shapley (1994), and Peck et al. (1992) for in-depth explanations of the formulation that we use in this paper.

termined solely by the buy-and-sell decisions of agents in the economy. The theory of SMG thus provides an elegant formulation that we believe appropriately captures the influence of strategic considerations on trade in world markets.

We extend the two-country two-commodity Ricardian framework, in the first instance, to a Cournotian setup. In §3.2, we present in a very general framework, which includes most utility functions that are used in the international trade literature, our main results (Theorems 1 and 2). Using a combination of fixed-point theorem, and Lagrange multiplier methods, we prove the existence of an equilibrium in which countries specialise based on comparative advantages in Theorem 1. In Theorem 2, we show that there simultaneously exists another equilibrium in which each country specialises in the production of its comparative-*dis*advantage good, though comparative advantages are clearly present. The intuition for this apparent anomaly is that market-clearing prices are sensitive to the market activities of each agent, such that the marginal price of a commodity is not equal to its average price—see, e.g., Tirole (1988: pp. 218-219). Hence, any effort by a worker to benefit from any price difference by shifting orders across markets (e.g, producing and selling more of the expensive commodity, while buying more of the cheaper one), alters the corresponding market-clearing prices adversely. The resulting net effect impacts the worker unfavourably, who therefore chooses to stay put. We emphasise that these results are achieved in a framework where *all of the original* assumptions of Ricardo–Haberler are retained—save price-taking agents when considering imperfectly-competitive markets.

We demonstrate that even when the number of agents increases without bounds, the RTCA still fails (Theorem 3). This is because these agents, though (countably) many in numbers, still perceive the influence of their market power on the market-clearing prices of the two goods. Thus, they use their ability to manipulate prices to their advantage. This possibility is also discussed in Toraubally (2018, 2019), where it is demonstrated that even in large economies with *many* agents, markets, and commodities, strategic behaviour may still persist.

In light of the above, we deem it important to extend our formulation and analyse what happens when the economy is populated only by a continuum of small, individually-insignificant workers. It is shown that in this setup, the predictions of the RTCA are restored, thereby establishing a strong link between the failure of the RTCA, and the strategic nature of agents. This conclusion may no doubt seem intuitive; yet, the analysis is necessary for a comprehensive exposition of the Ricardian market game which we propose. We show that in Ricardian economies with a continuum of price-taking agents, equilibrium takes (only one of) three forms (Proposition 2). The first is when both countries are completely specialised in their

respective comparative advantage goods. The second type consists of one country fully specialised in its comparative-advantage good, while the other produces both goods—a situation also known as an incomplete specialisation equilibrium. The third is when both countries completely specialise in producing only one and the same good. Now, it is a well-known fact that Nash equilibrium allocations of a market game need not be Walrasian, and vice versa. This is because a market game is transactions-oriented, and agents face liquidity constraints that cannot be ignored—the latter play no role whatsoever in Walrasian economies. In this light, it is worthwhile to provide a characterisation of when Nash equilibrium allocations of our game are also Walrasian, which we do in §5.1.

In §2, we present a brief literature review to better frame the extent of our contribution. In §3, we develop our model, and explain how interactions take place in the economy. We provide an analysis of the properties of equilibria, and present our main results in the form of various propositions and theorems. We also discuss the policy and welfare implications of the latter. In §§4 and 5, we extend the model in §3 to setups with, first, a countable infinity of agents, and subsequently, a continuum of agents, respectively. Our conclusions are summarised in the following section. For the reader's convenience, all the mathematical proofs derived in this paper have been relegated to an Appendix.

2 Literature Review

To put things into perspective, and to dispel any misconception right from the outset, we (very) briefly review a prominent strand of the trade literature, which has now come to be known as the "new trade theory." The most important works in this direction are those by Krugman (1979, 1980), who views monopolistic competition, increasing returns to scale (IRTS) and product differentiation as the main drivers of trade, Markusen (1981, monopoly/duopoly and IRTS), Brander (1981, IRTS, segmented markets, and transportation costs), and Brander and Krugman (1983, segmented markets and transportation costs). We emphasise that *none* of the aforementioned effects are present in the purely Ricardian model(s) that we propose in this paper. Moreover, in our framework, the domestic and foreign markets are integrated, as opposed to segmented. We also remark that although Markusen (1981) shows that even with constant returns to scale trade will still take place, his model relies on the existence of capital and labour as factors of production, and assumes that the market for the labour-intensive good is perfectly competitive—such that its price is given by Adam Smith's "invisible hand." Importantly, this means that his model also describes a theory of trade that is not driven *completely* and *uniquely* by strategic interactions amongst agents. Crucially, apart from Krugman (1979, 1980), the above are all partial equilibrium, as opposed

to general equilibrium models, which ours is. Last but not least, Krugman's (1979, 1980) analyses are carried out in monopolistically-competitive general equilibrium setups, such that individual agents are negligible (see also our discussion in the Introduction).

The paper most closely related to ours (in terms of the market game mechanism used) is Cordella and Gabszewicz (1997) (henceforth, C–G). C–G analyse how the use of market power by agents alters the predictions of the original Ricardian model. In their two-country, two-commodity framework, all individuals across both countries derive utility *only* from the consumption of the good in which they have a comparative disadvantage. They show that for a large class of Ricardian economies, despite there being free trade and clear comparative advantages, autarky is the *unique* oligopoly equilibrium on the world market. More precisely, at equilibrium, no trade at all takes place, and agents produce only the good in the production of which they have a comparative disadvantage. When the number of agents becomes "large enough," the Ricardian outcome is restored. Note that C–G only require that the number of agents in each country exceed some (obviously finite) upper bound, which they call $\bar{n}$, for the RTCA to hold again—i.e., agents do not have to be infinite in numbers, let alone price-takers, nor do the corresponding equilibria have to be Walrasian. In the framework that we propose, however, workers' utility functions depend on the consumption of all goods on the world market, as opposed to solely on the one in which they have a comparative disadvantage. Additionally, we show that even when the number of agents is countably infinite, the Ricardo–Haberlerian predictions still fail to obtain. This is because as in Toraubally (2018, 2019), agents still possess a non-negligible degree of market power. It is only when agents are price takers that specialisation (at a non-trivial equilibrium) as per the RTCA is guaranteed to take place. Also, it is interesting to note that while strategic interactions by agents in the model in C–G do affect the equilibrium exchange rate of goods, individuals still take world prices of the two commodities as given—i.e., there is no description of how commodity prices are formed. In this paper, however, prices for all commodities are determined purely endogenously, by agents' buy-and-sell decisions through a system of trading posts/markets.

3 The Ricardian market game Γ with finitely many agents

We consider two countries, F and H. The set of agents (workers) is $N = F \cup H$, where $|F|, |H| \geq 2$. There are 2 consumption goods that can be produced in the world economy, the technologies for producing which are possessed by both countries F and H. We let the set of both commodities which are produced, and tradeable be denoted by $\{1, 2\}$. There is also a 3^{rd} commodity, which we call m, that is not produced

but which, in addition to its role in utility, also acts as money.[7] The consumption set of each worker is therefore identified with $\mathbb{R}^3_+$.

Agents $n \in N$ do not have any initial endowments of goods 1 and 2, but are instead endowed with labour hours, with which to produce commodities 1 and 2.

Each country is characterised by its technology. The latter is exogenously fixed and exhibits constant returns to scale, and is denoted by $a^J_k,\ J = F, H$. More precisely, the technology in country J, a^J_k, determines how many labour hours it takes for a constituent worker in J to produce one unit of any commodity $k \in \{1, 2\}$. In line with Ricardo–Haberler's original formulation, we assume that $a^F_1/a^F_2 \neq a^H_1/a^H_2$. As in C–G, workers are taken to be homogeneous and perfectly mobile between the industries within a country, but immobile and heterogeneous across countries. In light of the above, each worker $n \in N$, is described by a preference relation representable by a utility function $u_n : \mathbb{R}^3_+ \to \mathbb{R}$, and initial endowments, of money $e_{n,m} \in \mathbb{R}_{++}$, and of labour hours $Q_n \in \mathbb{R}_{++}$.

Any commodity $k \in \{1, 2\}$ that workers choose to produce by offering labour hours, is *manufactured* at the corresponding trading post—i.e., no good can be produced without a trading post. At this common, fully integrated international trading post, workers also place bids (purchases) in terms of commodity m for every commodity $k \in \{1, 2\}$. Every worker can, in this way, consume vectors of each commodity produced. A worker may well buy back everything s/he has produced, but s/he can only do so through the respective trading posts. Formally, we define the strategy set of each $n \in N$ as:

$$S_n = \left\{ (b_n, l_n) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ : \sum_{k=1}^{2} l_{n,k} \leq Q_n; \sum_{k=1}^{2} b_{n,k} \leq e_{n,m} \right\},$$

where $b_n = (b_{n,1}, b_{n,2})$ and $l_n = (l_{n,1}, l_{n,2})$ represent vectors of bids placed, and labour hours offered across the various markets, respectively.

Throughout the rest of the paper, we will employ the following assumption:

Assumption 1. *Utility functions for all agents are concave, smooth, and differentiably strictly monotone.*[8]

3.1 Interactions in the economy

Given a strategy profile $(b, l) \in S \coloneqq \prod_{n \in N} S_n$, we define $B_k = \sum_{n \in N} b_{n,k}$, $\phi_k = \sum_{f \in F} \frac{l_{f,k}}{a^F_k} + \sum_{h \in H} \frac{l_{h,k}}{a^H_k}$, $B_{-n,k} = \sum_{i \in N \setminus \{n\}} b_{i,k}$, and $\phi_{-n,k} = \sum_{f \in F \setminus \{n\}} \frac{l_{f,k}}{a^F_k} + \sum_{h \in H \setminus \{n\}} \frac{l_{h,k}}{a^H_k}$. Consumption allocations of commod-

[7] The role of money does in no way affect the predictions of the original Ricardian model. As Haberler (1936: p. 131) argues, "all these [assumptions, including the introduction of money] give rise to annoying misunderstandings. Somebody or the other is always trying to show that the [RTCA] is valid only under the simple assumptions upon which it was originally formulated."

[8] i.e., for all $x \in \mathbb{R}^3_{++}$, $\frac{\partial u_n}{\partial x_k} > 0\ \ \forall k = 1, 2, m$.

ity $k \in \{1, 2, m\}$, for any worker $j \in J,\ J = F, H$, are then determined as follows:

$$x_{j,k} = \begin{cases} b_{j,k}\dfrac{\phi_k}{B_k} & \text{where } k \in \{1,2\}, \text{ and } B_k \cdot \phi_k \neq 0; \\ 0 & \text{where } k \in \{1,2\}, \text{ and } B_k \cdot \phi_k = 0; \end{cases}$$

$$x_{j,m} = e_{j,m} - \sum_{k=1}^{2} b_{j,k} + \sum_{k=1}^{2} \frac{l_{j,k}}{a_k^J}\frac{B_k}{\phi_k},$$

where we adopt the market game convention that any division by zero, including $\frac{0}{0}$, is equal to zero whenever it appears in any of the expressions above—see Peck et al. (1992) for an interpretation. According to the above mechanism, a worker $j \in J,\ J = F, H$, is allocated commodity $k \in \{1, 2, m\}$ in proportion to his/her bids and labour hours offered.

The trading post for good $k \in \{1, 2\}$ is said to be **active** if $B_k \cdot \phi_k > 0$. Observe that, as what agent j produces of good k is $\frac{l_{j,k}}{a_k^J}$, ϕ_k in fact refers to the total amount of good k that is produced and traded (offered for sale). Thus, when $B_k \cdot \phi_k > 0$, the fraction $p_k = \frac{B_k}{\phi_k}$ has a natural interpretation as the (market-clearing) price of commodity k. Henceforth, we will use the notation p_k and $\frac{B_k}{\phi_k}$ interchangeably, and refer to each as the price of k.

Note further that, as workers reap the full value of what they produce with their labour, the last term in $x_{j,m}$ may be interpreted as the total wages of worker j, just like in the traditional Ricardian model (see, e.g., Feenstra, 2015).

An equilibrium in our model is a profile constituting vectors of agents' bids (purchases) for commodities and labour hours offered, which forms a Nash equilibrium (N.E). And at an N.E, any worker $n \in N$ is viewed as solving the following problem:

$$\max_{(b_n, l_n) \in S_n} \left\{ v_n \left(\left(x_{n,k}\left(b_{n,k}, l_{n,k}, B_{-n,k}, \phi_{-n,k} \right) \right)_{k=1}^{2}, x_{n,m}\left(\left(b_{n,k}, l_{n,k}, B_{-n,k}, \phi_{-n,k} \right)_{k=1}^{2} \right) \right) \right\}. \tag{1}$$

3.2 Equilibrium Analysis, main results and policy implications

We may now introduce our first results. In particular, thanks to the allocation rule for all agents $n \in N$, our model provides the strongest incentives for trade to occur, an idea which is formalised in the following proposition:

Proposition 1. *At any N.E, the trading post for at least one good is active. More precisely, given that all players are rational and rationality is common knowledge, at any N.E,* ***every*** *agent buys and consumes* ***at least*** *one of*

goods 1 and 2.

Proposition 1 is nontrivial, and new to the market game literature. In particular, in standard SMG models, there always exists a trivial N.E, in which all agents make zero offers on *all* markets, such that zero trade results. If agents' endowments are already Pareto optimal, then this trivial N.E is logical and efficient. If endowments are Pareto-suboptimal, a trivial N.E is an inefficient outcome as agents would be better off reallocating resources. For our model, however, this no-trade scenario never materialises at equilibrium. This is because, contrary to extant models, in our case, agents are not endowed with any of goods 1 and 2. Yet, they can always play strategies such that they may, not only buy commodities 1 and 2, but also guarantee their initial endowment of money. Thus, every agent makes a non-zero net trade at any equilibrium.

