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Reestablishing Stability and Avoiding a Credit Crunch: Comparing Different Bad Bank Schemes

Achim Hauck*    Ulrike Neyer†    Thomas Vieten‡

August 2014

Abstract

This paper develops a model to analyze two different bad bank schemes, an outright sale of toxic assets to a state-owned bad bank and a repurchase agreement between the bad bank and the initial bank. For both schemes, we derive a critical transfer payment that induces a bank manager to participate. Participation improves the bank’s solvency and enables the bank to grant new loans. Therefore, both schemes can reestablish stability and avoid a credit crunch. An outright sale will be less costly to taxpayers than a repurchase agreement if the transfer payment is sufficiently low.

JEL classification: G21, G28, G30

Keywords: bad banks, financial crisis, financial stability, credit crunch

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INTRODUCTION

The worldwide financial crisis, which broke out in August 2007, led to severe losses in the financial sector. Banks suffered from so-called toxic assets in their balance sheets. Uncertainty about the "true value" of these assets and necessary depreciations, which significantly reduced the banks’ capital, raised concerns about the stability of the banking sector and about a possible significant reduction in credit supply.

In response to these developments, governments in several countries offered distressed banks to transfer their toxic assets to publicly sponsored special purpose vehicles, so-called bad banks. All implemented bad bank schemes have in common that they clean up the banks’ balance sheet at least temporarily. That is their main advantage over other regulatory interventions, like e.g. the mitigation of capital requirements or capital injections. In particular, they differ with respect to the risk-distribution between the distressed bank and the bad bank, and therefore, the taxpayers. In Germany, for example, the risk remains largely with the distressed bank, while in the US (Troubled Asset Relief Program) the bad bank scheme allows for a more or less complete risk transfer to the bad bank. To mitigate the financial crisis, a couple of other countries like Ireland (National Asset Management Agency) and Switzerland also adopted concepts similar to a bad bank scheme. Moreover, bad bank schemes were occasionally used prior to the worldwide financial crisis. Examples are the US-Savings & Loan crisis of the 1980s and the banking crisis in Sweden in the early 1990s.¹

Against this background, this paper develops a model which allows for a comparison of two different bad bank schemes. The first is characterized by a full transfer of the risk of toxic assets to the taxpayers. Under the second scheme, the risk of toxic assets remains with the distressed bank.\(^2\) In our analysis, we focus on two particular aspects. First, we investigate whether the different bad bank schemes are appropriate to stabilize the banking sector and to avoid a credit crunch. Second, we compare the different bad bank schemes with respect to their expected costs to taxpayers.

In our theoretical analysis, we consider a single commercial bank whose balance sheet consists of a risky asset that is funded by equity and deposits. Write-offs on the asset have led to a situation in which the bank's equity is just sufficient to meet a minimum capital requirement. Due to a high degree of uncertainty in the banking sector the bank is unable to attract new capital. Therefore, it is neither able to bear further possible depreciations of the toxic asset nor to grant new loans. In this situation, a risk-neutral bank manager has the opportunity to hive off the toxic asset to a bad bank. Concerning the risk allocation between the initial bank and the taxpayers, we consider two extreme cases. In the first case, the bank can make an outright sale of the toxic asset to a state-owned bad bank. As a consequence, the risk of the toxic asset is fully borne by the taxpayers. In the second case, the transfer of the toxic asset to the bad bank involves a repurchase agreement between the distressed bank and the bad bank implying that the risk of the toxic asset remains with the distressed bank. The idea of the second scheme is to give the bank some

\(^2\)The first scheme is similar to the one which has been implemented by the US Federal Reserve System to tackle the worldwide financial crisis (Federal Reserve Bank of New York, 2014). The second resembles the bad bank scheme which has been implemented by the German government in 2009 (Deutsche Bundesbank, 2009).
time to generate profits from its newly granted loans so that it will be able to bear possible losses from the toxic asset at a later date.

Our theoretical analysis reveals that under both bad bank schemes, the price, at which the toxic asset can be transferred to the bad bank, plays a crucial role. First, this transfer price must be high enough to induce the bank manager to participate in the bad bank scheme. Thus, there exists a minimum transfer price which has to be paid to stabilize the banking sector, since the banking sector will only become more stable if the manager transfers the toxic asset. Furthermore, the supply of new loans increases in the transfer price, i.e. if the danger of a credit crunch is high, the transfer payment must be sufficiently high to avert this threat.

From our theoretical analysis we conclude that if the transfer price is sufficiently high, a bad bank will stabilize the banking sector and avoid a credit crunch under both schemes, an outright sale as well as a repurchase agreement. Concerning the superiority of one scheme, the expected costs to taxpayers have to be considered. In case of an outright sale, the taxpayers can benefit from the potential returns on the toxic asset but do not reobtain the transfer payment. On the contrary, a repurchase agreement implies that the potential returns on the toxic asset remain at the distressed bank while the taxpayers reobtain the transfer price at least with positive probability. Therefore, an outright sale will be superior to a repurchase agreement only if the necessary transfer payment is relatively low. Otherwise, if the necessary transfer payment is relatively high, the repurchase agreement concept will involve less expected costs to the taxpayers.

The related literature on bad bank schemes can be divided into three groups. The first group examines bad bank schemes that were implemented prior to the worldwide financial crisis. White (1991) and Curry and Shibut (2000) explore the US-Savings & Loan crisis of the 1980s. Macey (1999) and Bergström, Englund, and
Thorell (2003) analyze the banking crisis in Sweden in the early 1990s. Particularly the implementation of bad banks in the Swedish banking crisis is often viewed as the textbook case of banking crises resolution. However, its applicability to the recent financial crisis is limited because the Swedish crisis was confined to a relatively small part of Europe while the world economy favored a quick recovery. Moreover, the banks’ toxic assets were predominantly book credits. Therefore, problems that are inherent in complex innovative financial products which were a main driving force of the financial crisis of 2007, did not exist. The second group discusses the pros and cons of bad bank schemes from a political economy perspective in the light of the worldwide financial crisis. Our paper is most closely related to the third group of the literature, which develops theoretical models to analyze governmental bank bailout policies. While the effects of different recapitalization plans for distressed banks are, in general, relatively well understood, the theoretical literature particularly focussing on bad bank schemes is still in its infancy. Tirole (2012) analyzes state-sponsored asset purchases to restart an illiquid market. Aghion, Bolton, and Fries (1999), Corbett and Mitchell (2000), Mitchell (1998, 2001), and Tanaka and Hoggarth (2006) investigate the effects of recapitalization plans on a bank manager’s incentive to misreport the amount of the bank’s loan losses either to avoid recapitalization or to realize excessive government support, respectively. Mailath and Mester (1994), Osano (2002, 2005), and Acharya and Yorulmazer (2008) study the risk of moral hazard inherent in governmental bailouts. While Mailath and Mester (1994) as well as Osano (2002, 2005) analyze the behavior of a single bank, Acharya and Yorulmazer (2008) look at the entire banking sector and the banks’ incentive to herd in their investment decisions to increase the risk that many banks may fail

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together. Several papers compare different forms of policy measures to stop a fall in loan supply following a banking crisis. Philippon and Schnabl (2010) argue that in a crisis capital injections are more efficient than asset purchases and debt guarantees. Elsinger and Summer (2010) support these results. Bhattacharya and Nyborg (2010) propose that capital injections and asset purchases are the most efficient forms of recapitalization. Dietrich and Hauck (2012) show that while debt or capital subsidies can lead to overinvestment and excessive risk taking, a sale of toxic assets to a bad bank does not generate adverse incentives but may have higher fiscal costs. Wilson (2012) point out that bad banks and capital injections both dominate state-sponsored purchases of preferred stock. Wilson and Wu (2012) show that these results are still valid when a policy maker tries to avoid risk shifting of a bank in financial distress. While these contributions compare a single bad bank scheme, which is similar to an outright sale, to other forms of public interventions, our paper is the first that explicitly compares different bad bank schemes in a unified framework. In particular, we investigate two bad bank schemes, an outright sale and a repurchase agreement, with respect to their appropriateness for reestablishing the stability of the banking sector and avoiding a credit crunch as well as with respect to their expected costs to taxpayers.

The paper is organized as follows. Section 1 develops the model and derives the critical transfer payment at which the bank manager is willing to participate in the respective bad bank schemes. Section 2 discusses policy implications, section 3 concludes the paper.

1 THE MODEL
1.1 Framework

We consider a risk-neutral, zero-interest-rate economy where the asset side of a commercial bank’s balance sheet consists of an illiquid risky asset.\footnote{Due to its complex structure, only a limited set of potential buyers is able to value this asset. Therefore, there is only limited liquidity, normalized to zero, to purchase the asset (Diamond and Rajan, 2011).} The commercial bank must back this asset with sufficient capital due to a minimum capital requirement. Write-offs on the asset have reduced the bank’s capital down to the minimum amount the bank must hold to fulfill this requirement.\footnote{Adding safe assets and/or further risky assets to the balance sheet would complicate the subsequent formal analysis without yielding much additional insight. Therefore, we stick to the simplest case in which the bank possesses a risky asset only.}

The bank faces new lending opportunities. As these loans are risky, they must also be backed with sufficient capital. However, the bank is unable to attract fresh outside capital. In the spirit of Diamond and Rajan (2001), this may be due to a potential hold-up problem between the bank manager and outside financiers, that can only be solved if the bank issues demandable deposits, which serve as a disciplinary device for the bank manager. Consequently, the bank cannot grant new loans unless it obtains outside help.

In this situation, the bank manager has the option to hive off the impaired asset to a government-owned bad bank. If he decides to do so, he can exchange the asset for safe government bonds. This transaction allows him to grant new loans since government bonds are not subject to a capital requirement.

No Transfer of the Toxic Asset, No New Loans

There are two dates $t = 0, 1$. At date $t = 0$ the bank possesses an impaired risky financial asset (toxic asset). The asset matures at date $t = 1$. At this date, it
yields a (gross) return \( \bar{K} \), which is equal to \( Y > 0 \) with probability \( \theta \) and zero with probability \( 1 - \theta \).