It is also worth noting that any equilibrium of our model is characterised by full employment of labour, in line with all Ricardian models. The intuition behind this conclusion in our model is simple: workers in $(F \cup H)$ derive no utility from leisure. Thus, their labour hours can always be "put to good use" on the market(s). Of course, in our model, producing and selling more of any commodity will drive down its price on the market. Yet, it suffices to note, for any worker $n \in N$ that their allocations of every good $k \in \{1, 2, m\}$ are all increasing in labour hours.

Note that Proposition 1 does not guarantee that both goods are traded at equilibrium, nor does it say which good is traded. A more meaningful result can be achieved through the imposition of a mild, natural assumption about workers' preferences. We simply require that agents' utility functions be strictly concave and satisfy the Inada condition $\lim_{x_{n,k}\to 0}(\partial u_n/\partial x_{n,k}) = +\infty, k = 1, 2$. For notational conciseness, let the class of utility functions satisfying this condition and Assumption 1 be denoted by $\mathscr{U}_{ACI}$. Note that $\mathscr{U}_{ACI}$ includes most utility functions used in international trade models, such as Constant Elasticity of Substitution (which nest Cobb–Douglas, Dixit–Stiglitz, and Mill–Graham preferences), additively-separable logarithmic, and Stone–Geary utility functions.

We are now in a position to present our first existence result. In Theorem 1, we prove that our model is "well-behaved," and admits equilibria as postulated by the RTCA.

Theorem 1. *If $u_n \in \mathscr{U}_{ACI}$ for each $n \in N$, there exists an equilibrium in which countries produce and export* ***only*** *the good in which they have a comparative advantage.*

Our next existence result below shows that our model includes configurations of international trade that are not possible in existing models. While specialisation in accordance with the RTCA does constitute

an equilibrium, there also *simultaneously* exist other equilibria at which both countries produce *only* their disadvantaged goods, even though comparative advantages are clearly present. Additionally, at such equilibria, active international trade takes place in both goods on the world market, and it is driven solely by strategic behaviour, without there being economies of scale, transportation costs or any other economic friction.

Theorem 2. *If $u_n \in \mathscr{U}_{ACI}$ for each $n \in N$, there also exists an equilibrium in which countries produce and export* ***only*** *the good in which they have a comparative* ***dis****advantage.*

Theorem 2 represents a severe, if not *the* severest, violation of the RTCA. Admittedly, that agents do not wish to deviate, shifting some of their labour hours towards the good they are relatively more efficient at producing, is most intriguing. Yet, the reason why this situation is tenable as an equilibrium is simple. Agents perceive the influence of their individual buy-and-labour-supply decisions on the world equilibrium exchange rate of goods, and they make strategic use of this market power. Once the price-taking assumption is abandoned, any unilateral perturbation in the amount bidden or the number of labour hours offered adversely alters the equilibrium prices and allocations for any worker. Consider any worker who wishes to shift his labour hours offered from the market for his comparative-disadvantage good, say, commodity 2, to that of commodity 1. In doing so, he automatically causes the price in the market for 1 to fall, in view of the increased supply for good 1. This implies that a lower price is now received, not only for the marginal unit, but for all units of 1 produced and sold. At the same time, the market-clearing price for good 2 soars, for there is now a relatively lower supply of the good. Again, this implies that a higher price needs to be paid, not only for the marginal unit, but for all units of 2 purchased. The upshot is that the worker's consumption of commodities 1, 2 and m will change in a way which is unfavourable to him. The same argument can be repeated for a contemplated shift of bids placed, or for a combined shift of bids and offers across the two markets. Hence, given the strategies played by the other workers, it pays to stay put.

The above conclusion holds true regardless of the size of the population, whether very large or very small, so long as no individual agent is *completely* insignificant relative to the market (see more in §4). Indeed, any individual agent who contributes ***nontrivially***, no matter how little, to the aggregate bids and/or offers and thus to the market-clearing prices, exerts a non-negligible degree of market power.[9]

[9] Novshek (1980) calls firms that are small but not insignificant relative to the market "non-infinitesimal firms." He shows that when the economy is populated by such firms, a ***Cournot*** market equilibrium exists. Unsurprisingly, this equilibrium is only *approximately* competitive, because any individual non-infinitesimal firm still has an effect on price. This is in line with our analyses in §§3 and 4.

Hence, the foregoing equilibrating apparatus remains in operation. A result of a similar nature is also proved in Toraubally (2018, 2019) where, what at a glance appears to be a profitable deviation, turns out not to be so for active market participants.

It is also relatively straightforward to deduce that the proof method we adopt does not preclude the existence of other equilibria which run counter to the predictions of the RTCA. For example, there are also exist equilibria at which each country produces both goods. For convenience, we collect this result in the following corollary.

Corollary 1. *If $u_n \in \mathscr{U}_{ACI}$ for each $n \in N$, there exists an equilibrium in which **each** country produces **both** goods on the market.*

3.3 Pareto rankability of imperfectly competitive N.E[10]

Peck et al. (1992) show that different offer-constrained equilibria can be Pareto ranked, although the authors do not analyse which equilibria Pareto dominate. In Goenka et al. (1998), the authors show in the context of an overlapping generations model with imperfect competition, that thicker markets (those with larger offers) lead to Pareto superior equilibria. In view of the existence results in §3.2, it therefore seems natural to wonder if there is a similar mechanism at work in our model.

We shall hereby enlighten this important matter by making extensive use of the Edgeworth box—like Shapley and Shubik (1977) also do. The result is a highly intuitive, descriptive yet general, analysis enabling us to visualise the qualitative effects of strategic behaviour on the Pareto optimality and rankability of various equilibria. Note that though simple, the Edgeworth box is powerful enough to portray almost all phenomena and properties of general equilibrium exchange economies (Mas-Colell et al., 1995).

Consider the model described in §3. Assume, for now, that neither F nor H has a comparative advantage in the production of any good. W.l.o.g., let the labour endowments be such that each country can produce one unit of either good, or any convex combinations thereof. Then country J's, $J = F, H$, production possibilities frontier (PPF) is described by

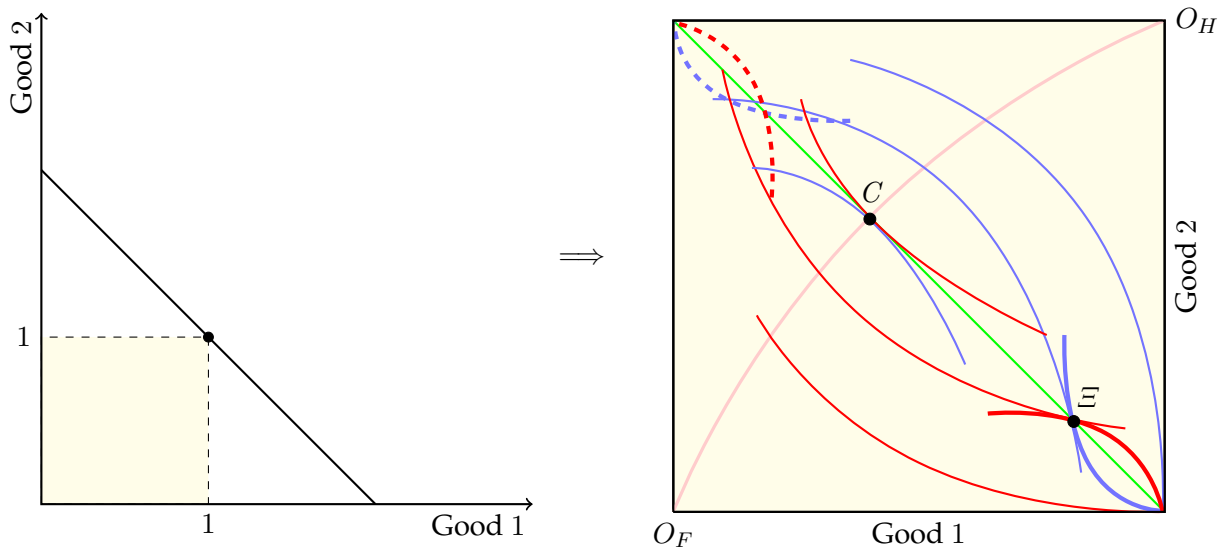
$$x_{J,1} + x_{J,2} = 1,$$

and the world PPF is described by

$$x_1 + x_2 = 2.$$

[10]It is a pleasure to thank Steve Spear, with whom the material in this section was developed over a long series of emails.

Figure 1: Strategic vs price-taking behaviour



(a) The world PPF

(b) The Edgeworth Box

This economy looks just like a simple pure exchange economy, with the endowments given by the choice of labour inputs on the country's PPF. Consequently, this can be modeled in a simple Edgeworth box, as in Panel (b) below. Assume that the total production of each good is one unit, so that the Edgeworth box we generate maximises the total possibilities for each good, and hence, is derived from the world PPF as in Panel (a).

In Panel (b), the green line shows the possible endowment points starting from complete specialisation in the lower right-hand corner—or, symmetrically in the upper left-hand corner. Given the assumption of heterogeneous preferences across countries, the pink curve is the set of Pareto optimal allocations.

For the competitive case, i.e., a continuum of price-taking agents in each country, the equilibrium allocation will be at the intersection of the pink curve and the green line, at C. Agents in this economy will be indifferent between any production allocations that yield endowments on the green line. In this case, if we now perturb the economy away from no comparative advantage to one where country F (H) has a sufficiently small comparative advantage in producing good 1 (2), the optimal outcome for the competitive case will be complete specialisation as per the RTCA, i.e., produce so that the endowment lies in the lower right-hand corner. This is because with the comparative-advantage perturbation, complete specialisation now maximises world output, and sharing the output of both goods dominates non-specialisation. Here, non-specialisation would manifest itself as a **change in the shape and size** of the Edgeworth box as we moved away from the specialisation corner. Hence, in a perfectly competitive environment, even

a minor perturbation away from the no-comparative-advantage case yields a discontinuous jump in the optimal production decisions.

The heavy (undotted) red and blue budget constraints/consumption possibilities frontiers[11] (CPF) show the situation for the imperfectly competitive economy (i.e., oligopolistic agents in each country), starting from the specialised endowment point. The N.E outcome occurs where the two curves intersect at Ξ, and we know that the N.E allocations are not Pareto optimal (Shapley and Shubik, 1977; Postlewaite and Schmeidler, 1978). Because of the inoptimality, agents in each country would be better off if they started trading from an endowment point at the N.E attained from pure specialisation—i.e., if each country started out producing at Ξ as depicted in Panel (b) and then exchanged goods, they would end up at a new N.E inside the lens-shaped region between the two indifference curves tangential to the initial CPF. In other words, they would be better off by *not* specialising before any redistribution of the social endowment takes place. Of course, as long as the N.E from whichever endowment point they start from is not Pareto optimal, the two countries are better off reallocating production. This only stops being the case at a Pareto optimal allocation. In fact, it is straightforward to see in Panel (b) that the allocations generated as N.E starting from endowments on the green line can be strictly Pareto ranked.

Now, as we did with the competitive case, perturb the production functions so that country F (H) has a sufficiently small comparative advantage in good 1 (2). Unlike the competitive case, we will not see a discontinuous jump to specialisation because by continuity, the Pareto rankability of the N.E allocations does not disappear, and hence, by continuity, the imperfectly competitive agents will prefer having slightly less goods available in order to maintain a higher welfare N.E allocation. Hence, we have an open set of examples where equilibria of the type described in Corollary 1 Pareto dominate those described in Theorem 1. One may, using this approach, also show the *generic*—in the sense of an open and dense set of N.E—Pareto superiority of the equilibria in Theorem 2 over those in Theorem 1 by simply noting that the intersection of the dashed CPF lies in the aforementioned lens-shaped region in Panel (b).

It should be possible to make a general argument via the fact that if each country (in the imperfectly competitive case) could produce a Pareto optimal endowment allocation in the no-comparative-advantage case, then under perturbations away from no comparative advantage, they will move continuously from a non-specialised production bundle to the specialised bundle as the extent of the comparative advantage gets larger. ♦

[11] Dubey and Shubik (1978) dub the CPF "holdings surface."

3.4 Policy Implications: the sino-US trade dispute, and Venezuela examples

A few preliminary comments, to better frame the policy implications of our results, are in order. First, as Ruffin (2003) argues,[12] "the folk theorem that under oligopoly, free trade increases the gains from trade [compared to the competitive case] through [a] pro-competitive effect is not true in general" (p. 332). The failure of this folk theorem is a manifestation of a more general problem: under oligopoly, which is embodied by a finite number of agents, a specific trader cannot be *completely* negligible, such that *complete* world production efficiency cannot obtain (see §3.3). Interestingly though, in such a scenario, **oligopolistic gains** help to **sustain/support** the domestic import-competing industry, akin to how import tariffs protect perfectly-competitive industries. Our results thus provide an alternative solution to the traditional import protection argument, without needing any of dynamic and static economies of scale, market segmentation, and decreasing marginal costs (see Krugman, 1984). This fact, together with our findings in §§3.2 and 3.3, shows that when comparative advantages are shifting, much like the current situation between China and the US, trade barriers should not always be the go-to measure. *Free* international trade is itself powerful enough for all (oligopolistic) parties involved to reap the gains from it, irrespective of where comparative advantages lie.

The above is substantiated by the empirical evidence. Arguably, the China–US trade war arose because of China's dominance in manufacturing, a sector in which the US has historically had a comparative advantage. Due to China's newfound comparative advantage in this sector, the US in 2017 ran a bilateral trade deficit in goods to the tune of $375 billion, or approximately $1.03 billion per day (Wall Street Journal, 2018). If one is to believe the textbook case of specialisation as per the RTCA being mutually beneficial to all engaged in it, why did these two economic powers erect trade barriers? The losses that both countries incurred as a result of the imposition of trade impediments make for compelling reading. Moody's Analytics (2019) found that the China–US trade war had cost the US economy nearly 300,000 jobs, with the monetary losses reported to be in the range of 0.3%–0.7% of real GDP (Bloomberg Economics, 2019). Bloomberg Economics (2019) predicted that the trade war would make the US economy $316 billion worse off by late 2020, while recent research from the Federal Reserve Bank of New York (2020) found that U.S. companies lost at least $1.7 trillion in stock prices as a result of US tariffs imposed on imports from China. Of course, China was also not immune from substantial economic losses, and

[12] Ruffin's (2003) results are driven by the use of Cobb–Douglas preferences, and sometimes by Mill–Graham preferences (identical Cobb–Douglas preferences with identical product shares). Thus, our result is valid for a larger class of utility functions than his. Note that the context in which Ruffin (2003) makes these observations is different to ours. Notwithstanding this fact, his conclusions are still useful for our purposes.

resorted to lowering tariffs for its other trading partners as it reduced its dependance on US markets (Brookings, 2020).