Figure 1 presents the balance sheet at \( t = 0 \) for the case that the bank manager does not hive off the toxic asset to a bad bank. Then, he will not be able to grant new loans. Accordingly, the asset side of the bank’s balance sheet consists of the risky asset only. Its book value is given by its expected gross return \( \theta Y \).\(^6\) The liability side consists of deposits \( D^{nB} \) and capital \( V_0^{nB} \) (the superscript \( nB \) indicates that the manager does not transfer the risky asset to a bad bank, the subscript 0 stands for date \( t = 0 \)). The balance sheet identity at \( t = 0 \) is therefore

\[
\theta Y = D^{nB} + V_0^{nB}. \tag{1}
\]

The bank’s capital just meets the capital requirement. It satisfies \( V_0^{nB} = r\theta Y \), where \( r \in (0, 1) \) denotes the minimum ratio of capital to risky assets. In conjunction with the balance sheet identity (1), this implies

\[
D^{nB} = (1 - r)\theta Y. \tag{2}
\]

\(^6\)Throughout the paper, we assume that any asset is valued either at expected gross return or by its initial nominal value, depending on which amount is smaller. Thus, the expected gross return of the toxic asset at \( t = 0 \) is smaller than its initial face value so that a write-off has become inevitable.
The balance sheet at \( t = 1 \) is shown in Figure 2. We assume that a full deposit insurance exists, so that depositors do not bear any losses.\(^7\) They receive \( D^{n_B} \) irrespective of the actual return on the risky asset. In contrast, the capital value and the insurance payment depend on the outcome of the risky asset at \( t = 1 \). With probability \( \theta \), the asset succeeds in which case its return \( \tilde{K} = Y \) suffices to repay \( D^{n_B} \) to depositors. Consequently, the bank will be solvent, the insurance must not pay anything and capital holders will receive the residual return \( Y - D^{n_B} \) (see the left hand side of Figure 2). With probability \( 1 - \theta \), the asset fails, \( \tilde{K} = 0 \). Then, the bank will be insolvent, the insurance must pay \( D^{n_B} \) to depositors and capital will be worthless. This case is shown on the right hand side of Figure 2. Accordingly, from a date \( t = 0 \) perspective, the bank is expected to be solvent with probability \( \theta \) and the expected value of bank capital satisfies

\[
E[\tilde{V}^{n_B}_1] = \theta(Y - D^{n_B}).
\]  

(3)

**Transfer of the Toxic Asset, New Loans**

At \( t = 0 \), the bank manager has the opportunity to hive off the risky asset to a government-owned bad bank. If he decides to do so, he will incur non-pecuniary stigma costs \( B \) which reflect a loss of reputation for the manager. Furthermore, the

\(^7\)For the sake of simplicity we abstain from modeling a deposit insurance premium.
bank will obtain safe government bonds worth $Z$ in exchange for the risky asset. This transfer payment $Z$ must satisfy

$$Z \geq D^{nB}.$$ (4)

Otherwise the bank would be bankrupt directly after having transferred its risky asset.

Figure 3 presents the resulting balance sheet at $t = 0$. The asset side consists of the newly obtained government bonds $Z$ and the volume $L_0$ of newly granted loans. These loans are valued at their nominal value $L_0$ since there have not been any write-offs yet. The bank can grant these loans because government bonds are not subject to capital requirements. The bank’s liabilities consist of deposits $D^B$ and capital $V_0^B$ (where the subscript $B$ indicates that the manager has transferred the risky asset to the bad bank) so that the balance sheet identity at $t = 0$ is

$$Z + L_0 = D^B + V_0^B.$$ (5)

Since the bank is unable to attract new capital and is not allowed to sell the government bonds, it must refinance new loans by acquiring new deposits. The supply of

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tr>
<td>Government Bonds</td>
<td>Deposits $D^B$</td>
</tr>
<tr>
<td>New Loans $L_0$</td>
<td>Capital $V_0^B$</td>
</tr>
</tbody>
</table>
fully insured deposits is totally elastic. Therefore, the total volume $D^B$ of deposits is given by the sum of "old" deposits $D^{nB}$ and the volume $L_0$ of new loans:

$$D^B = D^{nB} + L_0. \quad (6)$$

At $t = 1$, the return on the new loans is a random variable denoted by $\tilde{L}_1$. With probability $\theta_{new}$, the loans are successful and yield $(1+\alpha)L_0$, where $\alpha$ reflects the net rate of return on these loans. With probability $1 - \theta_{new}$, they fail and yield nothing. The newly granted loans are less risky than the toxic asset, $\theta_{new} > \theta$. Moreover, they have a positive expected net return per unit, $\theta_{new}(1 + \alpha) > 1$. The returns of the new loans and the toxic asset are uncorrelated.$^8$

The properties of the balance sheet at $t = 1$ depend on the concrete design of the bad bank scheme. We will analyze two different schemes. The first corresponds to an outright sale of the toxic asset to the bad bank. Under this scheme, the bank manager exchanges the risky asset for safe government bonds at $t = 0$. Thereafter, no further transaction takes place between the bank and the bad bank. That is, the bank neither bears further losses of the risky asset nor benefits from its potential profits. The second scheme resembles a repurchase agreement. While the impaired asset is still transferred to the bad bank at $t = 0$ in exchange for safe government bonds, the bank now agrees to buy the asset back at $t = 1$ and to return the government bonds at this date. Under this scheme, the bank still bears the risk of

$^8$This assumption does not affect our qualitative results.
the toxic asset but also participates in possible profits.\footnote{This scheme aims at allowing a participating bank to originate profitable loans by releasing equity that has previously been used to meet regulatory capital requirement for toxic assets. Future returns on these loans should enable the bank to potentially bear future losses from its toxic assets. The bad bank scheme implemented by the German government in 2009 has been designed in this spirit. It stipulates that while a bank can sell its toxic assets to a state-sponsored special purpose vehicle (the bad bank), it is still responsible for compensating losses incurred by the bad bank (Deutsche Bundesbank, 2009). However, the German scheme does not require the bank to build up reserves to cover these potential losses, see Institut der Wirtschaftsprüfer (2009) and the Accounting Standards Committee of Germany (2009), because the responsibility to cover actual losses is subject to the bank making profits. Moreover, the toxic assets on the bank’s balance sheet are replaced by safe government guaranteed bonds which are not subject to regulatory capital requirements.} We discuss the implications of both schemes for the balance sheet at \( t = 1 \) below.

**Preferences**

The bank manager aims to maximize his utility. When deciding on whether to transfer the toxic asset to the bad bank or not, he therefore compares his utility under both situations. If he does not transfer the asset, his utility \( U_{nB} \) will depend on the expected capital value \( E[\tilde{V}_{1}^{nB}] \) only. Instead, if he transfers the asset, his utility \( U_{B} \) will be determined by the expected capital value \( E[\tilde{V}_{1}^{B}] \) and the non-pecuniary stigma costs:

\[
U_{nB} = E[\tilde{V}_{1}^{nB}], \quad (7)
\]

\[
U_{B} = E[\tilde{V}_{1}^{B}] - B. \quad (8)
\]

Thus, we assume that the utility function is additively separable between the pecuniary expected capital value and the non-pecuniary stigma costs.

**1.2 New Lending**

If the bank manager participates in a bad bank scheme, he will be able to grant new loans. However, these loans are risky, so that the bank manager must back them
with capital. To keep the exposition simple, we assume that new loans are subject to the same minimum capital requirement as the toxic asset. Accordingly, to the minimum capital requirement, bank capital must satisfy $V_0^B \geq rL_0$. In conjunction with (2), (5) and (6), this directly leads to

**Lemma 1:** If the bank manager hives off the toxic asset to the bad bank at $t = 0$, the volume of new loans must satisfy

$$L_0 \leq \frac{1}{r}(Z - D^{nB}) = \theta Y + \frac{1}{r}(Z - \theta Y) =: L_{0,\text{max}}(Z).$$

(9)

The Lemma reveals that the minimum capital requirement imposes a restriction on the volume of new loans. According to (9), the maximum loan volume $L_{0,\text{max}}$ depends on the size of the transfer payment $Z$ relative to the book value $\theta Y$ of the toxic asset. To interpret this maximum loan volume, it is useful to distinguish between two effects that a bad bank scheme can have on the bank manager’s ability to grant new loans.

First, there will be an asset substitution effect (first term on the right hand side of (9)). The bad bank scheme allows the manager to replace his risky asset by safe government bonds. As long as the transfer payment is equal to the book value of the risky asset, $Z = \theta Y$, participation in the bad bank scheme leaves the bank’s capital unchanged. However, this capital, which has been used to back the risky asset, is now available for backing loans since government bonds do not require capital backing. Therefore, an amount equal to the book value of the toxic asset $\theta Y$ can be granted as new loans.

Second, there will be a capital change effect whenever the transfer payment $Z$ differs from the book value $\theta Y$ of the toxic asset (second term on the right hand side of (9)). This effect is due to the bank’s additional capital (in case of $Z > \theta Y$)
Figure 4: OS-scheme: Balance sheet at $t = 1$ if manager transfers the toxic assets.

or capital loss (in case of $Z < \theta Y$) when participating in a bad bank. If $Z > \theta Y$, the bank will receive additional capital. Multiplied by $\frac{1}{r} > 1$ we obtain the amount of new loans that can additionally be granted. If $Z < \theta Y$, the bank will "lose" capital with the transfer of the risky asset to the bad bank. Therefore, the amount of new loans is lower than the book value $\theta Y$ of the risky asset. Consequently, $L_0^{max}$ increases in $Z$ since either, there is an increase in additional capital or a decrease in lost capital: $\frac{\partial L_0^{max}}{\partial Z} = \frac{1}{r} > 0$.

### 1.3 Outright Sale of the Toxic Asset (OS-Scheme)

In this section, we analyze a bad bank scheme, which resembles an outright sale ($OS$) of the toxic asset. Under the OS-scheme, the bank manager can exchange the asset for safe government bonds worth $Z$ at $t = 0$. This transaction is irrevocable. Consequently, the bank neither has to bear any losses nor can benefit from any return of the toxic asset at $t = 1$. 
The Commercial Bank’s Balance Sheet at \( t = 1 \) and Expected Capital Value

The consequences of a participation in the OS-scheme for the bank’s balance sheet at \( t = 1 \) are shown in Figure 4. The asset side consists of the government bonds, the new loans and a possible payment from the deposit insurance while the liability side consists of deposits and capital. The figure distinguishes between two scenarios. If the government bonds and the return of the new loans cover the volume of deposits, \( Z + \tilde{L}_1 \geq D^B \), the bank will be able to meet its liabilities vis-a-vis depositors at \( t = 1 \). It is thus solvent. Therefore, it will pay \( D_B \) to depositors, the insurer will pay nothing, and capital holders will obtain the residual return \( Z + \tilde{L}_1 - D^B \) (see the left hand side of figure 4). On the contrary, if the total return \( Z + \tilde{L}_1 \) falls short of \( D^B \), the bank will be insolvent. In this case, the bank’s assets will be used to repay deposits, the insurance must settle the remaining claim \( D^B - (Z + \tilde{L}_1) \) of depositors, and the value of capital will be zero (see the right hand side of figure 4).

From the discussion of the bank’s balance sheet, it follows that the value of bank capital at \( t = 1 \) is equal to \( \max\{0, Z + \tilde{L}_1 - D^B\} \). At this date, there can be two states of the world. The new loans succeed with probability \( \theta_{\text{new}} \). Then, they yield a (gross) return \( (1 + \alpha)L_0 \) and the bank will be solvent.\(^{10}\) With probability \( 1 - \theta_{\text{new}} \), the new loans yield no return. In this case, the bank will be solvent only if \( Z \geq D^B \).

As a consequence, from the perspective of date \( t = 0 \), the expected date \( t = 1 \) capital value satisfies

\[
E[V^B_1] = \theta_{\text{new}}(Z + (1 + \alpha)L_0 - D^B) + (1 - \theta_{\text{new}}) \max\{0, Z - D^B\}. \tag{10}
\]

\(^{10}\)To see this, note that if the new loans succeed, it follows from (4) that \( Z + \tilde{L}_1 = Z + (1 + \alpha)L_0 \geq D^{\text{aB}} + (1 + \alpha)L_0 \) while (6) implies \( D^B = D^{\text{aB}} + L_0 < D^{\text{aB}} + (1 + \alpha)L_0 \). Accordingly, we have \( Z + \tilde{L}_1 > D^B \) so that the bank is solvent.
Inserting (6) in (10) yields

\[ E[V_1^B] = \theta_{new}(Z + \alpha L_0 - D_n^B) + (1 - \theta_{new}) \max\{0, Z - D_n^B - L_0\}, \tag{11} \]

Since the new loans have a positive expected net return, \( \theta_{new}(1 + \alpha) > 1 \), it follows from (10) that the expected capital value at \( t = 1 \) is increasing in the loan volume, \( \frac{\partial E[V_1^B]}{\partial L_0} > 0 \).

**The Bank Manager’s Optimizing Behavior**

If the bank manager transfers the toxic asset to the bad bank at \( t = 0 \), he will maximize the utility as given in (8). This utility is increasing in the expected capital value \( E[V_1^B] \) which again increases in \( L_0 \), so that it is optimal for the manager to grant the maximum possible amount of new loans \( L_0^{max} \) if he participates in the OS-scheme. We can infer from (6) and (9) that granting these loans requires \( D^B = Z + (1 - r)L_0^{max} \). Consequently, the bank will be insolvent if the new loans fail because then, the available government bonds worth \( Z \) will not suffice to satisfy the depositors’ total claim \( D^B \). Thus, the bank will be solvent at \( t = 1 \) with probability \( \theta_{new} \) if the bank manager hives off the toxic asset to the bad bank. Together with (6), (8) and (10), this implies that the bank manager’s utility \( U^B \) of participating in the OS-scheme will be

\[ U^B = \theta_{new}(Z + \alpha L_0^{max}(Z) - D_n^B) - B. \tag{12} \]

By contrast, if the manager decides against transferring the asset, it follows from (3) and (7) that the bank will be solvent with probability \( \theta \) so that his utility

\[ U^{nB} = \theta(Y - D_n^B). \tag{13} \]
The bank manager is willing to transfer the asset to the bad bank at $t = 0$ only if $U^B \geq U^{nB}$. Inserting (12) and (13) into this condition and rearranging terms yields:

$$\theta_{new}Z + \theta_{new}\alpha L^{\max}_0(Z) \geq \theta Y + (\theta_{new} - \theta)D^{nB} + B.$$

(14)

Condition (14) states that the bank manager will decide in favor of the bad bank scheme if his expected benefits are not outweighed by the expected costs. The left hand side of (14) reflects the manager’s expected benefits of transferring the toxic asset to a bad bank, the right hand side reflects his expected costs. The expected benefits stem from the government bonds $Z$ and the potential return $\alpha L^{\max}_0$ of the newly granted loans. The manager will benefit from both only if the new loans turn out to be successful, since otherwise the bank will be insolvent so that $Z$ will be used to repay depositors and the new loans yield nothing. Therefore, $Z$ and $\alpha L^{\max}_0$ have to be multiplied by $\theta_{new}$. The expected costs consist of the foregone expected (gross) return $\theta Y$ of the toxic asset, an increase in expected old liabilities $(\theta_{new} - \theta)D^{nB}$ and the stigma costs $B$. Expected old liabilities increase by $(\theta_{new} - \theta)D^{nB}$ because if the bank manager participates in the OS-scheme, the probability of bank solvency will increase from $\theta$ to $\theta_{new}$. That is, it becomes more likely that the bank will repay depositors without aid from the deposit insurer. After inserting (9) into the condition (14) and rearranging terms, we obtain

**Proposition 1:** Under the OS-scheme, the bank manager will transfer the toxic asset to the bad bank and the probability of the bank’s solvency will increase from $\theta$ to $\theta_{new}$ only if

$$Z \geq \theta Y + \frac{r}{\theta_{new}(\alpha + r)}[B - \hat{B}] =: Z_{OS}^*,$$

(15)
where \( \hat{B} \) is defined by

\[
\hat{B} = \theta_{\text{new}}\theta Y + \theta_{\text{new}}\alpha_\theta Y - \theta Y - (\theta_{\text{new}} - \theta)D^nB. \tag{16}
\]

The proposition states that the bank manager will only use the bad bank if he receives sufficient government bonds in exchange for the toxic asset. The transfer payment \( Z \) may not be smaller than the critical payment \( Z_{OS}^* \) because otherwise the manager’s expected costs of the transfer would exceed his expected benefits. According to (15), the threshold \( Z_{OS}^* \) is linearly increasing in the stigma costs \( B \). For \( B = \hat{B} \), it is equal to the toxic asset’s book value, \( Z_{OS}^* = \theta Y \). In this case, the bank manager will participate in the bad bank scheme even if the scheme has only an asset substitution effect without improving bank capital at \( t = 0 \). For \( B > \hat{B} \), the threshold \( Z_{OS}^* \) is larger than \( \theta Y \). Then, in order to induce the bank manager to participate, the scheme must not only allow for asset substitution but also increase the capital of the bank. This will give the bank manager the opportunity to grant a larger amount of new profitable loans and will therefore compensate him for the higher stigma costs. For \( B < \hat{B} \), the threshold \( Z_{OS}^* \) is smaller than \( \theta Y \) so that the bank manager will participate in the bad bank scheme even if this is detrimental for bank capital.

The threshold \( Z_{OS}^* \) as defined in (15) increases in the stigma costs since a higher \( B \) implies higher expected costs for the bank manager he must be compensated for:

\[
\frac{\partial Z_{OS}^*}{\partial B} = \frac{r}{\theta_{\text{new}}(\alpha + r)} > 0. \tag{17}
\]
If the stigma costs $B$ increases by one unit, expected costs will also increase by one unit (see the right hand side of equation (14)). Consequently, the transfer payment $Z_{\text{OS}}$ must rise until the manager’s expected benefit has also increased by one unit.

### 1.4 Repurchase of the toxic asset (RA-Scheme)

The bad bank scheme analyzed in this section is comparable to a repurchase agreement (RA). At $t = 0$, the bank manager can exchange the impaired asset against safe government bonds $Z$. However, at $t = 1$ the bank reobtains the asset and is obliged to repay $Z$ to the bad bank. Like the OS-scheme, the RA-scheme allows the bank manager to grant new loans at $t = 0$, as the government bonds are not subject to a capital requirement. However, unlike the OS-scheme, the RA-scheme ensures that the bank still participates in the risks and benefits of the toxic asset. The idea is that if the new loans turn out to be successful at $t = 1$, the profit can offset possible losses from the impaired asset. By transferring the asset to a bad bank, the bank thus only buys time under this scheme.

#### The Commercial Bank’s Balance Sheet at $t=1$ and Expected Capital Value

Figure 5 illustrates the commercial bank’s balance sheet at $t = 1$. If the total (gross) returns $\tilde{K} + \tilde{L}_1$ on the toxic asset and the new loans are sufficient to cover the claim $D^B$ of depositors, the bank can fully meet its liabilities. It is thus solvent. Therefore, the bad bank reobtains the transfer payment $Z$, the bank uses some of its investment returns to repay depositors and capital holders receive the residual proceeds, which are worth $\tilde{K} + \tilde{L}_1 - D^B$. This case is shown on the left hand side of Figure 5.
What will happen if the total investment returns $\tilde{K} + \tilde{L}_1$ fall short of $DB$? Then, the bank will be insolvent at $t = 1$ because its total liabilities $Z + DB$ will exceed its total assets $Z + \tilde{K} + \tilde{L}_1$. Accordingly, capital will be worthless and the order in which the claim of the bad bank and depositors are served becomes relevant. Suppose first that the claim of the bad bank is senior to deposits. We will refer to this variant of a repurchase agreement scheme as the $\text{RA}_S$-scheme. Under this scheme, the bad bank still obtains $Z$, and the investment returns $\tilde{K} + \tilde{L}_1$ are left for repaying deposits. As these proceeds do not suffice, the deposit insurer must bear the difference between the claim $DB$ of depositors and the investment returns $\tilde{K} + \tilde{L}_1$. The upper balance sheet on the right hand side of Figure 5 illustrates this scenario. Now, suppose that the bad bank’s claim is junior to deposits ($\text{RA}_J$). Then, bank capital is still worthless and the bad bank becomes the residual claimant (see the lower balance sheet on the right hand side of Figure 5). If the total assets $Z + \tilde{K} + \tilde{L}_1$ cover the claim $DB$ of depositors, the bank will repay depositors in full so that no assistance from the deposit insurer is needed. The bad bank receives the residual proceeds $Z + \tilde{K} + \tilde{L}_1 - DB$ in this case. Otherwise, if the total assets $Z + \tilde{K} + \tilde{L}_1$ are smaller...
than $D^B$, they are fully transferred to depositors, and the deposit insurer settles the depositors’ remaining claims. The bad bank does not receive any payment.