The findings in §3.2, together with the above figures, also shed light on another facet of the RTCA: its unsustainability and limited applicability in practical policy. Xie (2019) argues that the RTCA is too static in nature for policy based on it to reap success. While RTCA-driven economic policy may succeed in the short run, it is bound to fail in the long run. This is because the development model based on this approach has an inherent destructive mechanism which leads to the deterioration of a country's socio-politico-economic environment. Consider China's case. In the early 1990s, it adopted the traditional RTCA as its blueprint for growth and made significant economic progress. It exploited its comparative advantage in manufacturing based on cheap labour to usher in rapid economic growth. Yet, the 2007-8 financial crisis exposed the problematic nature of China's industrial structure: the latter is made up of technology-intensive and very large labour-intensive sectors pulling in opposite directions. This has created a vicious circle. China's international competitiveness in key, tech-heavy industries such as advanced manufacturing and large aircraft production is seriously inadequate, and is today exhibiting a downward trend. Hence, these sectors cannot be relied upon for maintaining, let alone stimulating, robust economic growth. To maintain comparative advantage, wages in labour-intensive sectors must therefore be tightly controlled and kept low at all times. This has widened the income gap between the tech- and labour-intensive sectors, exacerbating domestic inequality. At the same time, very cheap Chinese exports have adversely affected employment in the US, hence the change in US foreign policy, and the trade war. In turn, the massive fall in US demand for Chinese products and the ensuing substantial economic losses have had knock-on effects on domestic employment and growth in China. Thus, China finds itself in a quandary. On the one hand, its large domestic labour force is heavily dependent on the international market (especially demand) to keep employment levels healthy. On the other hand, if it persists with the traditional RTCA model, and no suitable countermeasures are adopted, there will be significant negative economic repercussions—e.g., the current trade war. Thus, the successful applicability and adaptability of RTCA-oriented trade policy hinges crucially on which stage of its development a country is at.

The vicious circle described above is not specific to China. There is a wealth of empirical evidence demonstrating the dangers of other economies blindly pursuing RTCA-driven specialisation. Prominent examples include extremely resource-abundant countries such as the oil states in the Gulf, Nigeria, Mexico and Venezuela (Sachs and Warner, 2001). Indeed, empirical growth studies classify natural resources as a robust explanatory variable for poor economic growth (Sala-i-Martin, 1997; Doppelhofer et

al., 2004). To drive home how poorly the RTCA fares absent the conditions for its successful applicability, let us delve deeper into the economic landscape of Venezuela. While the reasons underlying its economic underperformance are varied and not clear-cut, one which is ubiquitously encountered is the resource curse argument (Auty, 1983), which runs counter to the classical RTCA explanations of growth advantages. Venezuela is rich in—and has a comparative advantage in the production of—oil. In line with the postulates of the RTCA, it proceeded to lay heavy emphasis on developing its oil industry. The prioritisation of a single industry, alongside the lack of growth in non-resource sectors to diversify its industrial system (Venables, 2016), led Venezuela to experience one of the worst-ever cases of Dutch disease (see, e.g., Corden, 1984). Additionally, a key feature of resource-intensive economies is that they tend to have higher general price levels. Inasmuch as businesses in these countries use domestic inputs and sell the resulting products on international markets, their competitiveness suffers. These compounding factors led to Venezuela's gravely missing out on export-led growth (Sachs and Warner, 2001). Coupled with a volatile macroeconomic environment (Murillo, 2000), and drastic cuts in public spending, a currency devaluation, and inflation spiralling indefinitely, Venezuela went from being one of Latin America's oldest and most prosperous democracies, to one of its most impoverished nations (Naím and Toro, 2018). The empirical evidence thus reinforces the fact that the optimality of RTCA-induced specialisation depends not merely on comparative advantages, but a whole host of other equally, if not more, important economic factors.

Our results, especially the discussion in §3.3, hence make an important theoretical contribution to our understanding of current international trade issues. To wit, compared to the immediate post-World War II era, industries today are characterised by market power concentrated in the hands of a few corporations (Freund and Pierola, 2015). This is true in agriculture, finance, manufacturing, and non-financial services. Under the neoliberal trade consensus, the conventional wisdom around comparative advantage has allowed large companies to engage in costly lobbying for special (almost protected) positions in trade negotiations, often to the detriment of smaller, less powerful firms or less concentrated sectors. What the findings in this paper prove is that when firms have market power, trade negotiations need focus only on engaging in *free* international trade, without regard to issues of comparative advantage. ♦

It should by now be abundantly clear that strategic considerations by agents are a powerful determinant of the resulting equilibrium market structure. By extension, strategic behaviour thus drives the failure or prevalence of the RTCA. In the next section, we prove this point: even when the number of agents increases without bounds, the RTCA fails if agents behave **strategically**, such that *complete* world efficiency

is still not achieved. Notably, this is different to the approaches in C–G and Ruffin (2003). In particular, as alluded to in §2, C–G show that as the number of agents in their model becomes large enough (not necessarily infinite), the predictions of the RTCA are restored. That is, Ricardian specialisation turns out to be optimal even in imperfectly competitive/oligopolistic environments, where agents have market power. So, the analysis that follows is not about the well-known fact that perfect competition is not implied by a large number of agents. Rather, it is about the predictions of the RTCA not materialising so long as strategic behaviour persists.

4 The Ricardian market game Γ_∞ with a countable infinity of agents

We consider an economy featuring a purely atomic measure space of agents $(N, \mathcal{N}, \mu)$, where $N = F \cup H$, with $F = \{f_i\}_{i=1}^{\infty}$ and $H = \{h_i\}_{i=1}^{\infty}$. $\mathcal{N}$ is the collection of all μ-measurable subsets of N, and μ is a totally finite complete positive σ-additive measure on $\mathcal{N}$.[13] All our assumptions about the technologies and the mobility of workers are as described previously. Naturally, we require that the total endowments in the economy of labour hours, and commodity m, be such that $\sum_{n\in N} Q_n \cdot \mu(n) < \infty$, and $\sum_{n\in N} e_{n,m} \cdot \mu(n) < \infty$. The strategy set of each $n \in N$ is defined as:

$$S_n = \left\{(b_n, l_n) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ : \sum_{k=1}^{2} l_{n,k} \leq Q_n; \sum_{k=1}^{2} b_{n,k} \leq e_{n,m}\right\}.$$

Let $(\mathbb{R}^\omega, \tau_P)$ denote the space of all real-valued sequences endowed with the product topology, and let $S^\infty := \prod_{n\in N} S_n \subset (\mathbb{R}^\omega, \tau_P)$. Then, given a strategy profile $(b, l) \in S^\infty$, we let $B_k := \sum_{n\in N} b_{n,k} \cdot \mu(n)$, $\phi_k := \sum_{f\in F} \frac{l_{f,k}\cdot\mu(f)}{a^F_k} + \sum_{h\in H} \frac{l_{h,k}\cdot\mu(h)}{a^H_k}$, $B_{-n,k} := \sum_{i\in N\setminus\{n\}} b_{i,k} \cdot \mu(i)$, and $\phi_{-n,k} := \sum_{f\in F\setminus\{n\}} \frac{l_{f,k}\cdot\mu(f)}{a^F_k} + \sum_{h\in H\setminus\{n\}} \frac{l_{h,k}\cdot\mu(h)}{a^H_k}$. The consumption allocation of every commodity $k \in \{1, 2, m\}$, for any worker $j \in J$, $J = F, H$, is determined thus:

$$x_{j,k} = \begin{cases} b_{j,k}\dfrac{\phi_k}{B_k} & \text{where } k \in \{1,2\}, \text{ and } B_k \cdot \phi_k \neq 0; \\ 0 & \text{where } k \in \{1,2\}, \text{ and } B_k \cdot \phi_k = 0; \end{cases}$$

$$x_{j,m} = e_{j,m} - \sum_{k=1}^{2} b_{j,k} + \sum_{k=1}^{2} \frac{l_{j,k}}{a^J_k}\frac{B_k}{\phi_k},$$

where, as before, $\frac{B_k}{\phi_k} = \frac{\phi_k}{B_k} := 0$ whenever $B_k \cdot \phi_k = 0$. The trading post for good $k \in \{1, 2\}$ is said to be

[13] An atom of $(N, \mathcal{N}, \mu)$ is a set A such that $\mu(A) > 0$, and for any $B \subseteq A$, we either have $\mu(B) = 0$ or $\mu(A \setminus B) = 0$.

active if $B_k \cdot \phi_k > 0$. Given a profile $(b, l) \in S^\infty$, agents are viewed as solving the same problem as in (1).

We are now ready to state our main result for this section. This the same conclusion as in Theorems 1 and 2, but restated for a context with a countable infinity of agents. As remarked in the preamble to §4, in C–G's Ricardian market game, the RTCA no longer fails as the number of agents increases without bounds, whether or not these agents are large and/or small.[14] What simply drives, or stifles, trade in C–G's model is the *number* of agents, whereas here, it is their *nature* that matters. Moreover, to the best of our knowledge, this is the most general framework to date in which the RTCA has been shown to fail in purely Ricardian economies, with no frictions of any kind.

THEOREM 3. *If $u_n \in \mathscr{U}_{ACI}$ for each $n \in N$, there exists an equilibrium in which countries produce and export **only** the good in which they have a comparative advantage. There also simultaneously exists an equilibrium in which countries produce and export **only** the good in which they have a comparative **dis**advantage.*

A distinction is opportune: the set of equilibria in which the RTCA holds is a proper subset of the set of equilibria of frictionless economies with infinitely many agents. Indeed, as Theorem 3 makes clear, frictionless economies featuring large numbers of agents are not synonymous with the prevalence of the RTCA. Some further qualifications are needed for that (see the next section). To be mathematically precise, it is the *measure* of individual agents (or sets of agents) that matters. In particular, even with uncountably many agents, and a purely nonatomic measure, an equilibrium in which agents behave as per the predictions of the RTCA may still not exist—e.g., if the set of agents is the Cantor set, such that autarky then becomes the unique equilibrium.

To complete our analysis, we will extend the model(s) that we have analysed so far to a setup with an atomless continuum of workers in each country. In this context, workers have no market power whatsoever. We show that the RTCA is restored, and we establish the conditions under which N.E of our model are also Walrasian equilibria, thus providing a Cournotian foundation to the traditional RTCA.

5 The Ricardian market game $\Gamma_{\mathfrak{c}}$ with a continuum of agents

We let the set of agents be denoted by $N = F \cup H$, where $F = (0, 1]$ and $H = (1, 2]$. The collection of all half-open intervals in $(0, 2]$ defined by $\mathscr{S} = \{(a, b] : a, b \in N\}$, where $(a, b] = \emptyset$ if $b \leq a$, is a semiring. Let the set function $\nu : \mathscr{S} \to [0, \infty)$ be such that $\nu((a, b]) = b - a$. Then, ν is a σ-finite measure on $\mathscr{S}$, and

[14] In C–G and Ruffin (2003), there is a threshold number of Cournot traders below which there is world economic inefficiency, but above which complete world production efficiency prevails.

it generates a nonnegative extended real-valued set function μ, the Carathéodory extension of ν, defined on $\mathcal{P}(N)$. Let $\mathcal{N}$ denote the collection of all the μ-measurable subsets of N. Then, $\mathcal{N}$ is a σ-algebra, and μ is the Lebesgue measure when restricted to $\mathcal{N}$. Hence, the triple $(N, \mathcal{N}, \mu)$ is a complete, finite, and atomless measure space of agents. Any individual worker n—and more generally, any $\mathcal{W} \in \mathcal{N}$, such that $\mu(\mathcal{W}) = 0$—is without influence on the market(s), and will therefore, w.l.o.g., always be ignored. The strategy sets of agents are described by a measurable correspondence $S : N \rightrightarrows 2^{\mathbb{R}^4_+}$, such that

$$S(n) = \Big\{(b, l) \in \mathbb{R}^4_+ : \sum_{k=1}^{2} l_k(n) \leq Q(n); \sum_{k=1}^{2} b_k(n) \leq e_m(n)\Big\}.$$

A strategy profile is a pair of measurable mappings $b : N \to \mathbb{R}^2_+$ and $l : N \to \mathbb{R}^2_+$, such that $(b(n), l(n)) \in S(n)$ $a.e$ in N, i.e., a strategy profile is a measurable choice from the graph of the correspondence S, $Gr(S)$. It is easily seen that $S : N \rightrightarrows 2^{\mathbb{R}^4_+}$ has a measurable graph, and therefore such mappings exist, by *Aumann's Measurable Selection Theorem*. For a given strategy profile $(b, l) \in Gr(S)$, we define $B_k = \int_N b_k(n) d\mu(n)$, and $\phi_k = \frac{1}{a_k^F} \int_F l_k(f) d\mu(f) + \frac{1}{a_k^H} \int_H l_k(h) d\mu(h)$. As previously, transactions at each trading post clear through the price $p_k = B_k/\phi_k$. Consumption assignments of each commodity $k \in \{1, 2, m\}$, to any worker $j \in J$, $J = F, H$, are determined as shown:

$$x_k(j) = \begin{cases} b_k(j) \dfrac{\phi_k}{B_k} & \text{where } k \in \{1, 2\}, \text{ and } B_k \cdot \phi_k \neq 0; \\ 0 & \text{where } k \in \{1, 2\}, \text{ and } B_k \cdot \phi_k = 0; \end{cases}$$

$$x_m(j) = e_m(j) - \sum_{k=1}^{2} b_k(j) + \sum_{k=1}^{2} \frac{l_k(j)}{a_k^J} \frac{B_k}{\phi_k},$$

where as in the previous section, any division by zero, including $0/0$, is taken to equal zero. The trading post for good $k \in \{1, 2\}$ is said to be active if $B_k \cdot \phi_k > 0$.