We have seen that the distinction between the two variants of a repurchase agreement ($RA_S$ and $RA_J$) neither plays a role for the bank’s solvency nor for the value of bank capital at $t = 1$, which is equal to $\min\{\tilde{K} + \tilde{L}_1 - D^B, 0\}$. Therefore, this distinction is irrelevant for the behavior of the bank manager so that we will simply refer to the $RA$-scheme in the rest of this section. However, distinguishing between the two variants will be highly relevant for the discussion of the policy implications in section 2.

Let us now have a closer look at the bank’s solvency and capital at $t = 1$ and the value of bank capital at this date under the $RA$-scheme. The bank will be solvent if the total investment proceeds $\tilde{K} + \tilde{L}_1$ of the toxic asset and the new loans cover the volume of deposits $D^B$. The toxic asset yields $\tilde{K} = Y$ at $t = 1$ with probability $\theta$. Otherwise, it yields no return, $\tilde{K} = 0$. The return on the new loans at $t = 1$ is $(1 + \alpha)L_0$ with probability $\theta_{\text{new}}$ and 0 otherwise. The two investments are uncorrelated. Therefore, with respect to the total investment return and bank solvency at $t = 1$, we need to distinguish between four states of the world (see Figure 6). (a) With probability $\theta\theta_{\text{new}}$, both investments succeed. Then, the total return $Y + (1 + \alpha)L_0$ at $t = 1$ suffices to repay $D^B$ to depositors. The bank is thus solvent. (b) With probability $(1 - \theta)\theta_{\text{new}}$, only the new loans succeed. Then, the total return on the investments is equal to $(1 + \alpha)L_0$. Due to (6), this amount covers the liabilities $D^B$ vis-a-vis depositors only if

$$L_0 \geq \frac{D^B}{\alpha} =: \mathcal{T}_0.$$  \hspace{1cm} (18)
Both investments succeed: \( \theta \theta_{\text{new}} \)
- solvent
- solvent

Only new loans succeed: \( (1 - \theta)\theta_{\text{new}} \)
- insolvent
- solvent

Only toxic asset succeeds: \( \theta(1 - \theta_{\text{new}}) \)
- solvent
- insolvent

No investment succeeds: \( (1 - \theta)(1 - \theta_{\text{new}}) \)
- insolvent
- insolvent

<table>
<thead>
<tr>
<th>Probability of Solvency:</th>
<th>( \theta )</th>
<th>( \theta \theta_{\text{new}} )</th>
<th>( \theta_{\text{new}} )</th>
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<tbody>
<tr>
<td>L_0 \leq Y - D^nB =: L_0, \quad (19)</td>
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To avoid insolvency, the volume of new loans may thus not be too large. This is because the bank’s liabilities increase in the volume of the new loans and these liabilities also have to be covered by the proceeds of the toxic asset. (d) With probability \( (1 - \theta)(1 - \theta_{\text{new}}) \), both investments fail and yield no return at all implying that the bank is insolvent.

From the four states of the world, we can infer that the expected date \( t = 1 \) value of the bank capital in case of participation in the RA-scheme is:

\[
E[V^B_1] = \theta\theta_{\text{new}}(Y + (1 + \alpha)L_0 - D^B) + (1 - \theta)\theta_{\text{new}} \max\{(1 + \alpha)L_0 - D^B, 0\}
+ \theta(1 - \theta_{\text{new}}) \max\{Y - D^B, 0\} + (1 - \theta)(1 - \theta_{\text{new}})0.
\]
Due to (6), this can be rewritten to

\[
E[V_1^B] = \theta \theta_{new} (Y + \alpha L_0 - D_{nB}) + (1 - \theta) \theta_{new} \max\{\alpha L_0 - D_{nB}, 0\} \\
+ \theta (1 - \theta_{new}) \max\{Y - D_{nB} - L_0, 0\} + (1 - \theta)(1 - \theta_{new})0,
\]

so that we obtain \( \frac{\partial E[V_1^B]}{\partial L_0} > 0 \). The expected value of bank capital is thus increasing in \( L_0 \) under the RA-scheme.

The Bank Manager’s Optimizing Behavior

For the sake of simplicity, we restrict our subsequent analysis of the bank manager’s behavior to the plausible case\(^{11}\)

\[
(1 + \alpha) \leq \frac{1}{1 - (1 - r)\theta},
\]

which implies \( L_0 \leq \overline{L}_0 \). Consequently, figure 6 applies.

Recall from (13) that if the bank manager does not transfer the toxic asset to the bad bank, the probability of bank solvency will be \( \theta \), the bank will only survive if the toxic asset does not fail. If the manager decides to transfer the asset, the probability of the bank’s solvency may increase. Figure 6 illustrates that it will increase if \( L_0 > \overline{L}_0 \). If \( L_0 < \overline{L}_0 \), the probability of the bank’s solvency will not change or even decrease. As the government aims at improving the probability of

\(^{11}\)Assuming a minimum capital ratio of \( r = 0.08 \) and a probability of repayment of the risky asset \( \theta = 0.2 \), the return on the new loans had to exceed \( \alpha = 0.22 \) to violate (21). For a rising \( \theta \), the maximum \( \alpha \) rises as well.
the bank’s solvency (we will comment on this in section 2), we assume that the government offers a transfer payment\(^{12}\)

\[ Z \geq D^nB + \frac{rD^nB}{\alpha} =: \overline{Z}, \tag{22} \]

which implies \(L_0^{max} \geq \overline{L}_0\). Only with such a transfer payment, the RA-scheme has the potential to improve the bank’s solvency.

As the bank manager’s utility \(U^B\) is increasing in \(E[V_1^B]\), which in turn is increasing in \(L_0\), the manager will grant the maximum volume \(L_0^{max}\) under the RA-scheme. From this and (20) in conjunction with (8) and \(Z \geq \overline{Z}\), we can infer that the bank manager’s utility of participating in the RA-scheme satisfies

\[
U^B = \theta \theta_{new}(Y + \alpha L_0^{max}(Z) - D^nB) + \theta(1 - \theta_{new})0
\]

\[ + (1 - \theta)\theta_{new}(\alpha L_0^{max}(Z) - D^nB) + (1 - \theta)(1 - \theta_{new})0 - B. \tag{23} \]

The bank manager will hive off the toxic asset to the bad bank at \(t = 0\) only if \(U^B \geq U^nB\). Due to (13) and (23), this results in

\[
\theta_{new}\alpha L_0^{max}(Z) \geq \theta Y(1 - \theta_{new}) + (\theta_{new} - \theta)D^nB + B. \tag{24} \]

Analogously to (14), condition (24) states that the bank manager will opt for the RA-scheme if the expected benefits outweigh the expected costs. According to the left hand side of (24) the expected benefit of the RA-scheme stems from the expected return on the new loans as it is the case under the OS-scheme. Unlike the OS-scheme, however, the RA-scheme does not allow the manager to benefit from the government bonds because these bonds are either returned to the bad bank at

\(^{12}\)One obtains \(\overline{Z}\) by inserting (9) in (18) for \(L_0 = L_0^{max}\).
\( t = 1 \) or used to repay deposits. The right hand side of (24) reflects his expected costs. Like an outright sale, the RA-scheme is associated with stigma costs \( B \) and an increase in expected old liabilities \((\theta_{\text{new}} - \theta)D^{nB}\). Furthermore, the costs of the RA-scheme consist of a probably foregone potential return on the toxic assets \(\theta Y (1 - \theta_{\text{new}})\). While under the OS-scheme the expected return on the toxic asset is foregone with probability 1, it is foregone under the RA-scheme with probability \(1 - \theta_{\text{new}}\) only. With this probability, the new loans fail, in which case the bank is insolvent and the return on the toxic asset has to be used to repay depositors. In conjunction with (9) and the requirement \(Z \geq \bar{Z}\), (24) directly leads to

**Proposition 2:** Under the RA-scheme, the bank manager will transfer the toxic asset to the bad bank and the probability of bank solvency increases from \(\theta\) to \(\theta_{\text{new}}\) only if

\[
Z \geq \max\{\bar{Z}, Z_{RA}^*\}, \tag{25}
\]

where \(Z_{RA}^*\) is defined by

\[
Z_{RA}^* = \theta Y + \frac{r}{\theta_{\text{new}}\alpha} (B - \hat{B}). \tag{26}
\]

According to the proposition, the RA-scheme improves the bank's solvency provided that two preconditions are met. First, the transfer payment \(Z\) and the resulting volume of new loans must be sufficiently large so that the bank survives whenever the new loans succeed, \(Z \geq \bar{Z}\). Second, it must be sufficiently large to incite the bank manager to transfer the toxic asset to the bad bank, \(Z \geq Z_{RA}^*\). As it was the case under the OS-scheme, the threshold \(Z_{RA}^*\) is linearly increasing in the stigma costs \(B\) and equal to the book value \(\theta Y\) of the toxic asset for \(B = \hat{B}\). Consequently, the manager again is satisfied with pure asset substitution (without a change in
bank capital) if $B = \hat{B}$. He will require a capital increase, $Z_{RA}^* > \theta Y$, if $B > \hat{B}$, and accept a capital loss, $Z_{RA}^* < \theta Y$ if $B < \hat{B}$.

The critical stigma costs $\hat{B}$ relevant for the $RA$-scheme are identical to those relevant for the $OS$-scheme. They again reflect the manager’s expected pecuniary benefits less his expected pecuniary costs of transferring the toxic asset to the bad bank in exchange for a transfer payment $Z = \theta Y$.\(^{13}\) However, the threshold $Z_{RA}^*$ increases more in $B$ than the threshold $Z_{OS}^*$:

$$\frac{\partial Z_{RA}^*}{\partial B} = \frac{r}{\theta_{new\alpha}} > \frac{r}{\theta_{new}(\alpha + r)} = \frac{\partial Z_{OS}^*}{\partial B}. \quad (27)$$

If stigma costs increase by one unit, the transfer payment $Z^*$ must rise until the manager’s expected return has also increased by one unit. However, since under the $RA$-scheme, the manager never benefits from the proceeds of the government bonds $Z$, the marginal expected return of $Z$ is lower than in case of the $OS$-scheme. Consequently, the increase in $Z$ must be higher to compensate the manager for higher stigma costs, which becomes relevant below.

## 2 POLICY IMPLICATIONS

In the preceding section, we have investigated the incentives of a bank manager to hive off a toxic asset to a bad bank under two different bad bank schemes. Based on the results obtained there, this section takes a different perspective and discusses

\(^{13}\)According to (24), his expected pecuniary benefits are $\theta_{new} L_{\theta}^{\max}$, while his expected pecuniary costs are $\theta Y (1 - \theta_{new}) + (\theta_{new} - \theta) D_{\theta}^{\max} B$. For $Z = \theta Y$, the benefits will therefore be $\theta_{new} \alpha \theta Y$ (recall from (9) that $L_{\theta}^{\max} (Z = \theta Y) = \theta Y$) so that the critical stigma costs are given by $\hat{B} = \theta_{new} \alpha \theta Y - (1 - \theta_{new}) \theta Y - (\theta_{new} - \theta) D_{\theta}^{\max} B$, which is identical to (16). Intuitively, the $RA$-scheme is associated with a lower expected pecuniary benefit and lower expected pecuniary costs than the $OS$-scheme. The expected benefit differs by $\theta_{new} \alpha \theta Y$ because the bank manager does not obtain the proceeds of the government bonds under the $RA$-scheme. Expected costs differ by the same amount $\theta_{new} \theta Y$ because under the $RA$-scheme, the return on the toxic asset is foregone with probability $1 - \theta_{new}$ while it is foregone with certainty under the $OS$-scheme.
policy implications of our analysis. We will ask which bad bank scheme is optimal from the viewpoint of a policy maker who wishes to minimize the expected taxpayers’ costs. We proceed as follows. First, we clarify the costs of the different bad bank schemes. Then, we determine the cost minimizing scheme for the case that the policy maker aims at improving the stability of the banking sector in section 2.1, and avoiding a credit crunch in section 2.2.

2.1 Expected Costs to the Taxpayers

If the policy maker establishes a state-owned bad bank to relief a commercial bank from its toxic asset, the taxpayers may bear possible losses or may benefit from possible profits. The different bad bank schemes have different implications with respect to these losses and gains. Potential payments of the deposit insurer are not part of the taxpayers’ cost function since the deposit insurance is assumed to be privately-sponsored.\textsuperscript{14}

Under the \textit{OS}-scheme, the commercial bank sells the toxic asset to the bad bank. In return, the commercial bank obtains safe government bonds $Z$ from the bad bank. Since no further transaction takes place between the commercial bank and the bad bank, expected taxpayers’ costs are

$$E[C_{OS}] = Z - \theta Y.$$  \hfill (28)

\textsuperscript{14}Demirguc-Kunt, Karacaoglu, and Laeven (2005) show that many countries have implemented a privately-sponsored deposit insurance.
They consist of the price $Z$ the bad bank pays in form of government bonds for the toxic asset less its expected return $\theta Y$. Accordingly, the $OS$-scheme involves the possibility of future upside gains for the taxpayers.\(^{15}\)

If the policy maker implements the $RA$-scheme, the bank manager will still exchange the toxic asset against government bonds at $t = 0$. However, this transaction is reversed in $t = 1$. The risks and benefits of the toxic asset are thus left to the commercial bank. The costs of the $RA$-scheme to the taxpayers depend on whether the bad bank’s claim $Z$ at $t = 1$ is senior or junior to deposits. If it is senior ($RA_S$), the expected costs will be

$$E[C_{RA_S}] = 0. \quad (29)$$

The $RA_S$-scheme is thus costless to the taxpayers. This is because the bad bank will always reobtain the government bonds from the commercial bank at $t = 1$ under this scheme, irrespective of whether the commercial bank is solvent or not. The privately-sponsored deposit insurance will compensate the depositors if the bank fails.

If the claim of the bad bank is junior to deposits, the taxpayers will only incur no costs if the commercial bank is solvent and able to return $Z$ to the bad bank at $t = 1$.\(^{16}\) We know from Proposition 2 that the commercial bank will be solvent under the $RA$-scheme whenever the new loans succeed, which happens with probability $\theta_{new}$. They fail with probability $1 - \theta_{new}$. Then, the commercial bank is insolvent so

\(^{15}\)However, as pointed out by Beck, Coyle, Dewatripont, Freixas, and Seabright (2010, p. 44), there are strong incentives for politicians to exaggerate the likelihood of this outcome.

\(^{16}\)Note that if we assumed that the deposit insurance were government-sponsored, the two $RA$-schemes would be equivalent with respect to their expected costs. This is due to the fact that the bank only has two debtors — the government and the deposit insurer. These two parties share the costs of a bank’s insolvency depending on the priority of their claims. Assuming a government-sponsored deposit insurance, the distribution of costs is nonrelevant.
that the bad bank has a residual claim on the bank’s assets. Therefore, the expected
costs to the taxpayers under the RAJ-scheme will be given by

\[ E[C_{RAJ}] = \theta(1 - \theta_{\text{new}}) \min\{D^B - Y, Z\} + (1 - \theta)(1 - \theta_{\text{new}}) \min\{D^B, Z\}. \]  

The first term of the right hand side of (30) reflects the taxpayers’ costs if only the
toxic asset succeeds. Then, the return \( Y \) on the toxic asset will be used to repay
depositors. To settle the remaining claim \( D^B - Y \) of depositors the government
bonds will be used. Therefore, the bad bank will either lose government bonds
worth \( D^B - Y \) or it will lose its total claim \( Z \) on the commercial bank, depending
on which amount is smaller. The second term on the right hand side of (30) reflects
the costs if none of the assets succeeds. For this case, essentially the same argument
holds except that there are no returns on the toxic asset in this case. Recall from
the previous section that if the bank manager hives of the toxic asset to the bad
bank, he will always grant the maximum volume \( L_0^{max} \) of new loans. Due to (6) and
(9), we therefore obtain \( D^B = D^{nB} + L_0^{max} > Z \), so that (30) becomes

\[ E[C_{RAJ}] = \theta(1 - \theta_{\text{new}}) \min\{D^B - Y, Z\} + (1 - \theta)(1 - \theta_{\text{new}})Z. \]  

2.2 Reestabiling Stability of the Banking Sector

In the financial crisis which started in 2007, a major concern was that the failure
of large, systemically important banks might propagate through the entire financial
system causing substantial instabilities in the banking sector. A policy maker who
wishes to reduce the risk of such banking sector instabilities in times of crisis should
therefore adopt measures to improve the solvency of these systemically important
banks. Our model suggests that a bad bank scheme can be useful in this regard.
Therefore, this section asks which scheme the policy maker should apply if he wishes
to improve the solvency of a large bank at minimum costs to the taxpayers.

We know from Proposition 1 and 2 that the policy maker can improve the solvency of the bank by offering a bad bank scheme which is sufficiently favorable for the bank manager to participate. That is, the policy maker must offer sufficient government bonds $Z$ in exchange for the toxic asset, so that the bank manager makes use of the offer, $Z \geq Z_{OS}^*, Z_{RA}^*$. Moreover, in case of the $RA$-scheme, the volume of new loans must be large enough ($L_0^{max} \geq \bar{T}_0$) to avoid insolvency whenever only the new loans succeed which implies $Z \geq Z$. Provided that these conditions are met, the probability of bank solvency increases from $\theta$ to $\theta_{new}$ under both, the $OS$-scheme and the $RA$-scheme. Besides, once these conditions are met, any further increase of the offered transfer payment $Z$ has no effect on the probability of bank solvency. Therefore, as the expected costs to the taxpayers (weakly) increase in $Z$, the policy maker will always offer the smallest possible $Z$ consistent with the bad bank scheme applied. Denoting the weak (strict) preference relation by $\succeq$ ($\succ$), we obtain

**Proposition 3:** If the policy maker aims to improve the probability of solvency of the commercial bank, his preference order will satisfy

$$OS \succeq RA_S \succ RA_J \quad \text{if} \quad B \leq \hat{B},$$

$$RA_S \succ OS \succ RA_J \quad \text{if} \quad B \in (\hat{B}, \hat{\hat{B}}),$$

$$RA_S \succ RA_J \succeq OS \quad \text{if} \quad B \geq \hat{\hat{B}},$$

(32)

where $\hat{B}$ is defined in (16) and where $\hat{B} > \hat{B}$ denotes the critical stigma costs for which $E[C_{OS}(Z_{OS}^*)] = E[C_{RA_J}(\max\{Z, Z_{RA}^*\})]$.

**Proof:** See appendix.
The proposition reveals the policy maker’s preference order with respect to the different bad bank schemes. Let us now comment on this preference order.

(a) From the policy maker’s point of view the repurchase agreement in which the bad bank’s claim is senior to deposits (\(RA_S\)-scheme) is always superior to the one in which it is junior to deposits (\(RA_J\)-scheme). This is not surprising. As long as the claim of the bad bank has priority over deposits, the bad bank will reobtain \(Z\) at \(t = 1\) even if the commercial bank fails. Accordingly, the bad bank reobtains \(Z\) with certainty. In contrast, the repayment of \(Z\) is uncertain under the \(RA_J\)-scheme. As the claim of the bad bank is subordinated to deposits, the bad bank will reobtain less than \(Z\) (maybe even nothing) if the commercial bank is insolvent. Consequently, \(RA_S \succ RA_J\) for all \(B\).

(b) The stigma costs \(B\) become crucial for the preference order when taking an outright sale (\(OS\)-scheme) into account. At low stigma costs (\(B \leq \hat{B}\)), the policy maker prefers an outright sale of the toxic asset over both variants of the repurchase agreement (\(RA\)). The relatively low stigma costs imply that the transfer payment \(Z\) is also relatively low. This means that under the \(OS\)-scheme, the transfer is associated with an expected profit for the taxpayers (\(Z \leq \theta Y\)). This profit is out of reach under both \(RA\)-schemes since possible proceeds of the toxic asset remain with the commercial bank. Consequently, \(OS \succeq RA_S, RA_J\) for \(B \leq \hat{B}\).