Given a profile $(b, l) \in Gr(S)$, agents are viewed as solving the same problem as in (1). For the sake of completeness, we next define an N.E for our game.

Definition 1. *A strategy profile $(b^*, l^*) \in Gr(S)$ is an N.E of the market game iff*[15]

$$u_n\Big(x\big(b^*(n), l^*(n), B, Q\big)\Big) \geq u_n\Big(x\big(b(n), l(n), B, Q\big)\Big) \text{ for all } (b(n), l(n)) \in S(n), \ \mu\text{-}a.e, n \in N.$$

[15] As in Dubey and Shapley (1994), $u : N \times \mathbb{R}^3_+ \to \mathbb{R}$ given by $u(n, x) = u_n(x)$ is assumed to be measurable (where $N \times \mathbb{R}^3_+$ is equipped with the σ-field generated by the product of $\mathcal{N}$ and the Borel sets of $\mathbb{R}^3_+$).

5.1 Nash equilibria, Walrasian equilibria, and further comments

In this subsection, we will analyse the equilibrium properties of the market game with a continuum of agents. Our main result is formalised in the following proposition:

Proposition 2. *At any N.E of $\Gamma_{\mathfrak{c}}$ with positive bids and offers, the RTCA holds.*

The above conclusion, together with our analyses in §§3 and 4, establishes the fact that the failure of the RTCA operates in tandem with strategic behaviour. The RTCA holds in the current context because individual agents have no market power with which to influence the market-clearing prices and resulting allocations. Hence, price-taking behaviour prevails. This needs no further elaboration, as this conclusion is consistent with the findings of existing international trade models.

What needs to be elucidated, however, is the following crucial issue: does the fact that the economy is now populated *only* by price-taking agents mean that any N.E allocation of this model is also a perfectly-competitive outcome? As the reader may have already guessed, this need not be the case. If we are to establish a link between competitive (Walrasian) equilibria, and Nash equilibria when the latter are based on a commodity money which is fixed in supply, we must tackle the question of liquidity. This is simply because in a Walrasian equilibrium, it is only the relative prices of commodities that matter, such that liquidity constraints are irrelevant. On the other hand, a strategic market game's defining feature is that of fiscally-driven trade. As a result, if traders are bindingly financially-constrained at an N.E, then it is possible that the latter not be a competitive equilibrium (and vice versa). Thus, to circumvent this problem, traders must be given enough money. But what constitutes "enough" money? Borrowing from Dubey and Shapley (1994), we say that for an N.E to have prices and allocations that are competitive, all (except for possibly a measure-zero set of) traders must be "interior," i.e., they must *willingly* stop spending before their liquidity limits are reached.

In view of the above, following Dubey and Shapley (1994), we formalise this notion. Let $(b^*, l^*) \in Gr(S)$ be given. The resulting allocation x^* and price vector $p^* = (p_1^*, p_2^*, 1) \in \mathbb{R}^3_+$ are defined to constitute a Walrasian equilibrium if

1. $(b^*, l^*) \in Gr(S)$ is an active N.E of $\Gamma_{\mathfrak{c}}$, and;
2. $\mu\text{-}a.e., n \in N, \sum_{k=1}^{2} b_k^*(n) \ll e_m(n)$.

We next provide two examples which will shed light on all of the above points.

5.1.1 Example 1: An N.E which is not Walrasian

We let the set of agents be defined as in §5 above. Both countries F and H have differing constant-returns-to-scale technologies to produce goods 1 and 2, and the consumption set of each agent is a subset of $\mathbb{R}^3_+$. Every player μ-$a.e, n \in N$ possesses 10 units of commodity m, and each agent is endowed with labour hours as shown below:

$$Q(n) = 6,\ \mu\text{-}a.e, n \in N.$$

The technologies possessed by each country for the production of the two goods are as follows:

$$(a_1^F, a_1^H) = (\tfrac{3}{2}, 4);$$
$$(a_2^F, a_2^H) = (\tfrac{15}{7}, 5).$$

The preferences of the agents are represented by the following utility functions:

$$u_f(x) = \tfrac{9}{4}\ln(x_1(f)) + \tfrac{3}{2}\ln(x_2(f)) + 2\ln(x_m(f)), \quad \mu\text{-}a.e, f \in F;$$
$$u_h(x) = 94\ln(x_1(h)) + 16\ln(x_2(h)) + 30\ln(x_m(h)),\ \mu\text{-}a.e, h \in H.$$

In light of the above, it can be verified that the following profile of strategies constitutes an N.E:

$$\big(b_1(f), b_2(f), l_1(f)/a_1^F, l_2(f)/a_2^F\big) = (6, 4, 4, 0), \quad \mu\text{-}a.e, f \in F;$$
$$\big(b_1(h), b_2(h), l_1(h)/a_1^H, l_2(h)/a_2^H\big) = (\tfrac{94}{11}, \tfrac{16}{11}, 0, \tfrac{6}{5}),\ \mu\text{-}a.e, h \in H.$$

The corresponding market-clearing prices to the strategies above are:

$$(p_1, p_2) = (\tfrac{40}{11}, \tfrac{50}{11}),$$

and each agent ends up with consumption as follows

$$\big(x_1(f), x_2(f), x_m(f)\big) = (\tfrac{33}{20}, \tfrac{22}{25}, \tfrac{160}{11}),\ \mu\text{-}a.e, f \in F;$$
$$\big(x_1(h), x_2(h), x_m(h)\big) = (\tfrac{47}{20}, \tfrac{8}{25}, \tfrac{60}{11}),\ \ \mu\text{-}a.e, h \in H.$$

However, this economy has a unique Walrasian equilibrium, with

$$\begin{aligned}\big(x_1(f), x_2(f), x_m(f)\big) &= \big(\tfrac{3924}{1921}, \tfrac{1962}{2035}, \tfrac{4360}{293}\big),\ \mu\text{-}a.e, f \in F;\\ \big(x_1(h), x_2(h), x_m(h)\big) &= \big(\tfrac{3760}{1921}, \tfrac{96}{407}, \tfrac{1500}{293}\big),\ \ \mu\text{-}a.e, h \in H.\end{aligned}$$

and (relative) market-clearing prices

$$(p_1, p_2) = \big(\tfrac{9605}{1172}, \tfrac{10175}{879}\big).$$

Clearly, the N.E strategy profile presented in this example does not satisfy condition 2 as laid out in §5.1. As such, the resulting N.E allocation fails to be Walrasian. In particular, when an agent is bindingly financially-constrained, there exist other bundles which s/he would have preferred to the N.E allocation, but cannot afford. The agent "suffers" from a lack of liquidity. However, this does not mean that all agents are worse off at the non-Walrasian equilibrium than they are at the Walrasian one. In fact, this example has the following interesting characteristic: while trade at the N.E above takes place in accordance with the RTCA, a quick calculation shows that $\int_N u_n(x^{N.E})d\mu(n) > \int_N u_n(x^{Walras})d\mu(n)$—i.e., the gains from trade at the N.E are larger than they are in the Walrasian case. This reaffirms the well-known fact that while a Walrasian equilibrium is Pareto efficient, there is no reason to assume that this allocation is not distributionally unjust and highly inequitable. This point, while innocuous, must be borne in mind when comparing different equilibrium allocations. ♦

5.1.2 Example 2: An N.E which is also Walrasian

We let the set of agents be defined as in §5. Both countries F and H have differing constant-returns-to-scale technologies to produce goods 1 and 2, and the consumption set of each agent is a subset of $\mathbb{R}^3_+$. Every player μ-$a.e, n \in N$ possesses 10 units of commodity m, and each agent is endowed with labour hours as shown below:

$$Q(n) = 6,\ \mu\text{-}a.e, n \in N.$$

The technologies possessed by each country for the production of the two goods are as follows:

$$(a_1^F, a_1^H) = (\tfrac{3}{2}, 4);$$
$$(a_2^F, a_2^H) = (\tfrac{15}{7}, 5).$$

The preferences of the agents are represented by the following utility functions:

$$u_f(x) = 18\ln(x_1(f)) + 9\ln(x_2(f)) + 33\ln(x_m(f)),\ \mu\text{-}a.e, f \in F;$$
$$u_h(x) = 12\ln(x_1(h)) + 3\ln(x_2(h)) + 27\ln(x_m(h)),\ \mu\text{-}a.e, h \in H.$$

It may be verified that the following profile of strategies constitutes an N.E:

$$\begin{cases} \big(b_1(f), b_2(f), l_1(f)/a_1^F, l_2(f)/a_2^F\big) = (6, 3, 4, 0),\ \mu\text{-}a.e, f \in F; \\ \big(b_1(h), b_2(h), l_1(h)/a_1^H, l_2(h)/a_2^H\big) = (4, 1, 0, \tfrac{6}{5}),\ \mu\text{-}a.e, h \in H. \end{cases}$$

The corresponding market-clearing prices to the strategies above are

$$(p_1, p_2) = (\tfrac{5}{2}, \tfrac{10}{3}),$$

and each agent ends up with consumption

$$\begin{cases} \big(x_1(f), x_2(f), x_m(f)\big) = (\tfrac{33}{20}, \tfrac{22}{25}, \tfrac{160}{11}),\ \mu\text{-}a.e, f \in F; \\ \big(x_1(h), x_2(h), x_m(h)\big) = (\tfrac{47}{20}, \tfrac{8}{25}, \tfrac{60}{11}),\ \ \mu\text{-}a.e, h \in H. \end{cases}$$

(relative) prices and allocations which can be verified to be Walrasian as well. ♦

6 Conclusion

In this paper, we have provided a comprehensive treatment of the RTCA from a market-game perspective. We have shown that the law of comparative advantage does not always hold true. Free trade does not *necessarily* lead countries to specialise based on comparative advantages. We have also proved that even in large economies consisting of a countable infinity of agents, the RTCA still fails. Verily, the statements that "It is this principle [of comparative advantage] which determines that wine shall be made in France and Portugal, that corn shall be grown in America and Poland, and that hardware and other goods shall

be manufactured in England" (Ricardo, 1817: p. 90), and "Even with economies of scale and imperfectly competitive markets, for a wide variety of market structures, differences in the characteristics of countries are a major predictor of the pattern of trade" (Helpman and Krugman, 1986: p. 261), must be tempered: it is only when agents are price-taking *and* perfectly-competitive (so that equilibria are not trivial), as opposed to being strategic, that the RTCA prevails. We conclude that the failure of the RTCA is, therefore, intimately related to the introduction of even the slightest hint of strategic element in agents' decision-making process.

Certainly, our model of international trade is stylised, and we do not claim that it constitutes a unifying framework that accurately captures the intricacies of any particular type of trade between countries. Notwithstanding this fact, we believe that our simple formulation does still provide an alternative, but equally plausible explanation of the current goings-on in the international scene. More precisely, it makes a compelling case for countries to engage in, and benefit from free international trade, even when no specialisation as per the RTCA takes place.

In terms of future research, considering the finite- and infinite-agent contexts analysed in this work, a meaningful problem to address would be whether the failure of the RTCA as we have studied also applies to Ricardian frameworks with infinitely many goods of the kind proposed by Dornbusch et al. (1977). Another interesting avenue would be to, in view of our conclusions, test whether the RTCA also fails in a laboratory setting. As evidenced by the studies in Jiang et al. (2021), Ding and Puzzello (2019), Barreda-Tarrazona et al. (2018), and Duffy et al. (2011), the market game framework lends itself well to experiments (and international economics). The simplicity of the model we put forth should make this endeavour feasible.

Appendix

Proof of Proposition 1. Suppose that the trivial outcome of no bids being placed and no offers being made were an N.E for our market game, and let this N.E strategy profile be denoted by $\zeta = \{(b_n, l_n)\}_{n\in N} = (\mathbf{0}, \mathbf{0})$. Choose some $j \in J$, $J = F, H$, and let the strategies of all $n \in N \setminus \{j\}$ be given. Next, choose any sufficiently small $\epsilon > 0$, and consider for j, the strategy $\zeta_j^\epsilon = \left\{\left(b_{j,k}^\epsilon, l_{j,k}^\epsilon\right)\right._{k=1}^{2} = \left\{\left(\epsilon, a_k^J \cdot \epsilon\right)\right._{k=1}^{2}$. This strategy generates a market-clearing price $p_k = 1$ in all markets for commodities $k \in \{1, 2\}$. In particular, for $j \in J$, j's consumption of every good k is equal to ϵ, such that j's total bids, and total revenue from the labour hours offered are both equal to 2ϵ. So, for every $k \in \{1, 2\}$, we have that $x_{j,k}^{\zeta^\epsilon} > x_{j,k}^{\zeta} = 0$, while $x_{j,m}^{\zeta^\epsilon} = x_{j,m}^{\zeta} > 0$. Given that for all $x_j \in \mathbb{R}^3_{++}$, $\frac{\partial u_j}{\partial x_{j,k}} > 0$, $k = 1, 2, m$, it follows that for j, we must have

$u_j(x_j^{\zeta_\epsilon}) > u_j(x_j^{\zeta})$, thereby contradicting the claim that ζ was an N.E. So we know that there is a $k \in \{1,2\}$, the market for which is active at equilibrium.

We now show that ***every agent*** buys at least one $k \in \{1,2\}$ at equilibrium. There are 2 N.E scenarios to consider: (i) the market for, say, good 1 is active while no one trades in good 2, and; (ii) both markets are active. Consider Case (i). Suppose, by way of contradiction, that at an N.E, which we call ϑ, there were a worker $j \in J, J = F, H$, who did not trade at all. Then, for $\delta > 0$ and sufficiently small, by playing the strategy $(b_{j,1}^{\delta}, l_{j,1}^{\delta}) \coloneqq (\delta B_{-j,1}, \delta a_1^J \phi_{-j,1})$ in the market for good 1, and $(b_{j,2}^{\delta}, l_{j,2}^{\delta}) \coloneqq (\delta, a_2^J \cdot \delta)$ in the market for good 2, worker j can ensure that s/he gets $x_{j,k}^{\vartheta^\delta} > x_{j,k}^{\vartheta} = 0$, for every $k \in \{1,2\}$ while $x_{j,m}^{\vartheta^\delta} = x_{j,m}^{\vartheta} > 0$, such that $u_n(x_j^{\vartheta^\delta}) > u_n(x_j^{\vartheta})$, thus contradicting the claim that ϑ was an N.E. Consider Case (ii) now. Again, by way of contradiction, assume that at an N.E, ϑ, there were a worker j who did not trade at all. Then, for $\delta > 0$ and sufficiently small, by playing the strategy $\{(b_{j,k}^{\delta}, l_{j,k}^{\delta})\}_{k=1}^{2} = \{(\delta B_{-j,k}, \delta a_k^J \phi_{-j,k})\}_{k=1}^{2}$, worker j can ensure that s/he gets $x_{j,k}^{\vartheta^\delta} > x_{j,k}^{\vartheta} = 0$ for every $k \in \{1,2\}$, while $x_{j,m}^{\vartheta^\delta} = x_{j,m}^{\vartheta} > 0$, such that $u_n(x_j^{\vartheta^\delta}) > u_n(x_j^{\vartheta})$, a contradiction. Hence, at any N.E, all agents trade in at least one $k \in \{1,2\}$. □

While the proofs for Theorems 1 and 2 are very similar in essence, the *findings* in Theorem 2 are novel, to the extent that they may even appear "counterintuitive" at first glance. We therefore deem it more instructive to prove Theorem 2 first, providing an explicitly labelled step-by-step breakdown of the latter, which should help to dispel any reservation the reader may have. Theorem 1 can subsequently be proved using the same method. Below, we provide a non-technical roadmap to the proof of Theorem 2.

Our approach consists of first proving that all the conditions for successfully applying Kakutani's fixed-point theorem (KFT) in a partially offer-constrained market game are met. To this end, we begin by showing in Lemma 1 that the consumption possibilities set[16] (CPS) of each agent is convex. This allows us to demonstrate that each agent's optimisation problem is well-behaved, and the first-order conditions necessary and sufficient. That is, any N.E is ***completely characterised*** by the first-order conditions. Convexity of the CPS is also needed to prove that our correspondence of interest, Θ_n (which we define later in our proof) is convex-valued. In Lemma 2, we prove that the CPS of every agent is compact. This is needed to exhibit that a best-response to every agent's optimisation problem actually exists. We then derive and use the first-order conditions to prove that equilibrium bids for all agents are bounded away from zero. This result allows us to show that the payoff function for every n is continuous, and Θ_n upper hemicontinuous. We then use KFT to prove the existence of an active equilibrium. Our next task consists of showing that the constrained market game equilibrium obtained is also an N.E of the unconstrained

[16]This is just the budget set of the agent, i.e., the set of all feasible allocations, given the strategies of all the other players.

game. We do so through Lagrange multiplier methods. Our desired result is finally achieved by taking the limit of a sequence of N.E of partially offer-constrained games.

Lemma 1. *The consumption possibilities set (CPS) for each $n \in N$ is (strictly) convex.*

Proof. First, note that convexity of the CPS cannot be established as in standard market game models. This is because, as opposed to the latter, in which agents are endowed with at least one commodity $k \in \{1, 2\}$, in our formulation, agents are not endowed with any of goods 1 and 2. We will therefore adopt an approach different from those which are routinely used in the literature. So, let M_n denote the set of all feasible allocations for any $n \in N$. Let $x_n^{'}$ and $x_n^{''} \in M_n$, where $x_n^{'}$ and $x_n^{''}$ are obtained by the vectors of strategies $(b_n^{'}, l_n^{'})$ and $(b_n^{''}, l_n^{''})$, respectively. We need to show that, *given the strategies of all the other agents*, for all $\lambda \in [0, 1]$, $\lambda x_n^{'} + (1 - \lambda)\, x_n^{''} \in M_n$—i.e., if $x_n^* \equiv \lambda x_n^{'} + (1 - \lambda)\, x_n^{''}$, then we want to show that $\exists (b_n^*, l_n^*)$ feasible such that $x(b_n^*, l_n^*) = x_n^*$. So, first, fix any $\lambda \in [0, 1]$, which we will use *throughout the whole proof*. Consider next a commodity $k \in \{1, 2\}$, and define,

$$x_{n,k}^{'} = x(b_{n,k}^{'}, l_{n,k}^{'});$$
$$x_{n,k}^{''} = x(b_{n,k}^{''}, l_{n,k}^{''}).$$

In what follows, for ease of exposition, we drop the n subscript. Now, notice that the statement of Lemma 1 is trivially true if $x_k^{'} \cdot x_k^{''} = 0$. Hence, we prove it for the case when $x_k^{'} \cdot x_k^{''} > 0$. W.l.o.g, let $l_k^{''} \geq l_k^{'}$. Since x_k is increasing in l_k (holding b_k constant), it must be true that

$$x_k^{'} = x(b_k^{'}, l_k^{'}) \leq x(b_k^{'}, l_k^{''}) \equiv \bar{x}_k.$$

But x_k is strictly concave in b_k (holding l_k constant), such that we have

$$\begin{aligned} x_k^* &= \lambda x_k^{'} + (1 - \lambda)\, x_k^{''} \\ &\leq \lambda \bar{x}_k + (1 - \lambda)\, x_k^{''} = \lambda x(b_k^{'}, l_k^{''}) + (1 - \lambda)\, x(b_k^{''}, l_k^{''}) \\ &< x(\lambda b_k^{'} + (1 - \lambda)\, b_k^{''}, l_k^{''}) \equiv\ x(b_k^{\lambda}, l_k^{''}). \end{aligned}$$

x_k is continuous, and increasing in b_k (holding l_k constant). So, define a continuous surjection $\Omega_{l^{''}} : [0, b_k^{\lambda}] \to x(\cdot, l_k^{''})$.[17] Now, observe that $0 = \Omega_{l^{''}}(0) < x_k^* < \Omega_{l^{''}}(b_k^{\lambda})$, and hence, $\exists b_k^* \in (0, b_k^{\lambda})$ such that

[17] More precisely, $\Omega_{l^{''}} : [0, b_k^{\lambda}] \to x(\cdot, l_k^{''})$ is such that for any $\hat{b}_k \in [0, b_k^{\lambda}]$, we have $\Omega_{l^{''}}(\hat{b}_k) = x(\hat{b}_k, l_k^{''})$.

$\Omega_{l''}(b_k^*) = x_k^*$, by the *Intermediate Value Theorem* (IVT). Certainly, the same procedure can be followed for commodity $t \in \{1,2\} \setminus \{k\}$. We thus have that the set of feasible allocations of commodities produced is strictly convex. We next need to perform an appropriate analysis for x_m. This is because x_m *is already determined* by $\left((b_k, l_k)\right)_{k=1}^2$, and thus, the previous method (of applying the IVT) cannot be used for this part of the proof. Moreover, note that as opposed to standard market game models,[18] in our setup, x_m is not expressible as an explicit—but instead as an implicit—function of $(x_k)_{k=1}^2$.

So, consider $x^{'}, x^{''} \in M$, where $x^{'} = \left((x_k^{'})_{k=1}^2, x_m^{'}\right)$ is induced by $((b_k^{'}, l_k^{'}))_{k=1}^2$, and $x^{''} = \left((x_k^{''})_{k=1}^2, x_m^{''}\right)$ is induced by $\left((b_k^{''}, l_k^{''})\right)_{k=1}^2$, and $(x_k^{'})_{k=1}^2$, $(x_k^{''})_{k=1}^2$ are as in the first part of the proof. We need to show that given $x_m^{'}$ and $x_m^{''}$, then, for $\lambda \in [0,1]$ as in the first part of our proof, we have that $\lambda x_m^{'} + (1-\lambda)x_m^{''}$ is still attainable. In particular, since all $\left((x_k^{'})_{k=1}^2, x_m^{'}\right), \left((x_k^{''})_{k=1}^2, x_m^{''}\right) \in M$ are *surjective* and continuous in b and l, if we prove that $\lambda x_m^{'} + (1-\lambda)x_m^{''} \leq x_m^*$, where x_m^* corresponds to $(x_k^*)_{k=1}^2$ induced by $\left((b_k^*, l_k^*)\right)_{k=1}^2$ as above, then we will be done. So, let $x_m = e_m + \sum_{k=1}^2((l_k B_k / a_k^J \phi_k) - b_k)$ be denoted by $e_m + \sum_{k=1}^2 z_k(b_k, l_k)$, and define

$$z_k(b_k^{'}, l_k^{'}) = z_{m,k}^{'};$$
$$z_k(b_k^{''}, l_k^{''}) = z_{m,k}^{''}.$$

Again, w.l.o.g., consider first a commodity $k \in \{1,2\}$. In keeping with the analysis in the first part of our proof, we let $l_k^{''} \geq l_k^{'}$. As $z_{m,k}$ is increasing in l_k (holding b_k constant), we have that

$$z_{m,k}^{'} = z_k(b_k^{'}, l_k^{'}) \leq z_k(b_k^{'}, l_k^{''}).$$

Now, since $z_{m,k}$ is linear in b_k (holding l_k constant), we have that

$$\begin{aligned} z_{m,k}^{\lambda} \equiv \lambda z_{m,k}^{'} + (1-\lambda)\, z_{m,k}^{''} &= \lambda z_k(b_k^{'}, l_k^{'}) + (1-\lambda)\, z_k(b_k^{''}, l_k^{''}) \\ &\leq \lambda z_k(b_k^{'}, l_k^{''}) + (1-\lambda)\, z_k(b_k^{''}, l_k^{''}) \\ &= z_k(\lambda b_k^{'} + (1-\lambda) b_k^{''}, l_k^{''}) \equiv z_k(b_k^{\lambda}, l_k^{''}). \end{aligned}$$

The above set of relations can be summarised as $z_{m,k}^{\lambda} \leq z_k(b_k^{\lambda}, l_k^{''})$. Now, from the first part of our proof,

[18] See Dubey and Shubik (1978), Peck et al. (1992), and Toraubally (2018) for cases in which x_m is expressed as a concave function of $(x_k)_{k=1}^2$.

we have that $x_k^* = x(b_k^*, l_k'')$, where $b_k^* \in (0, b_k^\lambda)$. Since $z_{m,k}$ is decreasing in b_k (holding l_k constant),

$$z_{m,k}^\lambda = \lambda z_{m,k}^{'} + (1-\lambda)\, z_{m,k}^{''} \leq z_k(b_k^\lambda, l_k'') < z_k(b_k^*, l_k'').$$

Certainly, the same algorithm can be applied to good $t \in \{1,2\} \setminus \{k\}$. So:

$$\begin{gathered}
z_{m,k}^\lambda < z_k(b_k^*, l_k'') \;\; \forall k \in \{1,2\} \\
\Downarrow \\
\sum_{k=1}^{2} z_{m,k}^\lambda < \sum_{k=1}^{2} z_k(b_k^*, l_k'') \\
\Updownarrow \\
\lambda \sum_{k=1}^{2} z_{m,k} + (1-\lambda) \sum_{k=1}^{2} z_{m,k}^{''} < \sum_{k=1}^{2} z_k(b_k^*, l_k'') \\
\Updownarrow \\
\lambda \left(e_m + \sum_{k=1}^{2} z_k(b_k^{'}, l_k^{'}) \right) + (1-\lambda) \left(e_m + \sum_{k=1}^{2} z_k(b_k^{''}, l_k^{''}) \right) < e_m + \sum_{k=1}^{2} z_k(b_k^*, l_k'') \\
\Updownarrow \\
\lambda x_m^{'} + (1-\lambda) x_m^{''} < x_m^*.
\end{gathered}$$

With this, we have shown that the consumption possibilities set is indeed (strictly) convex. □

Given the concavity of utility functions and convexity of the CPS, we may switch to strategy space to derive the first-order necessary and sufficient conditions, which are as follows:

$$\left.\begin{aligned}
\frac{\partial u_n}{\partial x_{n,k}} \cdot \left(\frac{B_{-n,k}\phi_k}{(B_k)^2} \right) &= \lambda_n^b + \frac{\partial u_n}{\partial x_{n,m}} \cdot \left(\frac{\phi_{-n,k}}{\phi_k} \right) - \lambda_{n,k}^{b \geq 0}; \\
\frac{\partial u_n}{\partial x_{n,k}} \cdot \left(\frac{b_{n,k}}{a_k^J B_k} \right) &= \lambda_n^l - \frac{\partial u_n}{\partial x_{n,m}} \cdot \left(\frac{\phi_{-n,k}}{(\phi_k)^2} \cdot \frac{B_k}{a_k^J} \right) - \lambda_{n,k}^{l \geq 0},
\end{aligned}\right\} k \in \{1,2\} \tag{2}$$

where $J = F, H$, depending on which country n is a member of.

Lemma 2. *The CPS for each $n \in N$ is compact.*

Proof. We first prove that the CPS is closed. Consider any $n \in N$, let n's strategies be denoted by $s_n \in S_n$, and let the strategies of all agents in $N \setminus \{n\}$ be given. Next, consider any sequence of strategies $\{s_n^r\}_{r \in \mathbb{N}}$ in S_n and the corresponding allocations $\{x_n(s_n^r)\}_{r \in \mathbb{N}}$. For all $r \in \mathbb{N}$, we have that $s_n^r \in [0, e_{n,m}]^2 \times [0, Q_n]^2$, a compact subset of $\mathbb{R}_+^2 \times \mathbb{R}_+^2$. Hence, by extracting a subsequence if necessary, we have that $\lim_{r \to \infty} s_n^r =$

$s_n^* \in S_n$. But $x_n(\cdot)$ is continuous in s_n, such that $\lim_{r\to\infty} x_n(s_n^r) = x_n(s_n^*)$, which clearly lies in n's CPS. Thus, n's CPS is closed. Next, since S_n is compact and $x_n(\cdot)$ continuous in s_n, we have that $x_n(\cdot)$ is also bounded, by the *Boundedness Theorem*. □

Before moving on to our main proof, we introduce the following definition of a partially offer-constrained game. We do so by modifying Peck et al.'s (1992) (fully) offer-constrained game.