(c) For stigma costs being higher than \(\hat{B}\), the transfer payment \(Z^*\) must be higher than \(\theta Y\). In this case, an outright sale of the toxic asset to the bad bank leads to an expected loss to the taxpayers. Since the \(RA_S\)-scheme is costless to taxpayers (they reobtain the transfer payment \(Z\) with certainty), \(RA_S \succ OS\) for all \(B > \hat{B}\).

(d) For stigma costs being higher than \(\hat{B}\) but lower than \(\hat{\hat{B}}\), the \(OS\)-scheme is superior to the \(RA_J\)-scheme, while for stigma costs higher than \(\hat{\hat{B}}\), the policy maker prefers the \(RA_J\)-scheme over the \(OS\)-scheme. The explanation for this result
is as follows. From the policy maker’s point of view the advantage of the OS-scheme is that the taxpayers may benefit from potential proceeds of the toxic asset. However, the disadvantage is that the transfer payment $Z$ is lost, irrespective of the outcome of the toxic asset. In contrast, the bad bank does not participate in potential proceeds of the toxic asset under the RAJ-scheme but possibly reobtains $Z$. For relatively small stigma costs $B < \hat{B}$, $Z^*$ is that small that the advantage of the OS-scheme of participating in possible proceeds of the toxic asset outweighs the advantage of the RAJ-scheme of possibly reobtaining $Z^*$. Instead, for $B \geq \hat{B}$ the transfer payment $Z^*$ becomes that high that reobtaining $Z^*$ is more important so that the advantage of the RAJ-scheme outweighs the advantage of the OS-scheme. Consequently, $OS \succ RA_J$ for $B < \hat{B}$ and $RA_J \succeq OS$ for $B \geq \hat{B}$.

2.3 Avoiding a Credit Crunch

As the recent financial crisis unfolded, not only the stability of the banking sector was a major issue. There were also fears that the financial crisis might lead to a credit crunch (European Central Bank, 2007). We have seen in section 1 that a bad bank scheme can foster new lending. In doing so, it can serve as a measure to avoid a credit crunch. In this section, we ask which bad bank scheme a policy maker will apply if he aims at improving the solvency of a single commercial bank as well as fostering new lending to prevent a credit crunch at minimum expected costs.

A bad bank scheme relieves a commercial bank from its toxic asset at least temporarily. The commercial bank obtains safe government bonds $Z$ in exchange for its toxic asset. Unlike this toxic asset, the government bonds must not be backed with capital. Therefore, the bad bank scheme allows the bank manager to grant new loans with the maximum volume of new loans $L_0^{\max}$ increasing in $Z$. We have argued above that under both, the OS- and the RA-scheme, the bank manager will indeed
grant this maximum volume of new loans. Accordingly, if the policy maker has a target minimum loan volume \( L_{0}^{pm} \), it follows from (9) that he must offer a transfer price

\[
Z \geq \theta Y + r(L_{0}^{pm} - \theta Y) =: Z^{pm}.
\]

(33)

In addition, the offer of the policy maker must also satisfy \( Z \geq Z_{OS}^{*} \) or \( Z \geq \max\{Z_{RA}^{*}, \hat{Z}\} \) to make sure that the bank manager has an incentive to participate in the respective bad bank scheme and that the solvency of the bank improves. This leads us to

**Proposition 4:** If the policy maker aims to improve the solvency of the commercial bank and to ensure that the loan volume of the commercial bank does not fall short of \( L_{0}^{pm} \), his preference order will satisfy

1. If \( B \leq \hat{B} \)

\[
\begin{align*}
OS & \succeq RA_{S} > RA_{J} \quad \text{if} \quad L_{0}^{pm} \leq \theta Y, \\
RA_{S} & > OS \succeq RA_{J} \quad \text{if} \quad L_{0}^{pm} \in (\theta Y; \hat{L}_{0}^{pm}), \\
RA_{S} & > RA_{J} \succeq OS \quad \text{if} \quad L_{0}^{pm} \geq \hat{L}_{0}^{pm}.
\end{align*}
\]

(34)

2. If \( B \in (\hat{B}, \hat{\hat{B}}) \)

\[
\begin{align*}
RA_{S} & > OS \succeq RA_{J} \quad \text{if} \quad L_{0}^{pm} < \hat{L}_{0}^{pm}, \\
RA_{S} & > RA_{J} > OS \quad \text{if} \quad L_{0}^{pm} \geq \hat{L}_{0}^{pm}.
\end{align*}
\]

(35)

3. If \( B \geq \hat{\hat{B}} \)

\[
RA_{S} > RA_{J} > OS \quad \text{for all} \quad L_{0}^{pm},
\]

(36)

where \( \hat{B} \) is defined in (16), \( \hat{\hat{B}} \) denotes the critical stigma costs for which

\[
E[C_{OS}(Z_{OS}^{*})] = E[C_{RA_{J}}(\max\{\hat{Z}, Z_{RA}^{*}\})],
\]

and \( \hat{L}_{0}^{pm} \) denotes the critical amount of new loans that corresponds to the critical transfer payment \( \hat{Z} > \theta Y \) for which

\[
E[C_{OS}(\max\{\hat{Z}, Z_{OS}^{*}\})] = E[C_{RA_{J}}(\max\{\hat{Z}, Z, Z_{RA}^{*}\})].
\]
Proof: See appendix

The proposition states that the policy maker’s preference order depends on the stigma costs and the target loan volume $L_0^{pm}$. Like in Proposition 3, this result reflects the costs and benefits of the different bad bank schemes to the taxpayers, and, therefore, to the policy maker.

(a) The policy maker will always prefer the $RAS$-scheme over the $RAJ$-scheme as the former is costless while the latter implies a loss of the transfer payment $Z$ with positive probability.

(b) To determine the preference order with respect to the $RA$-schemes and the $OS$-scheme we have to distinguish between different levels of the stigma costs and the target loan volume. Under the $OS$-scheme, the policy maker loses the transfer payment with certainty but benefits from the return on the toxic asset. Accordingly, an outright sale will be most preferred whenever the transfer payment $Z$ is smaller than the expected return $\theta Y$ on the toxic asset, i.e. whenever the transfer implies a capital loss for the bank. Then, the $OS$-scheme allows for a profit for the policy maker, which is out of reach under a repurchase agreement. The policy maker will be able to make such a profit only if two conditions are met. First, the stigma costs $B$ of the bank manager must be sufficiently small, $B \leq \hat{B}$, so that he accepts the $OS$-scheme even if this leads to a capital loss for the bank. Second, the target loan volume $L_0^{pm}$ of the policy maker must be sufficiently small, $L_0^{pm} \leq \theta Y$, so that it can even be reached if the bank loses capital.

(c) If one of these just mentioned conditions is violated, the transfer payment under the $OS$-scheme must exceed the expected return on the toxic asset. In this case, the policy maker will prefer the costless $RAS$-scheme over the costly $OS$-scheme. Moreover, if both, the stigma costs $B$ and the target loan volume $L_0^{pm}$ are at
most intermediate, the transfer payment under the OS-scheme will be intermediate as well so that an outright sale leads to lower costs than the RA$_J$-scheme.

(d) Only if either the stigma costs or the target loan volume are large, the policy maker must offer a rather large transfer payment. Then, he prefers to reobtain the transfer payment with at least some probability under the RA$_J$-scheme over benefiting from the return on the toxic asset under an outright sale.

3 SUMMARY

The worldwide financial crisis that broke out in 2007 led to severe losses for banks caused by toxic assets. As a response, several governments implemented bad banks to relieve banks’ balance sheets from these assets.

In our paper, we have focussed on two different bad bank schemes and their appropriateness for achieving a policy maker’s objectives of reestablishing stability and avoiding a credit crunch. First, we have discussed an outright sale of the toxic asset to the bad bank. Second, we have analyzed a repurchase agreement. We have shown that under both schemes, there exists a critical transfer payment that induces the bank manager to participate in the bad bank. If the policy maker offers a transfer payment that is sufficiently large so that the bank manager will hive off the toxic asset, the bank’s probability of solvency will increase. Whenever the commercial bank is systemically important, this will improve the stability of the banking sector. Consequently, both bad bank schemes are appropriate instruments to reestablish stability. Moreover, we have shown that a transfer of the toxic asset to the bad bank will release bank’s equity. Therefore, the bank is able to grant new loans. The policy maker is able to control the amount of new loans by offering a
corresponding transfer payment. Consequently, both bad bank schemes are able to avoid a credit crunch.\textsuperscript{17}

However, the two schemes differ with respect to their expected costs to the taxpayers. On the one hand, an outright sale allows the policy maker to benefit from potential returns on the toxic asset. On the other hand, a repurchase agreement allows the policy maker to possibly reobtain the transfer payment which is lost under an outright sale. Therefore, only if the transfer payment is sufficiently low, e.g. caused by low stigma costs or a low target loan volume, the policy maker mostly prefers an outright sale. Otherwise, if the transfer payment is rather large due to high stigma costs or a target loan volume, a scheme with a repurchase agreement is preferred.

\textsuperscript{17}In our model, the implications of a bad bank scheme with a repurchase agreement are equivalent to revoking capital requirements. However, in practice the advantage of a bad bank scheme over a mitigation of capital requirements is that a bad banks scheme cleans banks’ balance sheets from toxic assets at least temporarily.
Bibliography


Proof of Proposition 3

We proceed in two steps. First, we determine the minimum expected costs $E[C_k^{\text{min}}]$ under the different bad bank schemes $k = OS, RA_S, RA_J$. Second, we derive the preference order of the policy maker by comparing the respective minimum expected costs.