Definition 2. For every $f \in F$, $h \in H$, and $a, b = 1, 2$, set $l_{f,a}$ to be such that $l_{f,a} \geq \mathscr{F}$, and $l_{h,b}$ to be such that $l_{h,b} \geq \mathscr{H}$, where $\mathscr{F}, \mathscr{H} \geq 0$, and $l_{n,1} + l_{n,2} \leq Q_n \; \forall n \in N$. Let $S_f(l_{f,a} \geq \mathscr{F}) = \{(b_f, l_f) \in \mathbb{R}^4_+ : \sum_{k=1}^2 b_{f,k} \leq e_{f,m}; \sum_{k=1}^2 l_{f,k} \leq Q_f; l_{f,a} \geq \mathscr{F}\}$, and $S_h(l_{h,b} \geq \mathscr{H}) = \{(b_h, l_h) \in \mathbb{R}^4_+ : \sum_{k=1}^2 b_{h,k} \leq e_{h,m}; \sum_{k=1}^2 l_{h,k} \leq Q_h; l_{h,b} \geq \mathscr{H}\}$. The partially offer-constrained market game $\Gamma(l_{f,a} \geq \mathscr{F}; l_{h,b} \geq \mathscr{H})$ is constructed from Γ by replacing the strategy set $\mathbb{R}^{4|N|}_+ \supset S = \prod_{f\in F} S_f \times \prod_{h\in H} S_h$ with the strategy set $\mathbb{R}^{4|N|}_+ \supset \prod_{f\in F} S_f(l_{f,a} \geq \mathscr{F}) \times \prod_{h\in H} S_h(l_{h,b} \geq \mathscr{H})$.

In the next result, we analyse a partially offer-constrained game, and we first demonstrate that at any N.E of such a game, every agent $j \in J, J = F, H$, purchases a strictly positive amount of each commodity $k \in \{1, 2\}$. We then make systematic use of a limiting argument to prove the more specific properties of the equilibria under consideration.

Proof of Theorem 2. In the first part of our proof, we will establish the existence of *an* equilibrium, the nature of which we will derive later. Along the way, we will sometimes use and modify as needed arguments from Dubey and Shubik (1978) and Spear (2003). For ease of exposition, we will introduce two mappings $\mathscr{A} : N \to \{1, 2\}$ and $\mathscr{D} : N \to \{1, 2\}$, where $\mathscr{A}$ and $\mathscr{D}$ map workers to the good in which they have a comparative advantage, and disadvantage, respectively. Consider the market game $\Gamma(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$, in which the labour hours offered by each $j \in J, J = F, H$, towards the good in which country J has a comparative disadvantage are constrained to be at least half their initial endowment of labour hours.

Step 1. Proving each n's best-response set is non-empty

Pick any worker $n \in N$, and let n's strategy set in $\Gamma(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$ be denoted by $\tilde{S}_n$. We will use the notation $\tilde{s}_n \in \tilde{S}_n$ to denote the strategies of worker n, and $\tilde{s}_{-n} = (\tilde{s}_1, \ldots, \tilde{s}_{n-1}, \tilde{s}_{n+1}, \ldots, \tilde{s}_{|N|}) \in \tilde{S}_{-n}$ the strategies of all agents but n. Let n's consumption possibilites set in $\Gamma(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$ be denoted by

$$\tilde{M}_n = \Big\{x_n \in \mathbb{R}^3_+ : x_{n,m} = e_{n,m} - \sum_{k=1}^2 b_{n,k} + \sum_{k=1}^2 \frac{l_{n,k}}{a_k^J} p_k \text{ and } x_{n,k} = \frac{b_{n,k}}{p_k}, k = 1, 2, \text{ where } (b_n, l_n) \in \tilde{S}_n\Big\}.$$

Before proceeding, let us prove that $n's$ best-response set is non-empty. This follows from *Weierstrass' Theorem*: $u(\cdot)$ is a continuous function of x_n, where $x_n \in \tilde{M}_n$, which by (a simple modification of) Lemma 2 above is compact. Thus, we know that a solution to n's maximisation problem exists.

Step 2. Proving all equilibrium bids are uniformly bounded above zero

- Case 1: Pick any $n \in N$, and suppose that n's liquidity constraint is slack, i.e., $b^*_{n,1} + b^*_{n,2} \ll e_{n,m}$.

We prove that for any $k \in \{1,2\}$, there exist positive constants D_1 and D_2, such that $p^*_k = \frac{B^*_k}{\phi^*_k} < D_k < \infty$, where p^*_k, B^*_k, ϕ^*_k represent, respectively, the price, aggregate bids, and aggregate offers, at equilibrium. To establish the existence of D_k, $k = 1,2$, consider w.l.o.g., any $f \in F$, and let us look at commodity $\mathscr{D}(f)$—which is in fact commodity $\mathscr{A}(h)$ for any $h \in H$. By the Inada condition, we know that for any $n \in N$, $b^*_{n,1} \cdot b^*_{n,2} > 0$. Thus, we may derive

$$p^*_{\mathscr{D}(f)} = \frac{B^*_{\mathscr{D}(f)}}{\phi^*_{\mathscr{D}(f)}} < \frac{\sum_{n\in N} e_{n,m}}{\phi^*_{\mathscr{D}(f)}} \leq \frac{2a^F_{\mathscr{D}(f)} \sum_{n\in N} e_{n,m}}{\sum_{f\in F} Q_f} \equiv D_{\mathscr{D}(f)},$$

where the second (weak) inequality follows from the fact that by construction, $l_{f,\mathscr{D}(f)} \geq \frac{Q_f}{2}$. Performing the same operation for commodity $\mathscr{A}(f)$ yields $D_{\mathscr{A}(f)} = (2a^H_{\mathscr{A}(f)} \sum_{n\in N} e_{n,m}) / \sum_{h\in H} Q_h$. Now, consider for n the following equation from (2):

$$\frac{\partial u_n}{\partial x_{n,k}} \cdot \left(\frac{B^*_{-n,k}\phi^*_k}{\left(B^*_k\right)^2} \right) = \frac{\partial u_n}{\partial x_{n,m}} \cdot \left(\frac{\phi^*_{-n,k}}{\phi^*_k} \right).$$

As $p^*_k < D_k$, simple manipulation of the above reveals that at ***any*** N.E with non-binding liquidity constraint, we must have

$$\frac{\partial u_n / \partial x_{n,k}}{\partial u_n / \partial x_{n,m}} < \frac{B^*_k D_k}{B^*_{-n,k}}.$$

Now, note that by the Inada condition, $\lim_{b_{n,k}\to 0} \frac{\partial u_n/\partial x_{n,k}}{\partial u_n/\partial x_{n,m}} = \infty$, while $\lim_{b_{n,k}\to 0} \frac{B^*_k D_k}{B^*_{-n,k}} = D_k < \infty$. Hence, there must exist a uniform lower bound $\underline{b}_{n,k} \gg 0$ on trader n's bid for k. The same argument can be repeated for commodity $t \in \{1,2\} \setminus \{k\}$. Thus, at equilibrium, $(b^*_{n,1}, b^*_{n,2})^\top \geq (\underline{b}_{n,1}, \underline{b}_{n,2})^\top \gg \mathbf{0}$, where $\top$ denotes transposition.

- Case 2: Suppose that n's liquidity constraint is binding, i.e., $b^*_{n,1} + b^*_{n,2} = e_{n,m}$.

Then from (2), we have that

$$\frac{\partial u_n}{\partial x_{n,k}} \cdot \left(\frac{B^*_{-n,k}\phi^*_k}{\left(B^*_k\right)^2} \right) - \frac{\partial u_n}{\partial x_{n,m}} \cdot \left(\frac{\phi^*_{-n,k}}{\phi^*_k} \right) \Bigg\{ = \frac{\partial u_n}{\partial x_{n,t}} \cdot \left(\frac{B^*_{-n,t}\phi^*_t}{\left(B^*_t\right)^2} \right) - \frac{\partial u_n}{\partial x_{n,m}} \cdot \left(\frac{\phi^*_{-n,t}}{\phi^*_t} \right).$$

Rearranging the above, we get

$$\frac{\partial u_n/\partial x_{n,k}}{\partial u_n/\partial x_{n,m}} \cdot \left(\frac{B^*_{-n,k}\phi^*_k}{\left(B^*_k\right)^2} \right) - \frac{\partial u_n/\partial x_{n,t}}{\partial u_n/\partial x_{n,m}} \cdot \left(\frac{B^*_{-n,t}\phi^*_t}{\left(B^*_t\right)^2} \right) \Bigg\{ = \frac{\phi^*_{-n,k}}{\phi^*_k} - \frac{\phi^*_{-n,t}}{\phi^*_t}.$$

It is easy to see that $\frac{\phi^*_{-n,k}}{\phi^*_k} - \frac{\phi^*_{-n,t}}{\phi^*_t} < 1$. Proceeding as in Dubey and Shubik's (1978) Lemma 2, we may derive, for any $k \in \{1,2\}$, positive constants C_1 and C_2, such that $0 < C_k < p^*_k = \frac{B^*_k}{\phi^*_k}$. Further manipulating the above equation then yields the following inequality, which must obtain at ***any*** N.E with binding liquidity constraint:

$$\Delta(b^*_{n,k}) \equiv \frac{\partial u_n/\partial x_{n,k}}{\partial u_n/\partial x_{n,m}} - \frac{\partial u_n/\partial x_{n,t}}{\partial u_n/\partial x_{n,m}} \cdot \left(\frac{B^*_k D_k}{B^*_{-n,k} C_t} \right) \Bigg\{ < \frac{B^*_k D_k}{B^*_{-n,k}}.$$

As $b^*_{n,k} \to 0$, we have that $b^*_{n,t} \to e_{n,m}$, such that eventually, $b^*_{n,t} > \frac{e_{n,m}}{2}$, implying $x_{n,t} > 0$ and therefore $\frac{\partial u_n}{\partial x_{n,t}} < \infty$. So, by the Inada condition, $\lim_{b^*_{n,k} \to 0} \Delta(b^*_{n,k}) = \infty$, while $\lim_{b^*_{n,k} \to 0} \frac{B^*_k D_k}{B^*_{-n,k}} = D_k < \infty$. Hence, there must exist a uniform lower bound $\underline{b}_{n,k} \gg 0$ on trader n's bid for k. The same argument can be repeated for commodity $t \in \{1,2\} \setminus \{k\}$. Therefore, at equilibrium, $(b^*_{n,1}, b^*_{n,2})^\top \geq (\underline{b}_{n,1}, \underline{b}_{n,2})^\top \gg \mathbf{0}$. Since n was arbitrarily chosen, we have thus ascertained that equilibrium bids are uniformly bounded away from zero $\forall n \in N$, and may now proceed.

Step 3. Proving continuity of payoff function, and upper hemicontinuity of Θ_n

Define $V_n(\tilde{s}_n, \tilde{s}_{-n}) = u_n(x_n(\tilde{s}_n, \tilde{s}_{-n}))$ to be the payoff function of each $n \in N$. Note that $V_n(\tilde{s}_n, \tilde{s}_{-n})$ is continuous, as it is the composition of continuous functions, given that equilibrium bids are bounded above zero. Let $\vartheta_n(\tilde{s}_{-n}) = \arg\max_{\tilde{s}_n \in \tilde{S}_n} V_n(\tilde{s}_n, \tilde{s}_{-n})$. Put $\tilde{S}_{n,\Xi} = [\underline{b}_{n,1}, e_{n,m} - \underline{b}_{n,2}] \times [\underline{b}_{n,2}, e_{n,m} - \underline{b}_{n,1}] \times [0, \frac{Q_n}{2}] \times [\frac{Q_n}{2}, Q_n]$, and let the correspondence $\Theta_n : \tilde{S}_{-n} \rightrightarrows \tilde{S}_{n,\Xi}$ be defined as $\Theta_n(\tilde{s}_{-n}) = \vartheta_n(\tilde{s}_{-n}) \cap \tilde{S}_{n,\Xi}$. It is straightforward to see that $\tilde{S}_{n,\Xi}$ is compact and convex, for each $n \in N$. We next show that Θ_n is upper hemicontinuous. Suppose that we have a sequence of strategies $\{\tilde{s}^r\}_{r \in \mathbb{N}}$ with $\lim_{r \to \infty} \tilde{s}^r = \tilde{s}^*$, and

a sequence of best-responses, $\{\tilde{\zeta}_n^r\}_{r\in\mathbb{N}}$ with $\lim_{r\to\infty}\tilde{\zeta}_n^r = \tilde{\zeta}_n^*$, such that we have

$$\tilde{\zeta}_n^r \in \Theta_n(\tilde{s}_{-n}^r) \text{ for all } r, \text{ and,}$$
$$\tilde{\zeta}_n^* \notin \Theta_n(\tilde{s}_{-n}^*).$$

As $\tilde{S}_{n,\Xi}$ is a compact set, it is easy to see that $\tilde{\zeta}_n^*$ is feasible. But since it is not a best-response to $\tilde{s}_{-n}^*$, it must be true, for an $\epsilon > 0$ and some other feasible unilateral deviation $\mathring{\zeta}_n$, that

$$V_n(\mathring{\zeta}_n, \tilde{s}_{-n}^*) \gg V_n(\tilde{\zeta}_n^*, \tilde{s}_{-n}^*) + 3\epsilon.$$

But then, since V_n is continuous, it must be true, for a sufficiently large $r \in \mathbb{N}$, that

$$V_n(\mathring{\zeta}_n, \tilde{s}_{-n}^r) \gg V_n(\mathring{\zeta}_n, \tilde{s}_{-n}^*) - \epsilon \gg V_n(\tilde{\zeta}_n^*, \tilde{s}_{-n}^*) + 2\epsilon \gg V_n(\tilde{\zeta}_n^r, \tilde{s}_{-n}^r) + \epsilon,$$

a contradiction, as $\tilde{\zeta}_n^r \in \Theta_n(\tilde{s}_{-n}^r)\ \forall r$.