First Step: Minimum Expected Costs of the Policy Maker

The policy maker will always offer the smallest possible $Z$, which is consistent with the respective bad bank scheme. By inserting these critical transfer payments as given in Proposition 1 and 2 in the corresponding expected cost functions as given in (28), (29) and (31), we obtain

$$E[C_{OS}^{\text{min}}] = E[C_{OS}(Z_{OS}^*)] = Z_{OS}^* - \theta Y = \frac{r}{\theta_{\text{new}(\alpha+\gamma)}}[B - \hat{B}],$$

(37)

$$E[C_{RA_S}^{\text{min}}] = 0,$$

(38)

$$E[C_{RA_J}^{\text{min}}] = E[C_{RA_J}(\max\{Z, Z_{RA_J}^*\})].$$

(39)

Before we proceed with the second step, it is useful to have a closer look at (39). Note that if the bank manager hives of the toxic asset to the bad bank, he will always grant the maximum volume $L_0^{\text{max}}$ of new loans. Inserting this and (6) in
(31), where we use $L_0$ as defined in (19) for the sake of a less complex presentation, yields

$$E[C_{RA_j}(Z)] = \theta(1 - \theta_{new}) \min\{L_0^{max} - L_0, Z\} + (1 - \theta)(1 - \theta_{new})Z. \quad (40)$$

As $E[C_{RA_j}(Z)]$ is increasing in $Z$ (note that $L_0^{max}$ is increasing in $Z$, see (9)), we can rewrite (39) to

$$E[C_{RA_j}^{min}] = \max\{E[C_{RA_j}(\overline{Z})], E[C_{RA_j}(Z^*_{RA})]\}. \quad (41)$$

Moreover, two properties of (40) will be important in the following. (a) If $Z = \overline{Z}$, it follows from (9) and (22) that $L_0^{max} = L_0$. Insertion of this and $\overline{Z}$ in (40) yields

$$E[C_{RA_j}(\overline{Z})] = \theta(1 - \theta_{new}) \min\{\overline{L}_0 - L_0, \overline{Z}\} + (1 - \theta)(1 - \theta_{new})\overline{Z} > 0. \quad (42)$$

(b) If $Z = Z^*_{RA}$, it follows from inserting (26) in (9) that $L_0^{max} = \theta Y + \frac{B - \hat{B}}{\theta_{new} \alpha}$. Insertion of this and (26) in (40) yields

$$E[C_{RA_j}(Z^*_{RA})] = \theta(1 - \theta_{new}) \min\{\theta Y + \frac{B - \hat{B}}{\theta_{new} \alpha} - L_0, \theta Y + \frac{r}{\theta_{new} \alpha} [B - \hat{B}]\} + (1 - \theta)(1 - \theta_{new})(\theta Y + \frac{r}{\theta_{new} \alpha} [B - \hat{B}]). \quad (43)$$

**Second step: Preference Order of the Policy Maker**

We now derive the preference order of the policy maker. As he aims at minimizing his expected costs, we obtain:

- He always prefers the $RA_S$-scheme over the $RA_J$-scheme, $RA_S \succ RA_J$, because (38), (41) and (42) imply $E[C_{RA_S}^{min}] = 0 < E[C_{RA_J}^{min}]$. 

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• He prefers the RA\textsubscript{S}-scheme over the OS-scheme, RA\textsubscript{S} \succ OS, only if 
\[ E[C_{min}^{RA_S}] < E[C_{min}^{OS}] \]. Due to (37) and (38), this condition results in \( B > \hat{B} \).

• He prefers the OS-scheme over the RA\textsubscript{J}-scheme, OS \succ RA\textsubscript{J}, only if 
\[ E[C_{min}^{OS}] < E[C_{min}^{RA_J}] \]. Due to (37) and (41), this condition is met if either

\[ E[C_{OS}(Z_{OS}^*)] = \mathcal{r}_{new}(\alpha + \mathcal{r}) (B - \hat{B}) < E[C_{RA_J}(Z)] \] \hfill (44)

or

\[ E[C_{OS}(Z_{OS}^*)] = \mathcal{r}_{new}(\alpha + \mathcal{r}) (B - \hat{B}) < E[C_{RA_J}(Z_{RA}^*)]. \] \hfill (45)

Now, note that the left hand side of (44) and (45) is equal to zero for \( B = \hat{B} \) and linearly increasing in \( B \) with

\[ \frac{\partial E[C_{OS}(Z_{OS}^*)]}{\partial B} = \mathcal{r}_{new}(\alpha + \mathcal{r}). \] \hfill (46)

Moreover:

– It follows from (42) and the definition of \( Z \) as given in (22) that the right hand side of (44) is positive and independent of \( B \). Accordingly, there exists a \( B_{Z}^{crit} > \hat{B} \) such that:

* the condition (44) is met if \( B < B_{Z}^{crit} \),

* the condition (44) is violated if \( B \geq B_{Z}^{crit} \),

so that we can already conclude that if \( B < \hat{B} < B_{Z}^{crit} \), it follows that \( OS \succ RA_J \).
Therefore, we can now restrict our attention to $B \geq \hat{B}$. It follows from (43) and the definition of $L_0$ that the right hand side of (45) is strictly positive for $B = \hat{B}$ and linearly increasing in $B$ with

$$
\frac{\partial E[C_{RAJ}(Z^*_{RA})]}{\partial B} = \begin{cases} 
(1 - \theta)_{new} \left( \frac{\theta + (1-\theta)r}{\theta_{new} \alpha} \right) & \text{if } B - \hat{B} < \frac{\theta_{new} \alpha L_0}{1-r} \\
(1 - \theta)_{new} \frac{r}{\theta_{new} \alpha} < \frac{\partial E[C_{OS}(Z^*_{OS})]}{\partial B} & \text{if } B - \hat{B} \geq \frac{\theta_{new} \alpha L_0}{1-r}.
\end{cases}
$$

(47)

Accordingly, there exists a $B_{Z,RA}^{crit} > \hat{B}$ such that

* the condition (45) is met if $B \in [\hat{B}, B_{Z,RA}^{crit})$,

* the condition (45) is violated if $B \geq B_{Z,RA}^{crit}$.

From the two cases, it directly follows that $OS > RA_J$ only if $B < \hat{B}$ with $\hat{B} := \max\{B_{Z_{RA}}^{crit}, B_{Z,RA}^{crit}\}$.

**Proof of Proposition 4**

We know from (33) that if the policy maker has a target minimum loan volume $L_{0}^{pm}$, he must offer government bonds worth

$$
Z \geq \theta Y + r(L_{0}^{pm} - \theta Y) =: Z^{pm}.
$$

(48)

There is thus a one-to-one relationship between $L_{0}^{pm}$ and $Z^{pm}$. For the sake of simplicity, we will use the minimum transfer payment $Z^{pm}$ instead of the target minimum loan volume $L_{0}^{pm}$ to prove the preference order of the policy maker.

We proceed in three steps. First, we clarify the minimum expected costs to the taxpayers, and thus the policy maker, under the different bad bank schemes. Second, we derive an intermediate result to simplify the proof. Third, we determine the preference order of the policy maker.
First Step: Minimum Expected Costs to the Taxpayers

If the policy maker wishes to ensure that the bank manager grants at least new loans \( L^{pm} \), it follows from (15), (25) and (48) that he must offer a transfer payment \( Z \geq \max\{Z_{OS}^*, Z^{pm}\} \) under the \( OS \)-scheme while he must offer a payment \( Z \geq \max\{Z, Z_{RA}^{*}, Z^{pm}\} \) under the \( RA \)-scheme. Therefore, we obtain from (28), (29) and (31) for the policy maker’s minimum expected costs

\[
E\left[C_{min}^{OS}\right] = E\left[C_{OS}(\max\{Z_{OS}^*, Z^{pm}\})\right], \tag{49}
\]

\[
E\left[C_{min}^{RA}\right] = 0, \tag{50}
\]

\[
E\left[C_{min}^{RAJ}\right] = E\left[C_{RAJ}(\max\{Z_{RA}^*, Z, Z^{pm}\})\right] > 0. \tag{51}
\]

Second Step: An Intermediate Result

Let us for now assume that the policy maker offers the same transfer payment \( Z \geq \theta Y \) under the \( OS \)-scheme and the \( RA_{J} \)-scheme. Then, there exists a \( \tilde{Z} > \theta Y \) such that

\[
E[C_{OS}(Z)] < E[C_{RAJ}(Z)] \quad \text{if} \quad Z \in [\theta Y, \tilde{Z}),
\]

\[
E[C_{OS}(Z)] \geq E[C_{RAJ}(Z)] \quad \text{if} \quad Z \geq \tilde{Z}. \tag{52}
\]

That is, for small \( Z \) the \( OS \)-scheme is always preferred over the \( RA_{J} \)-scheme while for large \( Z \) the preference order changes and the \( RA_{J} \)-scheme is preferred over the \( OS \)-scheme. This is because

- on the one hand, it follows from (28) that \( E[C_{OS}] = 0 \) if \( Z = \theta Y \) and that \( E[C_{OS}] \) is increasing in \( Z \) with \( \frac{\partial E[C_{OS}]}{\partial Z} = 1 \),

- on the other hand, inserting (6) and (9) in (31) yields

\[
E[C_{RAJ}] = \theta (1 - \theta_{new}) \min\{D^{nB} + \frac{1}{r}(Z - D^{nB}) - Y, Z\} + (1 - \theta)(1 - \theta_{new})Z
\]
implying \( E[C_{RAJ}] > 0 \) for all \( Z \geq \theta Y \) and that \( E[C_{RAJ}] \) is increasing in \( Z \) with

\[
\frac{\partial E[C_{RAJ}]}{\partial Z} = \begin{cases} 
\frac{\theta(1-\theta_{\text{new}})}{r} + (1 - \theta)(1 - \theta_{\text{new}}) & \text{if } Z < D^{nB} + \frac{\frac{r}{1-r}Y}{1} \\
(1 - \theta_{\text{new}}) < 1 & \text{if } Z \geq D^{nB} + \frac{\frac{r}{1-r}Y}{1}.
\end{cases}
\] (53)

**Third step: Preference Order of the Policy Maker**

We will now derive the preference order of the policy maker with respect to the OS-scheme, the RA\(_S\)-scheme and the RA\(_J\)-scheme. As a direct consequence of (50) and (51), we obtain \( RA_S \succ RA_J \). The preference order with respect to the OS-scheme, on the one hand, and the two RA-schemes, on the other hand, depends on the stigma costs \( B \). We will distinguish between three cases: \( B \leq \hat{B} \), \( B \in (\hat{B}, \hat{\hat{B}}) \), and \( B \geq \hat{\hat{B}} \).