Step 4. Exhibiting convex-valuedness of Θ_n

Observe that the utility function of each $n \in N$ is *strictly* concave in x, and that by a simple analogue of Lemma 1, $\tilde{M}_n$ is (strictly) convex, even with the additional constraint that $l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2}$. As such, given the strategies of all agents other than $n \in N$, the *Supporting Hyperplane Theorem* stipulates that if there is an allocation vector x_n^* that solves n's maximisation problem, then it must be ***unique***. Thus, Θ_n is convex-valued, as if $\tilde{\zeta}_n = (\tilde{b}_n, \tilde{l}_n) \in \Theta_n(\tilde{s}_{-n})$ and $\hat{\zeta}_n = (\hat{b}_n, \hat{l}_n) \in \Theta_n(\hat{s}_{-n})$ are arbitrary best-responses for n, then any convex combination of $\tilde{\zeta}_n$ and $\hat{\zeta}_n$ yields a convex combination of $x_n^* = x_n(\tilde{b}_n, \tilde{l}_n)$ and $x_n^* = x_n(\hat{b}_n, \hat{l}_n)$ itself. Consequently, any convex combination of $\tilde{\zeta}_n$ and $\hat{\zeta}_n$ is also a (feasible) best-response—apply the method in Lemma 1 once more for proof of, and an explanation of the geometry underlying, this fact.

Step 5. Applying Kakutani's fixed-point theorem

Finally, define $\tilde{S}_\Xi = \prod_{n\in N} \tilde{S}_{n,\Xi}$, and consider $\Phi : \tilde{S}_\Xi \rightrightarrows \tilde{S}_\Xi$, where

$$\Phi(\tilde{s}) = \prod_{n\in N} \Theta_n(\tilde{s}_{-n}), \quad \tilde{s} \in \tilde{S}_\Xi,$$

Φ satisfies the conditions of Kakutani's fixed-point theorem. Hence, $\exists \tilde{s}^* \in \tilde{S}_\Xi$, such that $\tilde{s}^* \in \Phi(\tilde{s}^*)$. Denote this fixed point by $(b_n^*, l_n^*)_{n\in N}$. This is clearly an N.E of $\Gamma(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$. Call this equilibrium $\Xi(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$.

Step 6. Proving N.E of constrained game is also N.E of unconstrained game

We next show that $\Xi(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$ is also an equilibrium of the completely unconstrained market game Γ. So, pick any agent $j \in J$, and recall that at $\Xi(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$, j is viewed as solving the following problem:

$$\begin{aligned} \max_{(b_j,l_j)\in\mathbb{R}^4_+} \; & \left\{ u_j\left((x_{j,k}(b_{j,k}, l_{j,k}, B_{-j,k}, \phi_{-j,k}))_{k=1}^2, x_{j,m}\left((b_{j,k}, l_{j,k}, B_{-j,k}, \phi_{-j,k})_{k=1}^2\right)\right)\right\} \\ \text{s.t.} \; & \sum_{k=1}^2 b_{j,k} \leq e_{j,m} \\ & \sum_{k=1}^2 l_{j,k} \leq Q_j \\ & l_{j,\mathscr{D}(j)} \geq Q_j/2. \end{aligned} \tag{3}$$

The non-degenerate constraint qualification (NDCQ) holds, such that $\Xi(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$ is characterised by the following first-order necessary and sufficient conditions:

$$\begin{aligned} \frac{\partial u_j}{\partial x_{j,k}} \cdot \frac{B^*_{-j,k}\phi^*_k}{\left(B^*_k\right)^2} &= \lambda^b_j + \frac{\partial u_j}{\partial x_{j,m}} \cdot \left(\frac{\phi^*_{-j,k}}{\phi^*_k}\right), \; k \in \{1,2\}; \\ \frac{\partial u_j}{\partial x_{j,\mathscr{A}(j)}} \cdot \frac{b^*_{j,\mathscr{A}(j)}}{a^J_{\mathscr{A}(j)}B^*_{\mathscr{A}(j)}} &= \lambda^l_j - \frac{\partial u_j}{\partial x_{j,m}} \cdot \frac{\phi^*_{-j,\mathscr{A}(j)}}{(\phi^*_{\mathscr{A}(j)})^2} \cdot \frac{B^*_{\mathscr{A}(j)}}{a^J_{\mathscr{A}(j)}} - \lambda^{l\geq 0}_{j,\mathscr{A}(j)}; \\ \frac{\partial u_j}{\partial x_{j,\mathscr{D}(j)}} \cdot \frac{b^*_{j,\mathscr{D}(j)}}{a^J_{\mathscr{D}(j)}B^*_{\mathscr{D}(j)}} &= \lambda^l_j - \frac{\partial u_j}{\partial x_{j,m}} \cdot \frac{\phi^*_{-j,\mathscr{D}(j)}}{(\phi^*_{\mathscr{D}(j)})^2} \cdot \frac{B^*_{\mathscr{D}(j)}}{a^J_{\mathscr{D}(j)}} - \bar{\lambda}; \end{aligned} \tag{4}$$

where λ^b_j, λ^l_j and $\bar{\lambda}$ are the Lagrange multipliers associated with the first, second and third constraints of the programme in (3), respectively, and $\lambda^{l\geq 0}_{j,\mathscr{A}(j)}$ is the multiplier associated with the non-negativity constraint on $l_{j,\mathscr{A}(j)}$. Of course, the remaining Kuhn–Tucker conditions are also satisfied, but have not been written out explicitly, to keep our argument unburdensome. We will prove that $\bar{\lambda} = 0$, such that the above F.O.C. will also satisfy the F.O.C. as in (2), for every $j \in J$. Trivially, if $l_{j,\mathscr{D}(j)} > Q_j/2$, then $\bar{\lambda} = 0$. We will now show that $\bar{\lambda} = 0$ even if the constraint is binding—i.e., $l_{j,\mathscr{D}(j)} = Q_j/2$. To this end, recall that at any equilibrium, there is full employment of labour, i.e., any solution to the programme in (3) must meet the constraint $\sum_{k=1}^2 l_{j,k} \leq Q_j$ with equality, as otherwise, total utility can always be increased.

We may therefore, by replacing $\sum_{k=1}^{2} l_{j,k} \leq Q_j$ with $\sum_{k=1}^{2} l_{j,k} = Q_j$, transform (3) into an equivalently-constrained problem. Using this fact, alongside the third constraint in (3), we see that $(\sum_{k=1}^{2} l_{j,k} = Q_j) \wedge (l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2}) \Leftrightarrow (\sum_{k=1}^{2} l_{j,k} = Q_j) \wedge (l_{j,\mathscr{D}(j)} \geq \frac{l_{j,\mathscr{A}(j)} + l_{j,\mathscr{D}(j)}}{2}) \Leftrightarrow (\sum_{k=1}^{2} l_{j,k} = Q_j) \wedge (l_{j,\mathscr{D}(j)} \geq l_{j,\mathscr{A}(j)})$. Thus, (3) may be reformulated as:

$$\begin{aligned} \max_{(b_j, l_j) \in \mathbb{R}^4_+} \; & \left\{ u_j \left(\left(x_{j,k}\left(b_{j,k}, l_{j,k}, B_{-j,k}, \phi_{-j,k}\right) \right)_{k=1}^{2}, x_{j,m}\left(\left(b_{j,k}, l_{j,k}, B_{-j,k}, \phi_{-j,k} \right)_{k=1}^{2} \right) \right) \right\} \\ \text{s.t.} \; & \sum_{k=1}^{2} b_{j,k} \leq e_{j,m} \\ & \sum_{k=1}^{2} l_{j,k} = Q_j \\ & l_{j,\mathscr{D}(j)} \geq l_{j,\mathscr{A}(j)}. \end{aligned} \tag{5}$$

The NDCQ holds again, such that $\Xi(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$ is also characterised by the following first-order necessary and sufficient conditions:

$$\begin{aligned} \frac{\partial u_j}{\partial x_{j,k}} \cdot \left(\frac{B^*_{-j,k}\phi^*_k}{\left(B^*_k\right)^2} \right) &= \lambda^b_j + \frac{\partial u_j}{\partial x_{j,m}} \cdot \left(\frac{\phi^*_{-j,k}}{\phi^*_k} \right), \; k \in \{1,2\}; \\ \frac{\partial u_j}{\partial x_{j,\mathscr{A}(j)}} \cdot \left(\frac{b^*_{j,\mathscr{A}(j)}}{a^J_{\mathscr{A}(j)} B^*_{\mathscr{A}(j)}} \right) &= \lambda^l_j - \frac{\partial u_j}{\partial x_{j,m}} \cdot \left(\frac{\phi^*_{-j,\mathscr{A}(j)}}{(\phi^*_{\mathscr{A}(j)})^2} \cdot \frac{B^*_{\mathscr{A}(j)}}{a^J_{\mathscr{A}(j)}} \right) - \lambda^{l \geq 0}_{j,\mathscr{A}(j)} + \hat{\bar{\lambda}}; \\ \frac{\partial u_j}{\partial x_{j,\mathscr{D}(j)}} \cdot \left(\frac{b^*_{j,\mathscr{D}(j)}}{a^J_{\mathscr{D}(j)} B^*_{\mathscr{D}(j)}} \right) &= \lambda^l_j - \frac{\partial u_j}{\partial x_{j,m}} \cdot \left(\frac{\phi^*_{-j,\mathscr{D}(j)}}{(\phi^*_{\mathscr{D}(j)})^2} \cdot \frac{B^*_{\mathscr{D}(j)}}{a^J_{\mathscr{D}(j)}} \right) - \hat{\bar{\lambda}}; \end{aligned} \tag{6}$$

where λ^b_j, λ^l_j, and $\lambda^{l \geq 0}_{j,\mathscr{A}(j)}$ represent the Lagrange multipliers of the same constraints as before—and they also take on the same optimal values, for recall that neither has the objective function changed, nor has the nature of any one of these constraints been manipulated—and $\hat{\bar{\lambda}}$ is the multiplier associated with the third constraint in (5). As before, the remaining Kuhn–Tucker conditions are also satisfied. So, suppose that at equilibrium, we had that $l_{j,\mathscr{D}(j)} = \frac{Q_j}{2} = l_{j,\mathscr{A}(j)}$, such that $\lambda^{l \geq 0}_{j,\mathscr{A}(j)} = 0$. The second equation in (4) along with the second equation in (6) then imply that $\lambda^l_j = \lambda^l_j + \hat{\bar{\lambda}}$, from which it follows that $\hat{\bar{\lambda}} = 0$. In turn, the third equation in (4) along with the third equation in (6) then imply that $\lambda^l_j - \bar{\lambda} = \lambda^l_j - \hat{\bar{\lambda}} = \lambda^l_j$, from which it follows that $\bar{\lambda} = 0$, such that (2) is then met for $\forall j \in J, J = F, H$. But this therefore means that $\Xi(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$ is also an equilibrium among unconstrained strategies in $S_n \; \forall n \in N$. Thus, $\Xi(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$ is an N.E of the unconstrained market game Γ.

Step 7. Applying a limiting argument

It can be verified that the above line of attack applies generally for the market game $\Gamma(l_{j,\mathscr{D}(j)} \geq \frac{\tau Q_j}{1+\tau})$, for ***any*** $\tau \in \mathbb{N}$. This is thanks to the facts that the utility function of every $n \in N$ is continuous in x, with x continuous in b and l (such that V_n is continuous), and each agent's CPS is compact and strictly convex—call this collective set of properties, $\mathbf{P}_{ccc}$. By $\mathbf{P}_{ccc}$, it follows that $\Theta_n(\tilde{s}_{-n})$ is upper hemicontinuous and convex-valued. So, denote the set of N.E strategy profiles of $\Gamma(l_{j,\mathscr{D}(j)} \geq \frac{\tau Q_j}{1+\tau})$ by $\mathcal{NE}(l_{j,\mathscr{D}(j)} \geq \frac{\tau Q_j}{1+\tau}) \equiv \mathcal{NE}^{\tau}$, and consider a sequence $\{\tau^r\}_{r=1}^{\infty}$ in $\mathbb{N}$ where $\tau^r \to \infty$ as $r \to \infty$, and a sequence of (sets of) N.E profiles $\{\mathcal{NE}^{\tau^r}\}_{r=1}^{\infty}$. By $\mathbf{P}_{ccc}$, it has to be the case that $\lim_{r\to\infty} \mathcal{NE}^{\tau^r} = \mathcal{NE}(l_{j,\mathscr{D}(j)} = Q_j) \neq \emptyset$, constitutes a set of N.E profiles of $\Gamma(l_{j,\mathscr{D}(j)} \geq Q_j)$. But as every $\mathcal{NE}(l_{j,\mathscr{D}(j)} \geq \frac{\tau^r Q_j}{1+\tau^r}), r \in \mathbb{N}$, is a set of N.E profiles of Γ,[19] the continuous differentiability of the utility functions implies that the Lagrange multiplier method developed above may again be utilised, alongside a limiting argument, to easily show that $\mathcal{NE}(l_{j,\mathscr{D}(j)} = Q_j)$ is also set of N.E profiles of Γ. Hence, an equilibrium in which each country specialises in its comparative-disadvantage good exists. □

Proof of Theorem 1. Consider the offer-constrained market game $\Gamma(l_{j,\mathscr{A}(j)} \geq \frac{Q_j}{2})$, and proceed as in the proof of Theorem 2 above. □

Proof of Theorem 3. Let the mappings $\mathscr{A} : N \to \{1,2\}$ and $\mathscr{D} : N \to \{1,2\}$ be as previously defined. Consider the market game $\Gamma_\infty(l_{j,\mathscr{D}(j)} \geq \frac{Q_j}{2})$, in which the labour hours offered by each $j \in J, J = F, H$, towards the good in which country J has a comparative disadvantage are constrained to be at least half their initial endowment of labour hours. S^∞—and by extension, $\tilde{S}^\infty$ and $\tilde{S}^\infty_\Xi$, the infinite-dimensional analogues of $\tilde{S}$ and $\tilde{S}_\Xi$ which appeared in the proof of Theorem 2—as described in §4, is a nonempty, compact, convex subset of a Hausdorff locally convex topological vector space. Utility functions are continuous in x, and $x : S^\infty \to \mathbb{R}^3_+$ is continuous in b and l in the product topology. Thus, the payoff function $V_n(\tilde{s}_n, \tilde{s}_{-n}) = u_n(x_n(\tilde{s}_n, \tilde{s}_{-n}))$ is continuous for every $n \in N$. Proceeding as in the proof of Theorem 2, one may use the Kakutani–Glicksberg–Fan fixed-point theorem to prove the existence of a fixed point $(b_n^*, l_n^*)_{n\in N}$. One may then adopt all the subsequent arguments in the proof of Theorem 2, while taking care to modify the first-order conditions to take into account the measure of each agent, to show that any N.E of $\Gamma_\infty(l_{j,\mathscr{D}(j)} \geq Q_j)$ is also an N.E of Γ_∞.

We now prove the existence of an equilibrium where countries specialise according to the RTCA. V_n

[19] This can be straightforwardly seen by combining (4) and (6) for any agent $j \in J, J = F, H$. It can be readily verified that the non-degenerate constraint qualification also holds.

is continuous for each $n \in N$, and each agent's CPS is compact and strictly convex—call this collective set of characteristics, $\mathbf{P}^{\infty}_{ccc}$. It can be verified, akin to Theorems 1 and 2, that the method used in the first part of this proof applies just as well for the market game $\Gamma_{\infty}(l_{j,\mathscr{A}(j)} \geq \frac{\tau Q_j}{1+\tau})$, for any $\tau \in \mathbb{N}$. So, as before, denote the set of N.E strategy profiles of $\Gamma_{\infty}(l_{j,\mathscr{A}(j)} \geq \frac{\tau Q_j}{1+\tau})$ by $\mathcal{NE}_{\infty}(l_{j,\mathscr{A}(j)} \geq \frac{\tau Q_j}{1+\tau}) \equiv \mathcal{NE}^{\tau}_{\infty}$, consider a sequence $\{\tau^r\}^{\infty}_{r=1}$ in $\mathbb{N}$ where $\tau^r \to \infty$ as $r \to \infty$, and a sequence of (sets of) N.E profiles $\{\mathcal{NE}^{\tau^r}_{\infty}\}^{\infty}_{r=1}$. It is straightforward to see that $\mathbf{P}^{\infty}_{ccc}$ implies convex-valuedness and upper hemicontinuity of $\Theta_n(\tilde{s}_{-n})$, where $\tilde{s}_{-n} \in \prod_{i \in N\setminus\{n\}} S_i \subset (\mathbb{R}^{\omega}, \tau_P)$, indicating that it therefore has to be the case that $\lim_{r\to\infty} \mathcal{NE}^{\tau^r}_{\infty} = \mathcal{NE}_{\infty}(l_{j,\mathscr{A}(j)} = Q_j) \neq \emptyset$, is a set of N.E profiles of $\Gamma_{\infty}(l_{j,\mathscr{A}(j)} \geq Q_j)$. But as every $\mathcal{NE}_{\infty}(l_{j,\mathscr{A}(j)} \geq \frac{\tau^r Q_j}{1+\tau^r}), r \in \mathbb{N}$, is also a set of N.E profiles of Γ_{∞}, the continuous differentiability of the utility functions implies that the technique developed in the second part of the proof of Theorem 2 may again be utilised, alongside a limiting argument, to easily show that $\mathcal{NE}_{\infty}(l_{j,\mathscr{A}(j)} = Q_j)$ is also an equilibrium of Γ_{∞}. Hence, an equilibrium in which each country produces only its comparative-advantage good exists. □

Proof of Proposition 2. First, let us remark that with an atomless measure space of agents as we consider, it is true that $B_{-n,k} = \int_{i\in N\setminus\{n\}} b_k(i)d\mu(i) = \int_{n\in N} b_k(n)d\mu(n) = B_k$, and $\phi_{-n,k} = \frac{1}{a^F_k}\int_{F\setminus\{n\}} l_k(f)d\mu(f) + \frac{1}{a^H_k}\int_{H\setminus\{n\}} l_k(h)d\mu(h) = \frac{1}{a^F_k}\int_F l_k(f)d\mu(f) + \frac{1}{a^H_k}\int_H l_k(h)d\mu(h) = \phi_k$. Next, given each agent's CPS is convex, and utility functions concave, we have that any N.E of $\Gamma_{\mathfrak{c}}$ with positive bids and offers is characterised by the following first-order (necessary and sufficient) conditions:

$$\left.\begin{aligned} \frac{\partial u_j(x)}{\partial x_k(j)} \cdot \frac{1}{p_k} &= \lambda^b_j + \frac{\partial u_j(x)}{\partial x_m(j)} - \lambda^{b\geq 0}_{j,k}; \\ \frac{\partial u_j(x)}{\partial x_m(j)} \cdot \left(\frac{p_k}{a^j_k}\right) &= \lambda^l_j - \lambda^{l\geq 0}_{j,k}, \end{aligned}\right\} k \in \{1,2\} \tag{7}$$

and the complementary slackness conditions (which have not been listed for conciseness) are also satisfied. We will make extensive use of the second equation in (7) to prove our proposition. W.l.o.g., let country F (resp. H) have a comparative advantage in the production of good k (t). There are then 9 cases to consider, which are as follows: (***1***) both F and H produce both goods; (***2***) F produces both, while H produces k only; (***3***) F produces both, while H produces t only; (***4***) F produces k only, while H produces both goods; (***5***) both F and H produce k only; (***6***) F produces k only, while H produces t only; (***7***) F produces t only, while H produces both goods; (***8***) F produces t only, while H produces k only; (***9***) both F and H produce t only.

Case (***1***): If F and H produce both goods, then from (7), we have for agents $f \in F$ that $\frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_k}{a^F_k}\right) =$

$\lambda_f^l = \frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_t}{a_t^F}\right)$, such that $\frac{p_k}{a_k^F} = \frac{p_t}{a_t^F} \Leftrightarrow \frac{p_k}{p_t} = \frac{a_k^F}{a_t^F}$. Similarly, for agents $h \in H$, we have that $\frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_k}{a_k^H}\right) = \lambda_h^l = \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_t}{a_t^H}\right)$, such that $\frac{p_k}{a_k^H} = \frac{p_t}{a_t^H} \Leftrightarrow \frac{p_k}{p_t} = \frac{a_k^H}{a_t^H}$. These imply that $\frac{a_k^F}{a_t^F} = \frac{a_k^H}{a_t^H}$, a contradiction, since F has a comparative advantage in k.

Case (***2***): If F produces both, while H produces k only, then for all $f \in F$, we have that $\frac{p_k}{p_t} = \frac{a_k^F}{a_t^F}$. On the other hand, for agents $h \in H$, we have that $\frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_k}{a_k^H}\right) = \lambda_h^l \geq \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_t}{a_t^H}\right)$, such that $\frac{p_k}{a_k^H} \geq \frac{p_t}{a_t^H} \Leftrightarrow \frac{p_k}{p_t} \geq \frac{a_k^H}{a_t^H}$. Combining these two conditions yields $\frac{p_k}{p_t} = \frac{a_k^F}{a_t^F} \geq \frac{a_k^H}{a_t^H}$, which, in light of the fact that F has a comparative advantage in k, gives rise to a contradiction.

Case (***3***): If F produces both, while H produces t only, then for all $f \in F$, we have that $\frac{p_k}{p_t} = \frac{a_k^F}{a_t^F}$. For agents $h \in H$, we have that $\lambda_h^l = \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_t}{a_t^H}\right) = \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_k}{a_k^H}\right) + \lambda_{h,k}^{l\geq 0}$. By construction, we know that $\frac{a_k^F}{a_t^F} < \frac{a_k^H}{a_t^H}$, such that in the current scenario, $\frac{p_k}{p_t} < \frac{a_k^H}{a_t^H} \Leftrightarrow \frac{p_t}{a_t^H} > \frac{p_k}{a_k^H}$. Thus, one may, by choosing $\lambda_{h,k}^{l\geq 0} > 0$ large enough, show that no mathematical impossibility obtains when the first two equations in (7) are considered, for any $n \in N$.

Case (***4***): If F produces k only, while H produces both goods, then we have for agents $f \in F$ that $\lambda_f^l = \frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_k}{a_k^F}\right) = \frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_t}{a_t^F}\right) + \lambda_{f,t}^{l\geq 0}$. On the other hand, for agents $h \in H$, we have that $\frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_k}{a_k^H}\right) = \lambda_h^l = \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_t}{a_t^H}\right)$, such that $\frac{p_k}{a_k^H} = \frac{p_t}{a_t^H} \Leftrightarrow \frac{p_k}{p_t} = \frac{a_k^H}{a_t^H}$. By construction, we know that $\frac{a_k^F}{a_t^F} < \frac{a_k^H}{a_t^H}$, yielding $\frac{a_k^F}{a_t^F} < \frac{p_k}{p_t} \Leftrightarrow \frac{p_k}{a_k^F} > \frac{p_t}{a_t^F}$. Thus, one may, by choosing $\lambda_{f,t}^{l\geq 0} > 0$ large enough, show that no mathematical impossibility obtains when the first two equations in (7) are considered, for any $n \in N$.

Case (***5***): If both F and H produce k only, then the markets for k and t are active and inactive, respectively. In particular, this means that $p_k > 0$, such that from (7) and Assumption (i), we have that $\lambda_f^l, \lambda_h^l > 0$. At the same time, it must be true that $p_t = 0 = b_t(n) = l_t(n)$, μ-$a.e, n \in N$. Thus, taking $\lambda_{n,t}^{b\geq 0} = \frac{\partial u_n(x)}{\partial x_m(n)} + \lambda_n^b$ and $\lambda_{n,t}^{l\geq 0} = \lambda_n^l$, it is easy to see that no mathematical impossibility is arrived at when the first two equations in (7) are considered, for any $n \in N$.

Case (***6***): If F produces k only, while H produces t only, then we have for agents $f \in F$ that $\lambda_f^l = \frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_k}{a_k^F}\right) = \frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_t}{a_t^F}\right) + \lambda_{f,t}^{l\geq 0}$. Similarly, for agents $h \in H$, we have that $\lambda_h^l = \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_t}{a_t^H}\right) = \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_k}{a_k^H}\right) + \lambda_{h,k}^{l\geq 0}$. Combining the equations for agents $f \in F$ and $h \in H$ gives $\frac{a_k^F}{a_t^F} \leq \frac{p_k}{p_t} \leq \frac{a_k^H}{a_t^H}$, which, given $\frac{a_k^F}{a_t^F} < \frac{a_k^H}{a_t^H}$, can be decomposed into three sets of inequalities, namely, Case (***6***a): $\frac{a_k^H}{a_t^H} > \frac{p_k}{p_t} = \frac{a_k^F}{a_t^F}$; Case (***6***b): $\frac{a_k^H}{a_t^H} = \frac{p_k}{p_t} > \frac{a_k^F}{a_t^F}$, and; Case (***6***c): $\frac{a_k^H}{a_t^H} > \frac{p_k}{p_t} > \frac{a_k^F}{a_t^F}$. Cases (***6***a) and (***6***b) are exactly the same as Cases (***3***) and (***4***), respectively, so we need only analyse Case (***6***c). It suffices to note, in view of the first two equations derived in Case (***6***) for agents $f \in F$ and $h \in H$, that by taking sufficiently large $\lambda_{f,t}^{l\geq 0} > 0$ and $\lambda_{h,k}^{l\geq 0} > 0$, no mathematical impossibility is arrived at when the first two equations in (7) are considered,

for any $n \in N$.

Case (***7***): If F produces t only, while H produces both goods, then using an argument similar to the one in Case (***2***), we have that $\frac{p_k}{p_t} = \frac{a_k^H}{a_t^H}$ μ-$a.e, h \in H$, while $\frac{p_k}{a_k^F} \leq \frac{p_t}{a_t^F} \Leftrightarrow \frac{p_k}{p_t} \leq \frac{a_k^F}{a_t^F}$ μ-$a.e, f \in F$, such that $\frac{a_k^F}{a_t^F} \geq \frac{a_k^H}{a_t^H}$, a contradiction.

Case (***8***): If F produces t only, while H produces k only, then we have for agents $f \in F$ that $\frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_k}{a_k^F}\right) \leq \lambda_f^l = \frac{\partial u_f(x)}{\partial x_m(f)} \cdot \left(\frac{p_t}{a_t^F}\right)$, such that $\frac{p_k}{a_k^F} \leq \frac{p_t}{a_t^F} \Leftrightarrow \frac{p_k}{p_t} \leq \frac{a_k^F}{a_t^F}$. On the other hand, for agents $h \in H$, we have that $\frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_k}{a_k^H}\right) = \lambda_h^l \geq \frac{\partial u_h(x)}{\partial x_m(h)} \cdot \left(\frac{p_t}{a_t^H}\right)$, such that $\frac{p_k}{a_k^H} \geq \frac{p_t}{a_t^H} \Leftrightarrow \frac{p_k}{p_t} \geq \frac{a_k^H}{a_t^H}$. These imply that $\frac{a_k^F}{a_t^F} \geq \frac{a_k^H}{a_t^H}$, a contradiction.

Case (***9***): If both F and H produce t only, then using an argument similar to the one in Case (***5***), it is easy to see that no mathematical impossibility is arrived at.

From the above, we see that only Cases (***3***), (***4***), (***5***), (***6***), and (***9***) are compatible with (Nash) equilibria of our model, which are the exact equilibria which the RTCA admits—see, e.g., Feenstra (2015: pp. 2-3). Notably, there is, ***at most***, only one country (and not both) that ever produces its comparative-disadvantage good at equilibrium, in stark contrast to the equilibria described in Theorems 2 and 3. □

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