**Case a: \( B \leq \hat{B} \)**

Suppose that \( B \leq \hat{B} \). Then (15) and (26) implies \( Z_{OS}^* \leq \theta Y \) and \( Z_{RA}^* \leq \theta Y \).

1. For \( Z_{pm} \leq \theta Y \), the transfer payment under the OS-scheme satisfies \( \max\{Z_{OS}^*, Z_{pm}\} \leq \theta Y \) so that (49), (50) and (51) in conjunction with (28) implies \( E[C_{\text{min}}^{\text{OS}}] \leq 0 = E[C_{RA_S}] < E[C_{RA_J}] \). This leads to the preference order \( OS \succeq RA_S \succ RA_J \).

2. For \( Z_{pm} > \theta Y \), it follows from (49) and (51) in conjunction with (28) that

\[
E[C_{\text{min}}^{\text{OS}}] = E[C_{OS}(Z_{pm})] > 0
\] (54)

\[
E[C_{\text{min}}^{\text{RA_J}}] = \begin{cases} 
E[C_{RA_J}(\bar{Z})] & \text{if } Z_{pm} \leq \bar{Z} \\
E[C_{RA_J}(Z_{pm})] & \text{if } Z_{pm} > \bar{Z}.
\end{cases}
\] (55)
Note that (50) and (54) directly lead to $E[C_{OS}^{\text{min}}] > E[C_{RA_S}^{\text{min}}]$ and thus $RA_S \succ OS$. To derive the preference order with respect to the $OS$-scheme and the $RA_J$-scheme for the case $Z^{pm} > \theta Y$, it is useful to distinguish between different levels of $Z^{pm}$.

(a) Suppose that $Z^{pm} \in (\theta Y, \bar{Z})$. Then, we have

$$E[C_{OS}^{\text{min}}] = E[C_{OS}(Z^{pm})] < E[C_{RA_J}(Z^{pm})] \leq E[C_{RA_J}^{\text{min}}].$$

The first (in)equality follows from (54), the second follows from (52) and the third follows from (55). Accordingly, we obtain $OS \succ RA_J$ in this case.

(b) Suppose that $Z^{pm} \geq \max\{\tilde{Z}, \overline{Z}\}$. Then, we have

$$E[C_{OS}^{\text{min}}] = E[C_{OS}(Z^{pm})] \geq E[C_{RA_J}(Z^{pm})] = E[C_{RA_J}^{\text{min}}].$$

Again, the first (in)equality follows from (54), the second follows from (52) and the third follows from (55). Accordingly, we obtain $RA_J \succeq OS$ in this case.

(c) Suppose that $Z^{pm} \in [\tilde{Z}, \max\{\tilde{Z}, \overline{Z}\})$, which is feasible only if $\tilde{Z} < \overline{Z}$.

Then, it follows from (54) and (55) in conjunction with (28) that

$$\frac{\partial E[C_{OS}^{\text{min}}]}{\partial Z^{pm}} = 1 \quad \text{and} \quad \frac{\partial E[C_{RA_J}^{\text{min}}]}{\partial Z^{pm}} = 0.$$

Consequently, we can conclude that there exists a $\hat{Z} > \theta Y$ such that

- $RA_S \succ OS \succ RA_J$ if $Z^{pm} \in (\theta Y, \hat{Z})$ and thus $L^{pm}_0 \in (\theta Y, \hat{L}^{pm}_0)$,
- $RA_S \succ RA_J \succ OS$ if $Z^{pm} \geq \hat{Z}$ and thus $L^{pm}_0 \geq \hat{L}^{pm}_0$. 

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Case b: $B \in (\hat{B}, \hat{\hat{B}})$

Suppose that $B \in (\hat{B}, \hat{\hat{B}})$. Then (15) and (26) implies $Z_{RA}^* > Z_{OS}^* > \theta Y$.

1. For $Z_{pm} \leq Z_{OS}^*$ the transfer payment under the OS-scheme satisfies
   \[ \max\{Z_{OS}^*, Z_{pm}\} = Z_{OS}^* \] and the transfer payment under the RA-schemes satisfies
   \[ \max\{\bar{Z}, Z_{RA}^*, Z_{pm}\} = \max\{\bar{Z}, Z_{RA}^*\}. \]
   Therefore, like in Proposition 3, we obtain $RA_S \succ OS \succ RA_J$.

2. For $Z_{pm} > Z_{OS}^*$, it follows from (49) and (51) in conjunction with (28) that
   \[
   E[C_{OS}^{min}] = E[C_{OS}(Z_{pm})] > 0 \tag{56}
   
   E[C_{RA_J}^{min}] = \begin{cases} 
   E[C_{RA_J}(\max\{\bar{Z}, Z_{RA}^*\})] & \text{if } Z_{pm} \leq \max\{\bar{Z}, Z_{RA}^*\} \\
   E[C_{RA_J}(Z_{pm})] & \text{if } Z_{pm} > \max\{\bar{Z}, Z_{RA}^*\} \end{cases} \tag{57}
   
   Note that (50) and (56) directly lead to $E[C_{OS}^{min}] > E[C_{RA_S}^{min}]$ and thus $RA_S \succ OS$. To derive the preference order with respect to the OS-scheme and the RAJ-scheme for the case $Z_{pm} > Z_{OS}^*$, it is useful to distinguish between different levels of $Z_{pm}$.

(a) Suppose that $Z_{pm} \in (Z_{OS}^*, \tilde{Z})$, which is feasible only if $\tilde{Z} > Z_{OS}^*$. Then, we have
   \[ E[C_{OS}^{min}] = E[C_{OS}(Z_{pm})] < E[C_{RA_J}(Z_{pm})] \leq E[C_{RA_J}^{min}]. \]

   The first (in)equality follows from (56), the second follows from (52) and the third follows from (57). Accordingly, we obtain $OS \succ RA_J$ in this case.

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(b) Suppose that \( Z_{pm} \geq \max\{\tilde{Z}, \max\{\bar{Z}, Z_{RA}^*}\} \). Then, we have

\[
E[C_{OS}^{\min}] = E[C_{OS}(Z_{pm})] \geq E[C_{RA_J}(Z_{pm})] = E[C_{RA_J}^{\min}].
\]

Again, the first (in)equality follows from (56), the second follows from (52) and the third follows from (57). Accordingly, we obtain \( RA_J \succeq OS \) in this case.

(c) Suppose that \( Z_{pm} \in [\tilde{Z}, \max\{\tilde{Z}, \max\{\bar{Z}, Z_{RA}^*\}\}] \), which is feasible only if \( \tilde{Z} < \max\{\bar{Z}, Z_{RA}^*\} \). Then, it follows from (56) and (57) in conjunction with (28) that

\[
\frac{\partial E[C_{OS}^{\min}]}{\partial Z_{pm}} = 1 \quad \text{and} \quad \frac{\partial E[C_{RA_J}^{\min}]}{\partial Z_{pm}} = 0.
\]

Consequently, we can conclude that there exists a \( \tilde{Z} > \theta Y \) such that

- \( RA_S \succ RA_J \succ OS \) if \( Z_{pm} \in (\theta Y, \tilde{Z}) \) and thus \( l_0^{pm} \in (\theta Y, \hat{l}_0^{pm}) \),
- \( RA_S \succ RA_J \succ OS \) if \( Z_{pm} \geq \tilde{Z} \) and thus \( l_0^{pm} \geq \hat{l}_0^{pm} \).

Case c: \( B \geq \hat{B} \)

Suppose that \( B \geq \hat{B} \). Then (15) and (26) implies \( Z_{RA}^* > Z_{OS}^* \succ \theta Y \).

1. For \( Z_{pm} \leq Z_{OS}^* \) the transfer payment under the \( OS \)-scheme satisfies

\[
\max\{Z_{OS}^*, Z_{pm}\} = Z_{OS}^*
\]

and the transfer payment under the \( RA \)-schemes satisfies \( \max\{\bar{Z}, Z_{RA}^*, Z_{pm}\} = \max\{\bar{Z}, Z_{RA}^*\} \). Therefore, like in Proposition 3, we obtain \( RA_S \succ RA_J \succeq OS \).

2. For \( Z_{pm} > Z_{OS}^* \), it follows from (49) and (51) in conjunction with (28) that

\[
E[C_{OS}^{\min}] = E[C_{OS}(Z_{pm})] > E[C_{OS}(Z_{OS}^*)] > 0 \quad \text{(58)}
\]

\[
E[C_{RA_J}^{\min}] = \begin{cases} 
E[C_{RA_J}(\max\{\bar{Z}, Z_{RA}^*\})] & \text{if } Z_{pm} \leq \max\{\bar{Z}, Z_{RA}^*\} \\
E[C_{RA_J}(Z_{pm})] & \text{if } Z_{pm} > \max\{\bar{Z}, Z_{RA}^*\}
\end{cases} \quad \text{(59)}
\]
Note that (50) and (58) directly lead to \( E[C^\text{min}_{OS}] > E[C^\text{min}_{RA}] \) and thus \( RA_S \succ OS \). To derive the preference order with respect to the \( OS \)-scheme and the \( RA_J \)-scheme for the case \( Z^{pm} > Z^{*}_{OS} \), it is useful to distinguish between different levels of \( Z^{pm} \).

(a) Suppose that \( Z^{pm} \leq \max\{\overline{Z}, Z^*_{RA}\} \). Then, we have

\[
E[C^\text{min}_{OS}] > E[C_{OS}(Z^{*}_{OS})] \geq E[C_{RA_J}(\max\{\overline{Z}, Z^*_{RA}\})] = E[C^\text{min}_{RA_J}].
\]

The first (in)equality follows from (58), the second follows from Proposition 3 and the third follows from (59). Accordingly, we obtain \( RA_J \succeq OS \) in this case.

(b) Suppose that \( Z^{pm} > \max\{\overline{Z}, Z^*_{RA}\} \). Then, we have

\[
E[C^\text{min}_{OS}] = E[C_{OS}(Z^{pm})] \geq E[C_{RA_J}(Z^{pm})] = E[C^\text{min}_{RA_J}].
\]

Again, the first (in)equality follows from (58), the second follows from Proposition 3 and the third follows from (59). Accordingly, we obtain \( RA_J \succeq OS \) in this case.

Consequently, we can conclude that \( RA_S \succ RA_J \succeq OS \).